Primordial Black Holes from Pre-Big Bang

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Why PBHs?

Firstly studied by Zeldovich & Hawking in '70 as objects formed by gravitational collapse of large inhomogeneities in the early universe.

- New access to the early universe.
- Important alternative to dark matter candidates.
  - Advantage of not needing (necessarily) new physics.

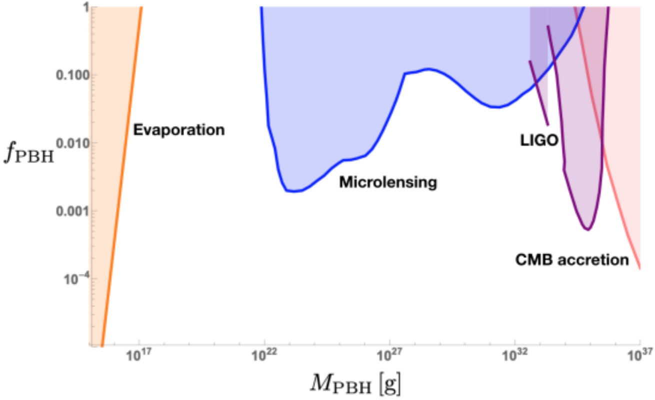


Figure: from arXiv:2112.05716.

- Pre-Big Bang Scenario

Different types of inflation

I: $a \sim t^{\beta},\ \beta > 1,\ t \to +\infty$. Power-low inflation, accelerated expansion and decreasing curvature:

$$\dot{a} > 0,\ \ddot{a} > 0,\ \dot{H} < 0\ ,$$

II: $a \sim (-t)^{\beta},\ \beta < 1,\ t \to 0_{-}$.

- **IIa** : **super inflation**: $\beta < 0$, accelerated expansion with growing curvature:

$$\dot{a} > 0,\ \ddot{a} > 0,\ H > 0,\ \dot{H} > 0\ .$$

- **IIb** : **accelerated contraction**: $0 < \beta < 1$, accelerated contraction with growing curvature:

$$\dot{a} < 0,\ \ddot{a} < 0,\ H < 0,\ \dot{H} < 0\ .$$

Pre-Big Bang Scenario

It is a scenario obtained from string theory[1]

$$g_s \ll 1\,, \quad H^2\alpha' \ll 1\,, \quad \alpha' = l_s^2/2\pi$$

$$\boxed{S = -\frac{1}{2l_s^{d-1}}\int d^{d+1}\sqrt{|g|}e^{-\phi}\Big[R + (\boldsymbol{\nabla}\phi)^2 - \frac{1}{12}\mathcal{H}^2_{\mu\nu\rho}\Big]}$$

where R is the Ricci scalar, ϕ is the dilatonic field and

$\mathcal{H}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$, $B_{\mu\nu}$ Ramond field

$$\mathcal{H}^{\mu\nu\rho} = \frac{e^\phi}{\sqrt{|g|}}\epsilon^{\mu\nu\rho\lambda}\partial_\lambda\sigma \qquad \text{in 4-D}$$

T-duality: given a universe with radius R, the theory is invariant under the transformation $R \to l_s^2/R$.

[1]M. Gasperini, G. Veneziano, *"The pre-big bang scenario in string cosmology"*, Physics Reports, Volume 373, 2003.

- Equations are invariant under time-inversion and **scale-factor duality**:

$$t \to -t \ , \qquad a_i \to \tilde{a}_i = a_i^{-1}$$

Thus we have 4 related solutions:

$$\{a(t),\ a(-t),\ a^{-1}(t),\ a^{-1}(-t)\}$$

.

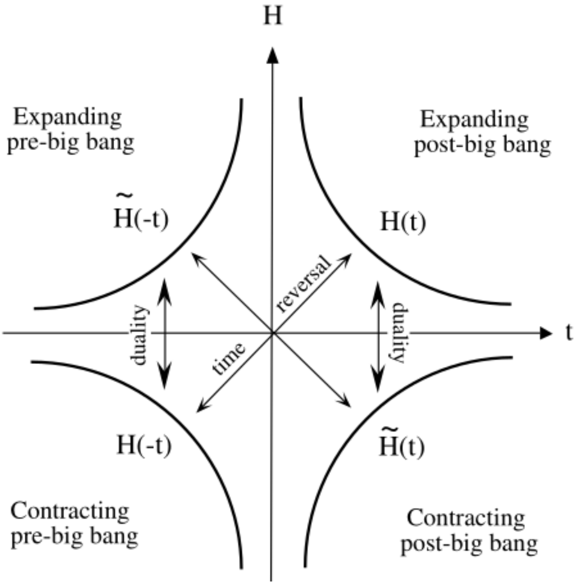
- We can move from the S-frame to the E-frame by a conformal transformation:

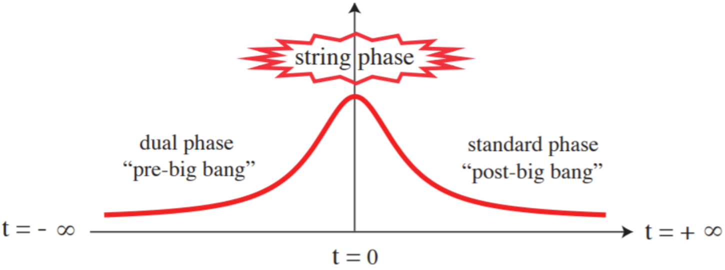
$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \ , \quad \Omega = e^{-\frac{\phi}{d-1}} \ , \quad \tilde{\phi} = \sqrt{\frac{2}{d-1}}\phi$$

in which the action takes the form:

$$S(\tilde{g}, \tilde{\phi}) = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|\tilde{g}|} \left[\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}\right] .$$

Expanding pre-big bang $a(t)$ **in S-frame** $\Longleftrightarrow$ **contracting pre-big bang** $\tilde{a(t)}$ **in E-frame**


H
t
Expanding
pre-big bang
Expanding
post-big bang
$\tilde{H}(-t)$
H(t)
reversal
time
duality
duality
H(-t)
$\tilde{H}(t)$
Contracting
pre-big bang
Contracting
post-big bang



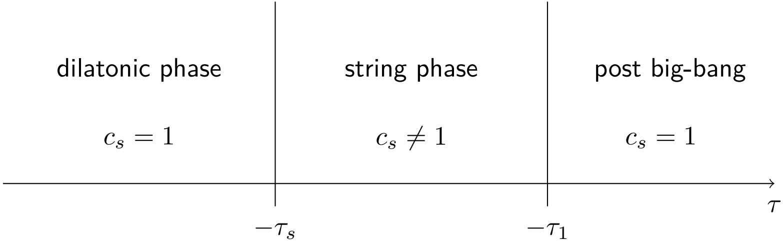
The Pre-Big Bang solution has a singularity in the future $(t \to 0_-)$; $H, g_s \to \infty$. The action needs to be corrected:

$$S = S_0 + S_{\alpha'} + S_{loop} \ ,$$

- α' corrections in the action, high derivative terms.
- loop corrections, as correction in powers of g_s.

Such corrections generate a non-trivial sound speed on the perturbations.

Two inflationary phases



Perturbations follow the Mukhanov-Sasaki equation in terms of the canonical variable $v = z\mathcal{R}$:

$$v'' - \left(c_s^2 \nabla^2 + \frac{z''}{z}\right) v = 0$$

We apply a matching procedure in the transition hypersurfaces:

$$\mathcal{R}_k^i(-\tau_t) = \mathcal{R}_k^{i+1}(-\tau_t) \quad \wedge \quad \mathcal{R}_k'^i(-\tau_t) = \mathcal{R}_k'^{i+1}(-\tau_t)$$

We can evaluate the power spectrum at re-entry $k = aH$:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \left|\mathcal{R}_k^3\right|^2_{|k\tau|=1}$$

Obtaining the complete spectrum:

$$\begin{aligned}
\mathcal{P}_{\mathcal{R}}(k) \quad & \sim \left(\frac{H_1}{M_P}\right)^2 \left(\frac{k}{k_1}\right)^{3-2|\nu_2|} c_s^{-1-2|\nu_2|}, && k_s/c_s < k < k_1/c_s, \\
& \sim \left(\frac{H_1}{M_P}\right)^2 \left(\frac{k_s}{k_1}\right)^{3-2|\nu_2|} \left(\frac{k}{k_s}\right)^{4}, && k_s < k < k_s/c_s \\
& \sim \left(\frac{H_1}{M_P}\right)^2 \left(\frac{k_s}{k_1}\right)^{3-2|\nu_2|} \left(\frac{k}{k_s}\right)^{3-2|\nu_1|} && k < k_s
\end{aligned}$$

Non-trivial sound speed c_s dependence on the high frequency band

$$\Downarrow$$

enhancement of the spectrum.

Pre-Big Bang Spectrum

- **axion spectrum**:

$$
\begin{aligned}
P_\sigma(\omega) &\simeq \frac{f^2}{2\pi^2}\left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega}{\omega_1}\right)^{3-|3+2\beta|} c_\sigma^{-1-|3+2\beta|} \quad , \quad \frac{\omega_s}{c_\sigma} < \omega < \frac{\omega_1}{c_\sigma}, \\
&\simeq \frac{f^2}{2\pi^2}\left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega_s}{\omega_1}\right)^{3-|3+2\beta|}\left(\frac{\omega}{\omega_s}\right)^4 \quad , \quad \omega_s < \omega < \frac{\omega_s}{c_\sigma}, \\
&\simeq \frac{f^2}{2\pi^2}\left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega_s}{\omega_1}\right)^{3-|3+2\beta|}\left(\frac{\omega}{\omega_s}\right)^{n_s-1} \quad , \quad \omega < \omega_s,
\end{aligned}
$$

- **dilaton spectrum**:

$$
\begin{aligned}
P_\phi(\omega) &\simeq \frac{1}{2\pi^2}\left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega}{\omega_1}\right)^{3-|3-2\beta|} c_\phi^{-1-|3-2\beta|} \quad , \quad \frac{\omega_s}{c_\phi} < \omega < \frac{\omega_1}{c_\phi}, \\
&\simeq \frac{1}{2\pi^2}\left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega_s}{\omega_1}\right)^{3-|3-2\beta|}\left(\frac{\omega}{\omega_s}\right)^4 \quad , \quad \omega_s < \omega < \frac{\omega_s}{c_\phi}, \\
&\simeq \frac{1}{2\pi^2}\left(\frac{H_1}{M_P}\right)^2\left(\frac{\omega_s}{\omega_1}\right)^{3-|3-2\beta|}\left(\frac{\omega}{\omega_s}\right)^3 \quad , \quad \omega < \omega_s,
\end{aligned}
$$

Parameter space of the model

Our parameter space can be expressed in terms of 2 parameters:

$$z_s = \frac{\tau_s}{\tau_1} = \frac{a_1}{a_s} = \frac{\omega_1}{\omega_s}, \qquad \text{time scale of string phase}$$

$$\frac{g_s}{g_1} = \left(\frac{\tau_s}{\tau_1}\right)^{-\beta} = z_s^{-\beta}. \qquad \text{evolution of dilaton field}$$

that should be constrained by typical constraints of the Pre-Big Bang scenario [2]

[2] M. Gasperini, "*Observable gravitational waves in pre-big bang cosmology: an update*", JCAP1612, 010 (2016).

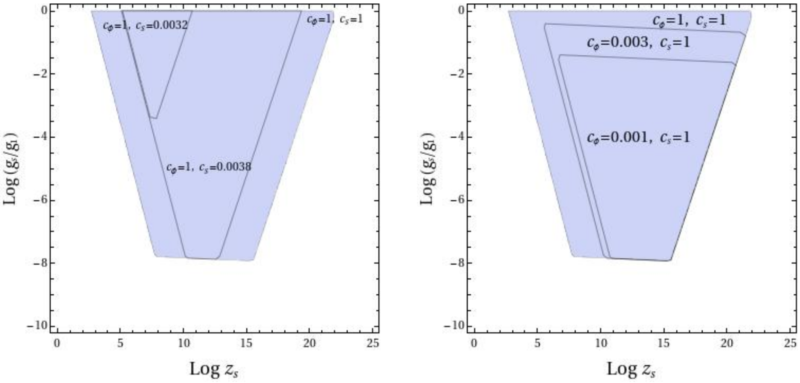


Figure: Parameter space at varying sound speed.

PBHs production

A fluctuation with frequency ω_M, which re-enters at the scale $H_M \rightarrow$ PBH with mass:

$$M \sim \frac{{M_P}^2}{H_M}.$$

We define the PBHs abundance: $$\beta \equiv \frac{\rho_{PBH}}{\rho_{tot}}\bigg|_{at formation}.$$

and we connect the PBHs with the dark matter abundance by the parameter f_{pbh}:

$$f_{pbh} \equiv \frac{\Omega_{pbh}}{\Omega_{cdm}} \qquad \Longrightarrow \qquad \begin{aligned} f_{pbh}^{RD} &\sim \beta \frac{\Omega_\gamma^0}{\Omega_{cdm}^0} \frac{T_k}{T_0} \\ f_{pbh}^{MD} &\sim \beta \frac{\Omega_\gamma^0}{\Omega_{cdm}^0} \frac{T_d}{T_0} \end{aligned}$$

Formation in radiation dominated era

$$\beta = \frac{2}{\sqrt{2\pi\sigma^2}} \int_{\delta_c}^{\infty} \exp\left\{ \frac{-\delta^2}{2\sigma^2} \right\} = \mathrm{Erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma} \right)$$

where the density contrast δ is related to $\mathcal{R}$ by

$$\delta = \frac{2(1+\omega)}{5+3\omega}\mathcal{R} \qquad \Rightarrow \qquad \sigma^2 \sim \mathcal{P}_\delta \sim \frac{16}{81}\mathcal{P}_\mathcal{R}$$

If DM is made of PBHs then $f_{pbh} \sim 1 \rightarrow$ constraints on the primordial spectrum:

$$f_{pbh} \sim 1 \quad \Rightarrow \quad \mathcal{P}_\mathcal{R} \gtrsim 10^{-2}$$

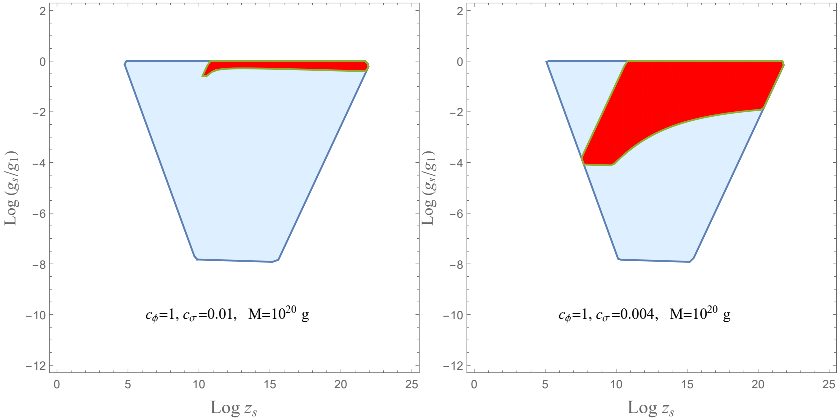


Figure: Production of Pbhs in RD era at varying axion sound speed.

Formation in matter era

When the collapse happens in matter era asphericities in the collapsing region should be taken into account [3]

$$\beta_0 \sim 0.056\sigma^5 \,, \qquad \sigma > 0.005$$

$$\beta_0 \sim 10^{-7}\sigma^2 \exp\left\{ \left(-\frac{0.15}{\sigma^{2/3}} \right) \right\}, \qquad \sigma < 0.005$$

where $\sigma < \sigma_{ang} \sim 0.005$ angular momentum of the collapsing region should be taken into account.

$$f_{pbh}^{MD} \sim \left(\frac{\beta_0}{5.5 \times 10^{-15}} \right) \left(\frac{T_d}{10^5 GeV} \right)$$

[3] T. Harada, C. Yoo, K. Kohri, and K. Nakao, Phys. Rev. D 96, 083517 (2017)

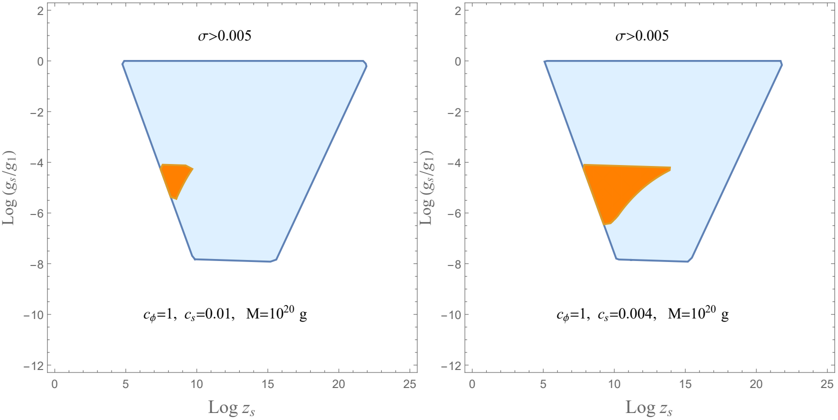


Figure: Production in matter era for $\sigma > \sigma_{ang}$.

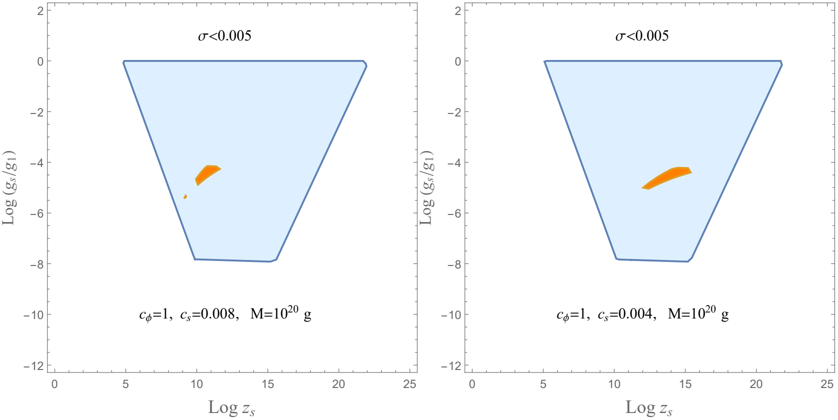
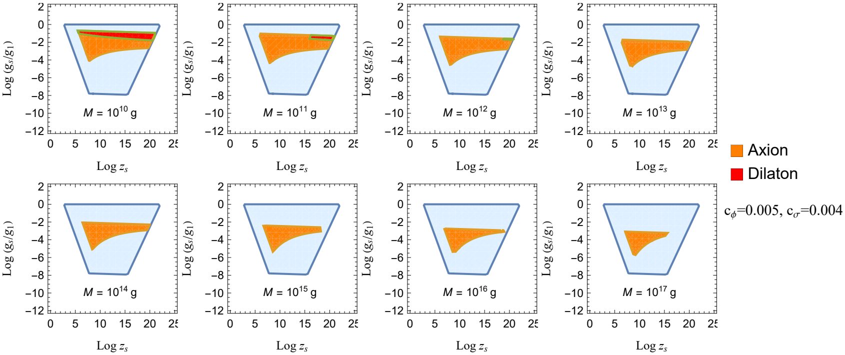


Figure: Production in matter era for $\sigma < \sigma_{ang}$

Light PBHs



Log (g_s/g_1)
Log z_s
M = 10^{10} g
M = 10^{11} g
M = 10^{12} g
M = 10^{13} g
M = 10^{14} g
M = 10^{15} g
M = 10^{16} g
M = 10^{17} g
Axion
Dilaton
c_ϕ=0.005, c_σ=0.004

Conclusions

- Non trivial sound speed dependence.
- A possibility of PBHs formations by this effect, requiring $c_s \ll 1$. In the particular Pre-Big Bang we obtain a suitable PBHs production in order to produce the dark matter if we require $0.003 < c_s < 0.01$.

Future prospects:

- Evaluate the case of $c_s(\tau)$ (motiveted by loop corrections).
- Light PBHs impact on the model (eventually how to avoid them).

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Thank you for the attention

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