

**At the crossroads of physics  
and mathematics : the joy of  
integrable combinatorics  
(Philippe60)**



**Rapport sur les  
contributions**

ID de Contribution: 1

Type: **Non spécifié**

## On the higher rank dimer and Ising models

*lundi 24 juin 2024 14:30 (45 minutes)*

Given a planar bipartite graph with a  $GL(n, \mathbb{R})$  local system, we define an associated Kasteleyn operator and show that its determinant enumerates certain objects ("n-multiwebs") generalizing the dimer model. Likewise on a nonbipartite graph with an  $Sp(2n)$  local system we show that the Pfaffian of an associated Kasteleyn-type matrix enumerates certain multiwebs generalizing Ising model configurations. This is based on joint work with D. Douglas, N. Ovenhouse, H. Shi, and H. Wu.

**Orateur:** KENYON, Richard (Yale University)

ID de Contribution: 3

Type: **Non spécifié**

## Random point processes in the plane and applications to birds of prey

*mercredi 26 juin 2024 09:30 (30 minutes)*

Random point processes including determinantal ones are popular models in ecology. In this talk I will put the two-dimensional Coulomb gas at general inverse temperature  $\beta \geq 0$  in a such a perspective. Away from the integrable point  $\beta=2$ , corresponding to the Ginibre ensemble of random matrices with complex normal entries, the Poisson point process at  $\beta=0$ , very little is known about the local statistics. We therefore resort to numerical simulations to determine the nearest and next-to nearest spacing to model data from biology. An alternative, approximate description is based on a  $2 \times 2$  random matrix  $\beta$ -ensemble. Annual ensembles of nests of three different birds of prey in the area of the Teutoburger Wald close to Bielefeld are modelled by such a simple random point process, in fitting an effective  $\beta$  to the data. In such a way repulsion strength can be quantified, comparing the inter and intra-species repulsion, as well as their change over time.

This is joint work with Adam Mielke, Patricia Paessler and the group of Oliver Krueger

**Orateur:** AKEMANN, Gernot (Bielefeld University & University of Bristol)

ID de Contribution: 4

Type: **Non spécifié**

## **Combinatorics of 3-coloured quadrangulations**

*lundi 24 juin 2024 11:45 (45 minutes)*

**Orateur:** BOUSQUET-MÉLOU, Mireille (CNRS, Université de Bordeaux)

ID de Contribution: 5

Type: **Non spécifié**

## Elliptic Calogero-Moser system and Gauge Theories

*mardi 25 juin 2024 09:30 (30 minutes)*

There is a remarkable correspondence between massive vacua of certain supersymmetric gauge theories and equilibrium position of mechanical integrable systems. Here I focus on the elliptic Calogero-Moser system, which can be seen as the interactions of particles on a two-dimensional torus. Despite its simplicity, the equilibria exhibit a surprisingly rich structure, with connections with modular forms, Galois theory, generalized symmetries, and phases of gauge theories.

**Orateur:** BOURGET, Antoine (IPhT)

ID de Contribution: 6

Type: **Non spécifié**

## The magic number conjecture for the $m=2$ amplituhedron and Parke-Taylor identities

*mardi 25 juin 2024 14:30 (45 minutes)*

The magic number conjecture says that the cardinality of a tiling of the amplituhedron  $A_{n,k,m}$  is the number of plane partitions which fit inside a  $k$  by  $(n-k-m)$  by  $m/2$  box.

(This is a generalization of the fact that triangulations of even-dimensional cyclic polytopes have the same size.) I'll explain how we prove the magic number conjecture for the  $m=2$  amplituhedron; we also show that all positroid tilings of the hypersimplex  $\Delta_{k+1,n}$  have the same cardinality. Along the way, we give volume formulas for Parke-Taylor polytopes in terms of circular extensions of cyclic partial orders, and we prove new variants of the classical Parke-Taylor identities. This is joint work with Matteo Parisi, Melissa Sherman-Bennett, and Ran Tessler.

**Orateur:** WILLIAMS, Lauren (Harvard University)

ID de Contribution: 7

Type: **Non spécifié**

## **Non-invertible symmetries in loop models**

*mercredi 26 juin 2024 16:00 (30 minutes)*

I will revisit some old results of Philippe, Jean-Bernard Zuber and I about degeneracies in  $O(n)$  CFTs, and interpret them using ideas of non-invertible symmetries as well as tools from the bootstrap.

**Orateur:** SALEUR, Hubert (IPhT)

ID de Contribution: 8

Type: **Non spécifié**

## **Playing with free probability in noisy many-body quantum systems**

*mercredi 26 juin 2024 11:15 (30 minutes)*

**Orateur:** BERNARD, Denis (LPENS, CNRS/École normale supérieure)



ID de Contribution: 9

Type: **Non spécifié**

## Integrable dynamics on polygons and the dimer integrable system

*lundi 24 juin 2024 17:00 (30 minutes)*

On the one hand, several discrete-time dynamical systems on spaces of polygons have been shown in the last twenty years to be integrable. On the other hand, Goncharov and Kenyon introduced ten years ago an integrable system associated with the dimer model on bipartite graphs on the torus. Building upon the notion of triple crossing diagram maps (introduced in recent works of Affolter, Glick, Pylyavskyy and myself), I will describe a framework which encompasses both the geometric dynamics on polygons and the dimer integrable system. This framework makes it possible in particular to identify the conserved quantities of both systems. I will illustrate this paradigm on the example of the pentagram map.

This talk is based on joint works with Niklas Affolter (TU Vienna), Terrence George (UCLA), Max Glick (Google) and Pavlo Pylyavskyy (University of Minnesota).

**Orateur:** RAMASSAMY, Sanjay (IPhT)

ID de Contribution: **10**

Type: **Non spécifié**

## **Mating of discrete trees and walks in the quarter plane**

*mardi 25 juin 2024 17:00 (30 minutes)*

I will exhibit a simple construction of planar maps using walks in the quarter plane. This allows to recover in a unified way several known bijections between walks and planar maps (possibly decorated by some combinatorial data) and also to find new bijections.

**Orateur:** BIANE, Philippe (LIGM, CNRS/Université Gustave Eiffel)

ID de Contribution: **11**

Type: **Non spécifié**

## Introduction

*lundi 24 juin 2024 09:40 (10 minutes)*

**Orateur:** PÉPIN, Catherine (IPhT)

ID de Contribution: 12

Type: **Non spécifié**

## **An overview of Philippe's contributions I**

*lundi 24 juin 2024 09:50 (55 minutes)*

We will give an informal overview of Philippe's contributions, told from our own perspectives.

- Jean-Bernard Zuber : Philippe @ Saclay (and elsewhere), the early years
- Emmanuel Guitter : The Philippe60 integer sequence
- Jérémie Bouttier : Philippe and the joyful integrable combinatorics of 2D quantum gravity

**Orateur:** THE ORGANIZERS

ID de Contribution: 13

Type: **Non spécifié**

## **An overview of Philippe's contributions II**

*lundi 24 juin 2024 11:15 (30 minutes)*

We will give an informal overview of Philippe's contributions, told from our own perspectives.

- Paul Zinn-Justin : PDF, ASM, DPP and TSSCPP
- Rinat Kedem : Philippe's Paths to positivity

**Orateur:** THE ORGANIZERS

ID de Contribution: 14

Type: **Non spécifié**

## Last passage percolation in a strip

*lundi 24 juin 2024 15:15 (30 minutes)*

I will present a method for computing the stationary measures of integrable probabilistic systems with boundaries. We will focus on the case of a model called last passage percolation, where the stationary measure can be determined with the help of variants of the Cauchy and Littlewood summation identities for Schur symmetric functions. The method works as well for other models and their associated families of symmetric functions, such as Whittaker functions or Hall-Littlewood polynomials. We will also discuss connections with the traditional approach for computing stationary measures of interacting particle systems between boundary reservoirs: the matrix product ansatz.

**Orateur:** BARRAQUAND, Guillaume (LPENS, CNRS/École normale supérieure)

ID de Contribution: 15

Type: **Non spécifié**

## Domino tilings of generalised Aztec triangles and determinant evaluations

*lundi 24 juin 2024 16:15 (45 minutes)*

Di Francesco introduced Aztec triangles as combinatorial objects for which their domino tilings are equinumerous with certain sets of configurations of the twenty-vertex model. He conjectured a closed form product formula for the numbers of these tilings, respectively of these configurations. The formula was proved by Christoph Koutschan using Zeilberger's holonomic Ansatz and heavy calculations using computer algebra.

In my talk I will generalise Di Francesco's construction of Aztec triangles. The main result is that also the number of domino tilings of these *generalised* Aztec triangles is given by a closed form product formula. The proof proceeds by translating the domino tilings into non-intersecting lattice paths and by using the Lindström-Gessel-Viennot theorem to obtain a determinant. The final - and "as usual" most difficult - step then consists in the evaluation of this determinant.

There is a whole zoo of determinant evaluations related to this problem.

This is joint work with Sylvie Corteel and Frederick Huang on the one hand, and with Christoph Koutschan and Michael Schlosser on the other hand.

**Orateur:** KRATTENTHALER, Christian (Universität Wien)

ID de Contribution: 16

Type: **Non spécifié**

## **Semiclassical limit of the spin Calogero-Moser system as a hybrid integrable system (online talk)**

*mercredi 26 juin 2024 12:00 (30 minutes)*

The talk will start with a brief outline of hybrid integrable systems. Such systems consist of an integrable classical background and a quantum “bundle” over the phase space of this classical system. The quantum dynamics of such a system is “driven” by the classical integrable dynamics. An example of such a system appears in the semiclassical limit of the spin Calogero-Moser system. We describe these hybrid systems over invariant tori of all possible dimensions.

**Orateur:** RESHETIKHIN, Nicolai (YMSC Tsinghua University)



ID de Contribution: 17

Type: **Non spécifié**

## **Integrable monopoles (online talk)**

*mardi 25 juin 2024 10:15 (30 minutes)*

A static monopole embedded in  $N=4$  super Yang-Mills theory constitutes a one-dimensional defect, a 't Hooft line and gives rise to a defect conformal field theory. We demonstrate how quantizing around the monopole background can be implemented via the solution of beautiful and exactly solvable quantum mechanical problems.

**Orateur:** KRISTJANSEN, Charlotte (Niels Bohr Institute, Copenhagen University)

ID de Contribution: **18**

Type: **Non spécifié**

## **The sinh-Gordon model and its excited states**

*mardi 25 juin 2024 11:15 (45 minutes)*

This talk returns to the old idea that excited states in integrable quantum field theories can be found by a process of analytic continuation. By focussing on the sinh-Gordon model at small coupling, evidence for a uniform structure is found which suggests that a complete description will be possible.

**Orateur:** DOREY, Patrick (Durham University)

ID de Contribution: 19

Type: **Non spécifié**

## **Branes and Supersymmetric Gauge Theories**

*mardi 25 juin 2024 12:00 (30 minutes)*

This talk will go over various brane constructions which arise in superstring theories and give rise to quiver gauge theories, emphasizing the combinatorial aspect of these physical systems. We will try to cover connections with mathematical topics in tropical geometry, brane tilings, cluster algebras, and symplectic singularities.

**Orateur:** HANANY, Amihay (Imperial College)

ID de Contribution: 20

Type: Non spécifié

## Many new conjectures on Fully-Packed Loop configurations

*mardi 25 juin 2024 15:15 (30 minutes)*

We deal with one of the favourite objects of Philippe: Fully-Packed Loop configurations, in domains where the Razumov–Stroganov conjecture holds. Recall that the RS conjecture relates FPL's and the steady state of the  $O(1)$  dense loop model. In short, it states that the refined enumeration of FPL's according to the (black) link pattern is proportional to the aforementioned steady state. The conjecture exists in two main flavours: “dihedral” (ASM, HTASM, QTASM on one side, and the DLM on the cylinder on the other side), and “vertical” (VSASM, UASM, UUASM, OSASM, OOASM on one side, and the DLM on the strip on the other side). Together with L. Cantini, we gave two proofs (in 2010 and 2012) of the conjecture in the dihedral cases, but, despite the efforts of ourselves and others, the vertical case is still unsolved.

We recently looked back at the FPL configurations pertinent to one of the unsolved “vertical” cases, namely the UASM (ASM on a  $2n \times n$  rectangle with U-turn boundary conditions on one of the long sides). We have looked at the refinement according to the black and white link patterns, and the overall number of loops. This doesn't seem to help in understanding the Razumov–Stroganov conjecture, but leads to many more conjectures, suggesting the existence of a remarkable deformation of Littlewood–Richardson coefficients, somewhat in the same spirit, but apparently by a completely different mechanism, to “FPL in a triangle” studied by P. Zinn-Justin and by Ph. Nadeau among others (including Philippe!).

Work in collaboration with L. Cantini.

**Orateur:** SPORTIELLO, Andrea (LIPN, CNRS/Université Sorbonne Paris Nord)

ID de Contribution: 21

Type: **Non spécifié**

## Arctic curve fluctuations in the square-ice model

*mardi 25 juin 2024 16:15 (45 minutes)*

We study the emptiness formation probability (EFP) in the six-vertex model with domain wall boundary conditions. At the ice point, i.e., when all the Boltzmann weights are equal, we are able to build an explicit, although still conjectural, expression for the EFP as the Fredholm determinant of some linear integral operator. As the geometric parameters of the EFP are tuned to the vicinity of the arctic curve arising in the scaling limit, the obtained representation turns into the GUE Tracy-Widom distribution.

Joint work with A. Pronko - ArXiv:2405.04358

**Orateur:** COLOMO, Filippo (INFN, Florence)

At the crossroads ... / Rapport sur les contributions

Cancelled

ID de Contribution: **22**

Type: **Non spécifié**

**Cancelled**

**Orateur:** OKOUNKOV, Andrei (Columbia University)

ID de Contribution: 23

Type: **Non spécifié**

## Incidence geometry and tiled surfaces

*mercredi 26 juin 2024 10:00 (45 minutes)*

We show that various classical theorems of real/complex linear incidence geometry, such as the theorems of Pappus, Desargues, Möbius, and so on, can be interpreted as special cases of a general result that involves a triangulation of a closed oriented surface, or a tiling of such a surface by quadrilateral tiles. This yields a general mechanism for producing new incidence theorems and generalizing the known ones.

This is joint work with Pavlo Pylyavskyy, see <https://arxiv.org/abs/2305.07728>.

**Orateur:** FOMIN, Sergey (University of Michigan)

ID de Contribution: 24

Type: **Non spécifié**

## **Non-unitary fermions and extended symmetry**

*mercredi 26 juin 2024 14:30 (30 minutes)*

I will present a long-range integrable model based on the Temperley-Lieb algebra at the free fermionic point. In spite of the lack of translational invariance, the model possesses an extended symmetry and a very simple spectrum.

**Orateur:** SERBAN, Didina (IPhT)



ID de Contribution: 25

Type: **Non spécifié**

## Ruijsenaars wavefunctions as modular group matrix coefficients

*mercredi 26 juin 2024 15:00 (30 minutes)*

The phase space of the Ruijsenaars integrable system can be identified with (a Poisson reduction of) the moduli space of  $GL_n$  local systems on a punctured torus. The latter admits a structure of a cluster Poisson variety. On the algebraic level, this leads to an injective homomorphism from a spherical subalgebra of the double affine Hecke algebra into the quantized algebra of global functions on the named cluster variety. From an analytic point of view, it allows for a unitary equivalence between Toda and Ruijsenaars quantum integrable systems. This in turn allows one to present the eigenfunctions of Macdonald operators as a matrix coefficient of an order 4 element in the mapping class group of a punctured torus. During this talk we will focus on the  $n = 2$  case when no Hamiltonian reduction is required. It will be based on a joint work with P. DiFrancesco, R. Kedem, S. Khoroshkin, and G. Schrader.

**Orateur:** SHAPIRO, Alexander (University of Edinburgh)

ID de Contribution: **26**

Type: **Non spécifié**

## Conclusion

*mercredi 26 juin 2024 16:30 (15 minutes)*