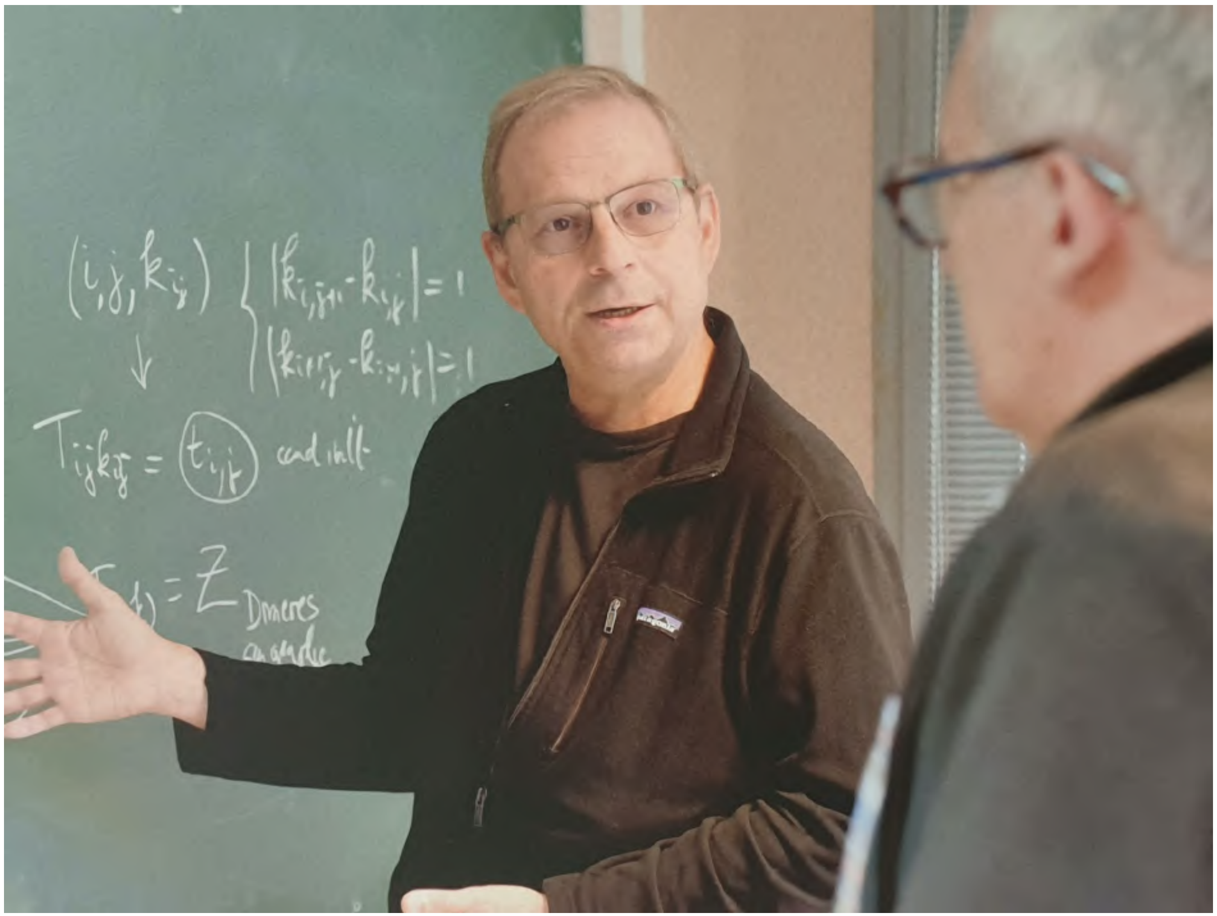
The Philippe60 integer sequence

(My collaboration with Philippe)



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THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

1 2 3 6 11 20 | Search | Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:1,2,3,6,11,20**

Displaying 1-10 of 34 results found. page 1 2 3 4

Sort: relevance | references | number | modified | created Format: long | short | data

A058214 Philippe Di Francesco's 60th birthday sequence +40
4

1, 2, 3, 6, 11, 20, 29, 37, 60, 96, 145, 214, 415 (list; graph; refs; listen; history; text; internal format)

1 2 3 6 11 20 29 37 60 96 145 214 415

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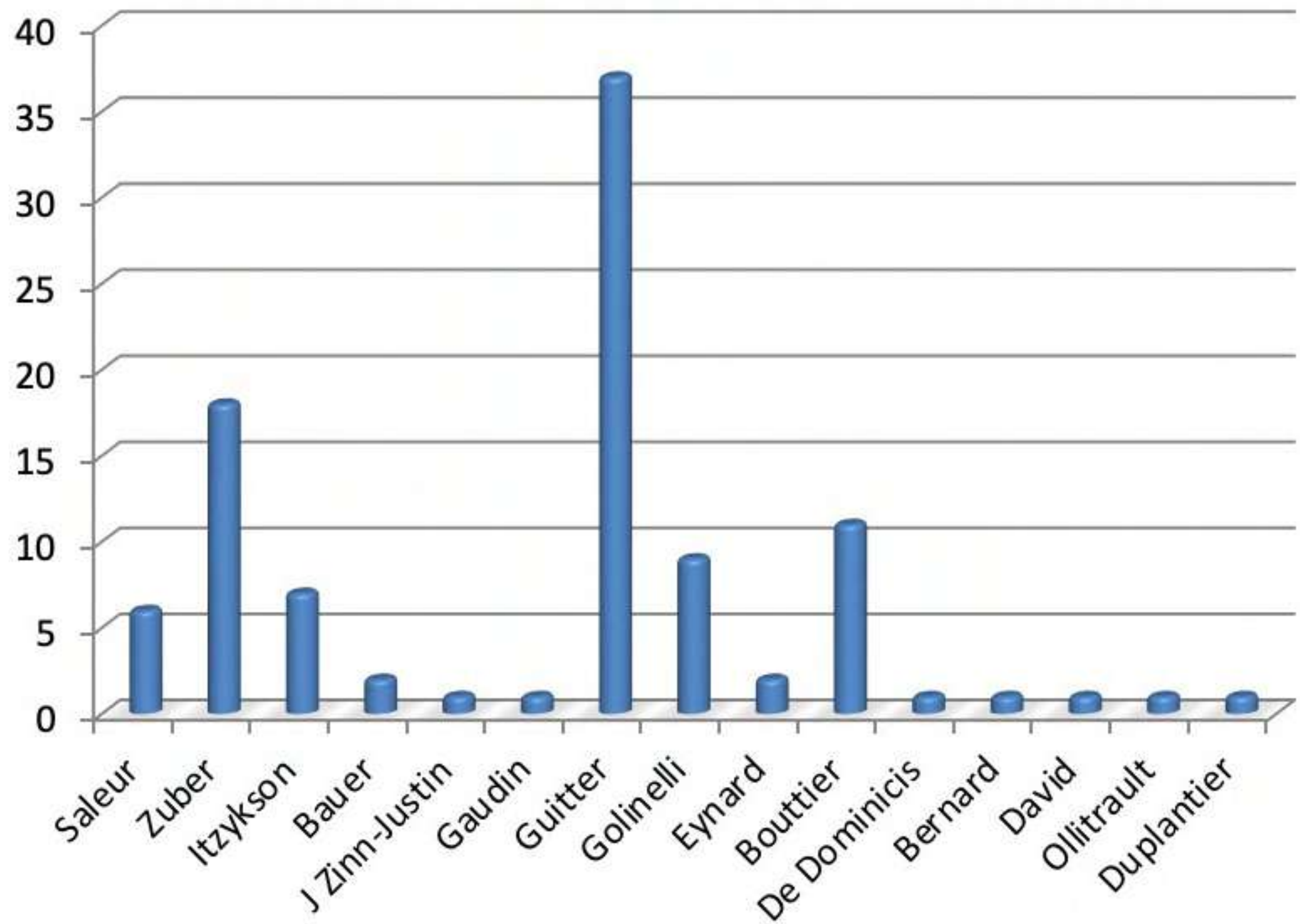
A058214 Philippe Di Francesco's 60th birthday sequence +40 4

1, 2, 3, 6, 11, 20, 29, 37, 60, 96, 145, 214, 415 (list; graph; refs; listen; history; text; internal format)

1 2 3 6 11 20 29 37 60 96 145 214 415

1 2 3 6 11 20 29 37 60 96 145 214 415

60 publications of Philippe with other members of IPhT

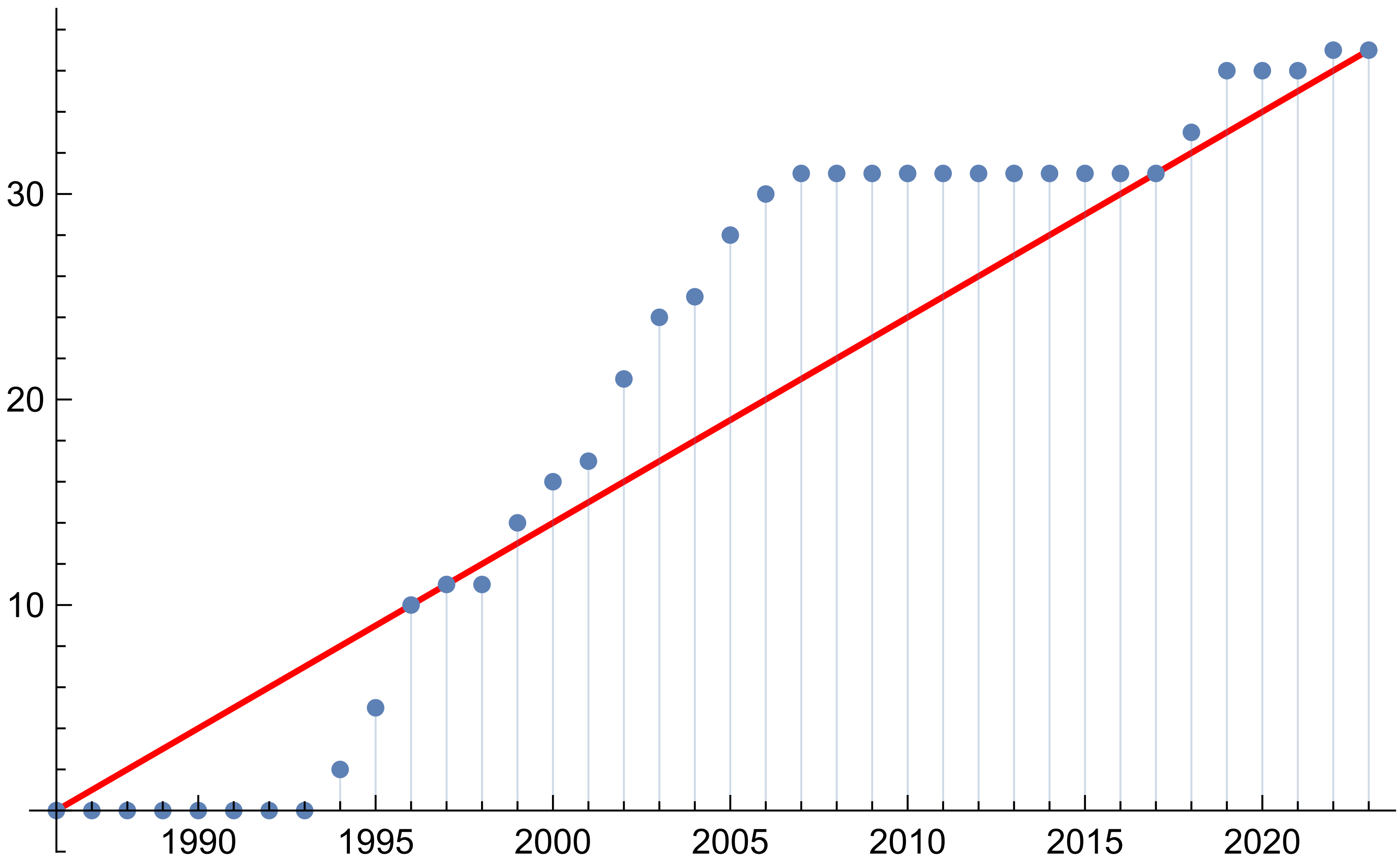


1 2 3 6 11 20 29 37 60 96 145 214 415

Joined the IPhT 37 years ago

1 2 3 6 11 20 29 37 60 96 145 214 415

Joined the IPhT 37 years ago, and have since written 37 papers together



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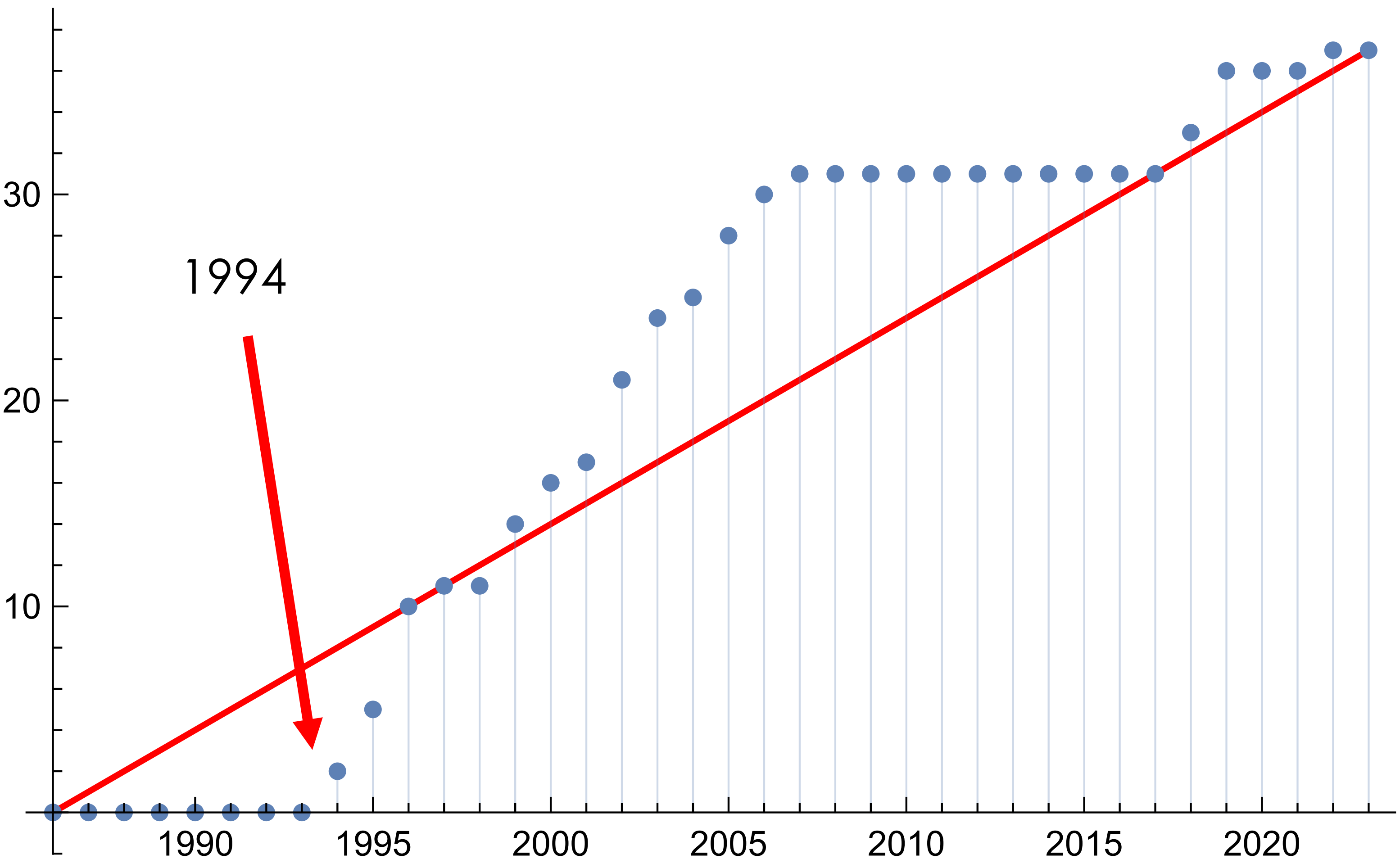

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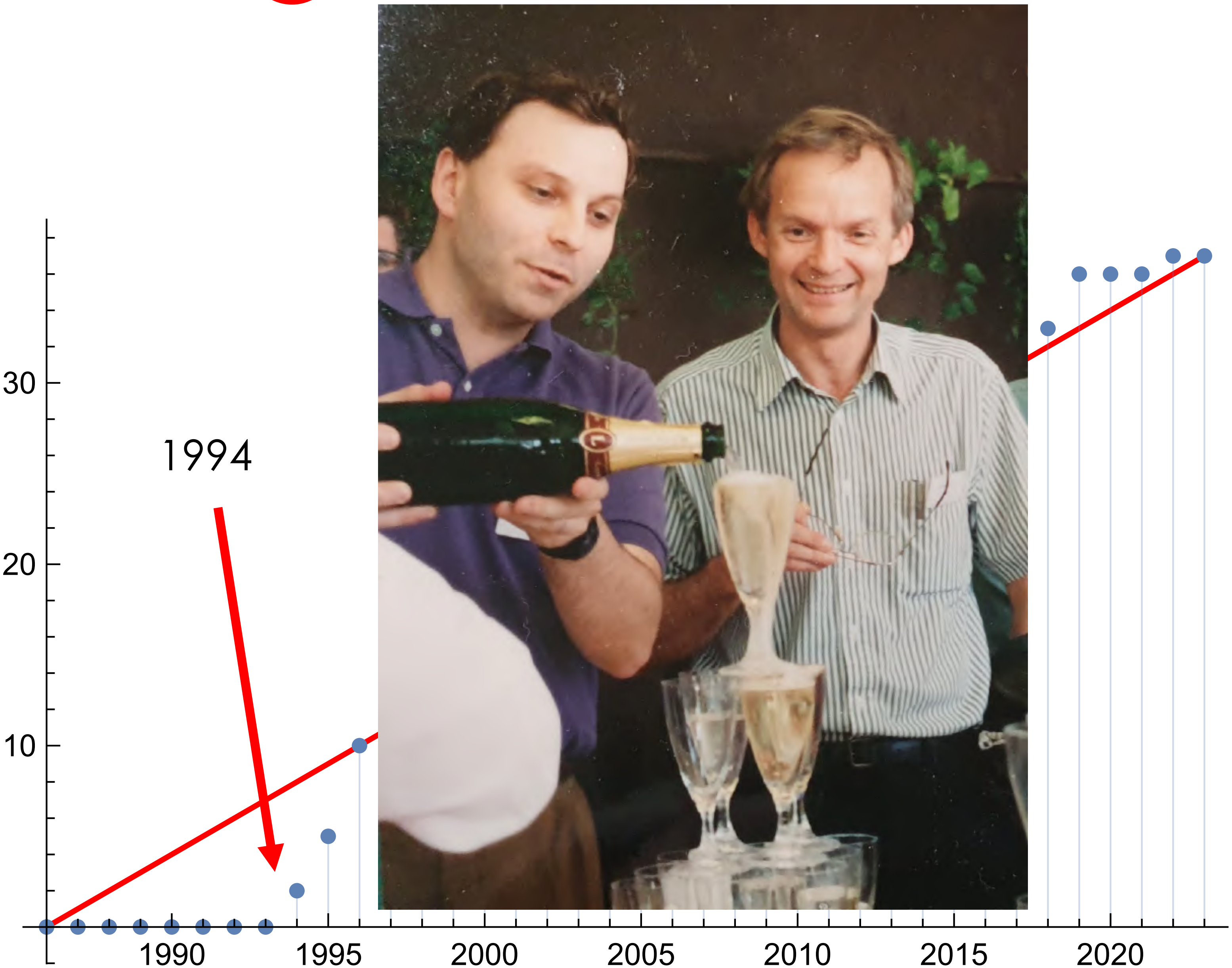

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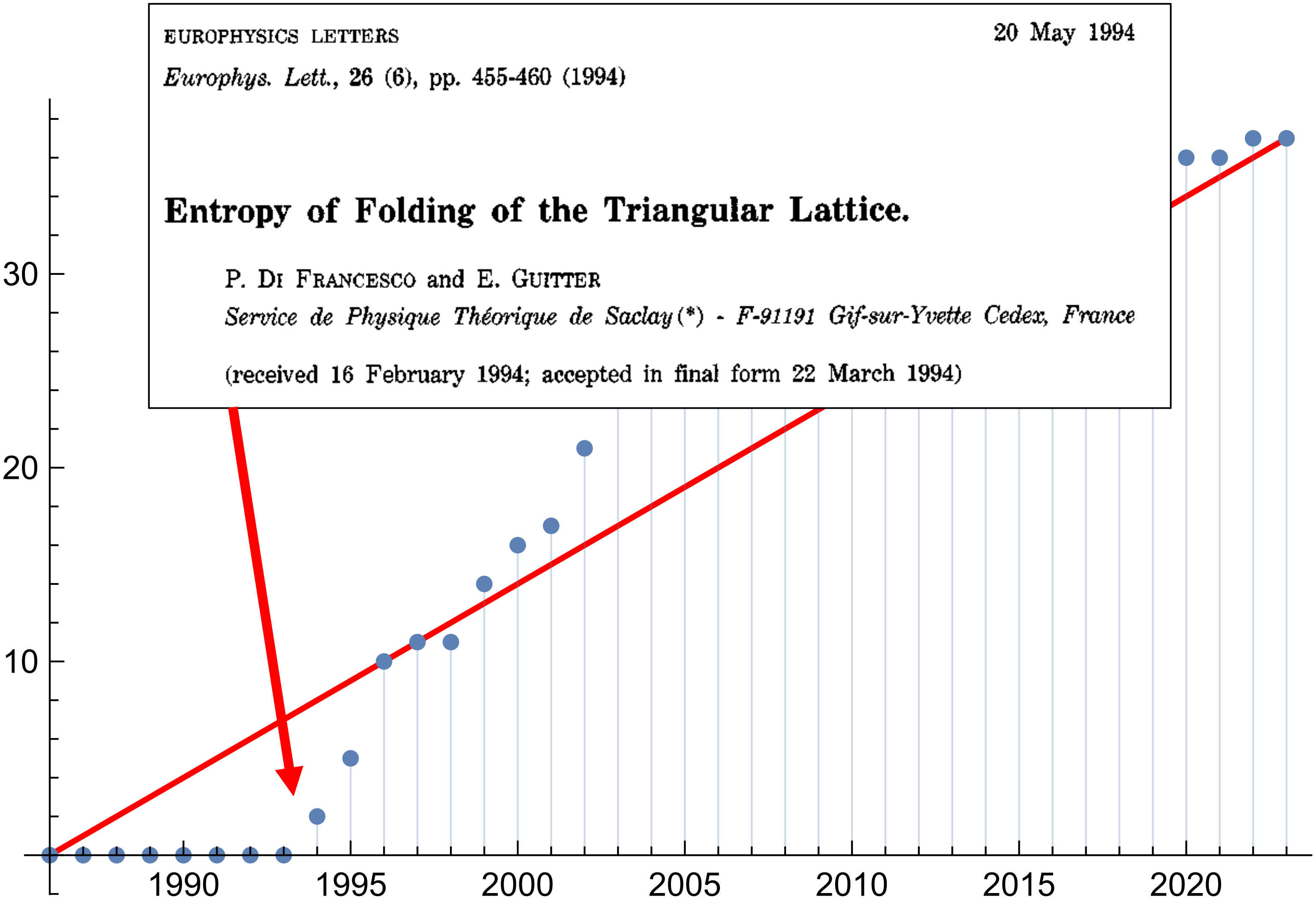


EUROPHYSICS LETTERS 20 May 1994

Europhys. Lett., **26** (6), pp. 455-460 (1994)

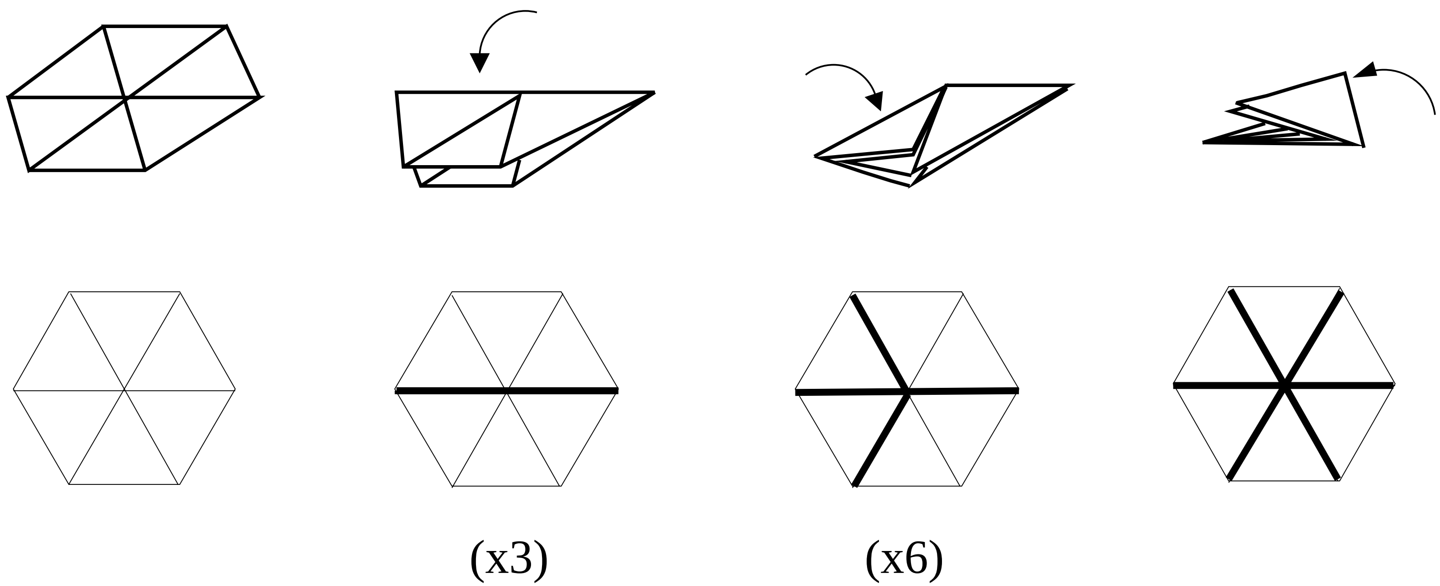
Entropy of Folding of the Triangular Lattice.

P. DI FRANCESCO and E. GUITTER

Service de Physique Théorique de Saclay (*) - *F-91191 Gif-sur-Yvette Cedex, France*

(received 16 February 1994; accepted in final form 22 March 1994)

1 2 3 6 11 20 29 37 60 96 145 214 415



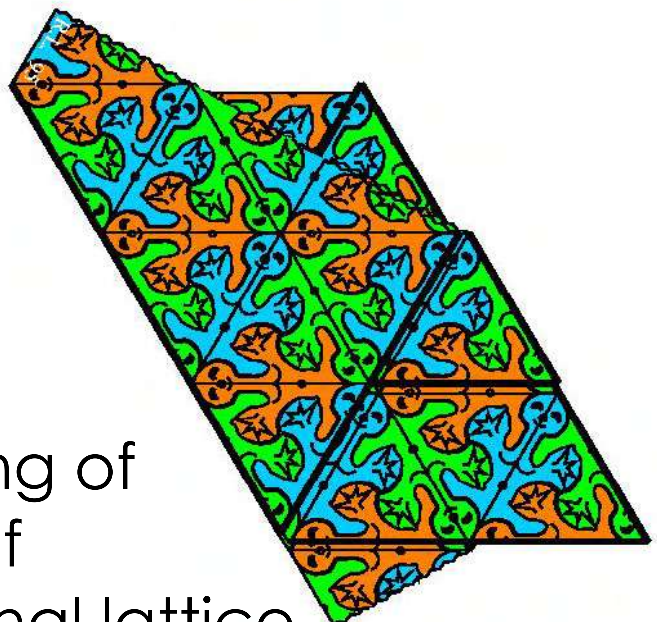
11 vertex model

Entropy ? N triangles: # foldings $\sim q^N$

$q \sim 1.21$ Kantor Jarić (1990)

1 2 3 6 11 20 29 37 60 96 145 214 415

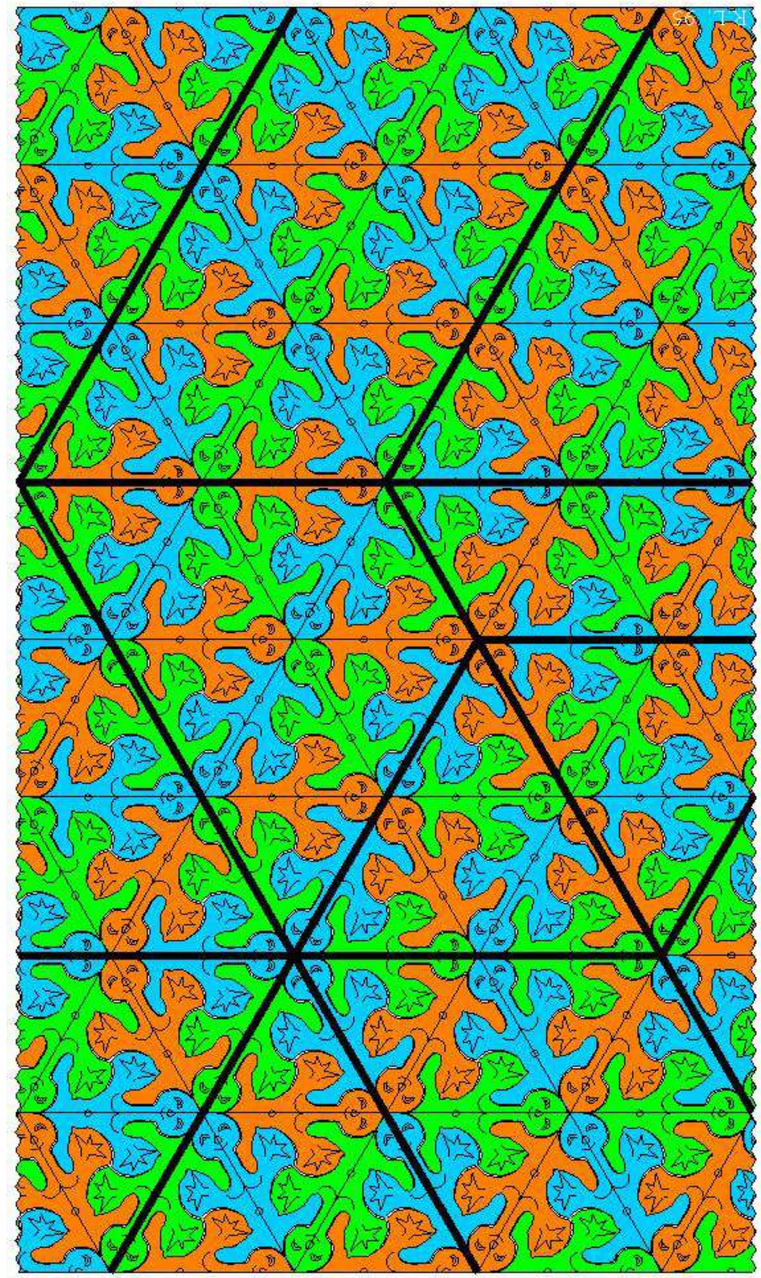
folding ⇔ 3-coloring of edges of triangular lattice



⇔ 3-coloring of edges of hexagonal lattice

$$q = \prod_{n=1}^{\infty} \frac{3n-1}{\sqrt{3n(3n-2)}} = 1.208717\cdots$$

Baxter (1970)



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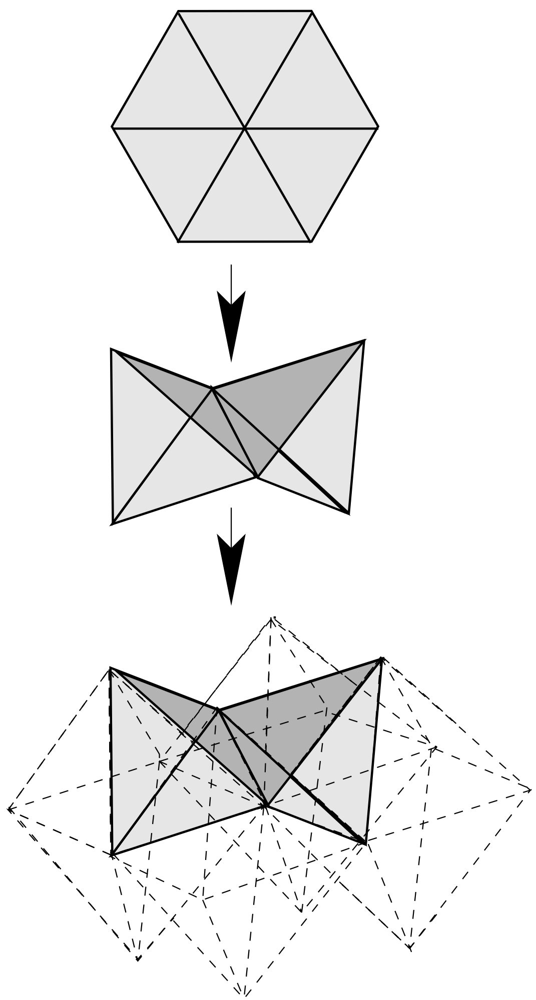
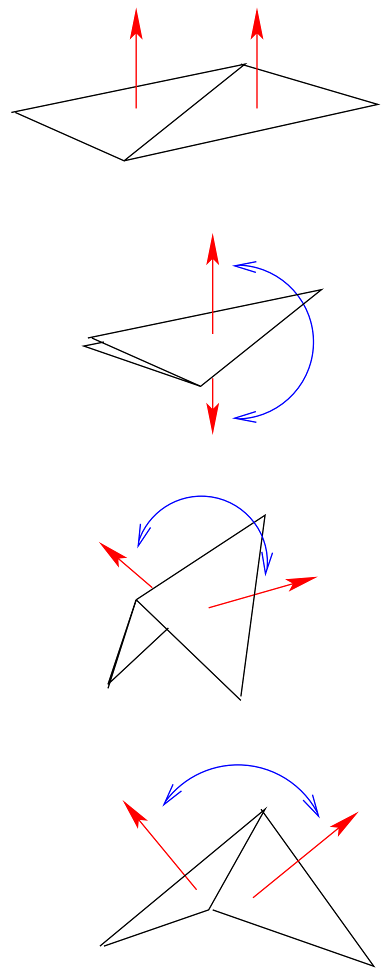




no fold

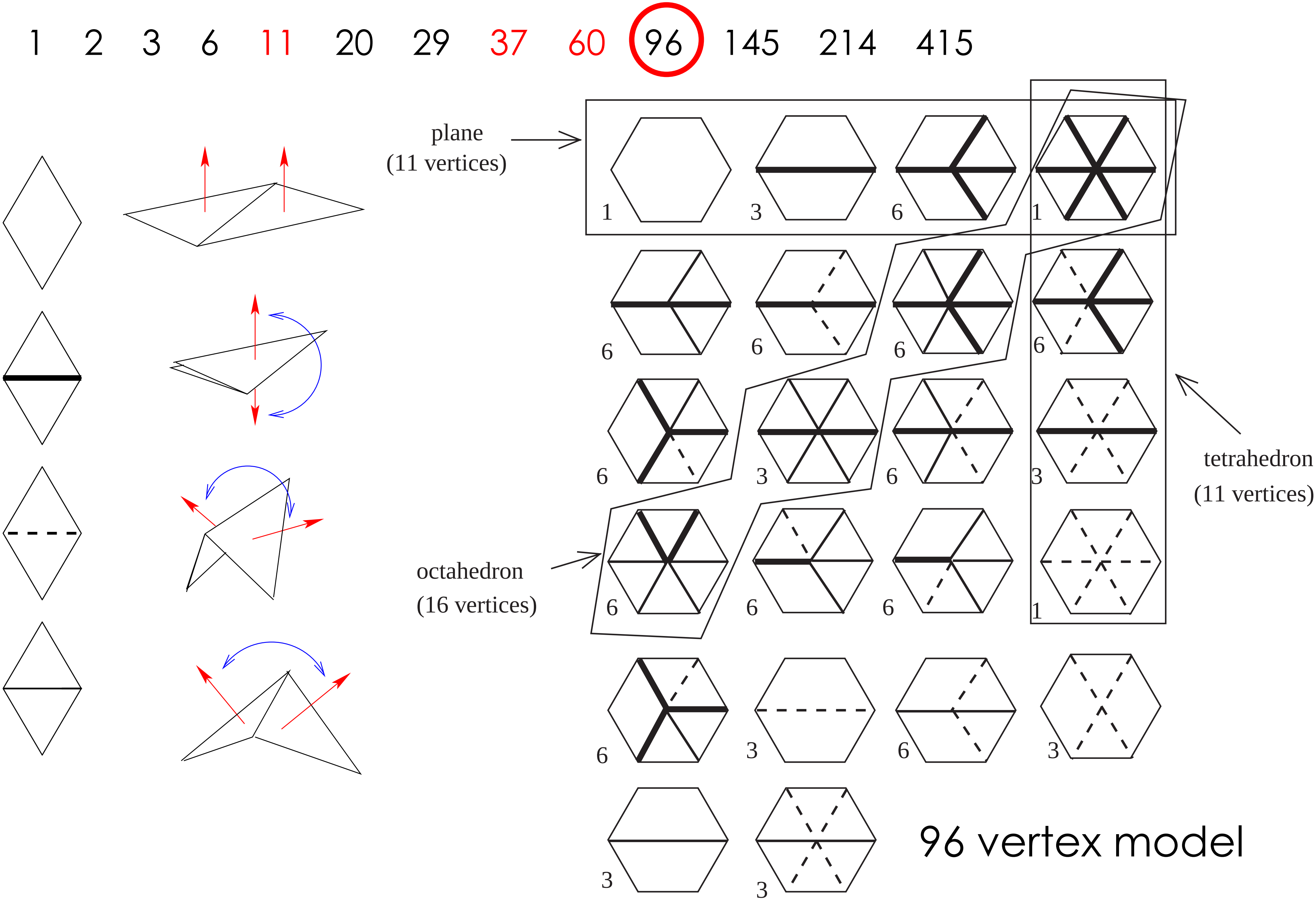
complete fold (180°)

acute fold (109°28')

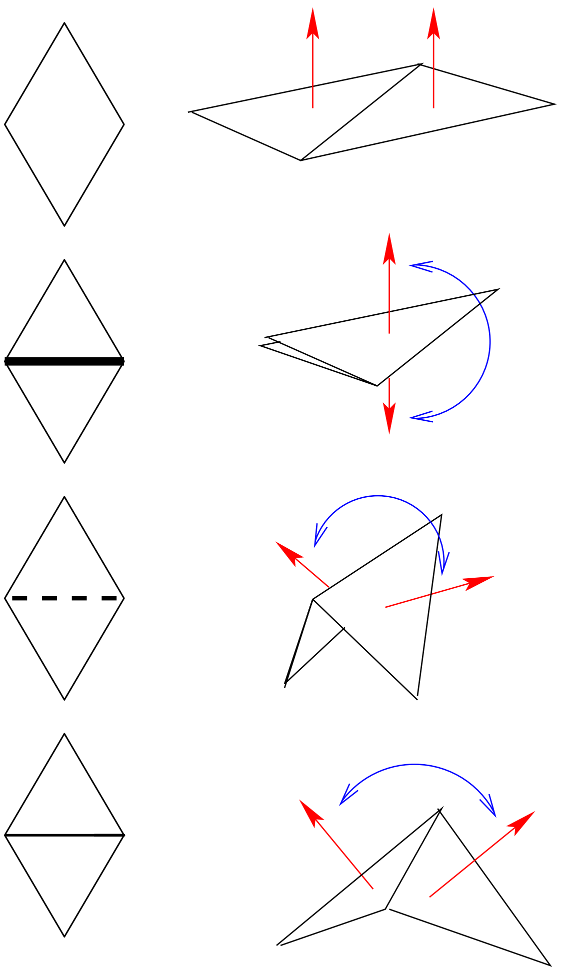
Obtuse fold (70°32')

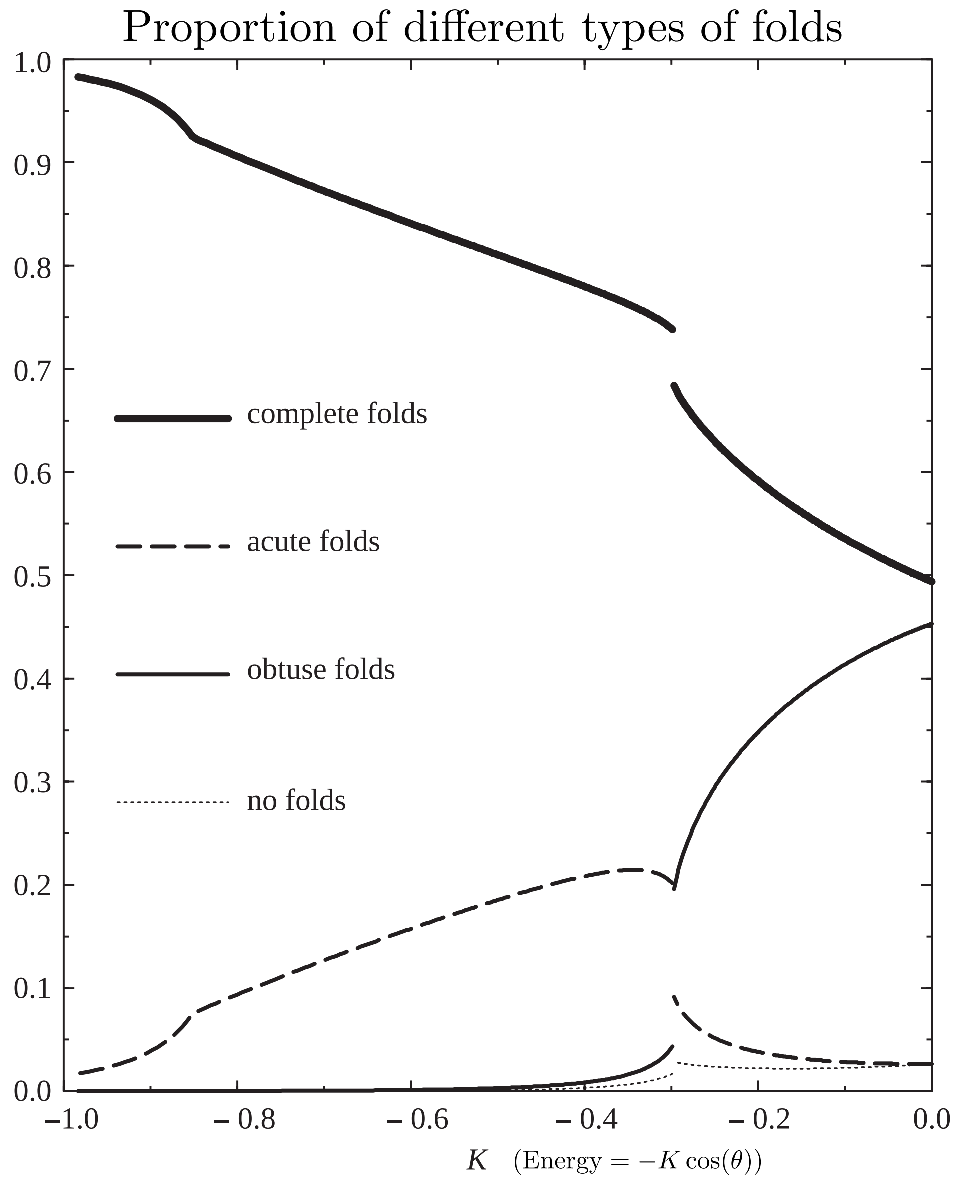
Folding on FCC lattice

with M. Bowick and O. Golinelli


1 2 3 6 11 20 29 37 60 96 145 214 415
plane
(11 vertices)
tetrahedron
(11 vertices)
octahedron
(16 vertices)
96 vertex model

1 2 3 6 11 20 29 37 60 96 145 214 415





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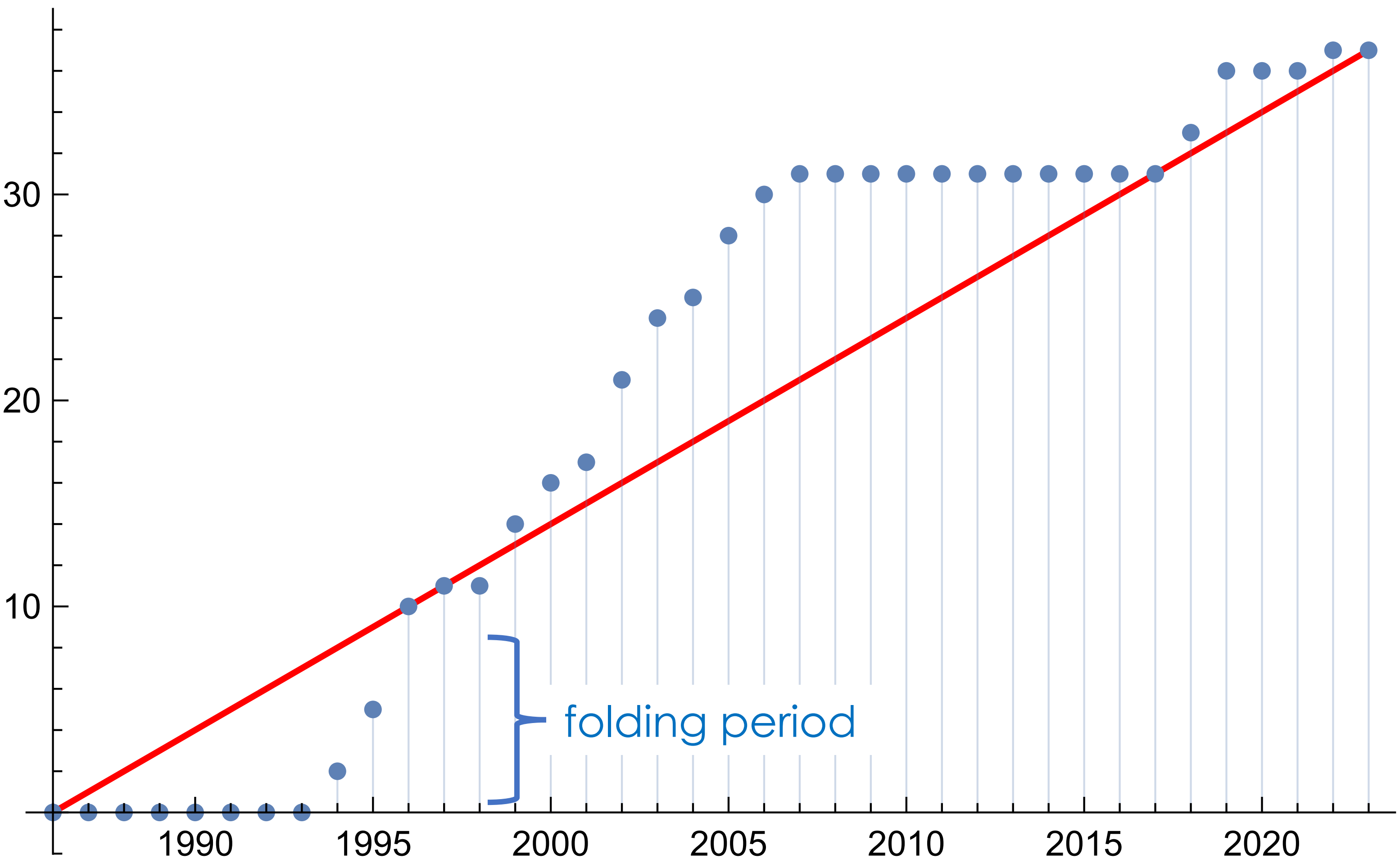

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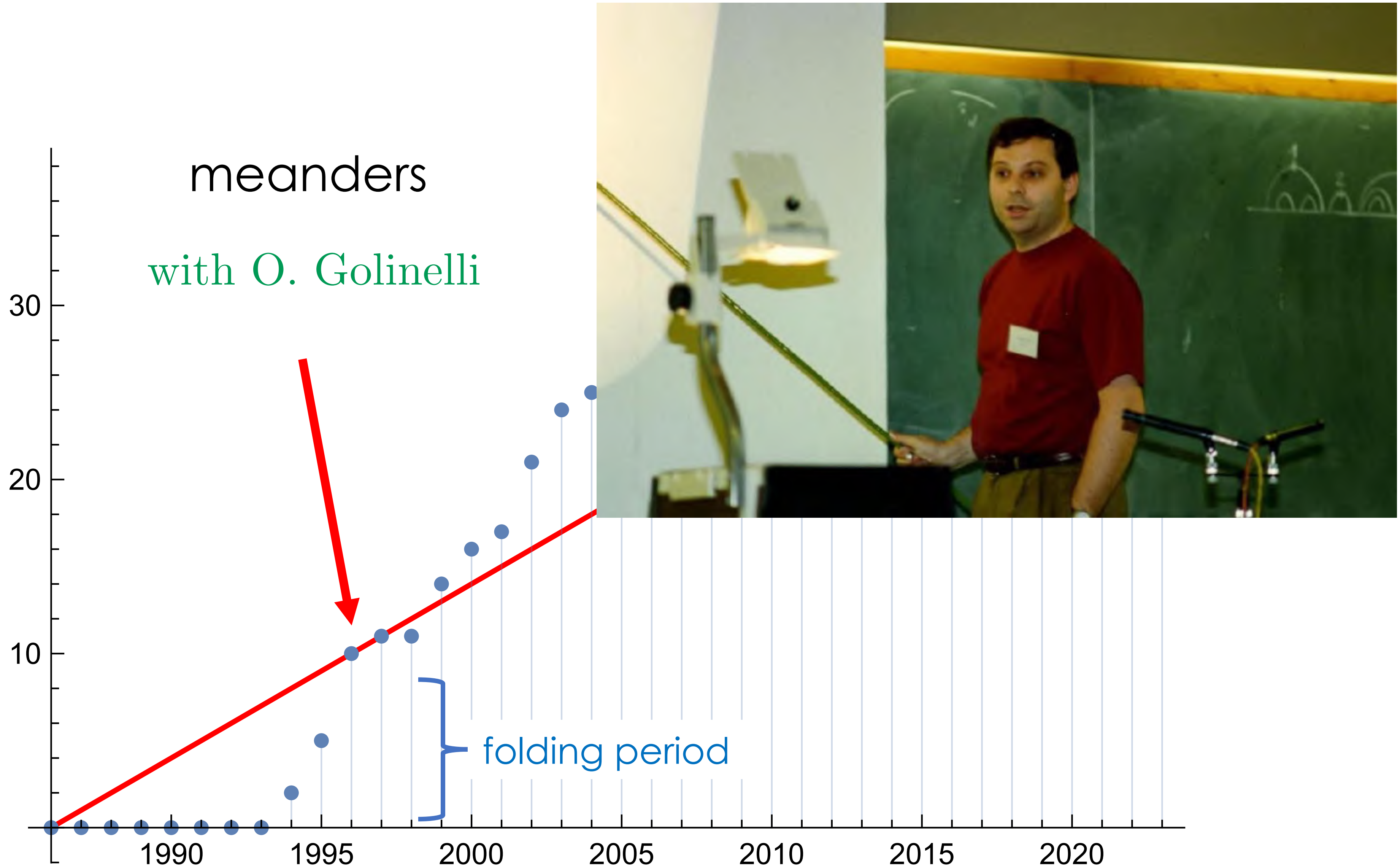

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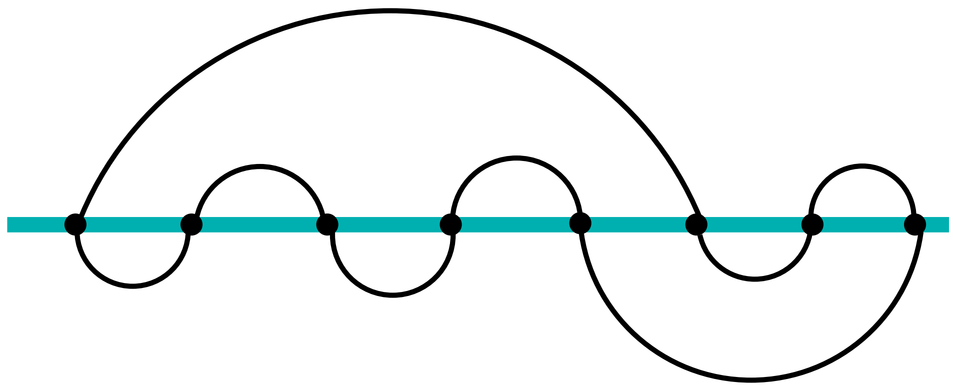
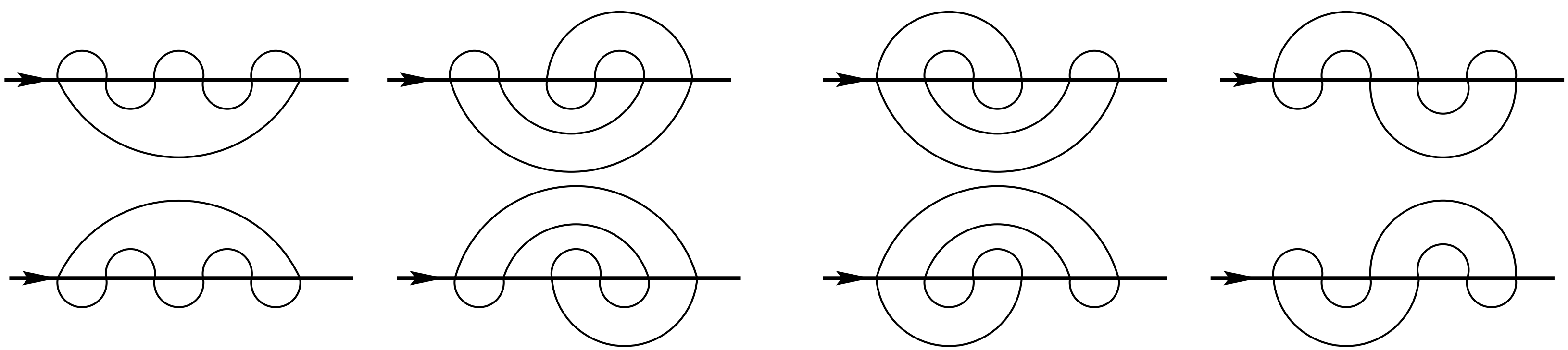
Meander: closed (racing) circuit crossing a river via $2n$ bridges





The $M_3 = 8$ configurations of meanders for $2n = 6$ bridges.

1 2 3 6 11 20 29 37 60 96 145 214 415

Selecta Mathematica Sovietica
Vol. 11, No. 2 (1992)

0272-9903/92/020117–28 $1.50 + 0.20/0

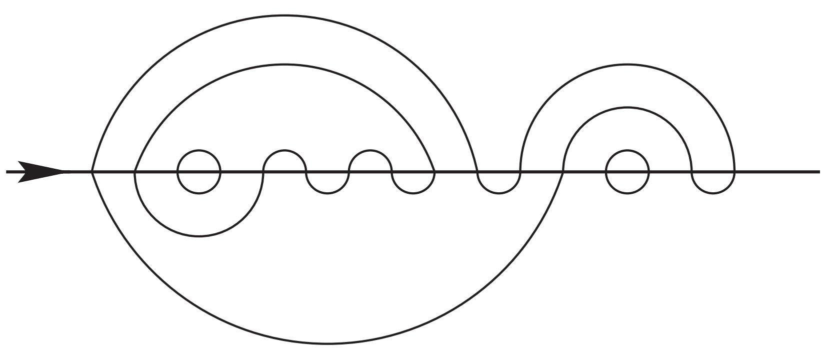
Meanders*

S. K. Lando and A. K. Zvonkin

Theorem. *There exists a number* A_M *such that, for any* $0 < A < A_M$, $A^n < M_n < (A_M)^n$ *for every large enough* n, *and* $A_M < (\pi/(4-\pi))^2$.

$$M_n \sim \text{const} \cdot (A_M)^n \cdot n^{-7/2} \quad \text{where } A_M = 12.26\cdots$$

- Weight q per connected component:



$$\rightarrow M_n(q) \sim \text{const.}(A_M(q))^n \; n^{-\alpha(q)}$$

1 2 3 6 11 20 29 37 60 96 145 214 415

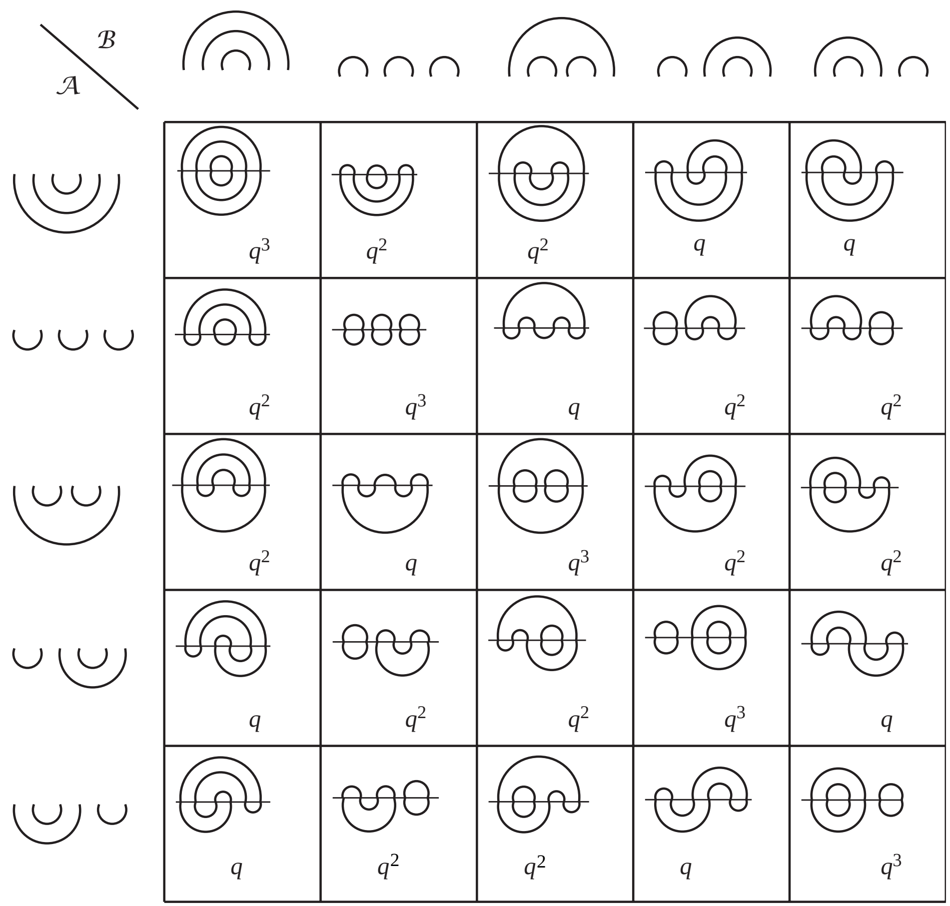
- $$\sqrt{A_M(q)} = 2\sqrt{q}\left(1 + \frac{1}{q} + \frac{3}{2q^2} - \frac{3}{2q^3} - \frac{29}{8q^4} - \frac{81}{8q^5} - \frac{89}{16q^6} + O\left(\frac{1}{q^7}\right)\right)$$

- « meander determinant »

$$\det = \prod_{1\leq \ell \leq i \leq n}\left(q - 2\cos\left(\pi\frac{\ell}{i+1}\right)\right)^{a_{n,i}}$$

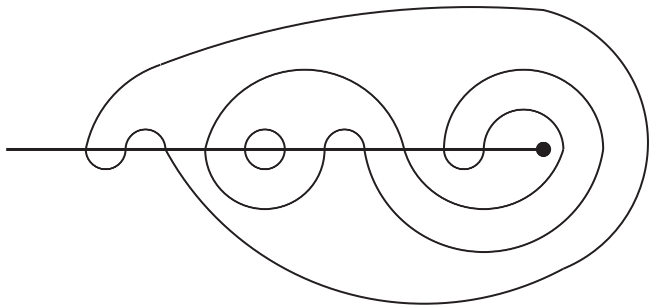
$$a_{n,i} = \binom{2n}{n-i} - 2\binom{2n}{n-i-1} + \binom{2n}{n-i-2}$$

NB: $\mathrm{tr}(\cdot)^2 = M_n(q^2)$



1 2 3 6 11 20 29 37 60 96 145 214 415

- Semi-meanders



- Numerics (based on exact enumerations)

O. Golinelli (1999) I. Jensen (1999)

$$M_n \sim \text{const.}(A_M)^n \; n^{-\alpha}$$

$$\alpha = 3.4208(6)$$

```
  PARAMETER (nmax = 14)
  INTEGER A(-nmax+1:nmax)
  INTEGER Sm(nmax)
  INTEGER n
  INTEGER j
  DATA n, Sm/0, nmax*0/
  A(0) = 1
  A(1) = 0
2 n = n + 1
  Sm(n) = Sm(n) + 1
  j = -n + 1
1 IF((n.EQ.nmax).OR.(j.EQ.n+1)) GOTO 3
  A(A(j)) = n+1
  A(n+1) = A(j)
  A(j) = -n
  A(-n) = j
  GOTO 2
3 A(A(-n+1)) = A(n)
  A(A(n)) = A(-n+1)
  j = A(n)+1
  n = n - 1
  IF (n .GT. 1) GOTO 1
  PRINT '(i3, i15)', (n, Sm(n), n = 1, nmax)
  END
```

1 2 3 6 11 20 29 37 60 96 145 214 415

Conjecture (2000)

$$\alpha = \frac{29 + \sqrt{145}}{12} = 3.42013\cdots$$

from KPZ for the coupling to gravity of a $c = -4$ CFT

1 2 3 6 11 20 29 37 60 96 145 214 415

Conjecture (2000)

$$\alpha = \frac{29 + \sqrt{145}}{12} = 3.42013\cdots$$

from KPZ for the coupling to gravity of a $c = -4$ CFT

I follow here the paper [7]. The problem of meanders "may be interpreted" as a model of "a pair of two fully packed loops". Now, a model of one fully packed loop has a "central charge" $c = -2$. "Therefore", the central charge for the model of two fully packed loops and, hence, for the model of meanders, is $c = -4$. There are reasons to believe that the model satisfies the "conformal invariance" property. If this is the case then, using the Knizhnik–Polyakov–Zamolodchikov equation [14], one may express the "string susceptibility" γ_{str} through the central charge:

$$\gamma_{str} = \frac{c - 1 - \sqrt{(25 - c)(1 - c)}}{12}.$$

Finally, the critical exponent α is expressed in terms of the string susceptibility as follows:

$$\alpha = 2 - \gamma_{str}.$$

For $c = -4$ all this gives

$$\alpha = \frac{29 + \sqrt{145}}{12} = 3.420132882\ldots$$

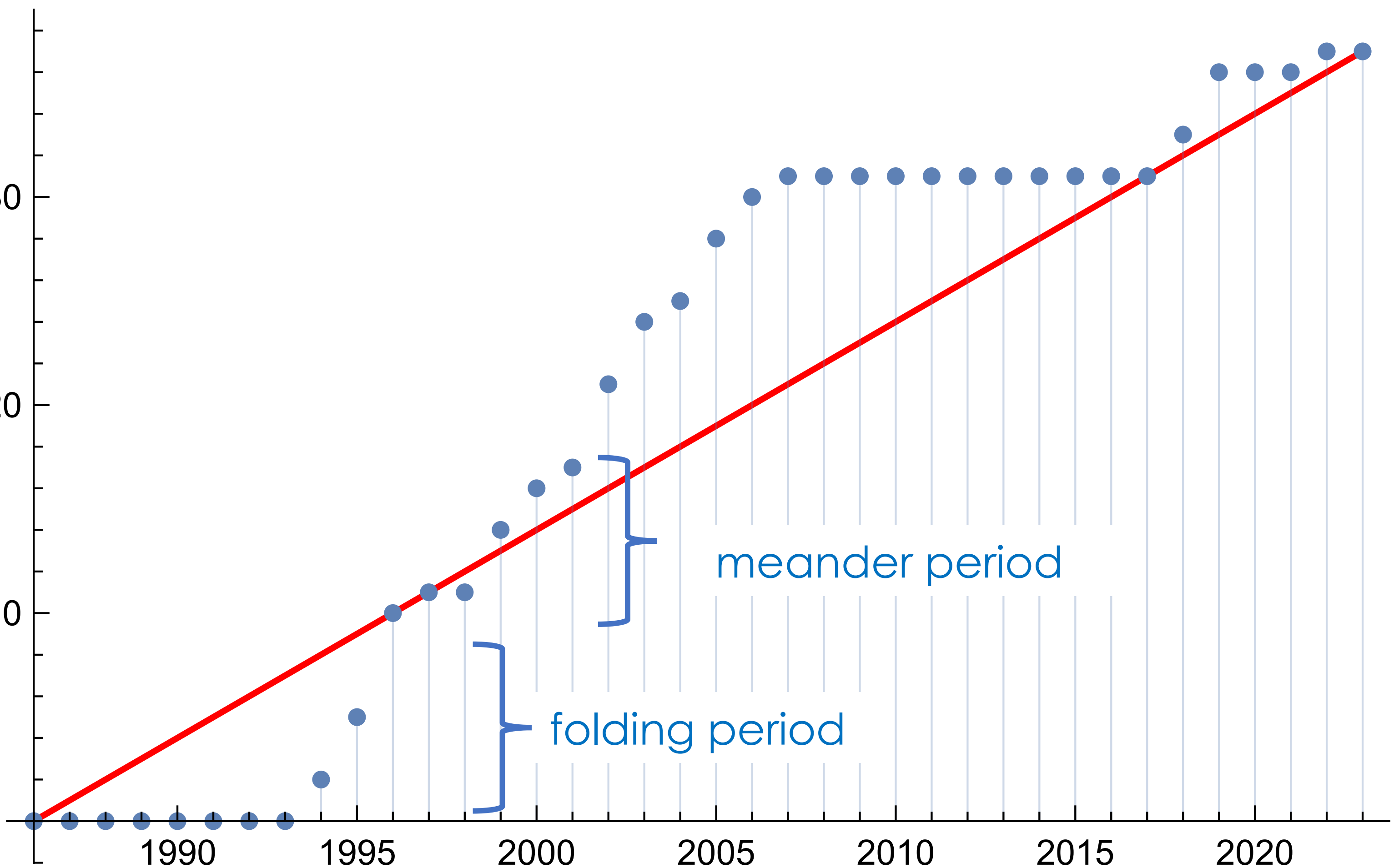
As one of our colleagues used to say, « cela ne s'invente pas ».

Meanders: A personal perspective to the memory of Pierre Rosenstiehl

Alexander K. Zvonkin

(2023)

1 2 3 6 11 20 29 37 60 96 145 214 415



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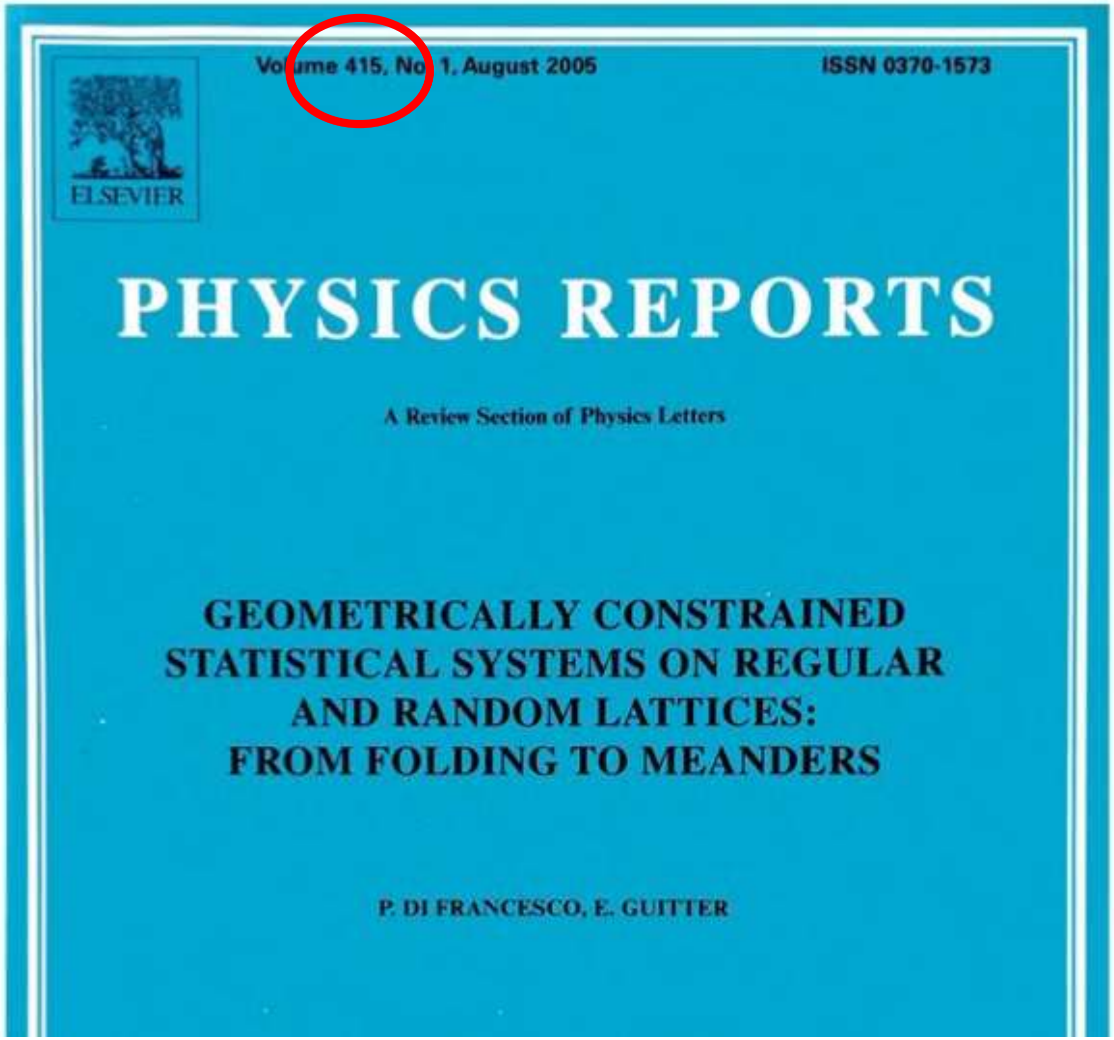

foldings, colorings, meanders are different facets of a same problem, that of fully packed loops on regular or random lattices

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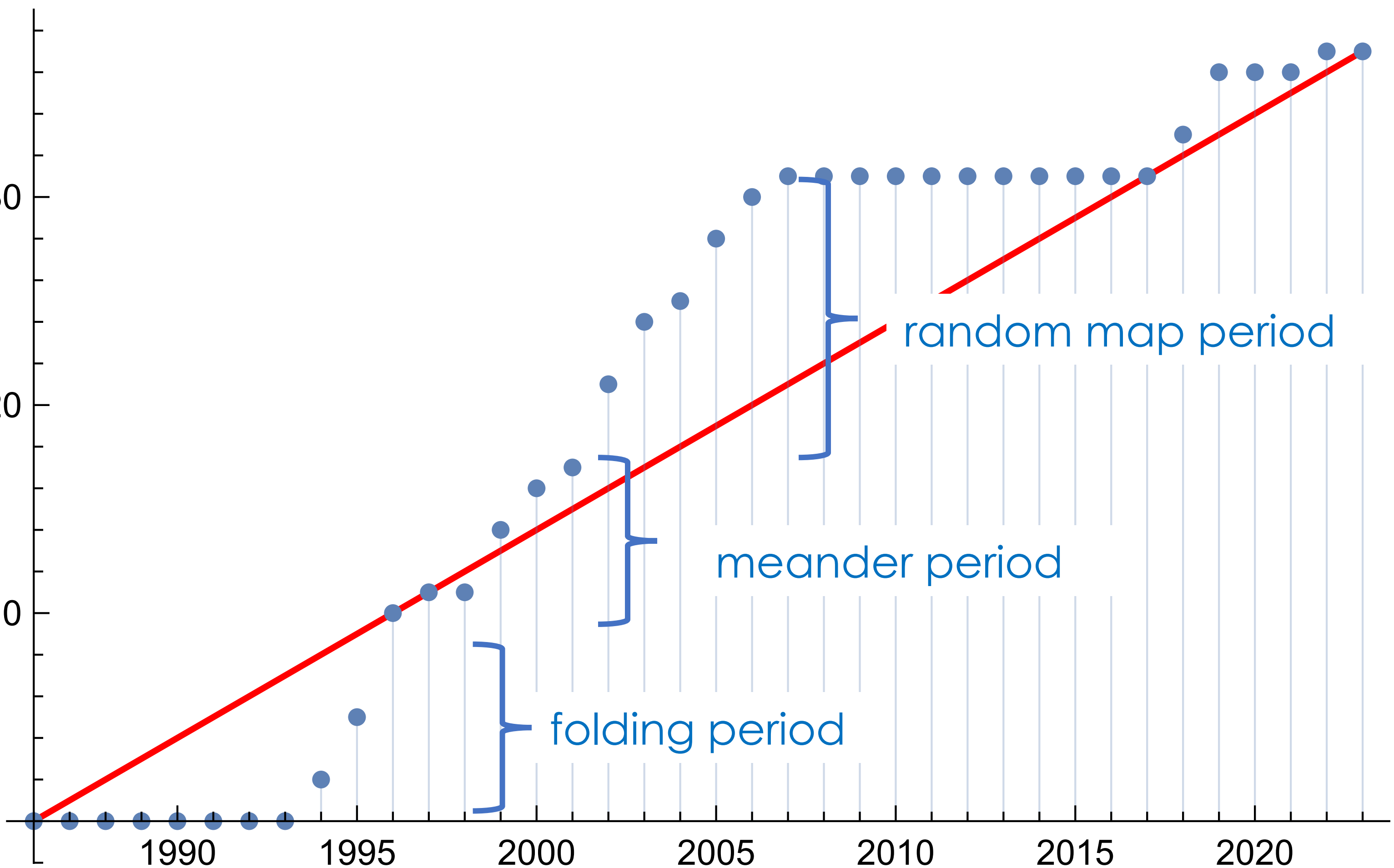


1 2 3 6 11 20 29 37 60 96 145 214 415

Coloring random triangulations

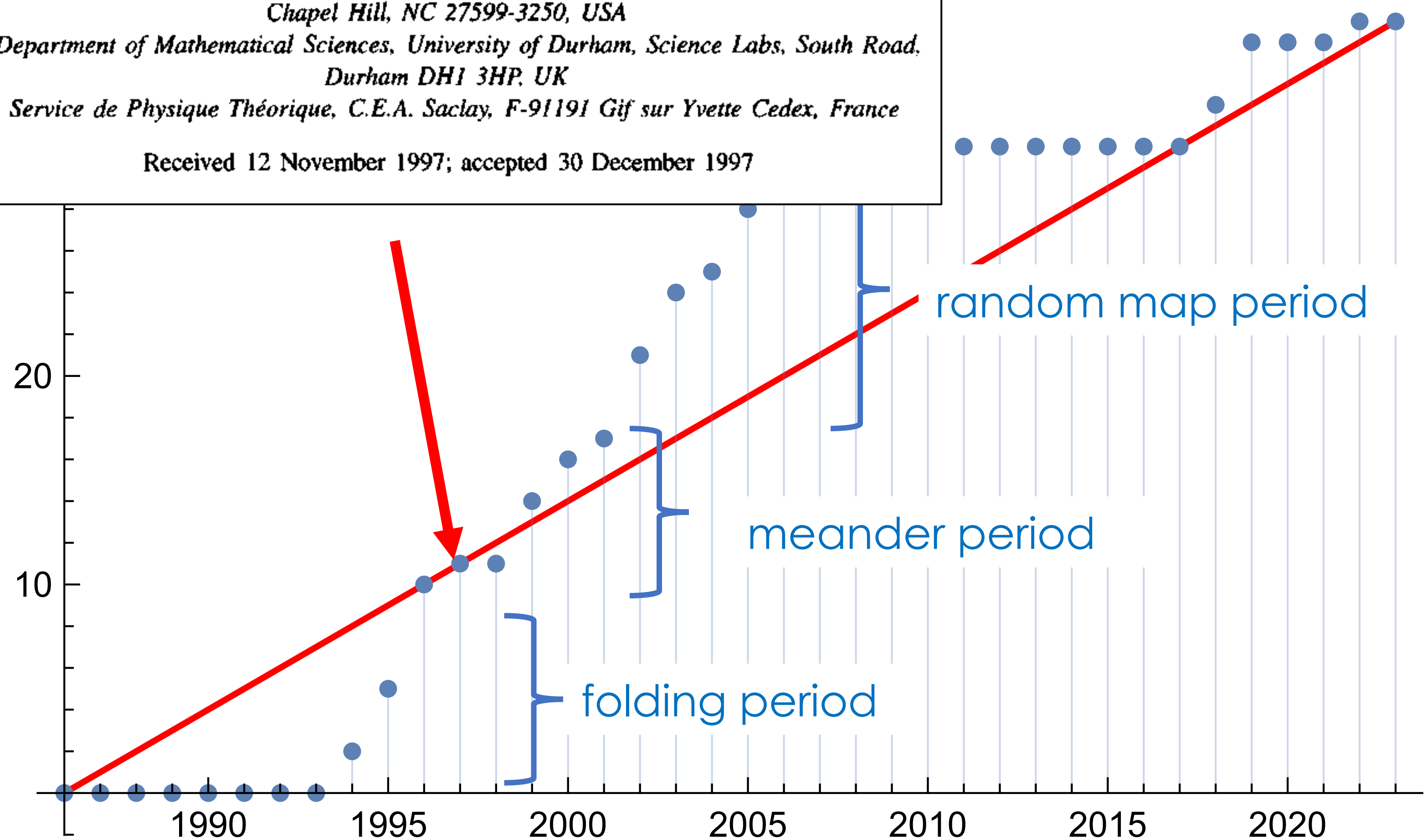
P. Di Francesco [a,1], B. Eynard [b,2], E. Guitter [c,3]

[a] *Department of Mathematics, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599-3250, USA*

[b] *Department of Mathematical Sciences, University of Durham, Science Labs, South Road, Durham DH1 3HP, UK*

[c] *Service de Physique Théorique, C.E.A. Saclay, F-91191 Gif sur Yvette Cedex, France*

Received 12 November 1997; accepted 30 December 1997



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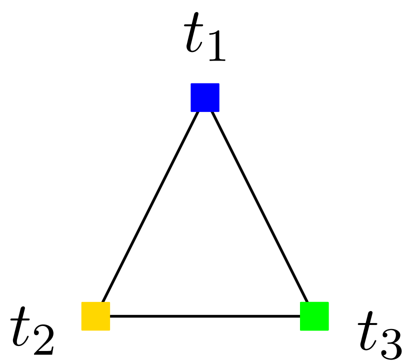
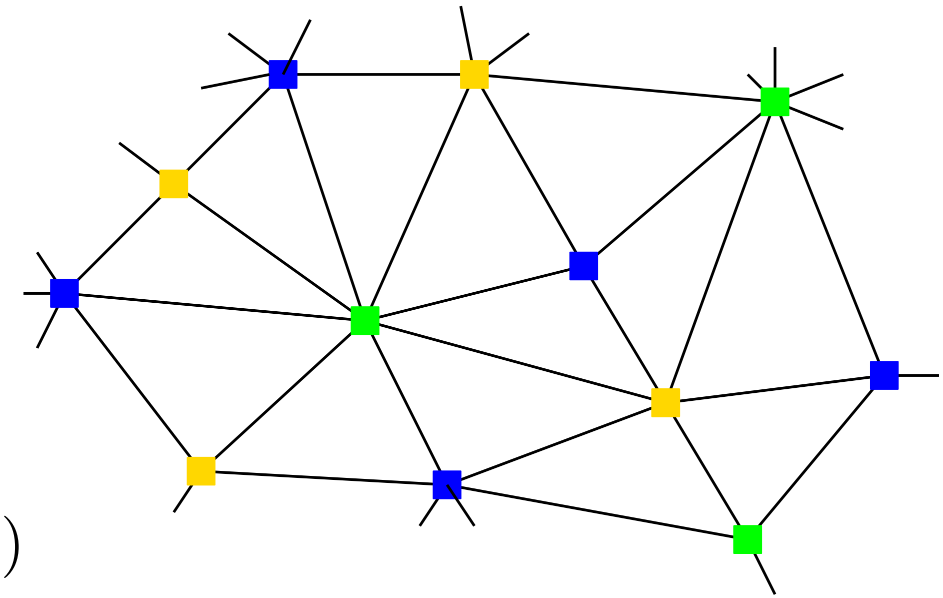
Counting planar Eulerian triangulations, i.e. triangulations which are vertex-colorable by 3 colors





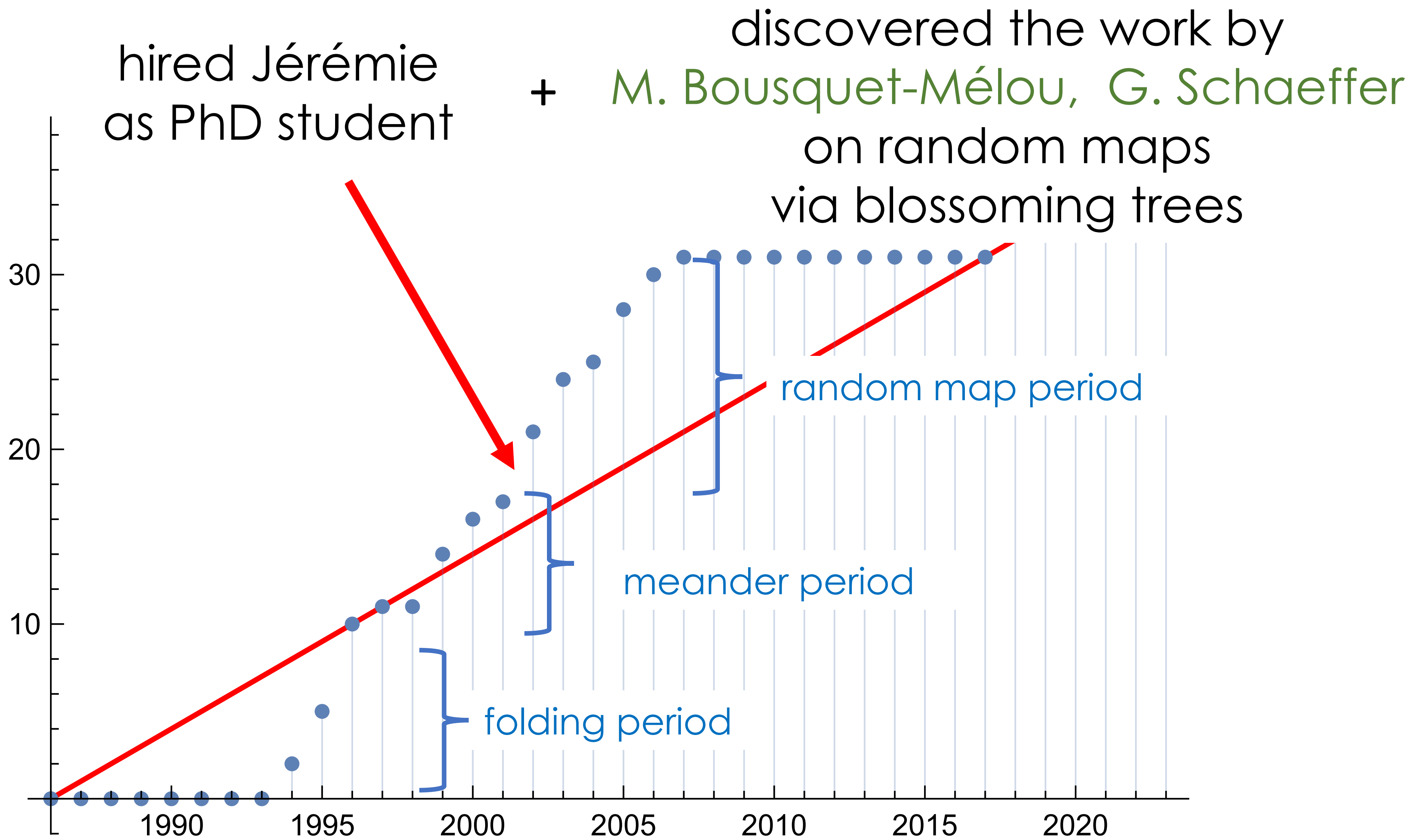
$$Z = U_1 U_2 U_3 (1 - U_1 - U_2 - U_3)$$

$$U_1 = \frac{t_1}{1 - U_2 - U_3}, \quad U_2 = \frac{t_2}{1 - U_3 - U_1} \quad U_3 = \frac{t_3}{1 - U_1 - U_2}$$

= tricolored tree generating functions

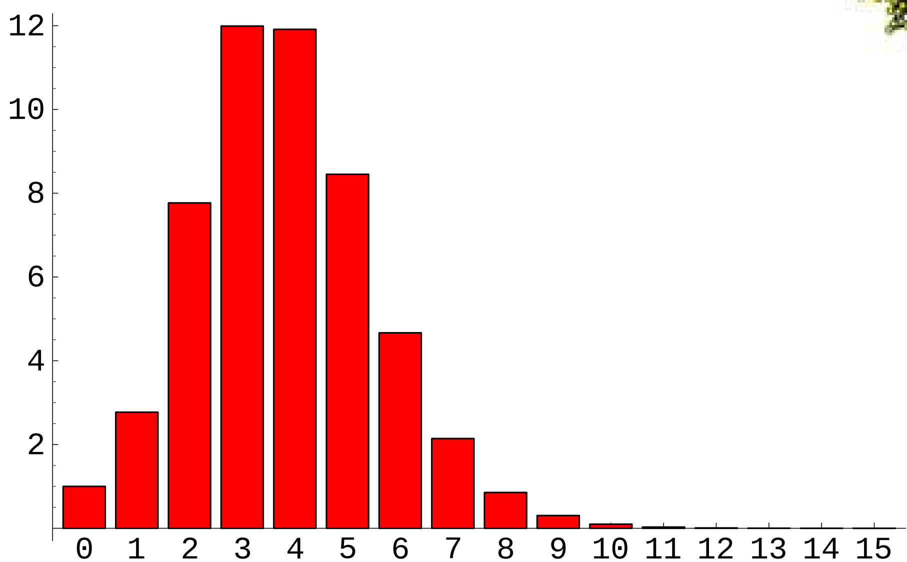
Where are the trees ?

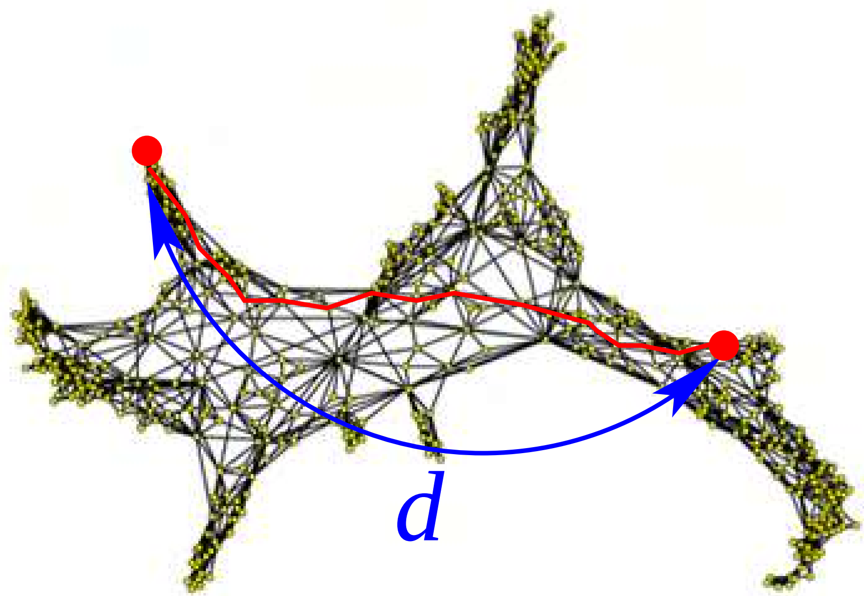
1 2 3 6 11 20 29 37 60 96 145 214 415



1 2 3 6 11 20 29 37 60 96 145 214 415

Distance profile = 2-point function





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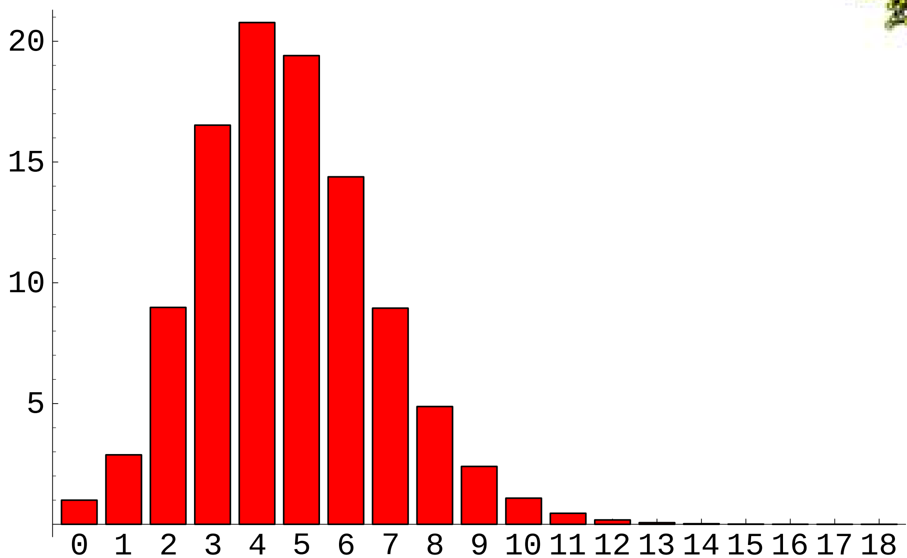

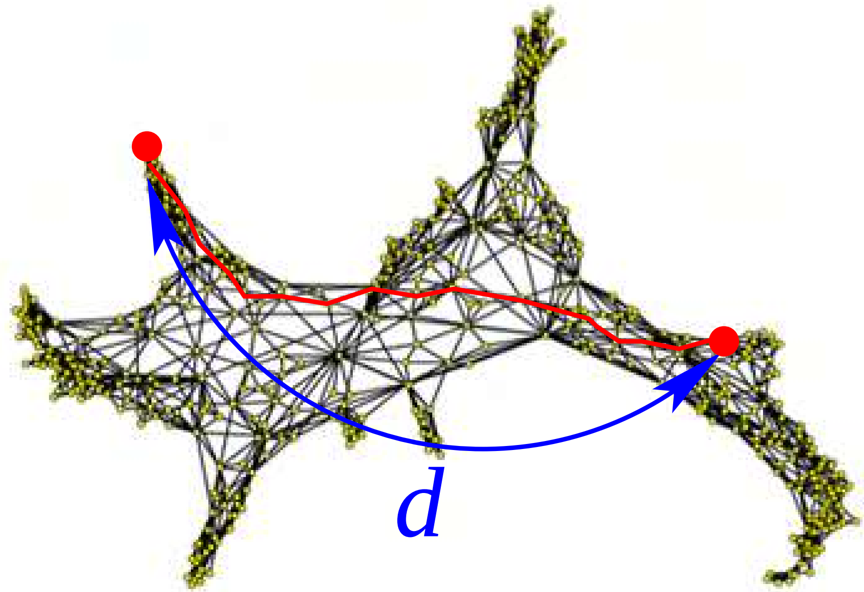

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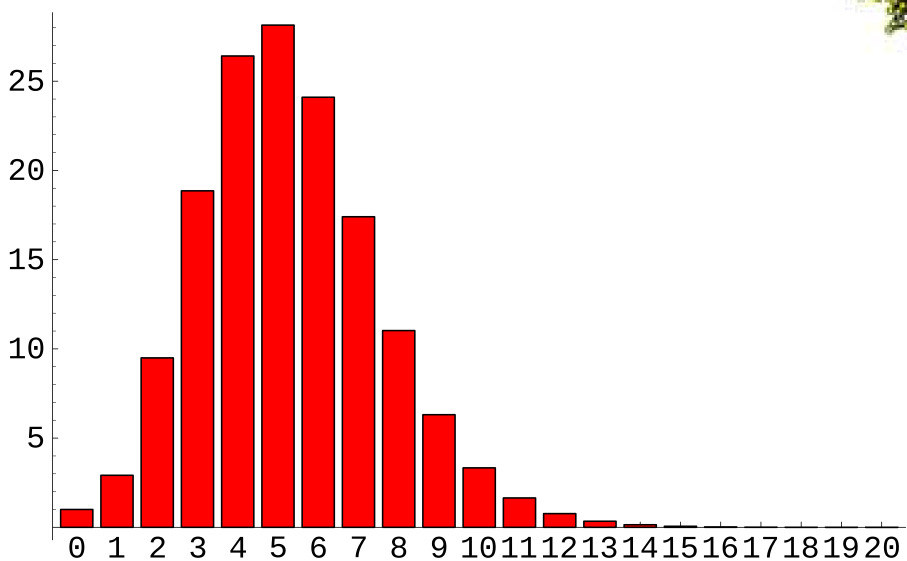

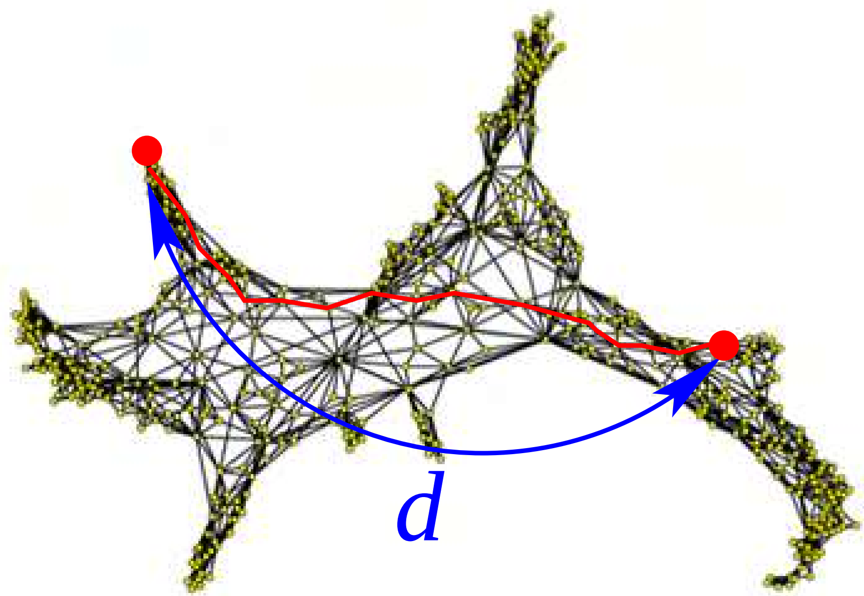

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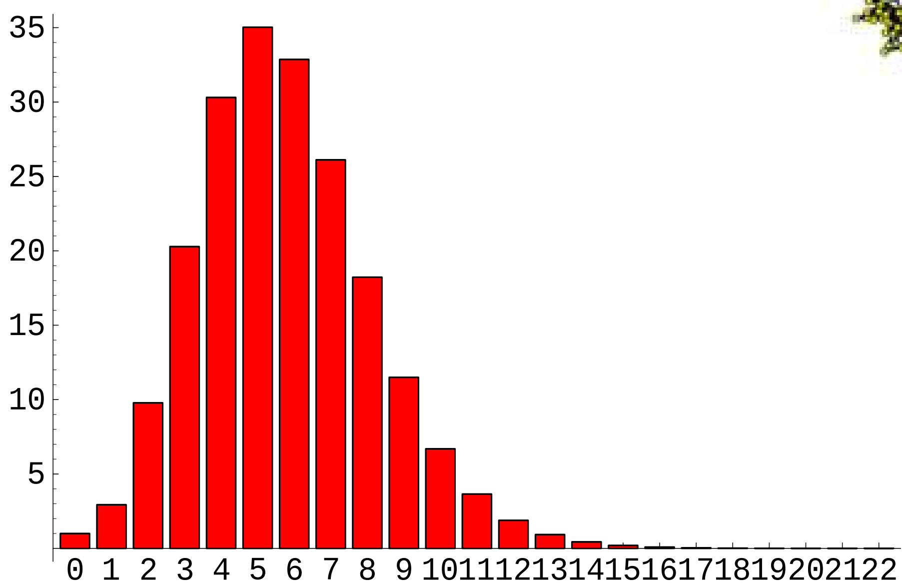

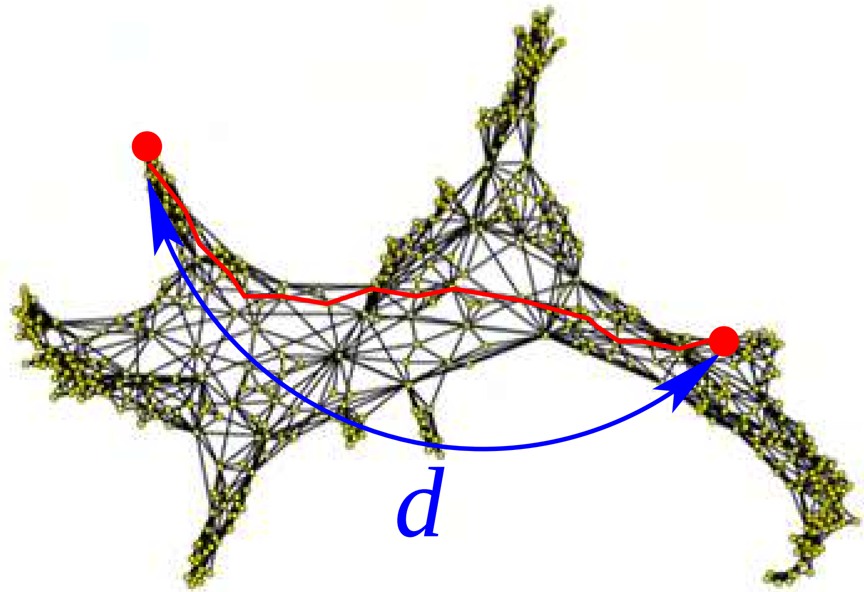

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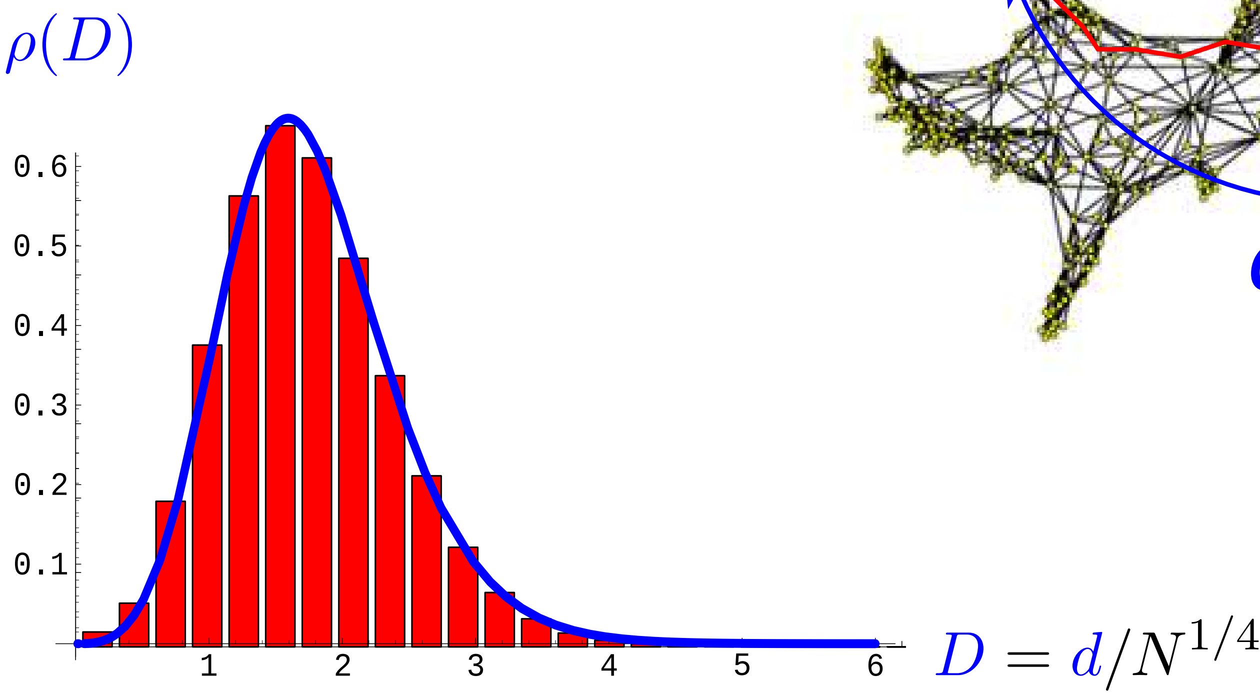

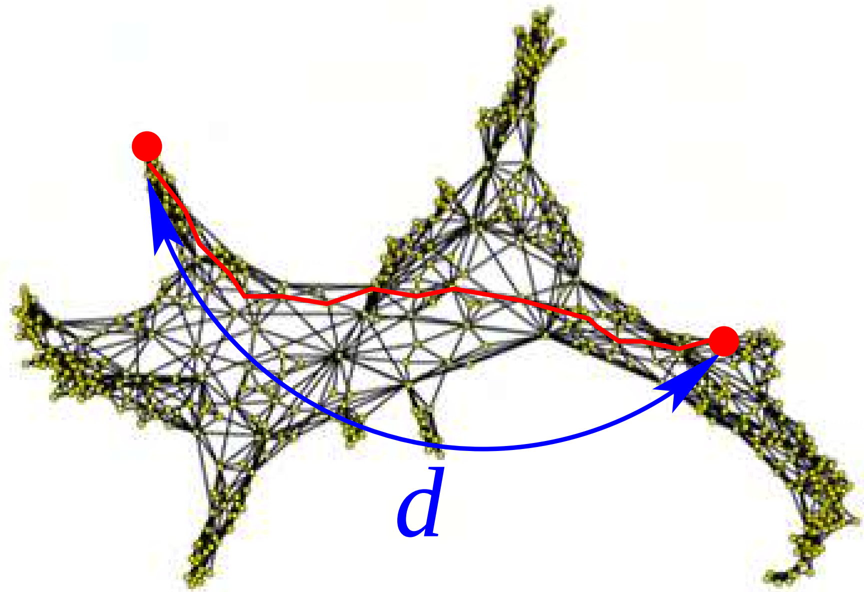

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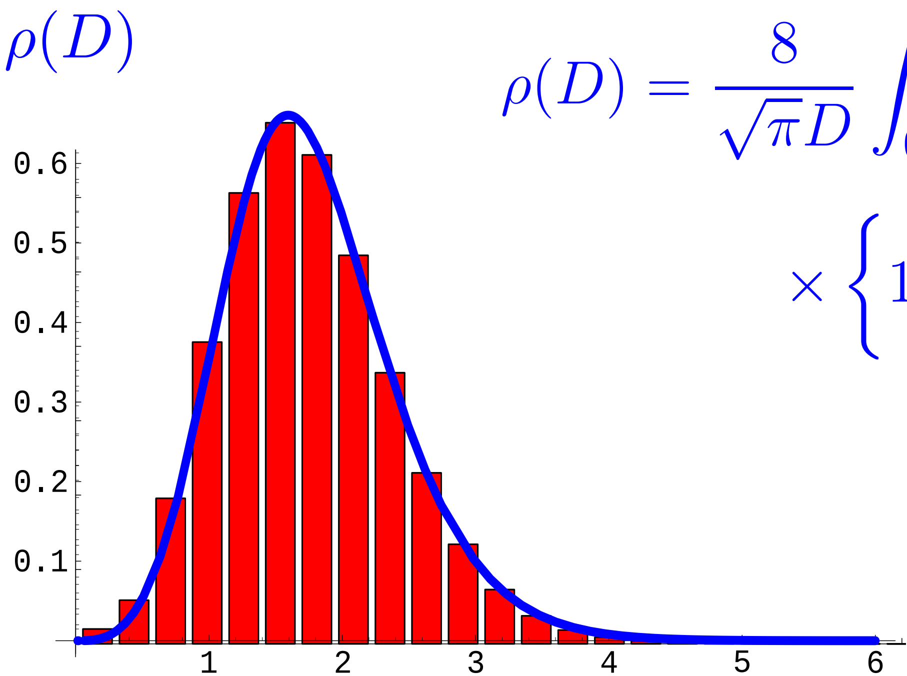
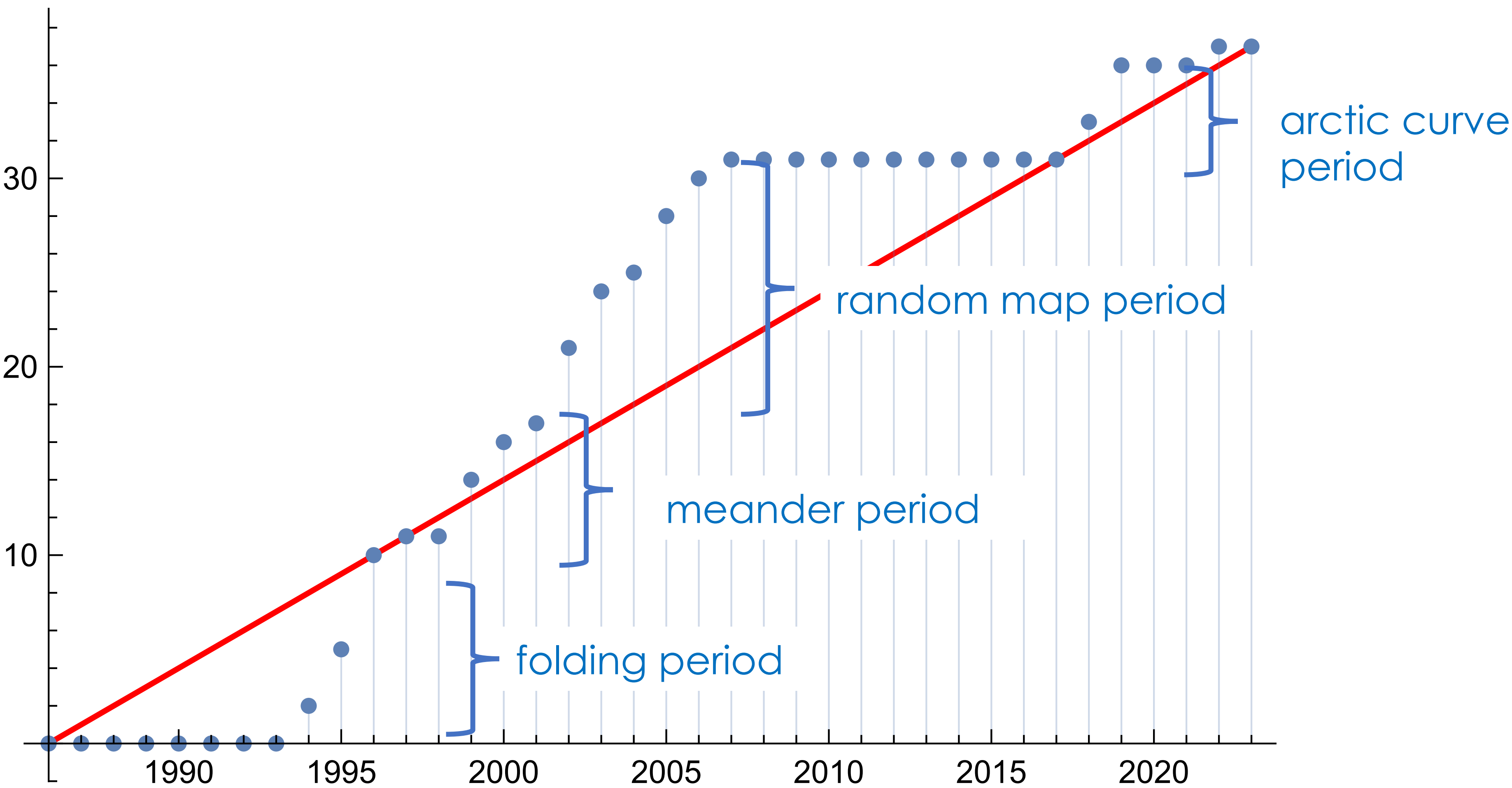


$$\rho(D) = \frac{8}{\sqrt{\pi} D} \int_0^\infty d\xi \xi^2 (2\xi^2 - 3) e^{-\xi^2}$$

$$\times \left\{ 1 - 6 \frac{1 - \cosh(\sqrt{3}\xi D)\cos(\sqrt{3}\xi D)}{(\cosh(\sqrt{3}\xi D) - \cos(\sqrt{3}\xi D))^2} \right\}$$

$$D = d/N^{1/4}$$

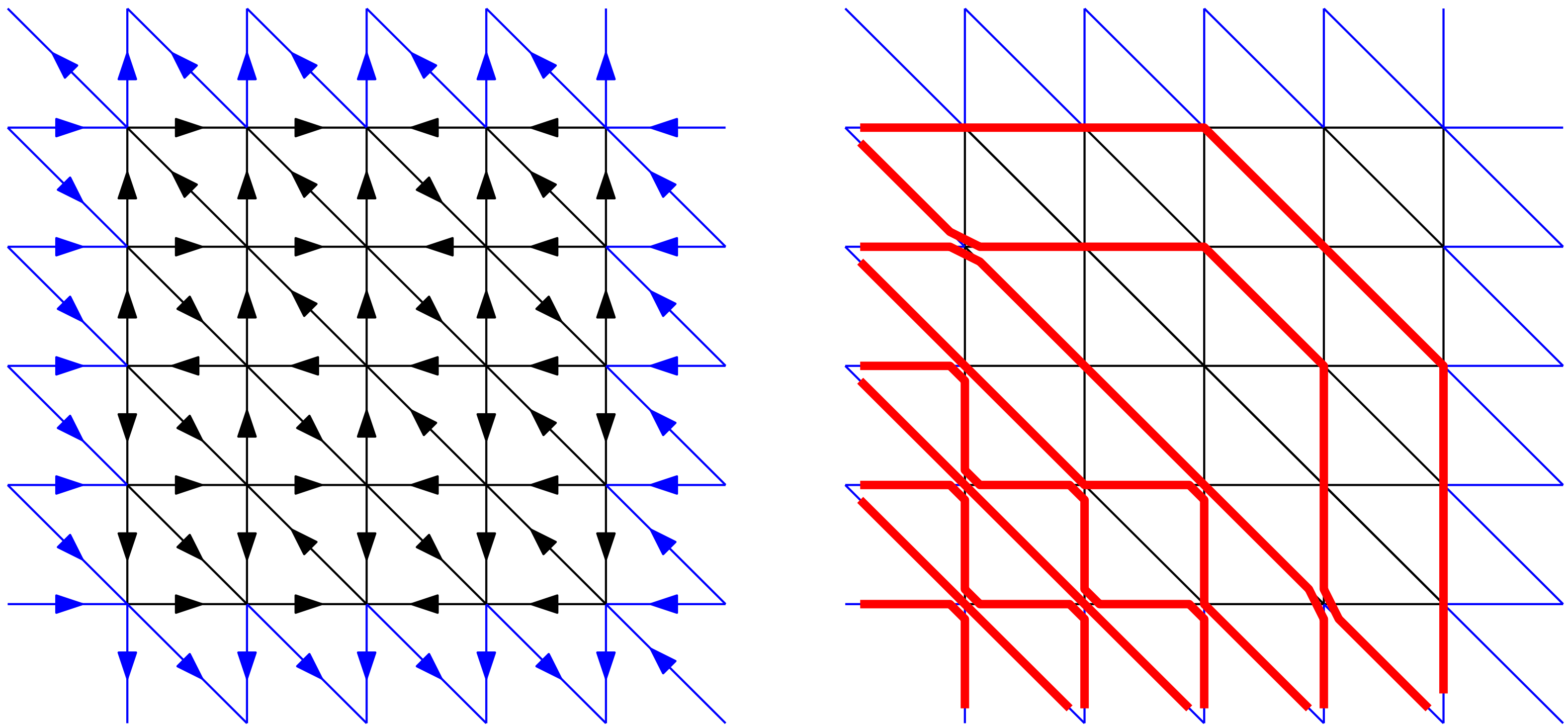
1 2 3 6 11 20 29 37 60 96 145 214 415



1 2 3 6 11 20 29 37 60 96 145 214 415

20 - vertex model = ice model on triangular lattice

with DWBC



= osculating lattice paths

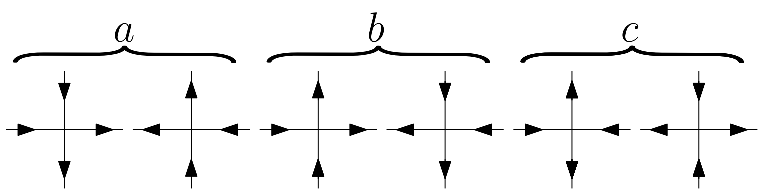
$$Z^{20V}(n) = 1,\ 3,\ 23,\ 433,\ 19705,\ 2151843,\ 561696335,\ 349667866305\ldots$$

1 2 3 6 11 20 29 37 60 96 145 214 415

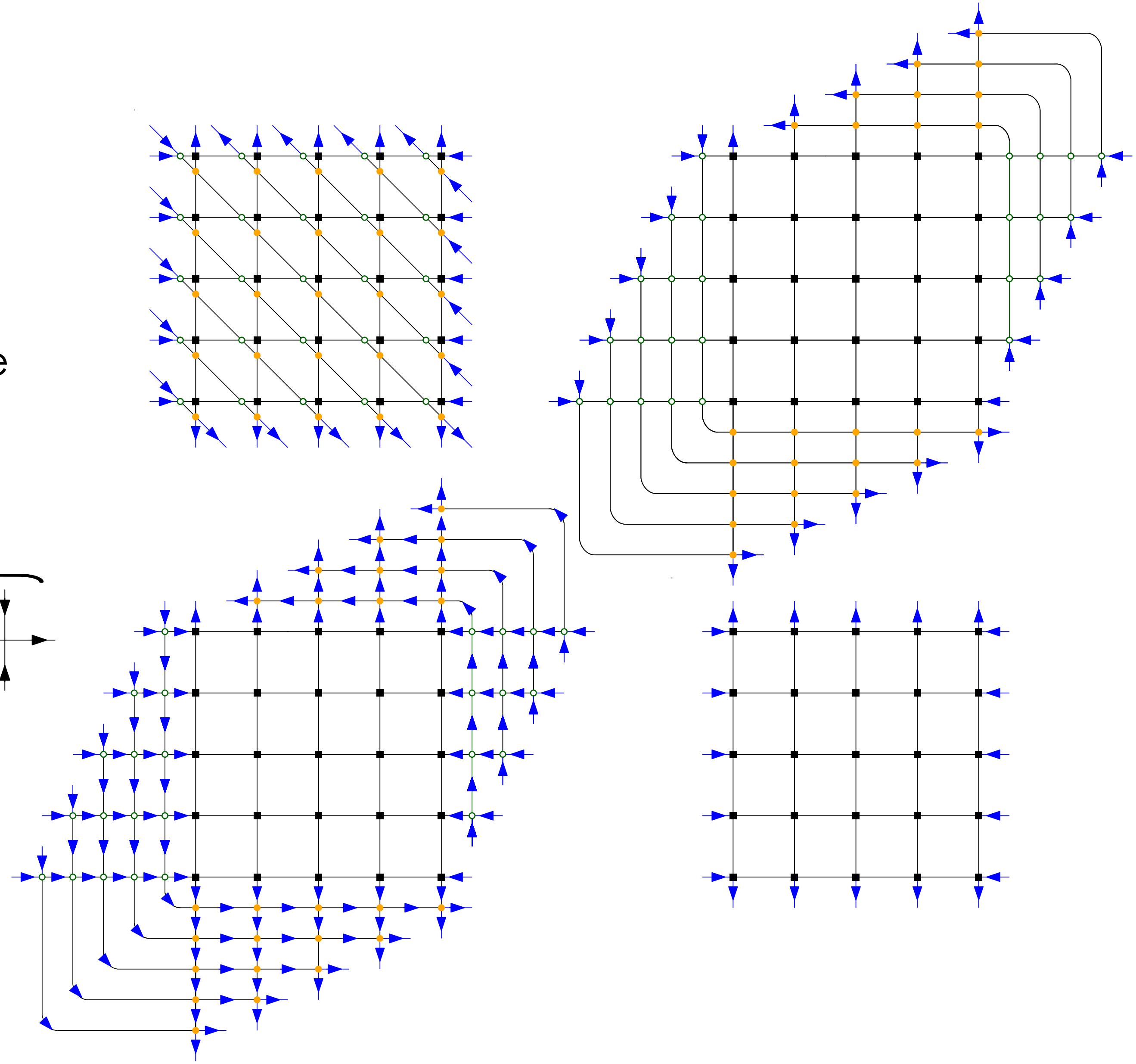
➢ $$Z^{20V}(n) = Z^{6V}_{[1,\sqrt{2},1]}(n)$$

6- vertex model
= ice model on square lattice

again with DWBC

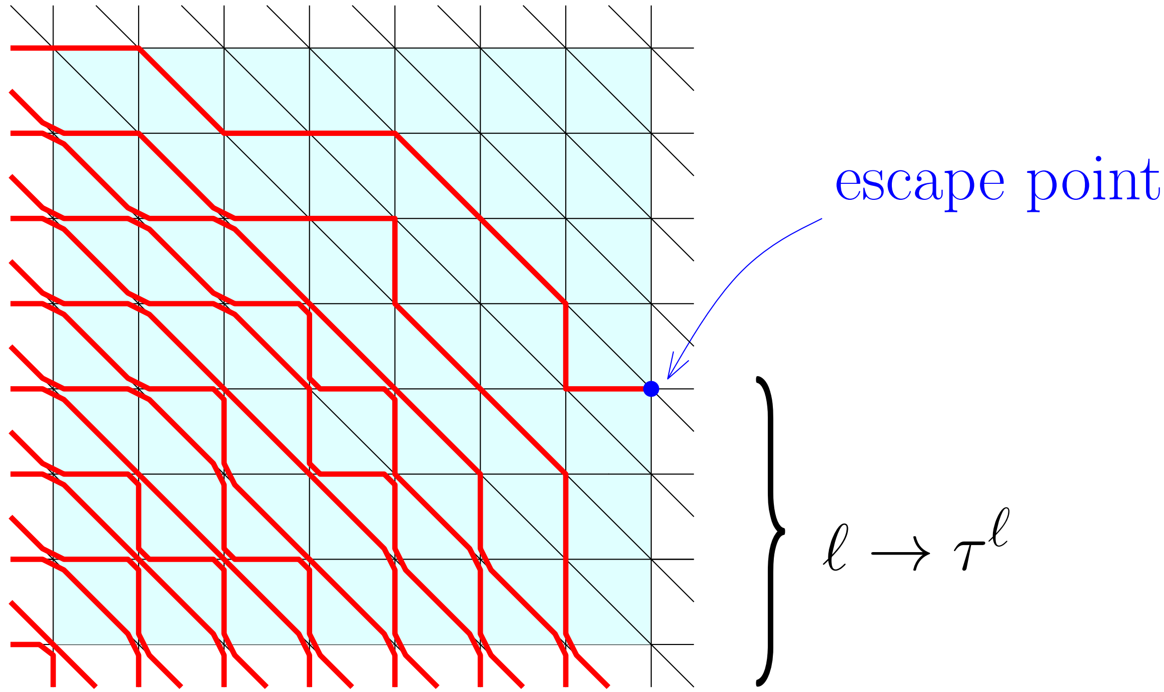


$$(a, b, c) = (1, \sqrt{2}, 1)$$



1 2 3 6 11 20 29 37 60 96 145 214 415

- also true for the 1-point function



$$Z^{20V}(n;\tau) = Z^{6V}_{[1,\sqrt{2},1]}\left(n;\frac{1+\tau}{2}\right)$$

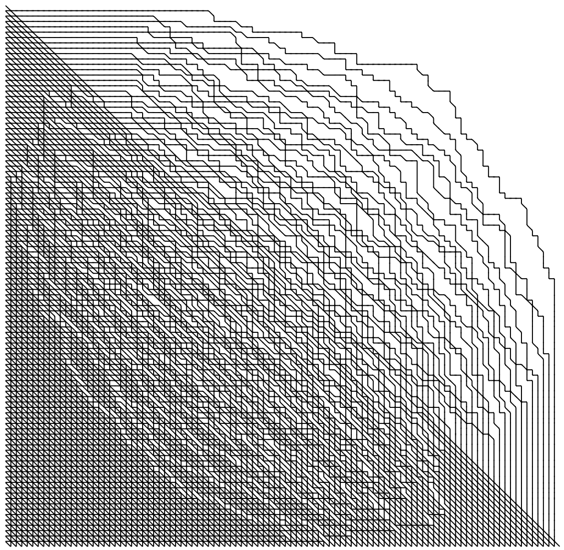
→ arctic curve

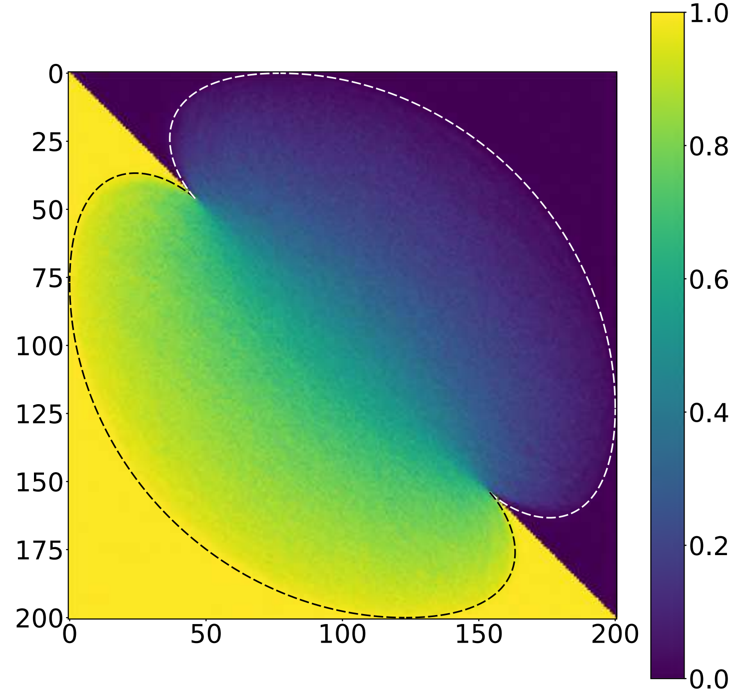
tangent method

F. Colomo, A. Sportiello (2016)

F. Colomo, A.G. Pronko (2010)

1 2 3 6 11 20 29 37 60 96 145 214 415

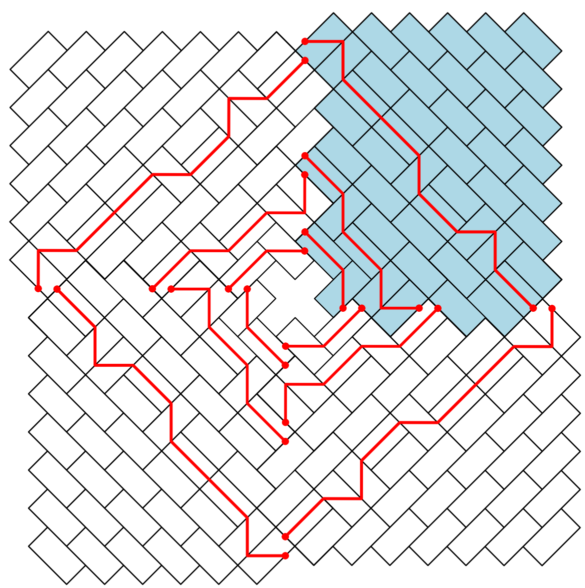




density of occupied diagonals

- we also have

$$Z^{20V}(n) =$$



Quater Turn symmetric Holey Aztec Domino Tiling

1 2 3 6 11 20 29 37 60 96 145 214 415

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