SFB 1283

LEVERHULME TRUST

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Random point processes in the plane and applications to birds of prey

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Philippe 60, 26.06.2024

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[J. Th. Biol. **509** (2021) 110475; J. Stat. Mech. (2024) 053501]

Outline

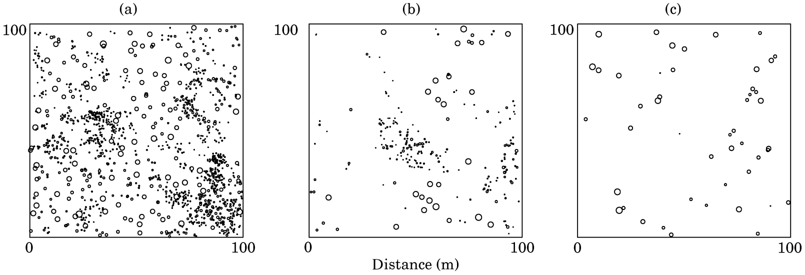
I) Examples for point processes from ecology

II) Mathematical description: Poisson, Ginibre, interpolation

III) Application to birds of prey in the Teutoburger Wald

IV) Discussion

I) Example spacial patterns of trees



- (a) beech (b) Norway spruce (c) silver fir in a 1-ha plot at Rothwald, Austria (circles $\sim$ d.b.h.)

  from [R. Law et al., J. Ecology **97** (2009) 616–628]

- quantify influence of **environment** (nutrition, terrain, etc.), **population** density, **interaction of species**

Example spacial patterns of nests of birds of prey

Left: nests of **Buzzard** in 2020 (data: Oliver Krüger's group)

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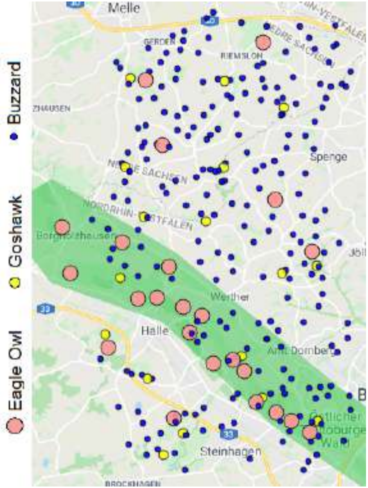
Left: nests of **Buzzard** in 2020 (data: Oliver Krüger's group)



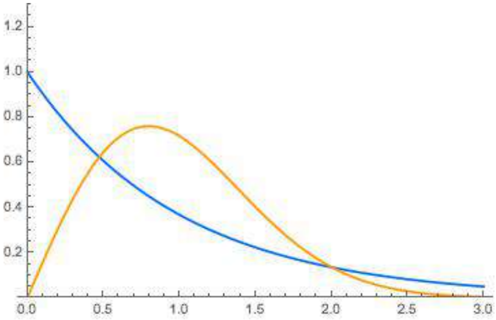
Right: include **Goshawks** and **Eagle Owl**

Interaction of species

- Buzzard, Goshawk and Eagle Owl [wikipedia]
- can we quantify their interaction (repulsion) based on the annual distribution of nests?

II) Math description: Independent points = Poisson

- **1D vs. 2D Poisson**: independent points in interval / disc
- **nearest neighbour spacing distribution**

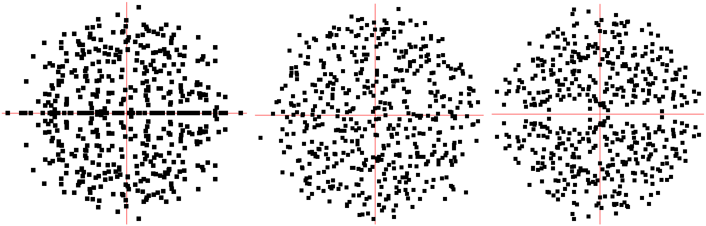


- $P_{2D}(S) = \frac{1}{2}\pi S\, e^{-\pi S^2/4}$ vs. $P_{1D}(S) = e^{-S}$

  2D repulsion $\sim S$ from area measure vs. 1D no repulsion

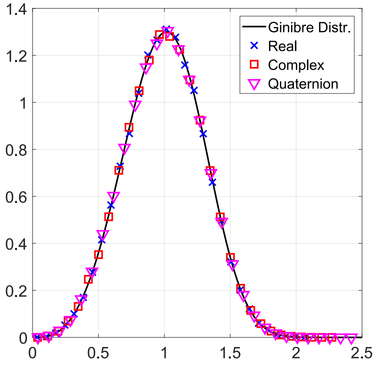
Random matrix eigenvalues: Ginibre ensembles

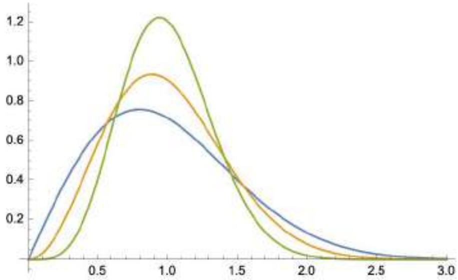
- $\boxed{P(J) \sim \exp\left[-\mathrm{Tr}\, JJ^*\right]}$ for $J_{ij} \in \mathbb{R}/\mathbb{C}/\mathbb{H}$ [Ginibre '65]



- complex eigenvalues: determinantal / Pfaffian process
- in bulk $\setminus\mathbb{R}$ **all** local eigenvalue correlations **equal** GinUE: GinOE [Borodin, Sinclair '08], GinSE [A, Kieburg, Mielke, Prosen '19]
- $\boxed{P_{\mathrm{Gin}}(s) = -\frac{\partial}{\partial s} \prod_{j=1}^{\infty} \frac{\Gamma(j+1,s^2)}{\Gamma(j+1)}} \sim s^{2+1}$ [Grobe, Haake, Sommers '88]

Bulk spacing: 3 Ginibre Ensembles vs. GO/U/SE





- 2D: numerical simulations of 3 Ginibre ensembles $\beta = 2$ (left)
- 1D: Wigner surmise for GOE, GUE, GSE $\beta = 1, 2, 4$ (right) with $\sim S^{\beta}$

Interpolating Poisson to correlated points

* Proposal 1: (e.g. [Møller et al. 2018])

  - **assume** point process is **determinantal**, kernel $K_N(z,u)$:

  $\boxed{R_k(z_1,\ldots,z_k) = \det_{k\times k}[K_N(z_i,z_j)]}$ for all k-th marginals:

  e.g. $R_1(z) = K_N(z,z)$ density

  - **fit kernel** $K_N(z_1,z_2)$ from data via connected $R_2(z_1,z_2)$

- **Poisson:** $\rightarrow$ diagonal Kernel $K_N(z_i,z_j) \sim \delta_{i,j} f(z_i)$:

  $\Rightarrow\ R_k(z_1,\ldots,z_k) = \prod_{i=1}^{k} K_N(z_i,z_i)$

* Proposal 2:

  - **assume** point process is **2D Coulomb gas** at $\beta > 0$

  - **fit** β from data

- **Poisson:** $\beta \rightarrow 0$
- successfully applied in non-Hermitian Quantum chaos $\rightarrow$ integrable transition [A, Kieburg, Mielke, Prosen '19]

2D Coulomb gas picture for complex eigenvalues

$$P_N(z_1, \ldots, z_N) = \frac{1}{Z_N(\beta)} \exp\left[\beta \sum_{j>k}^{N} \ln|z_j - z_k| - \sum_{i=1}^{N} |z_i|^2\right]$$

$$= \frac{1}{Z_N(\beta)} \prod_{j>k}^{N} |z_j - z_k|^{\beta} e^{-\sum_{i=1}^{N} |z_i|^2}$$

- **joint complex eigenvalue density:**
  complex Ginibre = static **2D Coulomb gas** at $\beta = \mathbf{2}$
- **unknown:** Selberg integral in the complex plane

$$\boxed{Z_N(\beta) = \int_{\mathbb{C}^N} d^2 z_1 \cdots d^2 z_N \prod_{j>k}^{N} |z_j - z_k|^{\beta} e^{-\sum_{i=1}^{N} |z_i|^2}}$$

$$Z_{N=2}(\beta) = (2\beta)! 2^{2\beta}; \; Z_{N=3}(\beta) = \frac{(6\beta)!}{2^{2\beta}} \sum_{k=0}^{2\beta} 3^{2k} \frac{\binom{2\beta}{k}^2}{\binom{6\beta}{2k}}, \; \beta \in \mathbb{N}$$

[Di Francesco, Gaudin, Itzykson, Lesage, '94]

- at $\beta \approx 142$ **condensation (!)** to Abrikosov lattice

  [Choquard, Clérouin '83; Cardoso, Stéphan, Abanov '20]

Known local correlations in 2D Coulomb at $\beta > 0$

- $\underline{\beta = 0}$: vicinity of zero $\beta \sim \kappa/N$: local statistics Poisson, universal [Lambert '21]

- $\underline{\beta = 2}$: Ginibre = integrable, determinantal point process [Ginibre '65, Haake et al. '88], universal

- $\underline{0 < \beta < 2}$: **numerical simulation**: nearest and next-to-nearest neighbour

  or **approximate**: **2D "surmise"** [A., Mielke, Päßler '22]

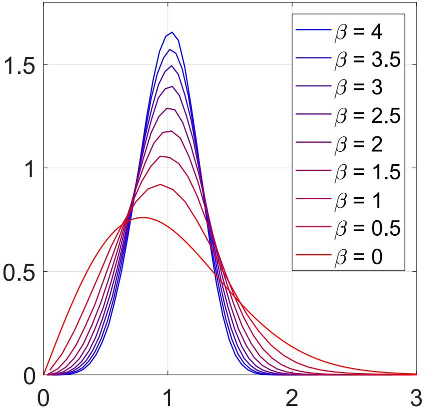
  $$\boxed{P_\beta(s) = \frac{2\alpha^{1+\beta/2}}{\Gamma(1+\beta/2)}\, s^{\beta+1} e^{-\alpha s^2}}\,,\ \alpha = \frac{\Gamma((3+\beta)/2)^2}{\Gamma(1+\beta/2)^2}$$

  heuristics matches

- 2D good approximation for β small
  ($\neq$ 1D Wigner surmise, for β large)

◦ global statistics: loop equations [Zabrodin, Wiegmann '06, Chekhov, Eynard, Marchal '11]

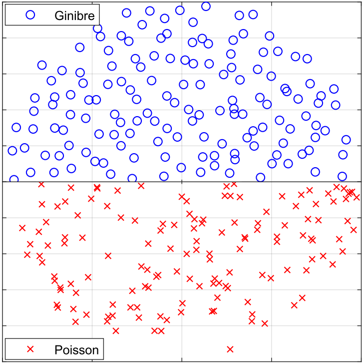
Numerical 2D Coulomb spacing distribution at $\beta \geq 0$



- Examples for nearest neighbour spacing distribution, do fits in steps 0.1

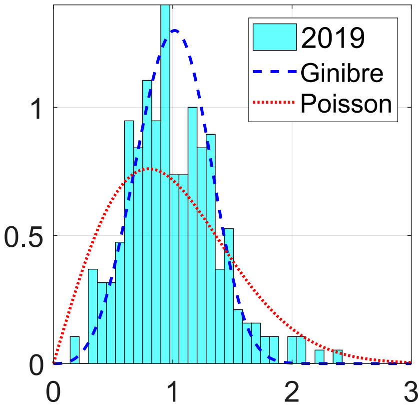
  [A, Kieburg, Mielke, Prosen '19]

III) Buzzard nest distribution: Poisson or Ginibre?



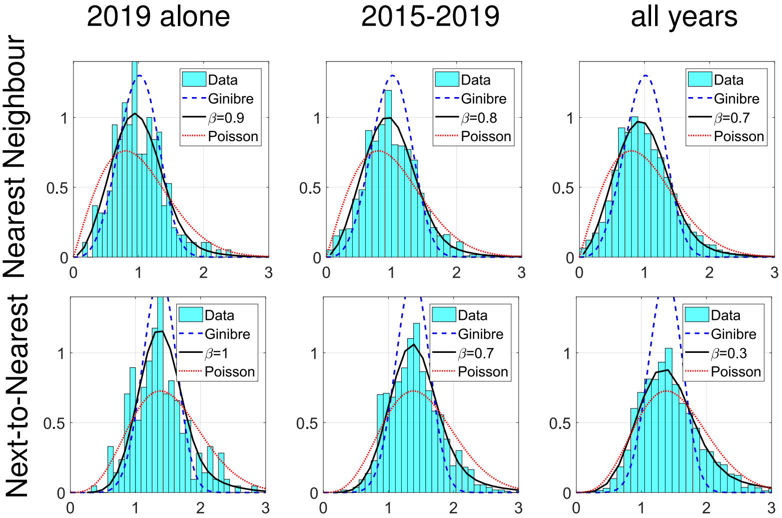
- Left: Top half eigenvalues of Ginibre ensemble vs. bottom half 2D Poisson
- Right: Buzzard nest distribution 2020

Buzzard Nests $\neq$ Poisson nor Ginibre



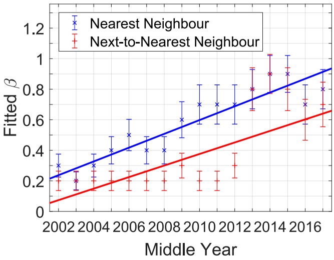
$\longrightarrow$ single parameter fit to β in 2D Coulomb gas

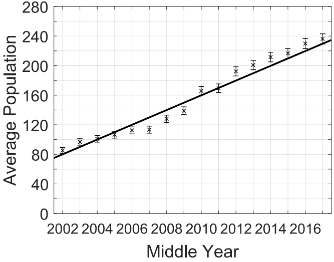
Time moving average: 1y, 5y, 20 y for Buzzards



- fit more years: β changes

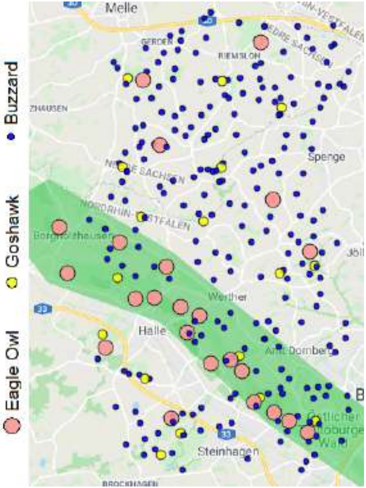
Time dependent repulsion vs. population size





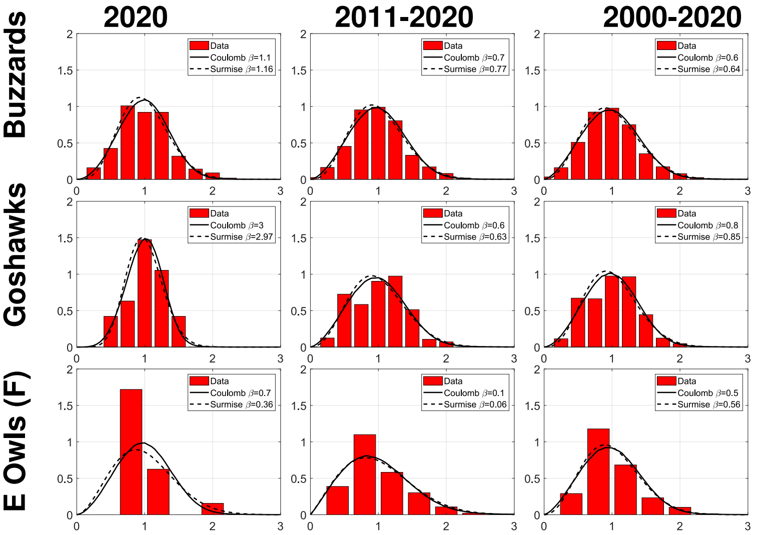
- β for NN and NNN (left) vs. population (right), both 5 y ave
- **linear increase** in both repulsion and population size
- **unfolding**: trivial decrease of spacing through increased density (population per area) is removed
- above population threshold: $\beta_{NN} \approx \beta_{NNN}$ comparable

Comparison and interaction among 3 species



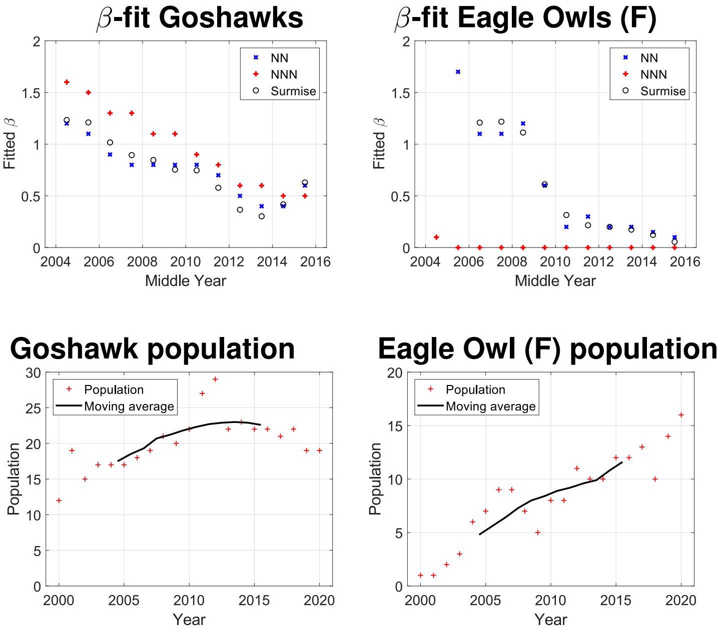
- fit repulsion **within each species** and compare
- fit repulsion **between each two species**

3 Species: 1y vs. 10 y vs 20 y Nearest neighbours

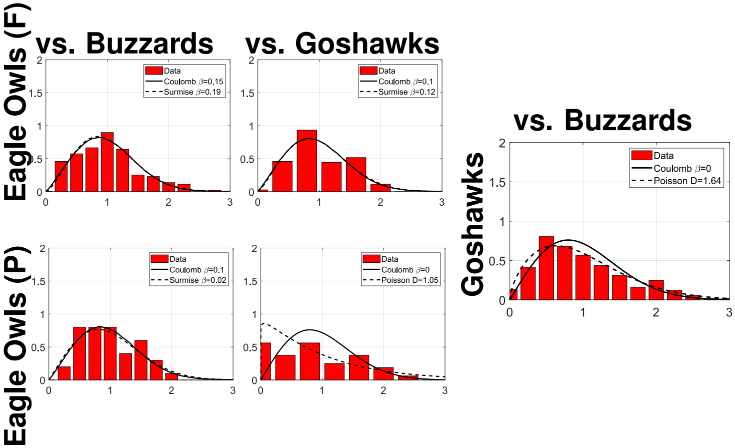


Buzzards $\beta \approx 0.6$ (3377 spacings), Goshawks $\beta \approx 0.8$ (423), Eagle Owls $\beta \approx 0.5$ (174)

Time dependent repulsion vs. population: 10 y ave

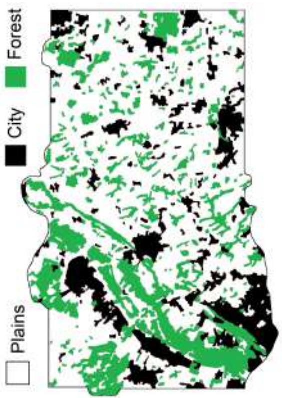


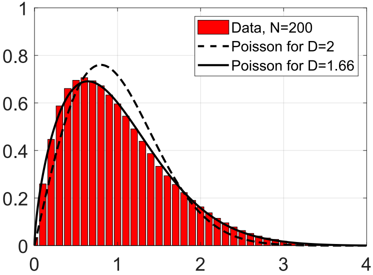
Repulsion between different species: all y average



- repulsion with Eagle Owls (F): 174 spacings $\beta = 0.1 - 0.15$
- repulsion with Eagle Owls (P): 40 spacings $\beta = 0 - 0.1$
- repulsion Goshawk to Buzzard: 423 spacings "$\beta < 0$"

Influence of forested terrain





- **fit dimension** D of Poisson process on forested area

$$P_D(S) = a_D S^{D-1} e^{-b_D S^D}$$

- uncorrelated points in forest "to the left" of Poisson in 2D

IV) Discussion

- fit of spacing between nests from 2D Coulomb gas
- β NOT a biological parameter, allows to

  - quantify relative repulsion within / between species

  - assess time / population dependence

  - range of interaction (qualitative):
  $\beta_{NN} > (<) \beta_{NNN}$ weaker (stronger) than 2D Coulomb
- influence of terrain: Poisson at $D_{\mathrm{eff}} \approx 1.66$

◦ other interaction (Yukawa), independence of points, spacing ratios

◦ find model from biology $\approx$ 2D Coulomb???

◦ development in next 10 y?

Bon Anniversaire Philippe !