

Uncertainty Quantification in Neural Networks: Methods and Considerations

Laurens Sluijterman - Mathematics department Eric Cator, Tom Heskes



#### Outline







# I. Why are neural networks uncertain?



# Why are neural networks uncertain? Methods: MVE, ensembling, dropout



- I. Why are neural networks uncertain?
- 2. Methods: MVE, ensembling, dropout
- 3. Out of distribution

# s uncertain? ng, dropout











# Epistemic / Aleatoric



Epistemic / Aleatoric



Epistemic / Aleatoric



#### Why are we uncertain?



#### Why are we uncertain?



# I. Uncertain about model parameters



# I. Uncertain about model parameters- Training data is random



# I. Uncertain about model parameters - Training data is random - Optimization is random



# I. Uncertain about model parameters - Training data is random - Optimization is random 2. Outcome is random



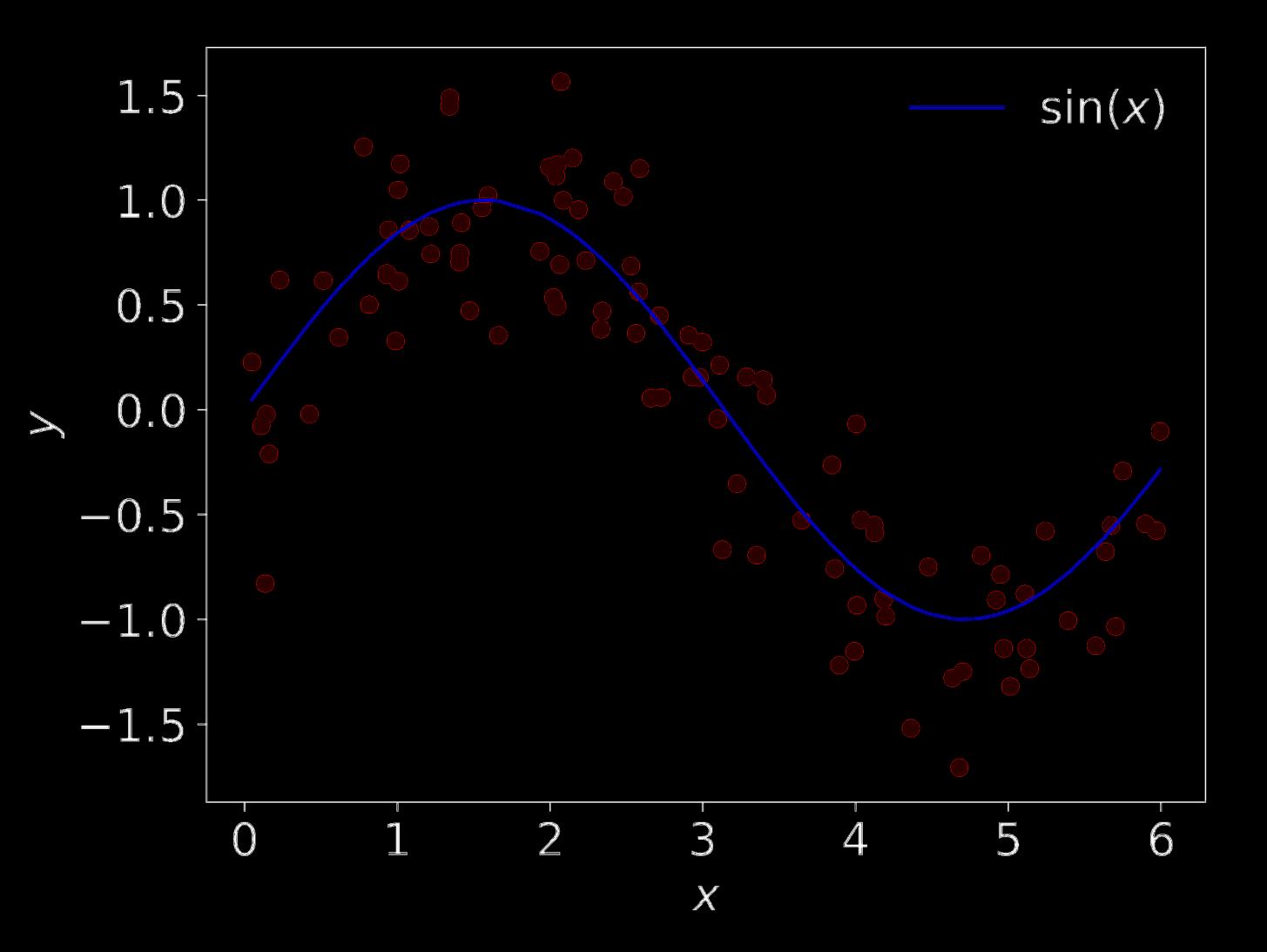
# I. Uncertain about model parameters - Training data is random - Optimization is random 2. Outcome is random 3. Uncertain about model



# I. Uncertain about model parameters - Training data is random - Optimization is random 2. Outcome is random

3. Uncertain about model

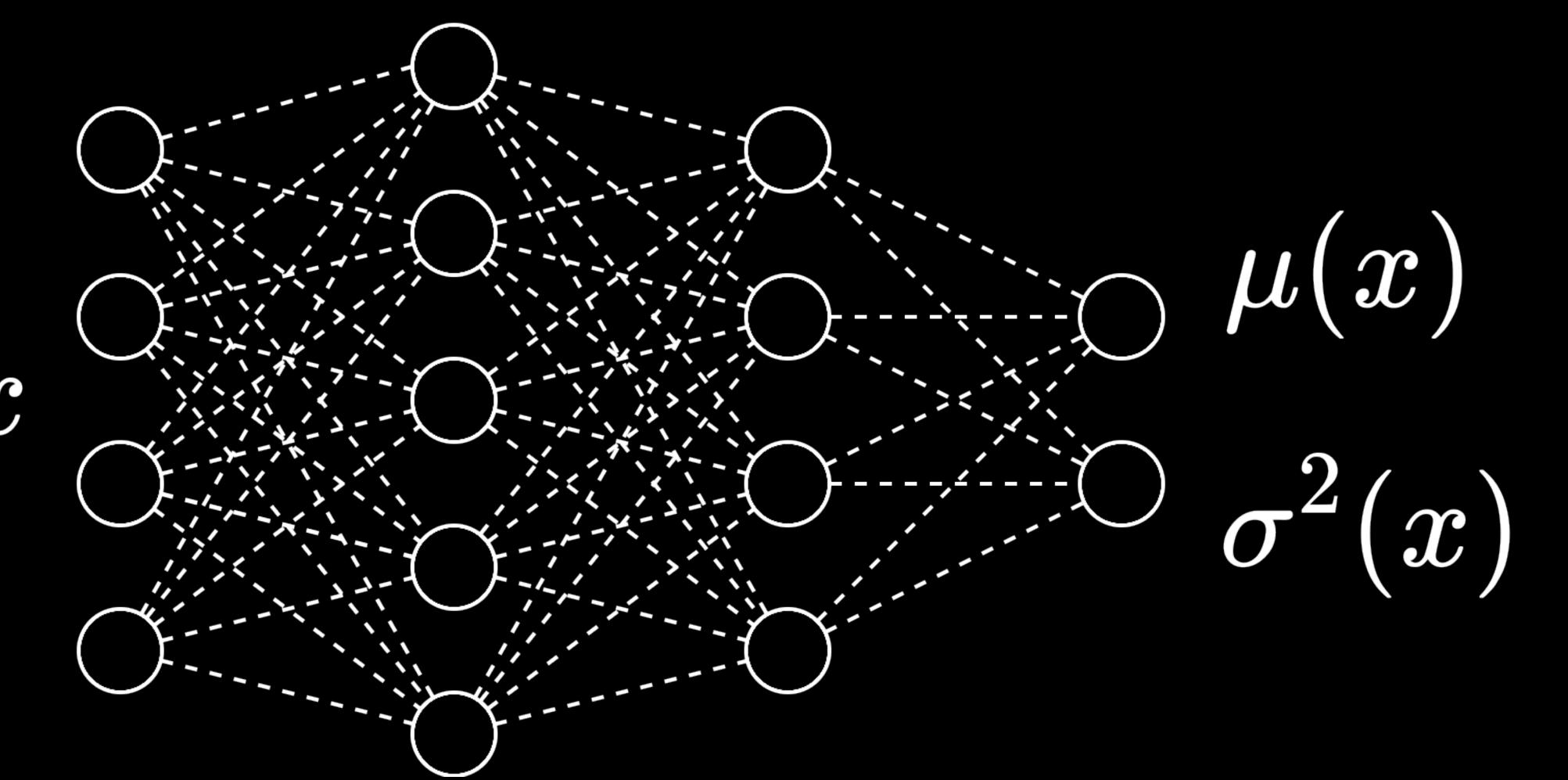






#### Learn the variance

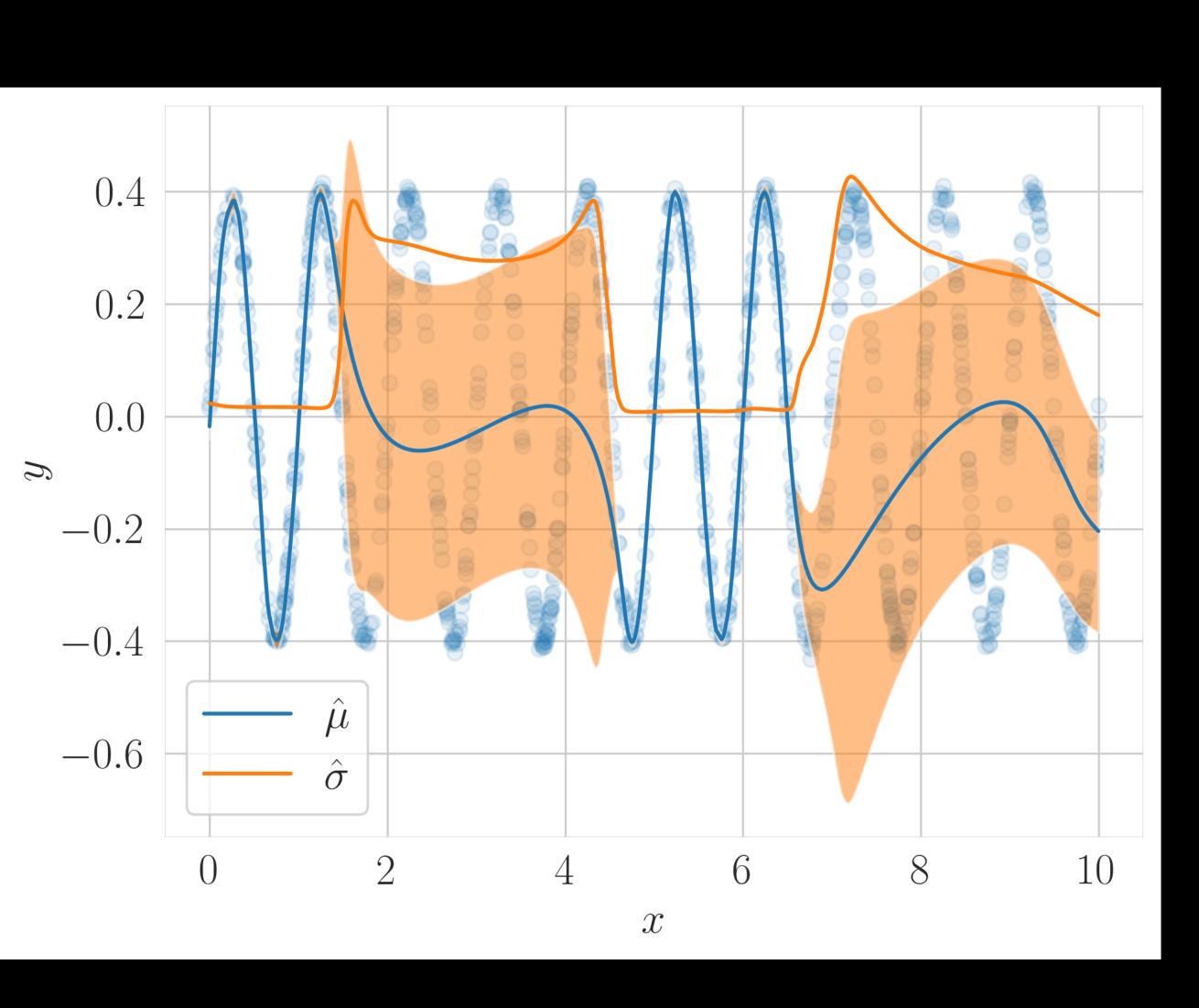




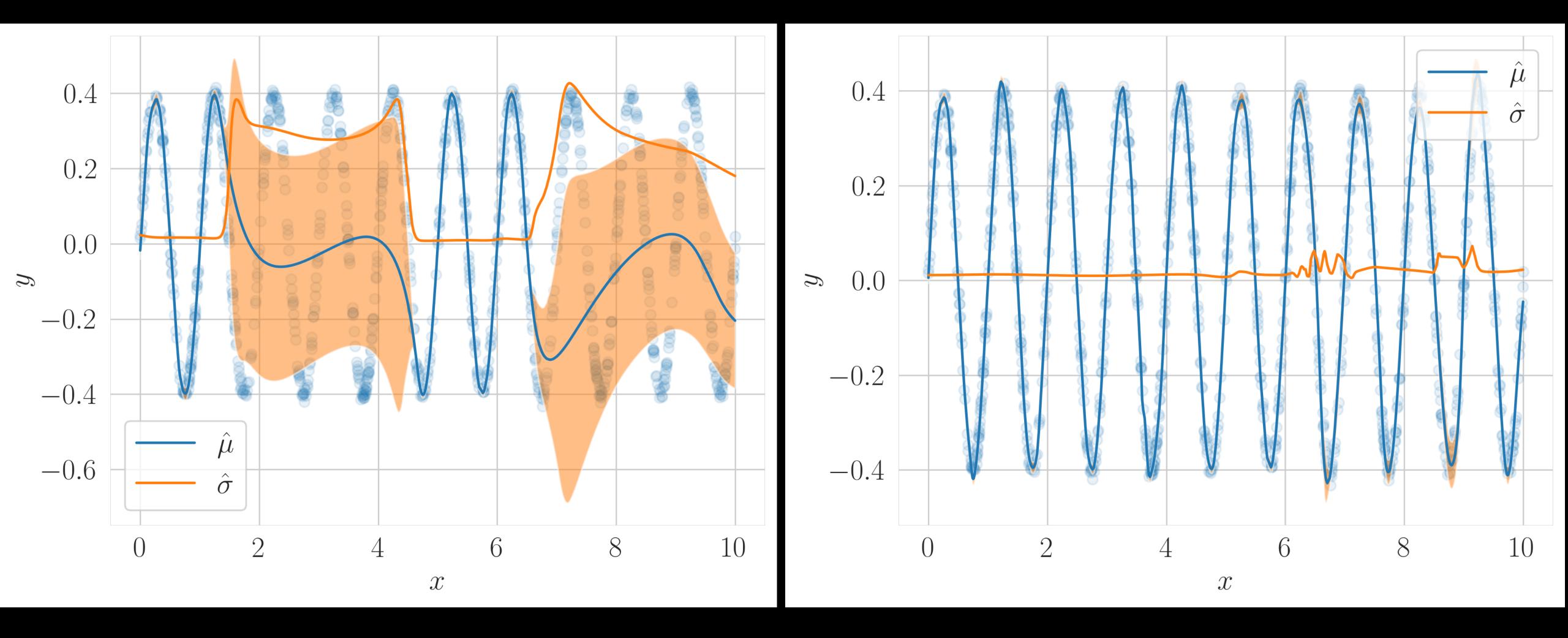


#### Learn the variance

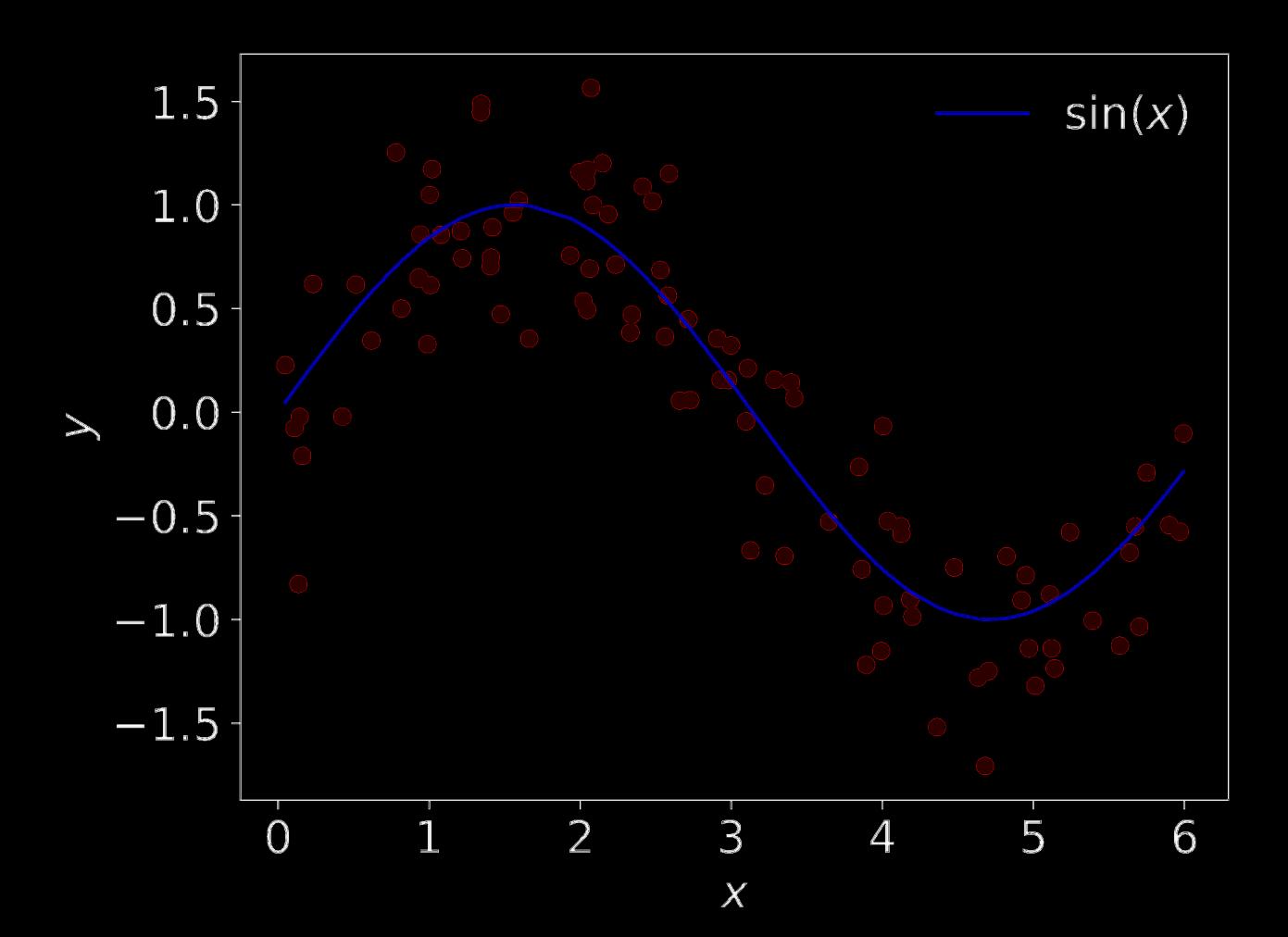
Sluijterman, Laurens, Eric Cator, and Tom Heskes. "Optimal Training of Mean Variance Estimation Neural Networks."



Sluijterman, Laurens, Eric Cator, and Tom Heskes. "Optimal Training of Mean Variance Estimation Neural Networks."

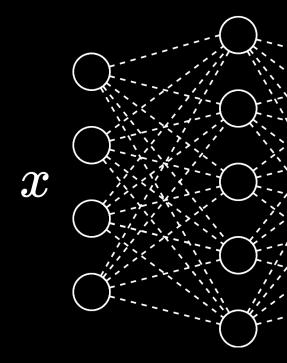


Sluijterman, Laurens, Eric Cator, and Tom Heskes. "Optimal Training of Mean Variance Estimation Neural Networks."

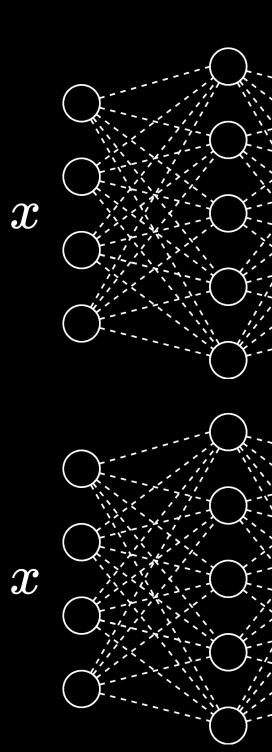


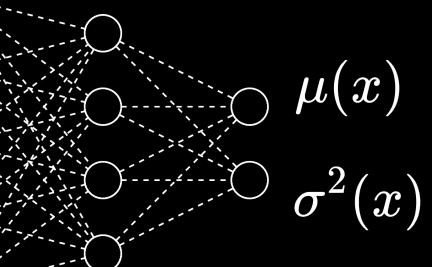


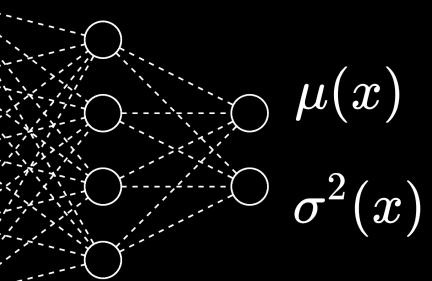


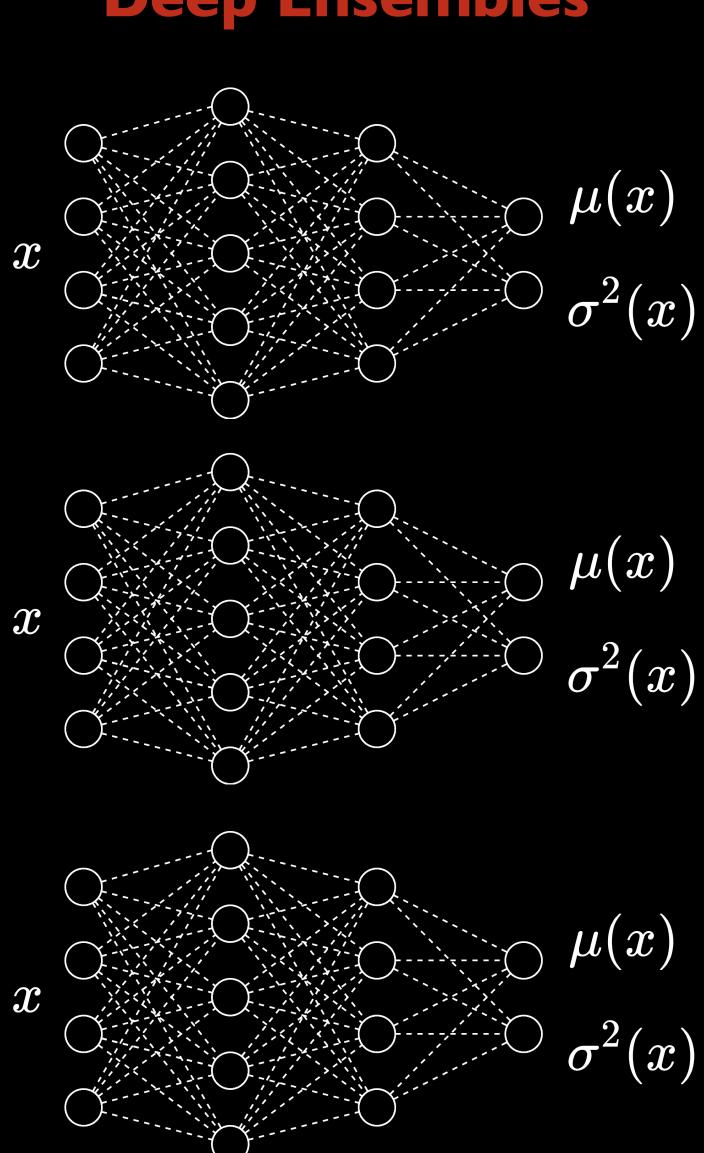


 $\mu(x)$  $\sigma^2(x)$ \_ \_ \_ (



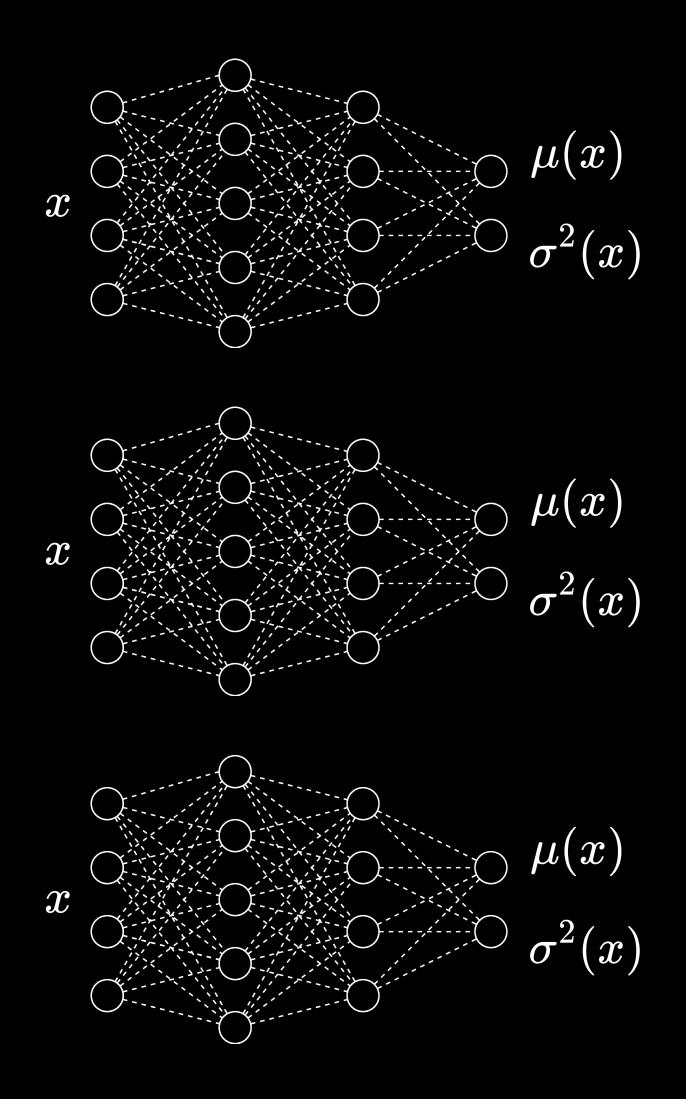




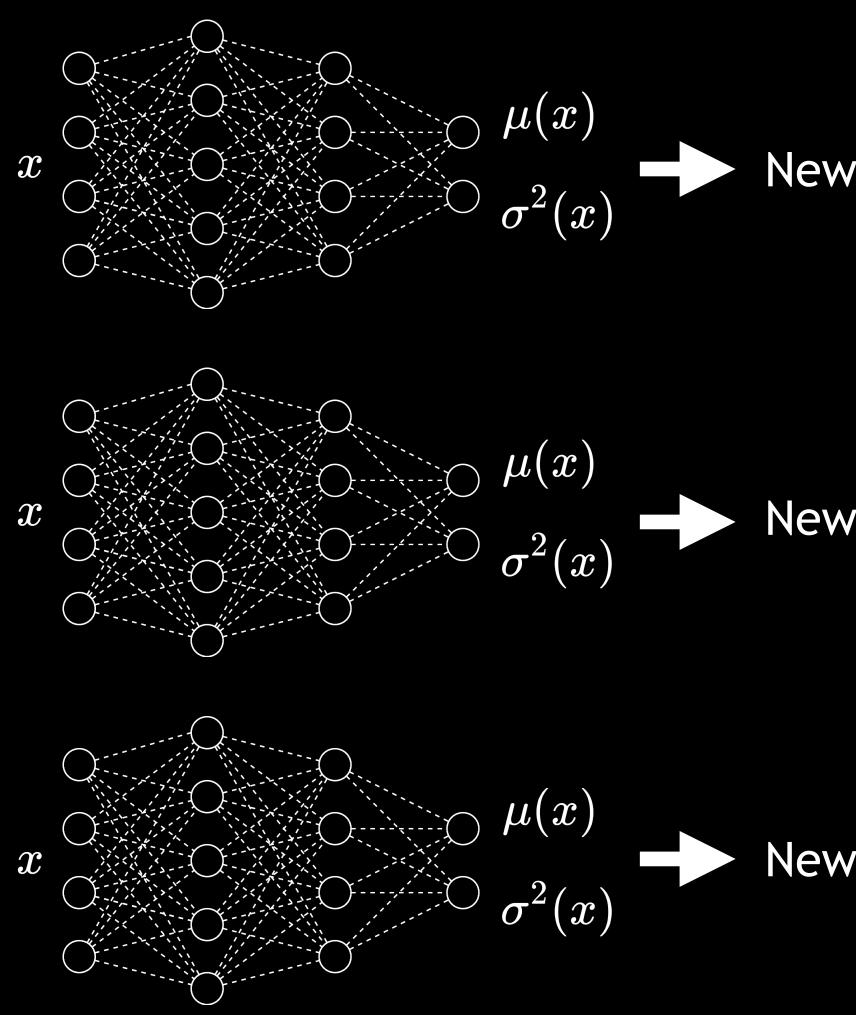


$$\mu(x)$$

#### Parametric bootstrap



#### Parametric bootstrap

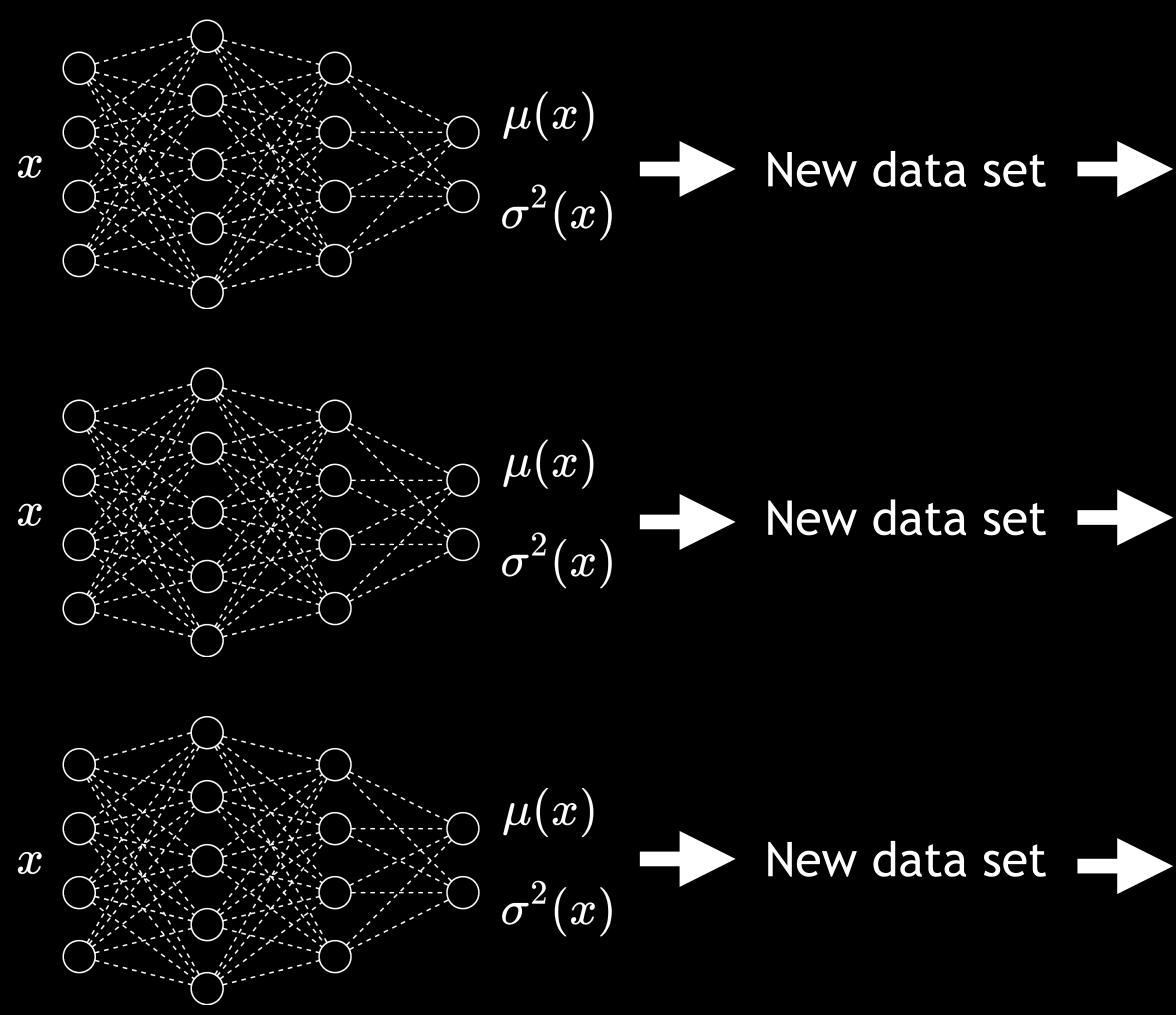


New data set

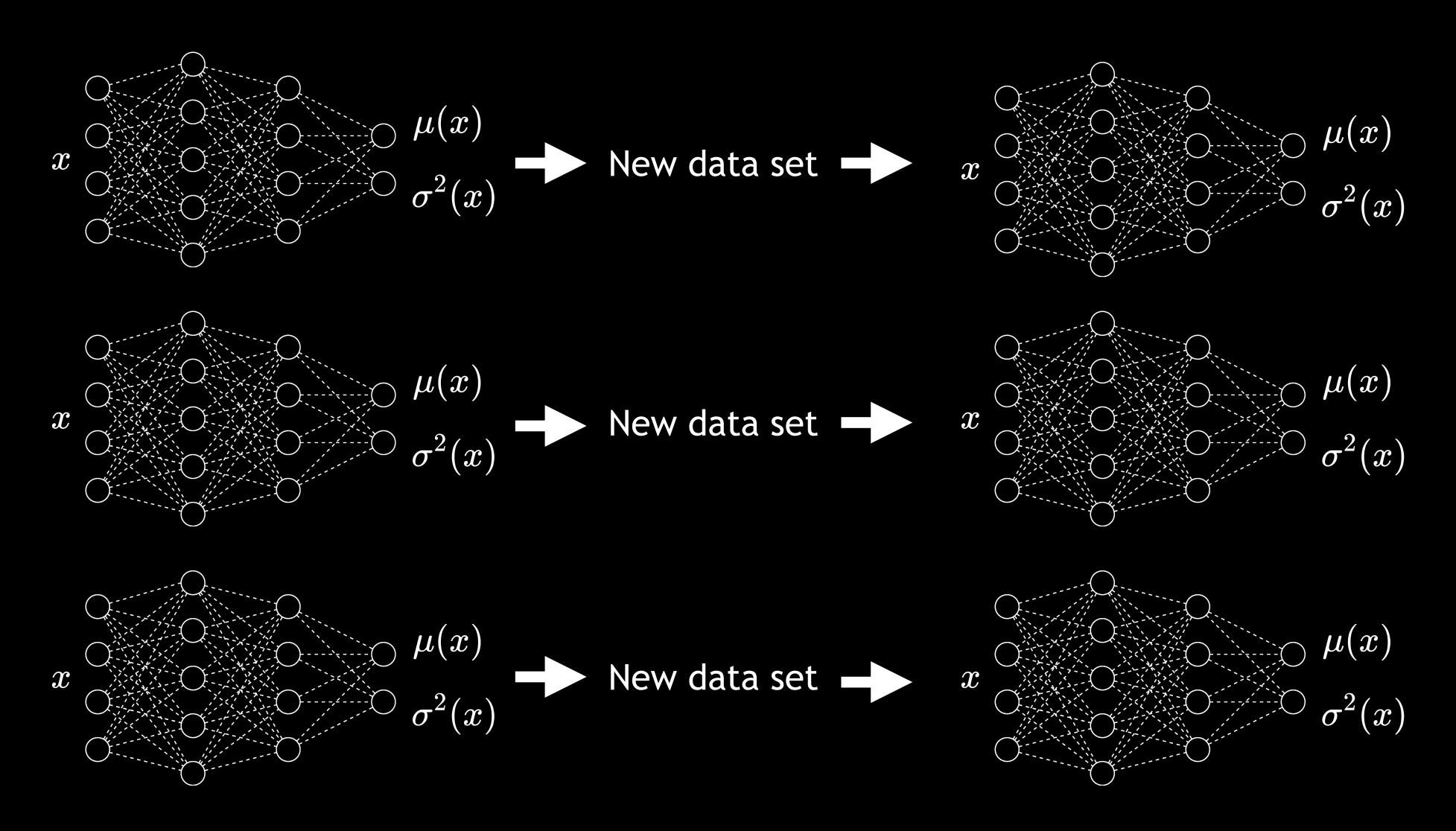
New data set

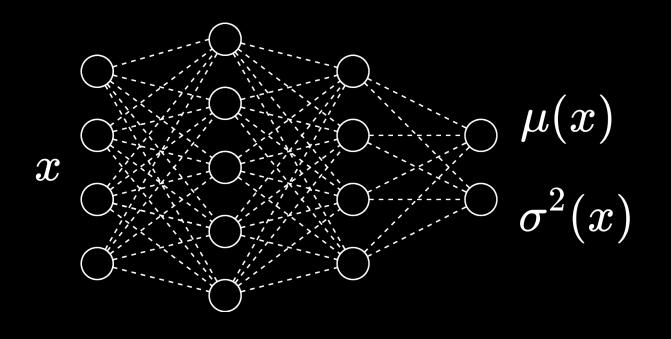
New data set

#### Parametric bootstrap

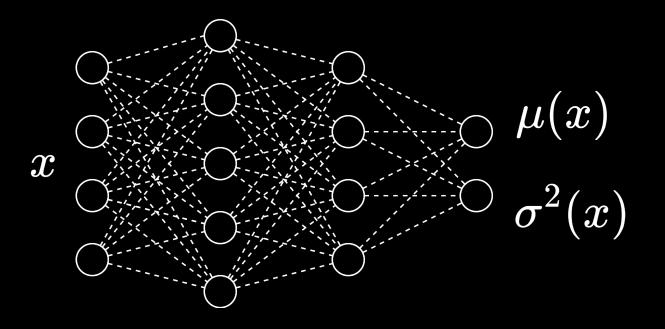


### Parametric bootstrap

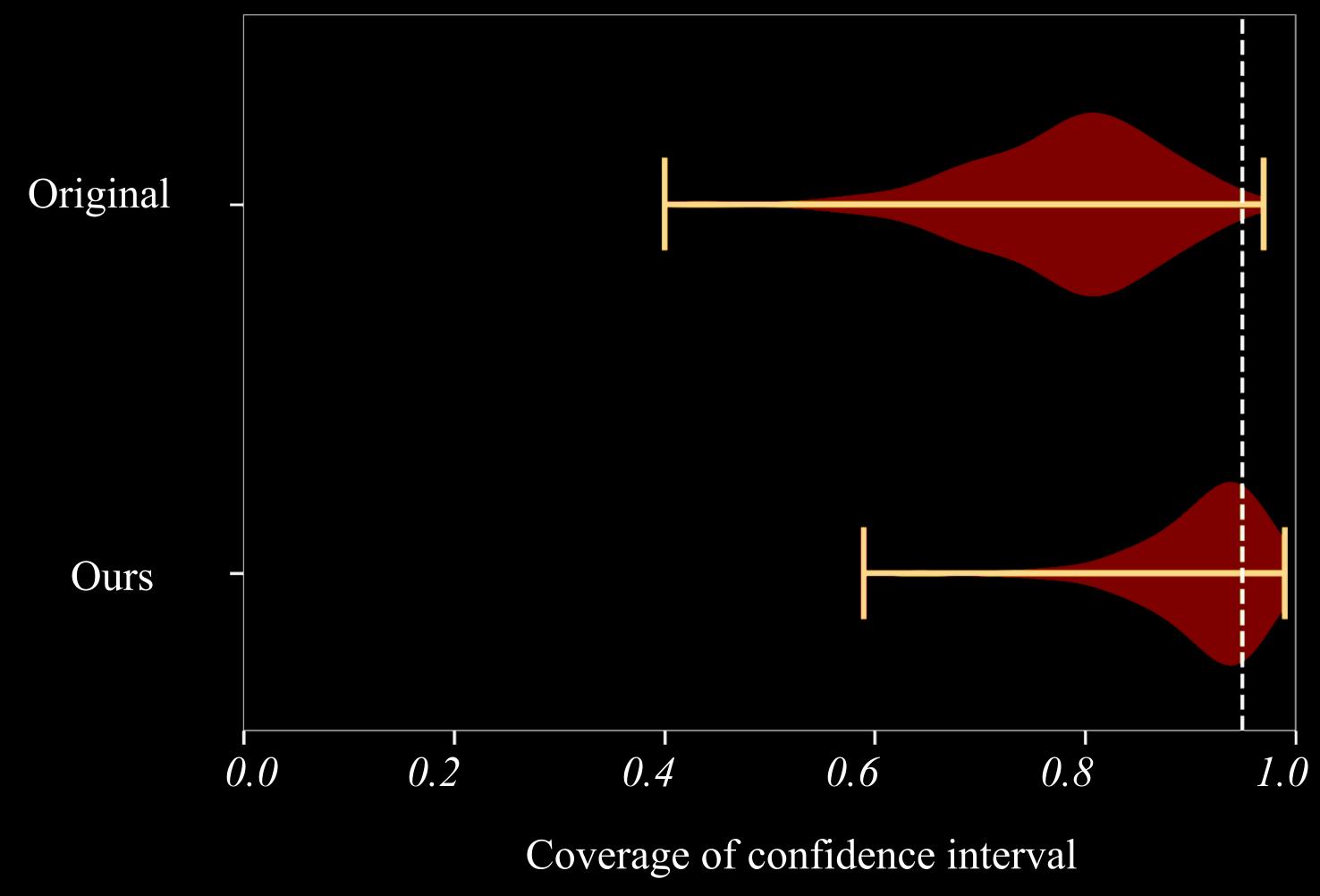




### Evaluate the difference before and after retraining



### Improved coverage



Sluijterman, Laurens, Eric Cator, and Tom Heskes. "Confident neural network regression with bootstrapped deep ensembles."



# $p(\theta \mid \mathcal{D}) = \frac{p(\mathcal{D} \mid \theta)p(\theta)}{p(\theta)}$ $p(\mathcal{D})$





# **I.Approximate** $p(\theta \mid \mathscr{D})$ with $q_m(\theta)$



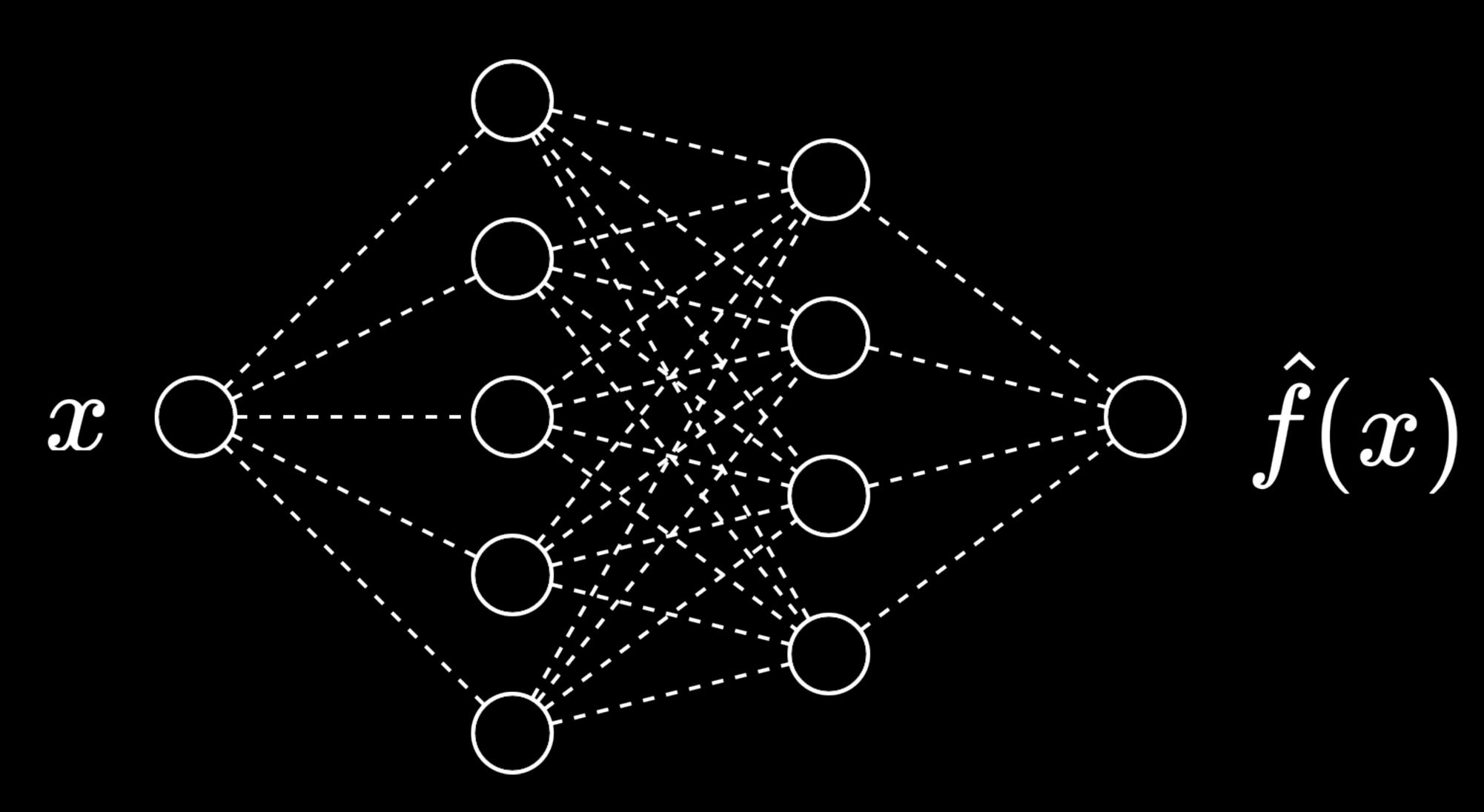
# I.Approximate $p(\theta \mid \mathcal{D})$ with $q_m(\theta)$ 2.Minimize $KL(q_m(\theta) \mid p(\theta \mid \mathcal{D}))$



# I.Approximate $p(\theta \mid \mathcal{D})$ with $q_m(\theta)$ 2.Minimize $KL(q_m(\theta) \mid p(\theta \mid \mathcal{D}))$

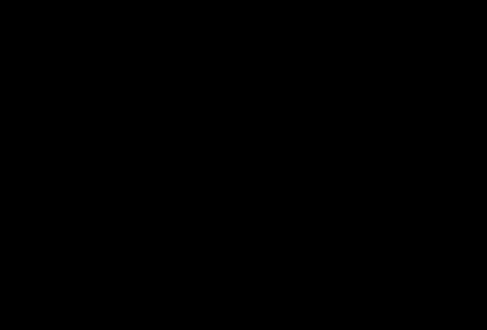


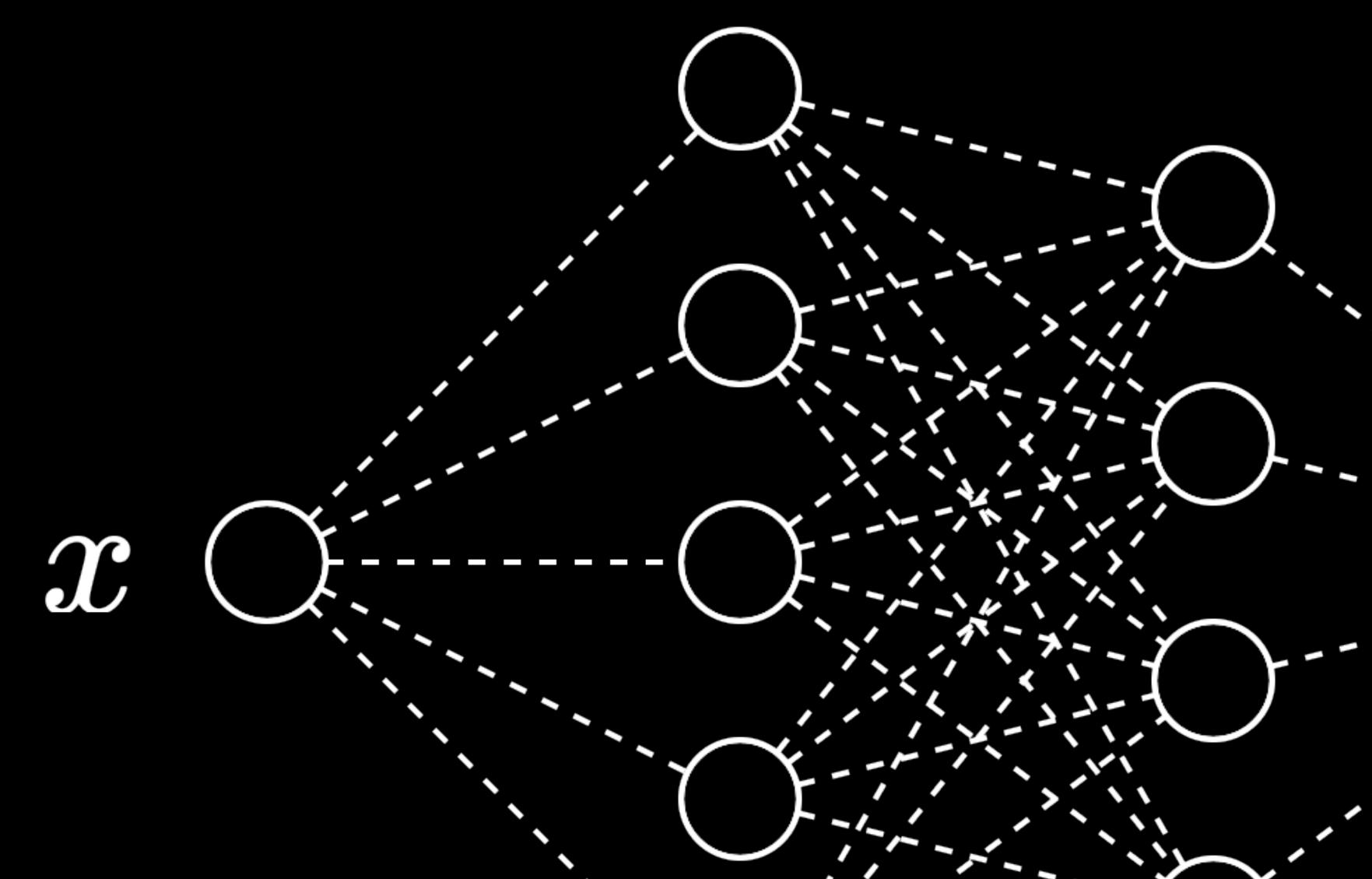
# I.Approximate $p(\theta \mid \mathcal{D})$ with $q_m(\theta)$ 2.Minimize $KL(q_m(\theta) \mid p(\theta \mid \mathcal{D}))$



### Dropout



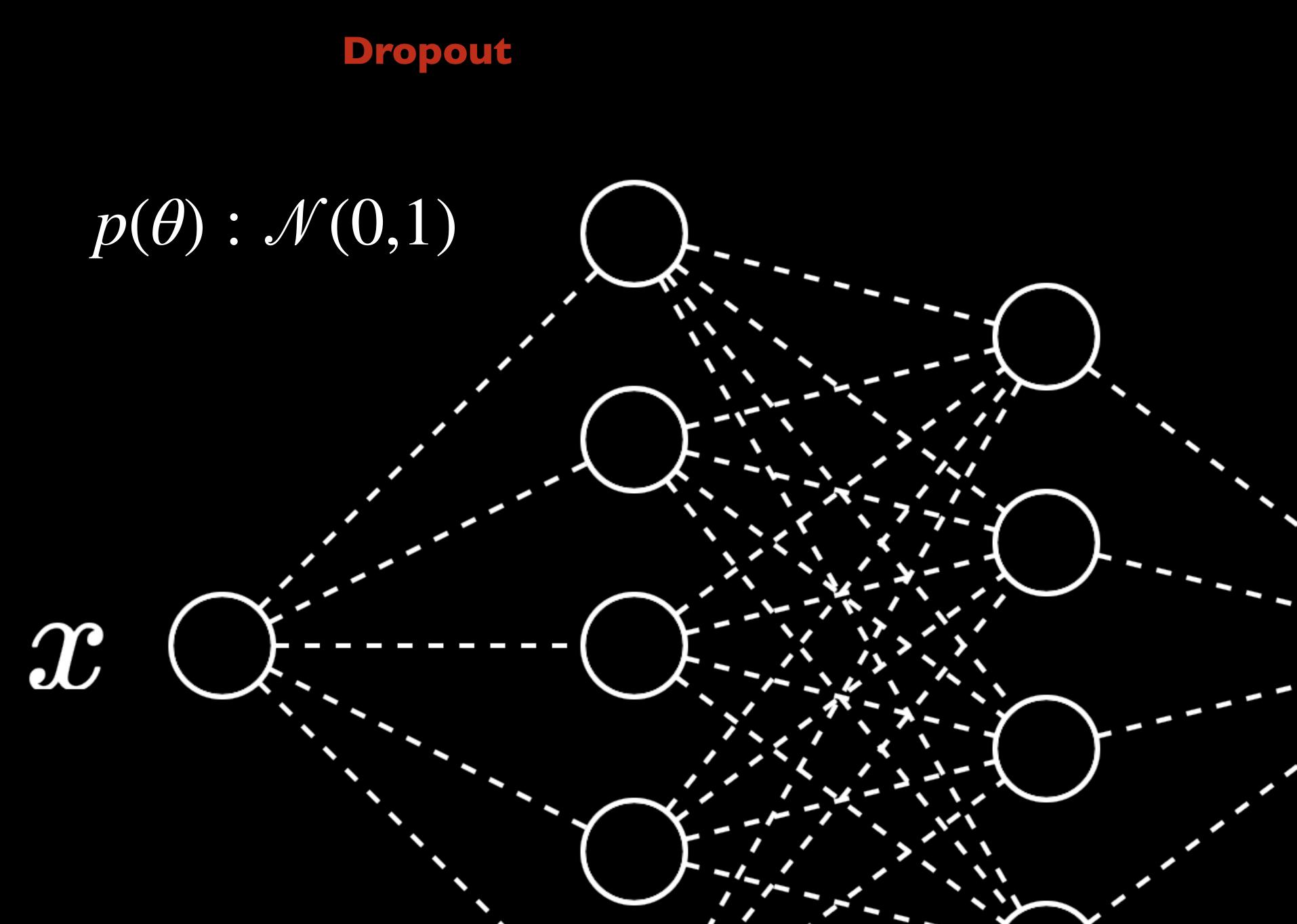




### Dropout

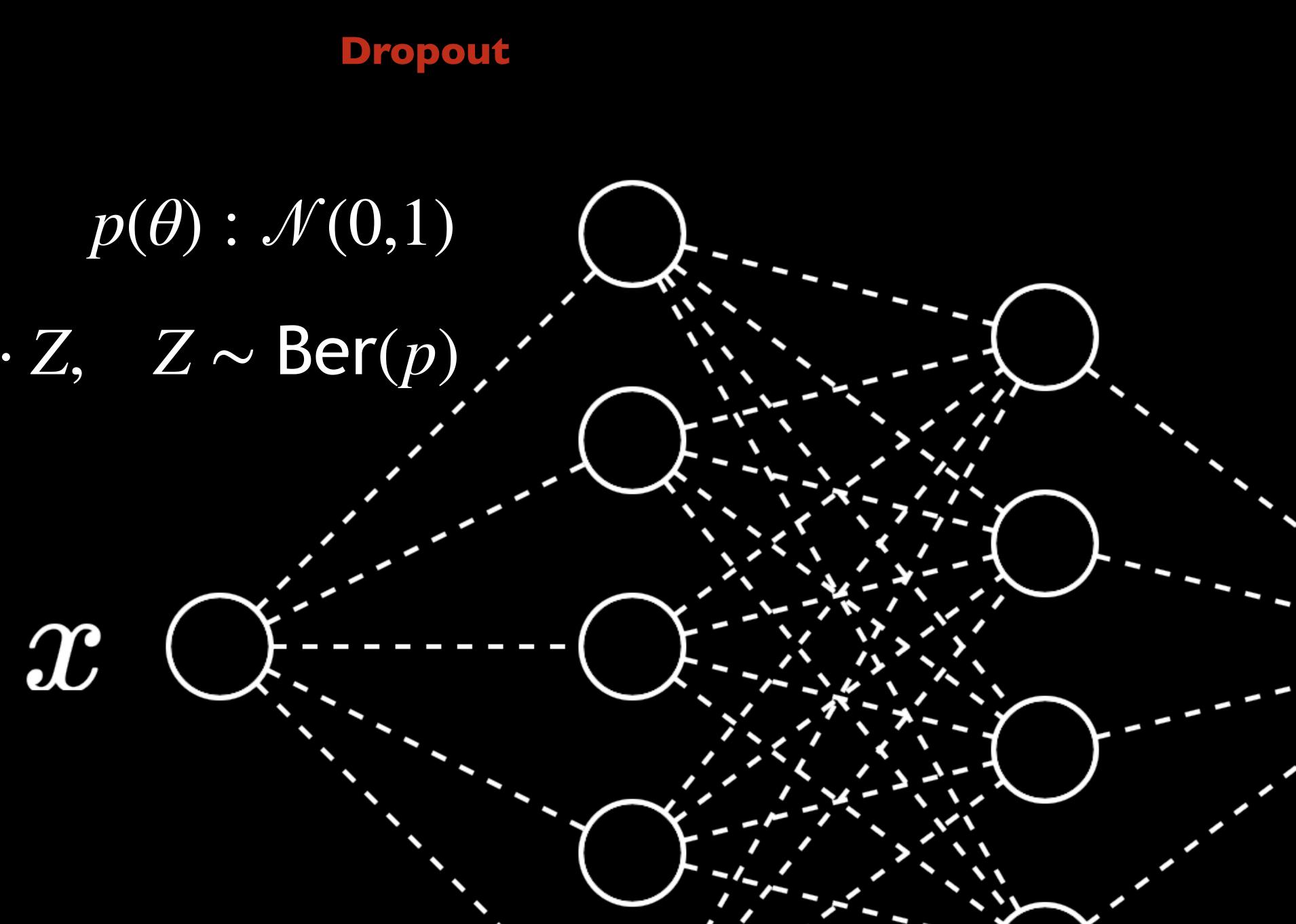








# $q_m(\theta): m \cdot Z, \quad Z \sim \operatorname{Ber}(p)$



### Interpretation



# I. What does a high-dimensional prior over weights mean?

I. What does a high-dimensional prior over weights mean?

## 2. The prior is chosen in order to get the result.



- I. What does a high-dimensional prior over weights mean?
- 2. The prior is chosen in order to get the result.
- 3. How good is the approximation?



- I. What does a high-dimensional prior over weights mean?
- 2. The prior is chosen in order to get the result.
- 3. How good is the approximation?
- 4. Random training

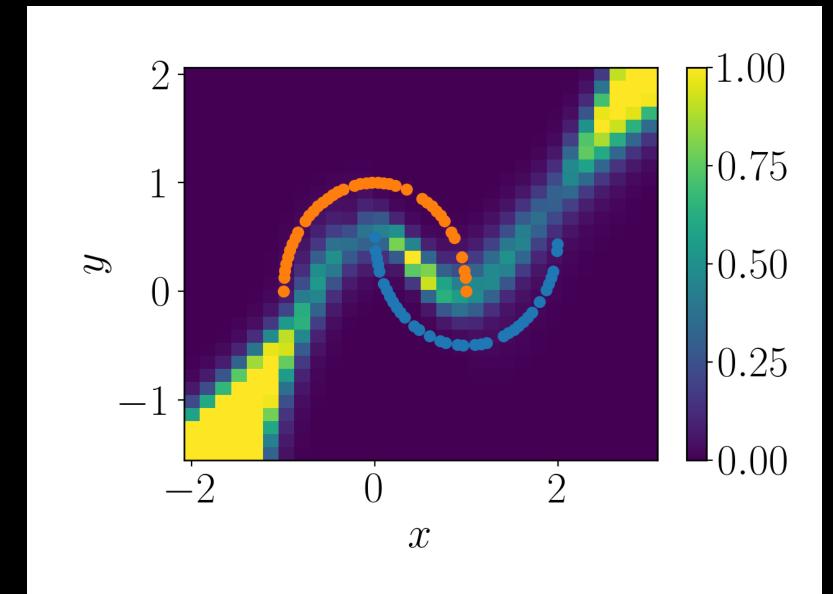


- I. What does a high-dimensional prior over weights mean?
- 2. The prior is chosen in order to get the result.
- 3. How good is the approximation?
- 4. Random training



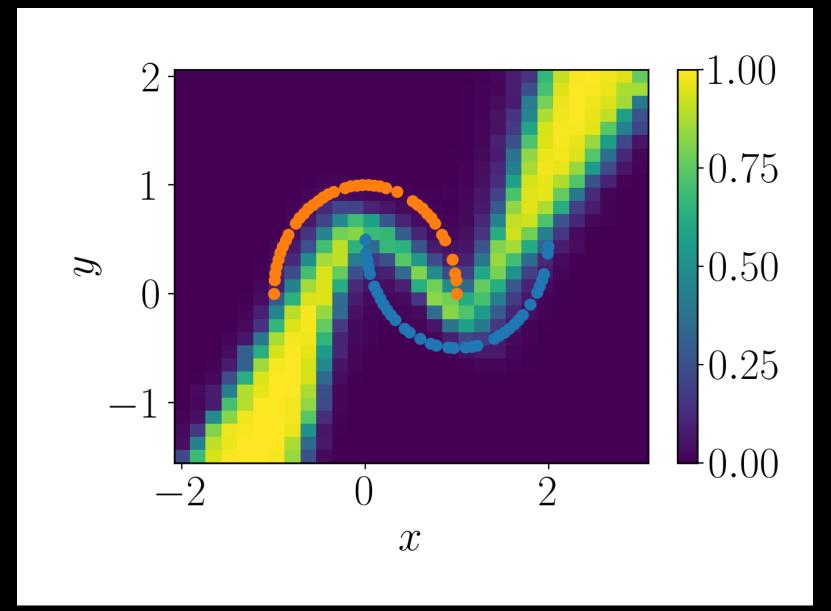


# Ensemble

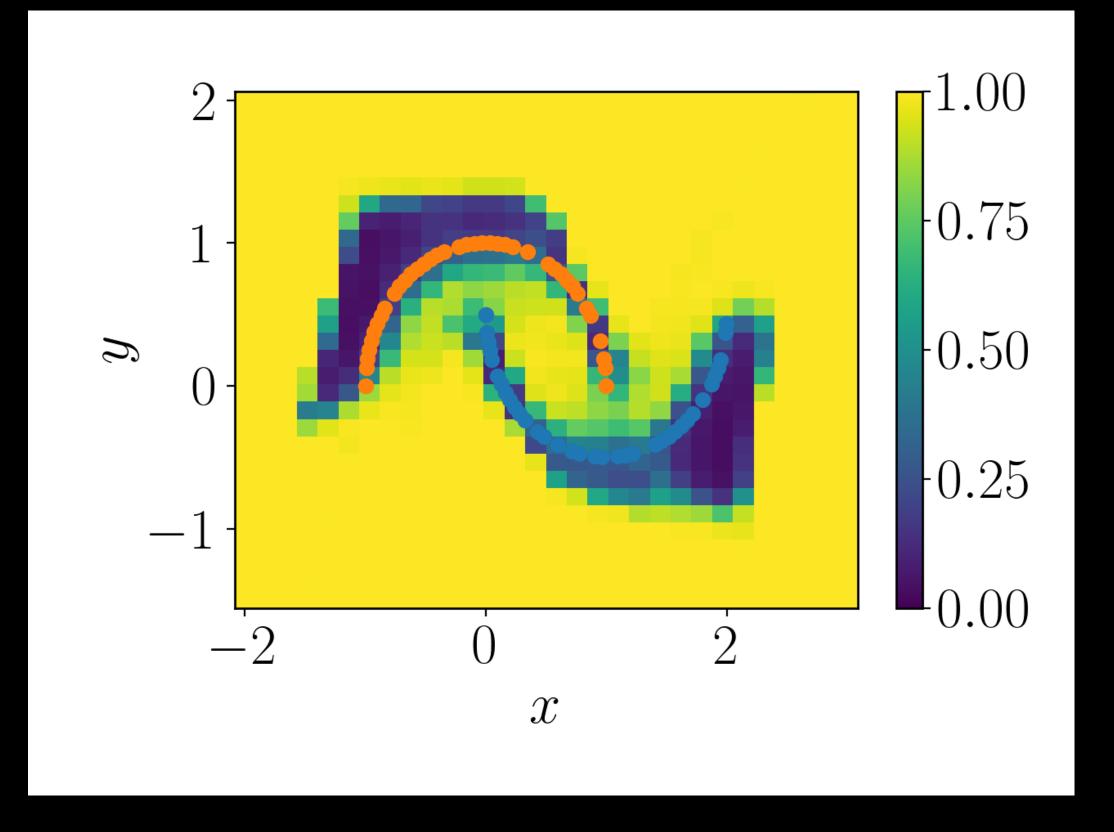


### **Out-of-distribution**

# Dropout







Sluijterman, Laurens, Eric Cator, and Tom Heskes. "Likelihood-ratio-based confidence intervals for neural networks."

# Likelihood ratio













## 2. Is our model correct?





### 2. Is our model correct?





### 2. Is our model correct?

# 3. MVE network and Out-of-distribution





### 2. Is our model correct?

# 3. MVE network and Out-of-distribution

