Results from the T2K + SK oscillation analysis

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on behalf of SK+T2K joint analysis

IRN Neutrino 2023, Karlsruhe
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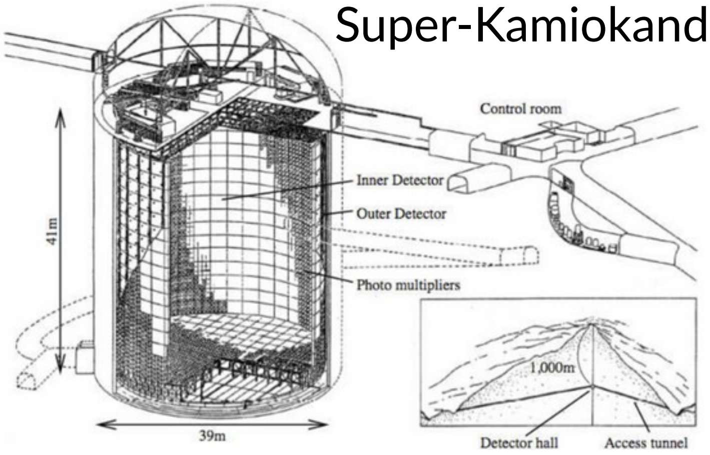
UNIVERSITY OF OXFORD

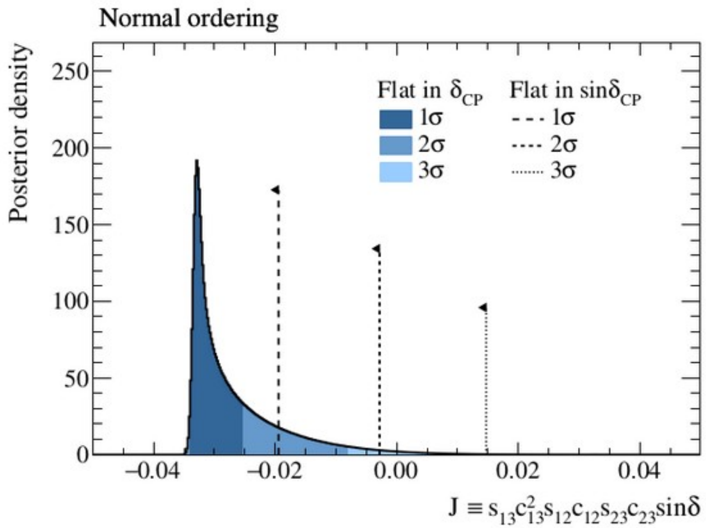
Outline

- Brief introduction to neutrino oscillations
- The T2K and SK experiments
- Why a joint analysis?
- Results
- The future

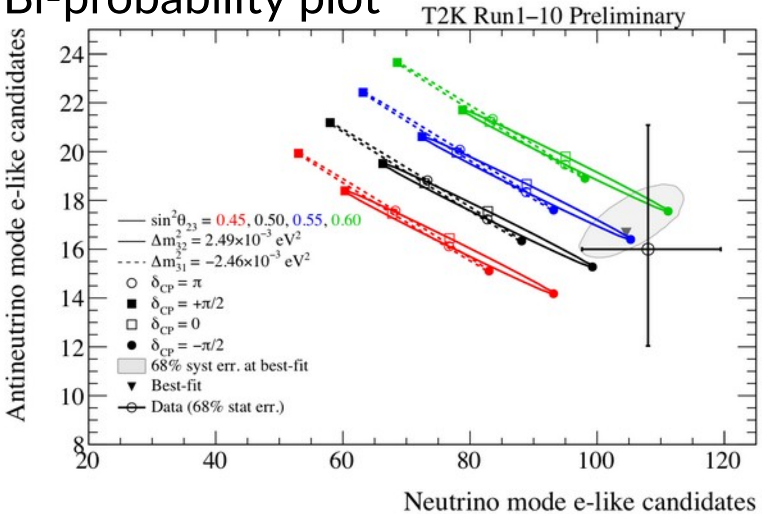
$$|\nu_i\rangle = \sum_{\alpha}^{n} U_{\alpha i}|\nu_\alpha\rangle$$

Super-Kamiokande





Bi-probability plot



Neutrino oscillations 101

Neutrino oscillations

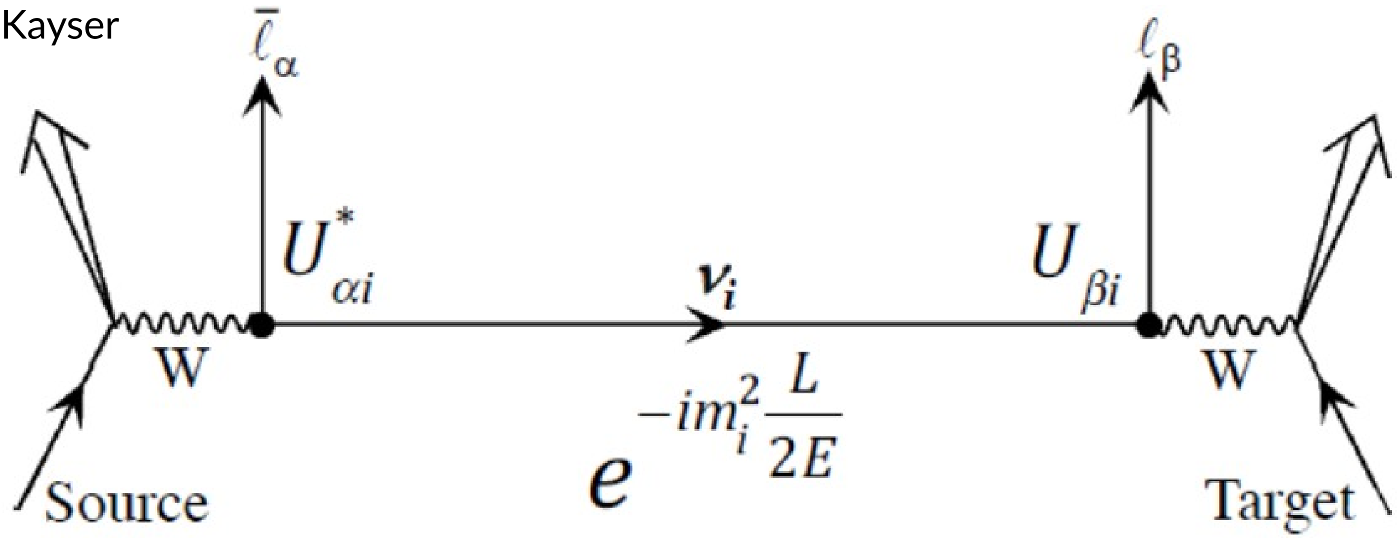
- Neutrino flavour and mass eigenstates are separated

$$|\nu_i\rangle = \sum_{\alpha}^{n} U_{\alpha i} |\nu_\alpha\rangle \qquad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Mass state — Mixing matrix — Flavour state

- Neutrinos propagate in **mass eigenstates**, but are born and detected in the **flavour eigenstate** via weak interaction

B. Kayser



- Results in oscillations of the detected flavour eigenstates

Neutrino oscillations

- Express probability to detect a neutrino with flavour α and energy *E*, as flavour *β* after it's travelled distance *L*

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$+ (-)2\sum_{i>j} Im\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin(\Delta m^2_{ij}\frac{L}{2E})$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

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Mixing angles

Mass² difference between eigenstate *i* and *j*

Experiment design

- Design of a neutrino oscillation experiment focusses on **L/E**
  - Determines sensitivity to mass squared splitting and mixing angles
  - Optimise L/E to match appearance/disappearance
  - Resolve neutrino energy adequately

Neutrino oscillations

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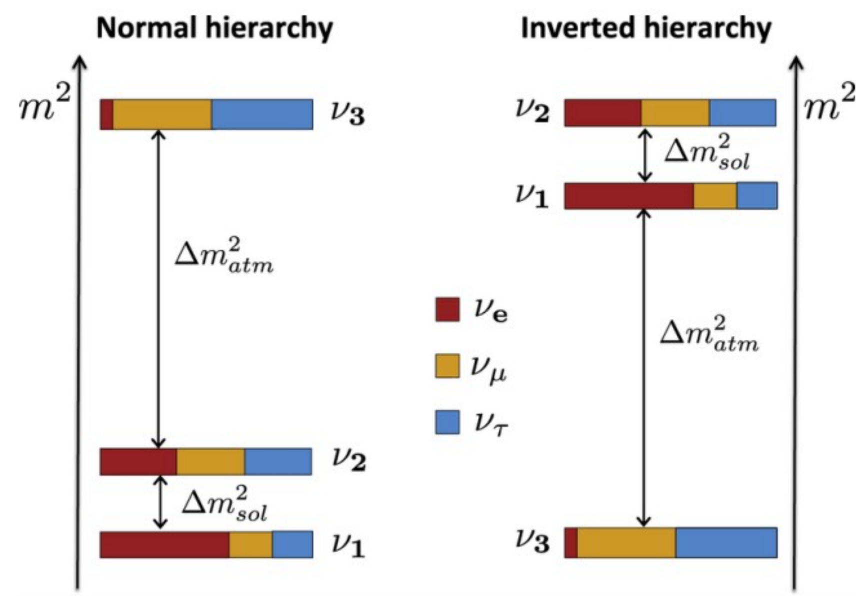
$$\Delta m^2_{ij} = m^2_i - m^2_j$$

Dominant effect from $\sin^2$ term leads to a unknown **mass hierarchy:** $\Delta m^2_{32}>0$?

sin term resolves mass hierarchy, and also enters through matter effects

Know $\Delta m^2_{21}>0$ from SNO experiment



Neutrino oscillations

- Express probability to detect a neutrino with flavour α and energy *E*, as flavour *β* after it's travelled distance *L*

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

$$+(-)2\sum_{i>j} Im\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin(\Delta m^2_{ij}\frac{L}{2E})$$

Measure differences in P(ν_μ→ν_e) and P(anti-ν_μ→anti-ν_e)
→ left with **single term**

$$\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$$

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = -16 J_{\alpha\beta} \sin\Delta_{12} \sin\Delta_{23} \sin\Delta_{31}$$

Sensitive to CP violating phase

Sensitive to mass hierarchy

$$J \equiv s_{12}c_{12}s_{23}c_{23}s_{13}c^2_{13}\sin\delta$$

Nunokawa et al, Prog. Part. Nucl. Phys. 60, 338

Neutrino oscillations

- The most general form of mixing matrix is seldom used; instead separate into three mixing matrices

$s_{ij} = \sin\theta_{ij}$
$c_{ij} = \cos\theta_{ij}$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atomspheric or "2,3" sector

Reactor, or "1,3" sector

Solar, or "1,2" sector

Neutrino oscillations

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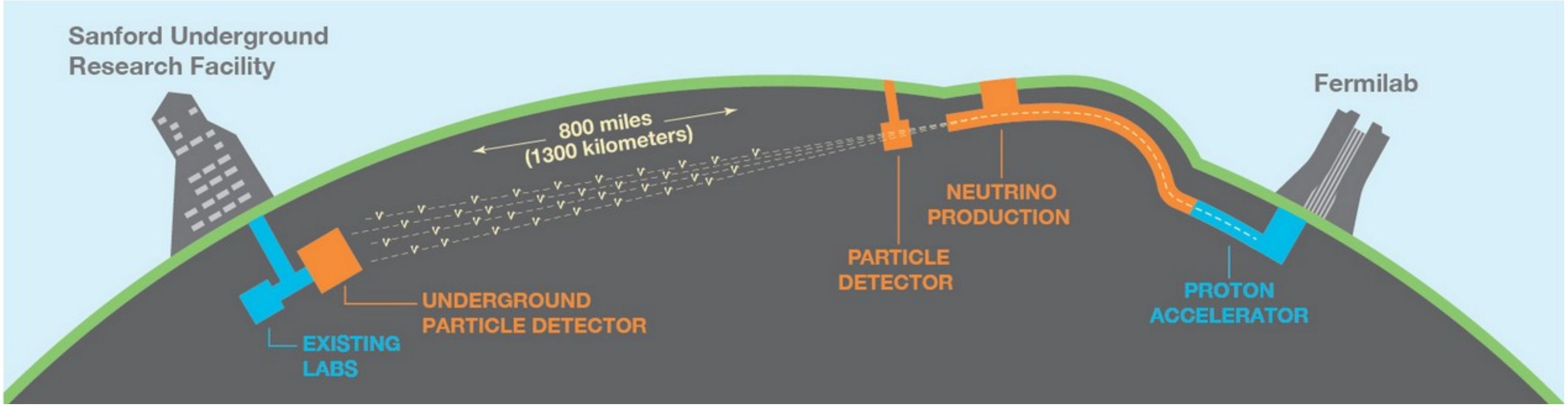
Atomspheric or “2,3” sector

Reactor, or “1,3” sector

Solar, or “1,2” sector

Long baseline experiments (K2K, T2K, NOvA, MINOS, DUNE, HK), atmospheric experiments (SK, IceCube)

L/E ~ 400-500km/GeV



From DUNE

Neutrino oscillations

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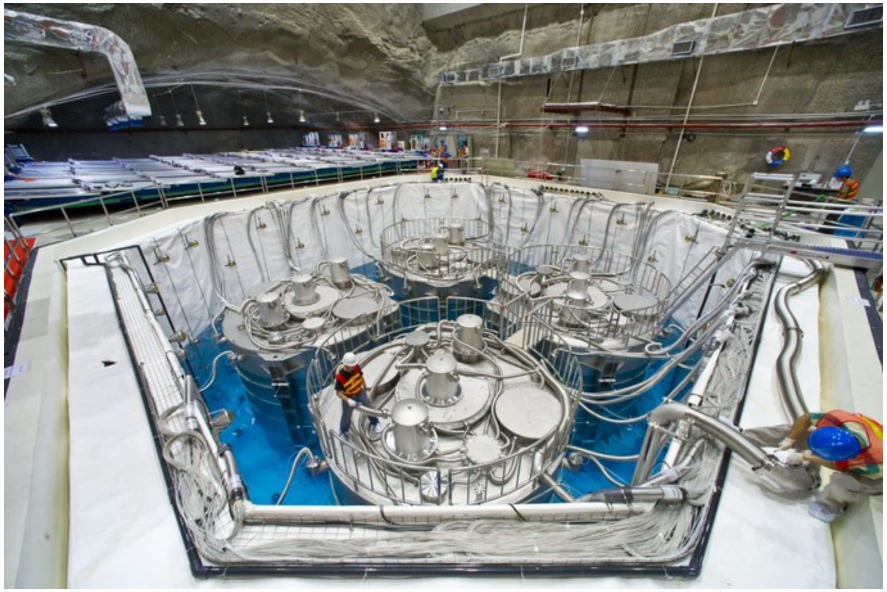
Atomspheric or "2,3" sector

Reactor, or "1,3" sector

Solar, or "1,2" sector

Reactor experiments (Daya Bay, RENO, Double Chooz)

L/E ~ 1km/MeV



From LBL

Neutrino oscillations

- The most general form of mixing matrix is seldom used; instead separate into three mixing matrices

$s_{ij} = \sin\theta_{ij}$
$c_{ij} = \cos\theta_{ij}$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atomspheric or "2,3" sector

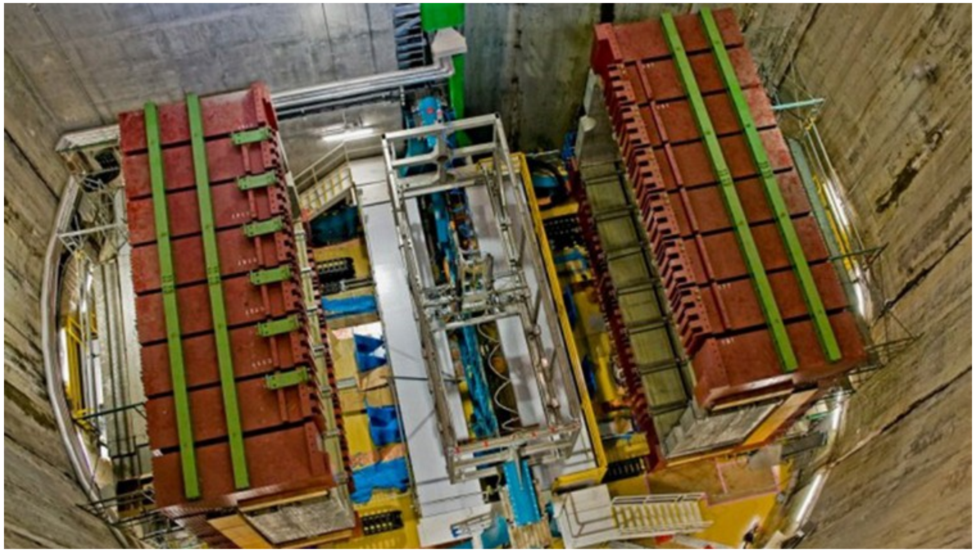
Reactor, or "1,3" sector

Solar, or "1,2" sector

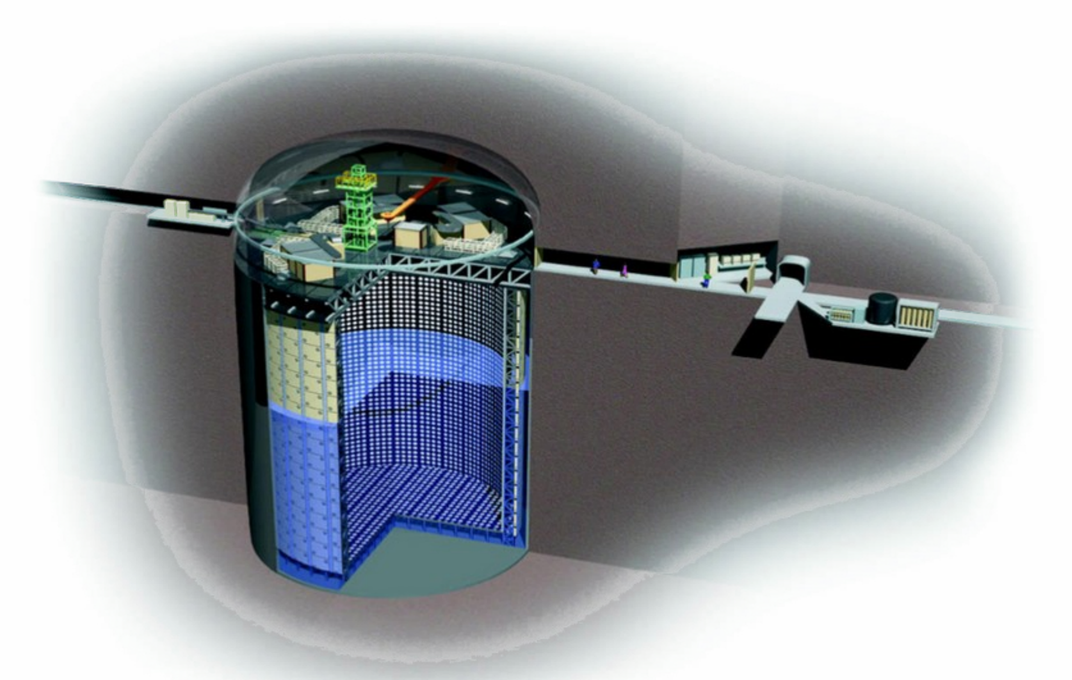
Solar experiments (SNO, SK) long baseline reactor experiments (KamLAND, JUNO)
L/E > 100km/MeV

From MIT

The T2K and SK experiments

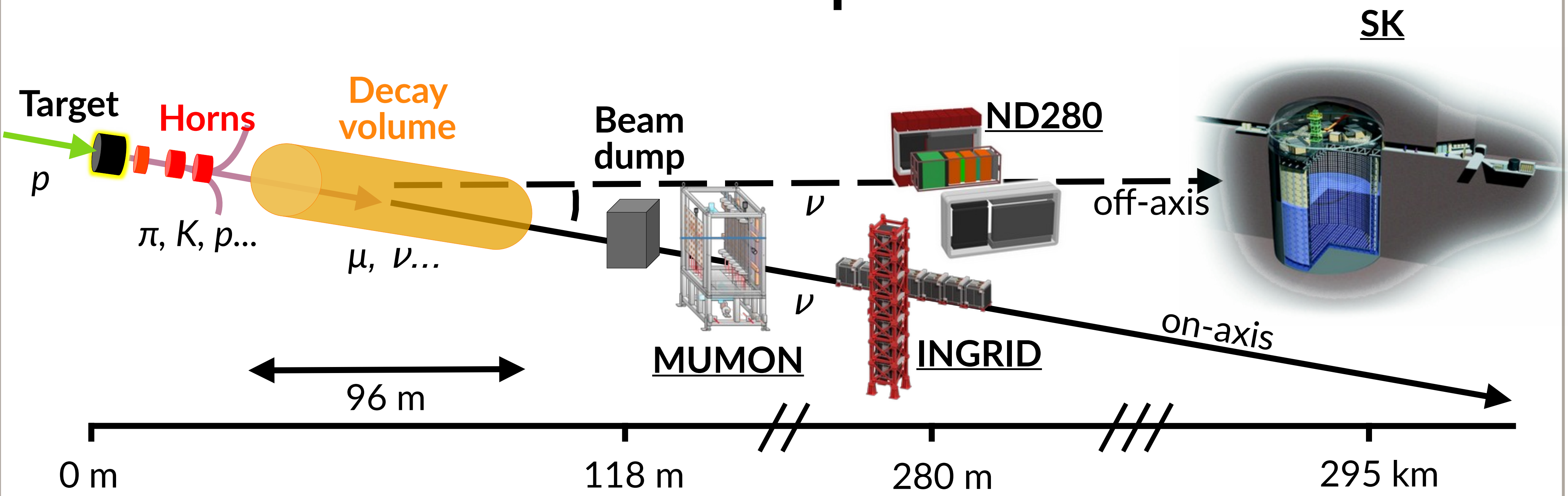


The "pit" 280m after the target station, housing ND280, INGRID, and other near detectors

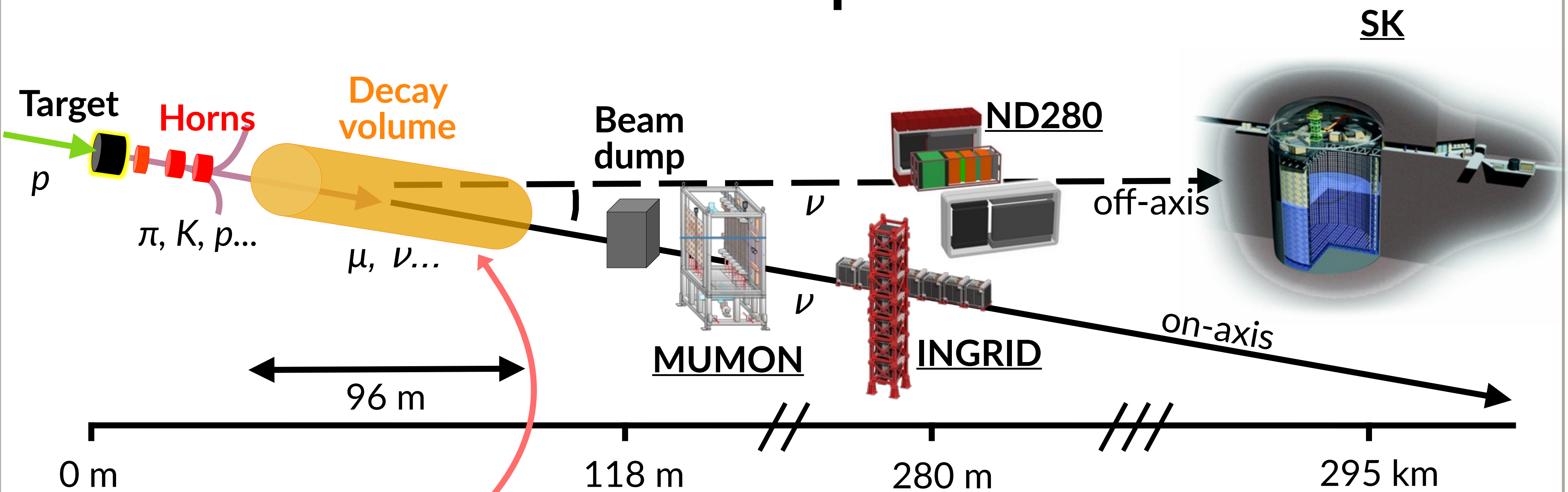


The SK detector: T2K's far detector and conducts its own atmospheric neutrino analysis

The T2K experiment

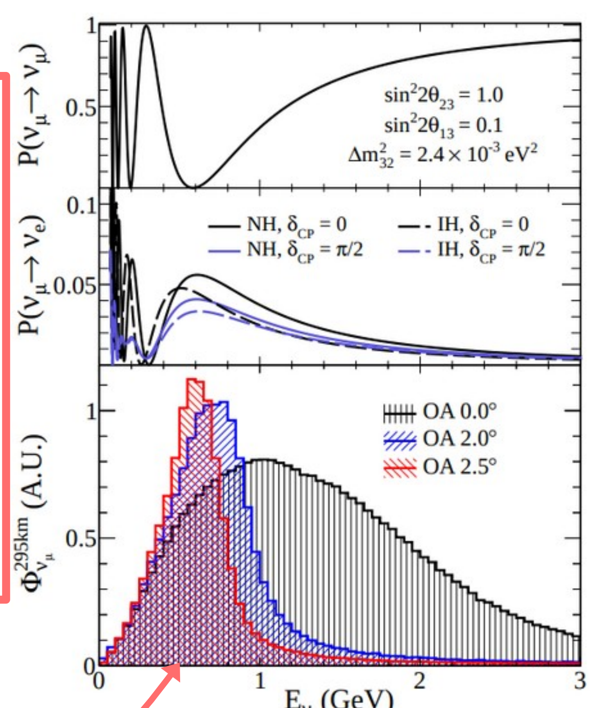


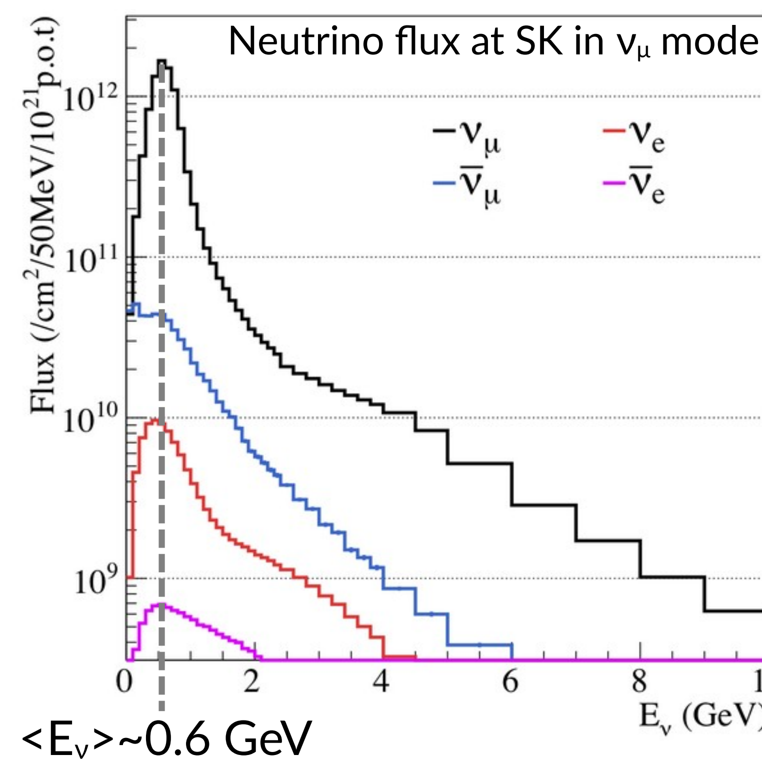
The T2K experiment



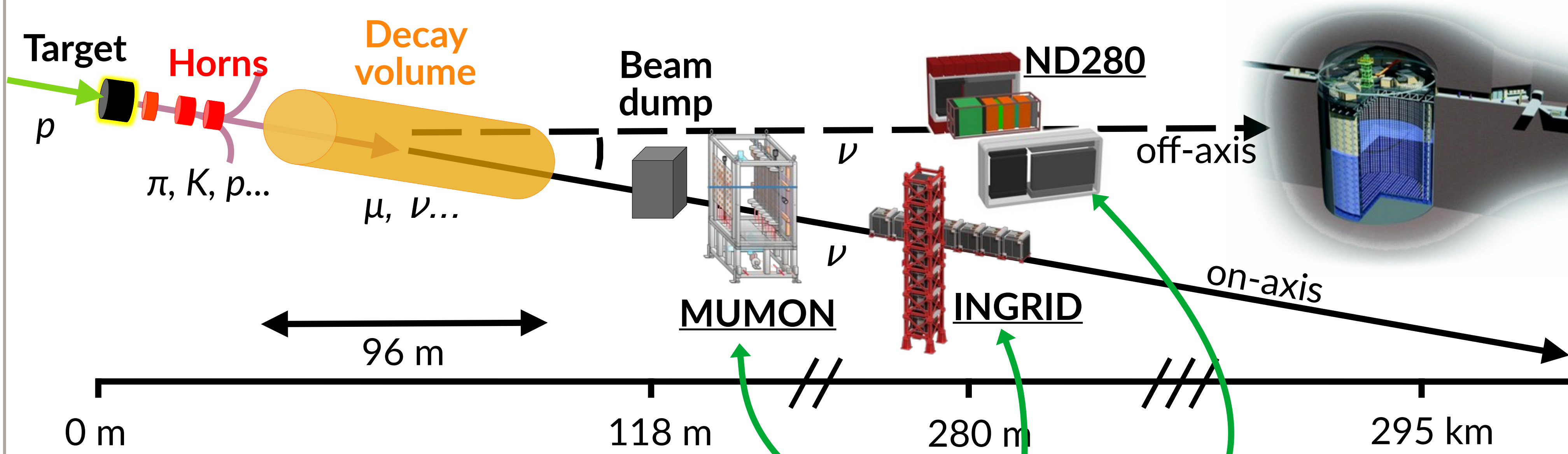
Start with predominantly ν_μ (anti-ν_μ) beam

Move off-axis for a narrow E_ν peak

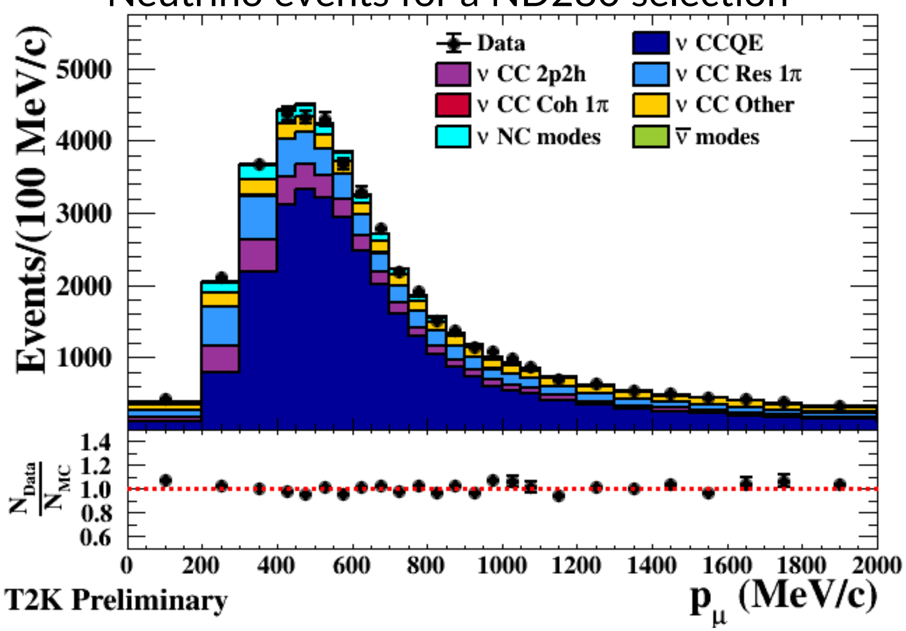




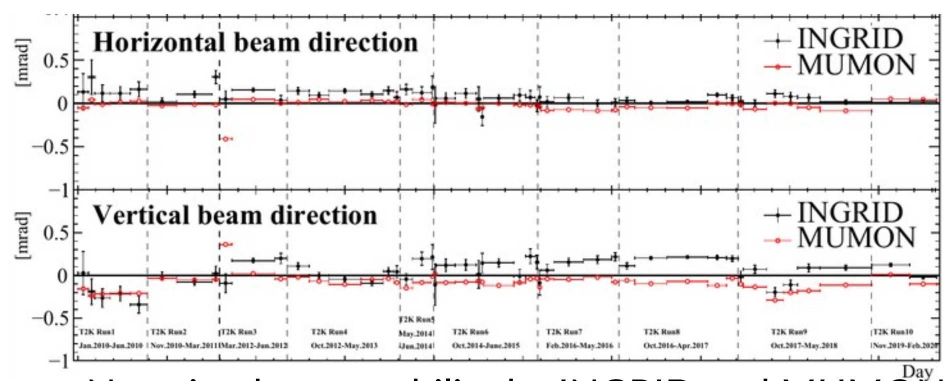
The T2K experiment



Neutrino events for a ND280 selection

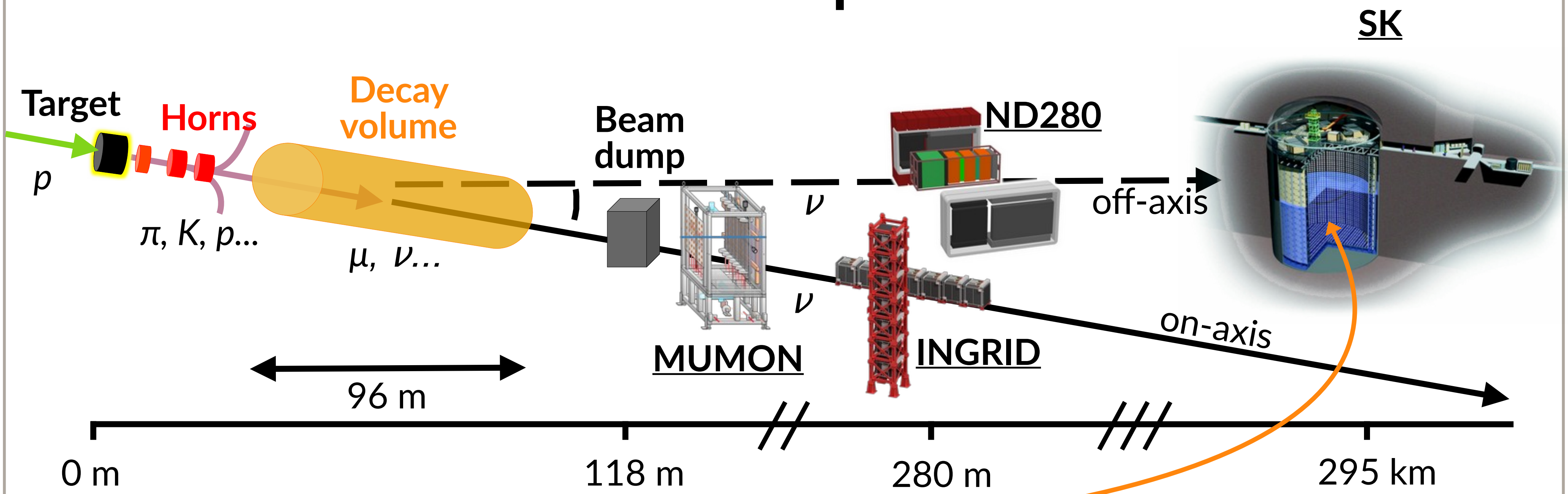


Characterise neutrino beam before long baseline oscillations



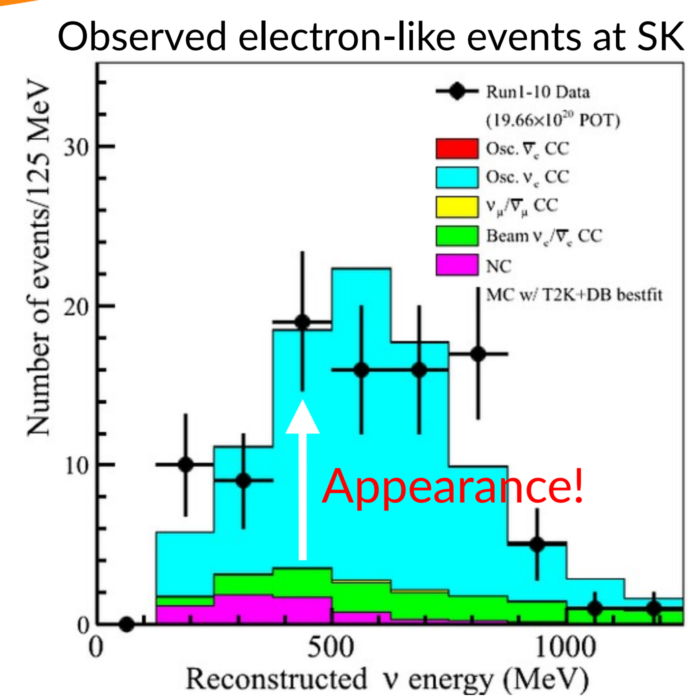
Neutrino beam stability by INGRID and MUMON

The T2K experiment



$\nu_{\mu}\rightarrow\nu_{\mu}$: How many ν_{μ} have ***disappeared*** in the beam measured at SK?

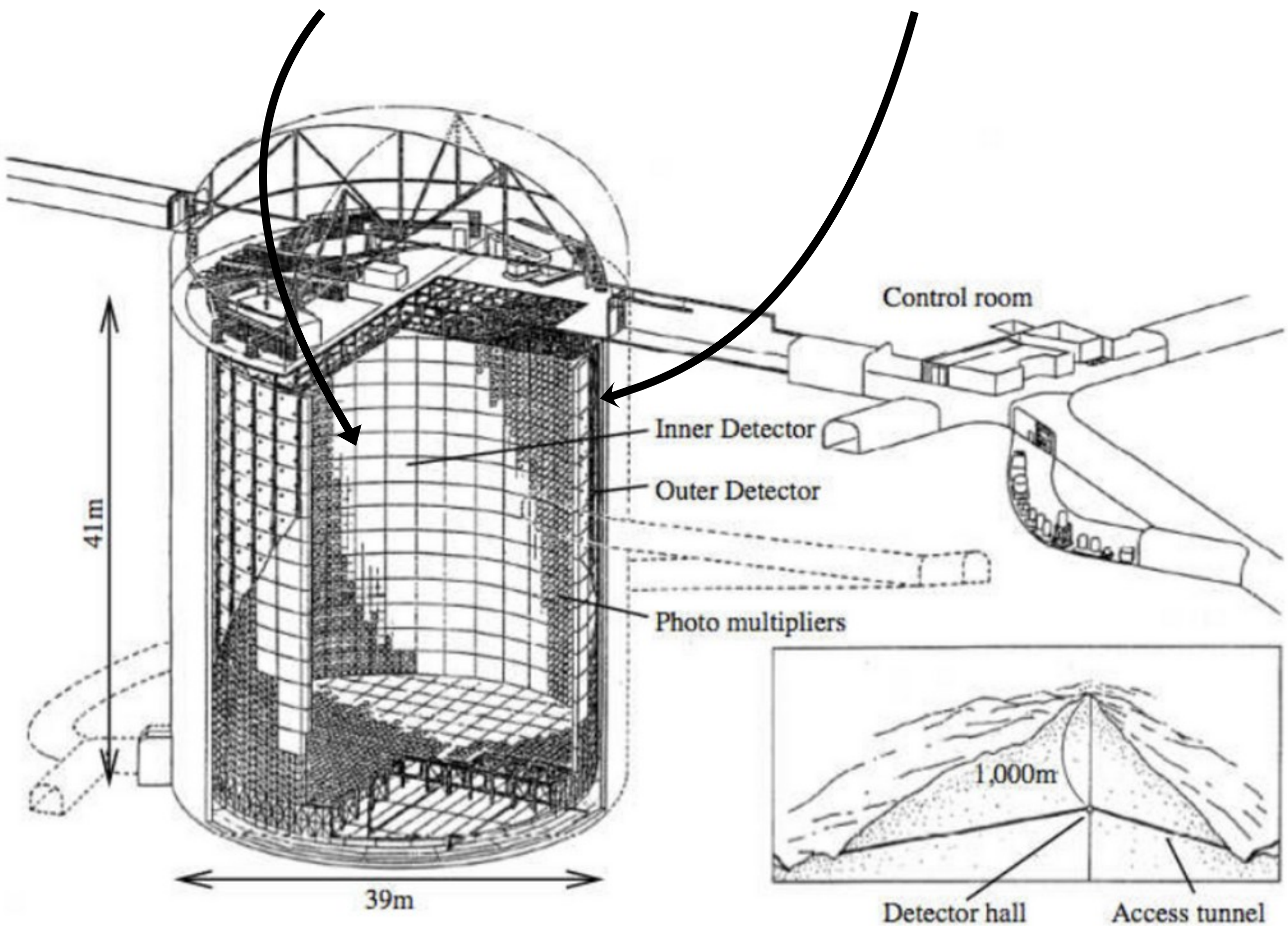
$\nu_{\mu}\rightarrow\nu_{e}$: How many ν_{e} have ***appeared*** in the beam measured at SK?



→ Measure long baseline oscillations

The SK detector

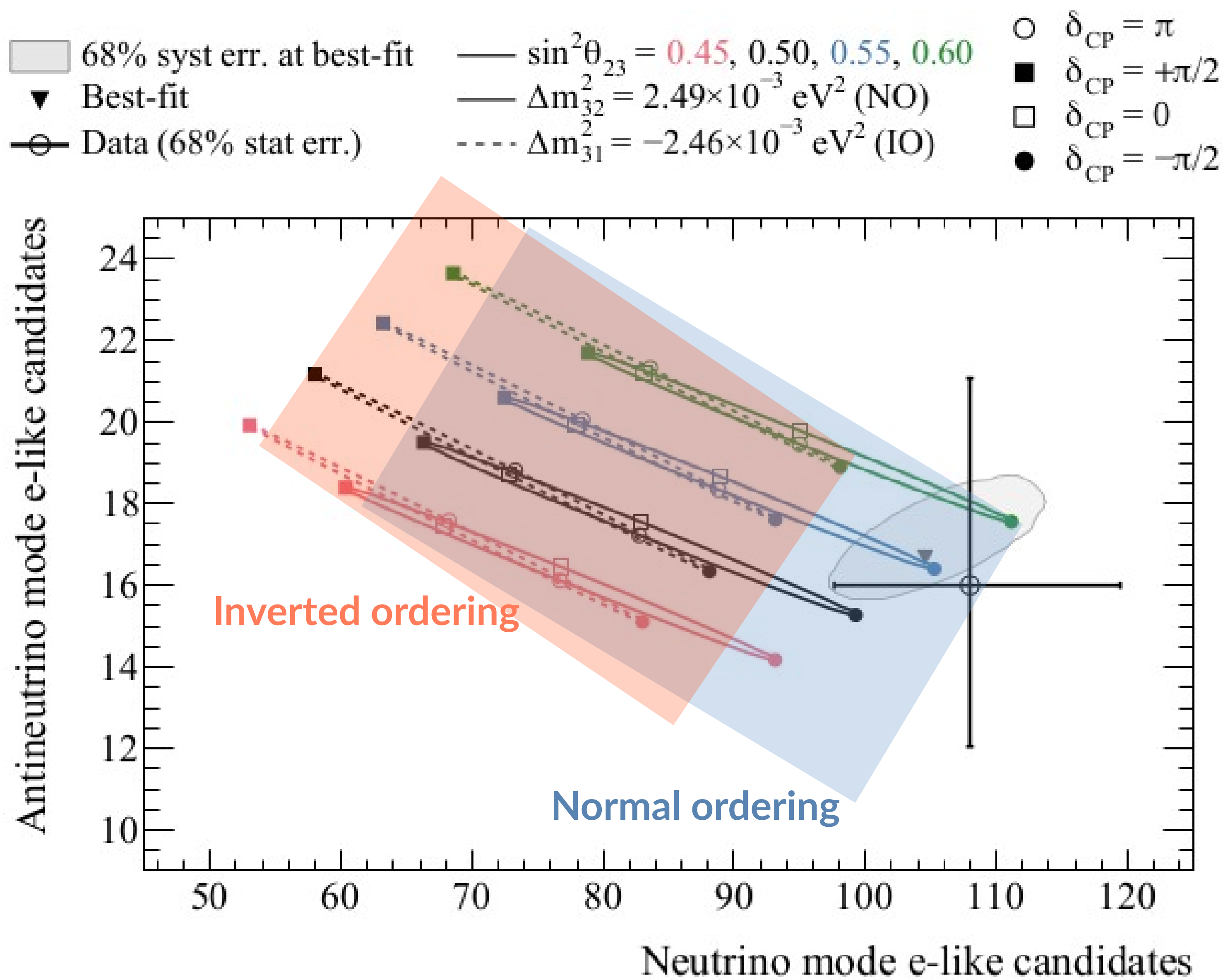
- 50kt water Cherenkov detector, 2.7 km water equivalent overburden
- Running since 1996, with latest upgrade to SK-V in 2018 relevant to this analysis (now doped with Gd!)
  - 2.5° off-axis with similar flux to ND280
- 11,146 20” PMTs in ID, 1,885 8” PMTs in OD – 40% PMT coverage



Why a joint analysis?

Why a joint analysis

- T2K has degeneracies with δ_{CP} and mass ordering



Why a joint analysis

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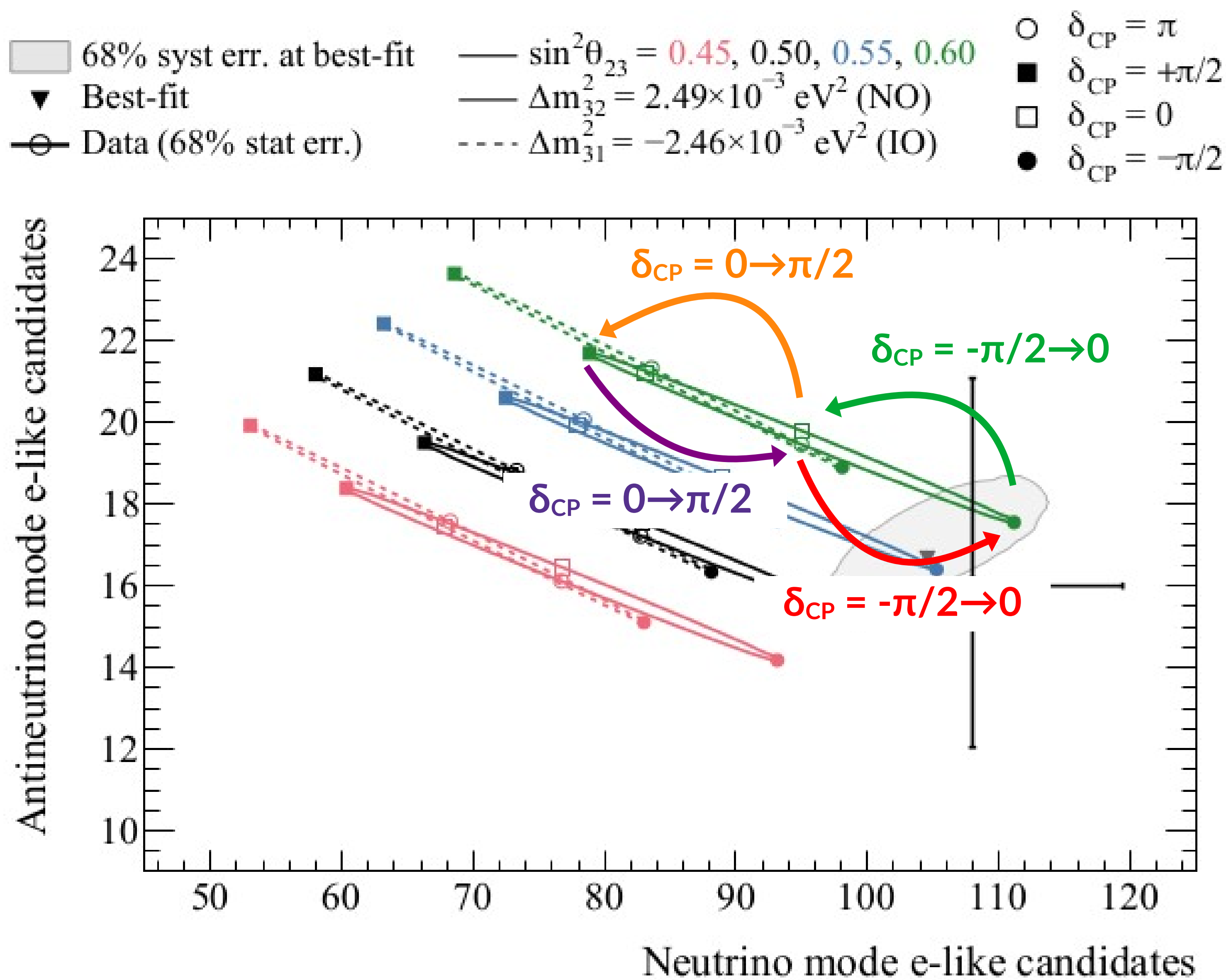
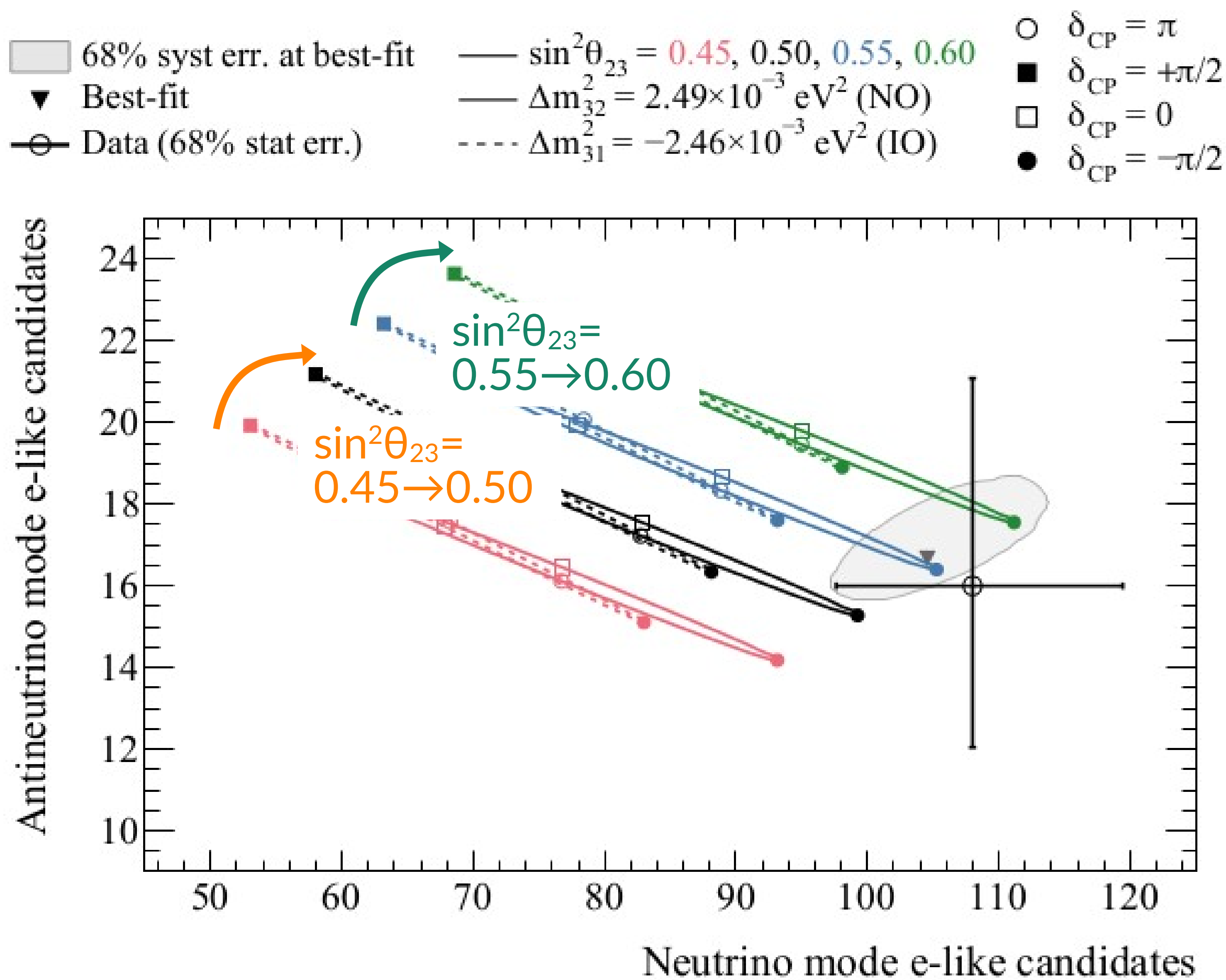


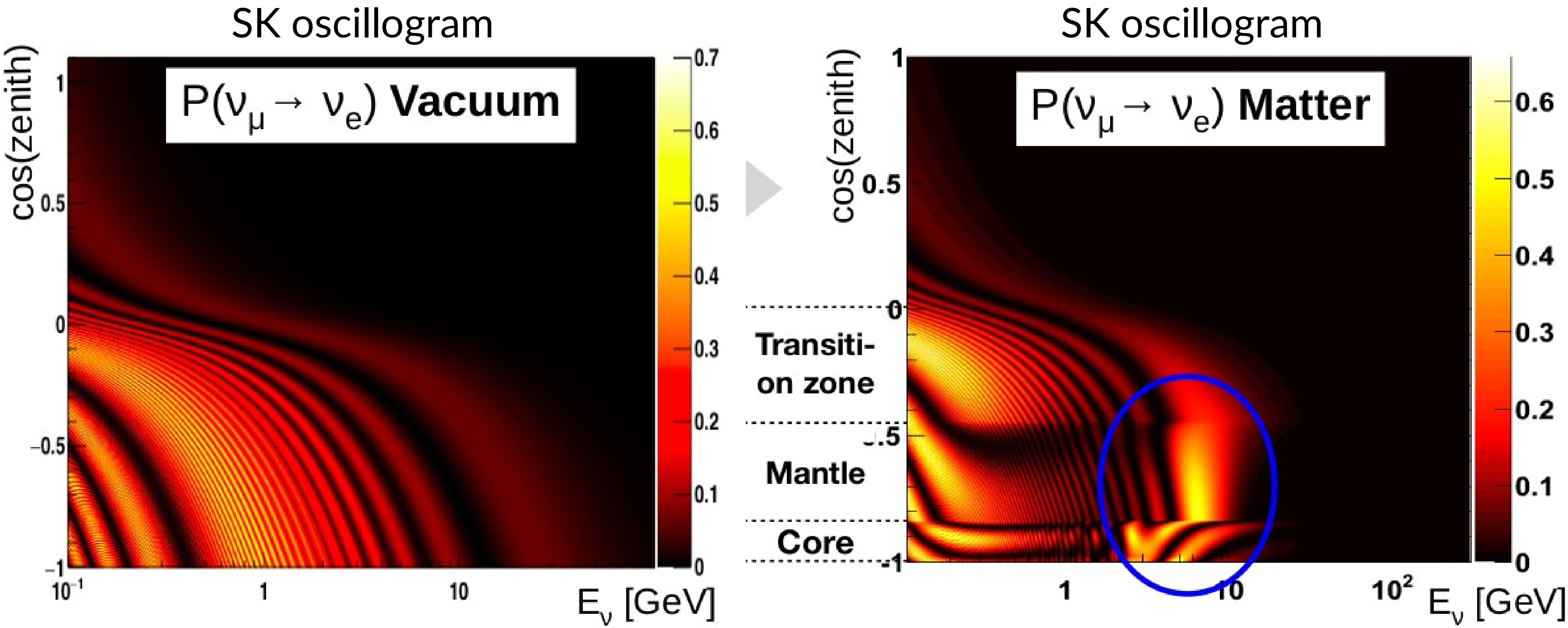
Why a joint analysis

- But, T2K has good sensitivity to mixing angle $\sin^2\theta_{23}$



Why a joint analysis

- Both experiments are sensitive to δ_{CP} from ν_e appearance
- T2K is not sensitive to mass ordering, but good constraint on δ_{CP}
- **SK has good constraint on mass ordering**, but barely on δ_{CP}: sees an average effect, due to energy resolution
  - T2K's $\sin^2\theta_{23}$ constraint helps reducing degeneracies in SK

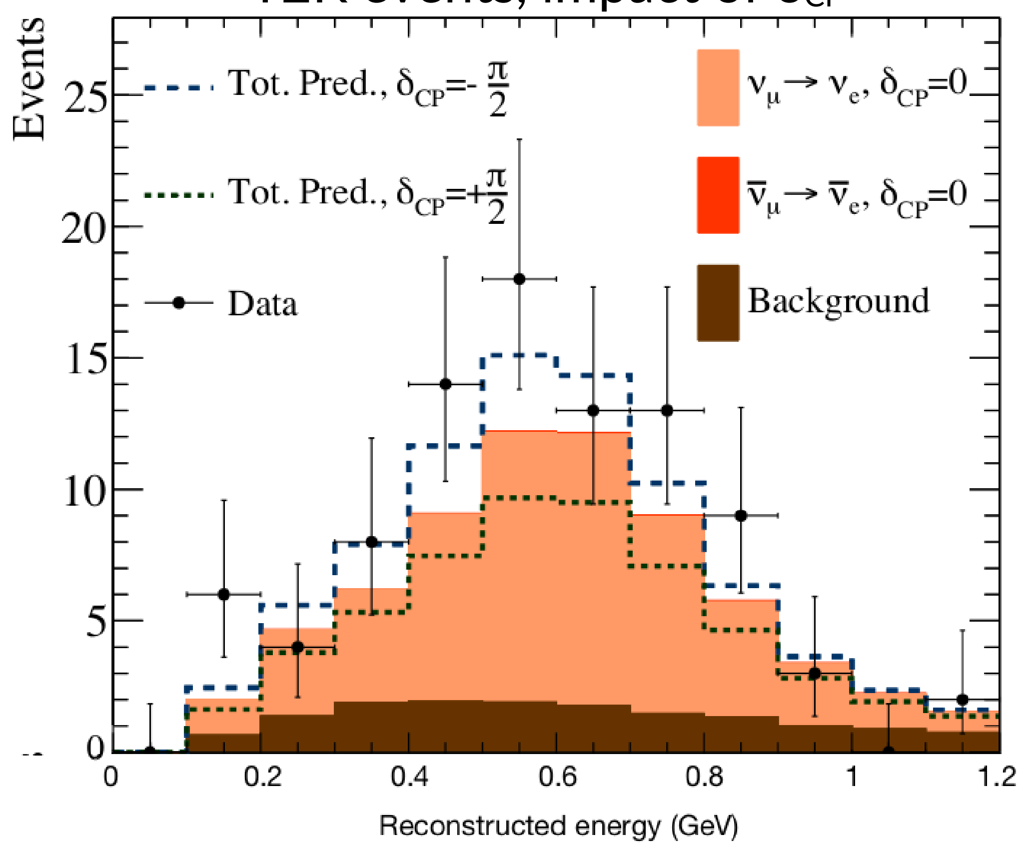


If **normal ordering**, **resonance** appears for **neutrinos**
If **inverted ordering**, **resonance** appears for **anti-neutrinos**

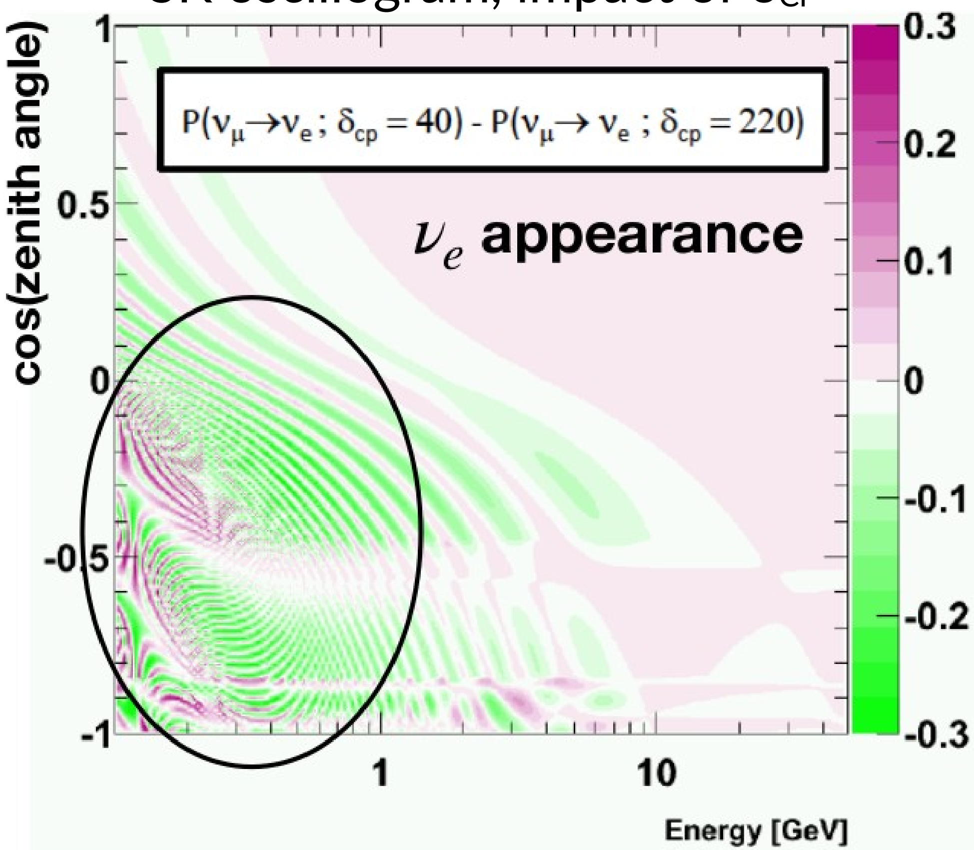
Why a joint analysis

- Both experiments are sensitive to δ_{CP} from ν_e appearance
- T2K is not sensitive to mass ordering, **but good constraint on δ_{CP}**
- SK has good constraint on mass ordering, **but barely on δ_{CP}**: sees an average effect, due to energy resolution
  - T2K's $\sin^2\theta_{23}$ constraint helps reducing degeneracies in SK

T2K events, impact of δ_{CP}

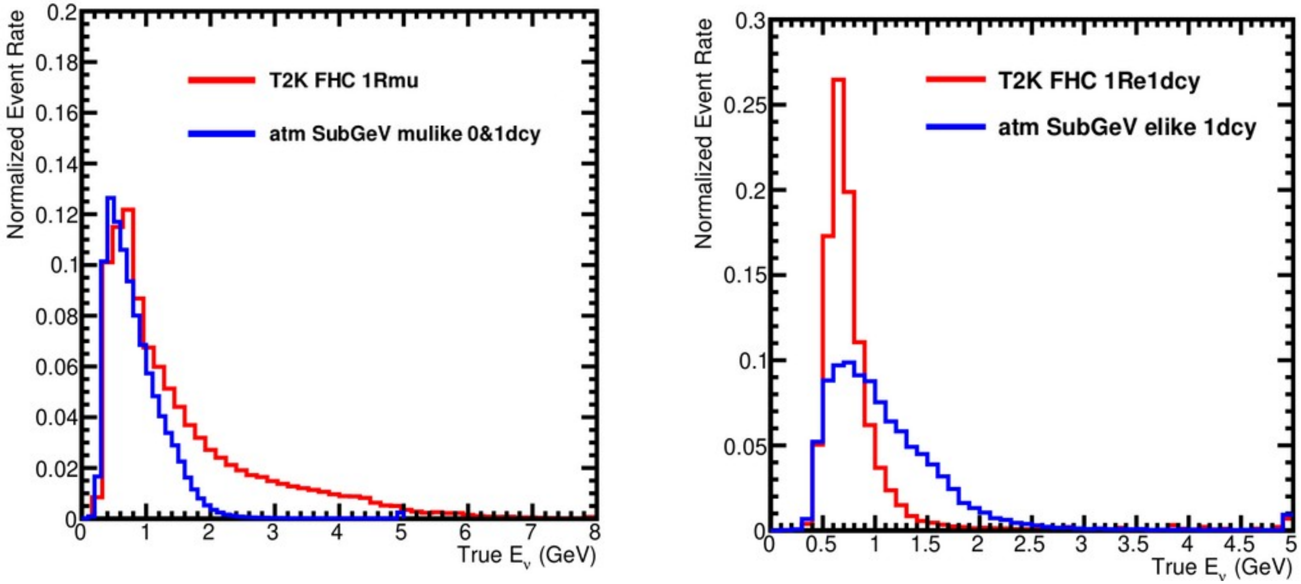


SK oscillogram, impact of δ_{CP}



Why a joint analysis

- SK sees multiple neutrino sources: here we use **atmospheric** neutrinos, and **beam neutrinos** from T2K



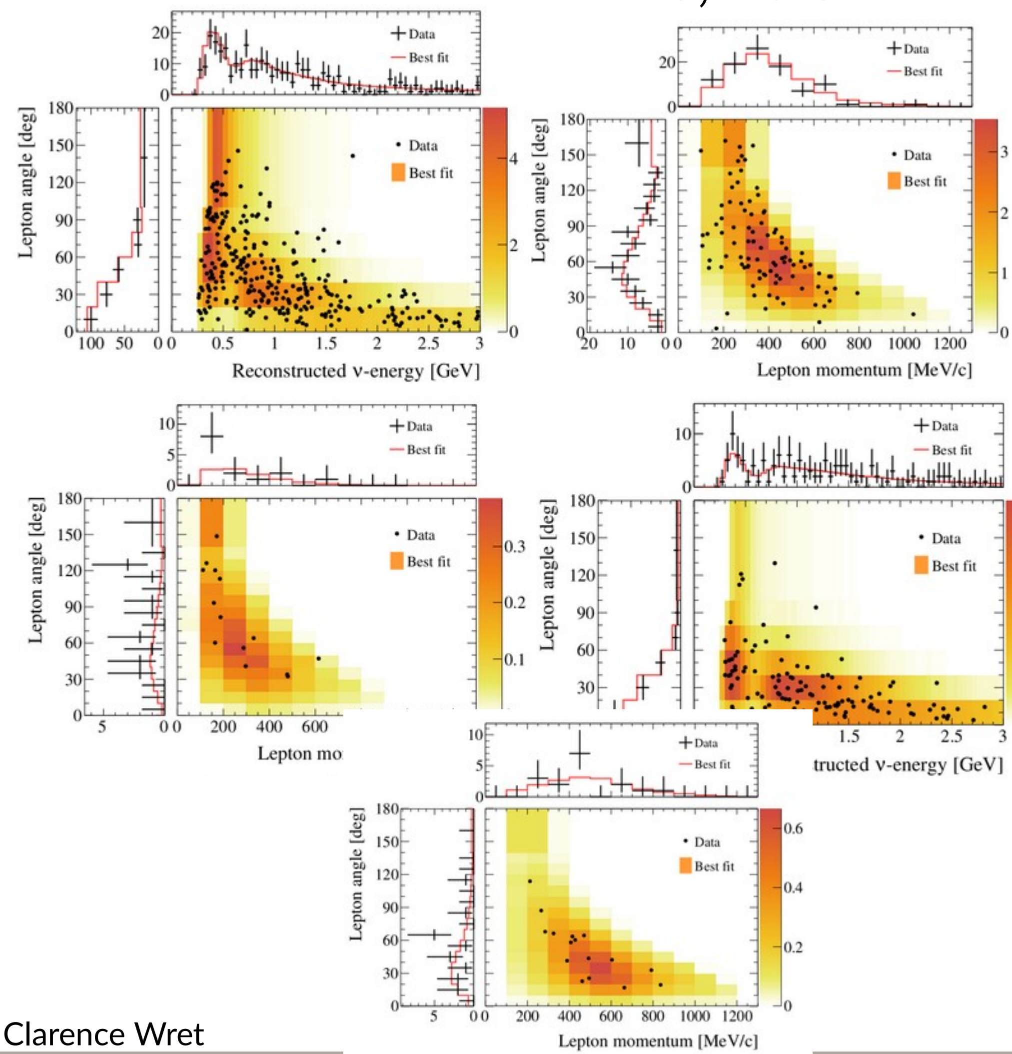
- Same detector, sometimes similar selections and fluxes
  - **Unify systematics and selections where possible**
  - Improved oscillation constraints through sharing systematics, and using high-statistics SK samples to inform T2K samples
  - Utilise high-statistics near-detector samples from T2K to constrain aspects of atmospheric selections: expose tensions
- Beam+atmospheric analysis may be required for Hyper-Kamiokande competitiveness with DUNE (depending on mass ordering and δ_{CP})

Selections

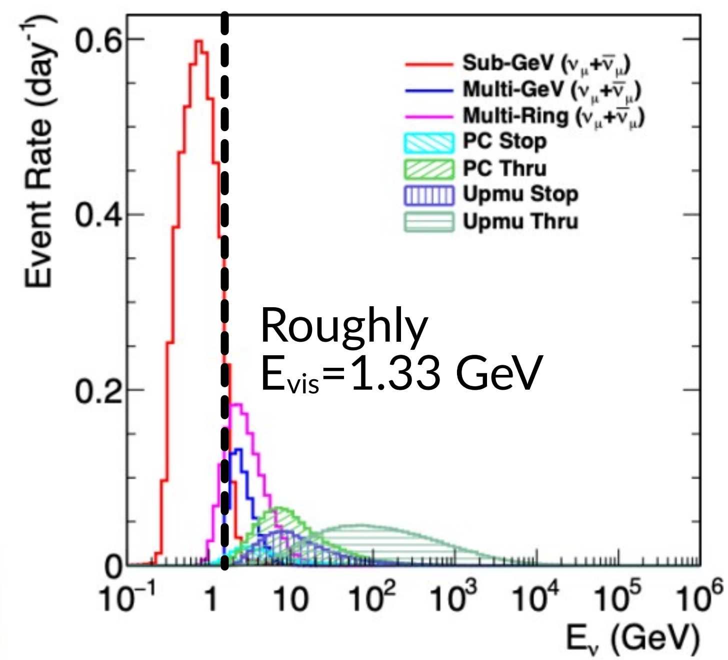
[Eur.Phys.J.C 83 (2023) 9, 782]

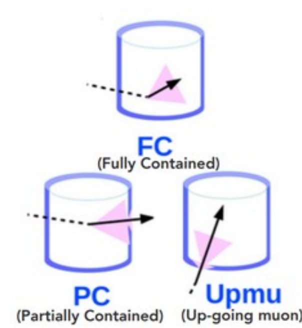
[PTEP 2019 (2019) 5, 053F01]

- T2K's 2020 analysis as basis
  - 5 samples: **single-ring** separated by **lepton flavour**, **Michel** electron, and **beam running mode**
  - POT: 19.7×10^{20} FHC, 16.3×10^{20} RHC



- SK's 2019 analysis as basis
  - **18 samples**, separated by **lepton flavour**, event **topology**, and **visible energy**
  - SK IV, before Gd-doping
  - 3244.4 days of atmospheric neutrino data





Shared systematics

- Utilise interaction model expertise from both experiments: unify low energy model and CCQE
- Apply T2K ND for relevant atmospheric selections
- Shared det. systematics
- No shared flux systematics

	Low-energy sub-GeV atm + beam	**High-energy** multi-GeV atm
CCQE	T2K model with ND280 constraint, correlated in low-E/highE (except for high-Q^2)	
	high-Q^2 params w/ND280	high-Q^2 params w/o ND
	add ν_e/ν_μ ratio unc. (CRPA)	
2p2h	T2K model w/ND280	SK model (100% error) + T2K-style shape
Resonant	T2K model w/ND280 + new pion momentum dial + NC1π0 uncertainties	SK model for 3 dials common with T2K, use more recent larger T2K priors
DIS	T2K model w/ND280	SK model
ν_τ	SK model (25% norm on top of other syst) for other systematics checked that we have no numerically unstable values	
FSI	T2K model w/ND280	T2K model w/o ND280 should be mostly same as SK model
SI	T2K model, correlated in low-E/high-E only applied to FC and PC for atm, PN not applied to atm	

Fake-data studies

- T2K uses "fake data" to gauge impact of missing interaction model features
  - How would a bias manifest if model X is true nature, but we fit it with our model
- Set "data" to be a model, redo near-detector analysis, propagate constraints from near detector to far detector, extract bias on oscillation parameters
- 14 different models tested: study impact on δ_{CP} and J, $\sin^2\theta_{23}$, mass ordering and Δm^2_{32} constraint
- Largest impact from Continuum Random Phase Approximation (CRPA) and the multiplicity of multi-pion events
  - Latest T2K analysis has uncertainties related to this, which we did not include in our analysis; hence a large impact
  - Smearing of Δm^2_{32} of 3.6×10^{-5} eV^2: **larger than overall syst uncertainty on Δm^2_{32}**

CRPA: [Phys. Rev. C 65, 025501], RPA: [Phys. Rev. C 83, 045501]

Results

Results

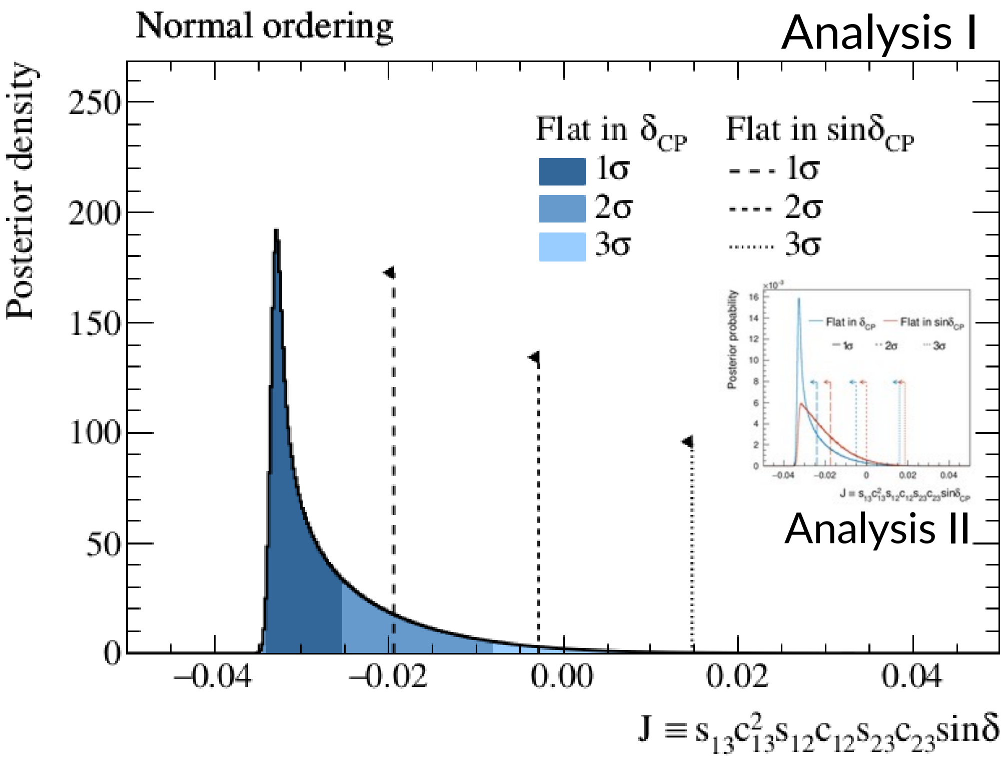
Reactor constraint on $\sin^2\theta_{13}$:
0.0218±0.0007 (PDG 2019)

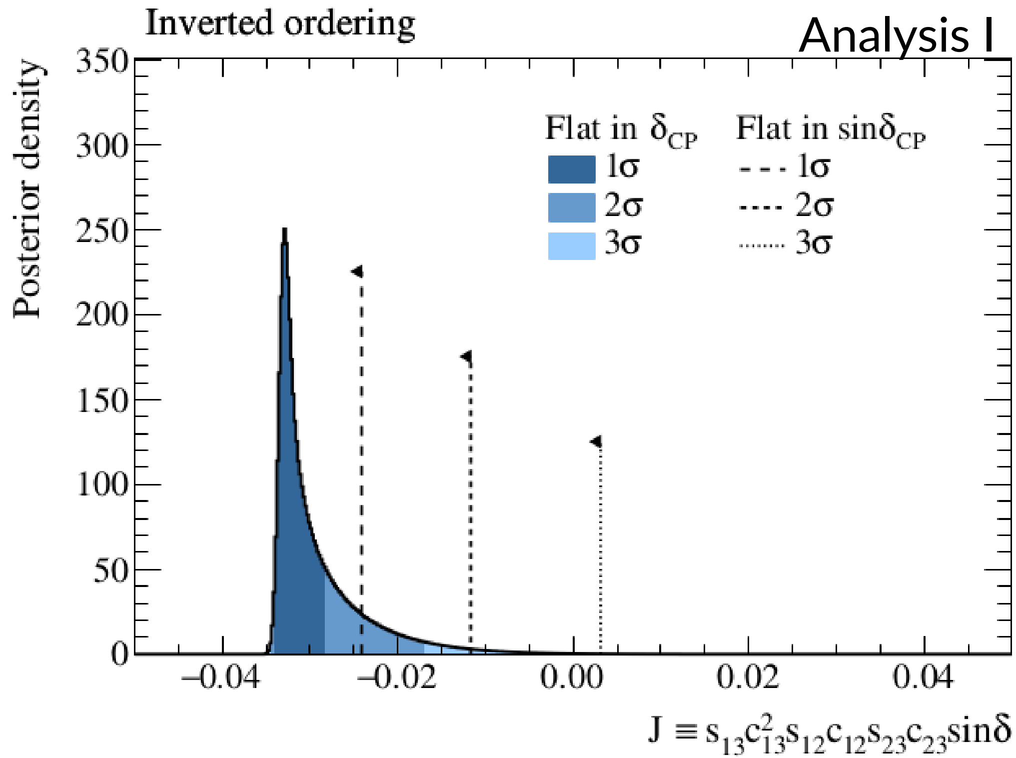
- Four analysis groups:
  - Two Bayesian MCMC analyses
  - One simplified frequentist analysis
  - SK's official frequentist analysis
- Here presenting results from the **two Bayesian MCMC** analyses, using different implementations

	Analysis 1	Analysis 2
Oscillation probability Systematic response	Binned	Event by event / Binned
T2K sample binning	$(E_{\mathrm{rec}}, \theta)$ for μ-like samples (p, θ) for e-like samples	(E_{rec}) for μ-like samples $(E_{\mathrm{rec}}, \theta)$ for e-like samples
T2K near detector constraint	Gaussian approximation (Sequential fit)	Full likelihood (Simultaneous fit)
Fast oscillation smearing	Semi-analytic averaging	Down-sampling finer to coarser grid
Earth density	Average density + deviations	Average density

Results, Jarlskog invariant

- If Jarlskog invariant is 0: no CP violation
- Test two choices of prior: flat in δ_{CP}, flat in $\sin\delta_{CP}$

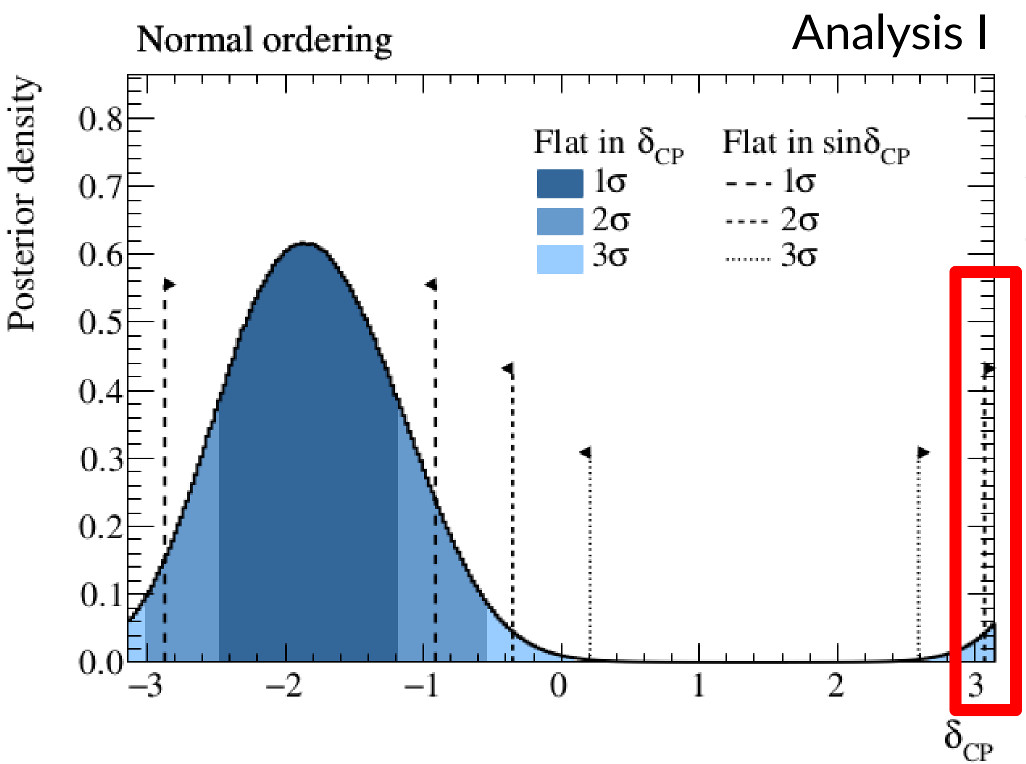


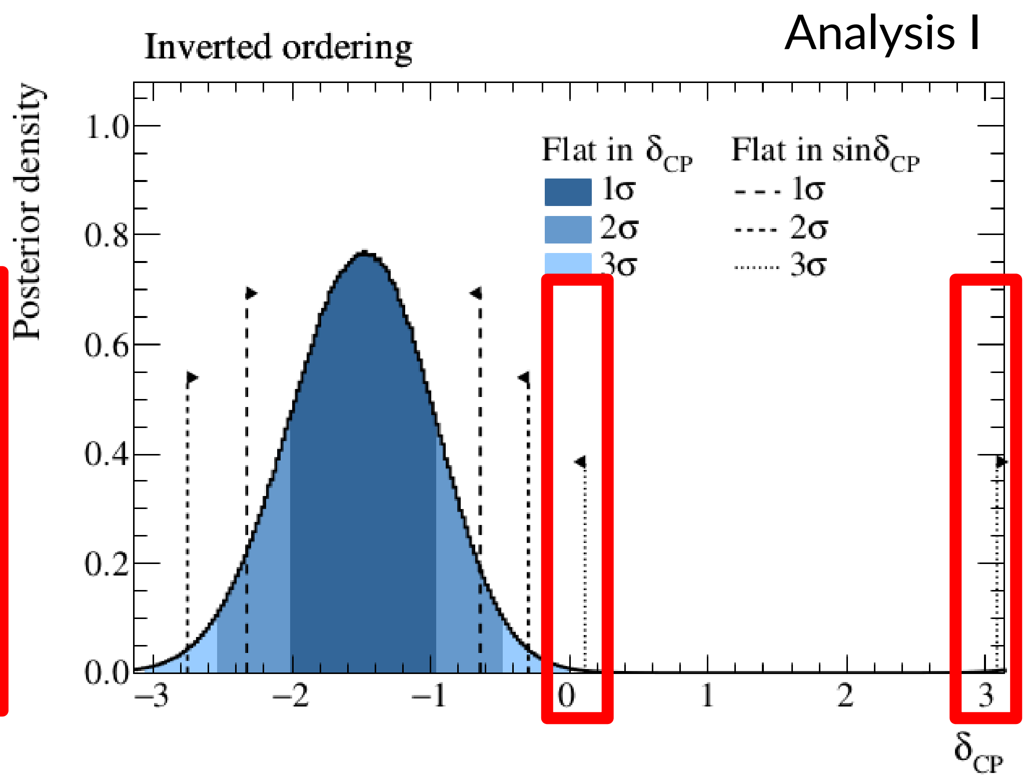


- >2σ exclusion of J=0 in normal ordering
- Nearly 3σ exclusion of J=0 in inverted ordering
- Similar (but weaker) exclusion for Analysis II

Results, CP-violating phase

- Similar results for δ_{CP} phase constraint
- $\delta_{CP}=\pi$ is just included in 2σ for normal ordering and a prior flat in $\sin\delta_{CP}$
- Inverted ordering nearly excludes $\delta_{CP}=0, \pi$ at 3σ for both prior choices





Summary table for CPV statements

Analysis	Variable	Prior	1σ	90%	2σ	3σ
Analysis 1	$\delta_{\rm CP}$	Flat in $\delta_{\rm CP}$	✓	✓	✓	×
		Flat in $\sin\delta_{\rm CP}$	✓	✓(×)	×	×
	$J_{\rm CP}$	Flat in $\delta_{\rm CP}$	✓	✓	✓	×
		Flat in $\sin\delta_{\rm CP}$	✓	✓	✓	×
Analysis 2	$\delta_{\rm CP}$	Flat in $\delta_{\rm CP}$	✓	✓	✓	×
		Flat in $\sin\delta_{\rm CP}$	✓	✓(×)	×	×
	$J_{\rm CP}$	Flat in $\delta_{\rm CP}$	✓	✓	✓	×
		Flat in $\sin\delta_{\rm CP}$	✓	✓	✓(×)	×

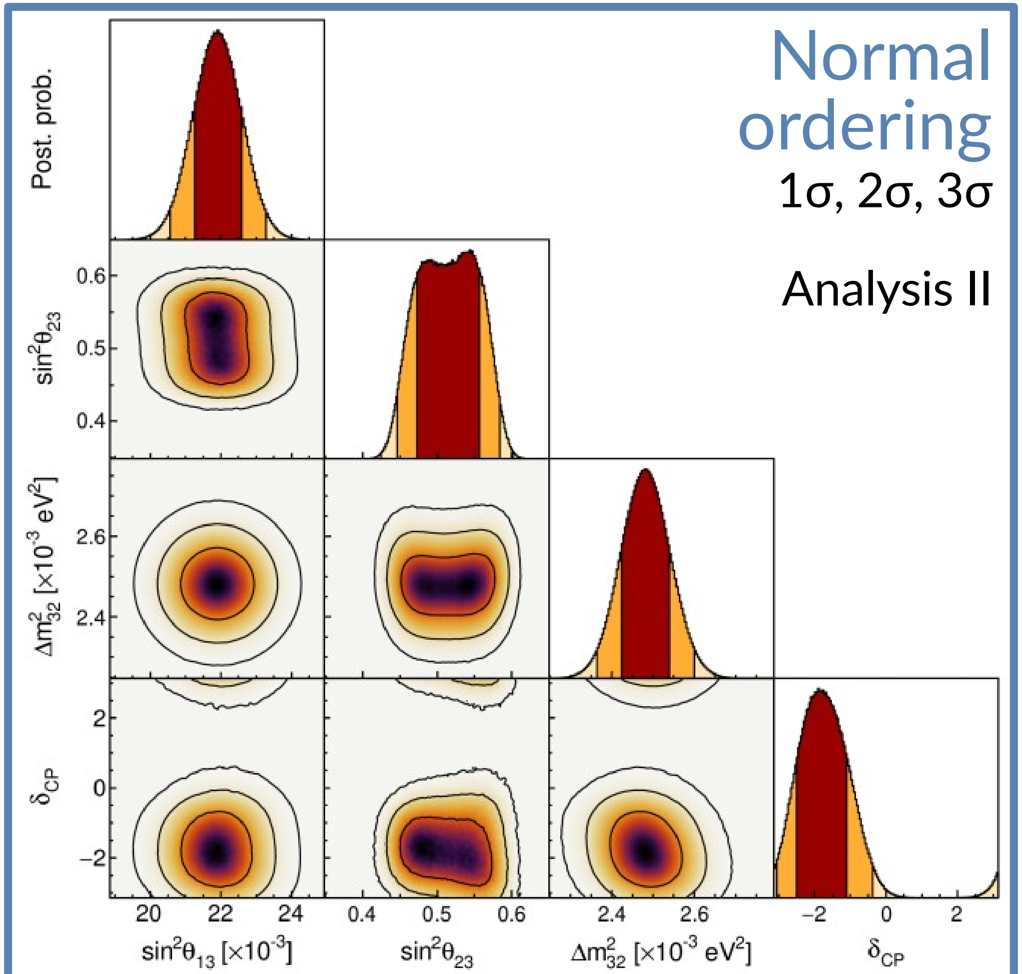
✓: excluded ×: not excluded

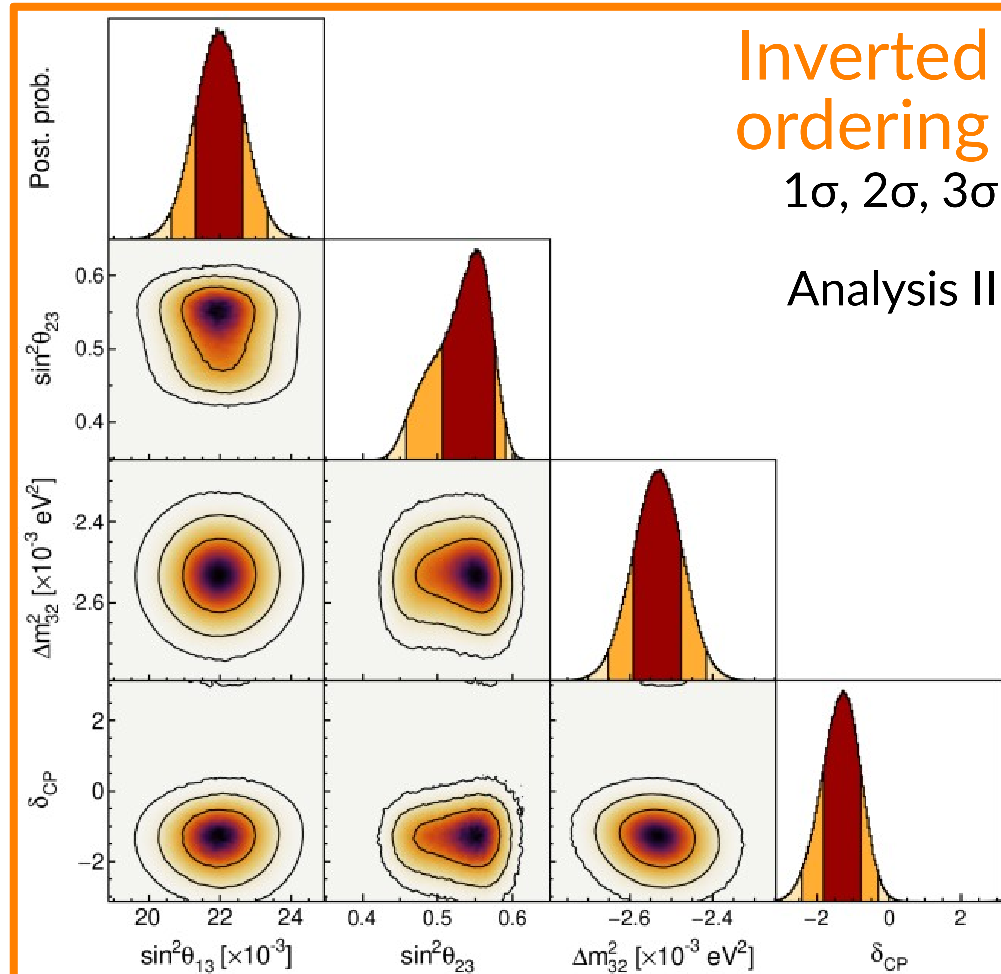
✓(×): excluded but may not be robust against the possible bias from an out-of-model effect

- 90% to 2σ exclusion of J=0 and δ_{CP}=0, π
- Dependent on prior choice, dependent on variable
- Analysis I and II are (mostly) consistent

Results, atmospheric

- Constraint on Δm^2 is weaker than T2K result due to fake-data studies
- Will improve with updated interaction modelling
- Normal ordering: weak upper octant preference
- Inverted ordering: stronger upper octant preference





Results, Bayes factors

- Express octant and ordering preferences as Bayes factors (ratios of posterior probabilities)

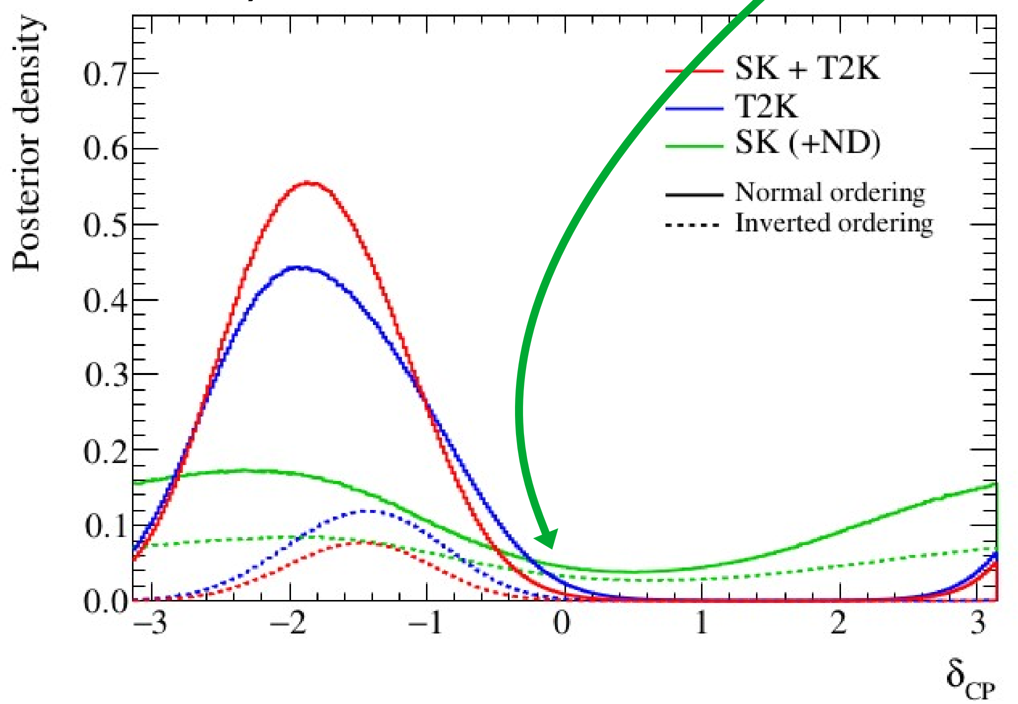
	T2K+SK		
	$\sin^2\theta_{23} < 0.5$	$\sin^2\theta_{23} > 0.5$	Line total
Normal ordering	0.367	0.533	0.900
Inverted ordering	0.022	0.078	0.100
Column total	0.389	0.611	1.000
MO Bayes factor $B(\mathrm{NO/IO})$		$\mathbf{8.98 \pm 0.06}$	
Octant Bayes factor $B(\mathrm{UO/LO})$		**1.57**	

- Moderate preference for normal ordering, weak preference for upper octant

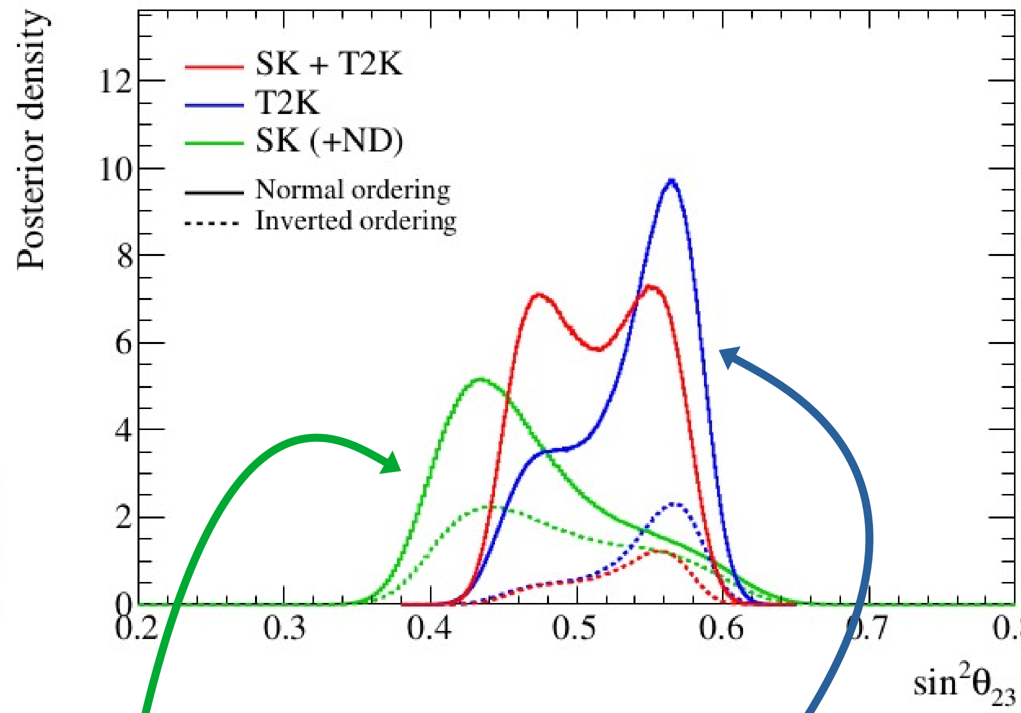
Results, comparing experiments

- T2K dominates δ_{CP} constraint, but SK has sizeable contribution around δ_{CP}=0

Analysis I



Analysis I

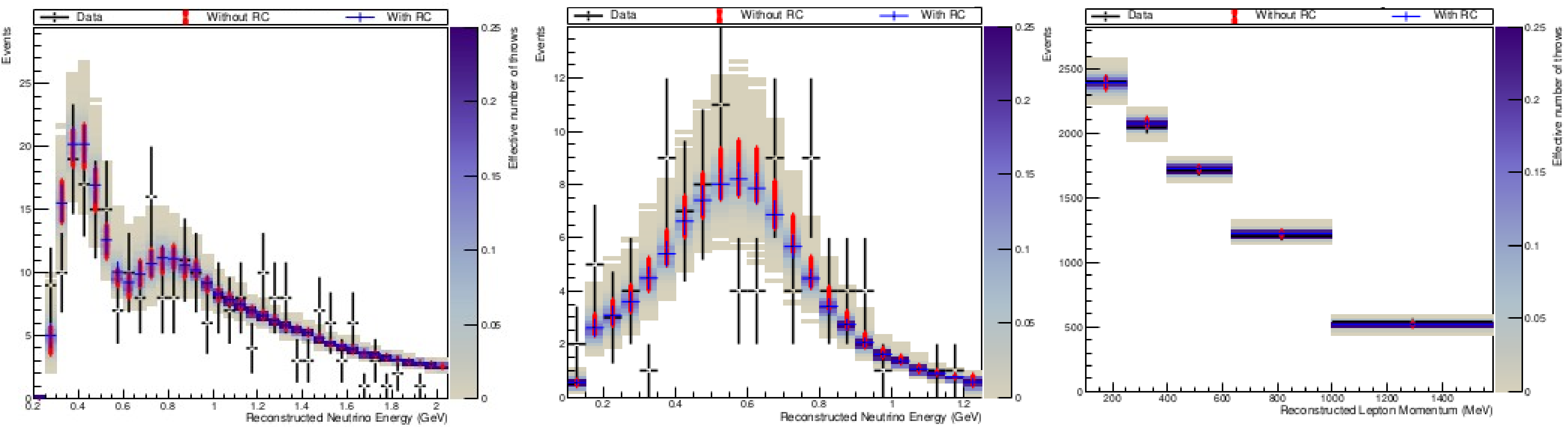


- Lower octant preferred by SK, upper octant preferred by T2K
  - Joint analysis has little octant preference

Results, p-values

- Construct posterior predictive distributions for all T2K and SK samples

T2K 1Rμ　　T2K 1Re　　SK Sub-GeV e-like 0de



- Can then construct Bayesian p-values for all T2K and SK samples
- Compatible p-values between analysis I and II, and with T2K 2020 results
  - p=**0.254** (shape), p=**0.202** (norm)

Future

- Writing short paper on oscillation analysis, expect soon!
- Long paper on method and model developments, including full oscillation result
- Two complementary frequentist oscillation analyses underway, one being the official SK atmospheric analysis
  - Will do Feldman-Cousins confidence intervals, and CL_s
- Interest from both collaborations to pursue another analysis
  - Have begun studying impact of more SK atmospheric data (SK I-III and later) and T2K beam data (still have another 1.7x to collect!)
  - Scope to deeper investigate flux correlations, develop near-detector selections targeted at atmospheric selections
  - … your ideas here!

Summary

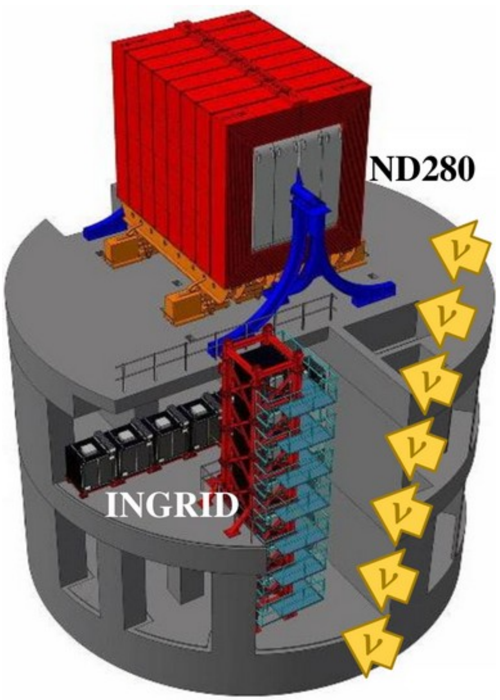
- Official simultaneous analysis between SK atmospheric and T2K beam neutrinos complete
  - First analysis to deep-dive into shared systematics!
- Numerous benefits: lifting oscillation parameter degeneracies, correlating systematics, sharing knowledge
  - A necessary exercise for future Hyper-Kamiokande experiment
- Teasing on 2σ exclusion of $J=0$; exclusion of CP violation between 90% and 2σ
- Preference for normal ordering, weak preference for upper octant

- Stay tuned for papers!

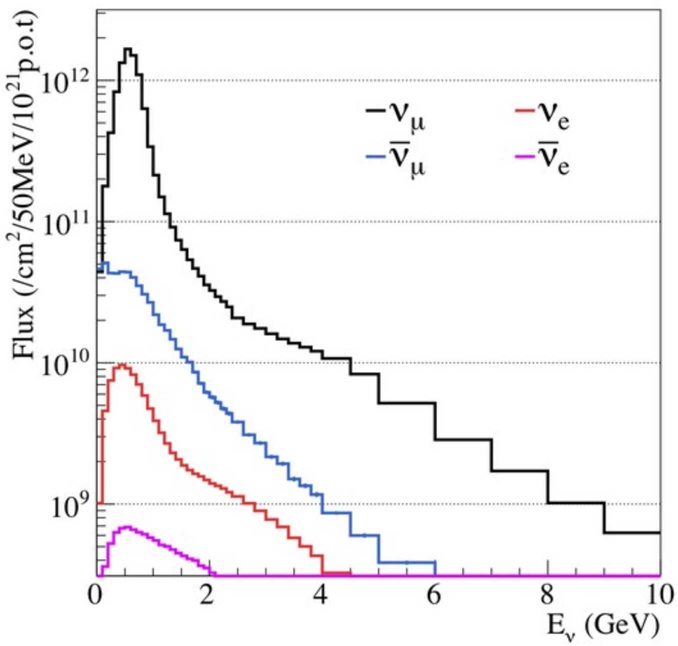
Backups

The T2K near detectors

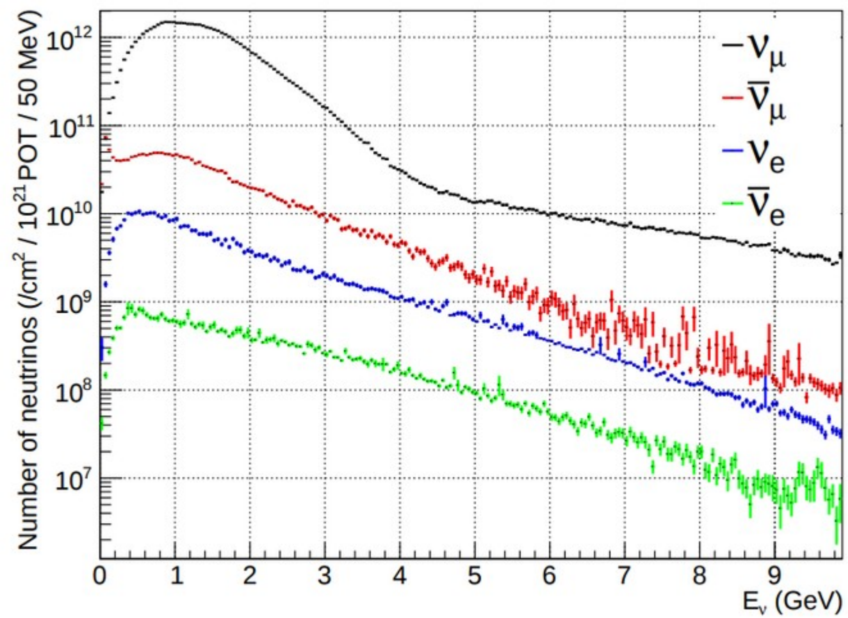
- Fluxes: ν_μ and anti-ν_μ dominated with different E_ν
  - ND280: 2.5° off-axis, 0.6 GeV narrow band – used in OA
  - INGRID: on-axis, 1.3 GeV wide band – used for monitoring



ND280 flux (off-axis)



INGRID flux (on-axis)



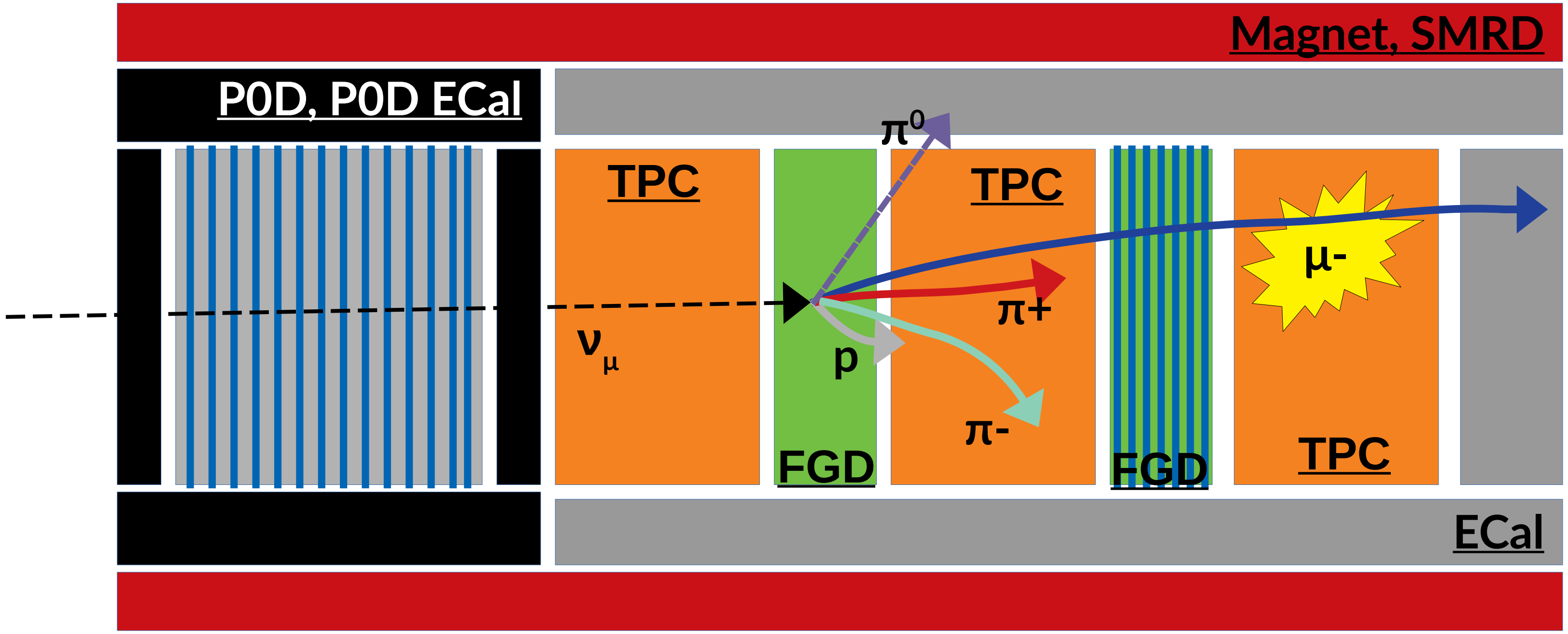
- Multiple targets in INGRID and ND280: C_8H_8, H_2O, Ar, Pb, Fe
- More detectors rolling into the ND280 pit, e.g. WAGASCI/BabyMIND, NINJA, proton and water modules

JINST 12 (2017) PTEP 2021 4 (2021) Phys. Rev. D 102, 072006 (2020)

The ND280 near detector

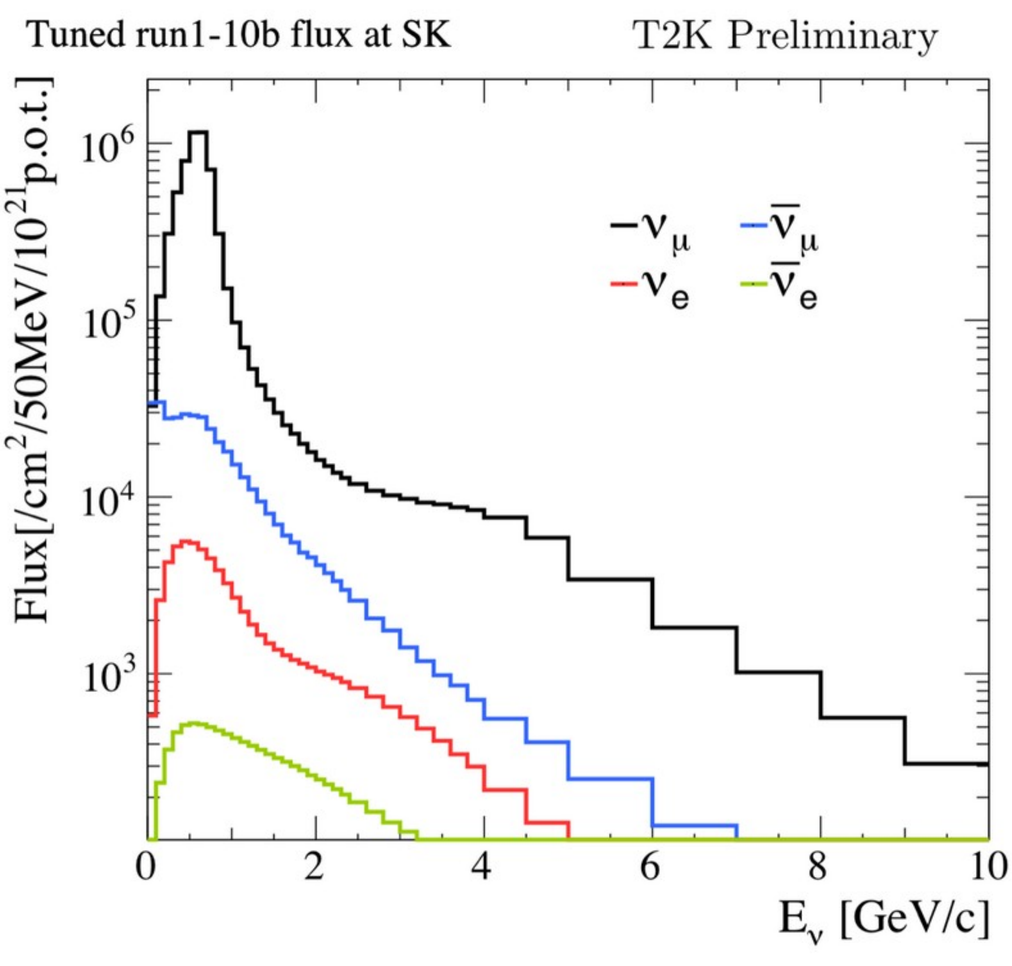
- Oscillation analysis utilises the FGD+TPC selections
  - Use FGD1 (CH) and FGD2 (CH, $\mathbf{H_2O}$) to constrain neutrino flux and interaction cross-section
  - Water target important, as it's the target in SK

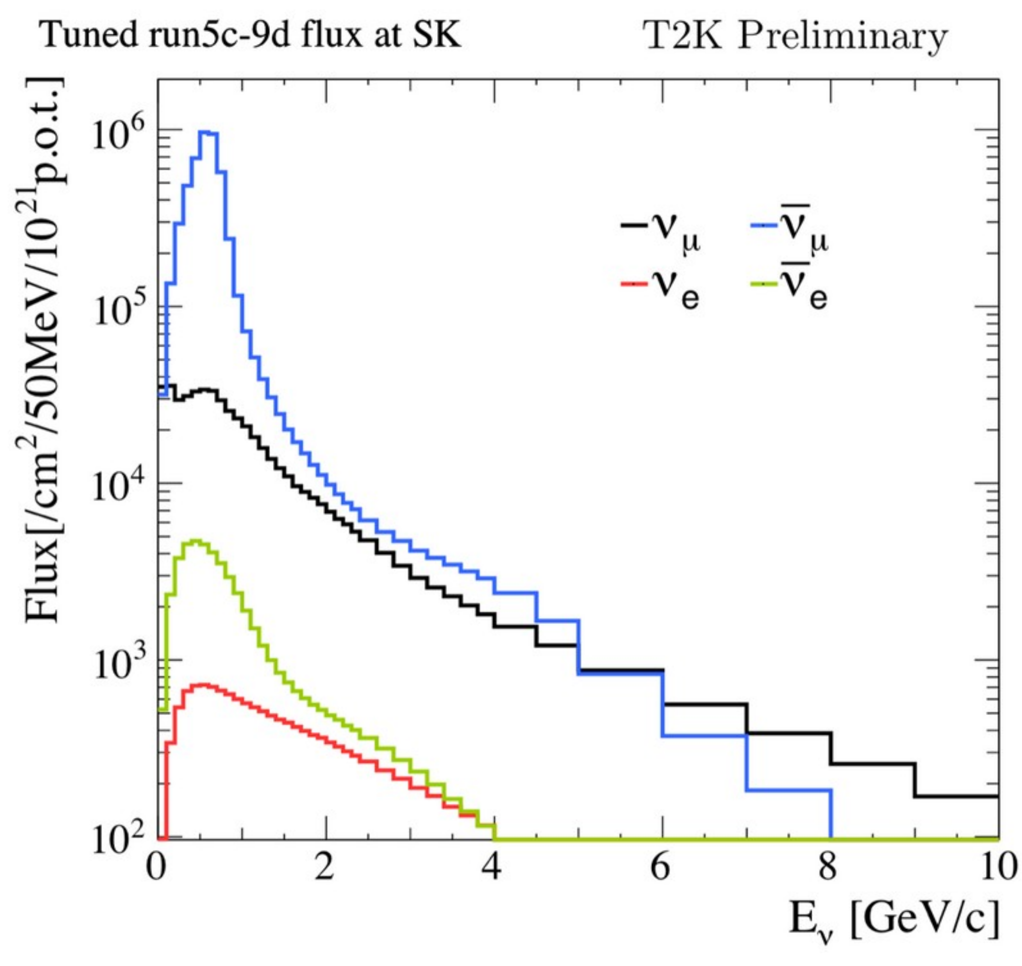
ND280 side-view



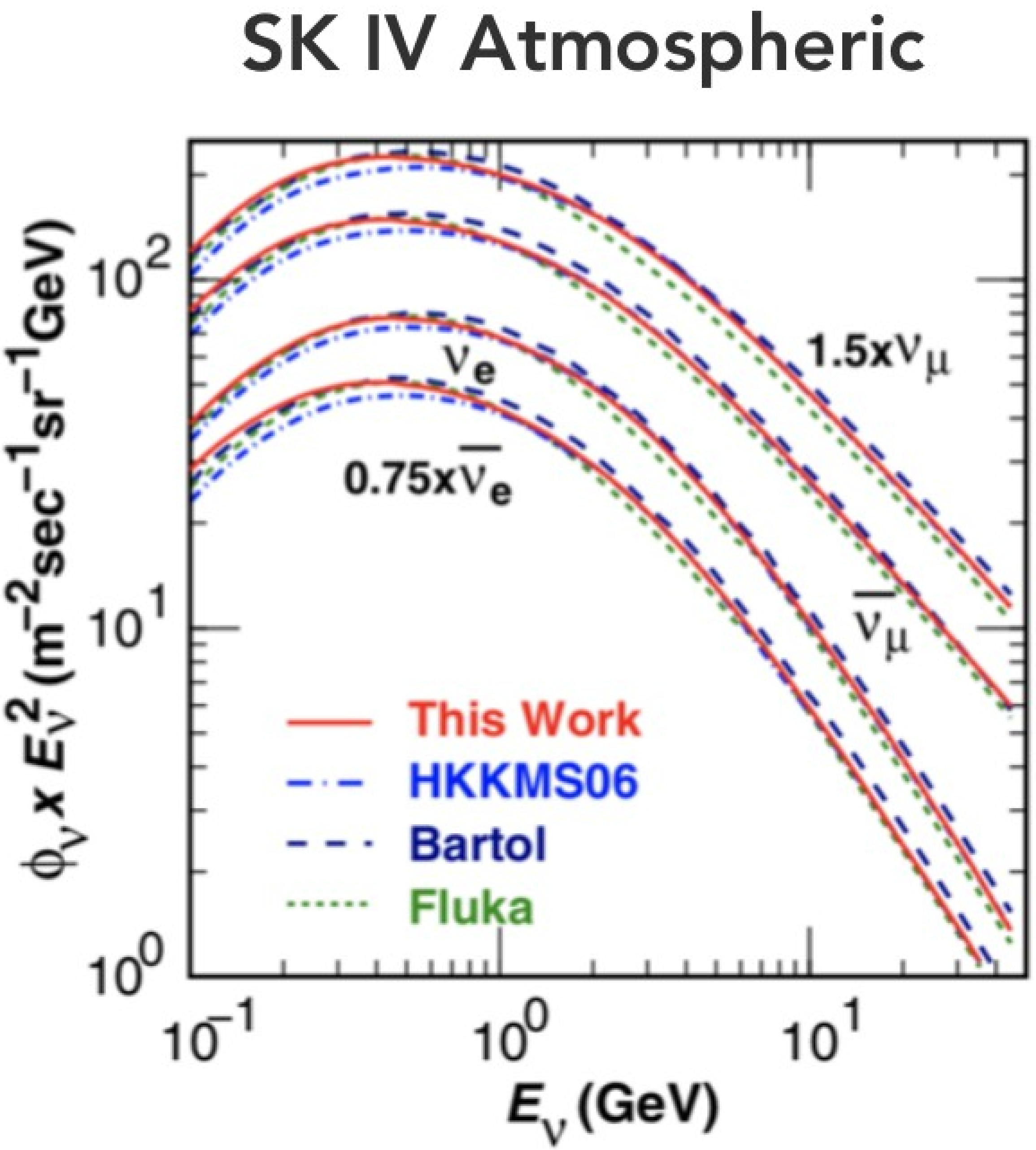
- Sign selection, ~8% MIP resolution in TPC; 0.2% μ/e confusion
  - Can constrain wrong-sign backgrounds in-situ

Flux at T2K SK

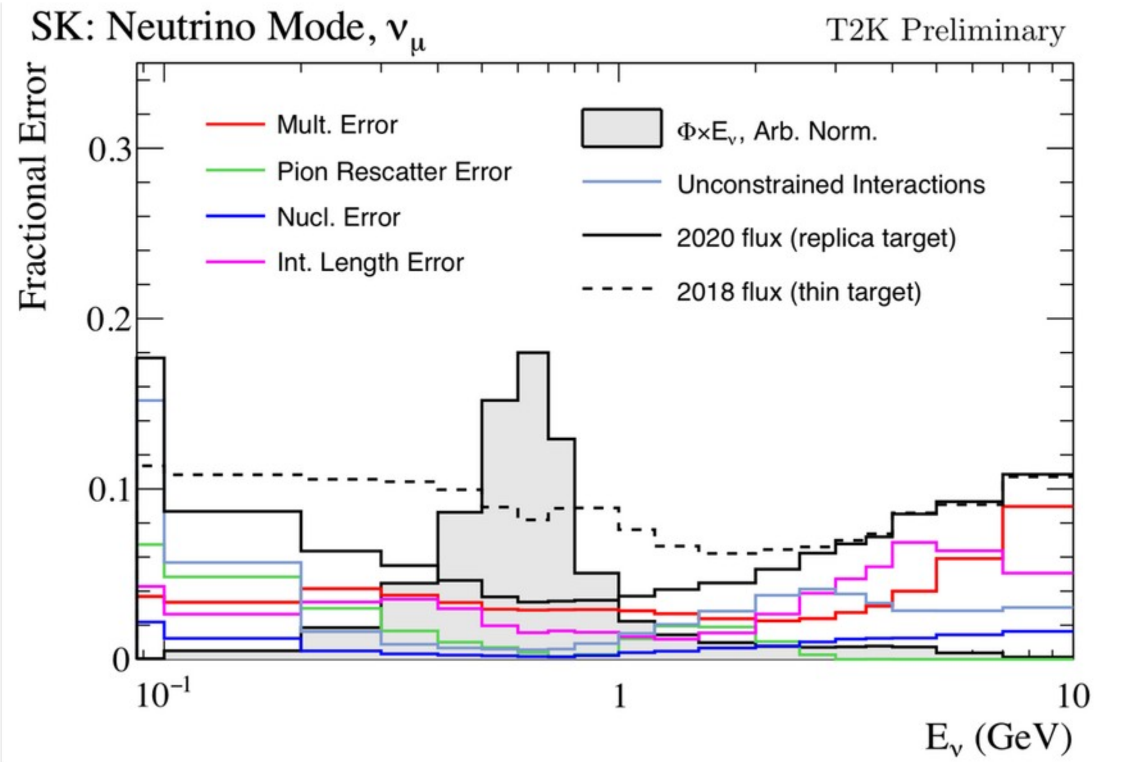


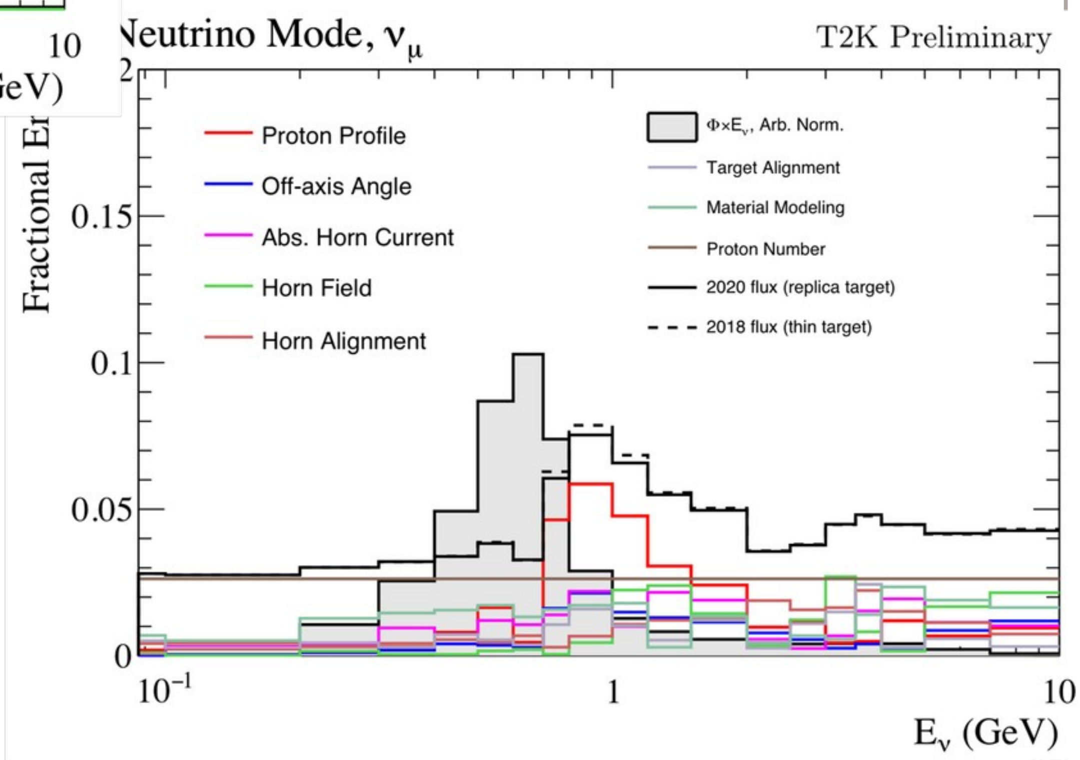


Flux at SK atmospheric



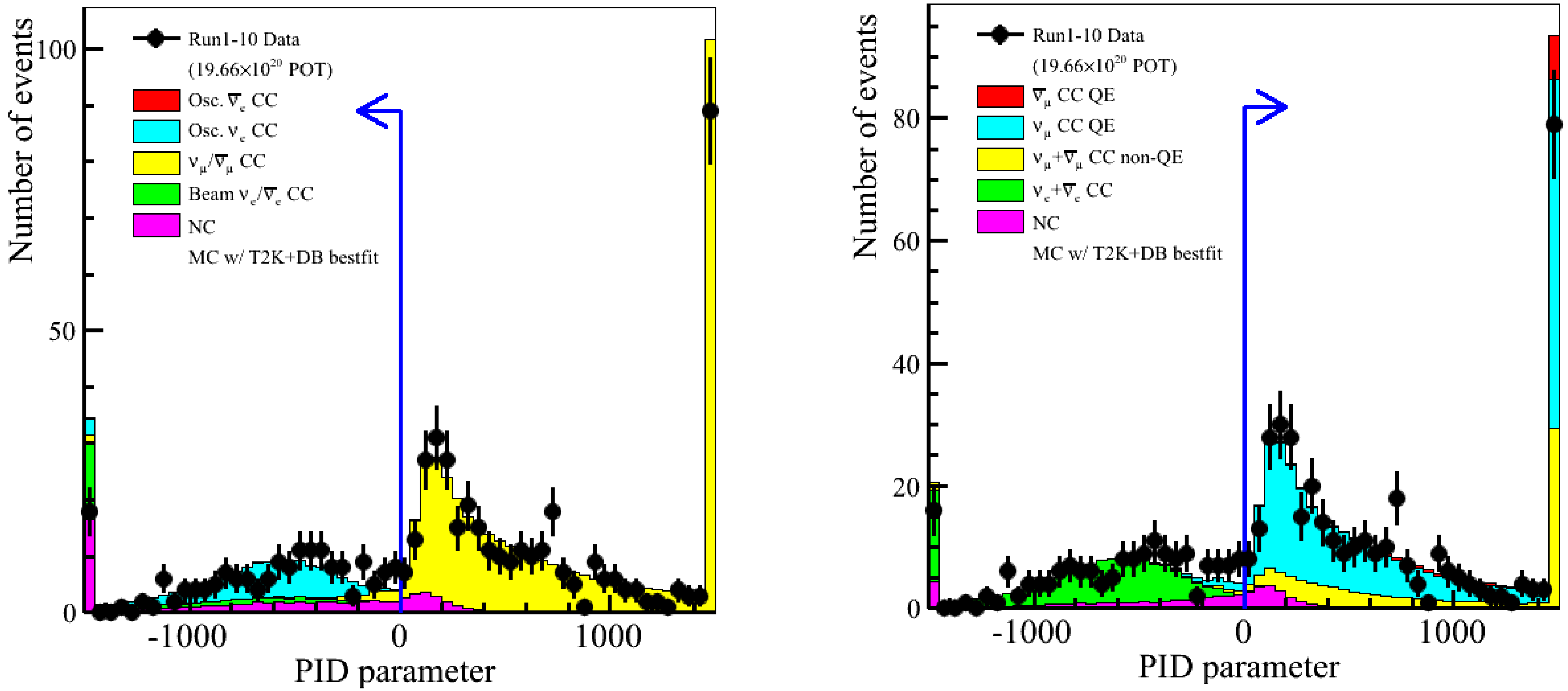
T2K flux uncertainties





The SK detector

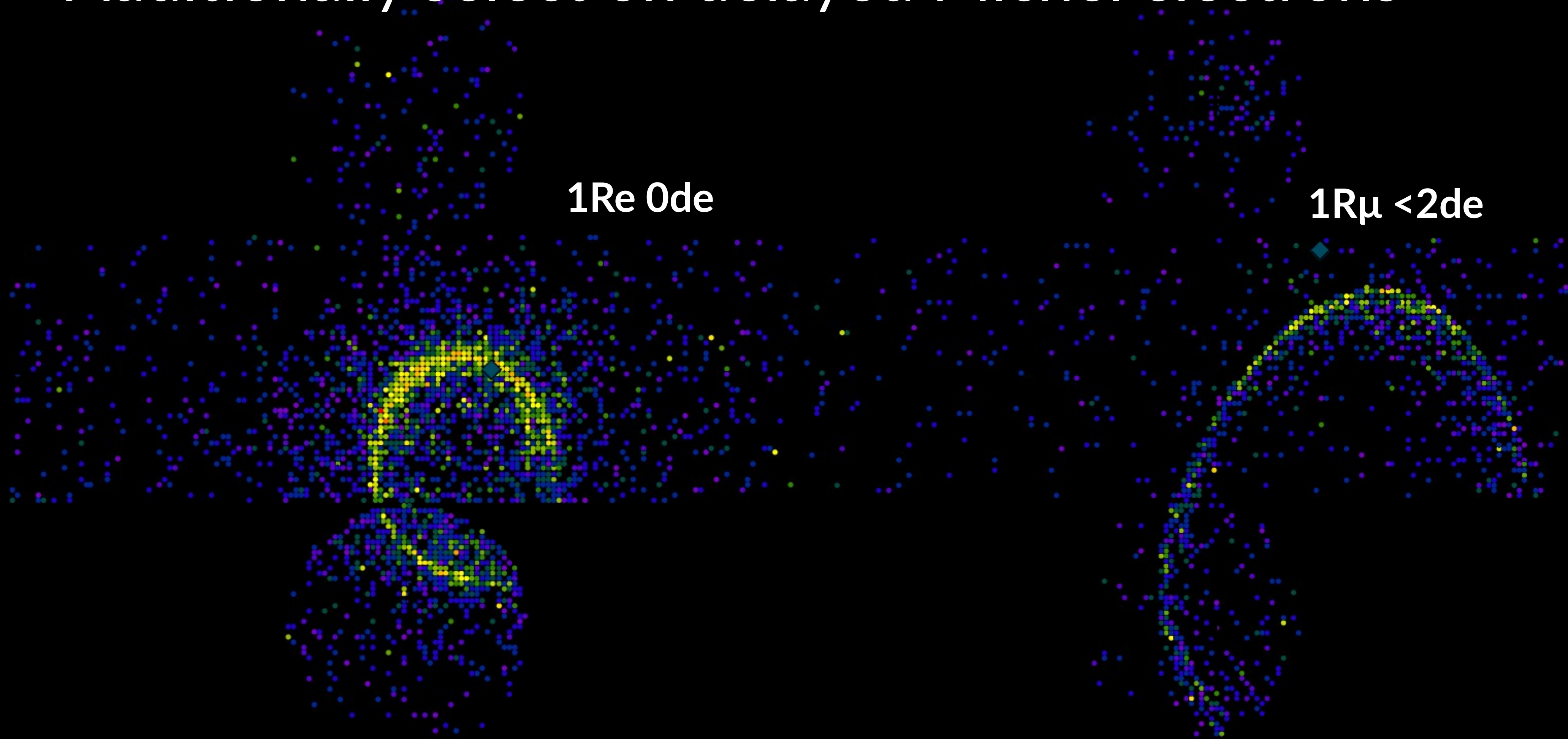
- Excellent μ/e separation: <1% mis-assign e as μ
- Reconstruction simultaneously fits all PMT hits, inspired by MiniBooNE



- Runs a multi-Cherenkov ring reconstruction, down-selects to single ring, and runs dedicated single ring fitter
  - Select number of rings and delayed Michel electrons
  - This analysis selects **single ring events**

The SK detector

- Cherenkov ring shape (sharp vs fuzzy) chiefly determines μ vs e
- Additionally select on delayed Michel electrons

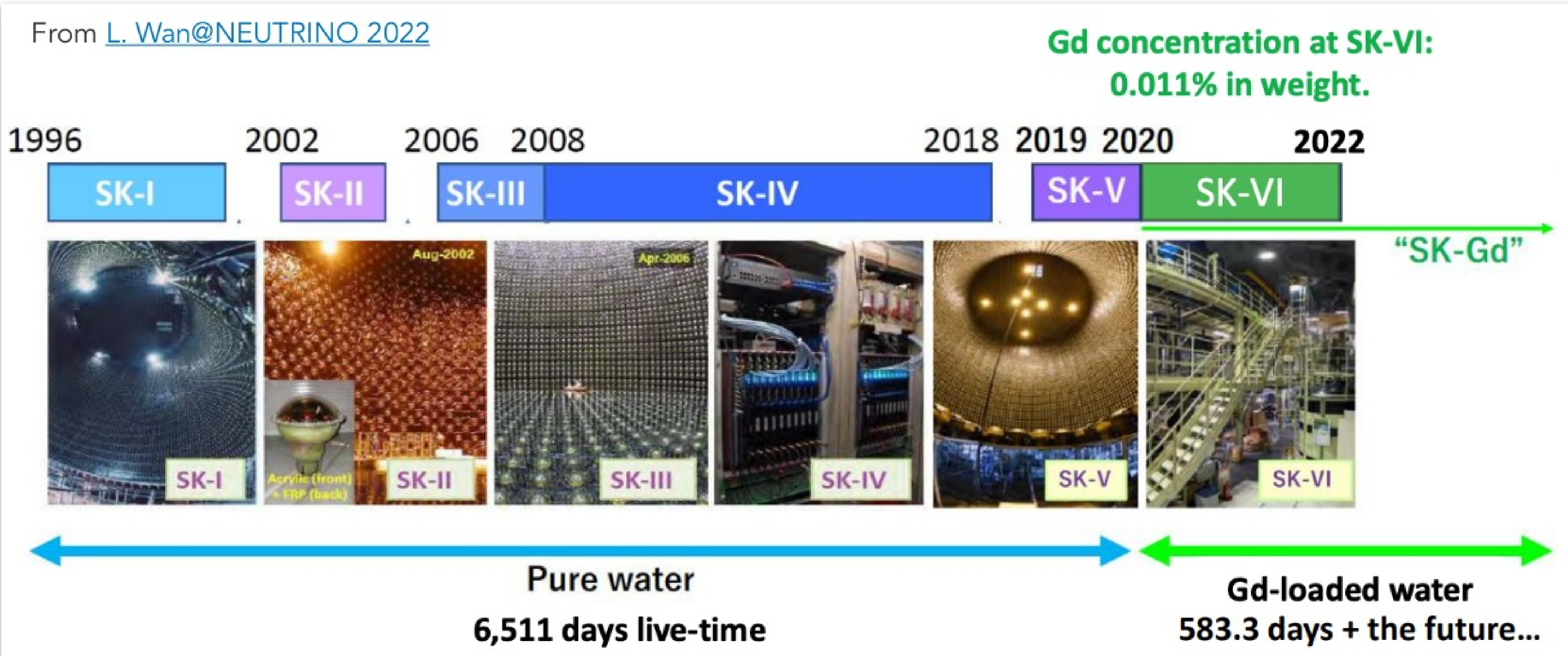


MoU

- SK and T2K signed memorandum of understanding (MoU) in late 2019
- Pursue joint oscillation analysis of SK atmospheric and T2K beam neutrinos
- Official effort from both experiments, with bi-weekly meetings and active consulting of experts

- **MoU** set out to use existing experiment techniques but also modify analyses **under supervision of experts** when necessary
- The analysis is **not just a statistical combination**, but leverages strengths of both experiments, e.g.
  - Use T2K's near-detector to constrain neutrino interaction model for SK atmospheric selections
  - Share parts of the interaction model where appropriate and feasible
  - Unify reconstruction and simulation of SK's beam and atmospheric neutrinos
  - Use high statistics SK atm. samples to understand features in T2K selections, e.g. 1Re1de and SubGeV e-like 1de
  - Develop earth model for neutrino oscillations
  - And many more!

SK running periods



Octant flip with(out) reactor constraint

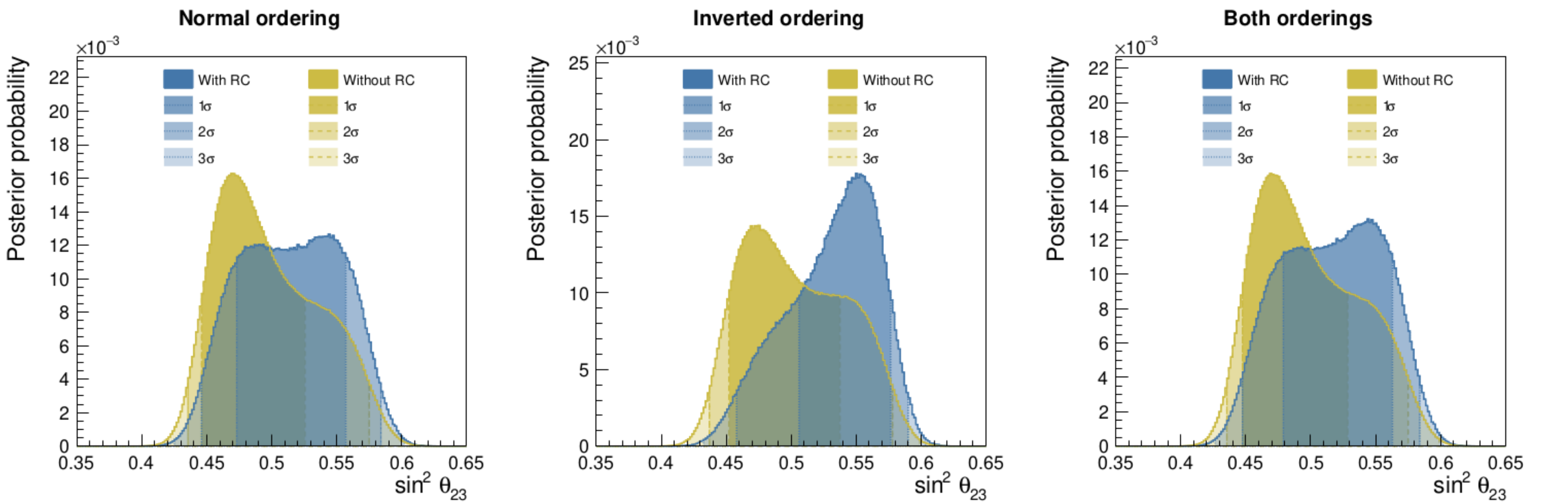


Figure 117: $\sin^2\theta_{23}$ from real data fit with (blue shaded) and without (yellow shaded) reactor constraint applied, for normal (left), inverted (center) and both (right) orderings.

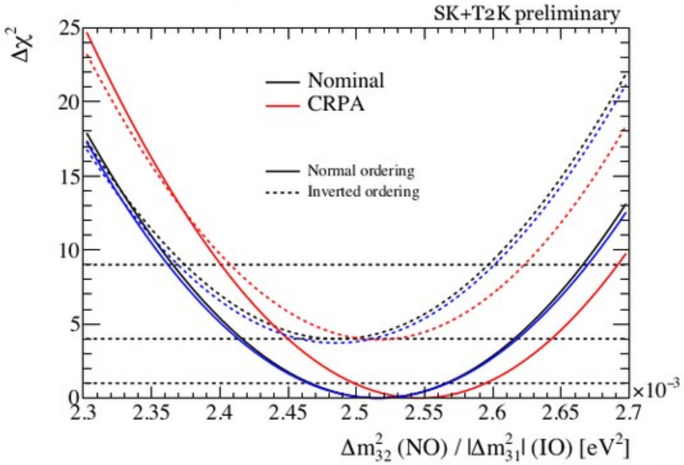
Bayes factors for each experiment

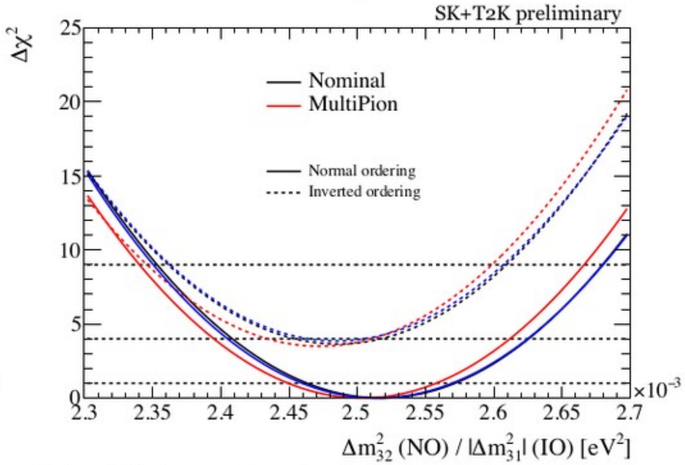
	T2K+SK			T2K			SK (+ND)		
	$\sin^2\theta_{23} < 0.5$	$\sin^2\theta_{23} > 0.5$	Line total	$\sin^2\theta_{23} < 0.5$	$\sin^2\theta_{23} > 0.5$	Line total	$\sin^2\theta_{23} < 0.5$	$\sin^2\theta_{23} > 0.5$	Line total
Normal ordering	0.367	0.533	0.900	0.190	0.642	0.832	0.468	0.186	0.654
Inverted ordering	0.022	0.078	0.100	0.025	0.142	0.168	0.214	0.132	0.346
Column total	0.389	0.611	1.000	0.215	0.785	1.000	0.682	0.318	1.000
MO Bayes factor $B(\mathrm{NO/IO})$	$\mathbf{8.98 \pm 0.06}$			$\mathbf{4.96 \pm 0.02}$			$\mathbf{1.886 \pm 0.008}$		
Octant Bayes factor $B(\mathrm{UO/LO})$	**1.57**			**3.65**			**0.47**		

FDS list

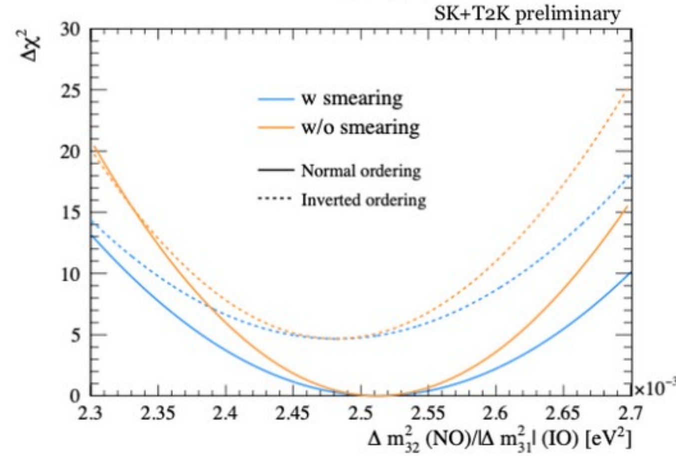
	Model component
Martini 2p2h	2p2h
ND280 data-driven pion kinematics	CC1π
CC0π non-QE alteration	CC0π
Removal energy	Nuclear Model
Axial form factors	CCQE
Pion SI bug fix	CC1π, CC$n\pi$
LFG	Nuclear model
CRPA	Nuclear model
Pion multiplicity	CC$n\pi$
Energy-dependent $\sigma_{\nu_e}/\sigma_{\nu_\mu}$	$\sigma_{\nu_e}/\sigma_{\nu_\mu}$
Xsec-only fit	Fit
Atmospheric down-going CC1π	CC1π
Atmospheric full-zenith CC1π	CC1π
No-migration energy scale fit	Fit

Example of the fake data fit results that showed large biases





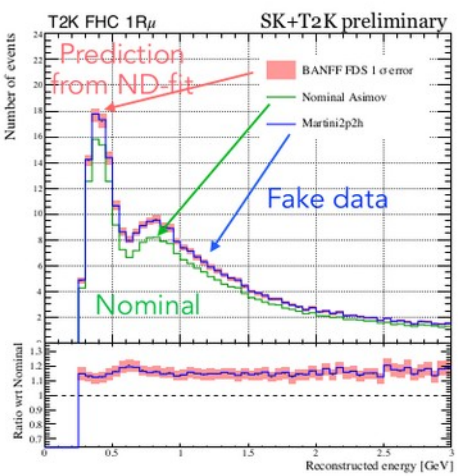
Gaussian smearing applied on data

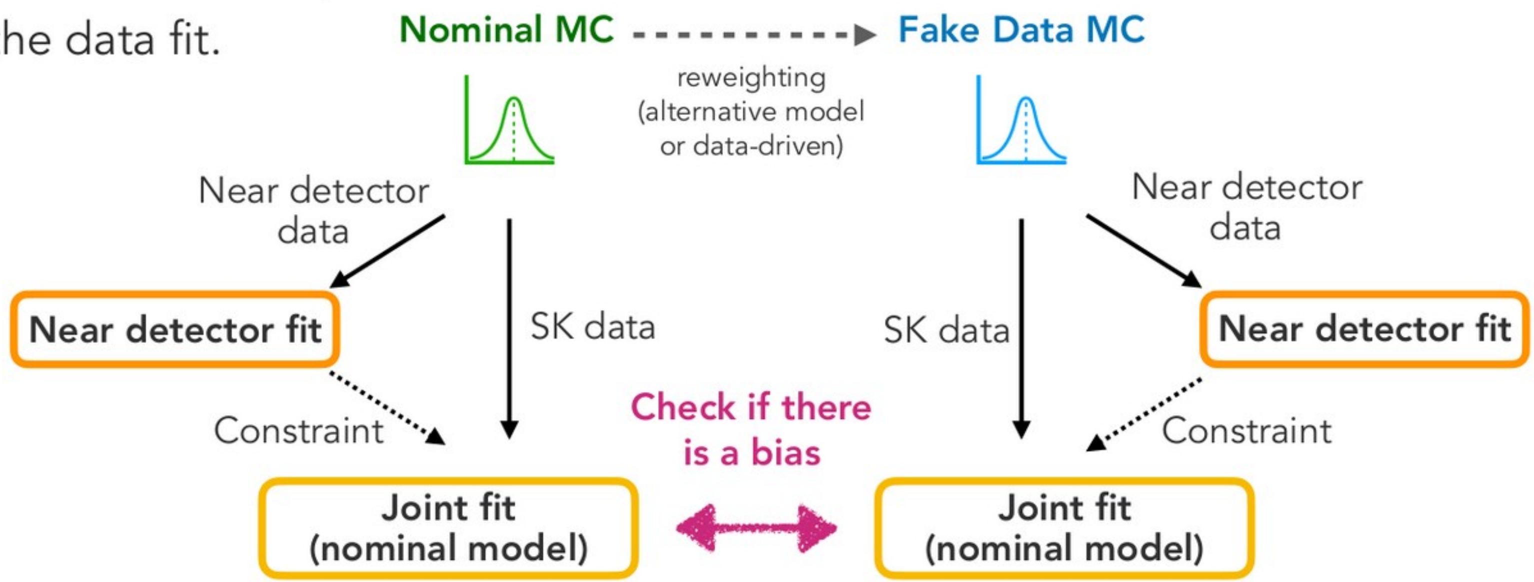


*Here $\Delta\chi^2$ = 1,4,9 lines are shown but it does not guarantee the correct coverage

FDS procedure

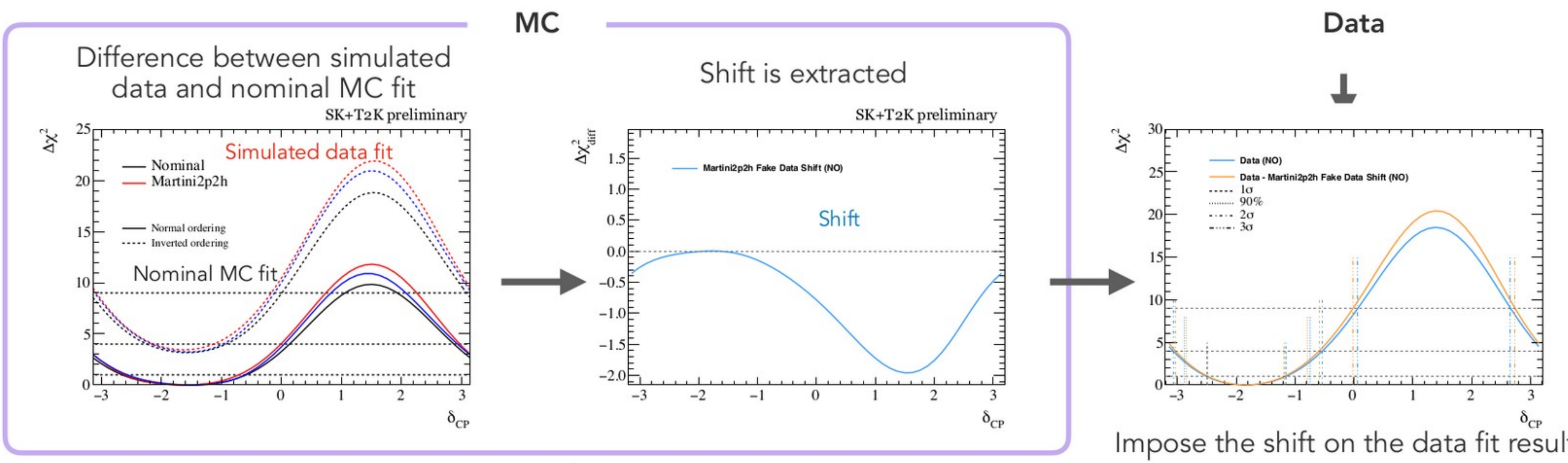
- We evaluate the possible bias in the oscillation parameter measurement due to the possible mis-modeling.
  - Generate a simulated data set using an alternative model and fit it with our nominal model.
  - If there is a significant bias, we update our model with additional systematics or apply smearing on the oscillation parameter.
  - This first step is done based on MC, before performing the data fit.





FDS procedure

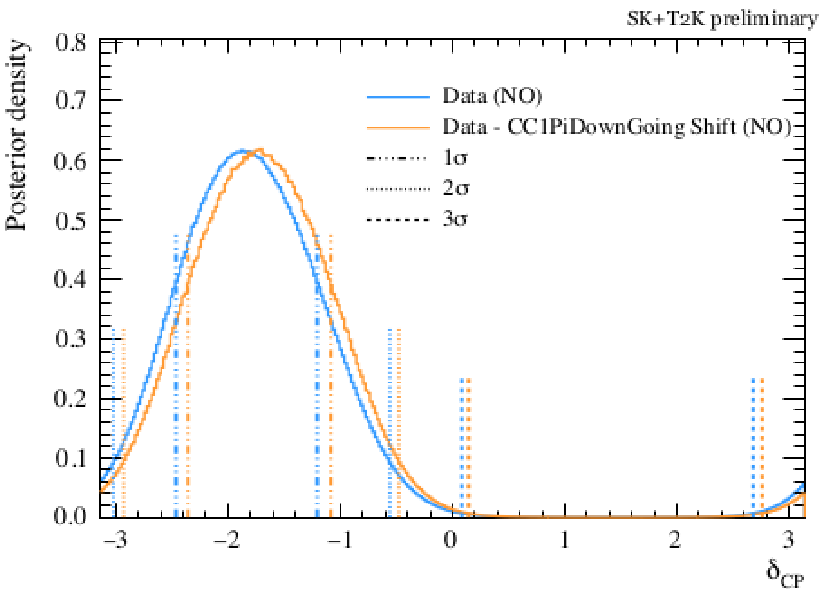
- The second step of the robustness test is done after the data fit.
  - We take the difference between nominal fit and simulated data fit results.
  - Impose this shift to the data fit to see if the bias in the interval edges can change our conclusion.
  - This effect is tested on δ_{CP} and Jarlskog invariant (relevant to our CP statement).

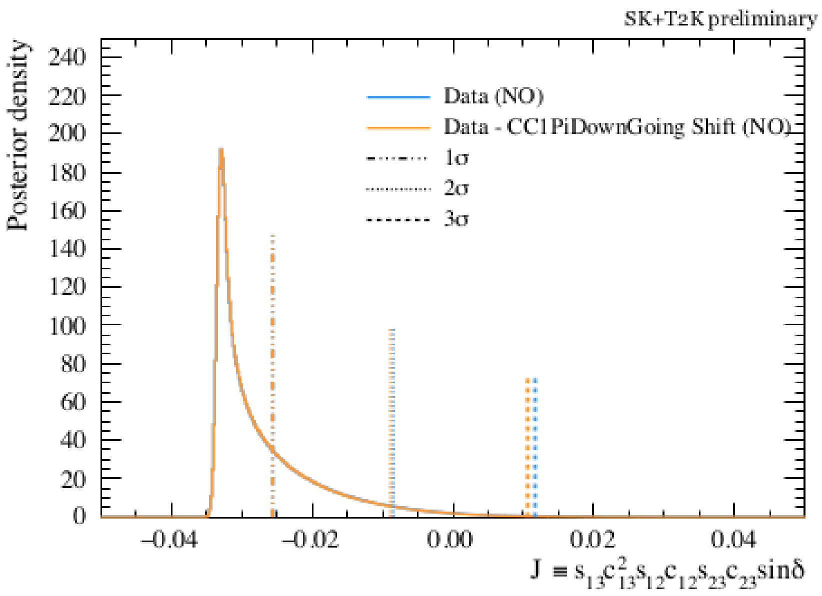


*Here $\Delta\chi^2 = 1,4,9$ lines are shown but it does not guarantee the correct coverage

FDS procedure

- We also tested whether it can change our conclusion on the significance of CP violation.
  - The size of the shift in the credible interval edges of δ_{CP} and Jarlskog invariant was checked.
  - None of them caused a shift of 2σ interval edges over the value of interest ($\delta_{\text{CP}} = 0, \pi$, $J_{\text{CP}} = 0$)
- Therefore it does not change our conclusion on CP violation around 2σ.





FDS procedure

Fake data	Reference	2σ interval ratio to Nominal $R_x^{2\sigma}$			Bias in the middle of 2σ interval $B_x^{\text{syst.}}$			Mass ordering
		δ_{CP}	Δm^2_{32}	$\sin^2\theta_{23}$	δ_{CP}	Δm^2_{32}	$\sin^2\theta_{23}$	Bayes factor ratio
Martini2p2h	Scaled Asiomv	0.972	0.989	0.982	-1.29%	-8.03%	-3.89%	1.09
DataDrivenPion	Normal Asimov	1.02	1	0.964	0.501%	18.4%	-4.57%	1.01
nonQECC0pi	Normal Asimov	0.948	0.904	0.995	-5.57%	-20.3%	-10.6%	1.00
Eb15MeV	Normal Asimov	1.01	1	1	1.49%	-22.5%	3.54%	0.96
Upper3CompCCQE	Scaled Asiomv	0.998	0.995	0.978	-1.22%	7.49%	-0.944%	0.97
PionSI	Normal Asimov	0.997	1.02	0.985	2.05%	44.8%	-0.91%	1.00
LFG	Normal Asimov	0.969	0.988	0.926	2%	38.4%	-4.76%	1.12
CRPA	Normal Asimov	0.991	0.969	1.02	-4.85%	154%	-10.5%	0.99
XSecOnly	Normal Asimov	1.01	0.989	0.991	5.02%	17%	-2.54%	1.10
MultiPion	Normal Asimov	1	0.999	1.02	2.35%	-63.5%	-4.53%	0.90
NueNumu	Scaled Asiomv	0.988	0.994	0.992	0.0401%	-5.82%	1.72%	1.08
CC1PiDownGoing	Normal Asimov	0.963	0.998	1	15.2%	-11%	8.45%	1.18
CC1PiFullZenith	Normal Asimov	0.961	0.998	1.01	-9.5%	-1.25%	12.8%	0.99
NoMigration	Normal Asimov	1.02	0.992	1.01	4.59%	-3.09%	0.943%	1.00

FDS procedure

		δ_{CP}	Δm^2_{32}	$\sin^2\theta_{23}$
AsimovA	Middle of the 1σ interval	-1.622	0.002517	0.5241
	1σ interval size: $1\sigma_{\text{tot.}}$	1.036	5.422e-05	0.04426
	1σ stat-only interval size: $1\sigma_{\text{stat.}}$	0.8142	4.983e-05	0.03771
	$1\sigma_{\text{syst.}} = \sqrt{(1\sigma_{\text{tot.}})^2 - (1\sigma_{\text{stat.}})^2}$	0.6403	2.138e-05	0.02318
	Middle of the 2σ interval	-1.639	0.002517	0.5185
	2σ interval size: $2\sigma_{\text{tot.}}$	1.773	0.0001085	0.07194
	2σ stat-only interval size: $2\sigma_{\text{stat.}}$	1.48	9.953e-05	0.06269
	$2\sigma_{\text{syst.}} = \sqrt{(2\sigma_{\text{tot.}})^2 - (2\sigma_{\text{stat.}})^2}$	0.9749	4.33e-05	0.03528

Highest posterior probability

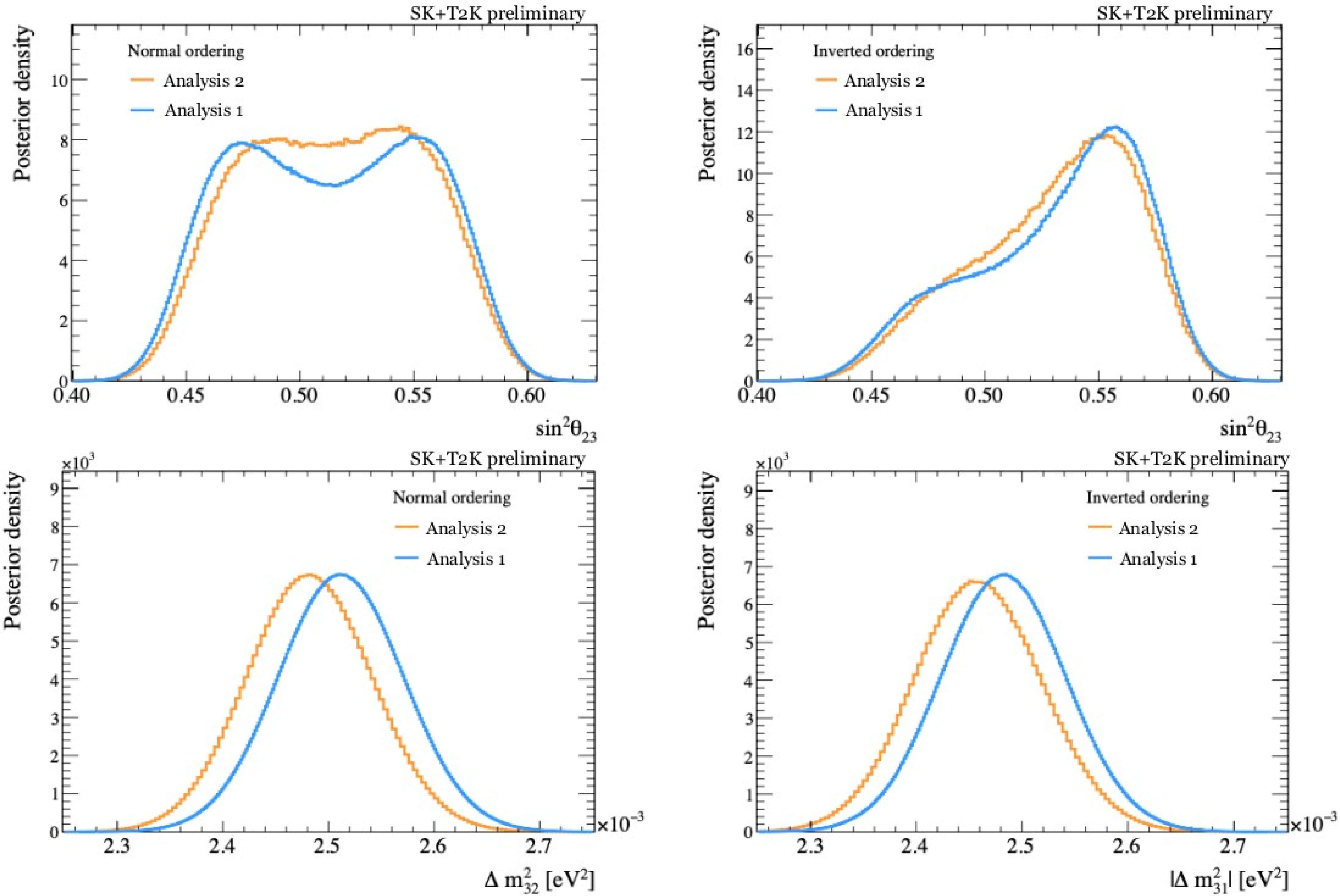
SK+T2K preliminary, Analysis 1

Normal ordering	$\sin^2\theta_{13}$	δ_{CP}	Δm^2_{32} [10^{-3} eV^2]	$\sin^2\theta_{23}$	J_{CP}
Most probable value	0.0219	-1.872	2.511	0.549	-0.033
1σ	[0.0212, 0.0226]	[-2.464, -1.205]	[2.452, 2.571]	[0.459, 0.505] and [0.521, 0.568]	[-0.034, -0.026]

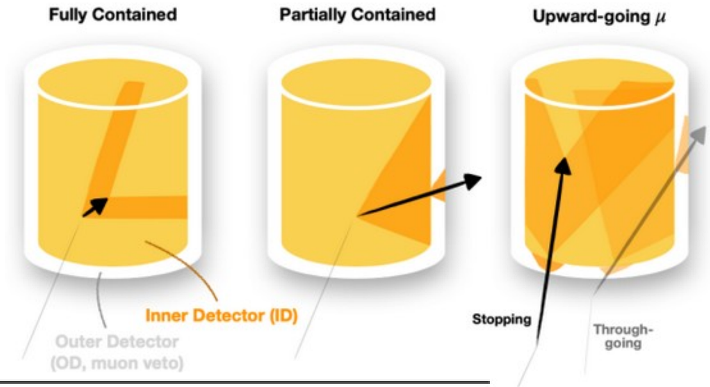
Inverted ordering	$\sin^2\theta_{13}$	δ_{CP}	Δm^2_{32} [10^{-3} eV^2]	$\sin^2\theta_{23}$	J_{CP}
Most probable value	0.0220	-1.476	2.484	0.558	-0.033
1σ	[0.0213, 0.0227]	[-2.003, -0.976]	[2.424, 2.541]	[0.508, 0.581]	[-0.034, -0.029]

Both ordering	$\sin^2\theta_{13}$	δ_{CP}	Δm^2_{32} [10^{-3} eV^2]	$\sin^2\theta_{23}$	J_{CP}
Most probable value	0.0219	-1.797	2.510	0.549	-0.033
1σ	[0.0212, 0.0226]	[-2.417, -1.159]	[2.449, 2.568]	[0.461, 0.503] and [0.520, 0.570]	[-0.034, -0.026]

Analysis I vs II



List of SK samples



Sample Name	Category	Selection			
SubGeV elike 0dcy	Fully Contained (FC)	Sub-GeV	Single-ring	e-like	0 decay-e
SubGeV elike 1dcy	Fully Contained (FC)	Sub-GeV	Single-ring	e-like	1 decay-e
SubGeV mulike 0dcy	Fully Contained (FC)	Sub-GeV	Single-ring	μ-like	0 decay-e
SubGeV mulike 1dcy	Fully Contained (FC)	Sub-GeV	Single-ring	μ-like	1 decay-e
SubGeV mulike 2dcy	Fully Contained (FC)	Sub-GeV	Single-ring	μ-like	≤ 2 decay-e
SubGeV pi0like	Fully Contained (FC)	Sub-GeV	Multi-ring	Two e-like rings	
MultiGeV elike nue	Fully Contained (FC)	Multi-GeV	Single-ring	e-like	≤ 1 decay-e
MultiGeV elike nuebar	Fully Contained (FC)	Multi-GeV	Single-ring	e-like	0 decay-e
MultiGeV mulike	Fully Contained (FC)	Multi-GeV	Single-ring	μ-like	
MultiRing elike nue	Fully Contained (FC)	Multi-GeV	Multi-ring	e-like	ν_e-like
MultiRing elike nuebar	Fully Contained (FC)	Multi-GeV	Multi-ring	e-like	$\bar{\nu}_e$-like
MultiRingOther 1	Fully Contained (FC)	Multi-GeV	Multi-ring	e-like	other
MultiRing mulike	Fully Contained (FC)	Multi-GeV	Multi-ring	μ-like	
PCStop	Partially Contained (PC)	No charge deposition in OD			
PCThru	Partially Contained (PC)	Charge deposition in OD			
UpStop mu	Up-going Muon (UpMu)	Stopping			
UpThruNonShower mu	Up-going Muon (UpMu)	Through-going Non-showering			
UpThruShower mu	Up-going Muon (UpMu)	Through-going Showering			

Results, comparing constraints

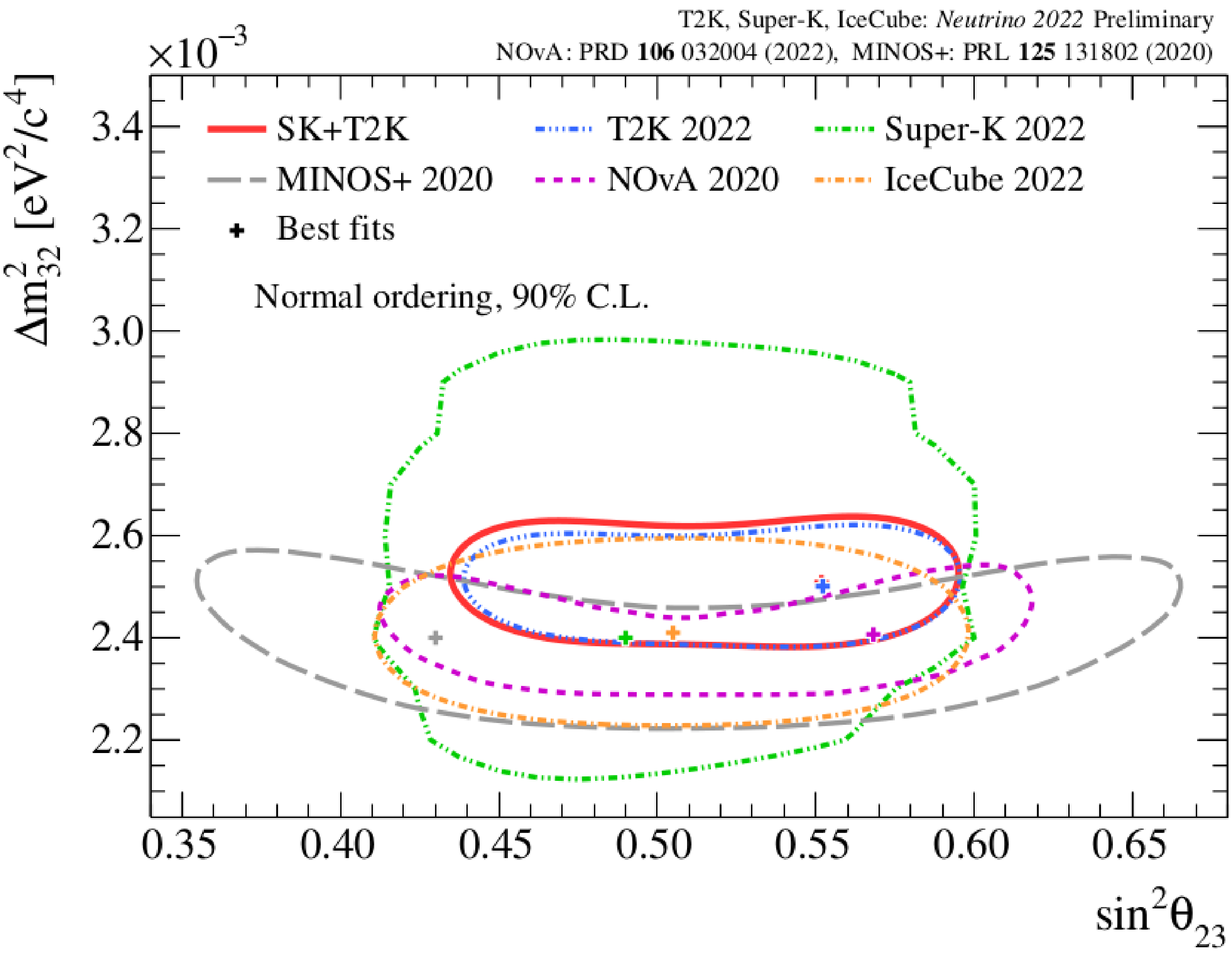
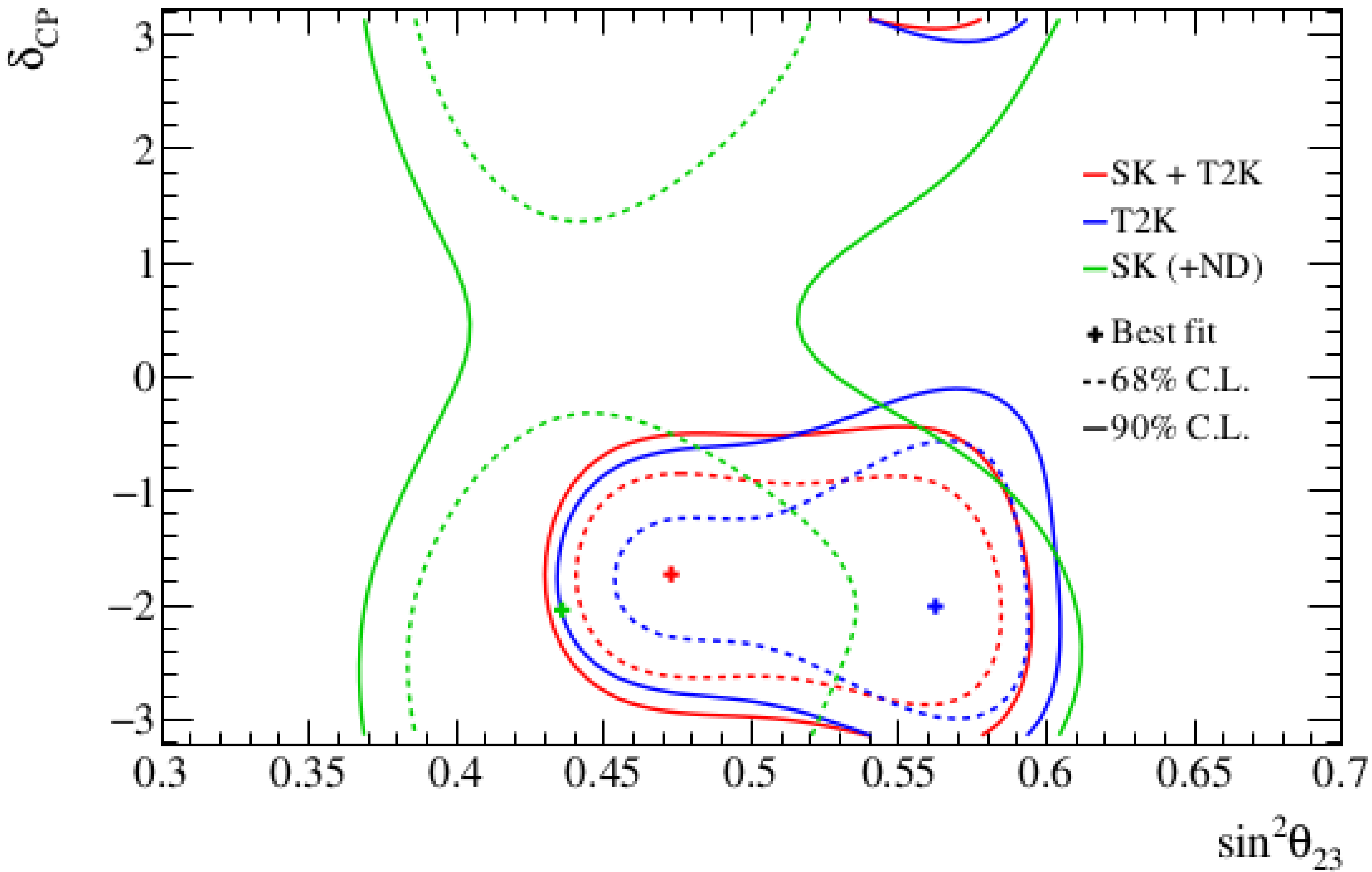


Figure 26. Comparison of 90% confidence regions in Δm^2_{32} vs. $\sin^2\theta_{23}$ in normal ordering, among SK+T2K (fixed-$\Delta\chi^2$), T2K (fixed-$\Delta\chi^2$), Super-K (fixed-$\Delta\chi^2$), MINOS [14], NOvA [15] (FC with global $\Delta\chi^2$ over both mass orderings), and IceCube (FC with fixed mass ordering).

Results, comparing constraints



(a) δ_{CP} vs $\sin^2\theta_{23}$

Uncertainty sources

Sample	SK atm. flux	T2K beam flux	SK det.	Cross sections	Total Syst.	Total
T2K FHC 1Rmu	0.00%	2.54%	2.39%	2.65%	2.81%	3.13%
T2K RHC 1Rmu	0.00%	2.64%	2.28%	3.58%	3.69%	3.80%
T2K FHC 1Re	0.00%	2.55%	2.69%	3.95%	3.99%	8.87%
T2K RHC 1Re	0.00%	2.72%	4.35%	4.37%	6.35%	14.75%
T2K FHC 1Re1de	0.00%	2.53%	7.87%	6.37%	9.21%	12.14%
SK SubGeV-elike-0dcy	3.53%	0.00%	2.55%	3.47%	1.42%	1.07%
SK SubGeV-elike-1dcy	3.00%	0.00%	3.39%	4.38%	3.03%	2.77%
SK SubGeV-mulike-0dcy	3.02%	0.00%	3.19%	2.72%	2.29%	2.16%
SK SubGeV-mulike-1dcy	3.08%	0.00%	2.58%	2.70%	1.28%	1.13%
SK SubGeV-mulike-2dcy	3.01%	0.00%	2.72%	4.34%	3.32%	3.28%
SK SubGeV-pi0like	2.72%	0.00%	2.39%	3.61%	2.32%	2.34%
SK MultiGeV-elike-nue	4.43%	0.00%	3.26%	7.09%	5.49%	5.61%
SK MultiGeV-elike-nuebar	4.23%	0.00%	3.21%	4.43%	2.97%	2.87%
SK MultiGeV-mulike	4.20%	0.00%	2.73%	4.23%	2.87%	2.90%
SK MultiRing-elike-nue	4.30%	0.00%	3.18%	4.26%	2.76%	2.73%
SK MultiRing-elike-nuebar	4.22%	0.00%	3.37%	4.24%	2.73%	2.63%
SK MultiRing-mulike	4.22%	0.00%	2.28%	4.12%	1.76%	1.75%
SK MultiRingOther-1	4.15%	0.00%	3.84%	5.03%	2.61%	2.56%
SK PCStop	4.37%	0.00%	4.80%	3.61%	4.45%	4.50%
SK PCThru	3.17%	0.00%	2.24%	3.82%	2.09%	2.09%
SK UpStop-mu	4.51%	0.00%	2.00%	3.77%	2.92%	2.99%
SK UpThruNonShower-mu	4.33%	0.00%	1.66%	3.90%	1.80%	1.76%
SK UpThruShower-mu	5.78%	0.00%	5.16%	3.72%	4.19%	4.22%

Bayesian prior choices for δ_{CP}

- Two widely accepted non-informative priors were tested in our analysis of CP violation.
  - Uniform δ_{CP}: closer to Jensen's prior for $U(3)$ Haar measure
  - Uniform $\sin\delta_{\text{CP}}$: closer to Jeffreys' prior ($\propto \sqrt{\det I_{\text{Fisher}}}$) for this analysis

Comparison of the two prior distributions in different parameter spaces