Composite Hybrid Inflation: Walking Dilaton and Waterfall Pions

Wanda Isnard

Institut de Physique des 2 Infinis de Lyon (IP2I)

With Giacomo Cacciapaglia, Dhong Yeon Cheong, Aldo Deandrea and Seong Chan Park

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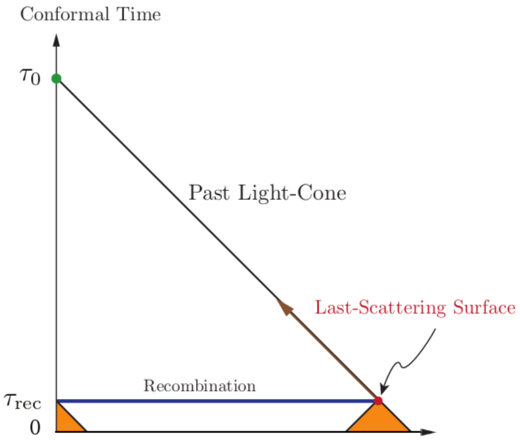
Content

Introduction : Inflation

Observations from the Cosmic Microwave Background (CMB) :

- Universe is very flat : $\Omega_k \sim 0$
  $\Longrightarrow$ Flatness problem

- Very small thermal fluctuations, but events are causally disconnected
  $\Longrightarrow$ Horizon problem



[Baumann]

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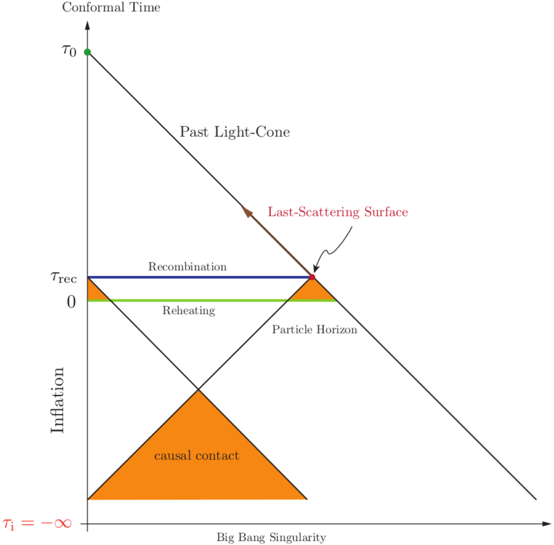
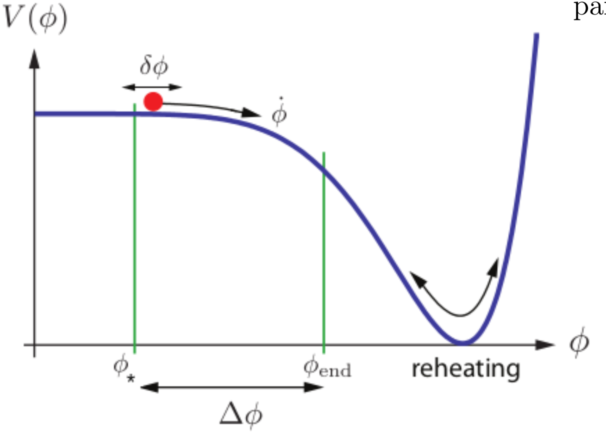
Inflation gives a simple explanation to both problems



[Baumann]

Introduction : Slow-roll Inflation

Scalar fields ϕ dominated by their potential $V(\phi)$ are good candidates to have an accelerated expansion :



Inflation characterized by slow-roll parameters :

$$\varepsilon = -\frac{\dot{H}}{H^2} \simeq \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1 \qquad (1)$$

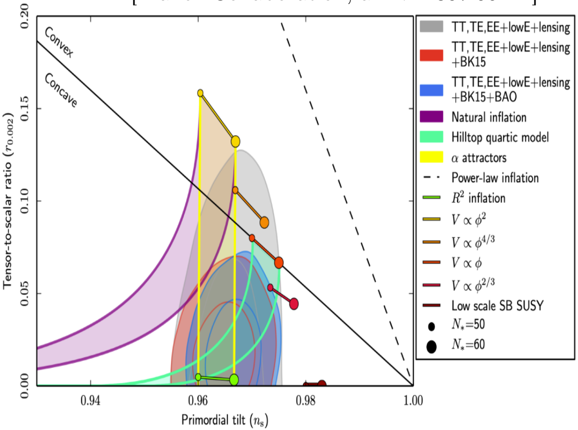
$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \simeq M_P^2 \frac{V''(\phi)}{V(\phi)} \ll 1 \qquad (2)$$

Introduction : Slow-roll Inflation

CMB/Planck observations lead to precision tests for inflationary models :

- Duration :
  $N = \ln\left(\frac{a_{\text{end}}}{a_*}\right) = \int_{\phi_*}^{\phi_{\text{end}}} \frac{\mathrm{d}\phi}{\sqrt{2\varepsilon}} \simeq 60$
- Scalar to tensor ratio : $r \simeq 16\varepsilon \leq 0.05$
- Spectral index :
  $n_s \simeq 1 - 6\varepsilon + 2\eta \simeq 0.965$
- Power spectrum :
  $\mathcal{P}_\mathcal{R} \simeq \frac{V(\phi_*)}{24\pi^2 M_P^4 \varepsilon} \simeq 2 \times 10^{-9}$

[Planck Collaboration, arxiv :1807.06211]



Questions

Can viable inflationary potential arise from BSM theories like composite models ?

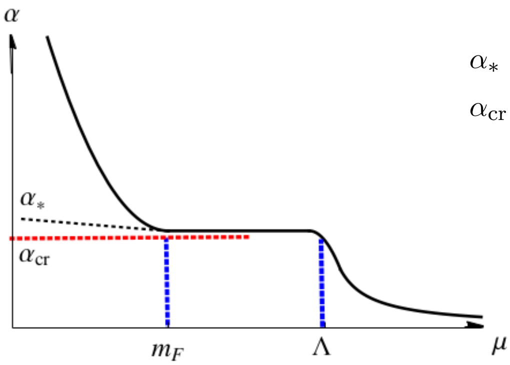
How inflation constrain such theories ? What is the phenomenology ?

The model

- SU(N_c) gauge theory, N_f Dirac fermions (Fundamental)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}_j \left(i\not{D} - m_0 \right) \psi_j \tag{3}$$

- Such theory can exhibit a walking regime :



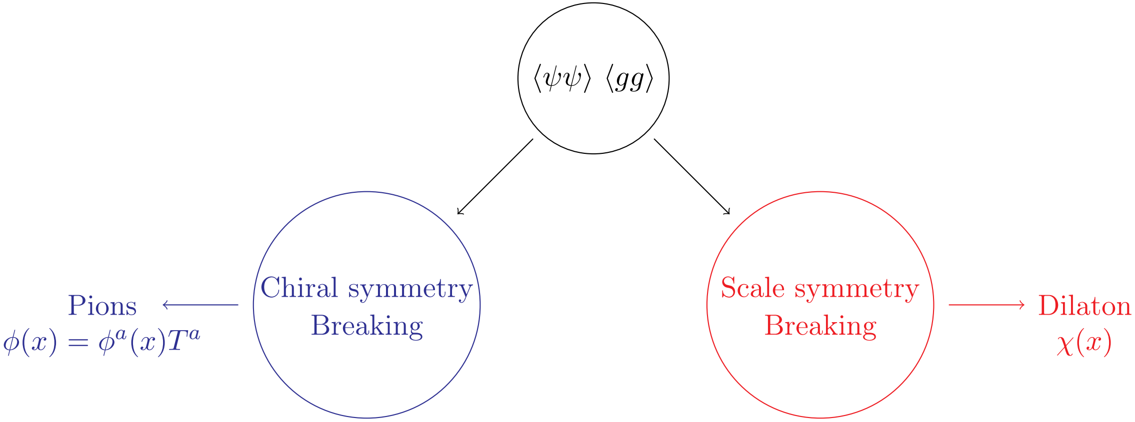
α_* : IR fixed point

$\alpha_{\rm cr} = \frac{\pi}{3C_2}$

[Matsuzaki,Yamawaki 05']

The model

- In the IR, $\mu \ll m_F$, a fermion and/or a gluon condensate $\langle\psi\psi\rangle$ ($\langle gg\rangle$) can form.



The model

- Chiral Lagrangian with $M_P = 1$, $U(x) = \exp\left(\frac{i\phi}{f_\phi}\right)$:

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset \frac{1}{2}R - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{f_\phi^2}{2}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] - \frac{\lambda_\chi}{4}\chi^4\left(\log\frac{\chi}{f_\chi} - A\right) + \frac{\lambda_\chi \delta_1 f_\chi^4}{2}\left(\frac{\chi}{f_\chi}\right)^{3-\gamma_m} \mathrm{Tr}\left[U + U^\dagger\right] + \frac{\lambda_\chi \delta_2 f_\chi^4}{4}\left(\frac{\chi}{f_\chi}\right)^{2(3-\gamma_{4f})} \mathrm{Tr}\left[(U - U^\dagger)^2\right] - V_0 \tag{4}$$

[Cacciapaglia, Pica, Sannino 20']

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Kinetic terms

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Kinetic terms — Dilaton potential (CW)

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Pion mass

[Cacciapaglia, Pica, Sannino 20']

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Pion mass

4 Fermi

[Cacciapaglia, Pica, Sannino 20']

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$$- V_0$$

- Lattice data : $0 < \gamma_m \leq 1$ and $0 < \gamma_{4f} \leq 1$ [Leung,Love,Bardeen 89'] [LatKMI Collaboration 14', 17']

- Composite theory : $\max(\delta_1, \delta_2) \lesssim \frac{1}{\lambda_\chi}\left(\frac{f_\phi}{f_\chi}\right)^4$

- De Sitter : $V_0 > 0$

The model

- Inflationary potential :

$$V(\phi,\chi) = -\lambda_\chi \delta_1 f_\chi^4 \left(\frac{\chi}{f_\chi}\right)^{3-\gamma_m} \cos\frac{\phi}{f_\phi} - \lambda_\chi \delta_2 f_\chi^4 \left(\frac{\chi}{f_\chi}\right)^{2(3-\gamma_{4f})} \sin^2\frac{\phi}{f_\phi} + \frac{\lambda_\chi}{4}\chi^4\left(\log\frac{\chi}{f_\chi} - A\right) + V_0 \tag{5}$$

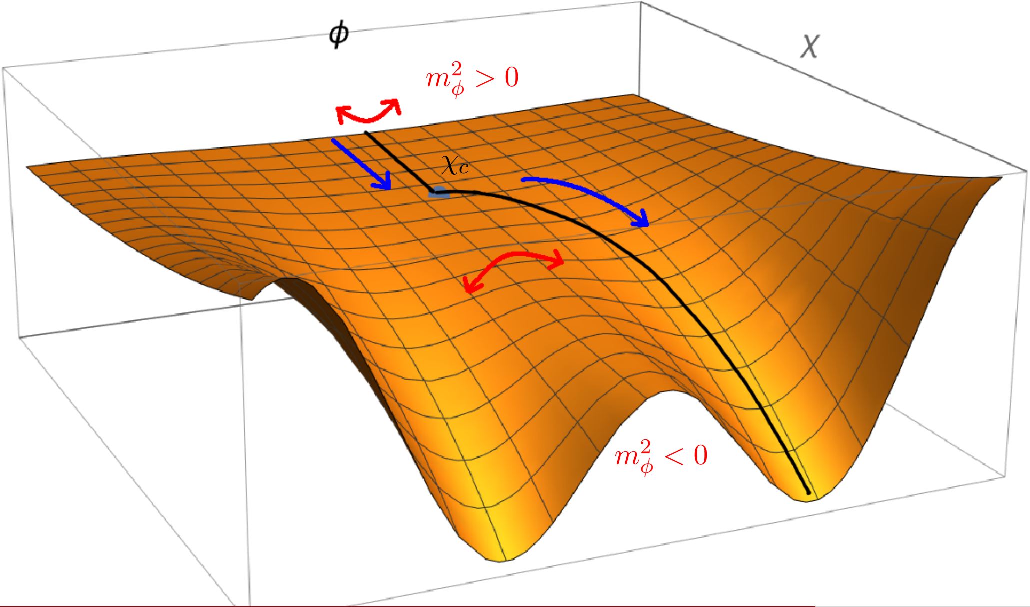
- VEV requirements : $V(\phi_0,\chi_0) = 0\,, \quad V_\chi(\phi_0,\chi_0) = 0 \quad \text{and} \quad V_\phi(\phi_0,\chi_0) = 0$

$$\chi_0 = f_\chi, \qquad \phi_0 = f_\phi \arccos\frac{\delta_1}{2\delta_2} \quad \text{for} \quad 0 < \delta_1 < 2\delta_2$$

$$V_0 = \frac{\lambda_\chi f_\chi^4}{16}\left[1 + 2\frac{\delta_1^2}{\delta_2}(2+\gamma_m-\gamma_{4f}) - 8\delta_2(1-\gamma_{4f})\right] \tag{6}$$

$$A = \frac{1}{4}\left[1 + 2\frac{\delta_1^2}{\delta_2}(\gamma_m-\gamma_{4f}) - 8\delta_2(3-\gamma_{4f})\right]$$

Inflationary Dynamics : Waterfall

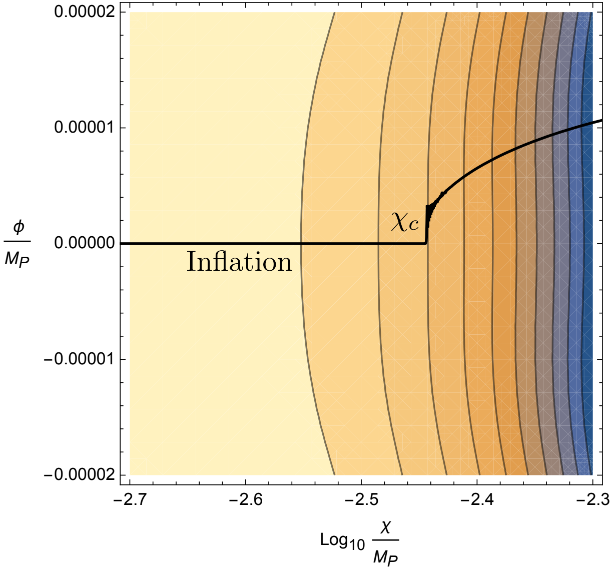


Inflationary Dynamics : Waterfall

- $\chi_c = \chi(m_\phi^2 = 0) = f_\chi \left(\frac{\delta_1}{2\delta_2}\right)^{\frac{1}{1+\gamma_m - 2\gamma_{4f}}}$

- $\phi(\chi < \chi_c) = 0$
- $\phi(\chi > \chi_c) = f_\phi \cos^{-1} \frac{\delta_1}{2\delta_2} \left(\frac{f_\chi}{\chi}\right)^{3+\gamma_m - 2\gamma_{4f}}$

Tachyonic instability terminates inflation at $\chi = \chi_c$



Constraints on the parameter space

- Rapid termination of inflation : $$\left| m_\phi^2 \right|_{\phi=0,\ \chi=\chi_c+\Delta\chi} \gg H^2$$

$\Longrightarrow$ Constrain : $f_\phi < f_\phi^{\text{waterfall}}$

- Composite consistency : $$\max(\delta_1, \delta_2) \lesssim \frac{1}{\lambda_\chi}\left(\frac{f_\phi}{f_\chi}\right)^4$$

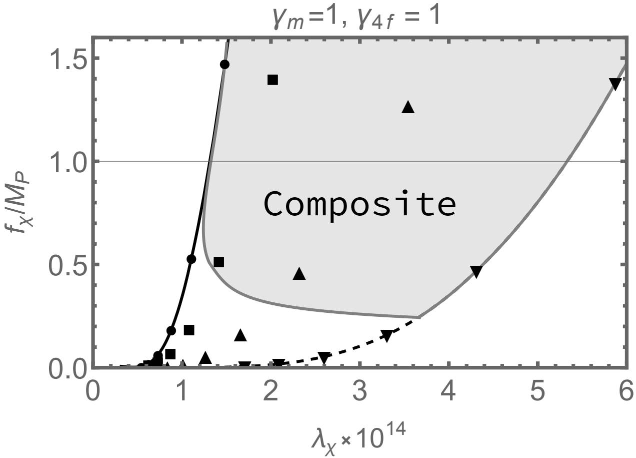
$\Longrightarrow$ Constrain : $f_\phi > f_\phi^{\text{composite}}$

- De Sitter : $V_0 > 0$

$\Longrightarrow$ Constrain : $\delta_2 < \delta_2^{\text{dS}}$

Constraints on the parameter space

- Setting $\gamma_m = \gamma_{4f} = 1$, we have 5 free parameters : δ_1, δ_2, χ_*, f_χ, λ_χ
- CMB observations $(n_s, N, \mathcal{P}_\mathcal{R})$ allow us to fix 3 paramaters : δ_1, δ_2, χ_*



- Predictions :

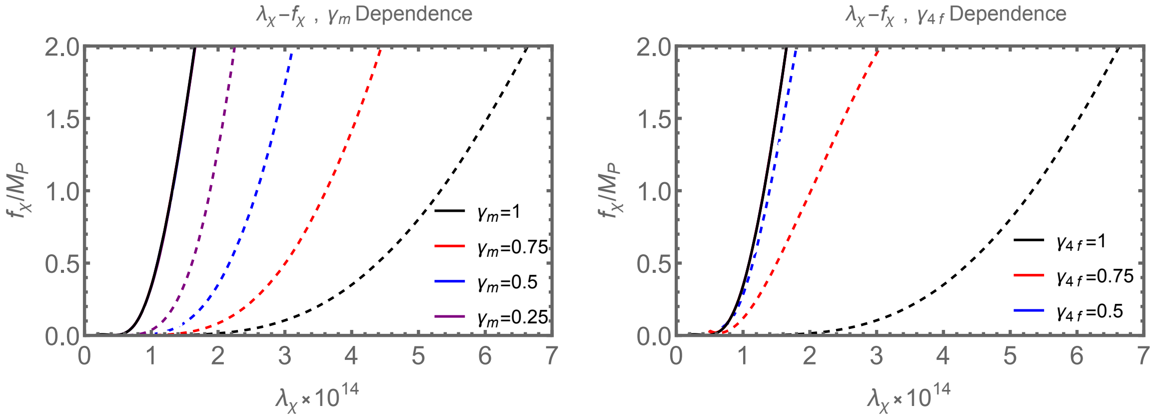
$$m_\chi|_{\chi_0,\phi_0} \sim 10^{11}\ \text{GeV}$$

$$m_\phi|_{\chi_0,\phi_0} \sim 10^{14}\ \text{GeV}$$

$$H_\text{inf} \sim 10^{10}\ \text{GeV}$$

Constraints on the parameter space

- Dependence on γ_m and γ_{4f} :



Conclusion

- Inflation provides a simple way to answer the Flatness and Horizon problems
- Inflation requires a particle to drive the accelerated expansion

$\Longrightarrow$ Test if BSM theories can provide a successful framework for inflation

- Study for a composite gauge theory

$\Longrightarrow$ Compatibility with CMB observations constrains a lot the parameter space of such theories

- Next : Reheating ? Topological defects ? Gravitational waves production ? Non-minimal coupling ?

Walking Regime

- Beta function : $\beta^{2\text{loop}}(\alpha) \equiv \mu \frac{\partial \alpha}{\partial \mu} = -b_0 \alpha^2 - b_1 \alpha^3$

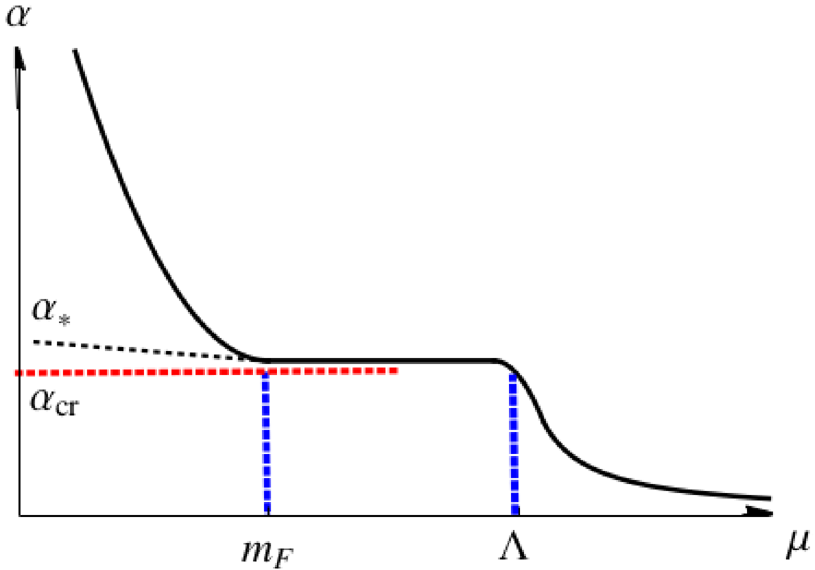
$$b_0 = \frac{1}{6\pi}(11N_c - 2N_f), \qquad b_1 = \frac{1}{24\pi^2}\left(34N_c^2 - 10N_cN_f - 3\frac{N_c^2-1}{N_c}N_f\right)$$

- IR fixed point : $\alpha_* = -\frac{b_0}{b_1} = \frac{4\pi(11-2n_f)}{N_c(13n_f-34)}$
- Intrinsic scale : $\Lambda \equiv \mu \exp\left(-\int^{\alpha(\mu^2)} \frac{d\alpha}{\beta^{2\text{loop}}(\alpha)}\right)$, leads to $\alpha(\mu^2) = \frac{\alpha_*}{1+W\left(\frac{1}{e}\left(\frac{\mu}{\Lambda}\right)^{b_0\alpha_*}\right)}$

$\Longrightarrow \quad \alpha(\mu^2)^{\text{IR}} \sim \frac{\alpha_*}{1+\frac{1}{e}\left(\frac{\mu}{\Lambda}\right)^{b_0\alpha_*}} \qquad \alpha(\mu^2)^{\text{UV}} \sim \frac{1}{b_0 \ln \frac{\mu}{\Lambda}}$

$\Longrightarrow$ Walking regime for $\mu \ll \Lambda$

Walking Regime



Walking Regime

- Miranski scaling : $m_F \simeq 4\Lambda \exp\left(\frac{-\pi}{\sqrt{\frac{\alpha}{\alpha_{\text{cr}}}-1}}\right)$

$\Longrightarrow \beta(\alpha) = \frac{\partial\alpha}{\partial \ln\Lambda} = -\frac{2\alpha_{\text{cr}}}{\pi}\left(\frac{\alpha}{\alpha_{\text{cr}}} - 1\right)^{\frac{3}{2}}$

- Solving this equation yields : $\alpha(\mu) = \alpha_{\text{cr}}\left(1 + \frac{\pi^2}{\ln^2 \frac{\mu}{m_F}}\right)$

$\Longrightarrow$ Walking regime for $\mu \gg m_F$

- Conclusion : Walking regime for $m_F \ll \mu \ll \Lambda$ where $\alpha \sim \alpha_* \sim \alpha_{\text{cr}}$

Single field inflation

- Corresponds to $\phi = 0$, $\delta_2 = 0$ such that :

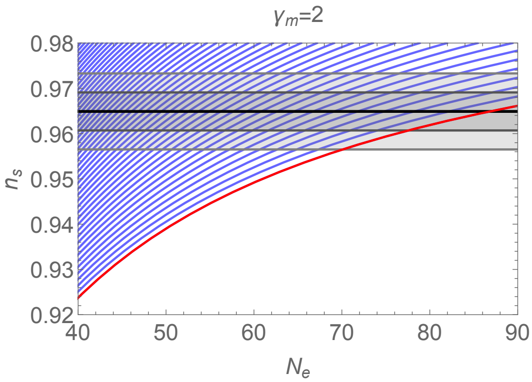
$$V(\phi = 0, \chi) = -\lambda_\chi \delta_1 f_\chi^4 \left(\frac{\chi}{f_\chi}\right)^{3-\gamma_m} + \frac{\lambda_\chi}{4}\chi^4 \left(\log \frac{\chi}{f_\chi} - A^{\text{single}}\right) + V_0^{\text{single}} \tag{7}$$

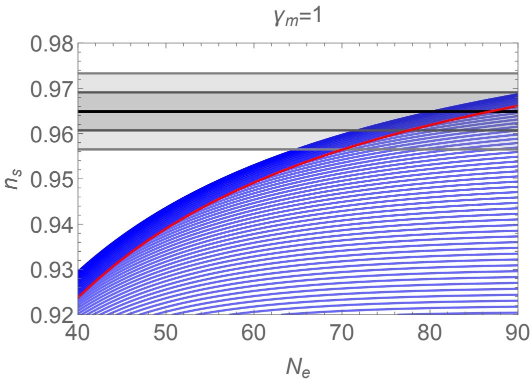
$$A^{\text{single}} = \frac{1}{4} + \delta_1(\gamma_m - 3) \tag{8}$$

$$V_0^{\text{single}} = \frac{1}{16}\lambda_\chi f_\chi^4 \left(1 + 4\delta_1(1 + \gamma_m)\right) \tag{9}$$

$\Longrightarrow$ CMB observations $(n_s, N, \mathcal{P}_\mathcal{R})$ fix $(\lambda_\chi, f_\chi, \chi_*)$

Single field inflation





Red : $\delta_1 = 0$

Blue : $\delta_1 \in [0, 1] \times 10^{-6}$