Asymptotic Grand Unification

Giacomo Cacciapaglia
IP2I Lyon, France

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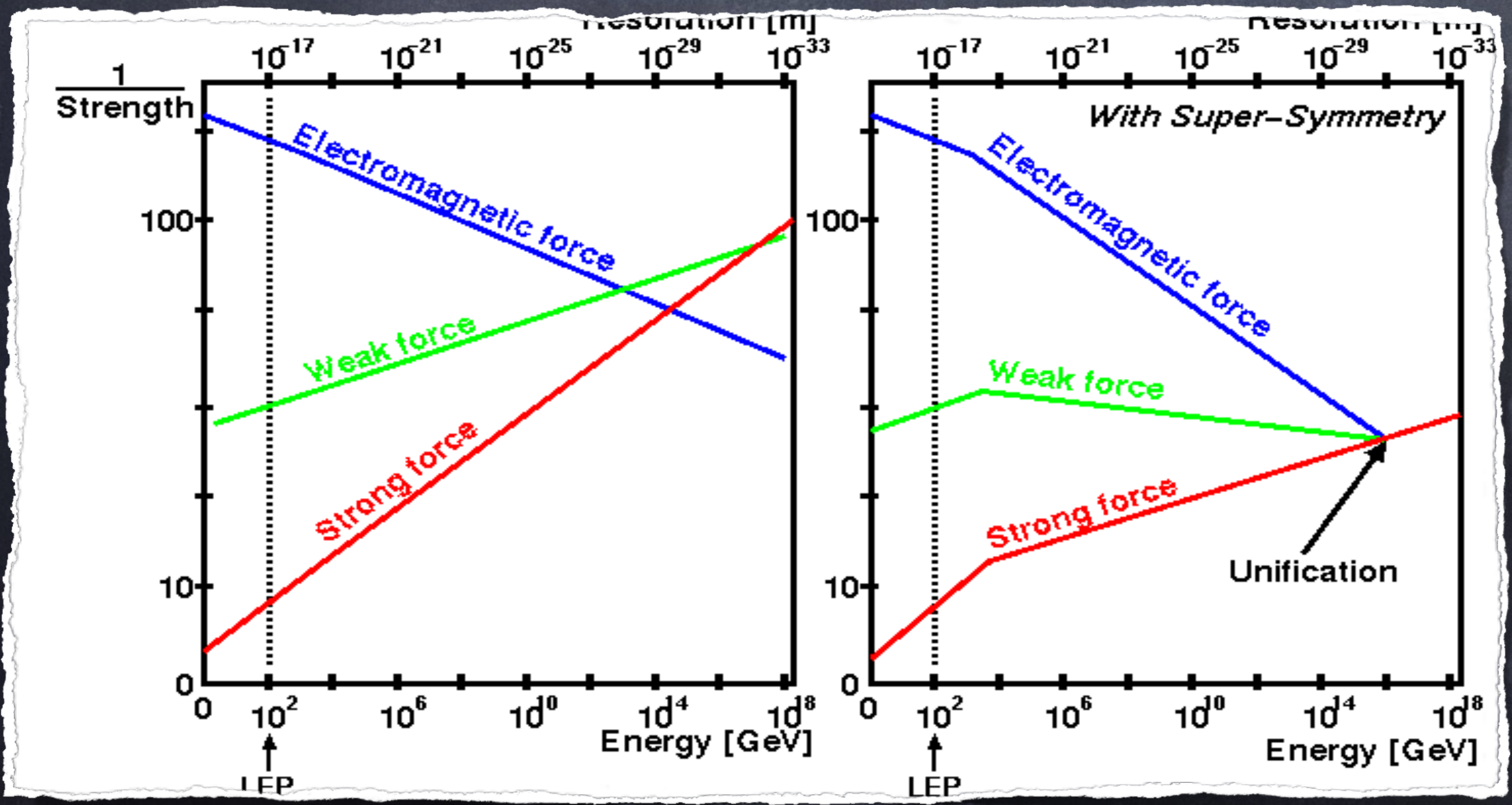
G.C., A.Cornell, A.Deandrea, C.Cot 2012.14732
G.C., A. Deandrea, R. Pasechnik, Z.W. Wang 2302.11671
G.C. 2309.10098

Traditional GUTs

- SM gauge couplings expected to be equal at the GUT scale
- supersymmetry helps building "realistic" models
- proton decay inevitable!



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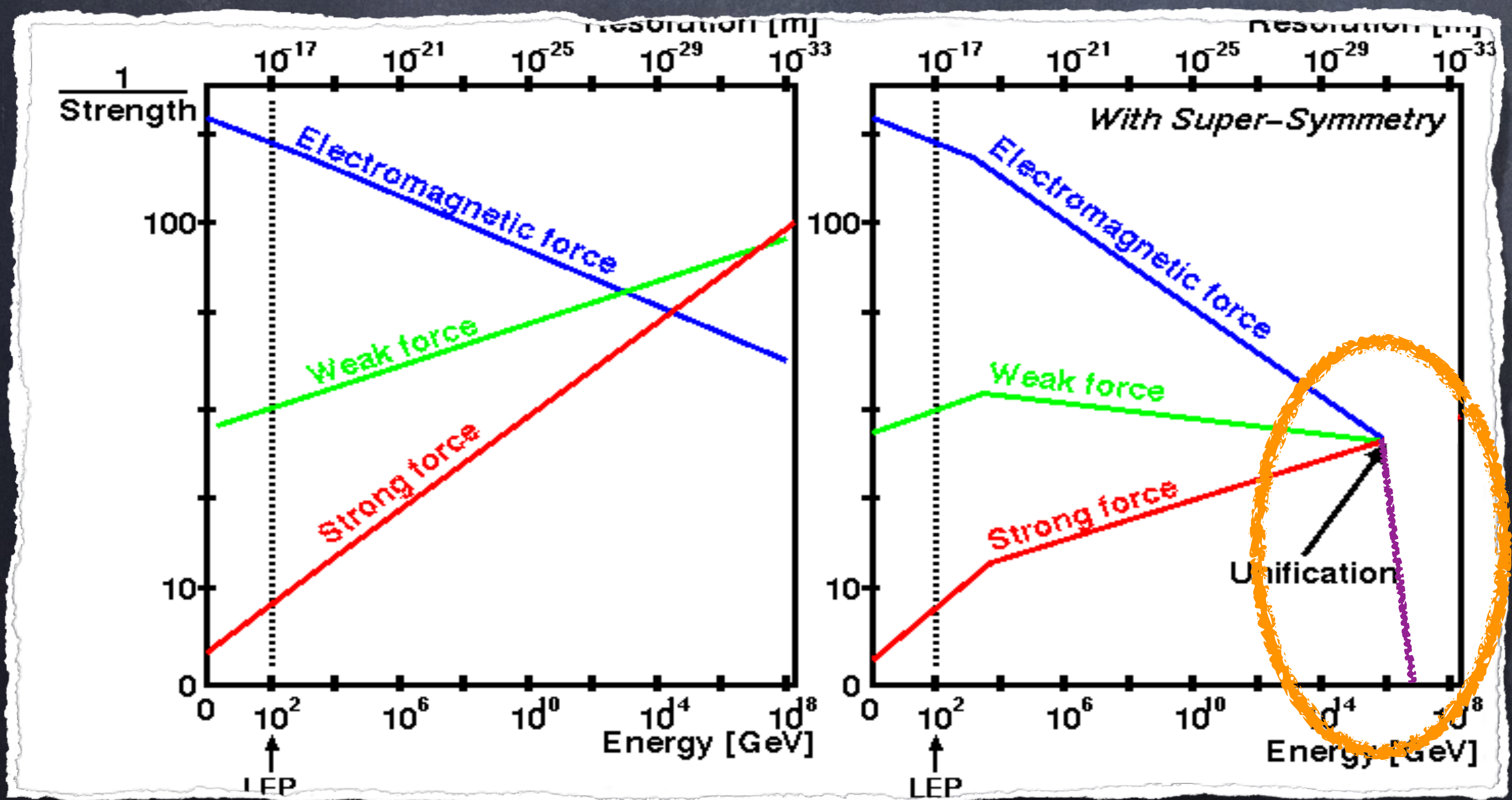
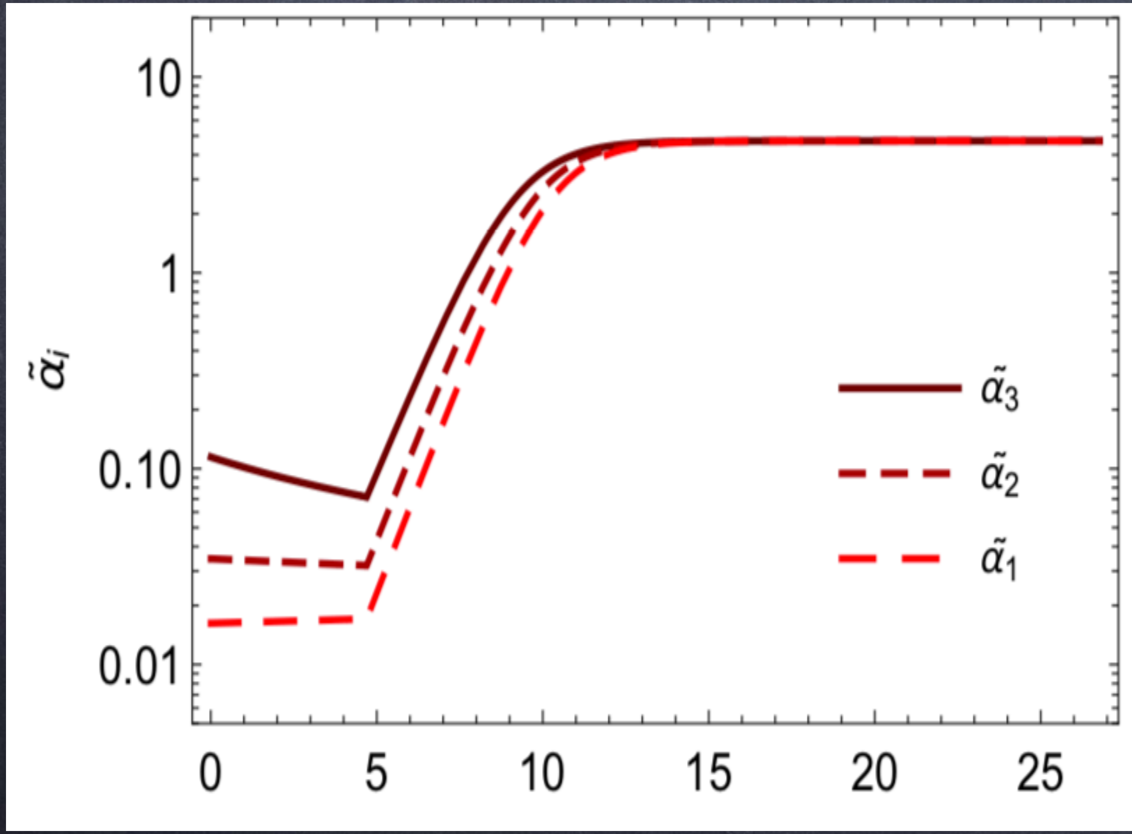


However:

- Large matter representations needed to break the gauge symmetry!
- Landau pole!!!!

asymptotic GUT (aGUT)

- Gauge couplings are never equal, but tend to the same UV fixed point!



A) Realised in asympt. safe theories

(via large Nf resum with intermediate Pati-Salam)

Molinaro et al, 1807.03669

(via perturbative fixed points and Susy)

Bajic et al, 1610.09681 and 2308.13311

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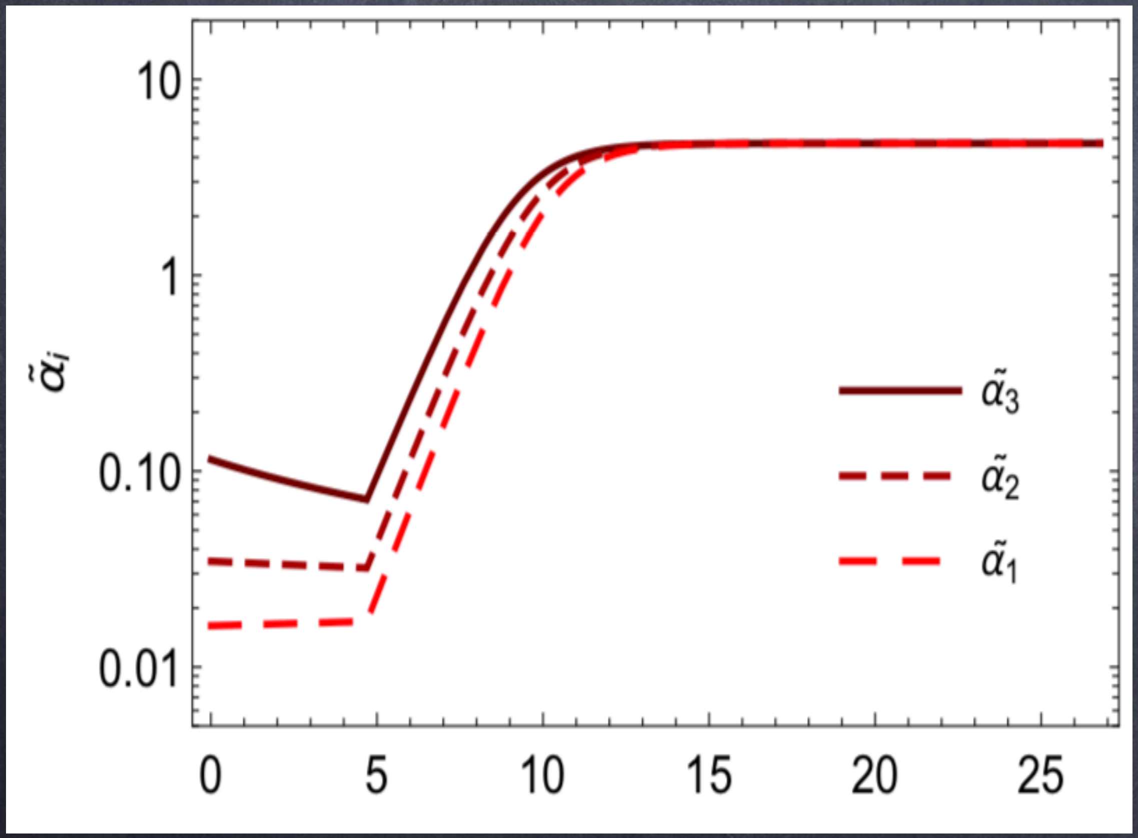


B) Extra compact dimensions

$$2\pi \frac{d\alpha}{d\ln\mu} = \mu R\, b_5\, \alpha^2$$

$\tilde{\alpha} = \mu R\, \alpha$ ('t Hooft coupling in 5D)

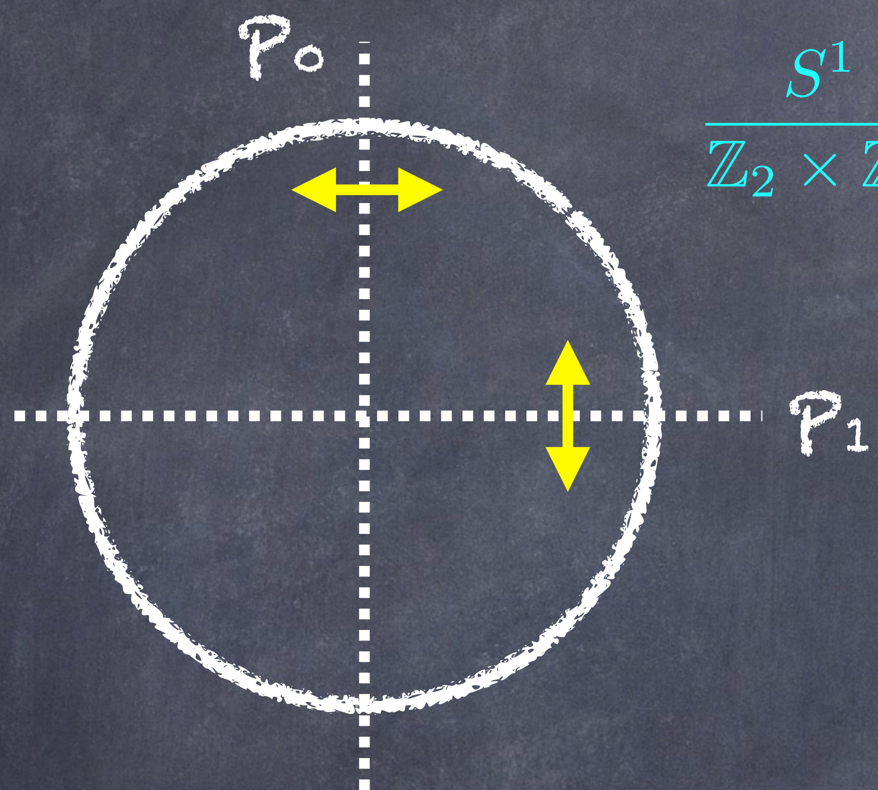
$$2\pi \left(\tilde{\alpha} + \frac{d\tilde{\alpha}}{d\ln\mu} \right) = b_5\, \tilde{\alpha}^2$$

$$\tilde{\alpha}_{UV} = -\frac{2\pi}{b_5}$$

Gies, PRD 68 (2003)
Morris, JHEP 01 (2005) 002

Minimal SU(5) aGUT

Cacciapaglia et al, PRD 104 (2021) 7



$$\frac{S^1}{\mathbb{Z}_2 \times \mathbb{Z}_2'}$$

$$(P_0) \Rightarrow \begin{cases} A^a_\mu(x,-y) = P_0 A^a_\mu(x,y) P_0^\dagger\,, \\ A^a_y(x,-y) = -P_0 A^a_y(x,y) P_0^\dagger\,, \end{cases}$$

$$(P_1) \Rightarrow \begin{cases} A^a_\mu(x,\pi R - y) = P_1 A^a_\mu(x,y) P_1^\dagger\,, \\ A^a_y(x,\pi R - y) = -P_1 A^a_y(x,y) P_1^\dagger\,, \end{cases}$$

$$P_0 = \left(+\,+\,+\,-\,-\right)\,,$$
$$P_1 = \left(+\,+\,+\,+\,+\right)\,.$$

$$\psi_5 = \begin{pmatrix} B^c \\ l \end{pmatrix} \quad \begin{matrix} (-\,+) \\ (+\,+) \end{matrix} \quad \begin{matrix} \\ \text{Lh zero} \end{matrix}$$

$$\psi_5 = \begin{pmatrix} b \\ L^c \end{pmatrix} \quad \begin{matrix} (-\,-) \\ (+\,-) \end{matrix} \quad \begin{matrix} \text{Rh zero} \\ \end{matrix}$$

- SU(5) broken in y=0 to the SM by boundary conditions
- SM fermions cannot be embedded in complete multiplets of SU(5)!!!

Yukawa non-unification

The most general bulk Lagrangian reads:

$$\mathcal{L}_{SU(5)} = -\frac{1}{4}F^{(a)}_{MN}F^{(a)MN} - \frac{1}{2\xi}(\partial_\mu A^\mu - \xi\partial_5 A_y)^2 + i\overline{\psi_5}\not{D}\psi_5 + i\overline{\psi_{\bar{5}}}\not{D}\psi_{\bar{5}} + i\overline{\psi_{10}}\not{D}\psi_{10}$$
$$+ i\overline{\psi_{\overline{10}}}\not{D}\psi_{\overline{10}} - \left(\sqrt{2}Y_\tau\,\overline{\psi_{\bar{5}}}\psi_{\overline{10}}\phi_5^* + \sqrt{2}Y_b\,\overline{\psi_5}\psi_{10}\phi_5^* + \frac{1}{2}Y_t\,\epsilon_5\,\overline{\psi_{\overline{10}}}\psi_{10}\phi_5 + \text{h.c.}\right)$$
$$+ |D_M\phi_5|^2 - V(\phi_5) + i\overline{\psi_1}\not{\partial}\psi_1 - \left(Y_\nu\,\overline{\psi_1}\psi_{\bar{5}}\phi_5 + \text{h.c.}\right),$$

- Yukawas DO NOT unify!
- Baryon and lepton numbers can be defined (no proton decay processes)

Indalo states

Multiplets	Fields	L	B	Q	Q_3
$\psi_{\bar{5}}$	B_R^c	1/2	1/6	1/3	0
	τ_L	1	0	-1	-1
	ν_L	1	0	0	1
ψ_5	b_R	0	1/3	-1/3	0
	$\mathcal{T}_L^c$	-1/2	1/2	1	1
	$\mathcal{N}_L^c$	-1/2	1/2	0	-1
ψ_{10}	T_R^c	1/2	1/6	-2/3	0
	$\mathcal{T}_R^c$	-1/2	1/2	1	0
	t_L	0	1/3	2/3	1
	b_L	0	1/3	-1/3	-1
$\psi_{\overline{10}}$	t_R	0	1/3	2/3	0
	τ_R	1	0	-1	0
	T_L^c	1/2	1/6	-2/3	-1
	B_L^c	1/2	1/6	1/3	1
ψ_1	N	1	0	0	0
ϕ_5	H	1/2	-1/6	-1/3	0
	ϕ^+	0	0	1	1
	ϕ_0	0	0	0	-1
A_X	X	1/2	-1/6	-4/3	-1
	Y	1/2	-1/6	-1/3	1

- Non-SM components carry unusual B and L charges
- Hence, they cannot decay into SM states
- States with mass 1/R stable

 = Indalo

- Prehistoric symbol found in Almería caves, Spain
- It means "creation" or "nature" in Zulu

Indalo-genesis

Multiplets	Fields	L	B	Q	Q_3
$\psi_{\overline{5}}$	B_R^c	1/2	1/6	1/3	0
	τ_L	1	0	-1	-1
	ν_L	1	0	0	1
ψ_5	b_R	0	1/3	-1/3	0
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ψ_{10}	T_R^c	1/2	1/6	-2/3	0
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$\psi_{\overline{10}}$	t_R	0	1/3	2/3	0
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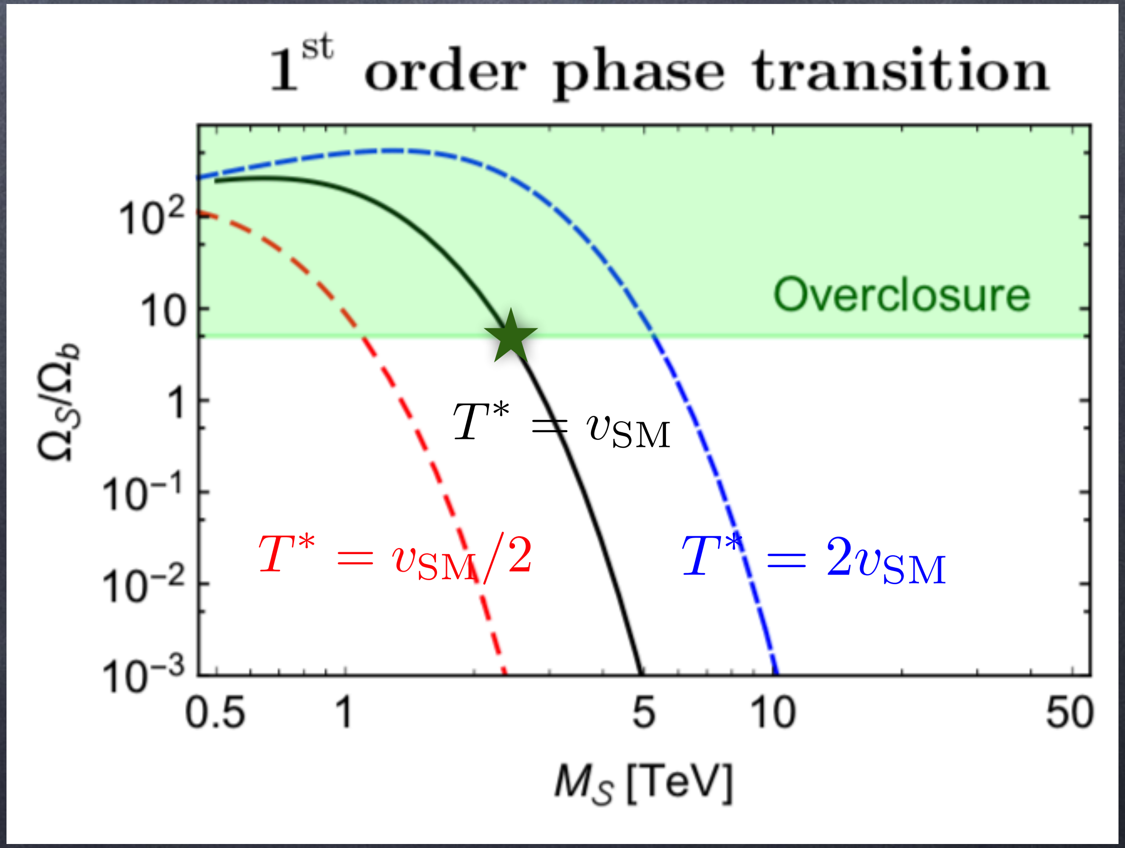
	T_L^c	1/2	1/6	-2/3	-1
	B_L^c	1/2	1/6	1/3	1
ψ_1	N	1	0	0	0
ϕ_5	H	1/2	-1/6	-1/3	0
	ϕ^+	0	0	1	1
	ϕ_0	0	0	0	-1
A_X	X	1/2	-1/6	-4/3	-1
	Y	1/2	-1/6	-1/3	1

- Baryogenesis could also produce an asymmetric abundance of Indalo states

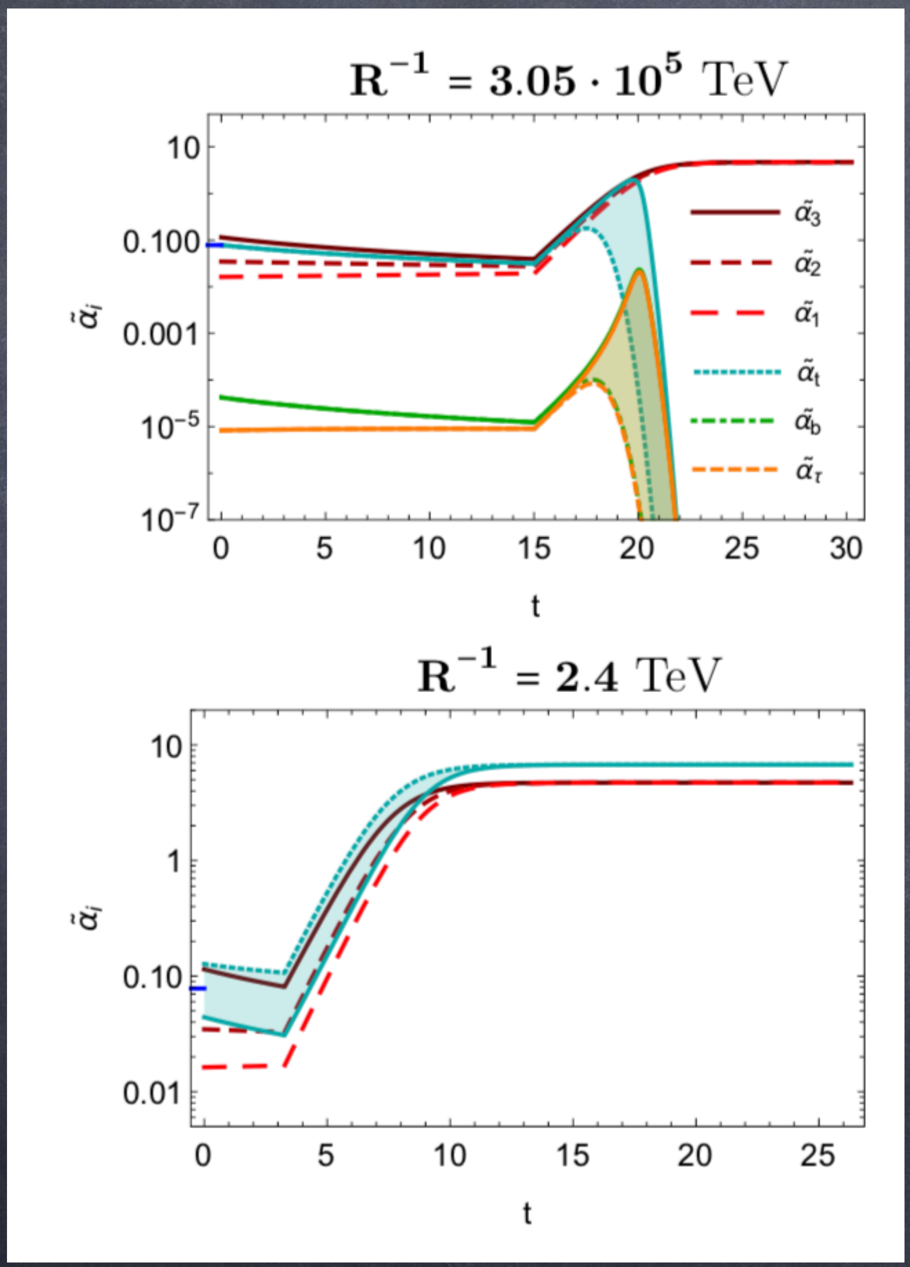
Dark Matter candidate!

$1/R = 2.4\ TeV$



The Yukawa sector runs into problems

Bulk Yukawas



For smaller values of the KK scale the Yukawas run to Landau poles

Localising all Yukawas except the top, allows for UV fixed point.

But hard to do: SO(10) is ruled out, in fact!

Khojali et al, 2210.03596

A classification is in order!

Cacciapaglia, 2309.10098

- Define a bulk gauge group $\mathcal{G} \supset \mathcal{G}_{SM}$, and parities breaking $\mathcal{G} \to \mathcal{H} \supset \mathcal{G}_{SM}$
- Find pairs $P_i \times P_j$ such that $\mathcal{H}_i \cap \mathcal{H}_j = \mathcal{G}_{SM} + X$ (minimality)
- Find minimal set of bulk fermions that contain SM zero modes and preserve UV fixed point
- Check running of the Yukawa couplings: do fixed point exist in the UV?
- Check if gauge-Higgs unification occurs, and if the model can be supersymmetrised (link Yukawa's to gauge couplings)

SU(6)

$$\mathcal{P}_1 = \text{diag}(+,+,+,+,+,-) \quad \mathcal{H}_1 = SU(5)\times U(1)_{Z1}\,,$$
$$\mathcal{P}_2 = \text{diag}(+,+,+,-,-,+) \quad \mathcal{H}_2 = SU(4)\times SU(2)\times U(1)_{Z2}\,,$$
$$\mathcal{P}_3 = \text{diag}(+,+,+,-,-,-) \quad \mathcal{H}_3 = SU(3)\times SU(3)\times U(1)_{Z3}\,.$$

The matrices $\mathscr{P}_i$ represent the intrinsic parity: an overall sign can be added for matter fields

Intrinsic parities:

Adjoint (**35**)	P_1	P_2	P_3
$(8,1)_{0,0}$	even	even	even
$(1,3)_{0,0}$	even	even	even
$(1,1)_{0,0}$	even	even	even
$(3,2)_{-5/3,0}$	even	odd	odd
$(\bar{3},2)_{5/3,0}$	even	odd	odd
$(1,1)_{0,0}$	even	even	even
$(3,1)_{-1/3,3}$	odd	even	odd
$(1,2)_{1/2,3}$	odd	odd	even
$(\bar{3},1)_{1/3,-3}$	odd	even	odd
$(1,2)_{-1/2,-3}$	odd	odd	even

F (**6**)	P_1	P_2	P_3
$(3,1)_{-1/3,1/2}$	even	even	even
$(1,2)_{1/2,1/2}$	even	odd	odd
$(1,1)_{0,-5/2}$	odd	even	odd

S (**21**)	P_1	P_2	P_3
$(3,2)_{1/6,1}$	even	odd	odd
$(6,1)_{-2/3,1}$	even	even	even
$(1,3)_{1,1}$	even	even	even
$(1,1)_{0,-5}$	even	even	even
$(3,1)_{-1/3,-2}$	odd	even	odd
$(1,2)_{1/2,-2}$	odd	odd	even

A (**15**)	P_1	P_2	P_3
$(3,2)_{1/6,1}$	even	odd	odd
$(\bar{3},1)_{-2/3,1}$	even	even	even
$(1,1)_{1,1}$	even	even	even
$(3,1)_{-1/3,-2}$	odd	even	odd
$(1,2)_{1/2,-2}$	odd	odd	even

A_3 (**20**)	P_1	P_2	P_3
$(\bar{3},2)_{-1/6,3/2}$	even	odd	odd
$(3,1)_{2/3,3/2}$	even	even	even
$(1,1)_{-1,3/2}$	even	even	even
$(3,2)_{1/6,-3/2}$	odd	odd	even
$(\bar{3},1)_{-2/3,-3/2}$	odd	even	odd
$(1,1)_{1,-3/2}$	odd	even	odd

SU(6)

Case $P_1 \times P_2$: one Higgs from gauge fields (GHU)

SM-like zero modes:

$$\mathbf{6}^{(-,-)} \supset d_R\,,\quad \bar{\mathbf{6}}^{(+,-)} \supset l_L + \nu_R\,,\quad \mathbf{15}^{(+,-)} \supset q_L + d_R\,,\quad \bar{\mathbf{15}}^{(-,-)} \supset l_L + u_R + e_R\,,$$
$$\mathbf{21}^{(+,-)} \supset q_L + d_R\,,\quad \mathbf{20}^{(-,-)} \supset q_L + u_R + e_R\,.$$

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$$\mathbf{21}^{(+,-)} \supset q_L + d_R\,,\quad \mathbf{20}^{(-,-)} \supset q_L + u_R + e_R\,.$$

Model 6A:

$$\Psi_{15}^{(+,-)} \supset q_L + d_R\,,\quad \Psi_{\bar{15}}^{(-,-)} \supset l_L + u_R + e_R\,,\quad \Phi_{15}^{(-,-)} \supset \varphi'_H\,.$$

d and e Yukawas from GHU

$$\mathcal{L}_{\rm Yuk} = -Y_u\, \overline{\Psi}_{\bar{15}} \Psi_{15} \Phi_{15} + \text{h.c.}$$

$$b_5 = \frac{61 - 16 n_g}{3}\,,$$

Gauge FP requires $n_g \leq 3$

SU(6)

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d and e Yukawas from GHU

$$\mathcal{L}_{\text{Yuk}} = -Y_u\, \overline{\Psi}_{\bar{15}} \Psi_{15} \Phi_{15} + \text{h.c.}$$

$$b_5 = \frac{61 - 16 n_g}{3}\,,$$

Gauge FP requires $n_g \leq 3$

Model 6S: 21 (symmetric) 105

$$b_5 = \frac{37}{3} - 8 n_g\,,$$

Gauge FP requires $n_g = 1$

Minimal SU(N) models
(not final)

Name	$\mathcal{G}_{\text{bulk}}$	Fermions	Scalars	Yukawas	n_g bulk	Higgs sector	Minimal?
$\mathcal{G}_{4D} = \mathcal{G}_{SM}$							
5A	SU(5)	$\Psi_5 \supset d_R\,,\ \Psi_{\bar{5}} \supset l_L\,,$ $\Psi_{10} \supset q_L\,,\ \Psi_{\overline{10}} \supset u_R + e_R$	$\Phi_5 \supset \varphi_H$	All bulk	≤ 3	SM-like	Yes
5S	SU(5)	$\Psi_5 \supset d_R\,,\ \Psi_{\bar{5}} \supset l_L\,,$ $\Psi_{15} \supset q_L\,,\ \Psi_{\overline{10}} \supset u_R + e_R$	$\Phi_5 \supset \varphi_H$ $\Phi_{\overline{45}} \supset \varphi'_H + \ldots$	All bulk	1	2HDM Type-II or flip	No
$\mathcal{G}_{4D} = \mathcal{G}_{SM} \times U(1)_Z$							
6A	SU(6)	$\Psi_{15} \supset q_L + d_R\,,$ $\Psi_{\overline{15}} \supset l_L + u_R + e_R$	Adj $\supset \varphi_H$ $\Phi_{15} \supset \varphi'_H$	d, e GHU u bulk	≤ 3	2HDM Type-II	Yes
6A flip	SU(6)	$\Psi_{20} \supset q_L + u_R + e_R\,,$ $\Psi_6 \supset d_R\,,\ \Psi_{\bar{6}} \supset l_L + \nu_R$	Adj $\supset \varphi_H$ $\Phi_{15} \supset \varphi'_H$	u GHU d, e bulk	≤ 3	2HDM Type-II	Yes
6S	SU(6)	$\Psi_{21} \supset q_L + d_R\,,$ $\Psi_{\overline{15}} \supset l_L + u_R + e_R$	Adj $\supset \varphi_H$ $\Phi_{105} \supset \varphi'_H + \ldots$	d, e GHU u bulk	1	2HDM Type-II	No
6A′	SU(6)	$\Psi_{15} \supset q_L + l^c_L\,,$ $\Psi_{\overline{15}} \supset u_R + e_R + d^c_R$	$\Phi_{15} \supset \varphi_H$	u bulk	≤ 3	SM-like	No
6S′	SU(6)	$\Psi_{21} \supset q_L + l^c_L\,,$ $\Psi_{\overline{15}} \supset u_R + e_R + d^c_R$	$\Phi_{105} \supset \varphi_H + \ldots$	u bulk	1	SM-like	No

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Minimal SU(N) models

(not final)

Name	$\mathcal{G}_{\text{bulk}}$	Fermions	Scalars	Yukawas	n_g bulk	Higgs sector	Minimal?
Yukawa Landau Poles!!! $\mathcal{G}_{4D} = \mathcal{G}_{SM}$							
~~5A~~	~~SU(5)~~	~~$\Psi_5 \supset d_R\,,\ \ \Psi_{\bar{5}} \supset l_L\,,$ $\Psi_{10} \supset q_L\,,\ \ \Psi_{\overline{10}} \supset u_R + e_R$~~	~~$\Phi_5 \supset \varphi_H$~~	~~All bulk~~	~~≤ 3~~	~~SM-like~~	~~Yes~~
5S	SU(5)	$\Psi_5 \supset d_R\,,\ \ \Psi_{\bar{5}} \supset l_L\,,$ $\Psi_{15} \supset q_L\,,\ \ \Psi_{\overline{10}} \supset u_R + e_R$	$\Phi_5 \supset \varphi_H$ $\Phi_{\overline{45}} \supset \varphi'_H + \ldots$	All bulk	1	2HDM Type-II or flip	No
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6S	SU(6)	$\Psi_{21} \supset q_L + d_R\,,$ $\Psi_{\overline{15}} \supset l_L + u_R + e_R$	Adj $\supset \varphi_H$ $\Phi_{105} \supset \varphi'_H + \ldots$	d, e GHU u bulk	1	2HDM Type-II	No
6A′	SU(6)	$\Psi_{15} \supset q_L + l_L^c\,,$ $\Psi_{\overline{15}} \supset u_R + e_R + d_R^c$	$\Phi_{15} \supset \varphi_H$	u bulk	≤ 3	SM-like	No
6S′	SU(6)	$\Psi_{21} \supset q_L + l_L^c\,,$ $\Psi_{\overline{15}} \supset u_R + e_R + d_R^c$	$\Phi_{105} \supset \varphi_H + \ldots$	u bulk	1	SM-like	No

Yukawa running

The bulk ERG for Yukawa couplings read:

$$2\pi \frac{d}{d \ln \mu} \tilde{\alpha}_y = \left(2\pi + \sum_{y'} c_{yy'} \tilde{\alpha}_{y'} - d_y \tilde{\alpha} \right) \tilde{\alpha}_y \,,$$

$$\tilde{\alpha}_y = R\mu \frac{y^2}{4\pi}$$

Fixed points exist iff the solutions

$$\tilde{\alpha}^*_y = \sum_{y'} c^{-1}_{yy'} \left(d_{y'} \tilde{\alpha}^* - 2\pi \right) ,$$

are all positive!

Yukawa running

6A	SU(6)	$\Psi_{15} \supset q_L + d_R$, $\Psi_{\overline{15}} \supset l_L + u_R + e_R$	Adj $\supset \varphi_H$ $\Phi_{15} \supset \varphi'_H$	d, e GHU u bulk	≤ 3	2HDM Type-II	Yes

$$\mathcal{L}_{\text{Yuk}} = -Y_u\, \overline{\Psi}_{\overline{15}} \Psi_{15} \Phi_{15} - Y_\nu\, \overline{\Psi}_1 \Psi_{\overline{15}} \Phi_{15} - Y_\chi\, \bar{\Psi}_{15} \Psi_{1'} \Phi_{15} + \text{h.c.}$$

We added two singlets: one for right-handed neutrinos, and the other for Indalo DM.

The other Yukawas (d and e) inherit the gauge fixed point thanks to GHU.

$$\tilde{\alpha}^* = \frac{6\,\pi}{13}\,, \quad \tilde{\alpha}_u^* = \frac{415\,\pi}{5538}\,, \quad \tilde{\alpha}_\nu^* = \tilde{\alpha}_\chi^* = \frac{122\,\pi}{923}\,.$$

Complete FPs found for $n_g = 3$!

Yukawa running

6A flip	SU(6)	$\Psi_{20} \supset q_L + u_R + e_R\,,$ $\Psi_6 \supset d_R\,,\ \ \Psi_{\bar{6}} \supset l_L + \nu_R$	$\text{Adj} \supset \varphi_H$ $\Phi_{15} \supset \varphi'_H$	u GHU d, e bulk	≤ 3	2HDM Type-II	Yes

$$\mathcal{L}_{\text{Yuk}} = -Y_d\, \overline{\Psi}_{20}\Psi_6\Phi_{15} - Y_l\, \overline{\Psi}_{\bar{6}}\Psi_{20}\Phi_{15} + \text{h.c.}$$

Singlets (right-handed neutrinos Indalo DM) embedded in the 6's.

The other Yukawas (u and ν) inherit the gauge fixed point thanks to GHU.

$$\tilde{\alpha}^* = \frac{6\,\pi}{13}\,, \quad \tilde{\alpha}^*_d = \frac{235\,\pi}{156}\,, \quad \tilde{\alpha}^*_l = \frac{235\,\pi}{5616}\,.$$

Complete FPs found for $n_g \leq 3$!

Minimal SU(N) models
(final)

Name	$\mathcal{G}_{\rm bulk}$	Fermions	Scalars	Yukawas	n_g bulk	Higgs	UV fixed points ($n_g = 3$)
$\mathcal{G}_{\rm 4D} = \mathcal{G}_{\rm SM} \times {\rm U}(1)_Z$							
6A	SU(6)	$\Psi_{15} \supset q_L + d_R\,, \Psi_1 \supset \nu_R\,,$ $\Psi_{\overline{15}} \supset l_L + u_R + e_R\,, \Psi_{1'}$	Adj $\supset \varphi_H$ $\Phi_{15} \supset \varphi'_H$	d, e GHU u, ν bulk	3	2HDM Type-II	$\tilde{\alpha}^* = \tilde{\alpha}^*_d = \tilde{\alpha}_l = \frac{6\,\pi}{13}$ $\tilde{\alpha}^*_u = \frac{415\,\pi}{5538}\,,\ \tilde{\alpha}^*_\nu = \tilde{\alpha}^*_\chi = \frac{122\,\pi}{923}$
6A flip	SU(6)	$\Psi_{20} \supset q_L + u_R + e_R\,,$ $\Psi_6 \supset d_R\,,\ \Psi_{\bar{6}} \supset l_L + \nu_R$	Adj $\supset \varphi_H$ $\Phi_{15} \supset \varphi'_H$	u, ν GHU d, e bulk	≤ 3	2HDM Type-II	$\tilde{\alpha}^* = \tilde{\alpha}^*_u = \tilde{\alpha}^*_\nu = \frac{6\,\pi}{13}$ $\tilde{\alpha}^*_d = 36\,\tilde{\alpha}^*_l = \frac{235\,\pi}{156}$

- Only two viable models found!
- Both have two Higgs doublets, one of which of gauge origins.
- Both models allow for 3 bulk generations with Baryon number conservation and Indalo DM!

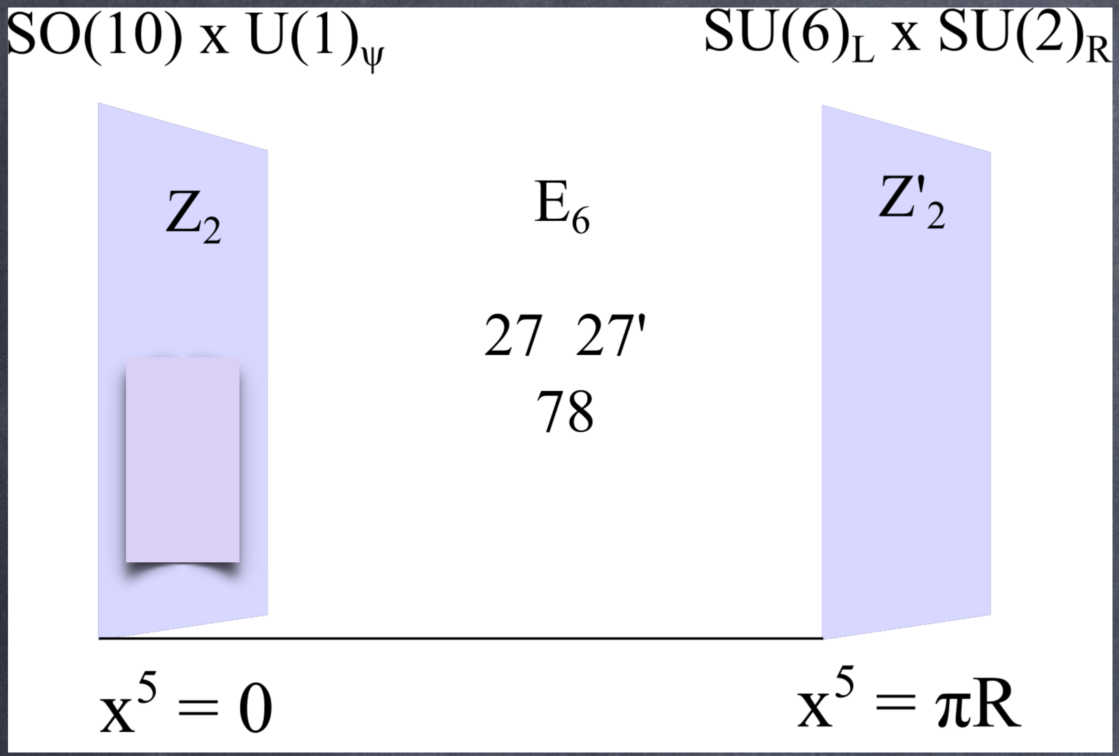
A more ambitious model

- Supersymmetry allows to generate fermions as gauge fields (gauginos)
- In E6, the adjoint 78 contains the right states (but in vector-like pairs)

See Kobayashi, Raby, Zhang, Nucl. Phys. B704, 3 (2005)

The exceptional case

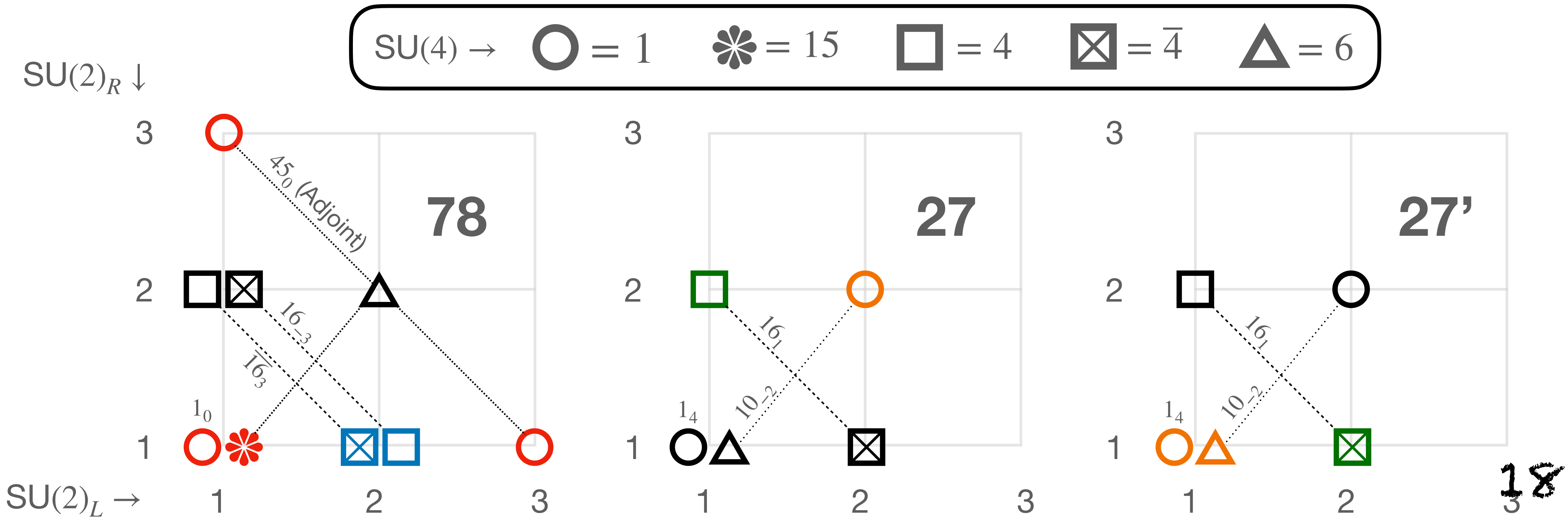
Cacciapaglia et al, 2302.11671



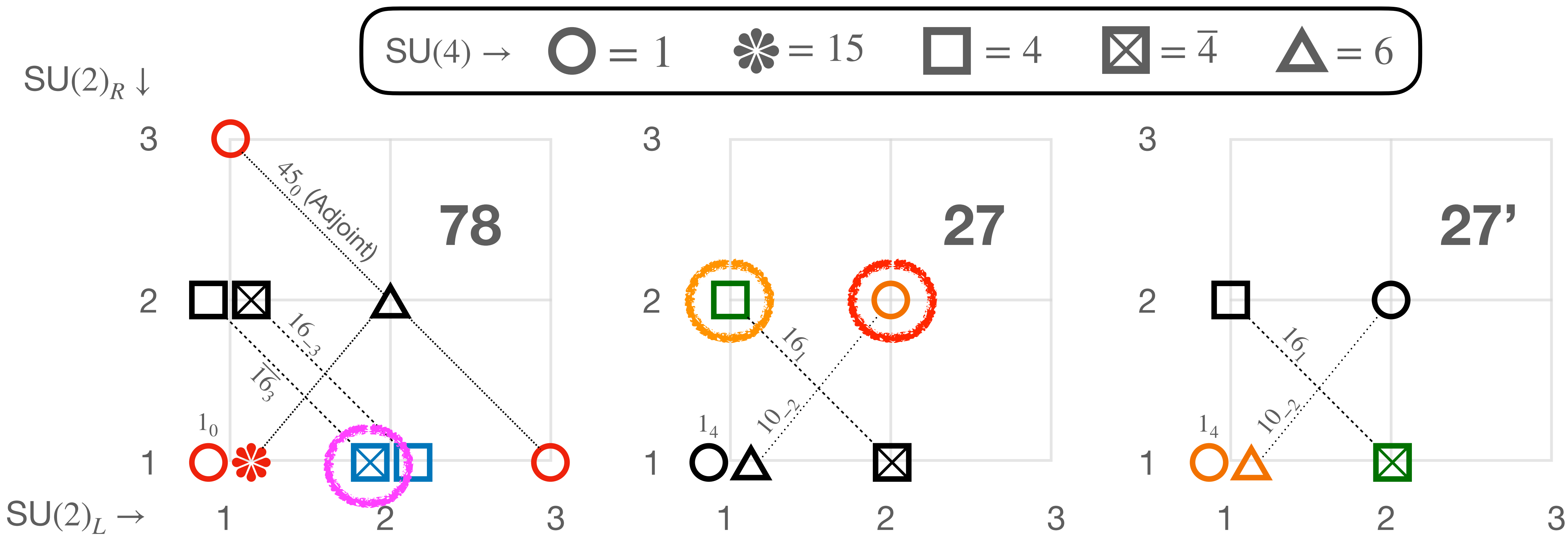
4D gauge symmetry:

$SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_\psi$

Bulk interactions limited To (SUSY) gauge!



The exceptional case



$$g\,\Phi^c_{27}\Phi_{78}\Phi_{27} \supset \frac{g}{\sqrt{2}}(\mathbf{1},\mathbf{2},\mathbf{2})_2(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-3}(\mathbf{4},\mathbf{2},\mathbf{1})_1$$

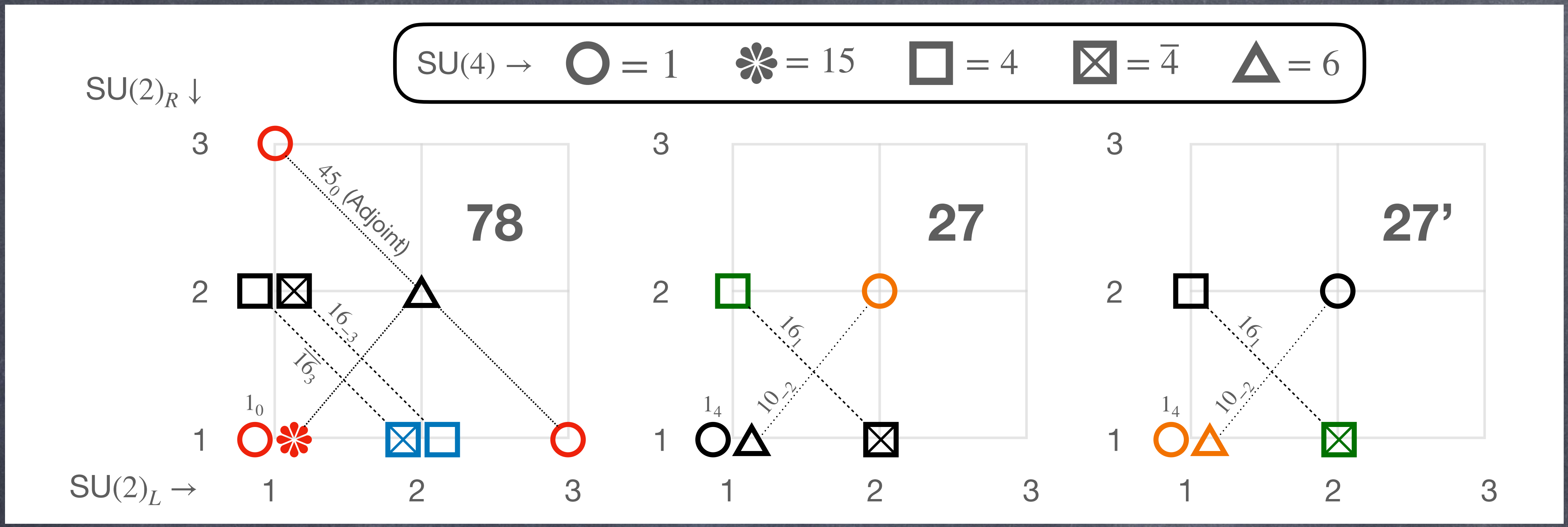
-> SM Yukawa couplings!

$$g\,\Phi^c_{27'}\Phi_{78}\Phi_{27'} \supset -\frac{g}{\sqrt{2}}(\mathbf{1},\mathbf{1},\mathbf{1})_{-4}\,(\mathbf{4},\mathbf{1},\mathbf{2})_3\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_1 + \frac{g}{\sqrt{2}}(\mathbf{6},\mathbf{1},\mathbf{1})_2\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{-3}\,(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_1$$

-> Gives mass to unwanted Chiral states via U(1) breaking

Bulk interactions preserve Baryon number!

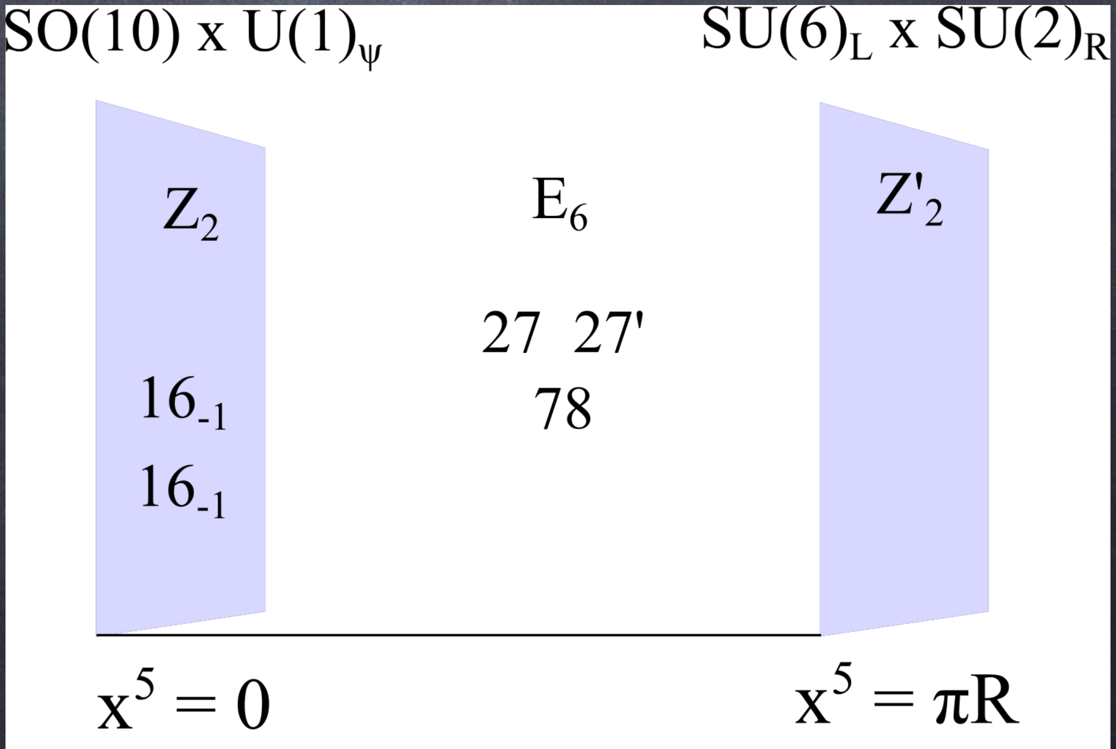
Predicting the light generations (from gauge anomalies)



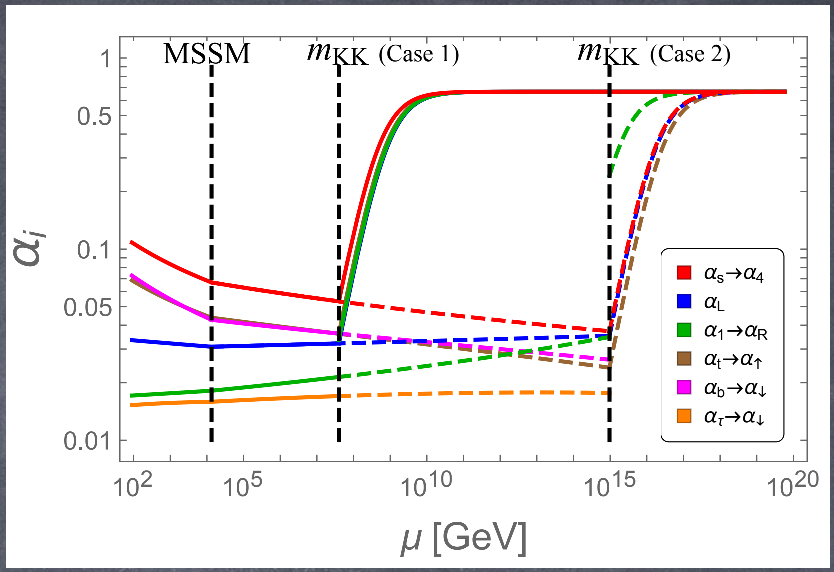
- The zero modes generate an anomaly for the U(1) gauge symmetry:

$$\mathscr{A}_{16_1} - \mathscr{A}_{10_{-2}+1_4} = 2\mathscr{A}_{16_1}$$

- Add exactly two generations on the SO(10) boundary!



Conclusions and perspectives



- Asymptotic GUT is a novel paradigm, avoiding many shortcomings of traditional GUTs
- 5D models are very constrained and successful cases can be classified
- The SU(N) kinship only allows for two minimal models
- SO(N), Sp(N) and exceptional groups under way
- Non-minimal cases also interesting: e.g. SUSY E6 model with complete unification for one generation

Bonus tracks

The Yukawa sector runs

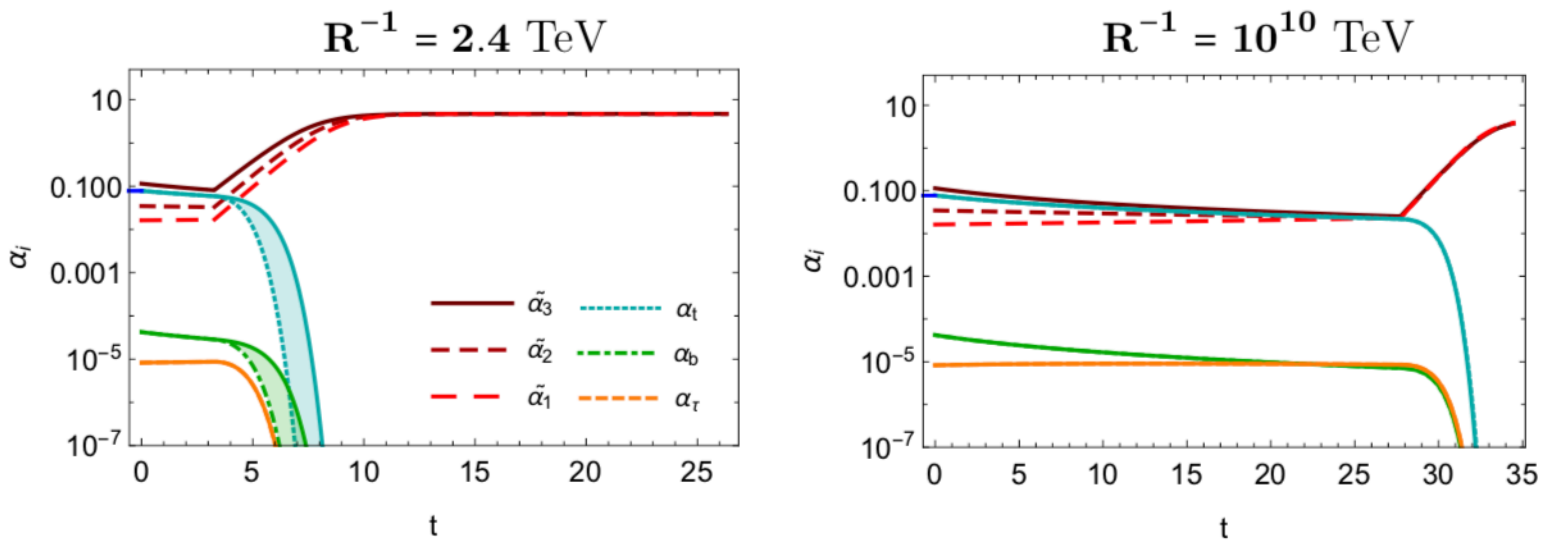


Figure 3. Running of the localized Yukawa couplings compared to the bulk gauge ones for two sample values of the compactification scale. The bands indicate the uncertainty related to KK gauge couplings (see text). The largest value of t corresponds to the 5D Planck mass value.

Localised Yukawas - SU(5) brane

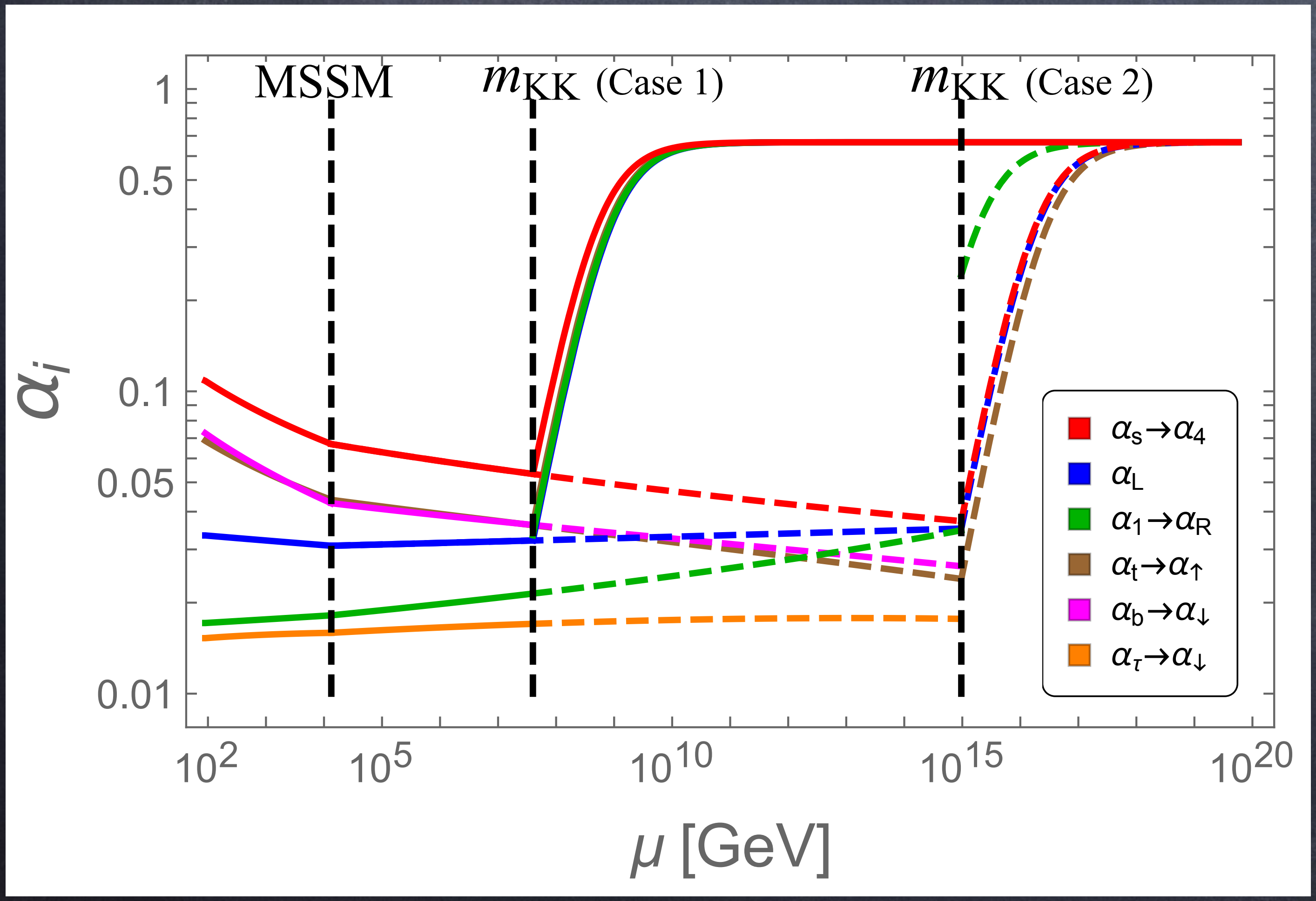
The fixed point

$$b_5 = -\frac{\pi}{2}\left(C(G) - \sum_i T_i(R_i) \right) = -3\pi$$

$$C(G) = 12 \quad T(27) = 3$$

$$\tilde{\alpha}^* = \frac{2}{3}$$

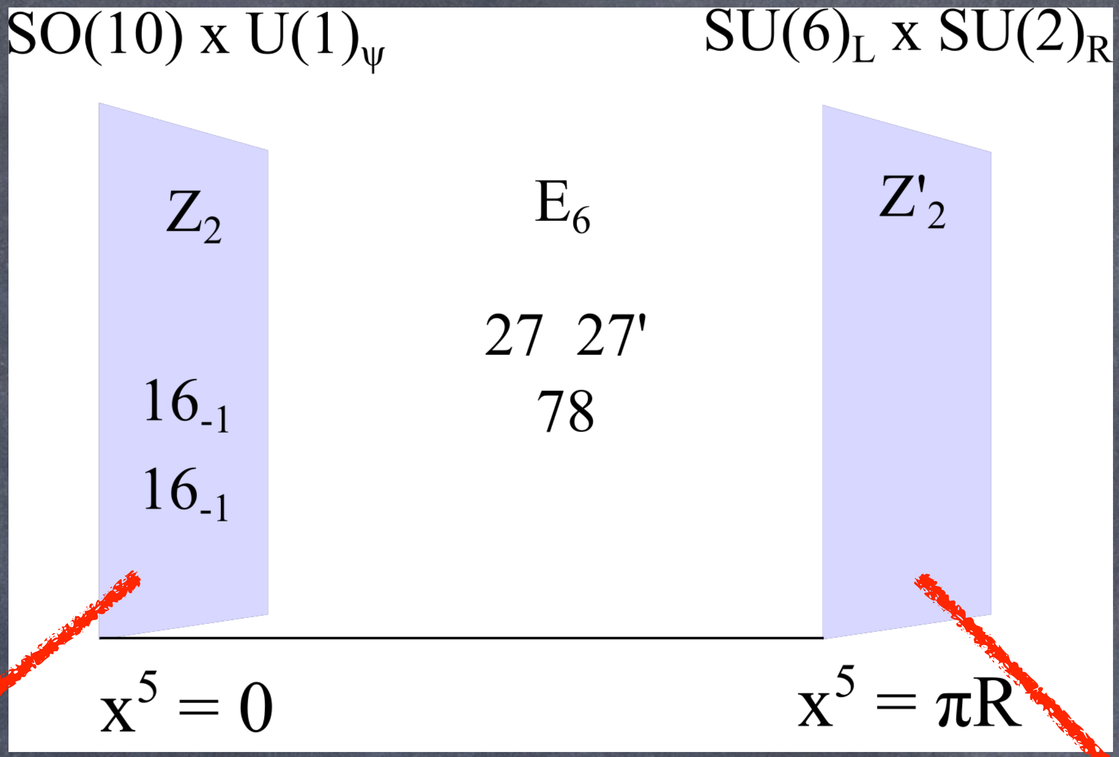
No more than one generation allowed in the bulk!



- PS breaking due to a gauge-scalar
- U(1) breaking by singlet in 27′
- SUSY breaking to be studied

Two model avenues:

SO(10) gens



One gen in (15,1)+(6,2)

Model 1 :

- Predicts 2 generations
- "Usual" SO(10) model building allowed
- Scale pushed high by proton decay

Model 2 :

- Light generations preserve baryon number
- Number of generations not predicted
- Scale can be lowered (1000's TeV) from PS breaking