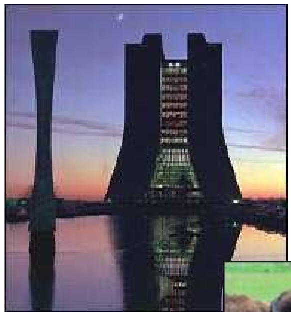
Top Mass Measurement with the Matrix Element Method at TeVatron



Workshop on Top Physics:
from the TeVatron to the LHC

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Outline

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  - Mass Measurement and Statistical Uncertainty Calibration
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- The Dilepton Channel
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Brief Genesis

- In methods using templates for events with missing energy (for instance $W^- \rightarrow e^- \nu$), the transverse variables are mainly used in hadronic collisions. Using the full kinematics would be better
- The matrix element method has been developed to make use of the full kinematics combined with the dynamics of the process in an event probability
- This method has been discussed for the first time for the top mass measurement by K. Kondo in 1990
- The matrix element method has been used for the first time in hadronic collisions to rederive the top mass with the Run I data at D0 and has improved the statistical uncertainty by 50%

The Matrix Element Method

- A signal probability per event is computed using the theoretical description of the $t\bar{t}$ production and decay (matrix element), for a given hypothesis m_t, and a set of measured quantities x

$$P_{sig}(x, m_t) = \frac{1}{N} \int_{y, q_1, q_2} d\Phi_6 \left| M_{t\bar{t}}(m_t, y) \right|^2 W(x, y) f_{PDF}(q_1) f_{PDF}(q_2) dq_1 dq_2$$

  – Normalization N: observed cross-section in the detector acceptance
  – Leading order matrix element for the $t\bar{t}$ production and decay into the partonic state y
  – Transfer function (TF): probability to measure a set of quantities x when the partonic final state is y (take into account the detector resolution and hadronization for the final partons). For well measured quantities, the TF is a delta function
  – Parton distribution functions
- All final state particles 4-momenta are needed to compute this probability, while the system is underconstrained: the remaining unknown variables are integrated over

The Matrix Element Method (2)

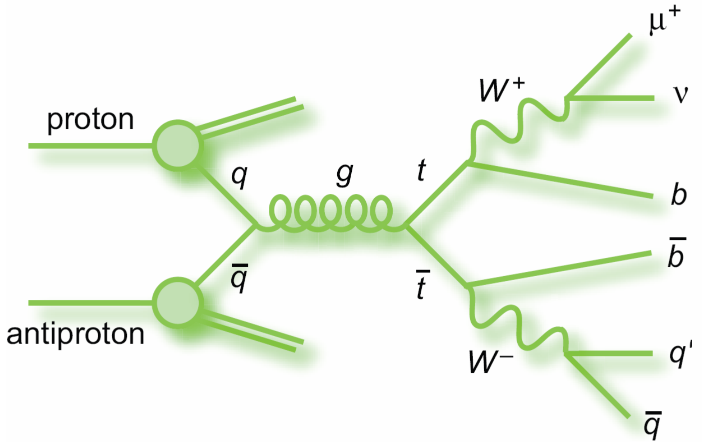
- The backgrounds are taken into account in a generalized probability per event, where the different fractions f are fixed by the selection or fitted

$$P_{evt}(x,m_t) = f_{sig}P_{sig} + f_{bkg_1}P_{bkg_1} + \ldots + f_{bkg_n}P_{bkg_n}$$

- Finally, the measured top mass is obtained minimizing the likelihood with respect to the top mass m_t:

$$\mathcal{L}(m_t) = -\ln \prod_{events} P_{evt}(x,m_t)$$

- All possible jet permutations are added in the event probability computation (12 for the lepton+jet channel !)



The Matrix Element Method (3)

- In each analysis, several assumptions are made to simplify the computation:
  - the lepton angles are supposed to be perfectly measured (the transfer functions are delta functions)
  - the angles of the jets are supposed to be perfectly measured (the transfer functions are delta functions)
  - light quark masses are 0
  - depending on the analysis, additional assumptions can be made

- Multidimensional integration is performed by the Monte Carlo method (VEGAS)
- To better control the integrations and to reduce integration time, a variable change is done. Usually variables are chosen to have the narrowest range to save integration time, such as W (from the top decay) mass for example

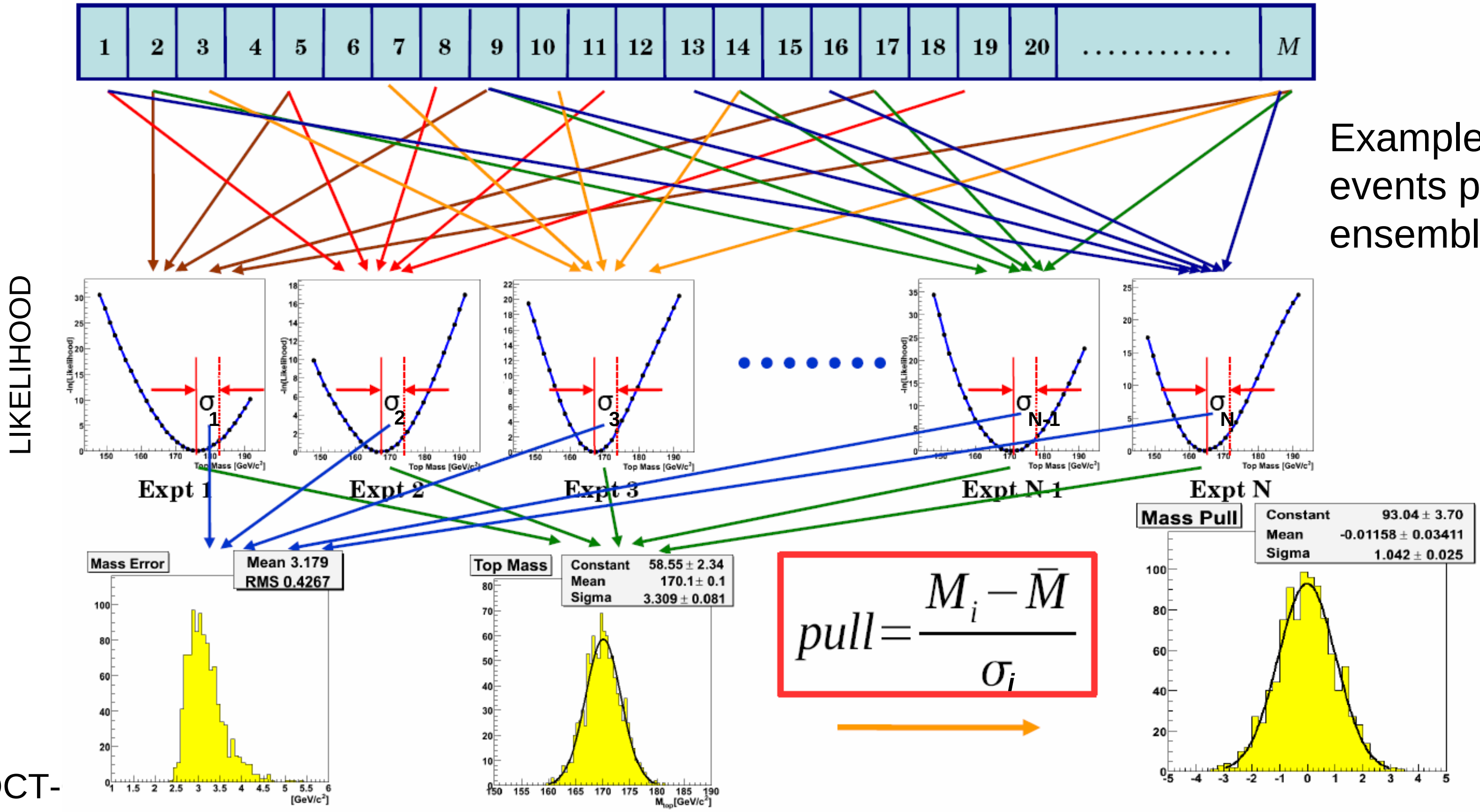
Method Steps

- For each integration point:
  - The 4 momenta of the top pair decay products are calculated using the values of the integration variables and the measured jet and lepton angles (and the electron energy if no TF is applied)
  - The matrix element is calculated
  - The PDF are evaluated, summing over all possible quark flavor
  - The probability to observe the measured jet energies and muon momentum is evaluated using the transfer functions
  - The jacobian determinant of the variable change is included in the computation

Ensemble Testing

- This method is used to calibrate both the mass measurement and the statistical uncertainty
- N ensembles of n events are formed from the M available MC events (n should correspond to the selected number of data events)
  - Each MC event can be used several times in each ensemble and in several ensembles

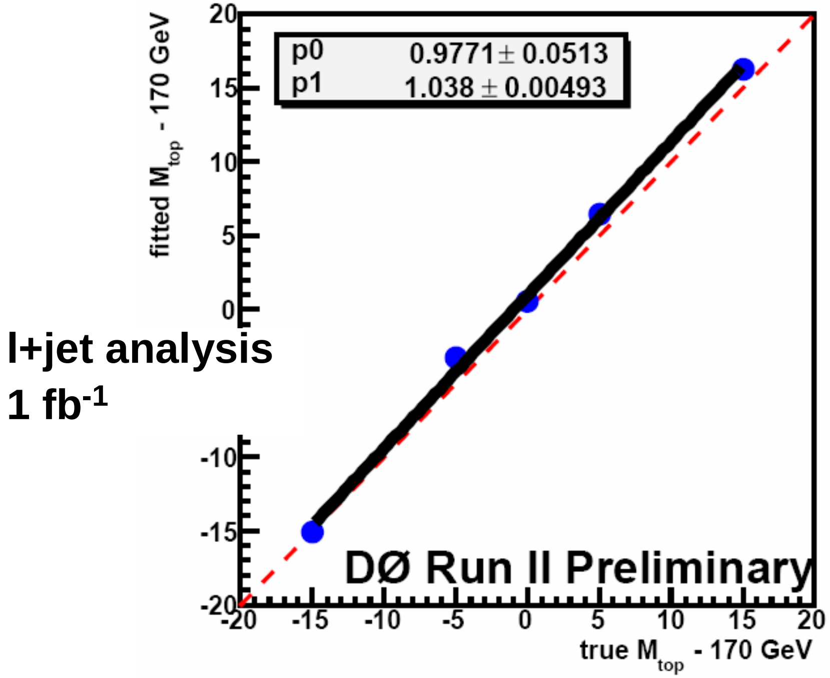
For a given input mass m_t



Example: 5 events per ensemble

Mass Measurement Calibration

- For possible biases correction (due to the simplification hypothesis made in the computation of the probability), we need to calibrate the response with MC
- The calibration is obtained by extracting the top mass from a large number of ensembles for different input masses, and plotting the fitted mass wrt the input mass
- A fit gives the offset p0 (taken in this example at m_t = 170 GeV/c^2) and the slope p1 of the calibration

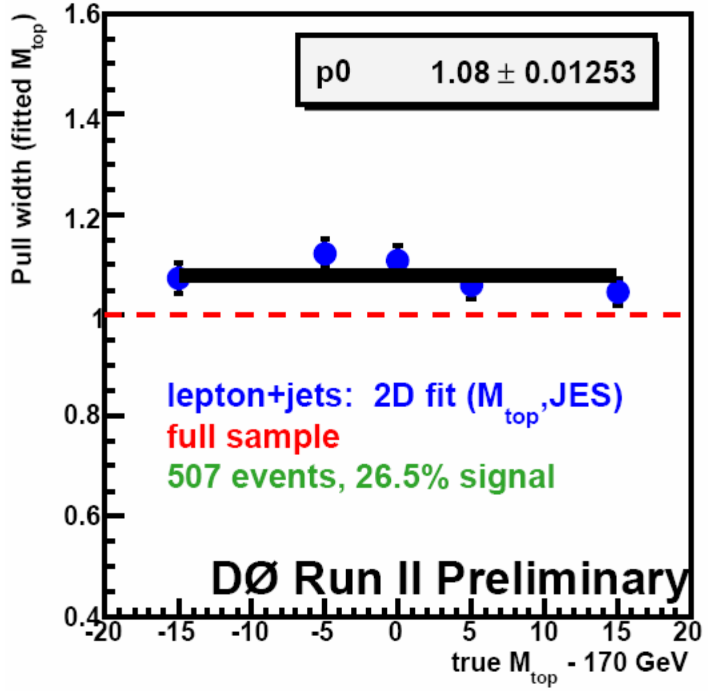


A correction factor is then applied to the measurement performed in data

Statistical Uncertainty Calibration

- For the same reasons, the statistical error extracted from the likelihood can be biased
- From the MC events, the pull width for each input top mass is plotted as a function of the input top mass, and fitted.
- In this example, the statistical error is underestimated by 8%

$$pull = \frac{M_i - \overline{M}}{\sigma_i}$$



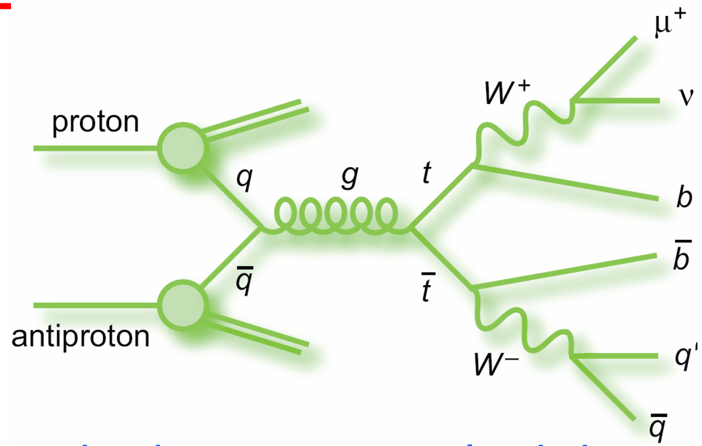
A correction factor is then applied to the statistical error obtained in data

Assets & Drawbacks

- Main assets of the Matrix Element Method:
  - This method uses all the measured quantities in the event
  - The major asset of the matrix element method over other mass measurement methods is the statistical uncertainty improvement mainly due to the per-event probability that gives a higher weight to the better measured events

- But…
  - The matrix element method implies huge CPU consumption due to the multiple integrations: for example, D∅ lepton+jets 1 fb^{-1} analysis uses ~ 400 * 2GHz computers during 1 month !

The Lepton+Jet Channel Specificities



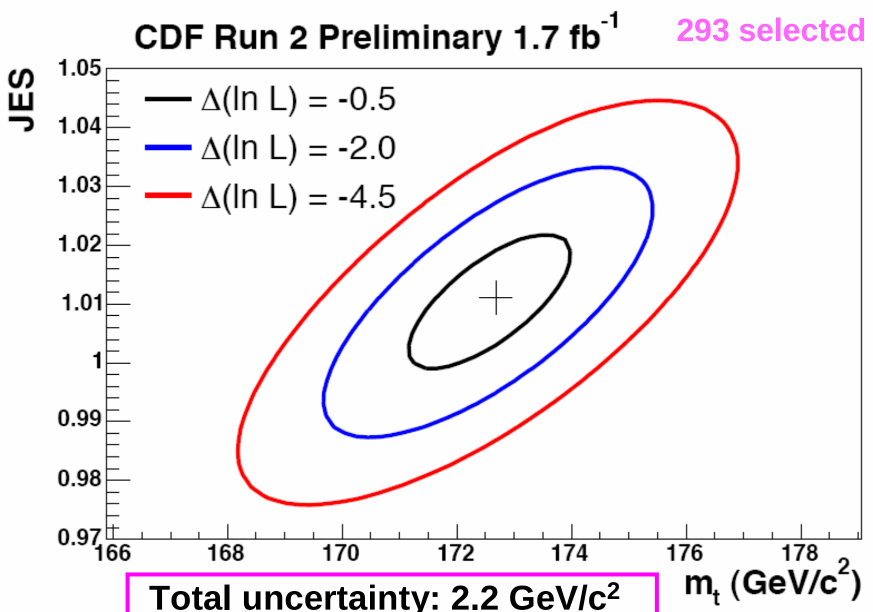
- The b-tagging information is used during the selection
- Exactly 4 jets are required
- To reduce the main systematic uncertainty due to the jet energy scale, it is calibrated in-situ using the W mass from the hadronic branch as a constraint. A 2D likelihood is then computed with an additional tested parameter, JES: a multiplicative factor applied to the jet energy
- During the probability computation, a weight w^j is applied to each jet-parton assignment j, taking into account for the b-tagging information

$$P_{sig}(x, m_t, JES) = \sum_{permut_j} w^j P^j_{sig}(x, m_t, JES)$$

- The top quark mass is obtained by minimizing a 1D likelihood
  - CDF: taking the minimum $\mathcal{L}$ value along the JES axis for each m_t $\quad \mathcal{L}(x, m_t) = \min_{j \in JES} \mathcal{L}(x, m_t, JES)$
  - D∅: projecting the 2D $\mathcal{L}$ on the m_t axis $\quad \mathcal{L}(x, m_t) = \int \mathcal{L}(x, m_t, JES) d(JES)$

CDF Measurement in the Lepton+Jets Channel

- Additional hypothesis: the hadronic b quark mass is neglected in the leptonic branch, but not in the hadronic one, lepton momenta are perfectly measured
- Chosen set of variables to integrate over:
  - 2 squared top masses and 2 squared W masses
  - $\beta=\log(p_1/p_2)$ where p_1 and p_2 are the quarks momenta from the hadronic W decay
  - $p_T(t\bar{t})$: 2 dimensional transverse momentum



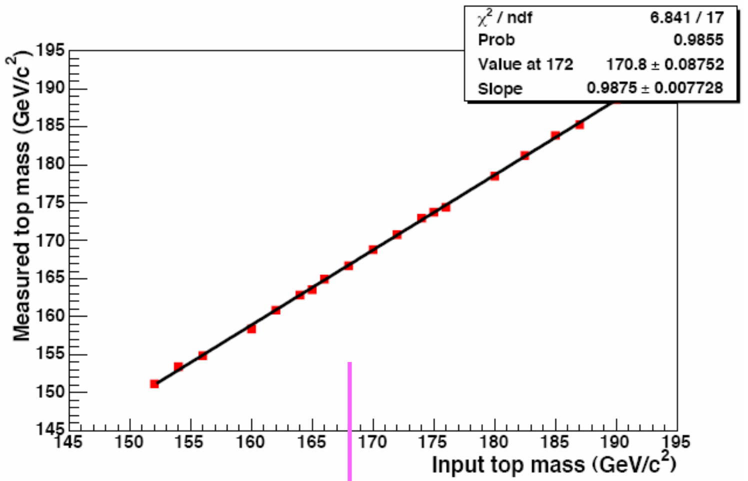
Total uncertainty: 2.2 GeV/c^2
Relative uncertainty: 1.3%

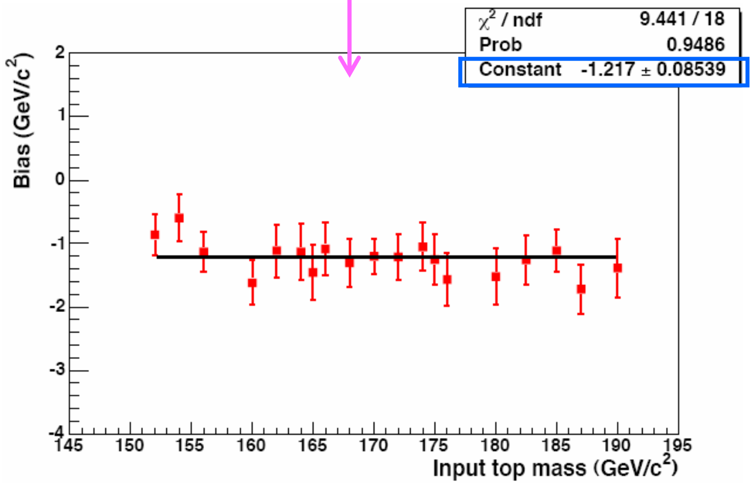
Systematic source	Systematic uncertainty (GeV)
Calibration	0.09
MC generator	0.19 ± 0.36
ISR	0.26 ± 0.37
FSR	0.13 ± 0.38
Residual JES	0.53
b-JES	0.36
Lepton P_T	0.11
Permutation weighting	0.03
Multiple interactions	0.05
PDFs	0.25
Background: fraction	0.33
Background: composition	0.39
Background: average shape	0.31
Background: Q^2	0.07 ± 0.20
Gluon fraction	0.14
b-tag E_T dependence	0.16
Total	1.16

$$\boxed{M_{\rm top} = 172.7 \pm 1.3\ (\text{stat.}) \pm 1.2\ (\text{JES}) \pm 1.2\ (\text{syst.})\ \text{GeV}/c^2}$$

Method Calibration

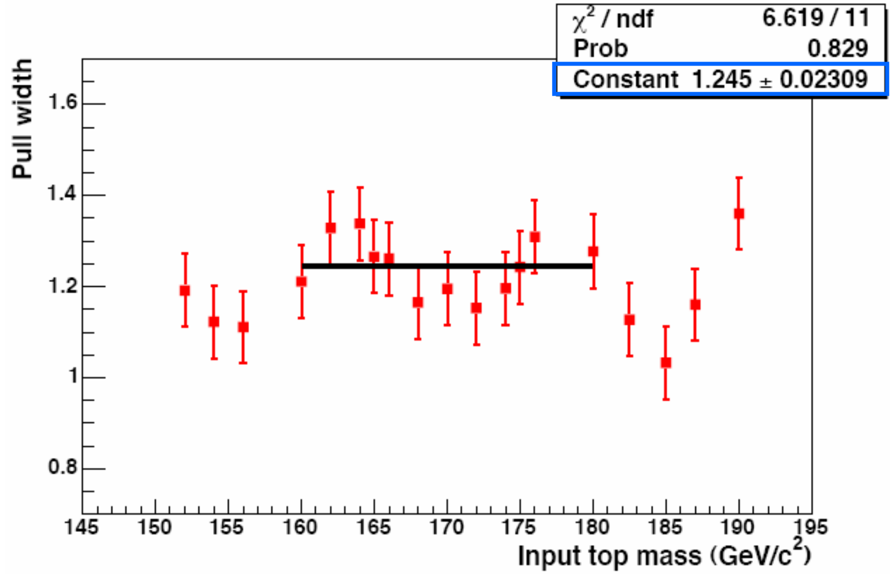
- Mass measurement calibration





The mass measurement is biased by -1.22 GeV/c^2

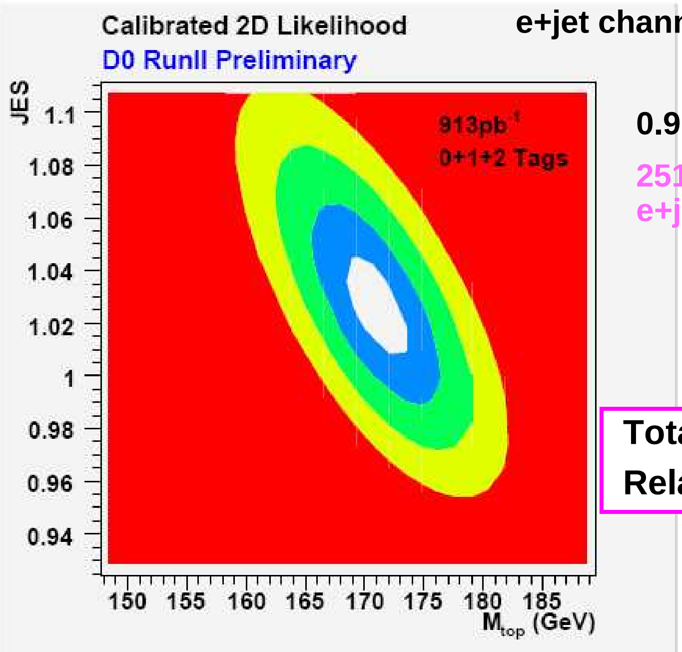
- Statistical uncertainty calibration



The measured statistical uncertainty is scaled by a factor of 1.245

DØ Measurement in the Lepton+Jets Chanel

- Additional hypothesis: the lepton resolutions are integrated over
- Chosen set of variables to integrate over:
  - 2 squared top masses and the squared hadronic W mass
  - the momentum of one of the quarks from the hadronic W decay
  - longitudinal momentum of the (b+ν) system from the leptonic branch



0.9 fb^{-1}

251 selected events in the e+jets channel

Total uncertainty: 2.7 GeV/c^2
Relative uncertainty: 1.6%

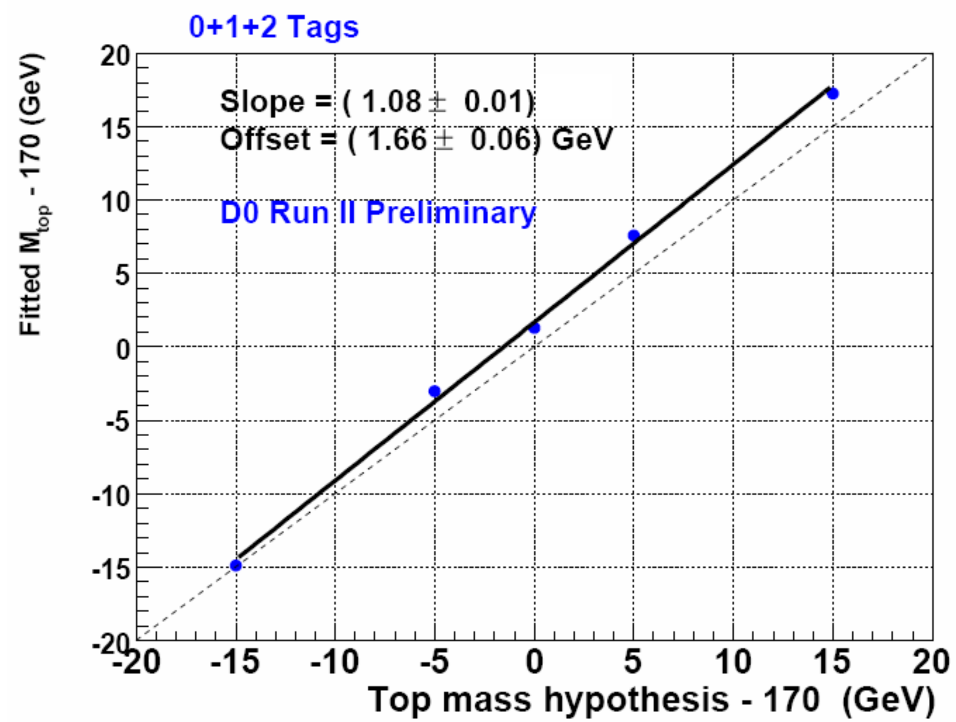
Error Source	Topological Analysis	b Tagging Analysis
statistical error and jet energy scale	± 2.5	± 2.4
physics modeling:		
signal modeling	± 0.98	± 0.45
background modeling	± 0.47	± 0.15
PDF uncertainty	$+0.26\ -0.40$	$+0.26\ -0.40$
b fragmentation	± 0.14	± 0.54
b/c semileptonic decays	$+0.06\ -0.07$	± 0.05
detector modeling:		
JES p_T dependence	± 0.14	± 0.23
b response (h/e)	± 0.71	± 0.57
trigger	± 0.08	± 0.08
method:		
signal fraction	± 0.15	$+0.53\ -0.24$
QCD contamination	± 0.16	± 0.21
MC calibration	± 0.06	± 0.07
b-tagging		± 0.29
total systematic error	± 1.4	± 1.2
total error	± 2.9	± 2.7

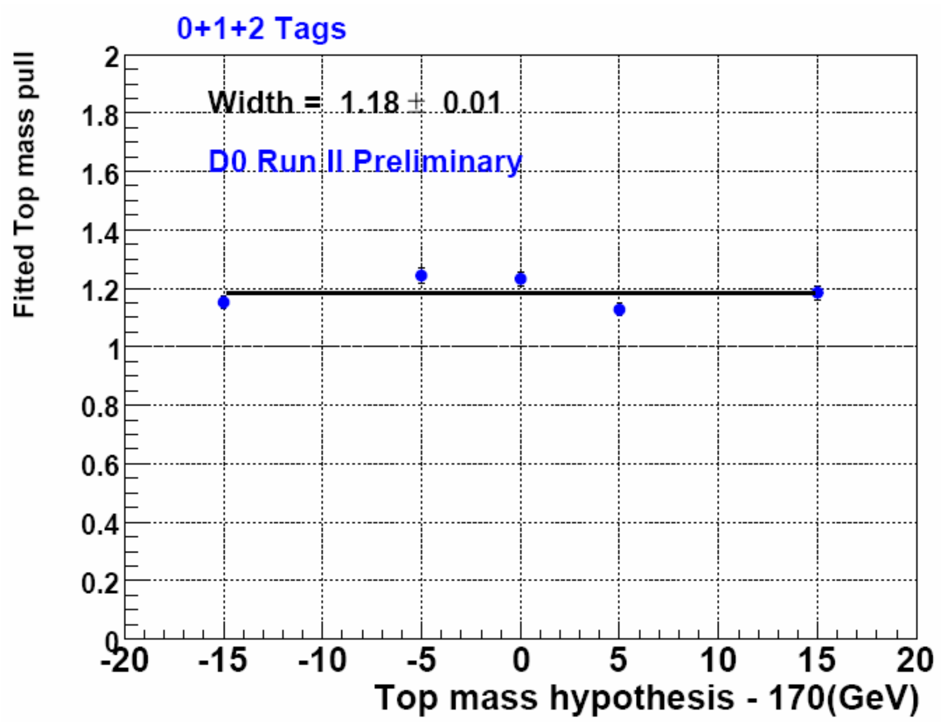
$$m_{\rm top}(b\text{–tag}) = 170.5 \pm 1.8({\rm stat}) \pm 1.6({\rm JES}) \pm 1.2({\rm syst})\ {\rm GeV}$$

Method Calibration

e+jet channel

- Mass measurement calibration
- Statistical uncertainty calibration



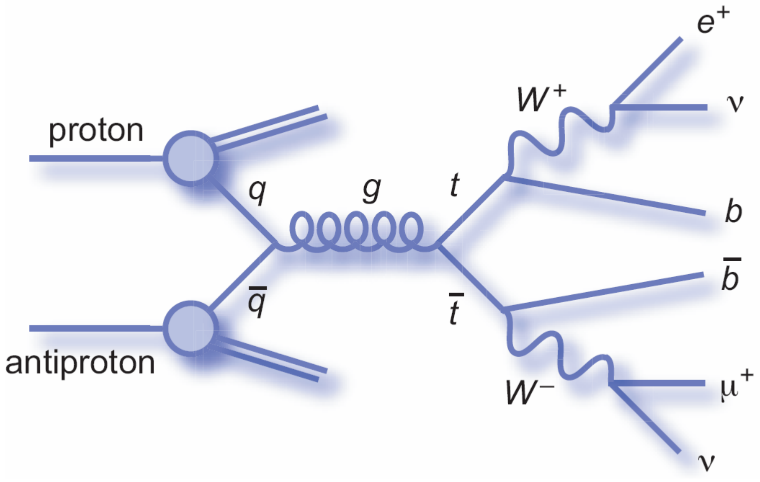


The mass measurement is corrected by the fitted parameters

The measured statistical uncertainty is scaled by a factor of 1.18

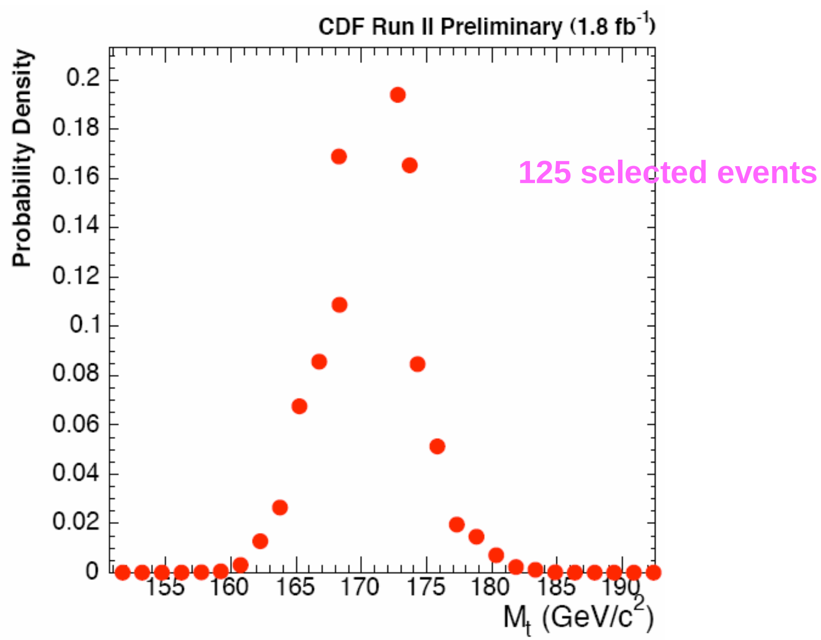
The Dilepton Channel Specificities

- This channel has a smaller branching ratio than the lepton+jets one, but lower backgrounds
- There are only 2 jets in the events, but no hadronic W decay $\rightarrow$ no JES fitting possibility
- 2 neutrinos in the final state $\rightarrow$ more integrations are required



CDF Measurement in the Dilepton Channel

- Additional hypothesis: the 2 leading jets come from the b-quarks, and all lepton momenta are perfectly measured
- Chosen set of variables:
  - 2 squared top masses and 2 squared W masses
  - transverse momentum of the $t\bar{t}$ pair (2 components)



Source	Size (GeV/c^2)
Jet Energy Scale	2.6
Lepton Energy Scale	0.1
Generator	0.6
Method	0.7
Sample composition uncertainty	0.4
Background statistics	0.7
Background modeling	0.3
FSR modeling	0.3
ISR modeling	0.3
PDFs	0.5
Total	3.0

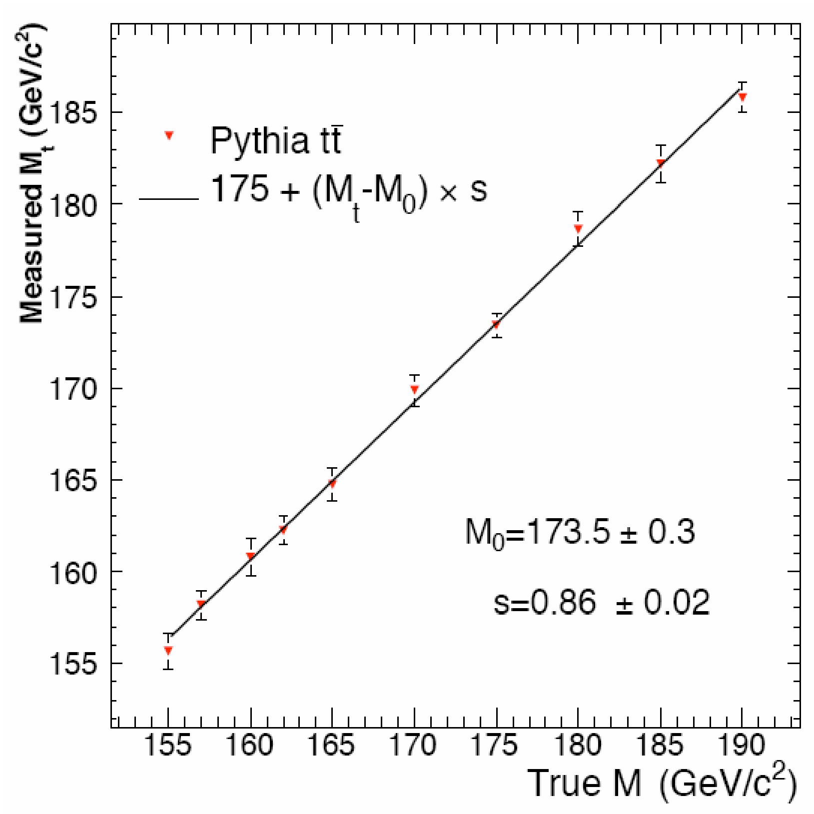
Summary of systematic errors

$$M_{top} = 170.4 \pm 3.1(\text{stat.}) \pm 3.0(\text{syst.})\ \text{GeV}/c^2$$

Total uncertainty: 4.3 GeV/c^2
Relative uncertainty: 2.5%

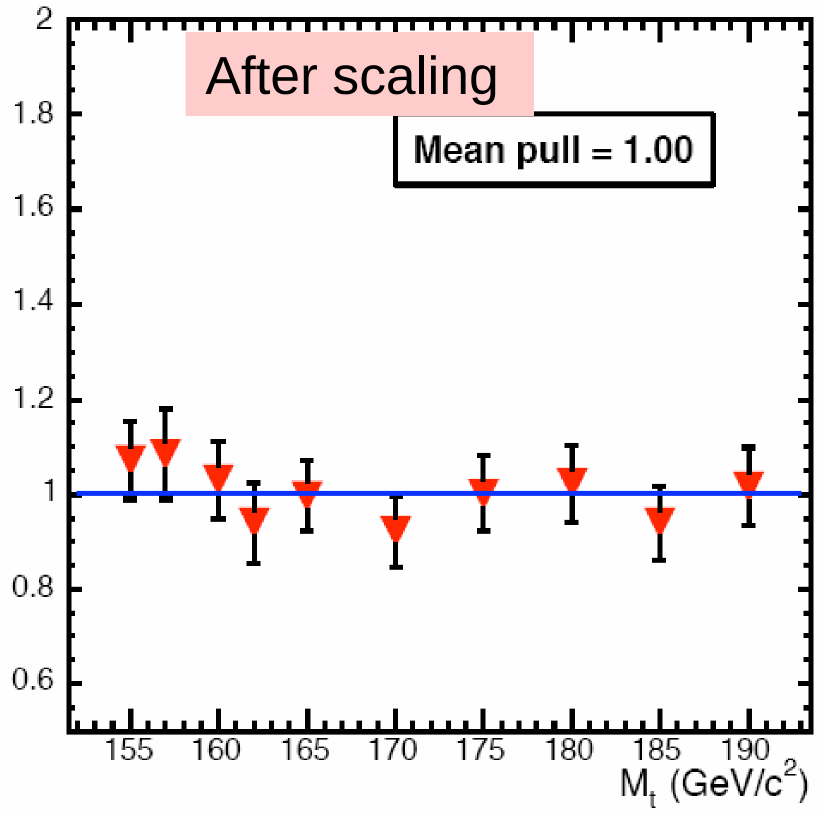
Method Calibration

- Mass measurement calibration



The mass measurement is corrected by the fitted parameters

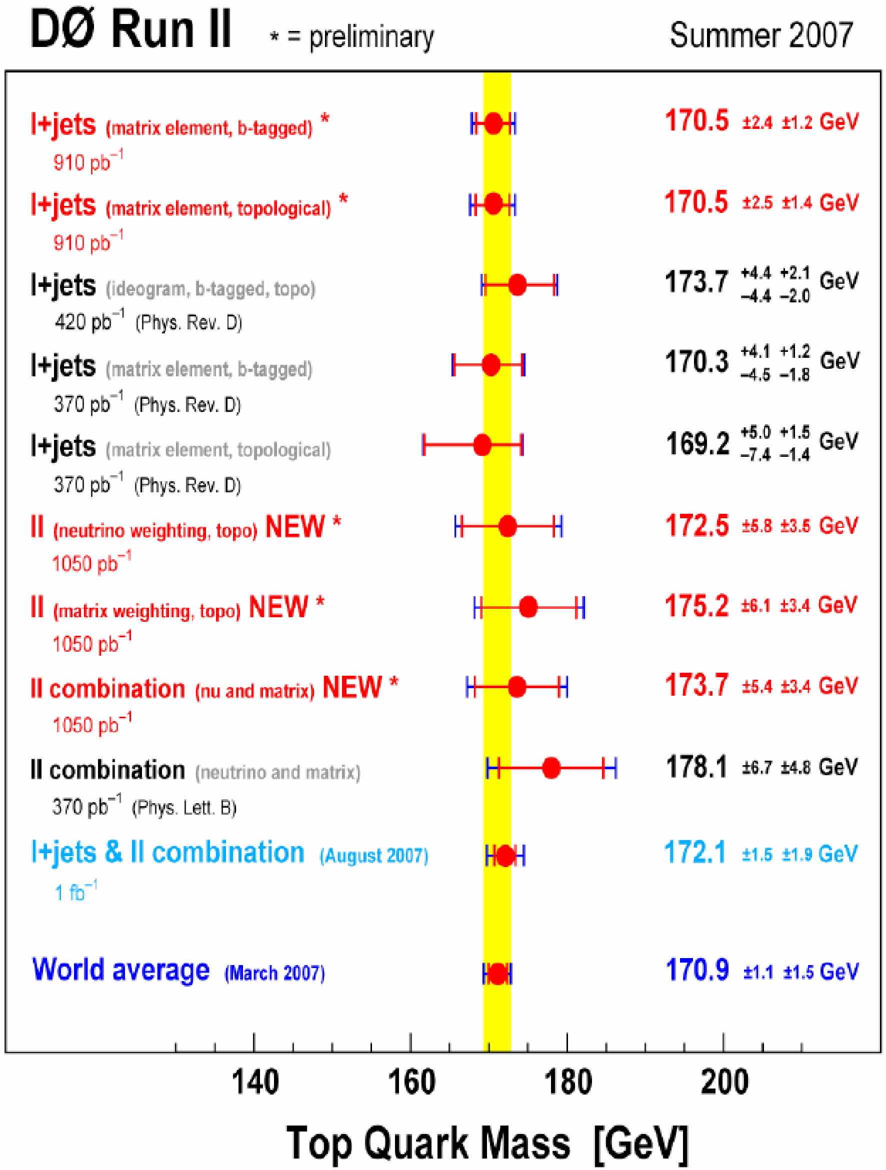
- Statistical uncertainty calibration

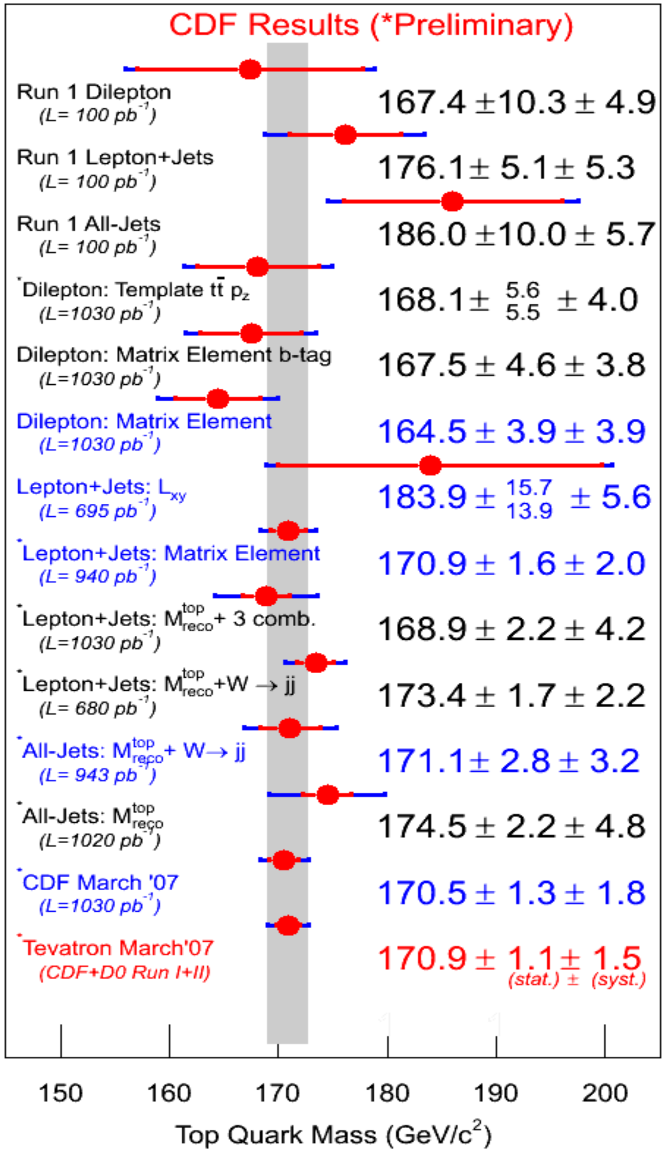


The measured statistical uncertainty is scaled by a factor of 1.11

TeVatron Top Mass Measurements

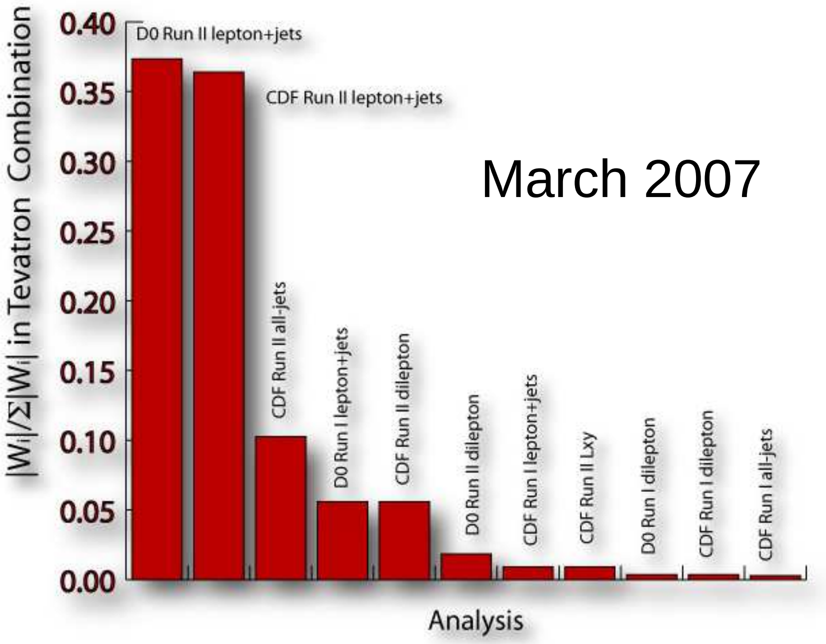
- Matrix Element Method provides the best individual measurements

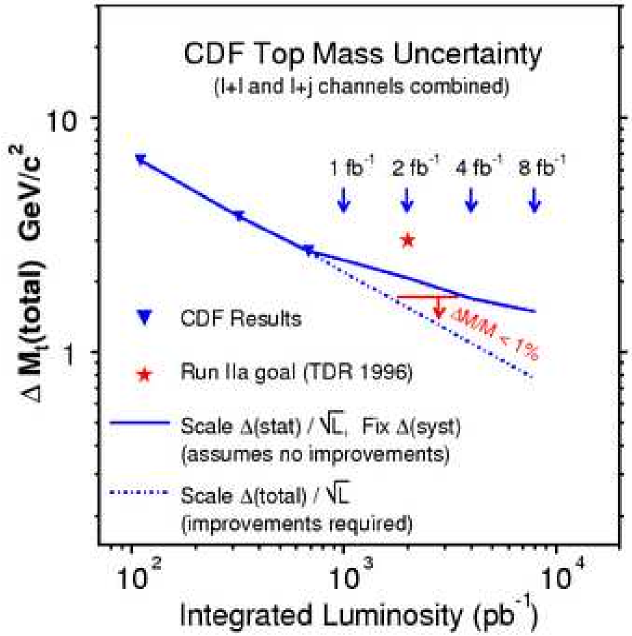




Conclusion

- The matrix element method is a powerful method to measure the top quark mass, but requires to integrate over 7/8 variables (needs a lot of CPU time)
- The systematic uncertainty is reduced in the lepton+jets channel due to the in-situ jet energy scale calibration
- Assuming no improvements, the total uncertainty on the combined dilepton and lepton+jets channel could be ~1.6 GeV/c^2 with ~4 fb^{-1}





Backup: Jet Transfer Function

- The jet TF for JES=1 is (E_x: measured energy, E_y: quark energy)

$$W_{\text{jet}}(E_x, E_y; JES=1) = \frac{1}{\sqrt{2\pi}(p_2 + p_3 p_5)} \left[\exp\left(-\frac{((E_x - E_y) - p_1)^2}{2p_2^2} \right) + p_3 \exp\left(-\frac{((E_x - E_y) - p_4)^2}{2p_5^2} \right) \right]$$

  with p_i functions of the quark energy

$$p_i = a_i + E_y \cdot b_i$$

- The parameters are determined from MC events in 4 regions in η and for 3 different quark varieties: (u,d,s,c), b with an associated soft muon, and all other b

- If JES≠1, the jet TF is

$$W_{\text{jet}}(E_x, E_y; JES) = \frac{W_{\text{jet}}(\frac{E_x}{JES}, E_y; 1)}{JES}$$

Backup: Muon Transfer Function

- The muon TF is given by:

$$W_\mu\left((q/p_T)^{\mu,x},(q/p_T)^{\mu,y}\right)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{(q/p_T)^{\mu,x}-(q/p_T)^{\mu,y}}{\sigma}\right)^2\right)$$

  with q the charge of generated (y) muon or its reconstructed (x) track

- The resolution is obtained from muon tracks in simulated events

$$\sigma=\begin{cases}\sigma_0 & \text{for } |\eta|\leq\eta_0\\ \sqrt{\sigma_0^2+\left[c\left(|\eta|-\eta_0\right)\right]^2} & \text{for } |\eta|>\eta_0\end{cases}$$

Backup: Electron Transfer Function

- Electron TF:

$$W_e\left(E_x, E_y\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{E_x - E_y}{\sigma}\right)^2\right]$$

where

$$\begin{aligned}
E_x &= \text{reconstructed electron energy} \\
E_y &= 1.0002 \cdot E_{\text{true}} + 0.324 \\
\sigma &= \sqrt{(0.028 \cdot E_y)^2 + (S \cdot E_y)^2 + (0.4)^2} \\
S &= \frac{0.164}{\sqrt{E_y}} + \frac{0.122}{E_y} \exp\left(\frac{p_1}{\sin\left\{2 \arctan\left[\exp(-\eta_e)\right]\right\}}\right) - p_1 \\
p_1 &= 1.35193 - \frac{2.09564}{E_y} - \frac{6.98578}{E_y^2}.
\end{aligned}$$