# AcerMC for Top Physics and ISR/FSR Studies

## Top Physics Workshop, Grenoble 2007

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http://cern.ch/Borut.Kersevan/AcerMC.Welcome.html

- **Top-involved processes in AcerMC:** some facts
- **Single top PS+ME matching:** a glimpse into the theory
- **ISR/FSR studies and comparisons :** generator/truth level evaluation

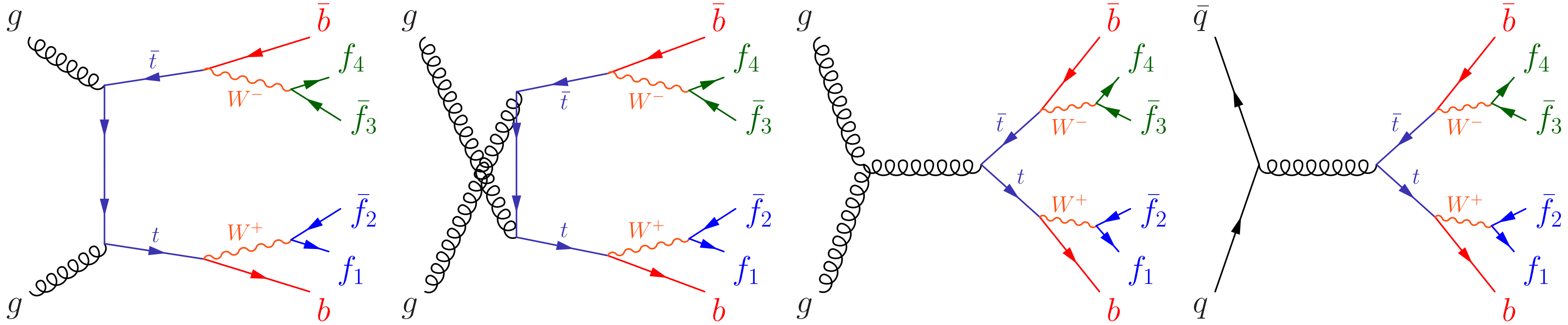
## Currently implemented processes:

| Process | Description |
|---|---|
| 1 | $gg \to t\bar{t}b\bar{b}$ |
| 2 | $q\bar{q} \to t\bar{t}b\bar{b}$ |
| 3 | $q\bar{q} \to W(\to f\bar{f})b\bar{b}$ |
| 4 | $q\bar{q} \to W(\to f\bar{f})t\bar{t}$ |
| 5 | $gg \to Z/\gamma^*(\to f\bar{f})b\bar{b}$ |
| 6 | $q\bar{q} \to Z/\gamma^*(\to f\bar{f})b\bar{b}$ |
| 7 | $gg \to Z/\gamma^*(\to f\bar{f}, \nu\nu)t\bar{t}$ |
| 8 | $q\bar{q} \to Z/\gamma^*(\to f\bar{f}, \nu\nu)t\bar{t}$ |
| 9 | $gg \to (Z/W/\gamma^* \to)t\bar{t}b\bar{b}$ |
| 10 | $q\bar{q} \to (Z/W/\gamma^* \to)t\bar{t}b\bar{b}$ |
| 11 | $gg \to (t\bar{t} \to)f\bar{f}bf\bar{f}b$ |
| 12 | $q\bar{q} \to (t\bar{t} \to)f\bar{f}bf\bar{f}b$ |
| 13 | $gg \to (WWb\bar{b} \to)f\bar{f}f\bar{f}b\bar{b}$ |
| 14 | $q\bar{q} \to (WWb\bar{b} \to)f\bar{f}bf\bar{f}b\bar{b}$ |
| 15 | $gg \to t\bar{t}t\bar{t}$ |
| 16 | $q\bar{q} \to t\bar{t}t\bar{t}$ |
| 17 | $qb \oplus qg \to qt \oplus b \to qbf\bar{f} \oplus b$ (100+101) |
| 18 | $bb \oplus bg \to Z^0 \oplus b \to f\bar{f} \oplus b$ (96+97) |
| 19 | $qq \to tb \to bf\bar{f}b$ |
| 20 | $gb \oplus gg \to (WWb \oplus \bar{b} \to)f\bar{f}f\bar{f}b \oplus \bar{b}$ (13+105) |
| 21 | $gb \to tW \to bf\bar{f}f\bar{f}$ |
| 22 | $qq \to Z^{0\prime} \to t\bar{t} \to b\bar{b}f\bar{f}f\bar{f}$ |

## Various types:

- Processes involving top pair production
- The single top processes
- The Z-prime decay to tops
- The $(A + B)$ denote PS+ME matched processes.
- All processes with decayed tops include full spin information.

**The top pair production (signal)':** The usual diagrams implemented:




- Full spin correlations, top and W widths and so on...
- The same one can get from any generator using full ME, like MadGraph etc...
- There are however some improvements:
  - Good sampling efficiency - fast.
  - Custom decay switches to make life easier.
  - QCD corrections to the top and W width: factors $\alpha_s/\pi$, automatically included in Pythia or Herwig, since tops are decayed separately from the hard process but missing in simple ME. A few percent effect but still...

There are also two processes expanding on this, including all the diagrams with $b\bar{b}WW$ intermediate state (34 diagrams instead of 4). Small contribution in terms of cross-section but possibly relevant for specific cases/measurements.

**The single top production processes:** The single top production processes are now implemented in the **AcerMC** :

- the associated Wt production process $gb \rightarrow tW \rightarrow bf\bar{f}f\bar{f}$,
- the s-channel production process $qq \rightarrow tb \rightarrow bf\bar{f}b$ and
- the t-channel production process $qb \oplus qg \rightarrow qt \oplus b \rightarrow qbf\bar{f} \oplus b$,

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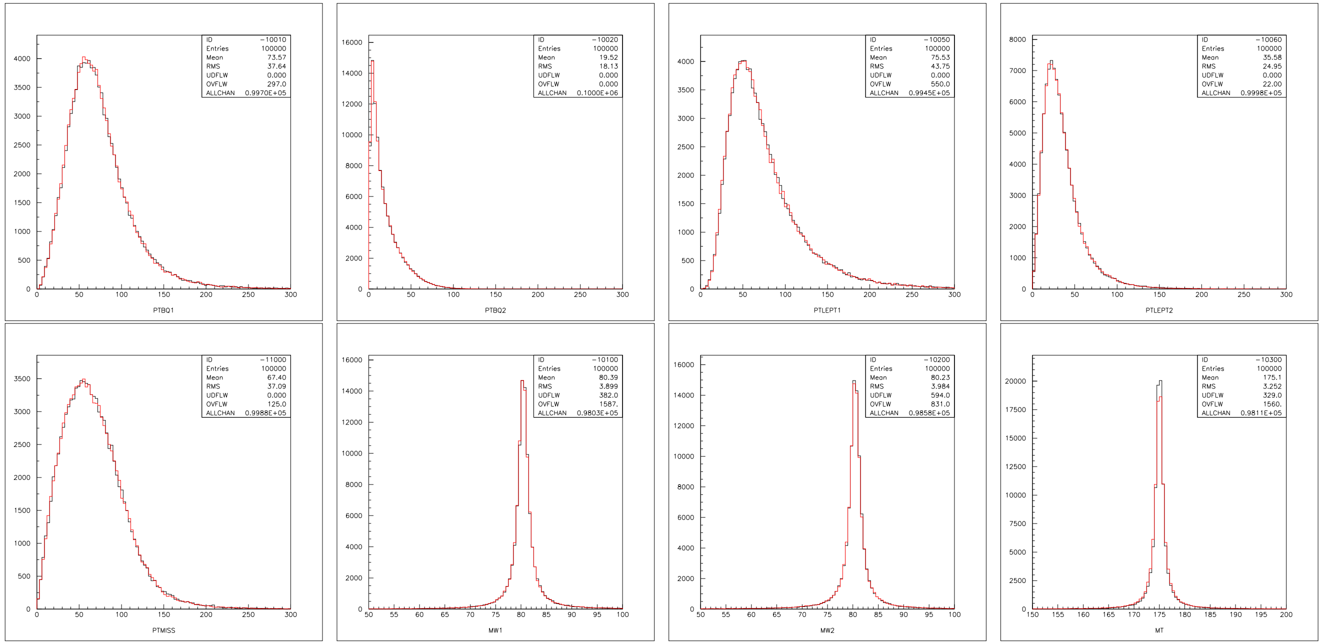
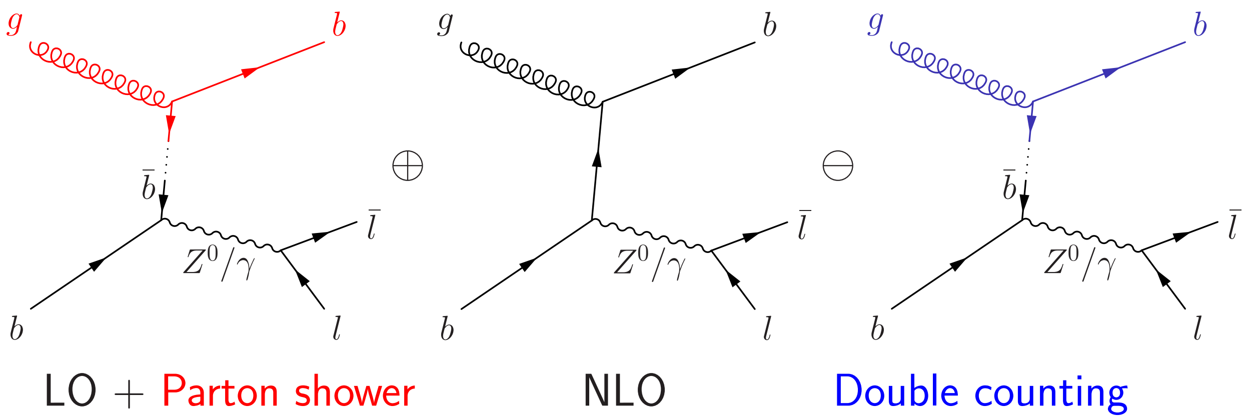
- The single top production processes have been validated by comparison to **TopRex** by A. Lucotte and myself.
- Reasonable agreement is found between the **AcerMC** exact and **TopRex** approximate implementations.
- Special care is needed for the t-channel process: $\pm 1$**weights!**
- Examples show comparison between the two generators for the tW process (**TopRex** is in **red**):



## Parton shower and ME matching in AcerMC: Theoretical basics

- In a series of papers Collins *et. al.* have derived an consistent procedure of combining a few processes, e.g. the Drell-Yan $q\bar{q} \to Z$ with $gq \to Z\ q$: Consistent meaning it reproduces the Compton part of the NLO differential cross-section *exactly, by paper calculation*. [hep-ph/0110257,hep-ph/0001040,hep-ph/0105291]
  - An related issue is that actually the PDFs need to be modified [hep-ph/0204127]
- The second point is the treatment of quark mass - the ACOT papers [hep-ph/9312318,hep-ph/9312319] provide a method of incorporating the massive quarks into the factorisation theorem (actually done for DIS and c quark). There is a lot of work done on this, for the impact on LHC have a look at [hep-ph/9712494] and papers citing it.
- We tried to merge these two into the LHC case.
- To give a picture of what we would like to do:



## The Subtraction Prescription

- the prescription developed in the papers above is in a sense 'standard' and we generalized it: Introduce a counter-event (implemented by modifying the MC weight) that removes the double-counting between a Sudakov-showered LO process and corresponding NLO process in terms of $\alpha_s$ (with identical particles in initial and final states).
  - NLO in tree-level terms only.
- The counter-term can be obtained by fixed-order $\alpha_s$ expansion of the Sudakov exponent:

$$S_a = \exp\left\{ -\int_{\mu^2}^{\mu_0^2} \frac{d\mu'^2}{\mu'^2} \frac{\alpha_s(\mu'^2)}{2\pi} \times \sum_c \int_{\xi_c}^{1} \frac{dz}{z} P_{a\to c}(z) \frac{f_{a/I}(\frac{\xi_c}{z}, \mu'^2)}{f_{c/I}(\xi_c, \mu'^2)} \right\}.$$

giving the subtraction weight:

$$d\sigma_s(NLO) \sim \frac{\alpha_s(\mu_0^2)}{2\pi} P_{a\to c}(z) \times d\sigma(LO)$$

.

- In principle such a weight should cancel the NLO contribution in the collinear limit and the LO + shower contribution in the opposite case (close to the factorisation scale):

$$d\sigma(LO) \times \mathrm{PS} = \frac{\alpha_s(\mu)}{2\pi} \mathrm{P_{a\to c}(z) S_a}(\mu) \times \mathrm{d}\sigma(\mathrm{LO}) \to (\mu \sim \mu_0) \to \mathrm{d}\sigma_s(\mathrm{NLO})$$

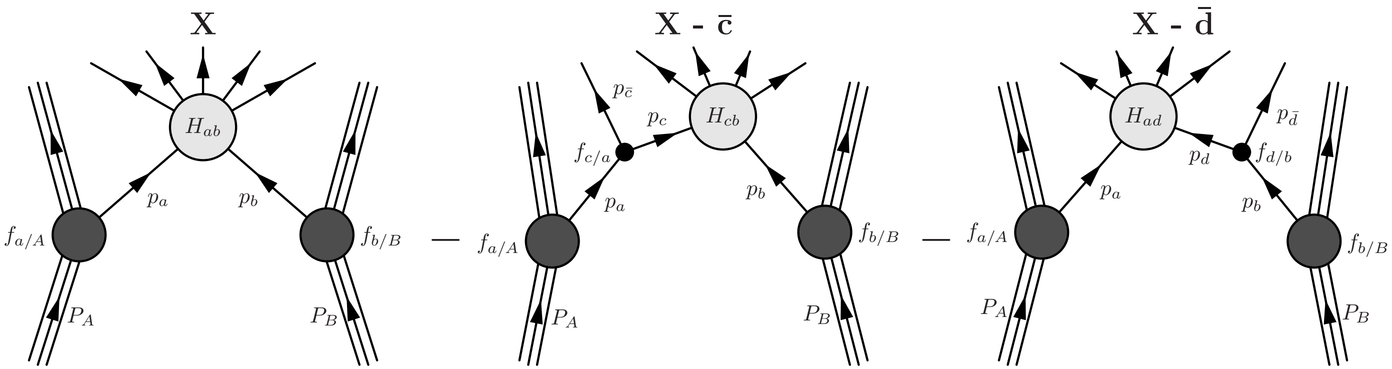
since $S_a(\mu \sim \mu_0) \simeq 1$;

- Similar to what is done e.g. in MC@NLO - The devil is in the details, of course...

**The principle issue/difference is the heavy particle/parton mass treatment:**

- It traces back to the treatment of masses in the factorisation theorem:
  - All the partons in 'usual' NLO calculations are generally treated as massless.
  - This becomes a problem in case of gluon splitting to heavy partons like $b$ or $c$ quarks.
  - The heavy partons in the final state need to have masses to accurately describe the observable jet kinematics.
- Back to basics again. . . to see that the factorisation theorem is actually derived using the light-cone coordinates $p^\mu = (p^+, \vec{p}^T, p^-)$ where $\mathrm{p}^\pm = \frac{1}{\sqrt{2}}(\mathrm{p}^0 \pm \mathrm{p}^3)$, which can incorporate particle massess.
- This is what the series of ACOT (M. A. G. Aivazis, J. C. Collins, F. I. Olness and W. K. Tung) and many derived papers solved for cross-section calculations. . . and we adapted it into a Monte-Carlo algorithm fitting the proton-proton collisions.
- This traslates to modified kinematics in factorisation w.r.t. massless: $p_a = (p_a^+, \vec{0}^T, p_a^-) = (\xi_a P_A^+, \vec{0}^T, \frac{m_a^2}{2\xi_a P_A^+})$ and $p_b = (p_b^+, \vec{0}^T, p_b^-) = (\frac{m_b^2}{2\xi_b P_B^-}, \vec{0}^T, \xi_b P_B^-)$ for the colliding a and b partons.

## The Collins Showering Prescription



- The kinematic mapping when performing parton showers is in itself not unique; what Collins proposed and we implemented by adding masses (at present we use only do the ISR $g \to b\bar{b}$) is:

1. Incoming hadron A is moving in the $\mathrm{z}$ direction and hadron B in the $-\mathrm{z}$ direction, carrying momenta $\mathrm{P_A}$ and $\mathrm{P_B}$ with the center-of-mass energy $\sqrt{\mathrm{s}}$, whereby one can neglect the hadron masses at LHC energies.
2. The incoming partons with momenta $\mathrm{p_a}$ and $\mathrm{p_b}$ have the momentum fractions $\mathrm{p_a^+} = \mathrm{x_a P_A^+}$ and $\mathrm{p_b^-} = \mathrm{x_b P_B^-}$ relative to the parent hadrons and the center-of-mass energy $\sqrt{s_n}$.
3. The split propagator (i.e., particle $\mathrm{c}$) virtuality $p_c^2 = (p_a - p_{\bar{c}})^2 = t_{n-1}$ gives the shower evolution parameter $\mu^2$ value from $t_{n-1} - m_{\bar{c}}^2 = -\mu^2$.
4. All incoming and outgoing particles (partons) are on mass shell.
5. The splitting parameter of the evolution kernel is $\mathrm{z} = \frac{\xi_\mathrm{c}}{\xi_\mathrm{a}}$.
6. The rapidity $\mathrm{y} = \frac{1}{2} \ln \left( \frac{\mathrm{k_{n-1}^+}}{\mathrm{k_{n-1}^-}} \right)$ of the subsystem $(X - \bar{c})$ is preserved in the translation.

## The Collins Prescription Cont'd

- Actually Collins *et. al.* developed this kinematic prescription with the corrollary that the correct way to implement the showering in an MC algorithm is to modify the parton density functions for quarks!
- In order to match the derived prescription with the explicit $\overline{\mathrm{MS}}$ NLO result for Z + jet production *on paper* new PDFs need to be defined:

$$
\begin{aligned}
z\, f_{i/I}^{\mathrm{JCC}}(z,\mu^2) \;&=\; z\, f_{i/I}^{\overline{\mathrm{MS}}}(z,\mu^2) \\
&+\; \frac{\alpha_s(\mu^2)}{2\pi} \int_z^1 d\xi \frac{z}{\xi} f_{g/I}^{\overline{\mathrm{MS}}}(\xi,\mu^2) \left[ P_{g\to i\bar{i}}(\frac{z}{\xi}) \ln\left(1-\frac{z}{\xi}\right) + \frac{z}{\xi}\left(1-\frac{z}{\xi}\right)\right] \\
&+\; \mathcal{O}(\text{first-order quark terms}) \;+ \mathcal{O}(\alpha_s^2)
\end{aligned}
$$

- This simple form is *particular* to the proscribed kinematic mapping/showering, it is *not general*!
- Side comment: This means that the discussion of NLO vs LO PDFs for showering is actually more complicated.
- These new distributions can in a reasonably straightforward manner be obtained by numerical integration using e.g. CTEQ functions as input.

## Implementation in AcerMC

- The same kinematic translations (rapidity conservation) need to be applied both to the Sudakov showering of the corresponding LO event (We thus wrote our own..) and to the NLO subtraction term.
- The final result is thus an implemented prescription for the combination of ISR and pQCD calculations in case of massive colliding partons.
- The final event unweighting results in $\pm$ 1 weights.

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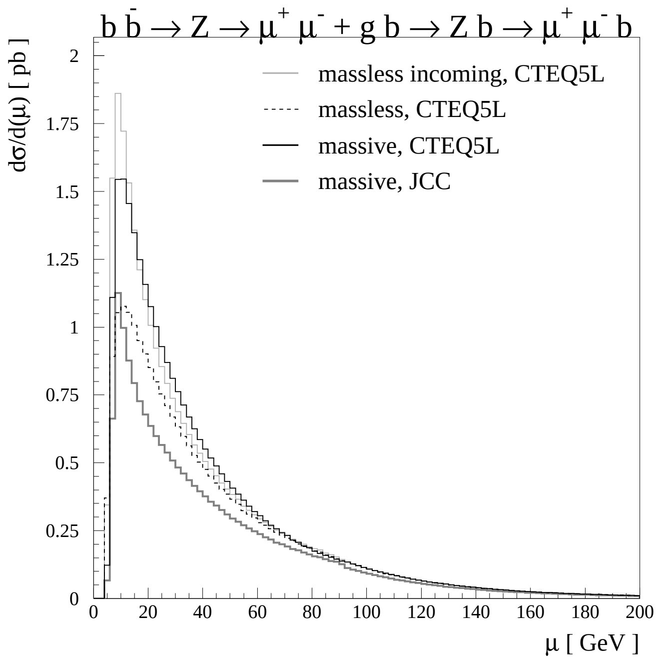
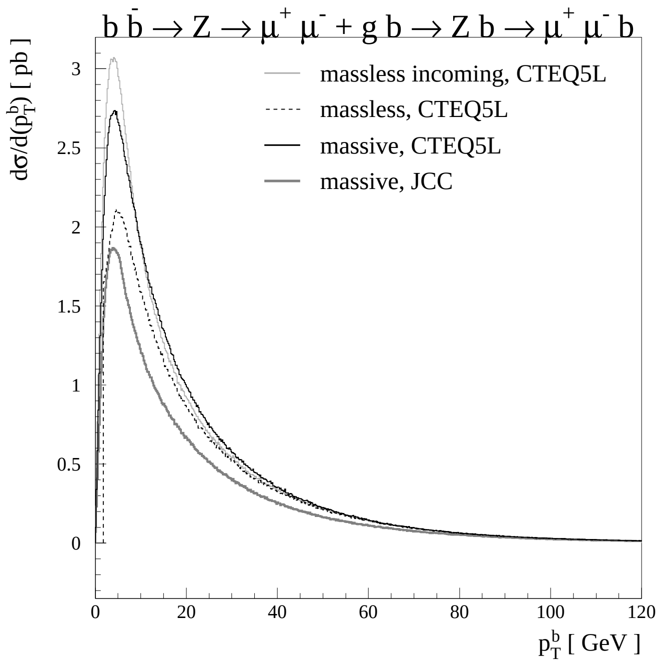
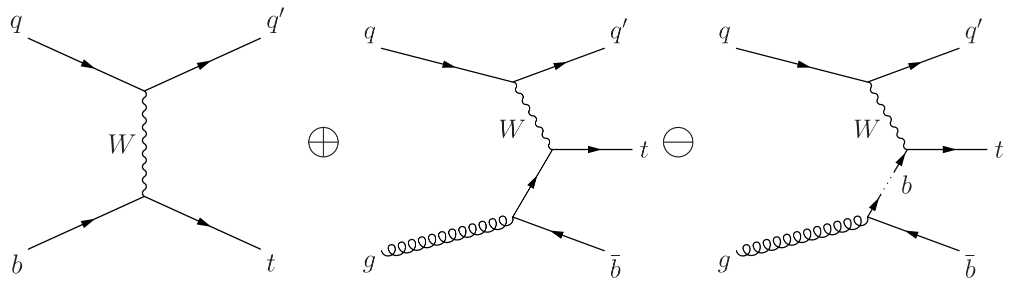
- The impact of mass treatment and PDF is not negligible:



## t-channel single top production:

- The t-channel process is the combined production of the $qb \to qt$ and $qg \to qtb$ W-exchange processes.
- One needs to remove the double counting between the ISR $g \to b\bar{b}$ splitting and the next-order $\alpha_S$ process $qg \to qtb$.
- In fact the t-channel single top production involves the full matrix element including top decays.

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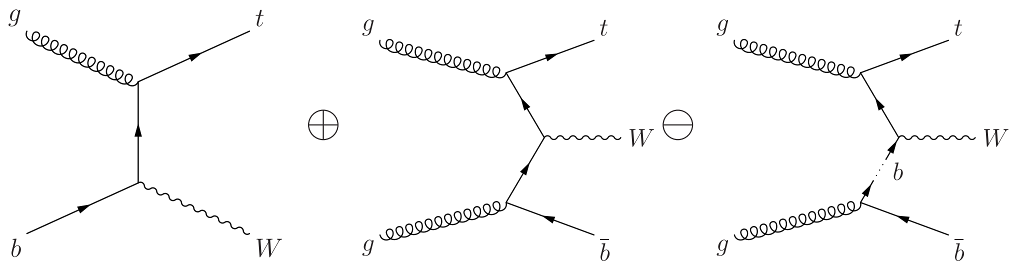
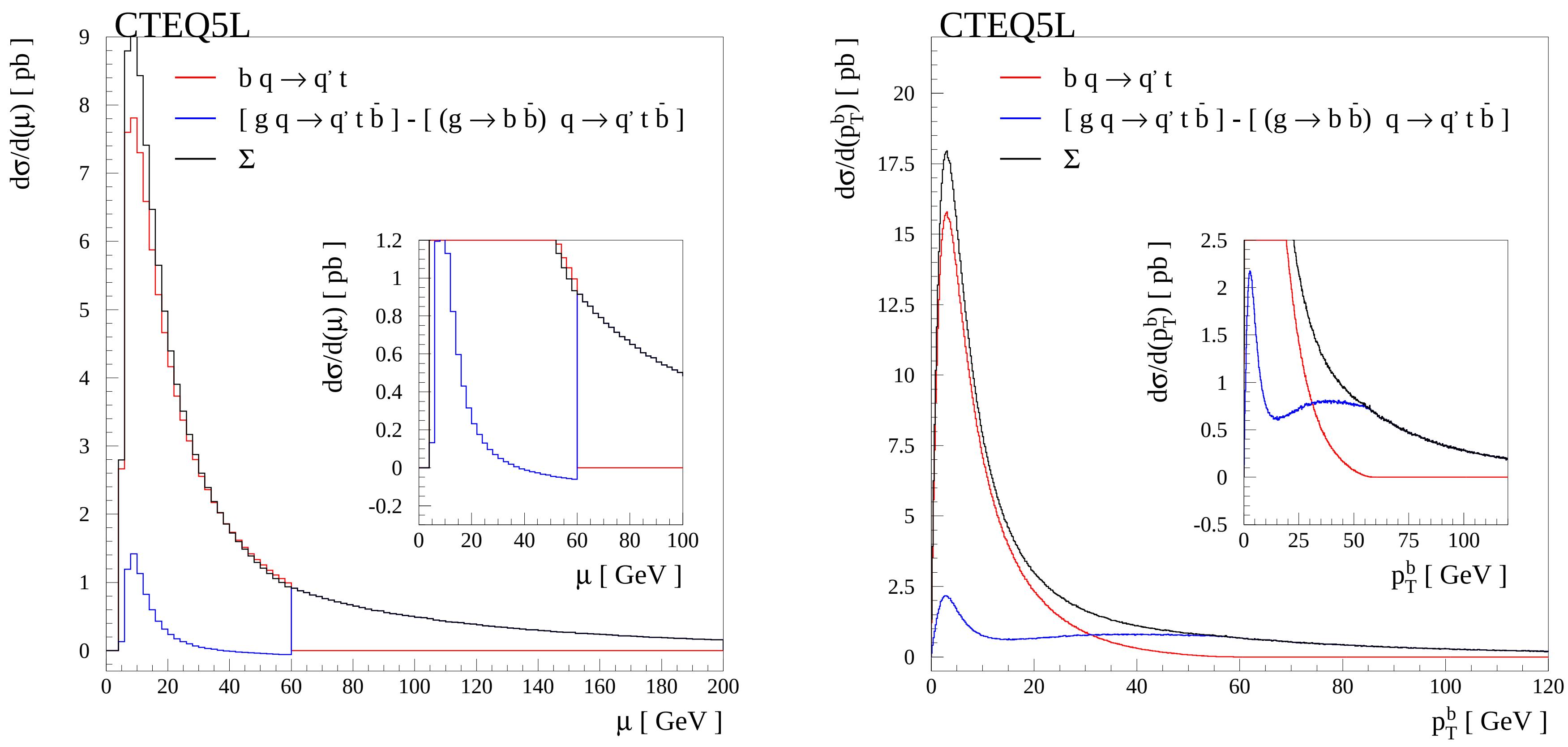
**tW-channel single top production:** Similar case, it double counts the $tWb$ diagrams.



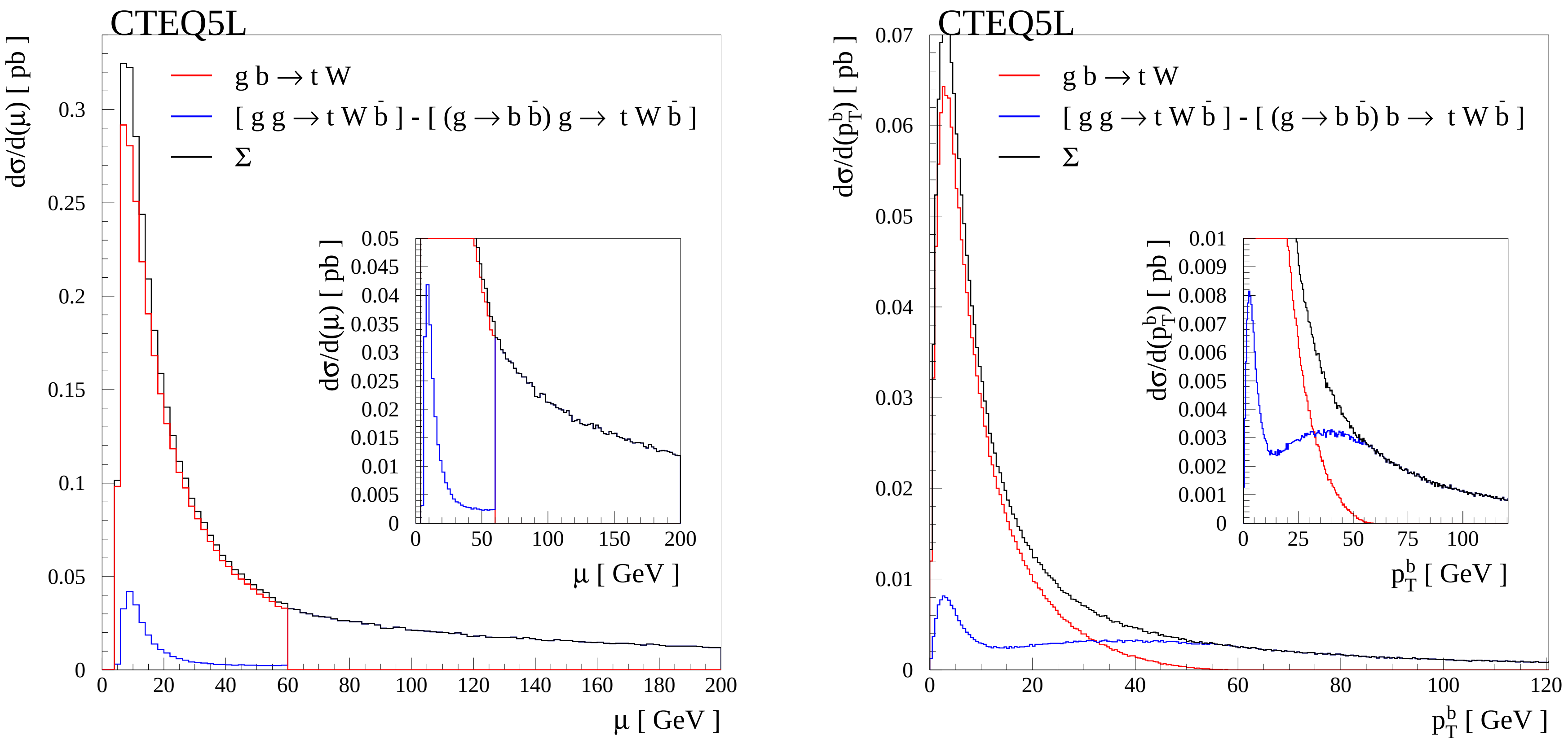
# Kinematic distributions for t-channel single top :

- Note that a smooth continuation in the b-quark virtuality is achieved irrespectively of the matching value (factorisation/showering scale).
- The $p_T$ distribution is a result of non-trivial contributions.



## Kinematic distributions for tW-channel single top:

- Note that a smooth continuation in the b-quark virtuality is again achieved.
- The $p_T$ distribution again a result of non-trivial contributions.
- The plots serve as a cross-check; in AcerMC process 20 the procedure is applied to the $WWb\bar{b}$ $(2 \rightarrow 6)$ process 13 which includes the $tWb$ intermediate states among its 31 diagrams.



## Conclusions:

- The described procedure has been shown to work...
- For details please consult [hep-ph/0603068] or JHEP 0609:033,2006.
- In case one wants to check this in practice:

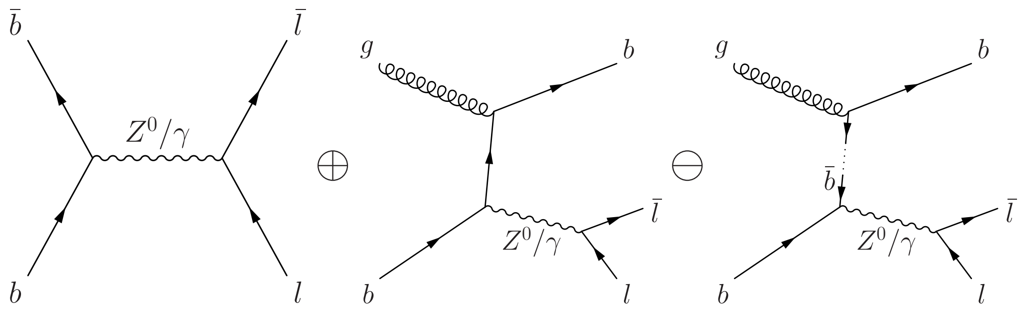
  `http://cern.ch/Borut.Kersevan/AcerMC.Welcome.html`

  - This procedure is recursive, so it could be implemented for arbitrary number of splits (ISR/FSR) and possibly a CKKW-like procedure could be achieved.
  - Needs work and time..

- Some comparisons with e.g. MC@NLO single top are still to be done within ATLAS...
  - Second part of my talk: the ISR/FSR systematic studies:
  - FORTRAN $\rightarrow$ C++
  - PostScript $\rightarrow$ JPEG
  - LaTex $\rightarrow$ PPT

## Drell-Yan Z + b production:

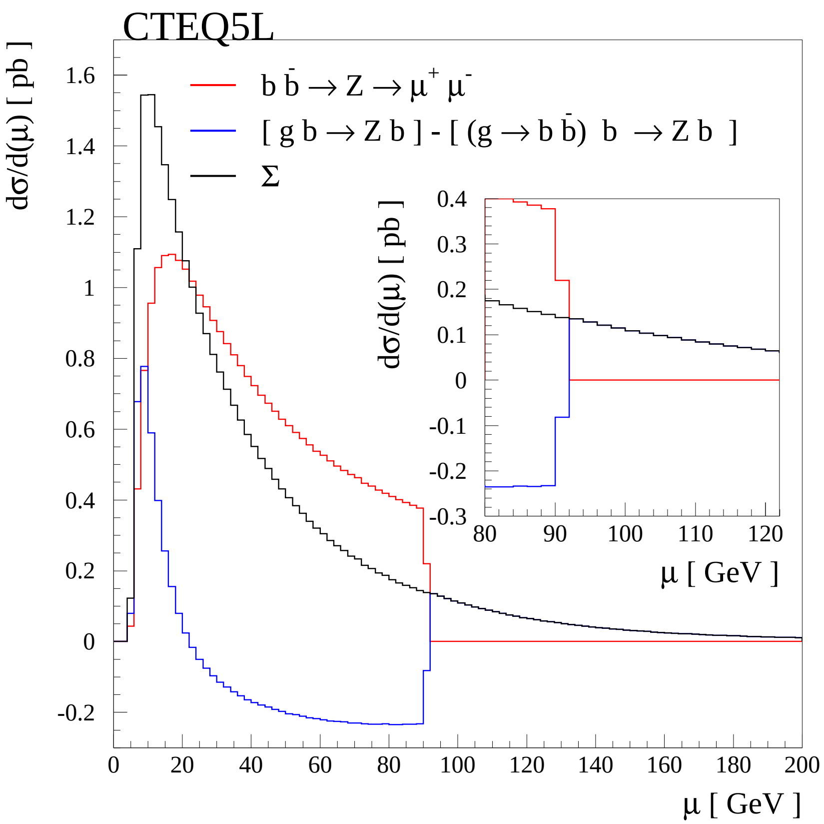
- The double counting is in this case two-fold: either $b$ or $\bar{b}$ can originate in gluon splitting.
- In fact the Drell-Yan case has been implemented with the full matrix element including photon interference.

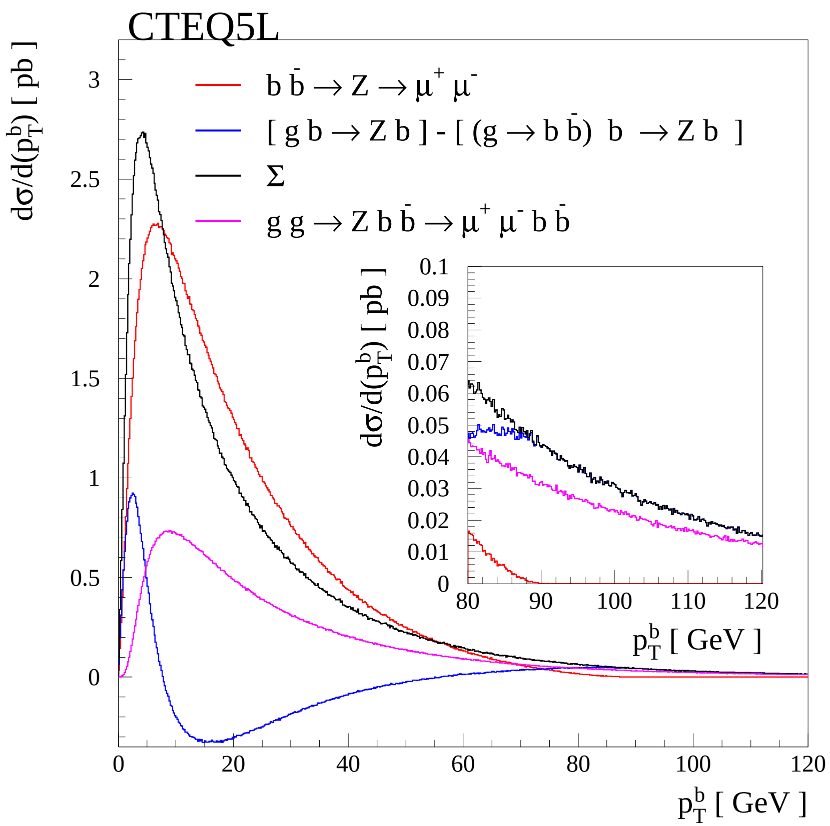



| Process | $\sigma_{\mathrm{CTEQ5L},\mu_0=\mathrm{m_Z}}$ [pb] | $\sigma_{\mathrm{JCC},\mu_0=\mathrm{m_Z}}$ [pb] |
|---|---|---|
| $bb \to Z \to \mu^+\mu^-$ | 57.9 | 39.9 |
| $gb \to Zb \to \mu^+\mu^- b$ | 72.1 | 60.0 |
| $(g \to bb)b \to Zb \to \mu^+\mu^- b$ | 73.3 | 60.9 |
| $\Sigma$ | 56.7 | 39.0 |
| $gg \to Zbb \to \mu^+\mu^- bb$ | 22.8 | 22.8 |

# Drell-Yan Z + b production cont'd:

- Note that a smooth continuation in the b-quark virtuality is achieved regardless of the matching point/factorisation scale.
- The $p_T$ distribution is a result of non-trivial contributions in this case.



**Factorisation theorem:** The factorisation theorem in hadron-hadron (proton-proton) collisions is usually formulated within the following expression:

$$|\mathcal{M}_{AB\to X}|^2 = \sum_{a,b} f_{a/A} \otimes H_{ab\to X} \otimes f_{b/B} = \sum_{a,b} \int \frac{d\xi_a}{\xi_a} \int \frac{d\xi_b}{\xi_b} f_{a/A}(\xi_a, \mu_F)\, f_{b/B}(\xi_b, \mu_F)\, H_{ab\to X}(\xi_a, \xi_b, \mu_F \ldots),$$

where $H_{ab\to X}$ is the the hard ('short time') part of the squared amplitude and the soft contributions are absorbed into the parton distribution functions $\mathrm{f_{i/I}}(\xi_\mathrm{i}, \mu_\mathrm{F})$ with $\mu_\mathrm{F}$ being the (factorization) scale at which the two parts were separated.

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In case all partons are considered massless the flux factor in the partonic cross-section expression is $\hat{\mathrm{s}} = \xi_\mathrm{a}\xi_\mathrm{b}(2\mathrm{s})$ with $(2s)$ being the hadronic flux and the equivalent expression is given by:

$$\sigma_\mathrm{AB\to X} = \int \mathrm{d}\xi_\mathrm{a} \int \mathrm{d}\xi_\mathrm{b} \mathrm{f_{a/A}}(\xi_\mathrm{a}, \mu_\mathrm{F})\, \mathrm{f_{b/B}}(\xi_\mathrm{b}, \mu_\mathrm{F})\, \sigma^\mathrm{hard}_\mathrm{ab\to X}(\hat{\mathrm{s}}, \mu_\mathrm{F})$$

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These expressions are as such useful, however in experiments we observe events:

- The PDF-s $\mathrm{f_{i/I}}(\xi_\mathrm{i}, \mu_\mathrm{F})$ sum all possible 'histories' that produced the parton i.
- How does this manifest itself in actual observable events?

**Short derivation of the subtraction terms:**

The appropriate subtraction terms can actually be derived from the factorisation theorem itself by using DGLAP at the parton level and doing power counting of $\alpha_s$:

- The pQCD squared amplitude $|\mathcal{M}_{\mathrm{ab}\to\mathrm{X}}|^2$ involving initial state partons $\mathrm{a}, \mathrm{b}$ is subject to the same factorization theorem:

$$|\mathcal{M}_{ab\to X}|^2 = \sum_{c,d} f_{c/a} \otimes H_{cd\to X} \otimes f_{d/b},$$

- At zero-th order in $\alpha_s$:

$$f^{(0)}_{i/j}(\xi) = \delta^i_j \delta(\xi - 1)$$

- and hence:

$$|\mathcal{M}^{(0)}_{ab\to X}|^2 = H^{(0)}_{ab\to X}.$$

  Subsequently, at first order in $\alpha_s$:

$$f_{i/j}(\xi) = f^{(0)}_{i/j}(\xi) + f^{(1)}_{i/j}(\xi) = f^{(0)}_{i/j}(\xi) + \frac{\alpha_s(\mu_F)}{2\pi} P^{(0)}_{j\to i}(\xi) \ln\left(\frac{\mu_F^2}{m^2}\right),$$

- and thus at this order:

$$|\mathcal{M}^{(1)}_{ab\to X}|^2 = H^{(1)}_{ab\to X} + \sum_c f^{(1)}_{c/a} \otimes H^{(0)}_{cb\to X} + \sum_d H^{(0)}_{ad\to X} \otimes f^{(1)}_{d/b},$$

- The last equation can thus be inverted to give:

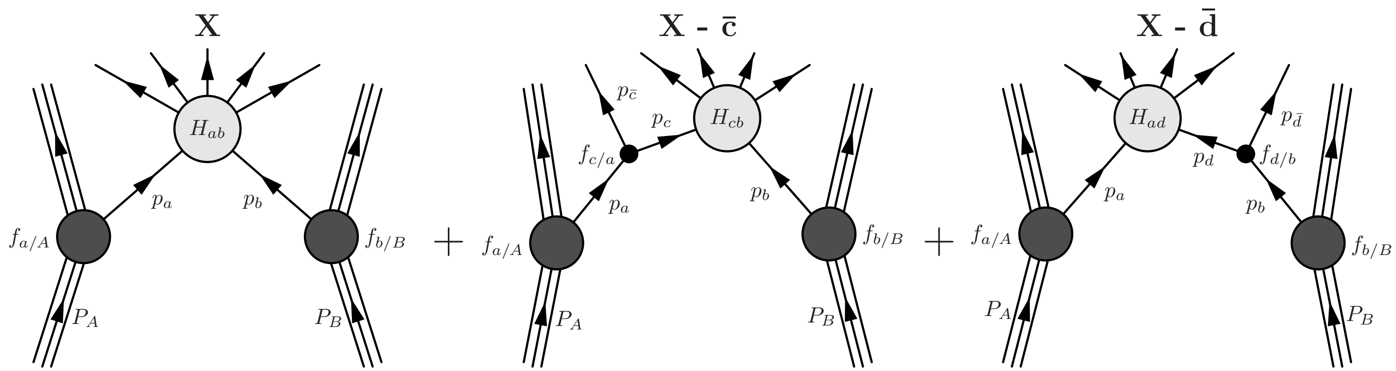
$$H^{(1)}_{ab\to X} = |\mathcal{M}^{(1)}_{ab\to X}|^2 - \sum_c f^{(1)}_{c/a} \otimes |\mathcal{M}^{(0)}_{cb\to X}|^2 - \sum_d |\mathcal{M}^{(0)}_{ad\to X}|^2 \otimes f^{(1)}_{d/b}$$

- Putting it back into the factorisation theorem expression:

$$|\mathcal{M}_{AB\to X}|^2 = |\mathcal{M}^{(0)}_{AB\to X}|^2 + |\mathcal{M}^{(1)}_{AB\to X}|^2 - |\mathcal{M}_{AB\to X}|^2_{\mathrm{s}},$$

- with the subtraction terms given by:

$$|\mathcal{M}_{AB\to X}|^2_{\mathrm{s}} = \sum_{a,b} f_{a/A} \otimes \sum_c f^{(1)}_{c/a} \otimes H^{(0)}_{cb\to X} \otimes f_{b/B} + \sum_{a,b} f_{a/A} \otimes \sum_d H^{(0)}_{ad\to X} \otimes f^{(1)}_{d/b} \otimes f_{b/B}.$$

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The kinematic transforms are however far from simple...