Making Kerr quasinormal mode frequency computation robust

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with Leo Stein (Univ. of Mississippi)

Atelier API "Ondes gravitationnelles et objets compacts"

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- Determining QNM frequencies is essential for GW data analysis.
- **Objective:** work towards improving the spectral variants of Leaver's method of *[1410.7698 (Cook & Zalutskiy)]* and *[1908.10377 (Leo Stein)]* (*qnm* python package of *BHPToolkit*).

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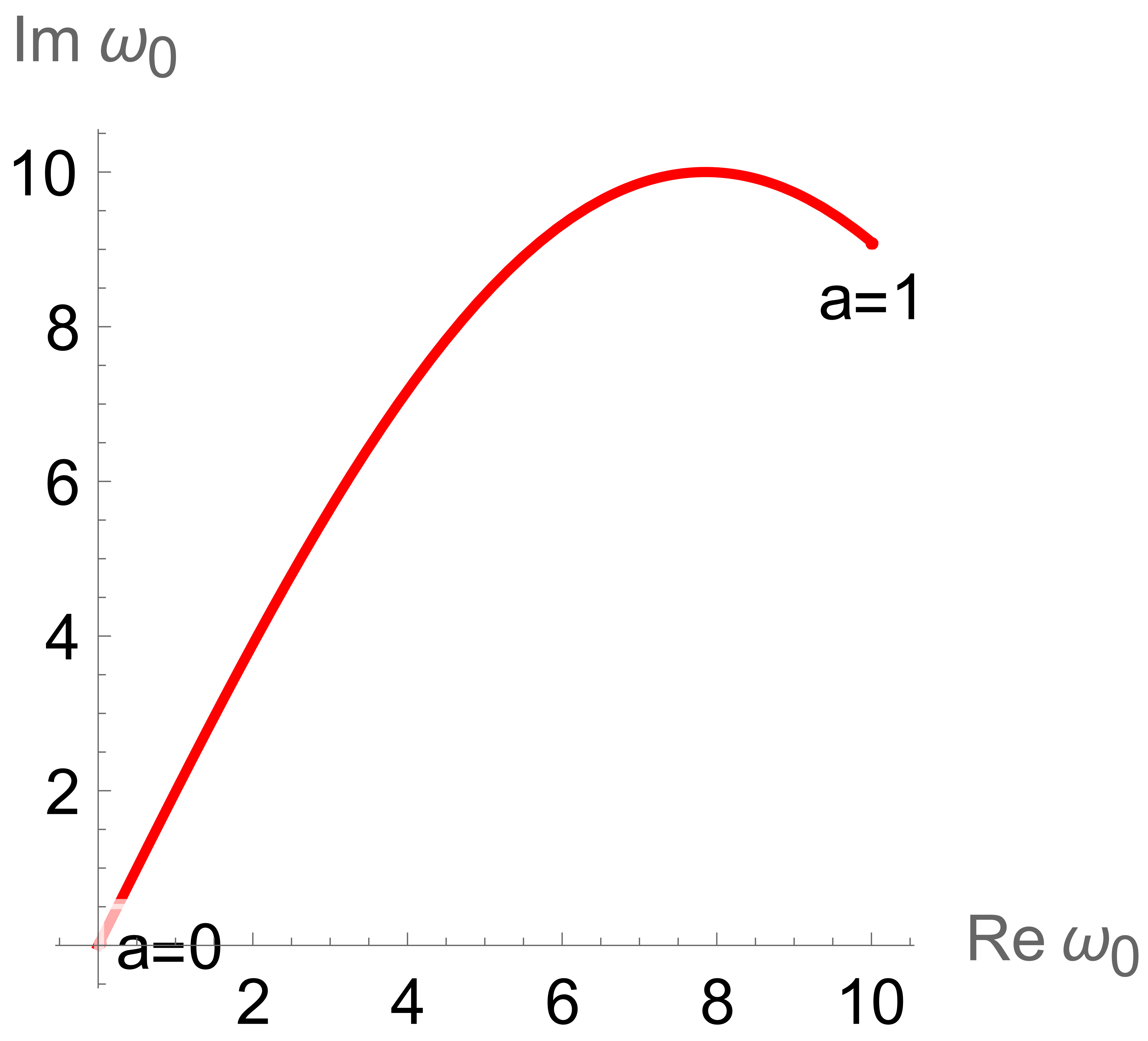

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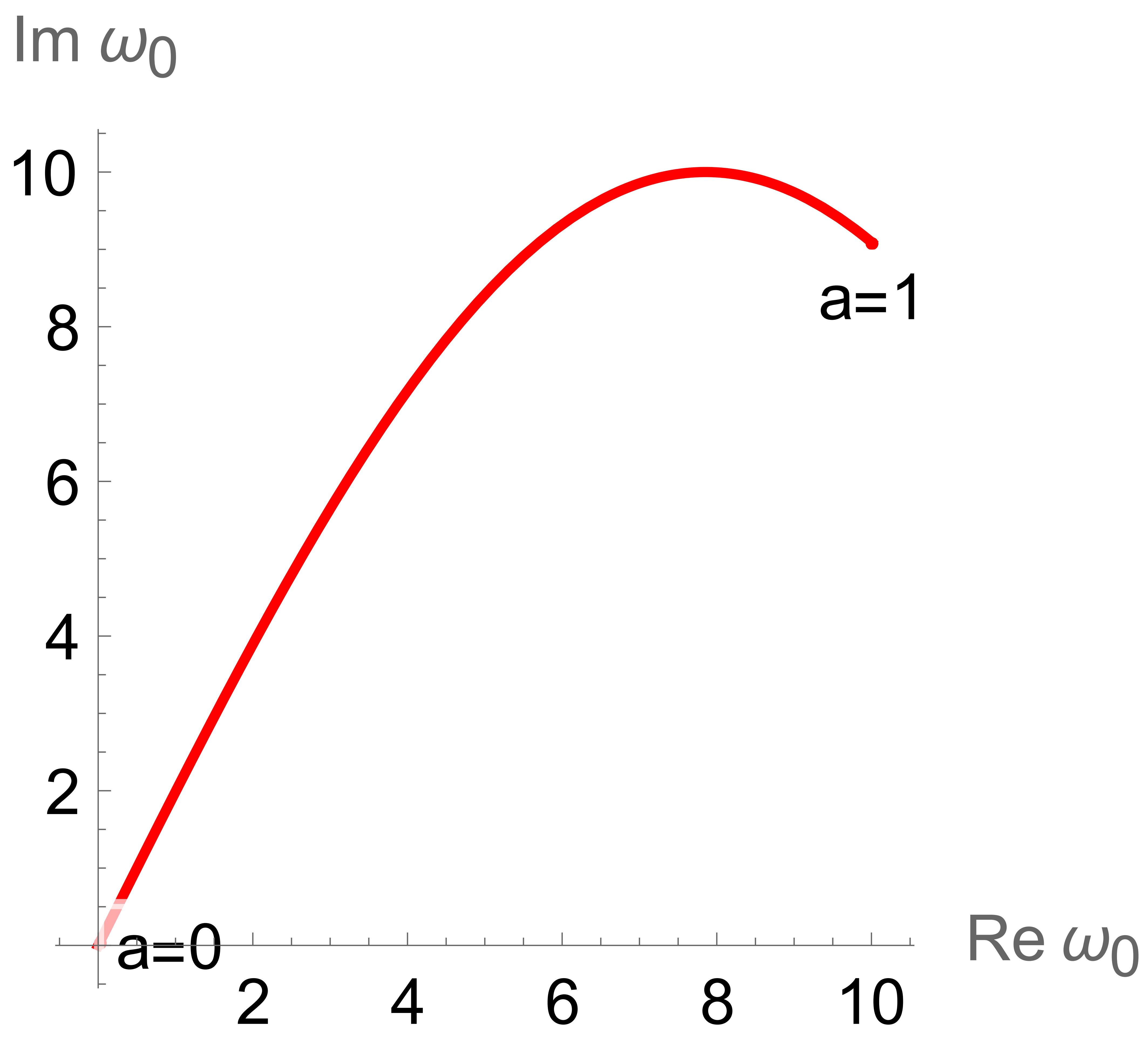

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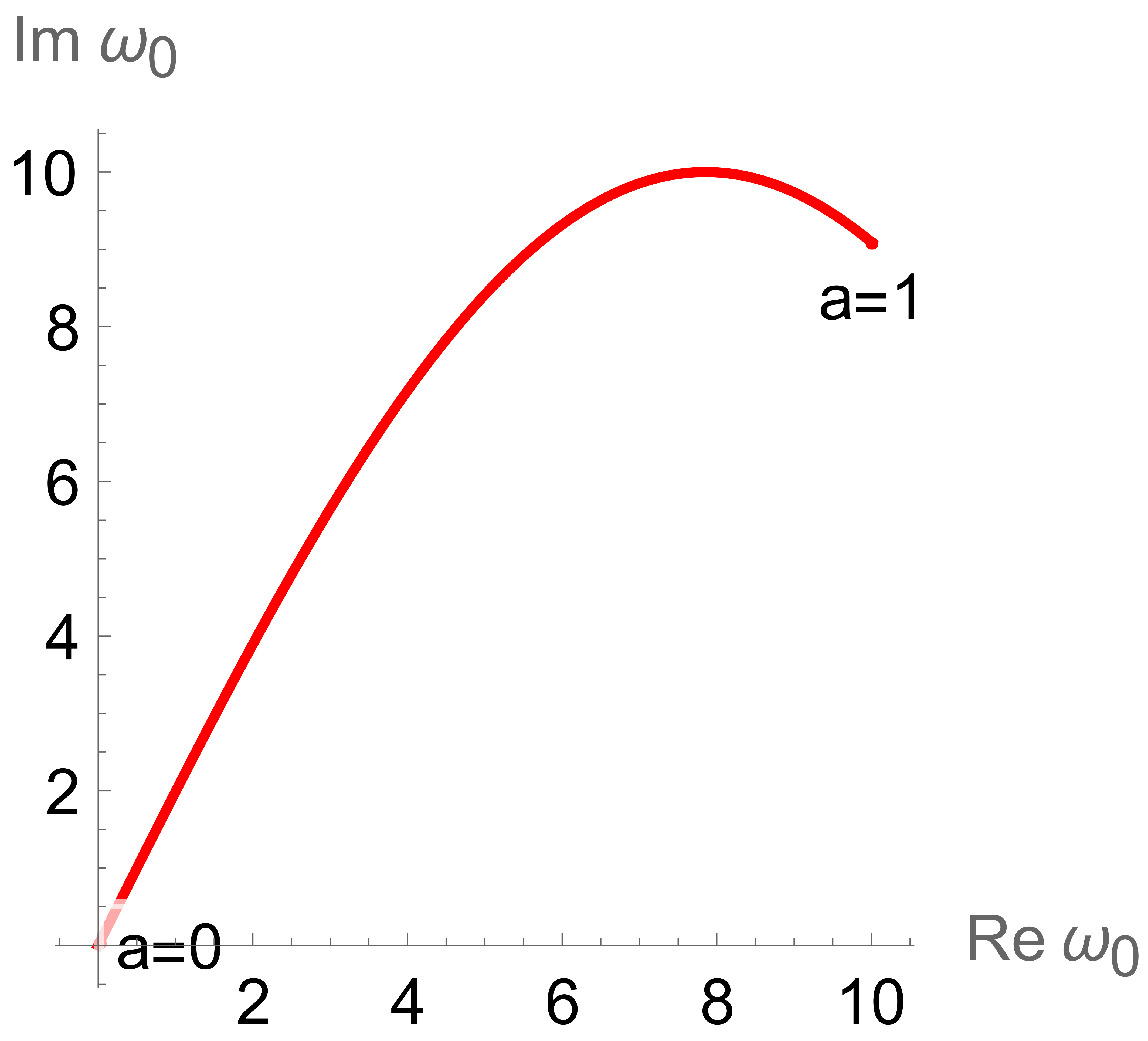
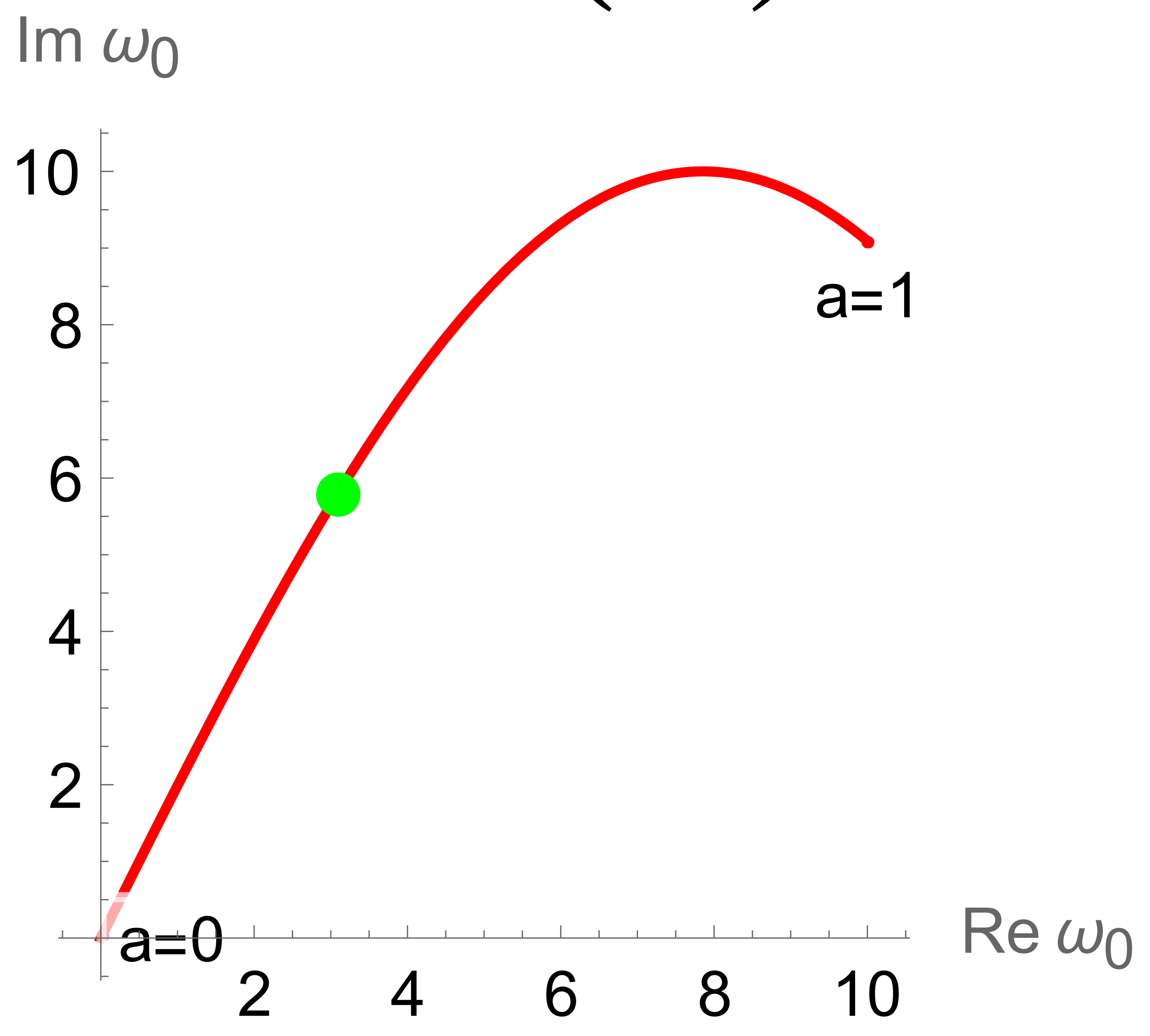
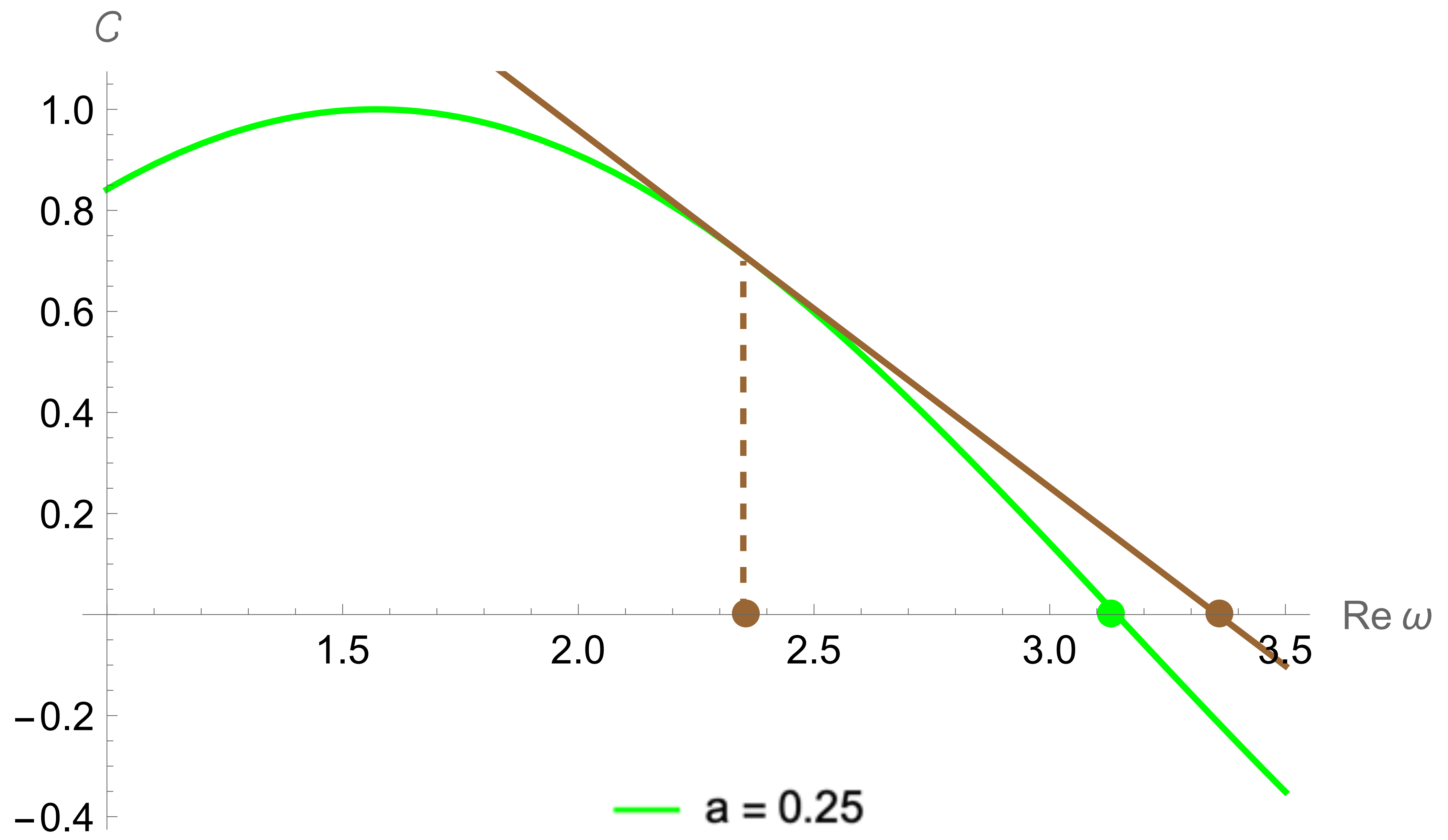
- **Note:** We will use fake QNM curves for simplicity.



QNM frequency ω_0: root of $\mathscr{C}(\omega) = 0$





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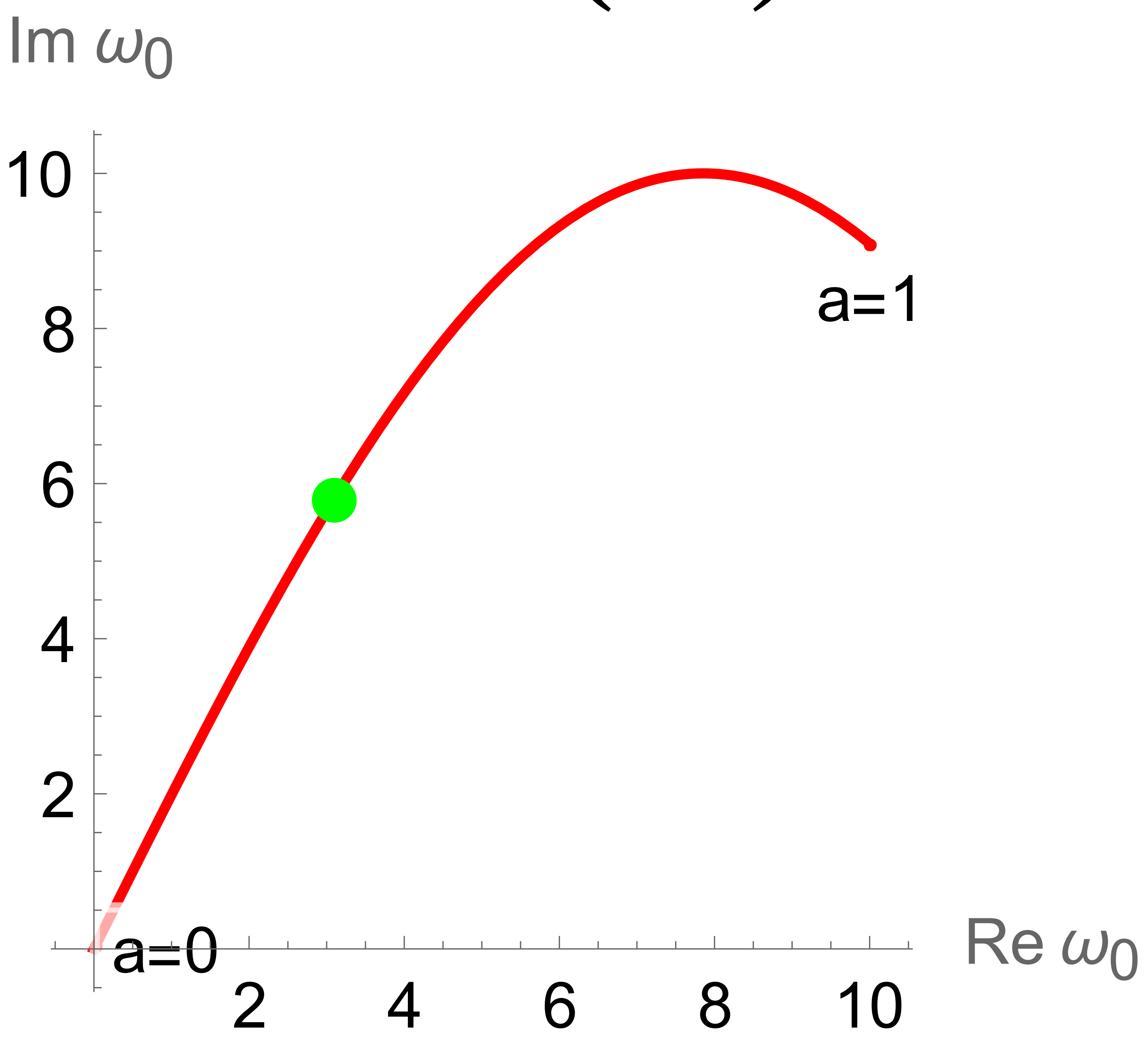

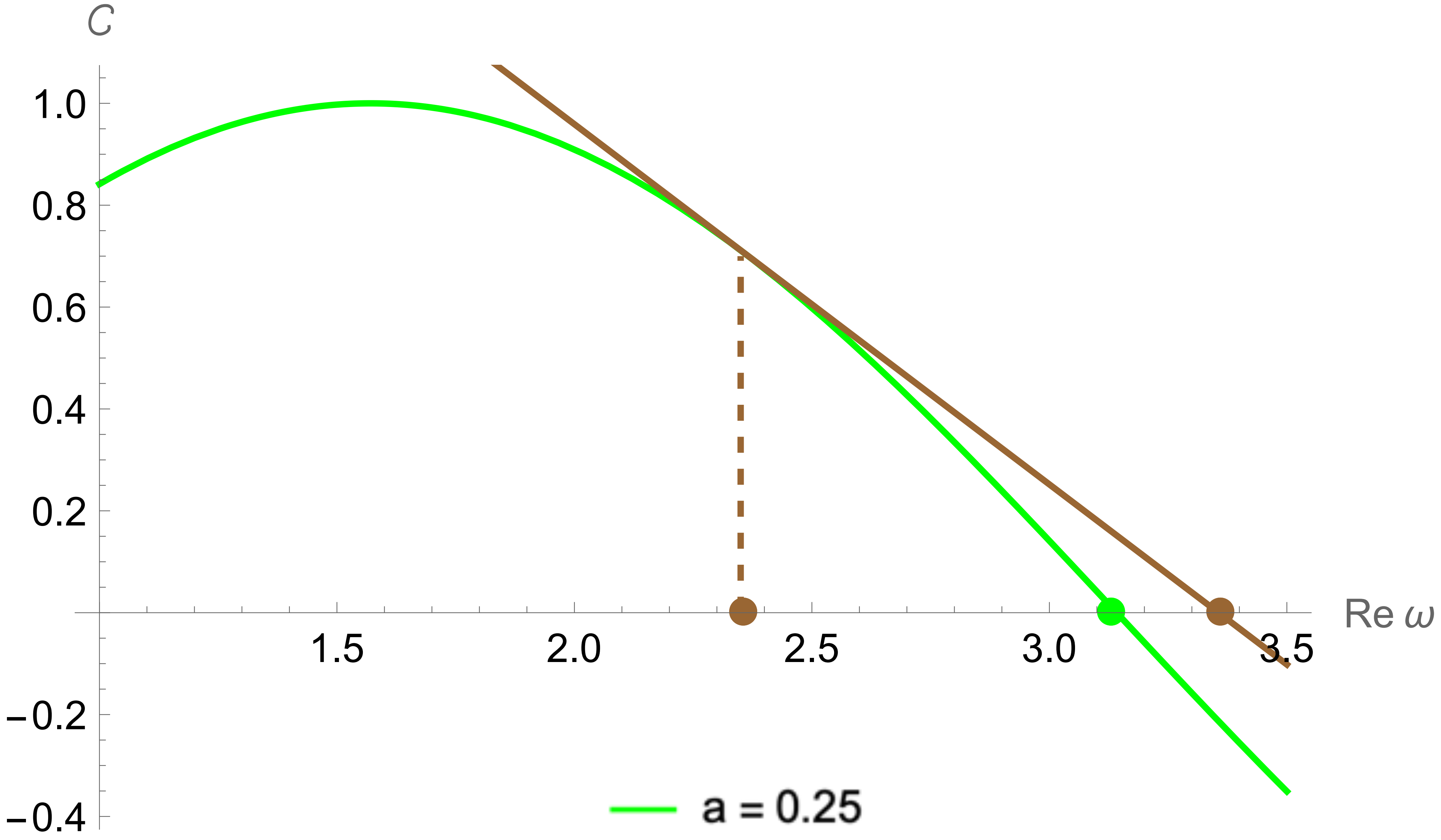

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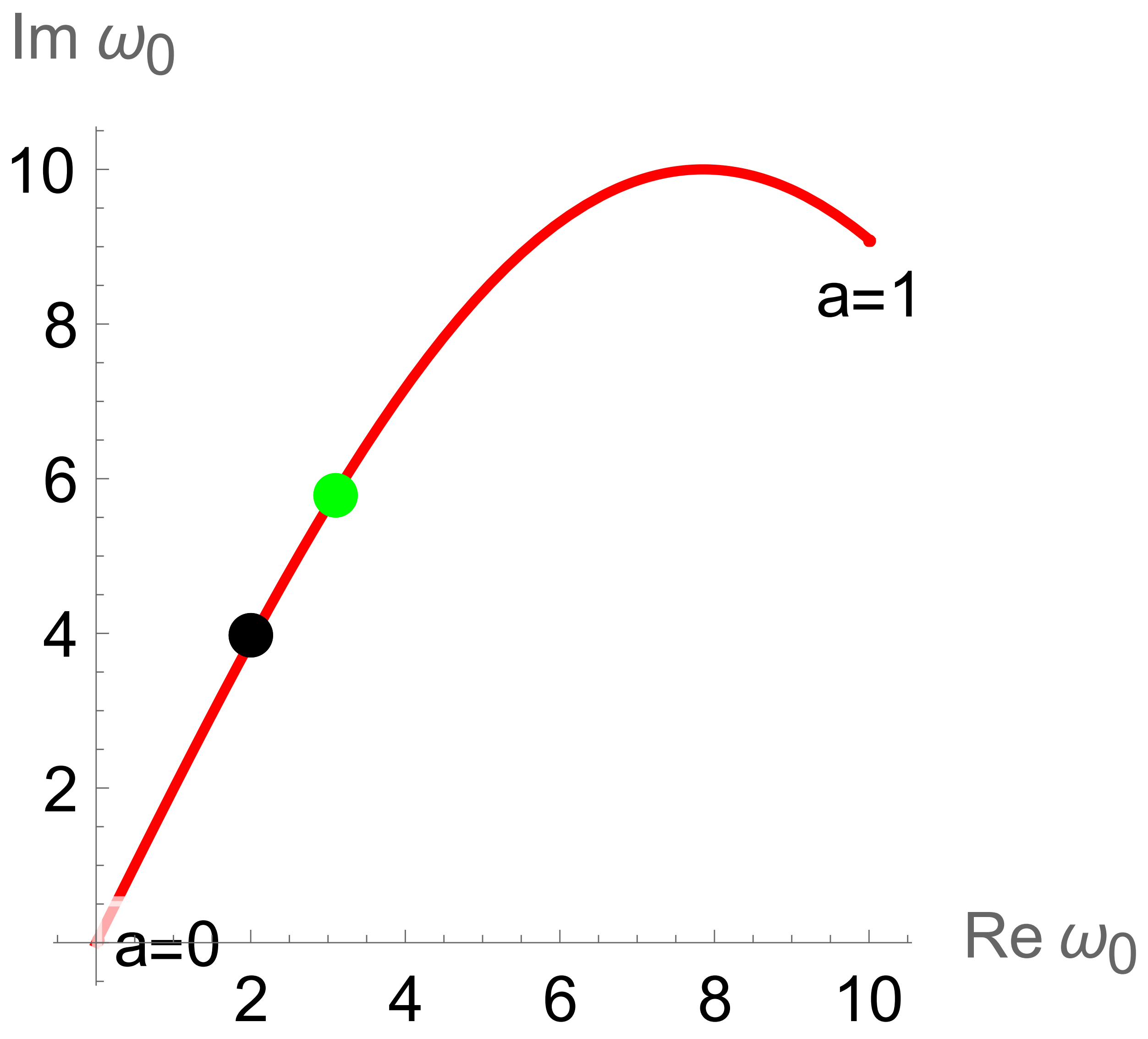

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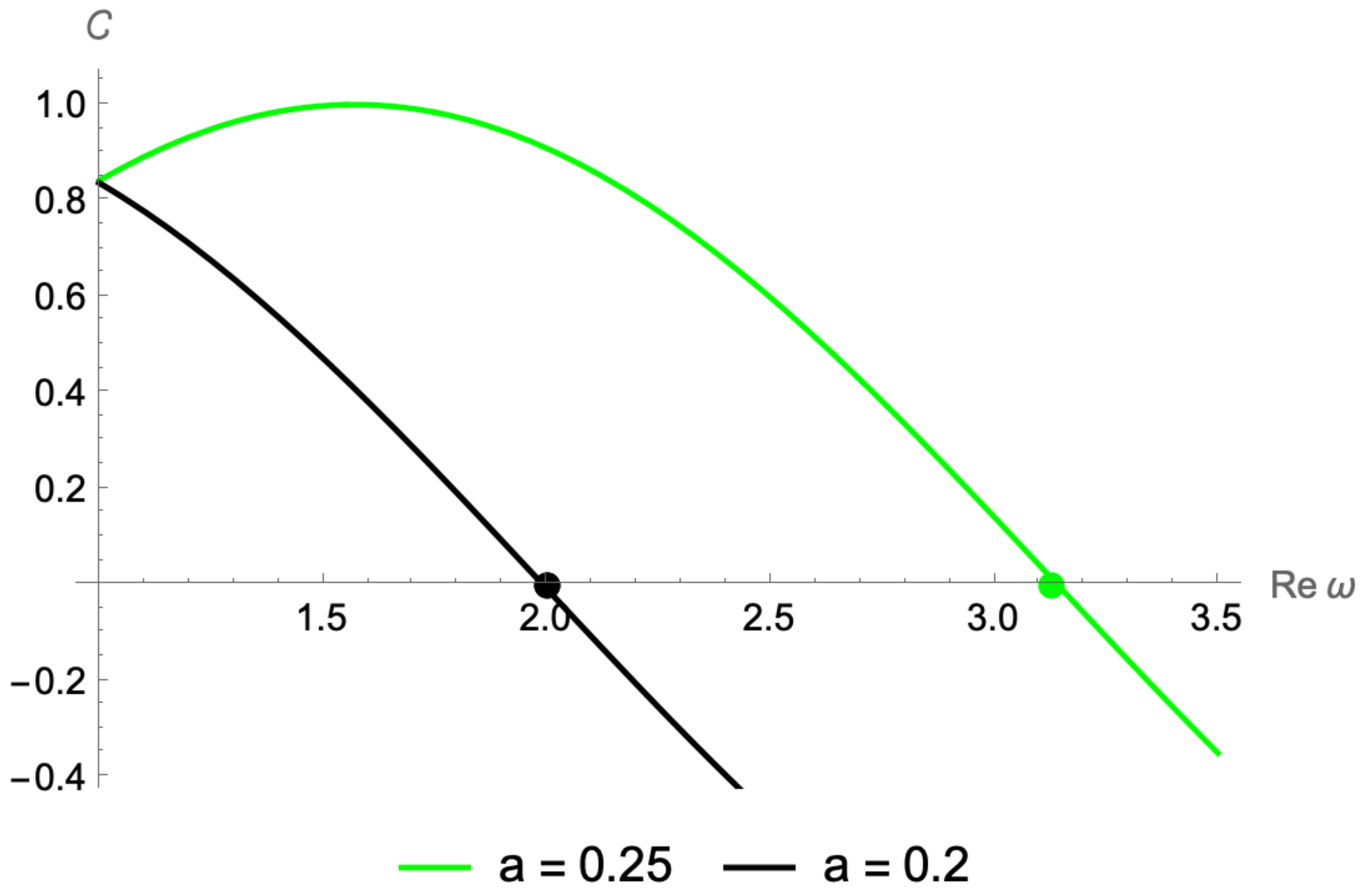

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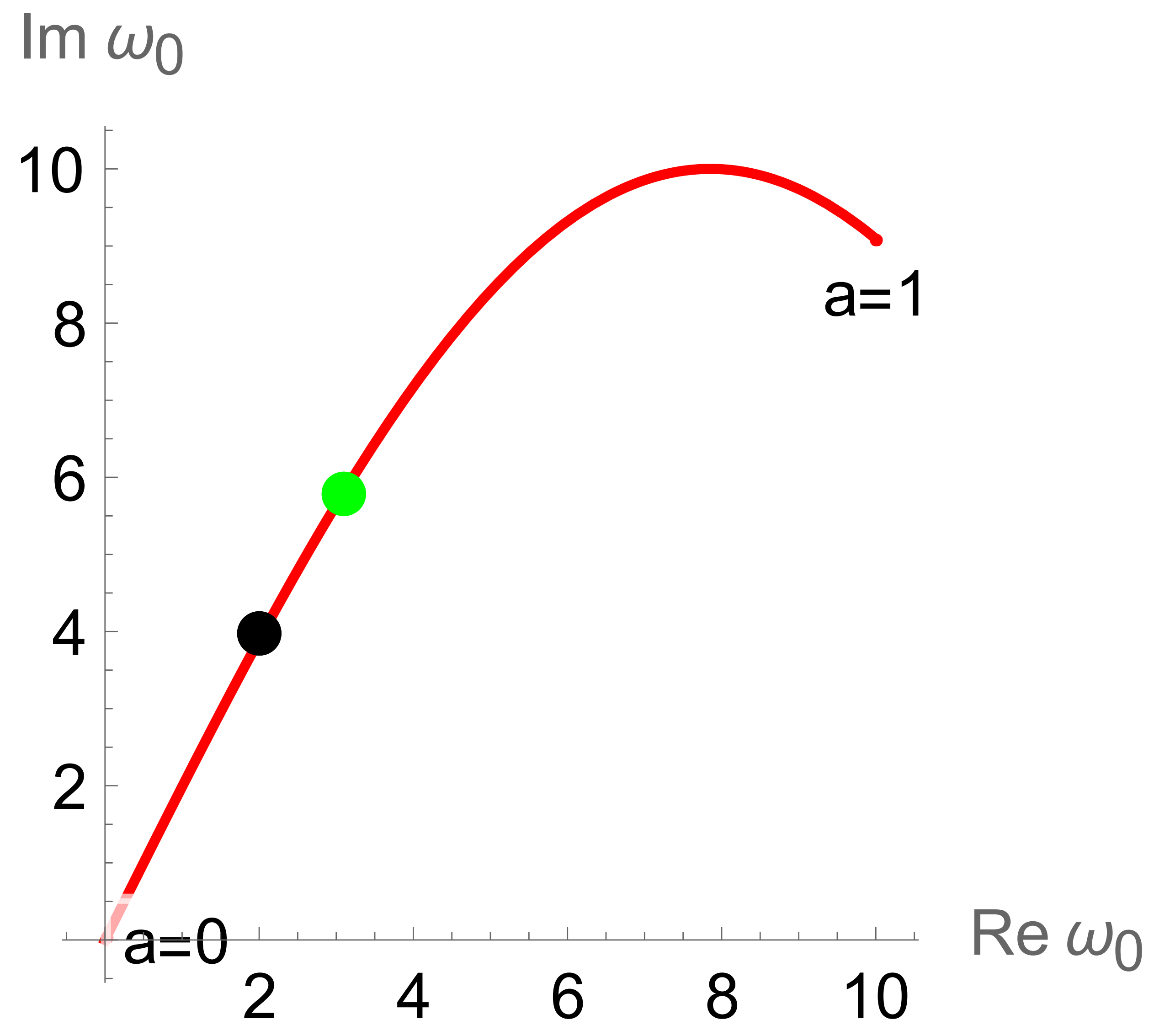
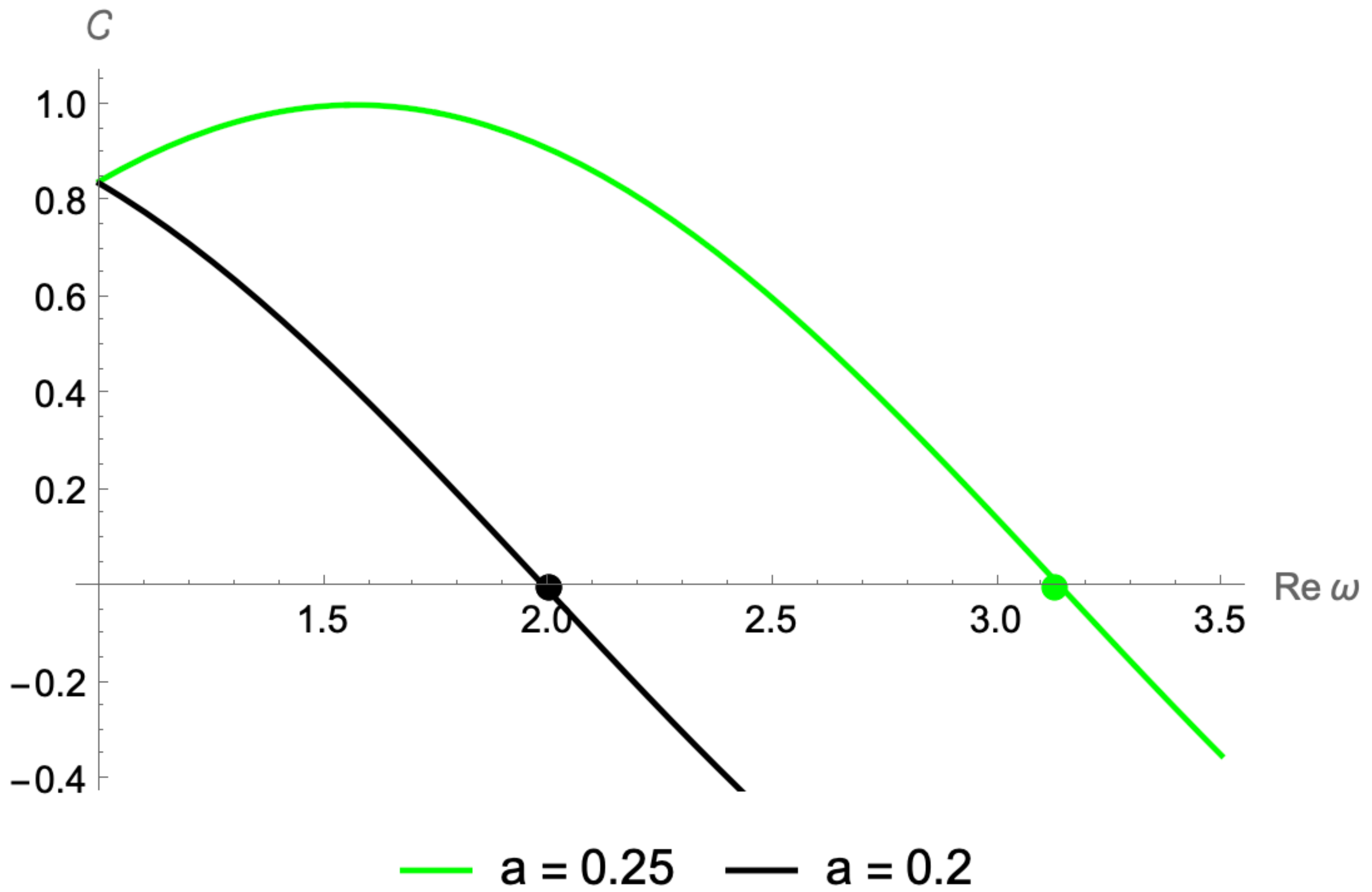
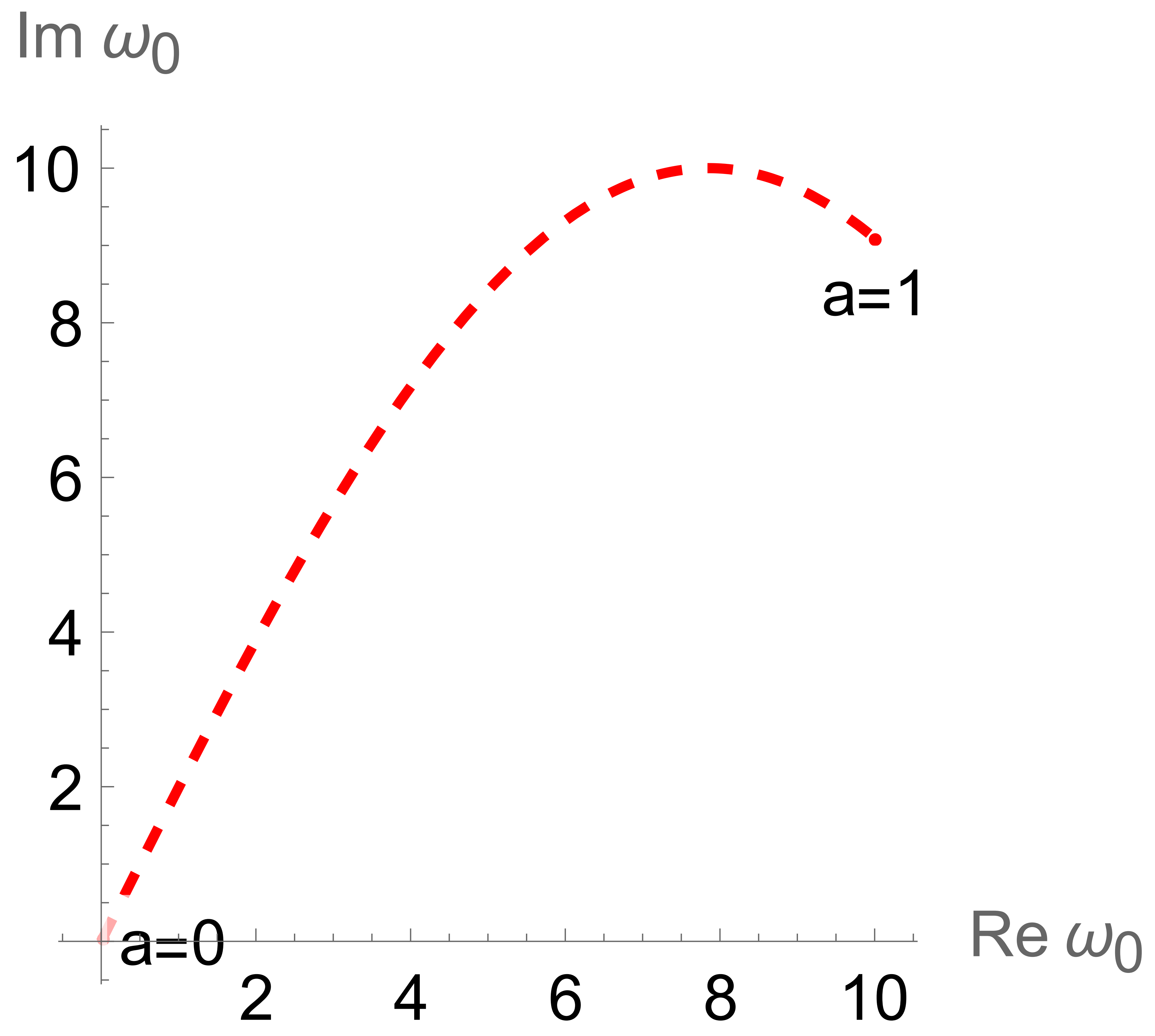
- **Important:** Distinguish b/w $\mathscr{C}(\omega, a)$ *(bottom)* and ω_0 *(up)*.





$\omega_0(a_0)$ as a starting guess to find $\omega(a_0 + da)$



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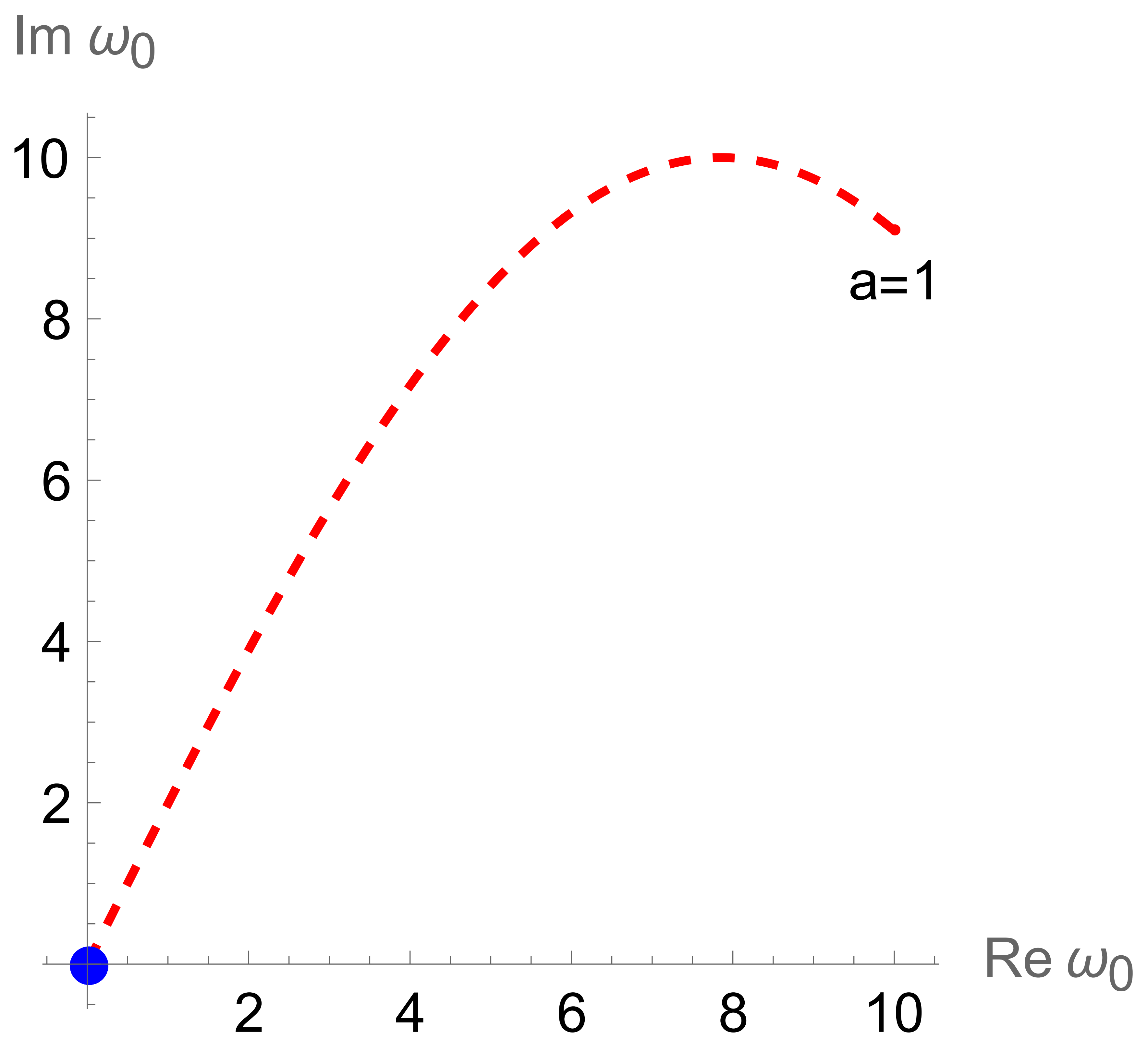

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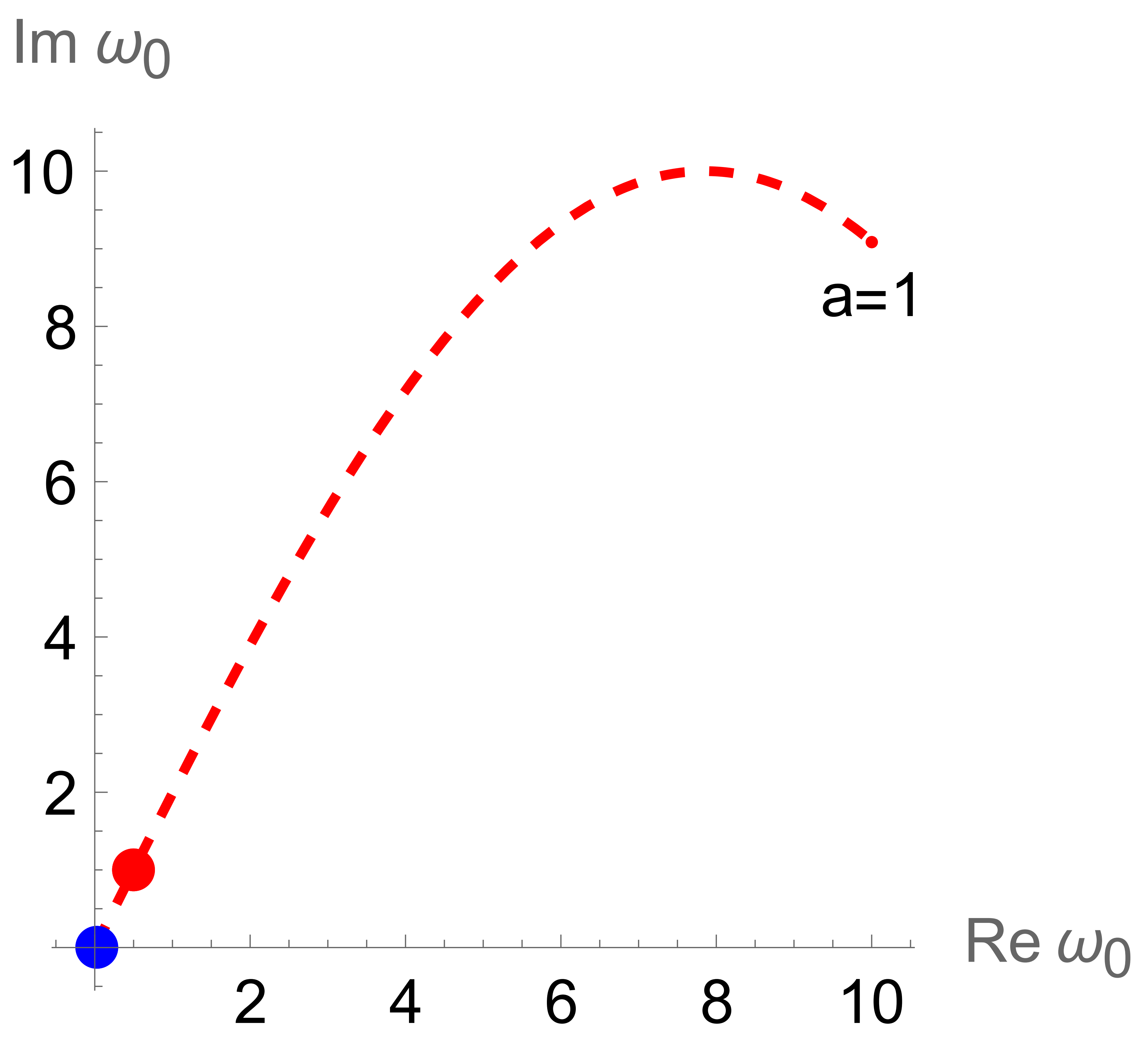

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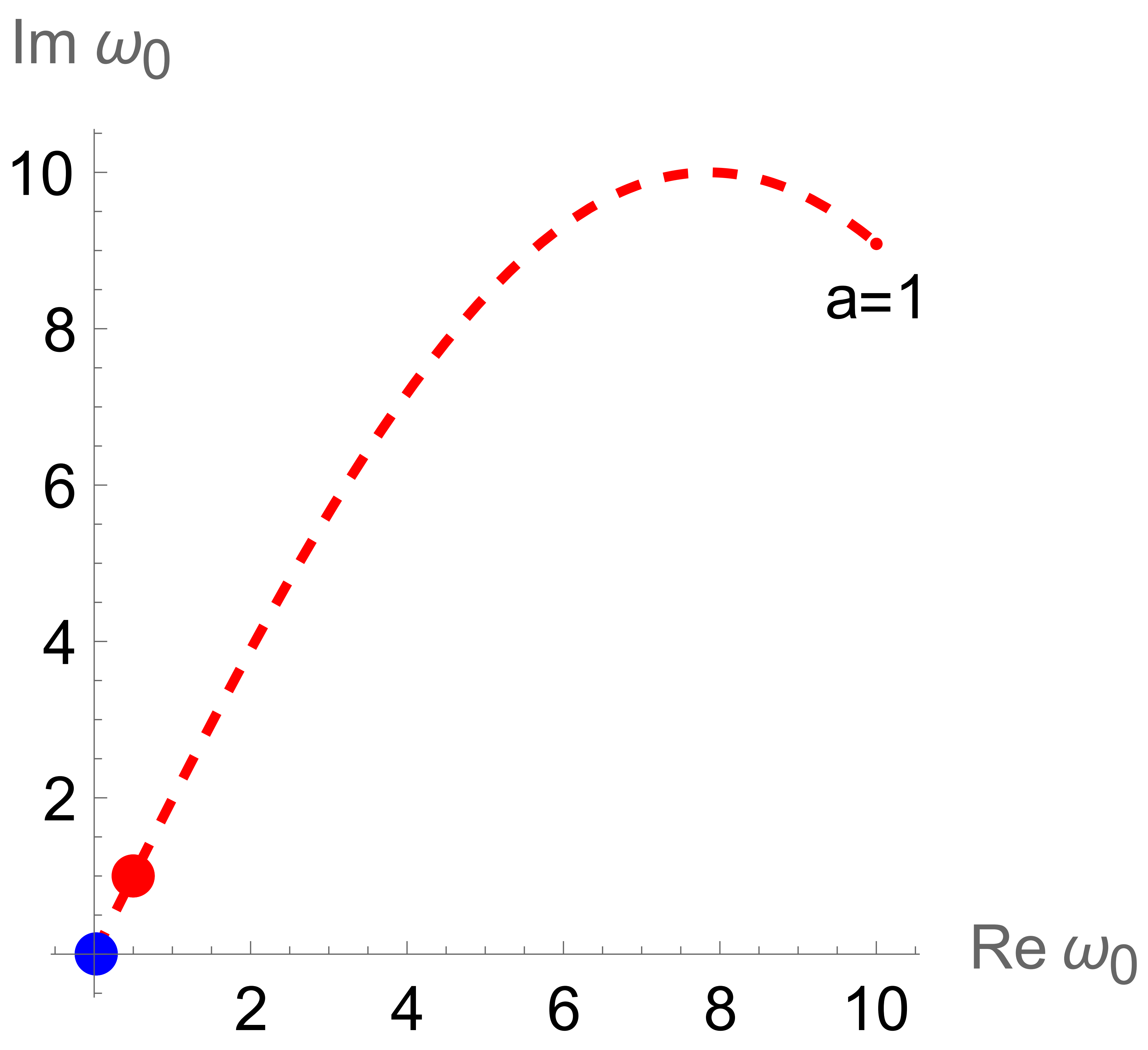

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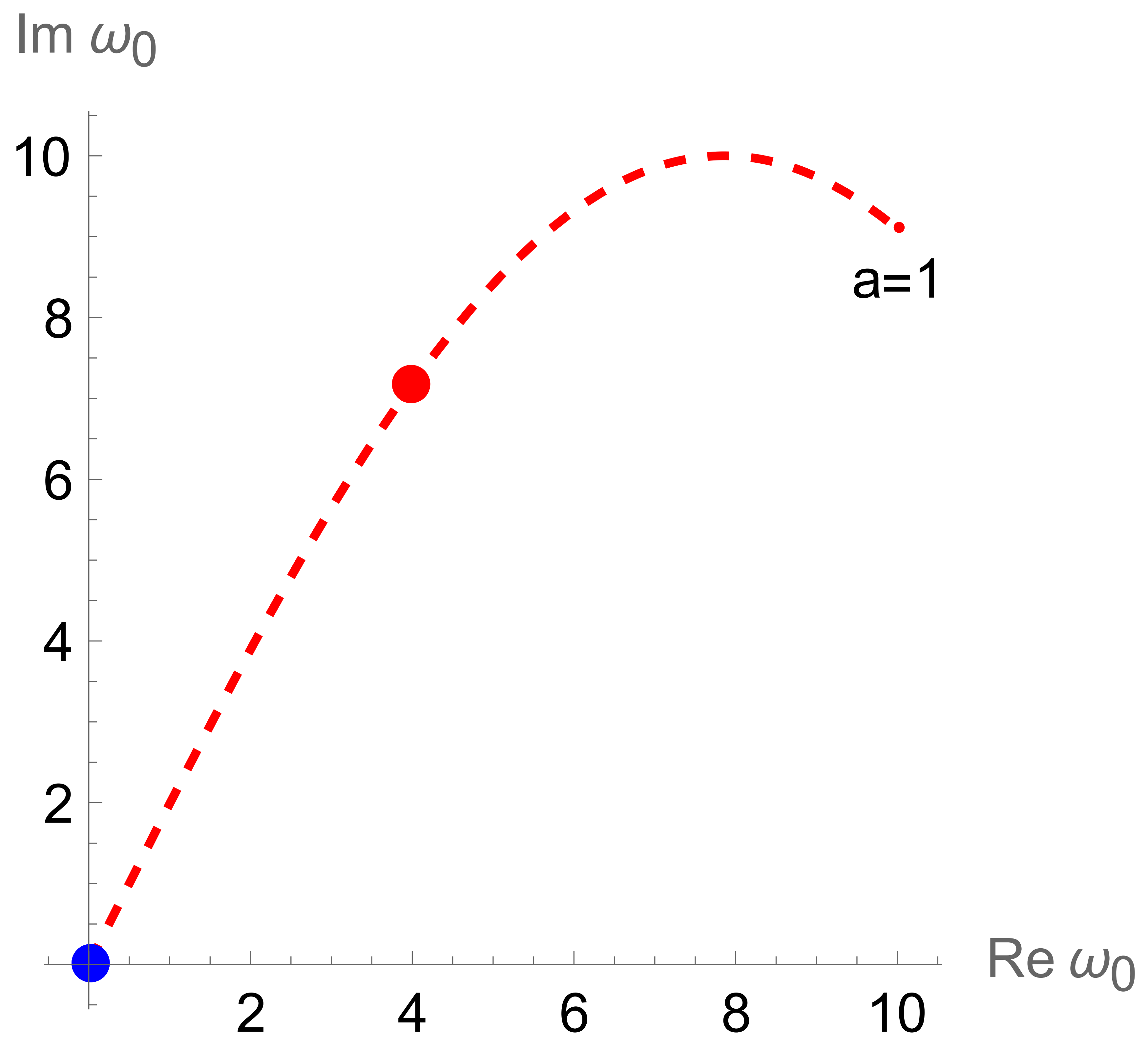

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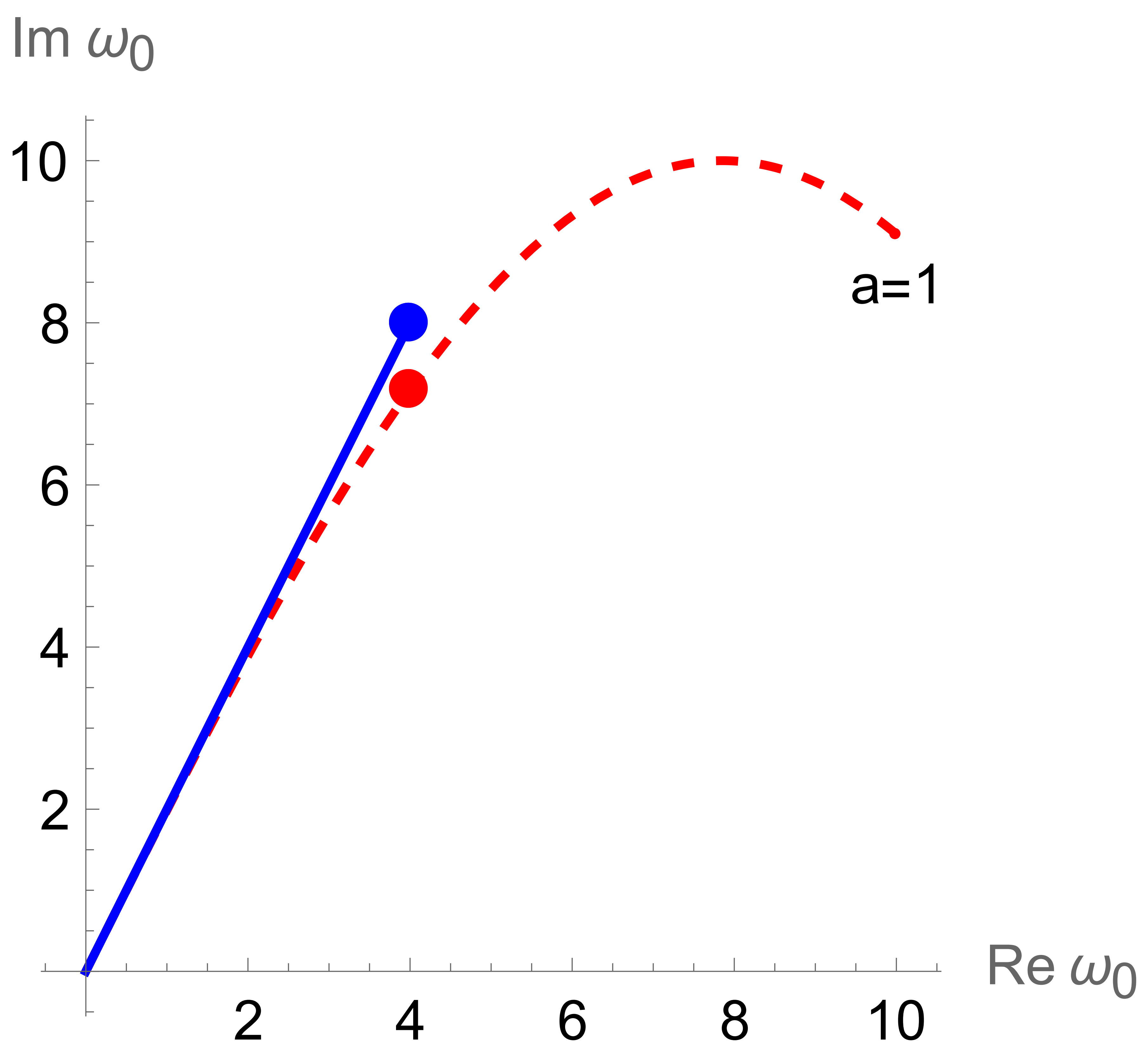

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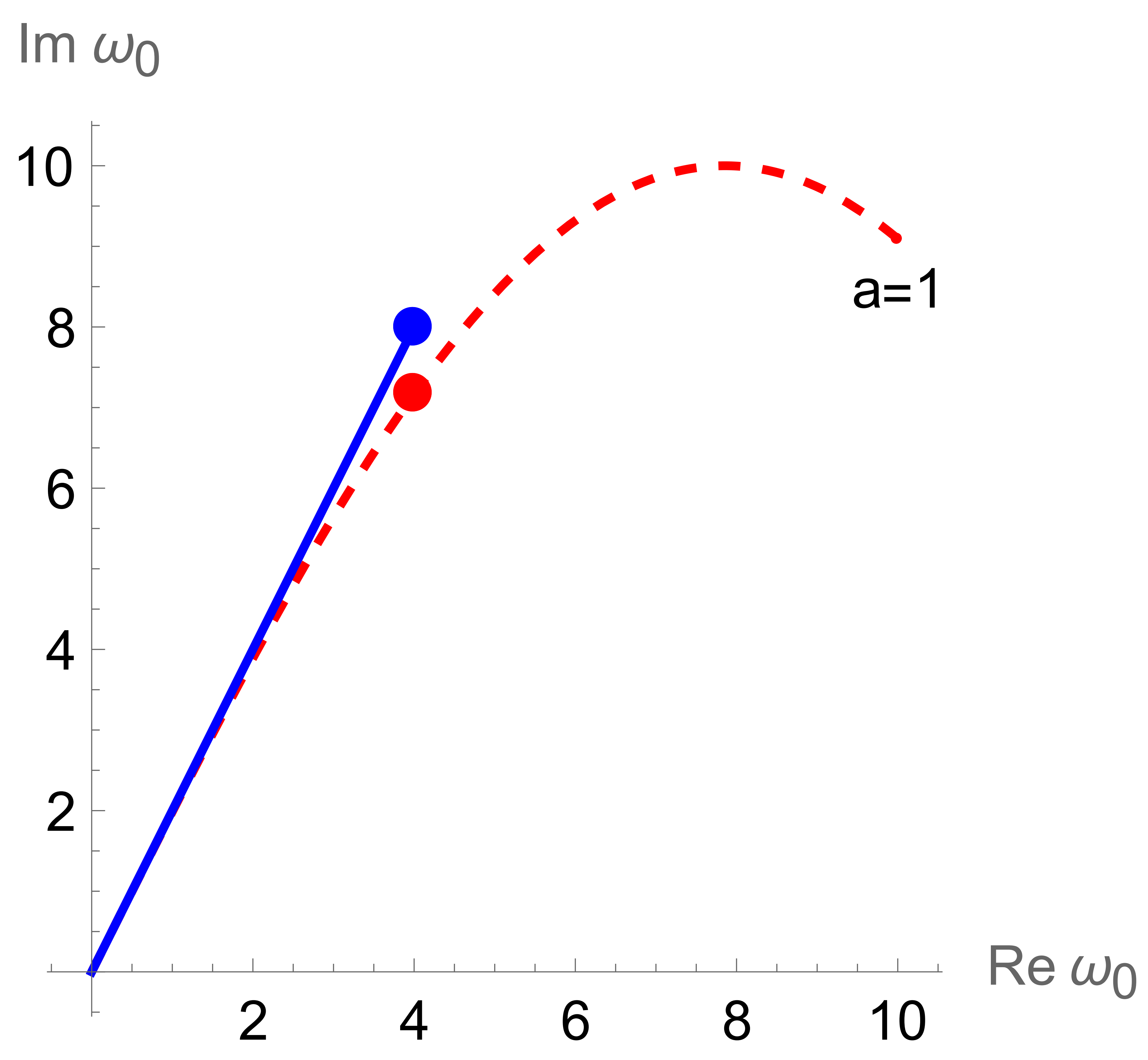
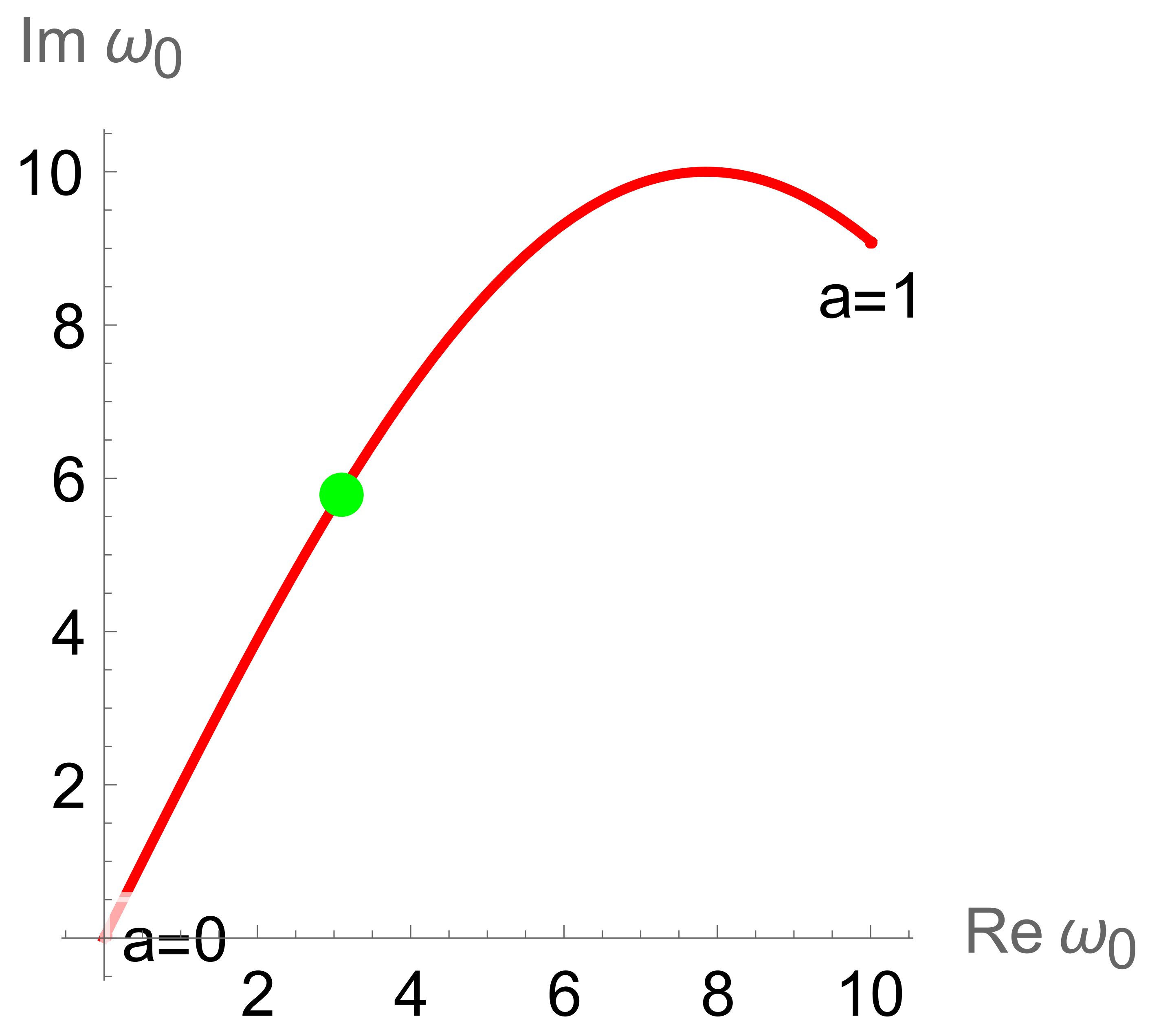
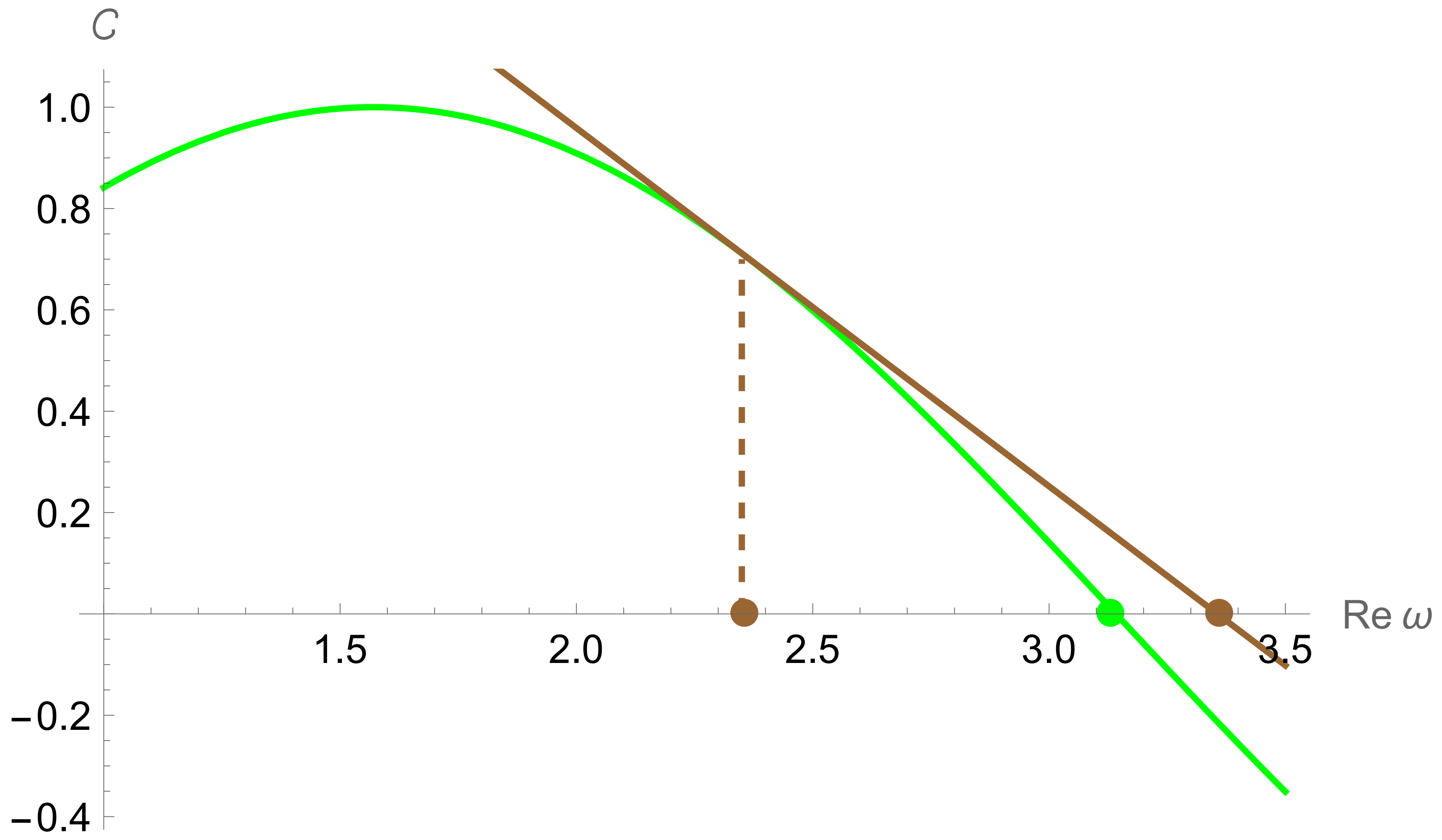


Recall...


Im ω_0
10
8
6
4
2
a=1
a=0
2
4
6
8
10
Re ω_0


C
1.0
0.8
0.6
0.4
0.2
−0.2
−0.4
1.5
2.0
2.5
3.0
3.5
Re ω

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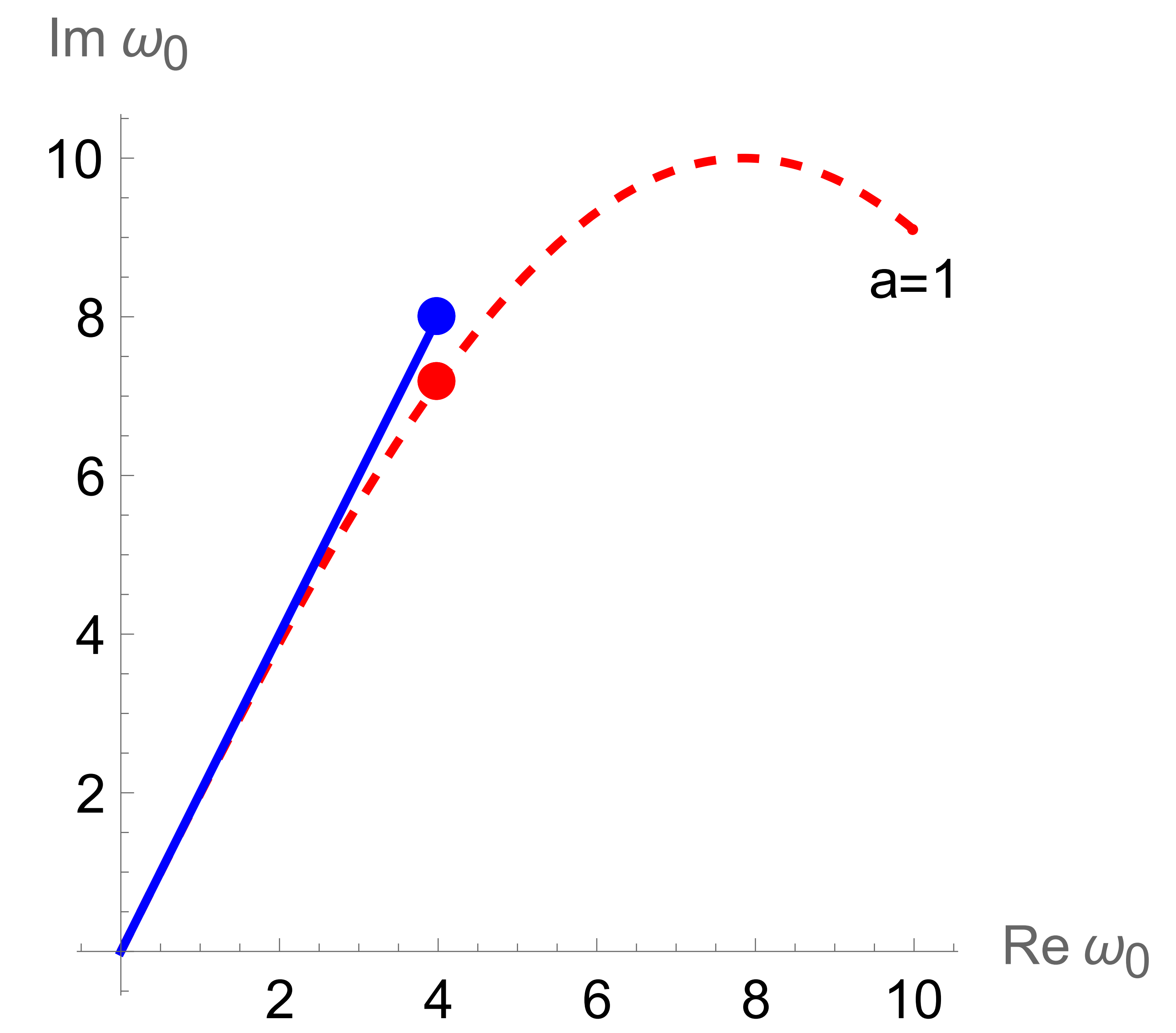

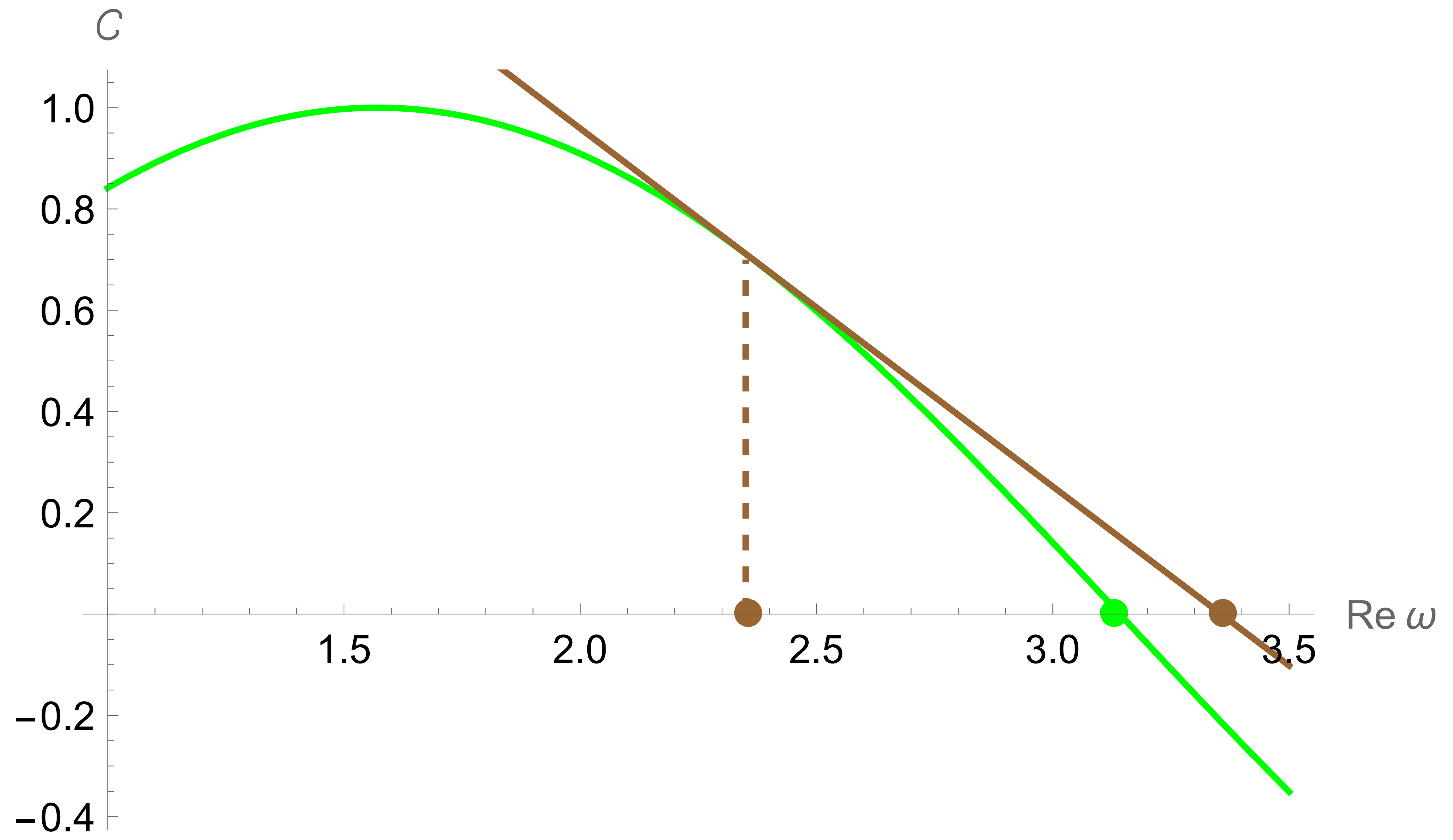

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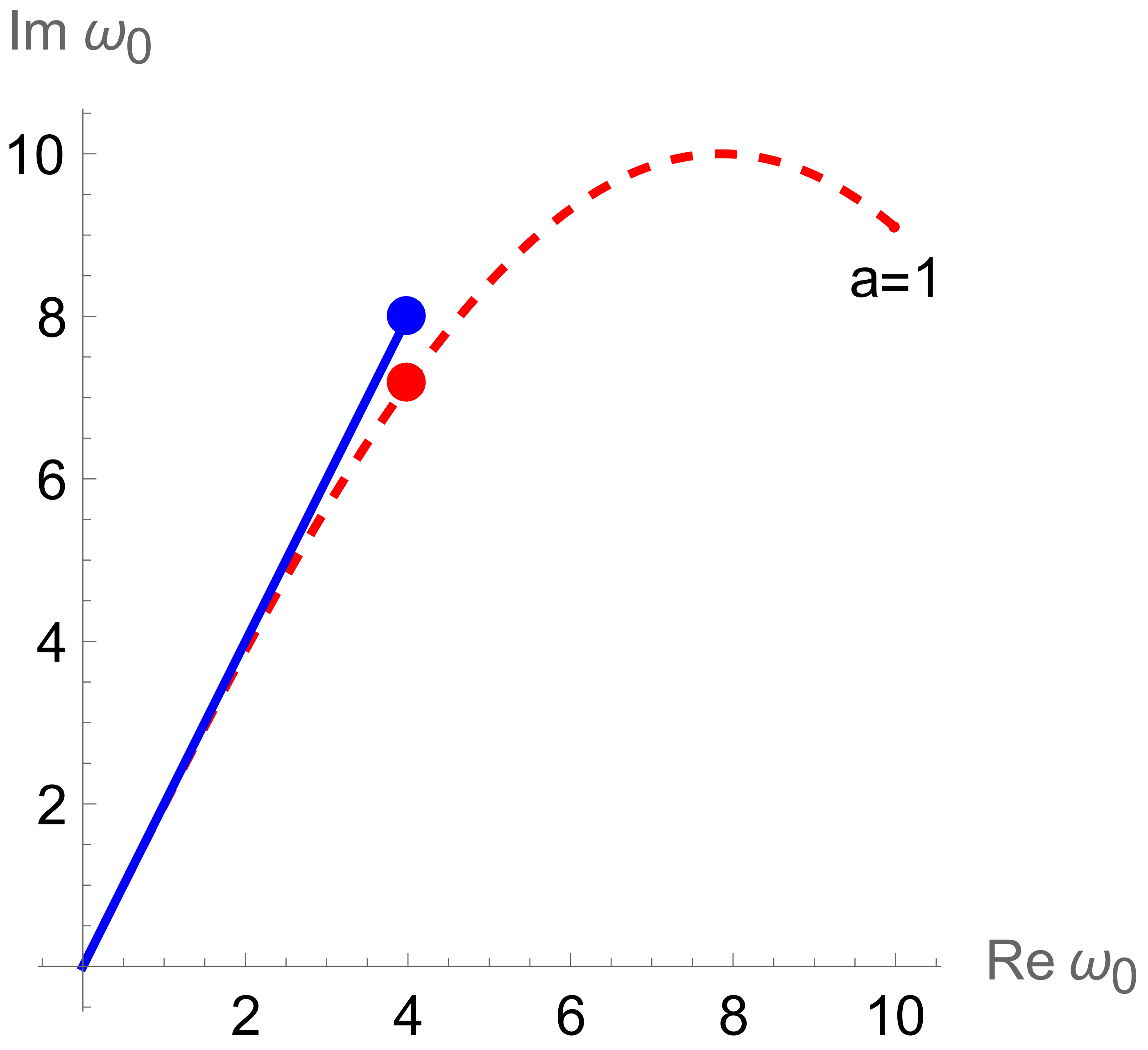
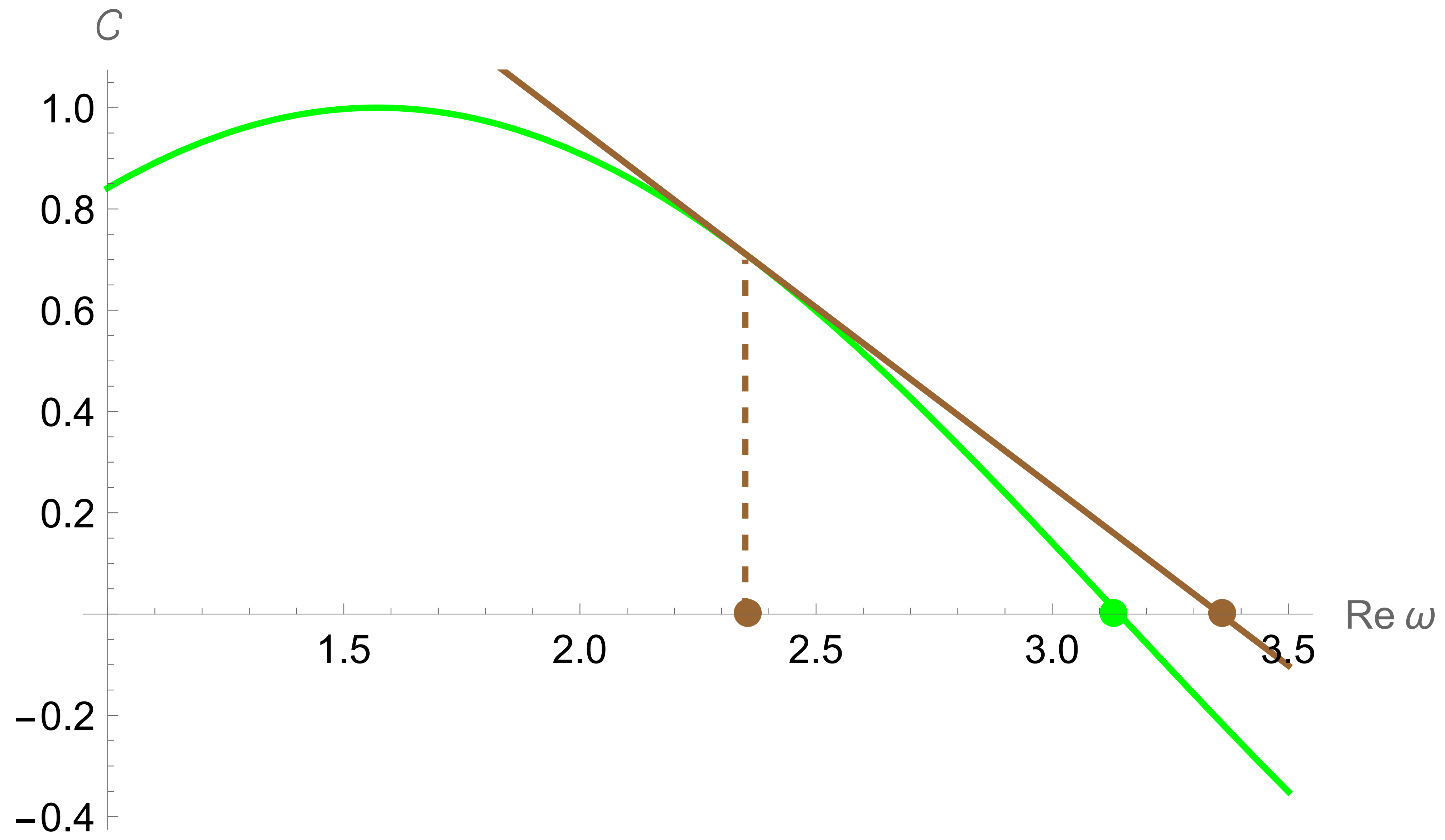
- **Result:** we provide $d\mathscr{C}/d\omega$ for Newton-Raphson analytically.
- Analytical derivatives preferred over numerical ones *(see Secs. 5.7, 9.4, 9.6, and 9.7 of Numerical Recipes in C).*





Another purpose of derivatives

QNM curves intersect *[1410.7698 (Cook & Zalutskiy)]*

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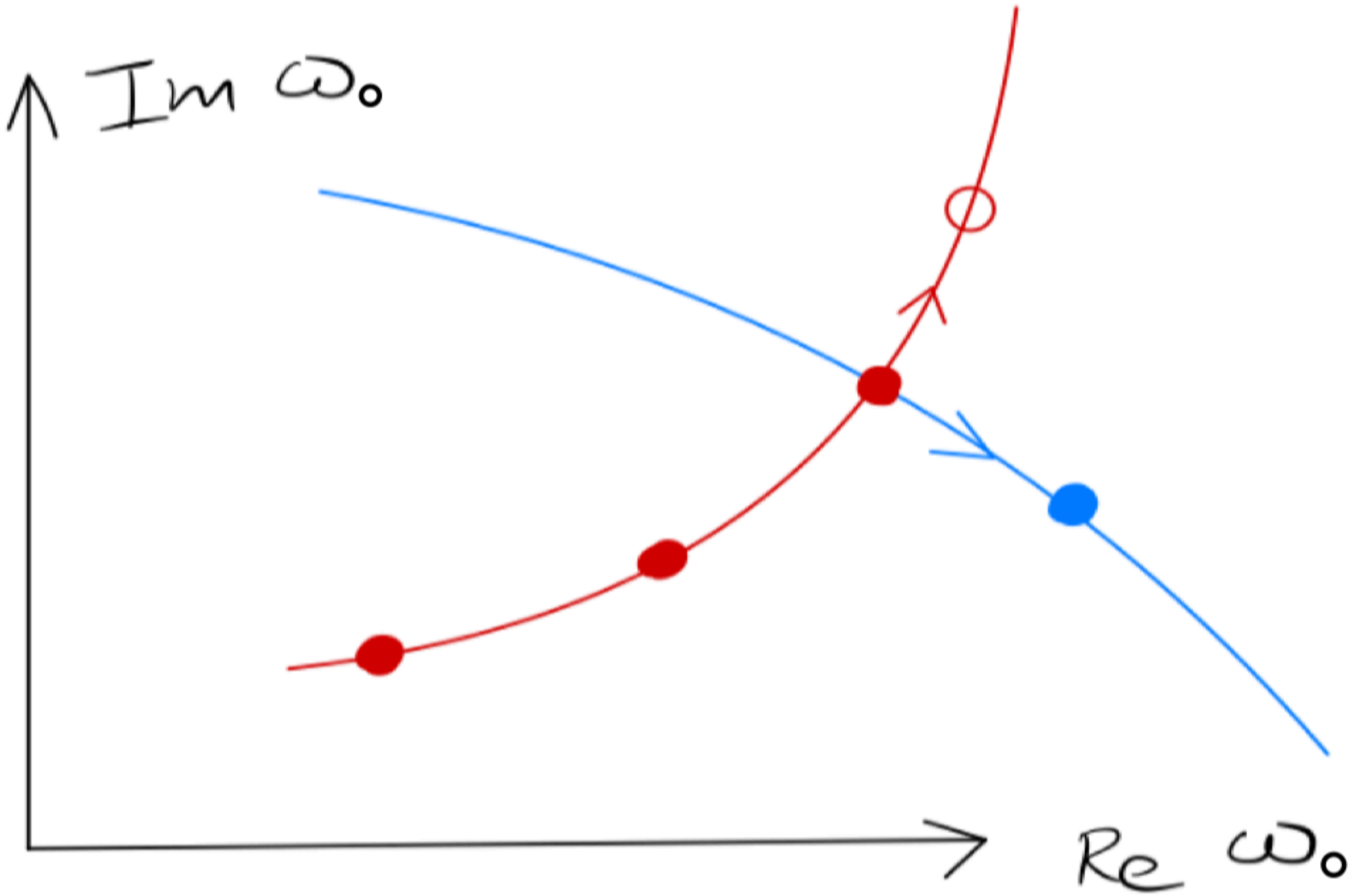
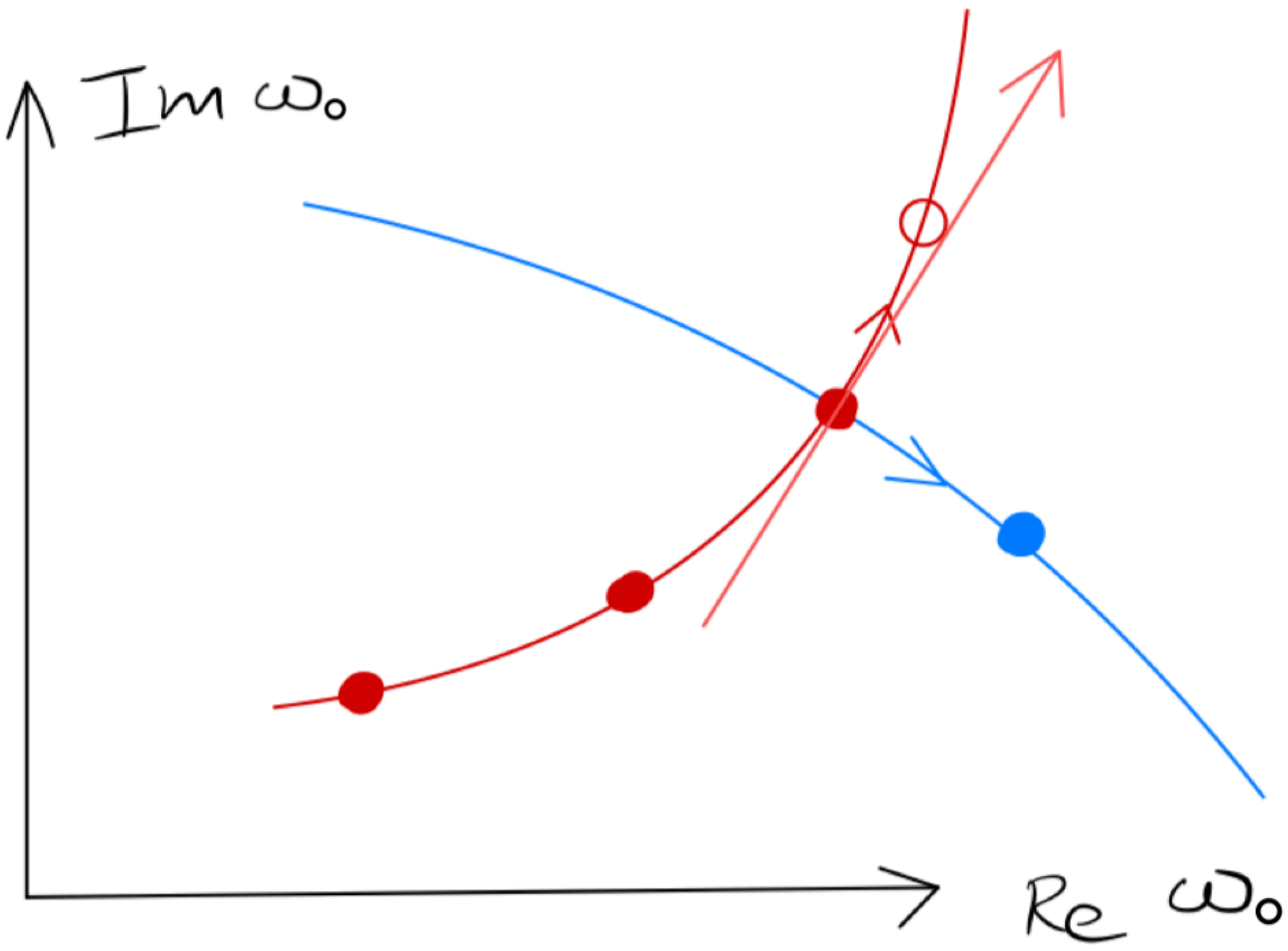


While following red curve, we do not want to land on the blue curve.

Another purpose of derivatives



$d\omega_0/da$ allow us to follow the red QNM curve.

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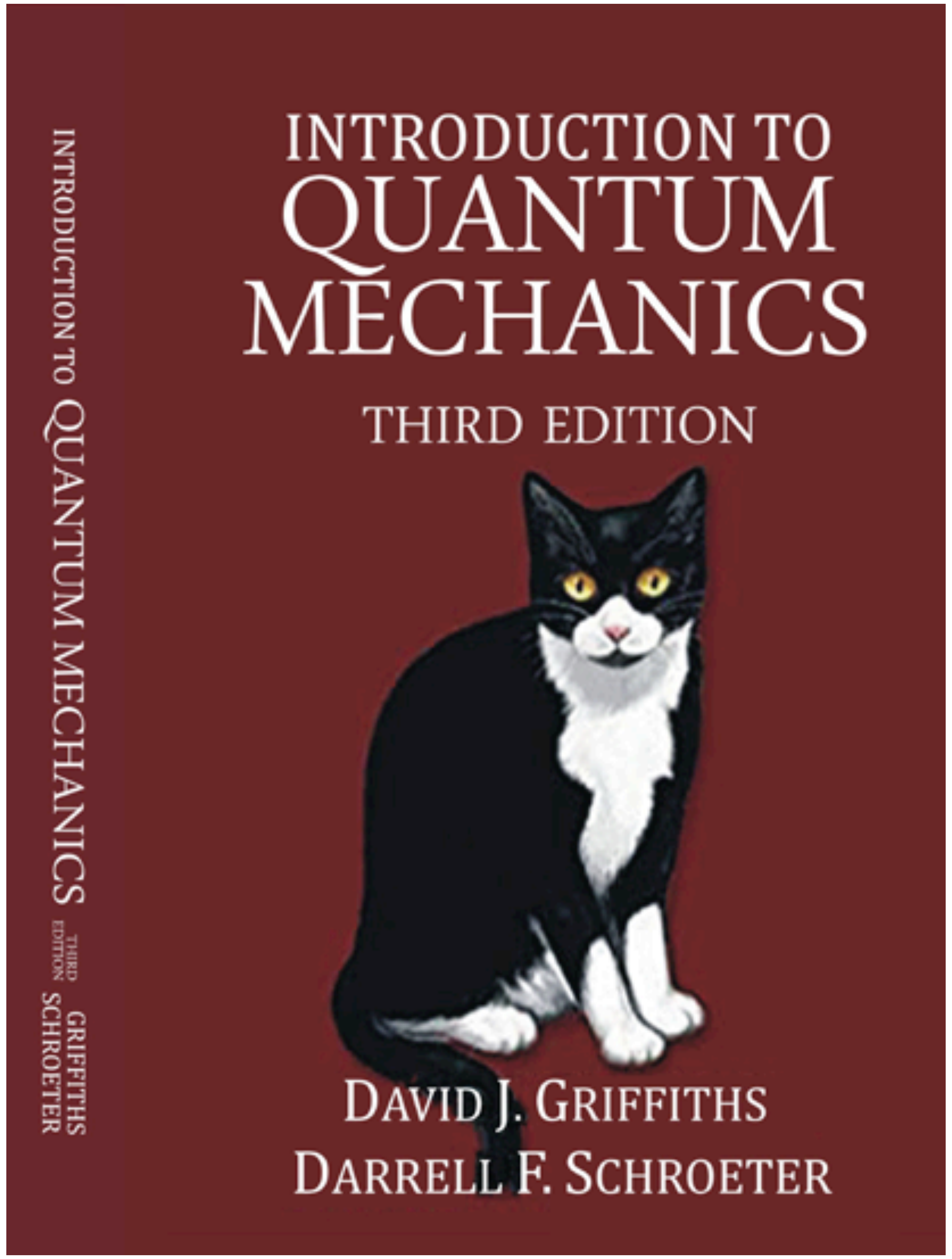

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- I lied; we have 2 equations in 2 unknowns (ω_0, A); $A =$ eigenvalue.
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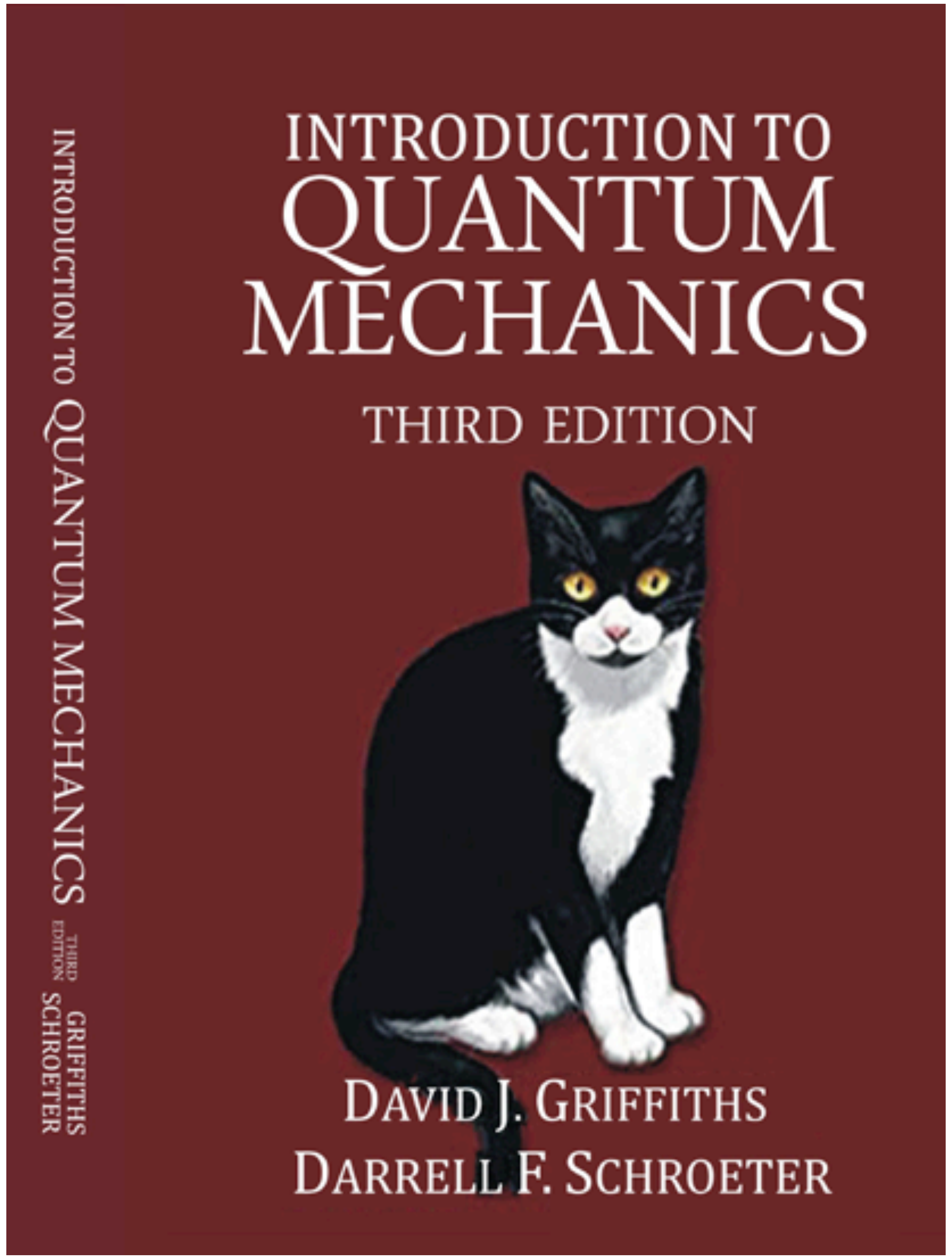
- Redo Griffiths' quantum mechanical perturbation theory with a *non-Hermitian* matrix.



Summary

- **Result:** we provided derivatives ($d\omega_0/da$, $d\mathscr{C}/d\omega$ & dA/da) to make QNM frequency computation more efficient and robust.

- **Result:** $d\omega_0/da$ lets us take larger step sizes $da \sim 0.02 \rightarrow 0.25$.

- **Future:** Calculate and incorporate $d^2\omega_0/da^2$; can let us take $da \sim 0.65$.

- **Future:** apply this method to beyond Kerr QNMs (within GR) and beyond GR; method seems adaptable.

- **Refs:** *arXiv: 2210.03657, github.com/sashwattanay/qnm*
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