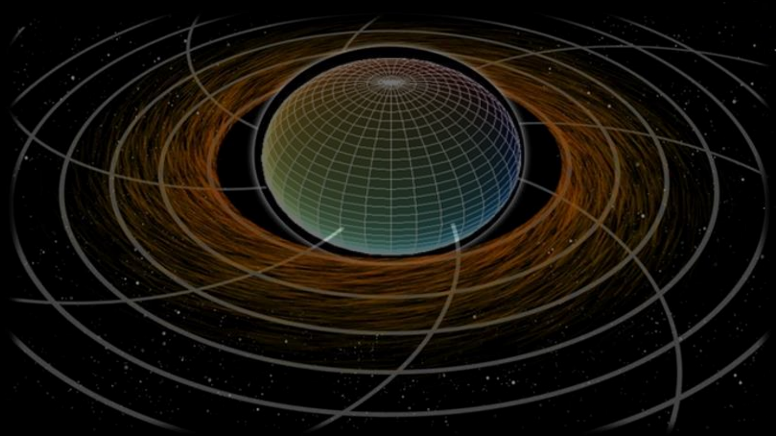
RELATIVISTIC EFFECTS ON THE ORBITS OF THE CLOSEST STARS TO THE BLACK HOLE AT THE CENTER OF THE GALAXY

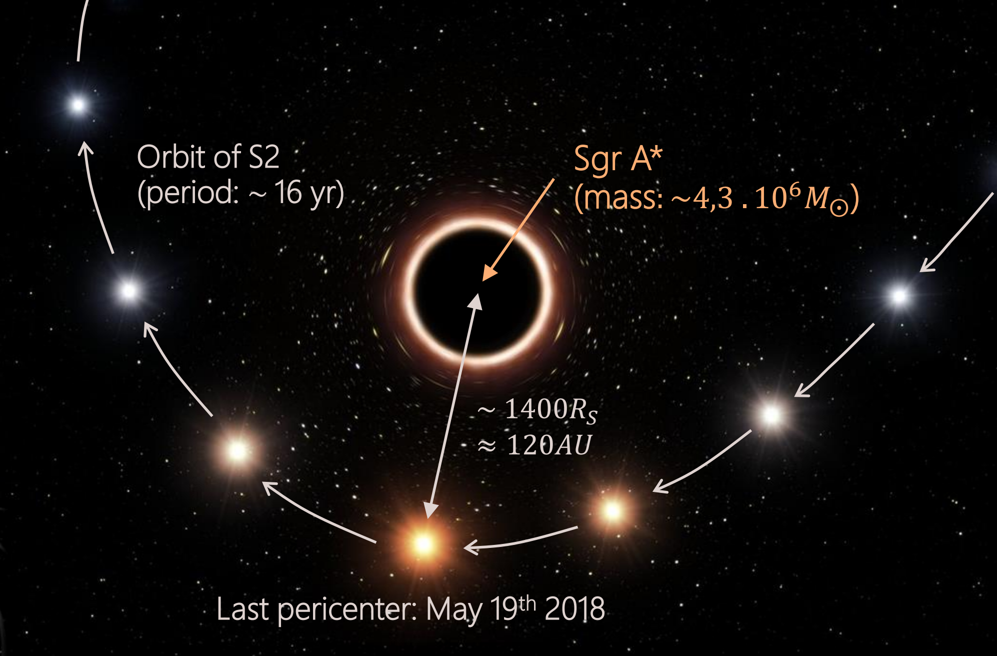
API: 17/11/2023

Karim ABD EL DAYEM

Supervisors: Frédéric VINCENT, Thibaut PAUMARD, Guy PERRIN

CONTEXT & AIMS





Curved space-time ⟷ Mass

Spinning space-time ⟷ Spin

Oblate spheroid ⟷ Quadrupole moment

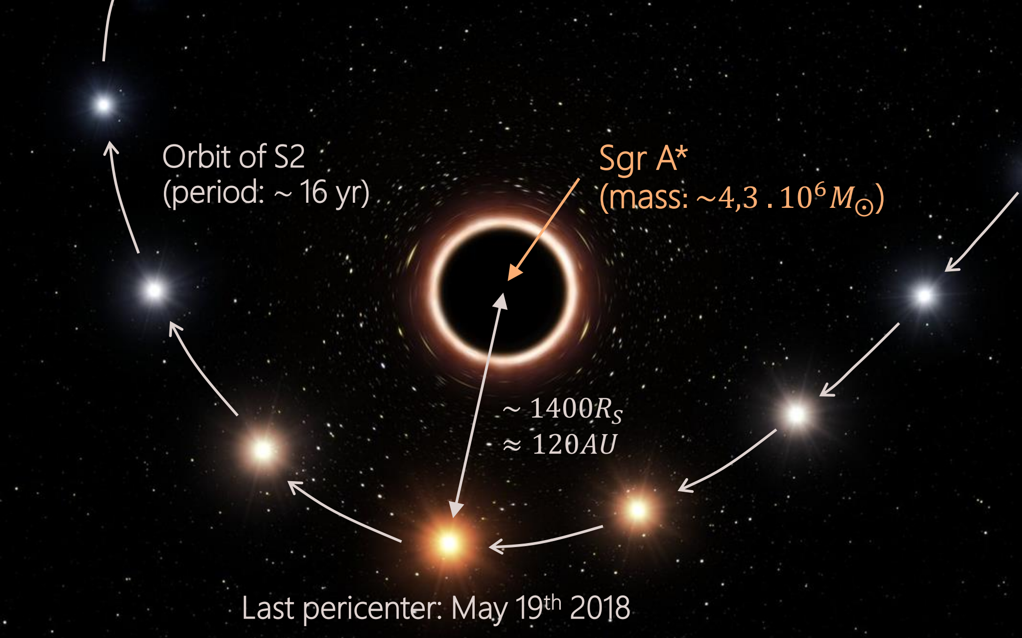
No hair-theorem:

Mass

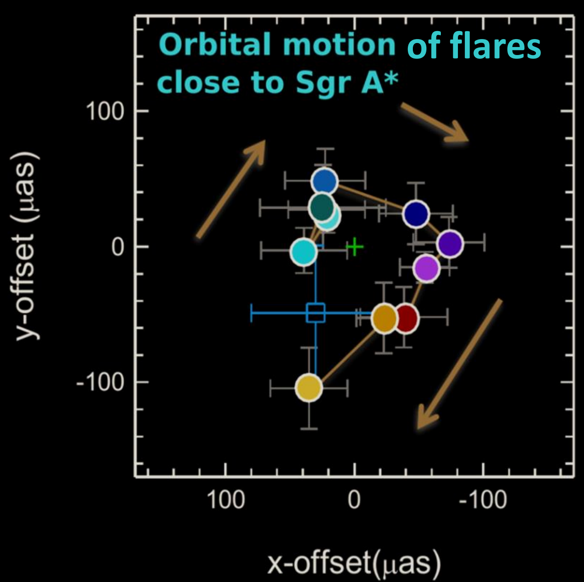
Spin ⟷ Quadrupole moment

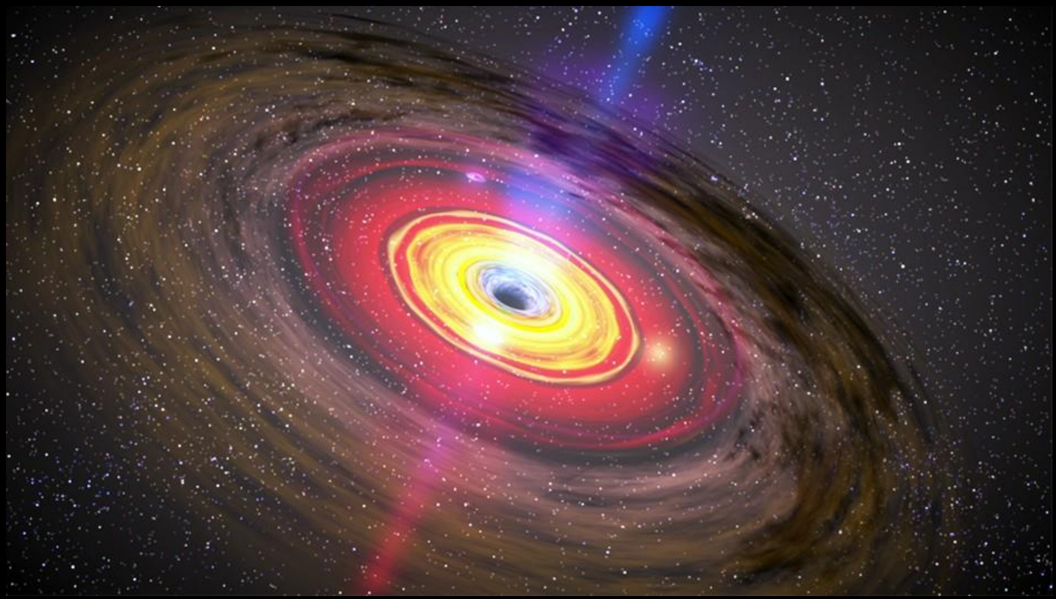
CONTEXT & AIMS

GRAVITY Collaboration (2018, 2020) and EHT (2022) results:



2018





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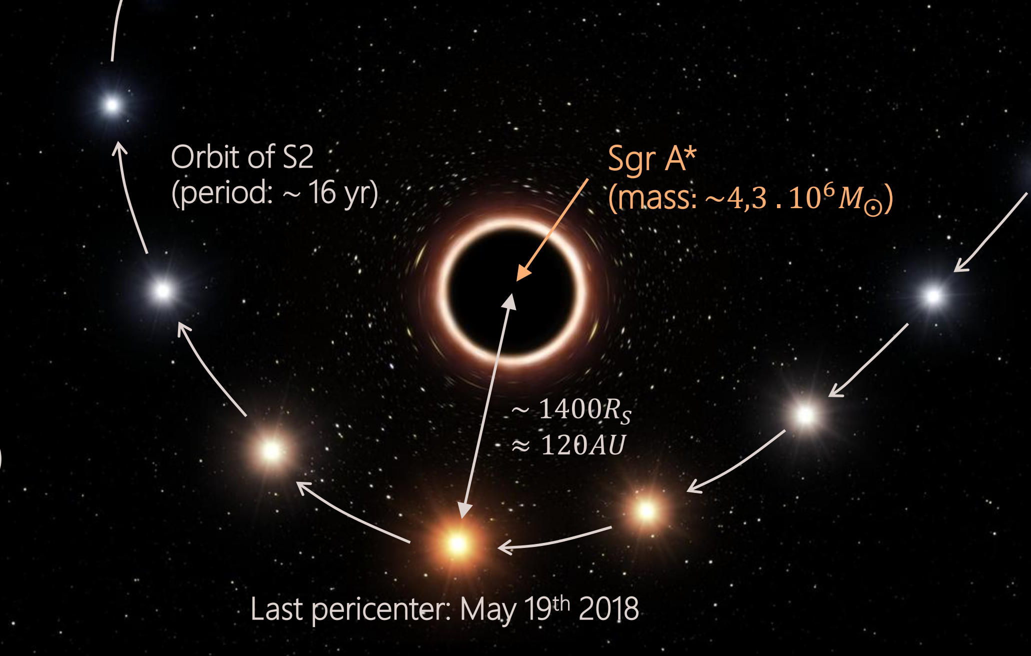

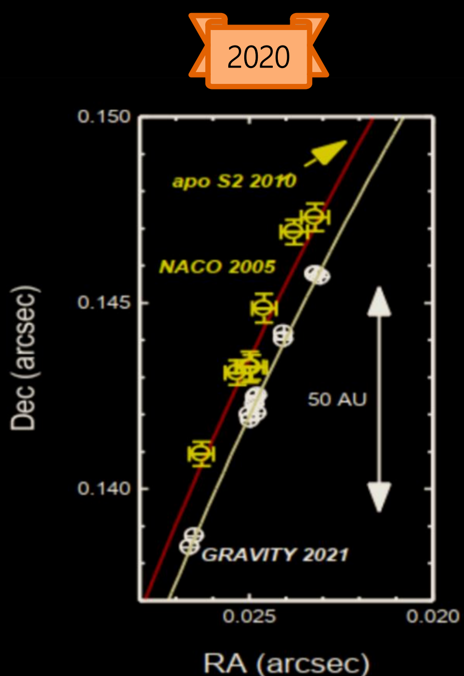

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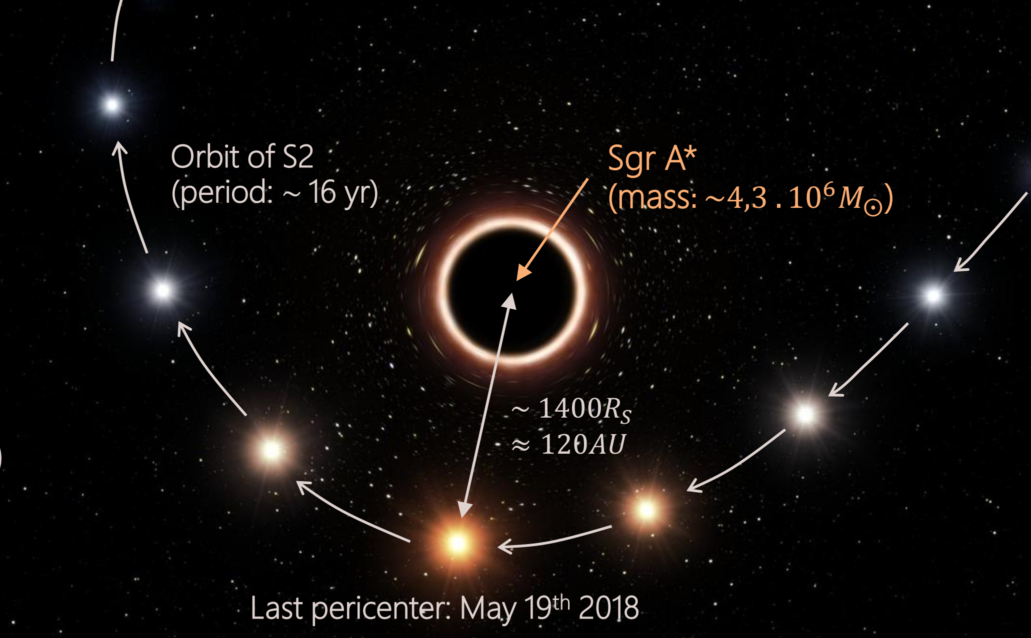

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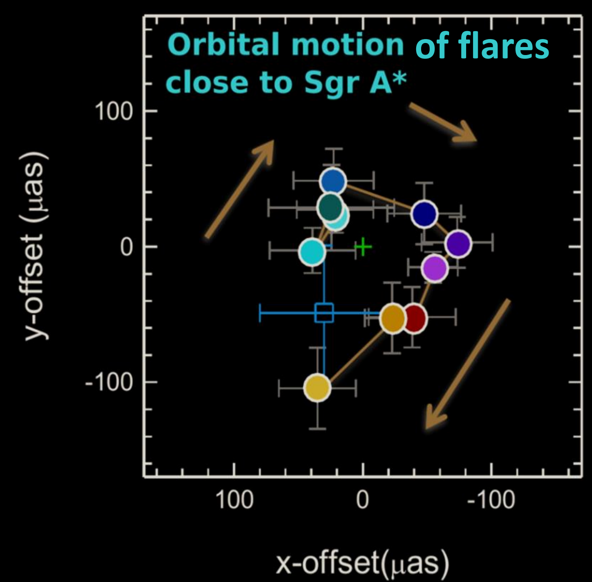
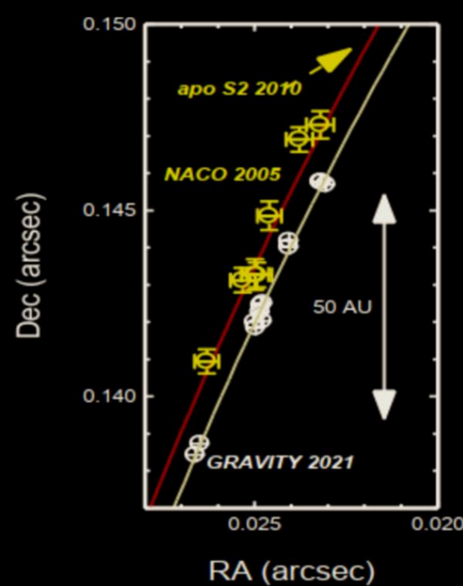
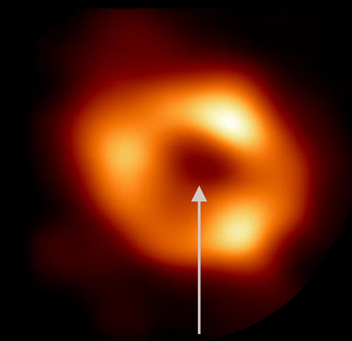


2020



2022



Mass = 4.3 × $10^6 M_\odot$
Spin = ?
Quadrupole moment = ?

Maximal accuracy	Astrometric (µas)	Spectroscopic (km/s)	State
SINFONI (VLT)	–	≈ 10	Decommissioned
NIRSPEC (Keck)	–	≈ 10	Operational
GRAVITY (VLT)	≈ 10	–	Operational
ERIS (VLT)	–	≈ 10	Operational in 2022+
GRAVITY+ (VLT)	≈ 10	–	Operational in 2024+
MICADO (E-ELT)	≈ 50	≈ 1	Operational in 2028+

DEPEND ON THE FLUX OF THE INSTRUMENT

INSTRUMENTATION

VLT (Paranal) :

- GRAVITY (Interferometer)

$m_K \leq 19$

- SINFONI (Spectropgraph)

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GRAVITY+ CAN MULTIPLY THE NUMBER OF PHOTONS UP TO A FACTOR 20!

-Better temporal coverage

-Less systematic errors

-Less photon noise

DEPEND ON THE FLUX REACHING THE INSTRUMENT

POSSIBILITY OF DETECTING OTHER STARS

INSTRUMENTATION

VLT (Paranal) :

- GRAVITY → GRAVITY + (Interferometer)

$m_K \leq 19$ → $m_K \leq 22$

- ERIS (Spectropgraph)

RELATIVISTIC EFFECTS IN THE VICINITY OF SGR A*

Roemer Effect	Kepler / Minkowski	Developed with analytical approximations
Schwarzschild Precession	Schwarzschild	Developed with analytical approximations
Shapiro Effect	Schwarzschild / Kerr	Developed only in Schwarzschild
Relativistic Redshifts	Schwarzschild / Kerr	Developed only in Schwarzschild
Gravitational Lensing	Schwarzschild / Kerr	Developed only in Schwarzschild
Lense-Thirring Effect	Kerr	Not developed with analytical approximations

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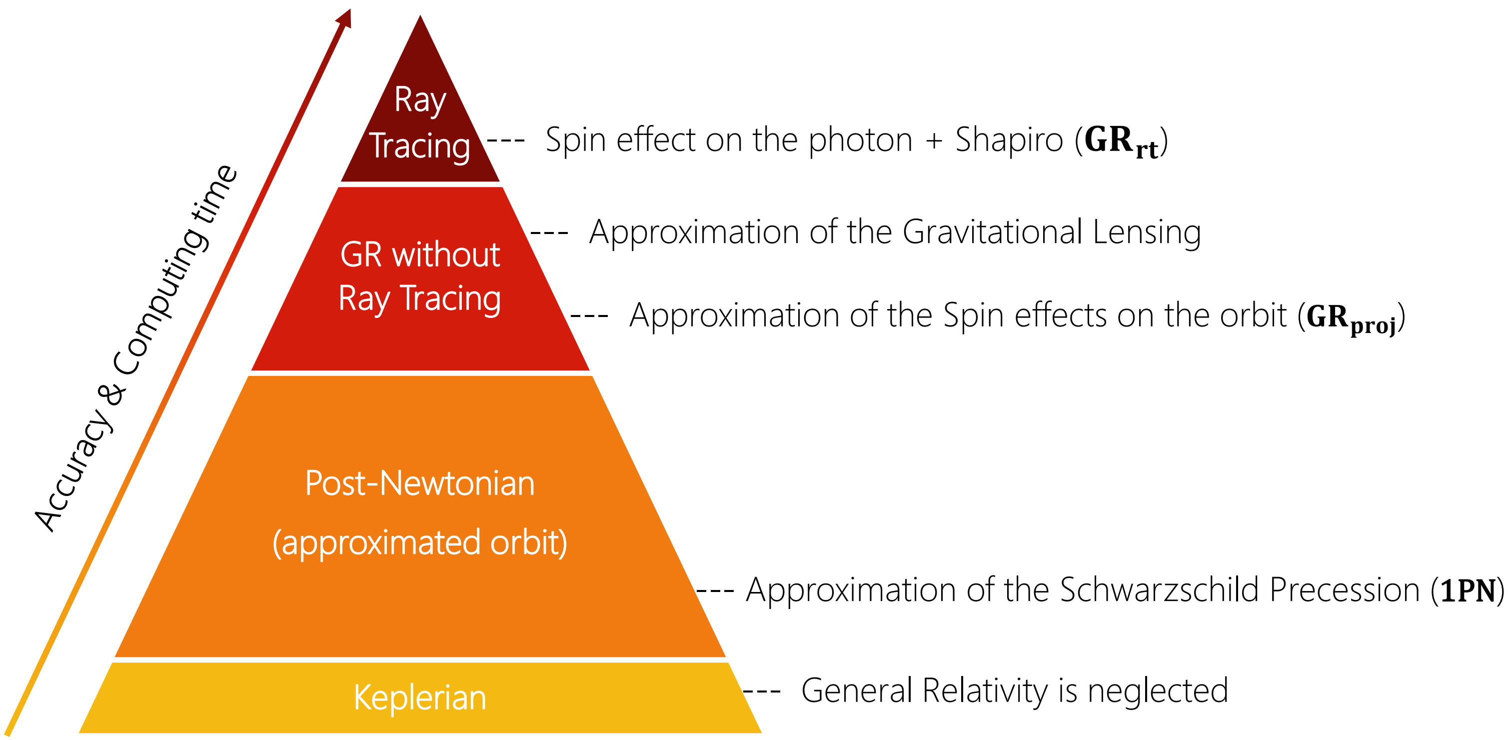
TOOLS: GYOTO AND PALETTE OF RELATIVISTIC MODELS

GYOTO

- Very modular C++ code
- Compute null and time-like geodesics
- Integrates the radiative transfer equation



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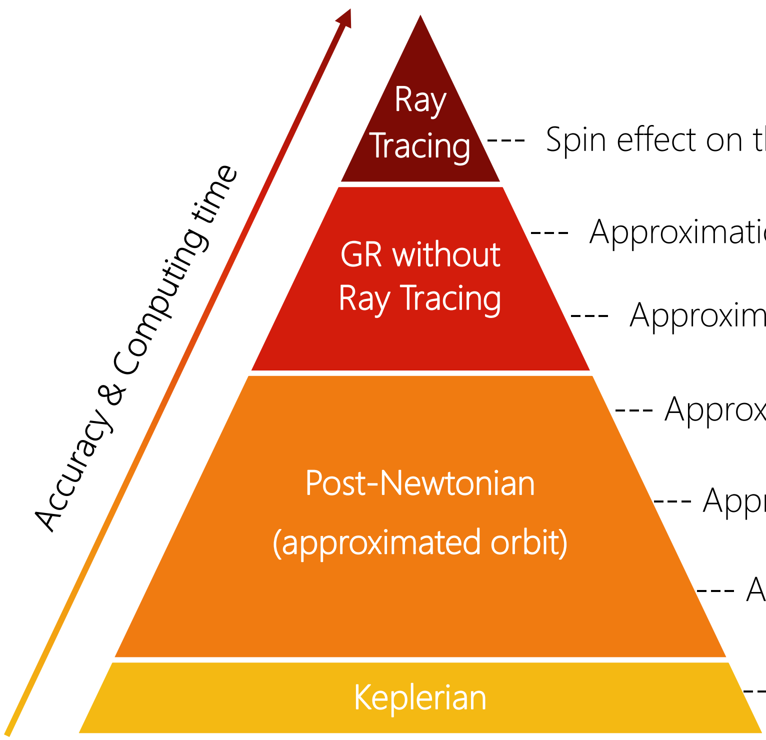


--- Spin effect on the photon + Shapiro ($\mathbf{GR_{rt}}$)

--- Approximation of the Gravitational Lensing

--- Approximation of the Spin effects on the orbit & RV ($\mathbf{GR_{proj}}$)

--- Approximation of the Quadrupole effects on the orbit ($\mathbf{2PN_Q}$)

--- Approximation of the Lense-Thirring effects on the orbit ($\mathbf{1.5PN}$)

--- Approximation of the Schwarzschild Precession ($\mathbf{1PN}$)

--- General Relativity is neglected

↳ Development of the $\mathbf{1.5PN}$, $\mathbf{GR_{proj}}$ and $\mathbf{2PN_Q}$ models

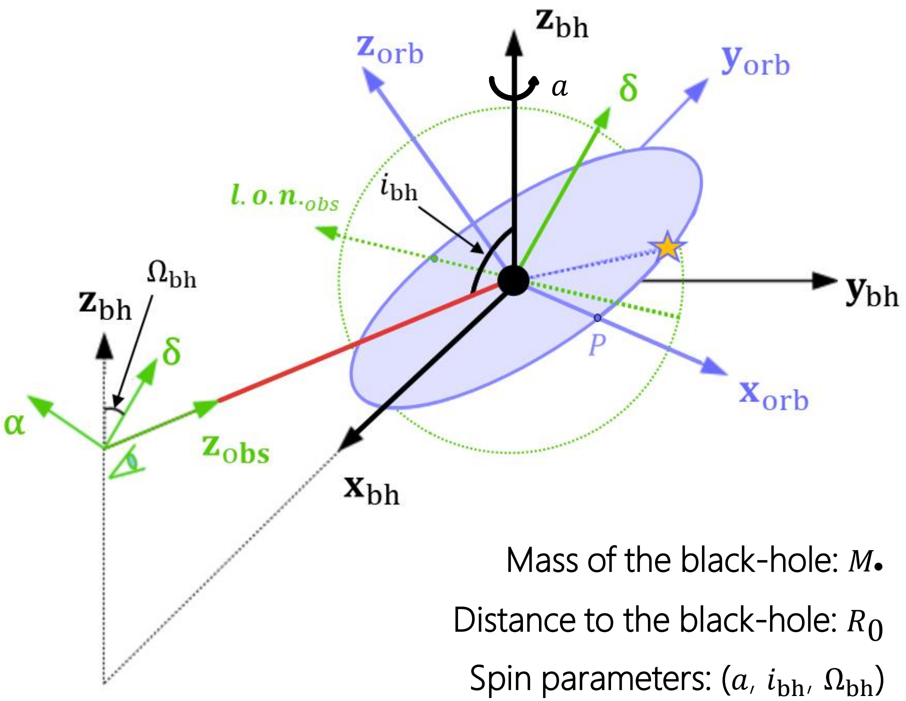
Karim

FRAMES OF REFERENCE AND OSCULATING ELEMENTS

Black-hole frame of reference: (x_{bh}, y_{bh}, z_{bh})

Orbit frame of reference: $(x_{orb}, y_{orb}, z_{orb})$

Observer's frame of reference: $(\alpha, \delta, z_{obs}) = (\mathrm{RA}, \mathrm{DEC}, z_{obs})$



Mass of the black-hole: $M_\bullet$

Distance to the black-hole: R_0

Spin parameters: $(a, i_{\mathrm{bh}}, \Omega_{\mathrm{bh}})$

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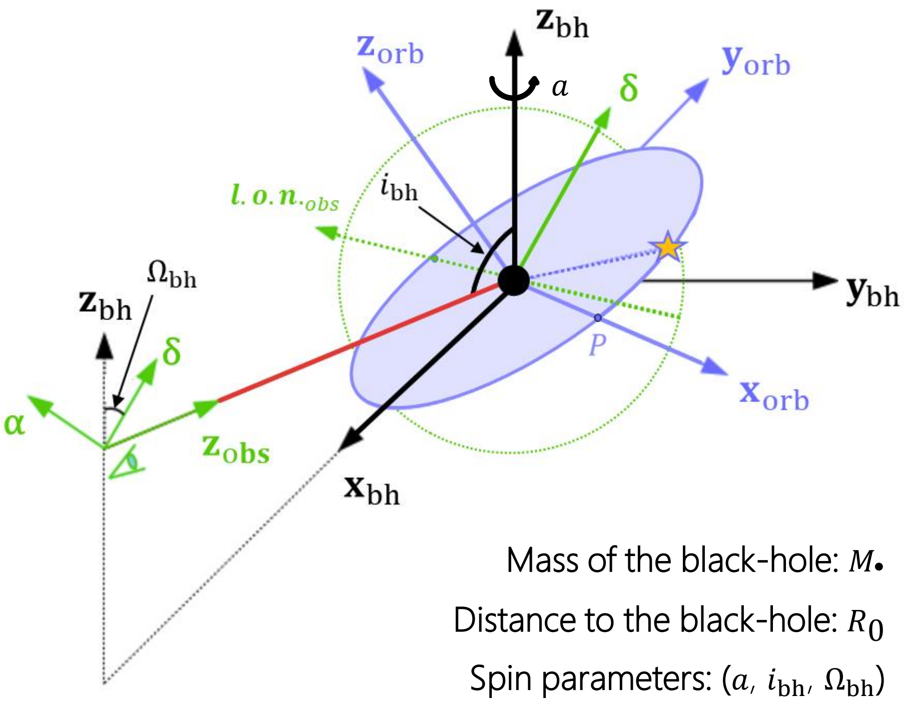

Mass of the black-hole: $M_\bullet$

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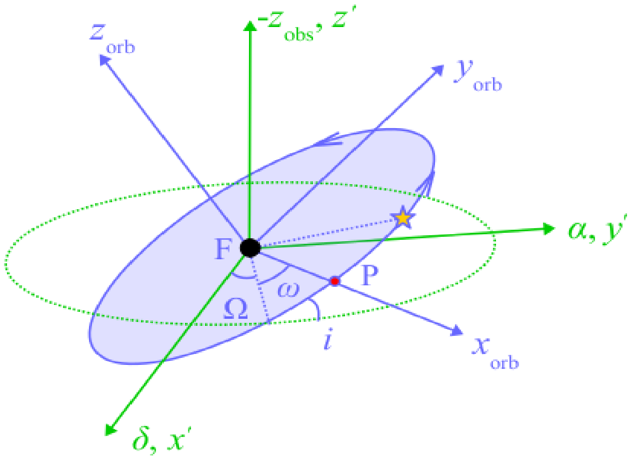
Spin parameters: (a, i_{bh}, Ω_{bh})



Symbol	Orbital elements
a_{sma}	Semi-major axis
e	Eccentricity
i	Inclination
ω	Argument od periastron
Ω	Longitude of the ascending node
P	Period
t_p	Time of periastron passage

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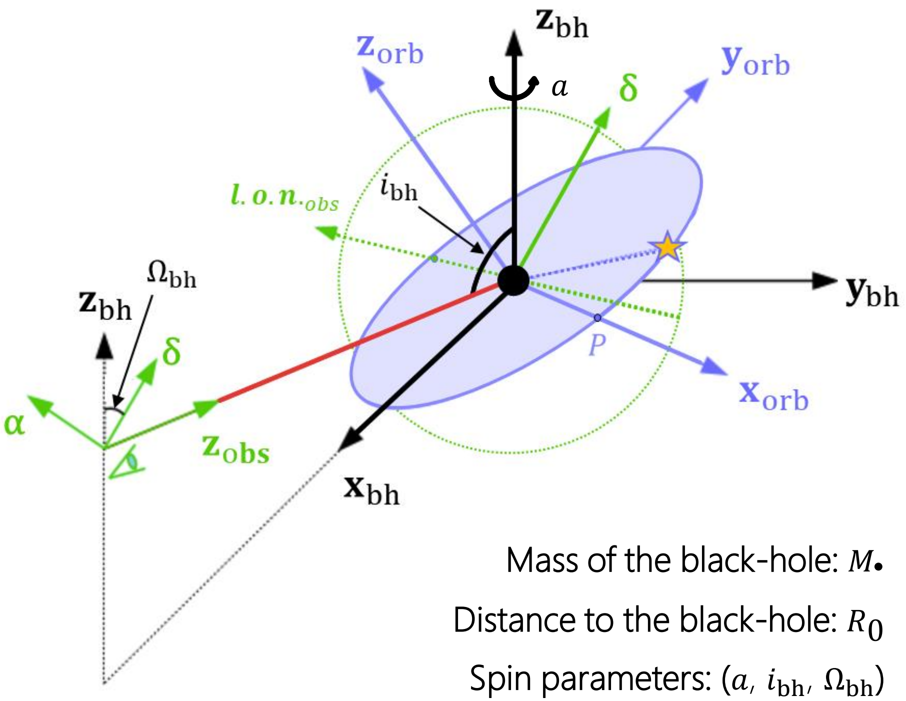

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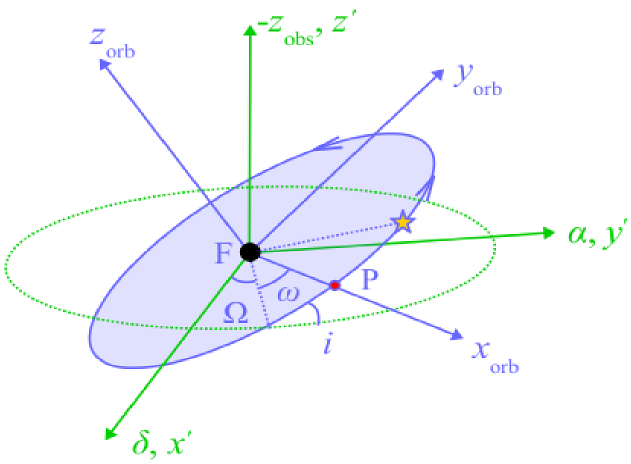

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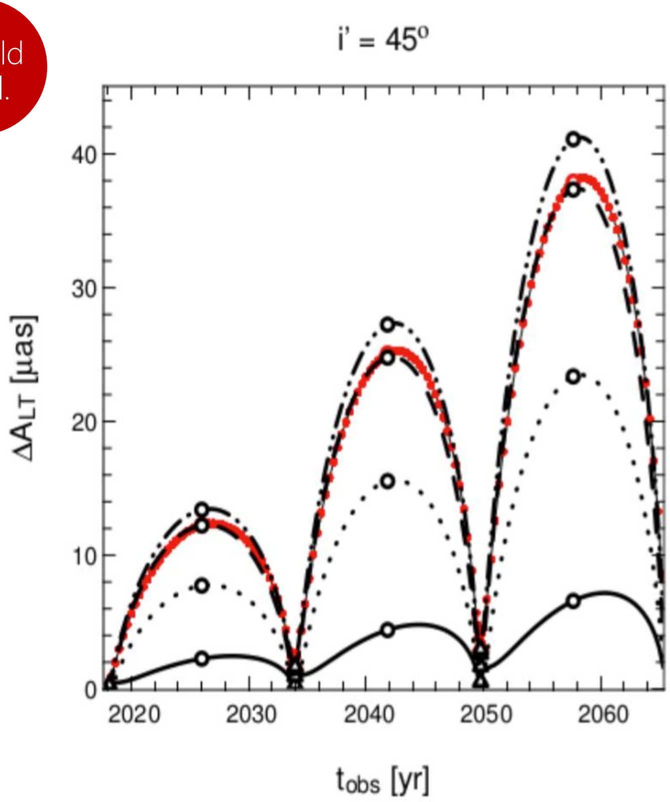
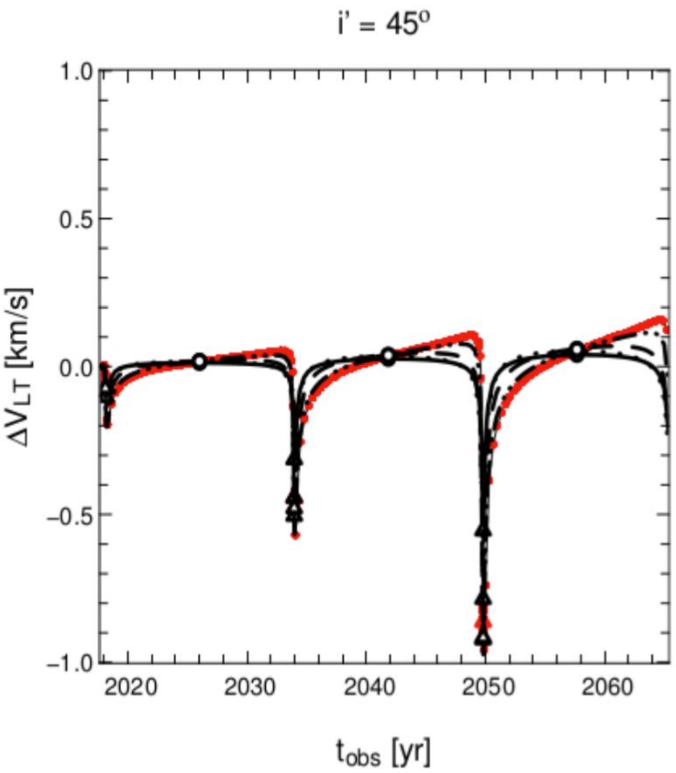
t_p	Time of periastron passage
t_{osc}	Osculating time

STATE OF THE ART ON THE DETERMINATION OF THE SPIN OF SGR A*

Grould et al.



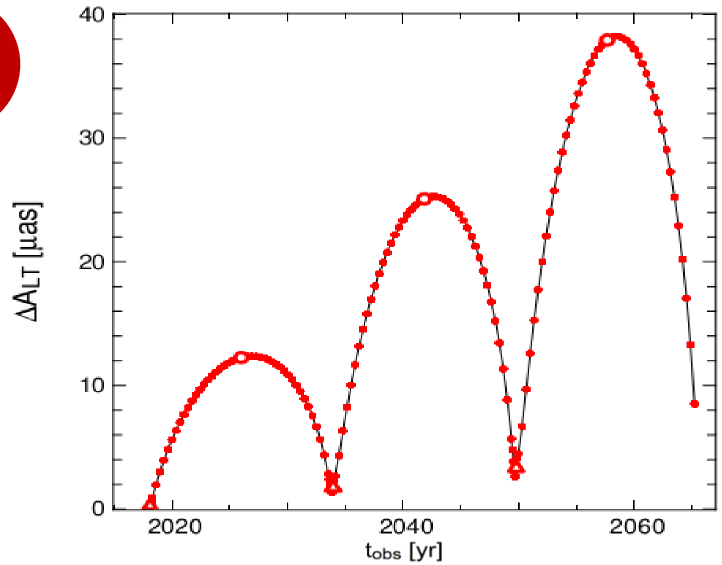


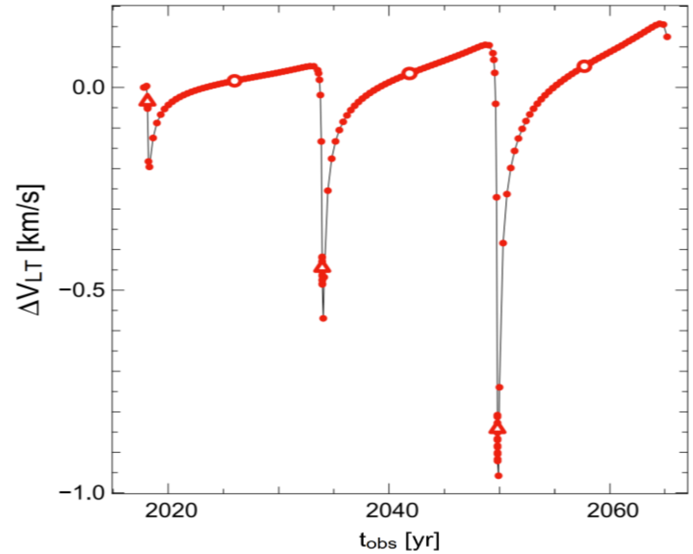
Spin effects on S2 for different spin orientations: Ω′=0°, 45°, 90°, 135°, 160° in solide lines, dotted, dashed, dash-dot-dotted and red dotted respectively

$$\Delta A_{LT} = \sqrt{\left(RA_{spin=0.99} - RA_{spin=0}\right)^2 + \left(DEC_{spin=0.99} - DEC_{spin=0}\right)^2} \quad ; \quad \Delta V_{LT} = RV_{spin=0.99} - RV_{spin=0}$$

CONFRONTATION OF THE RESULTS ON S2 WITH THE LITERATURE

Grould et al.





Spin can be detected with S2 after 47yr with an error of $1\sigma = 0,1$

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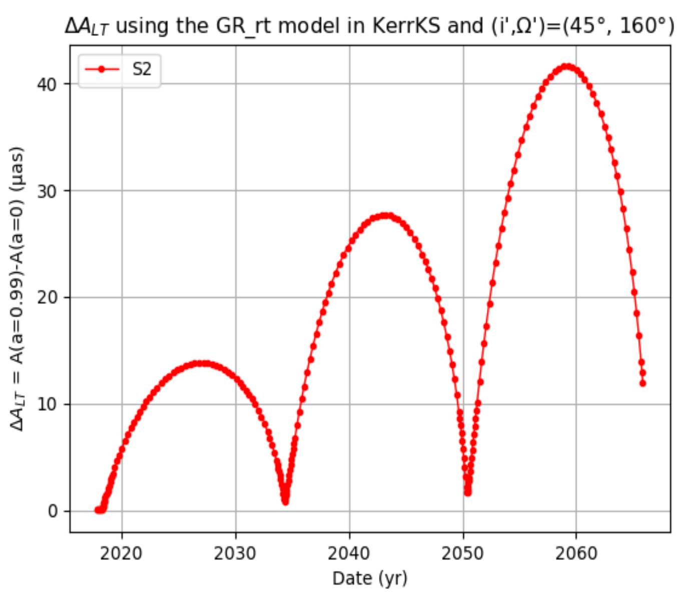
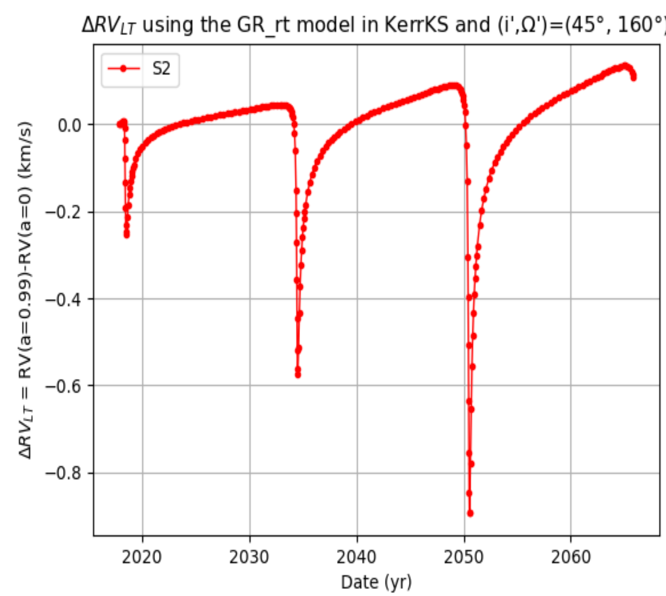
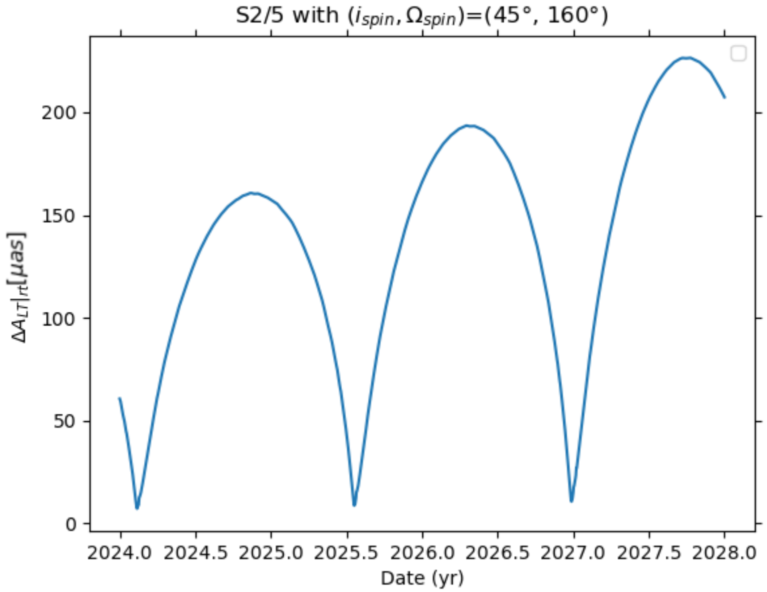
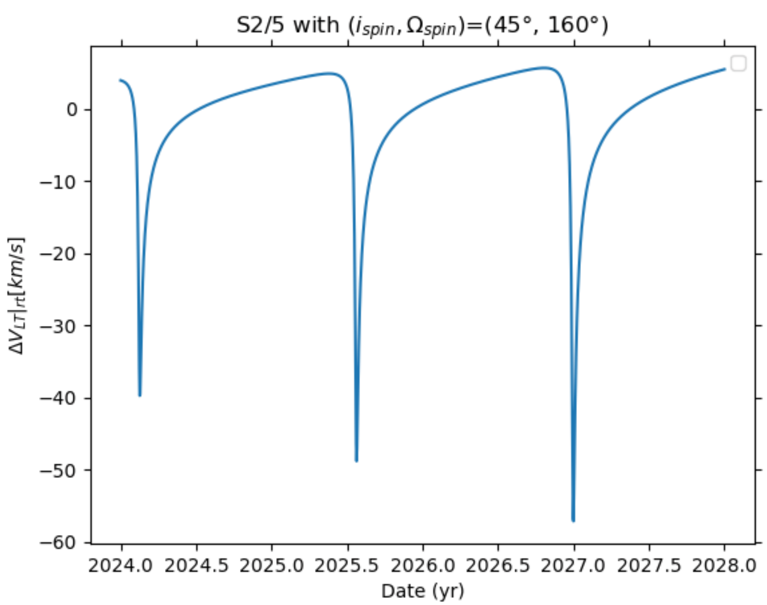




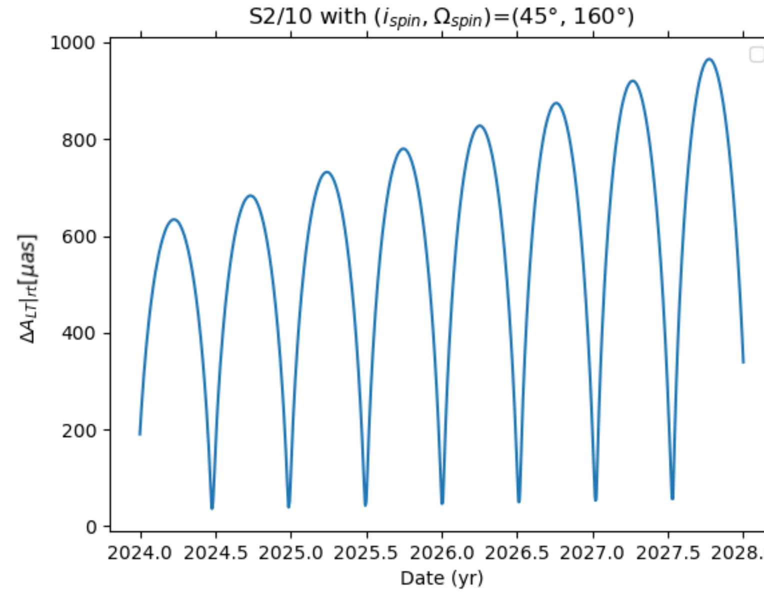
S2/5



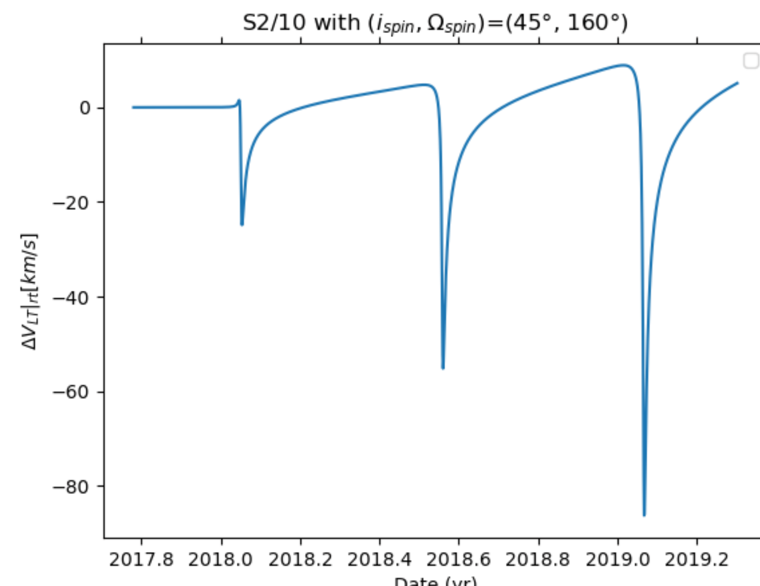
Detectable in 17 months



S2/10

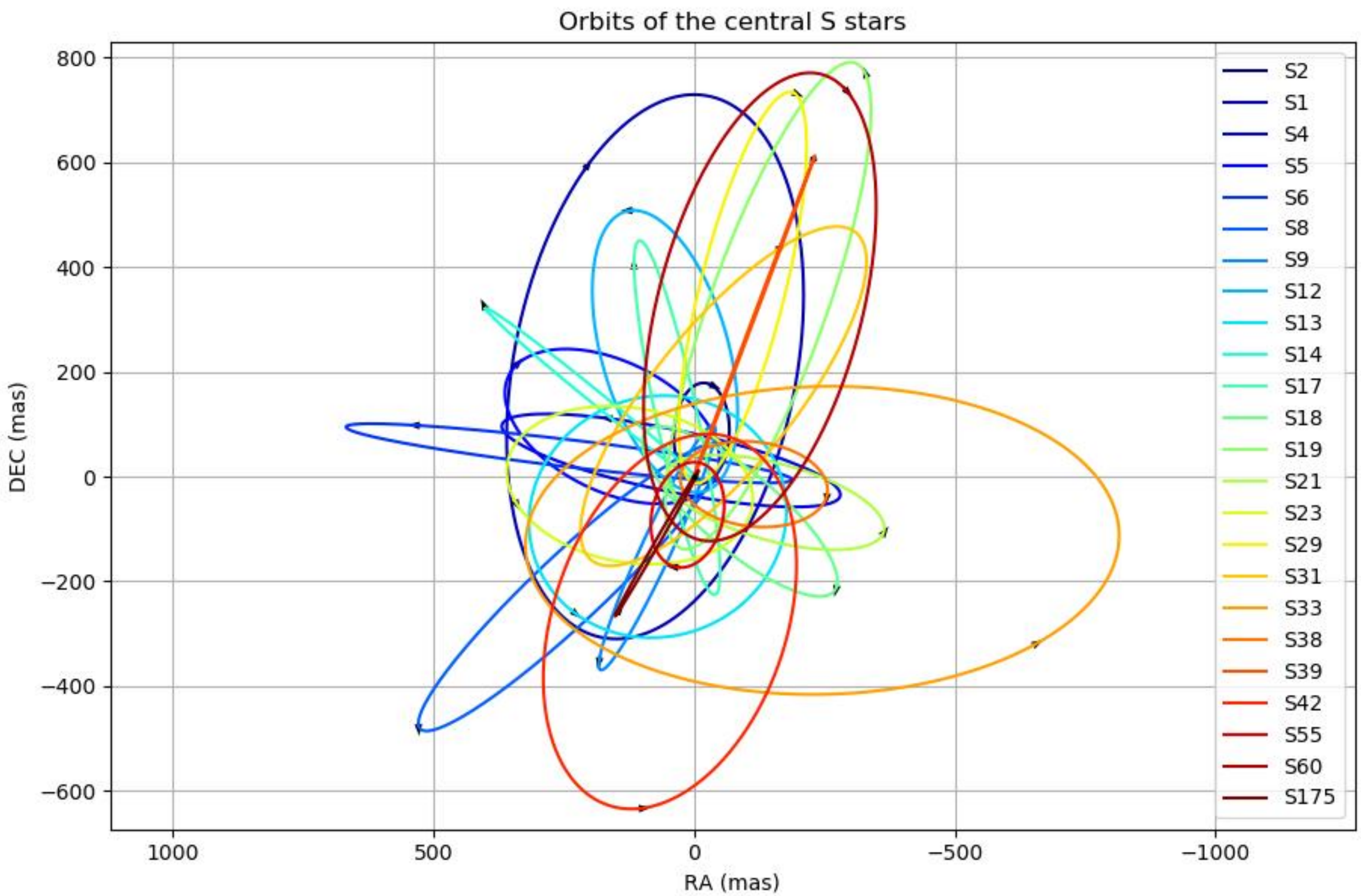


Detectable in 6 months



What to do if no closer-in stars were to be found?

⇨ Explore the possibility of a spin detection from a collection of eccentric orbits?



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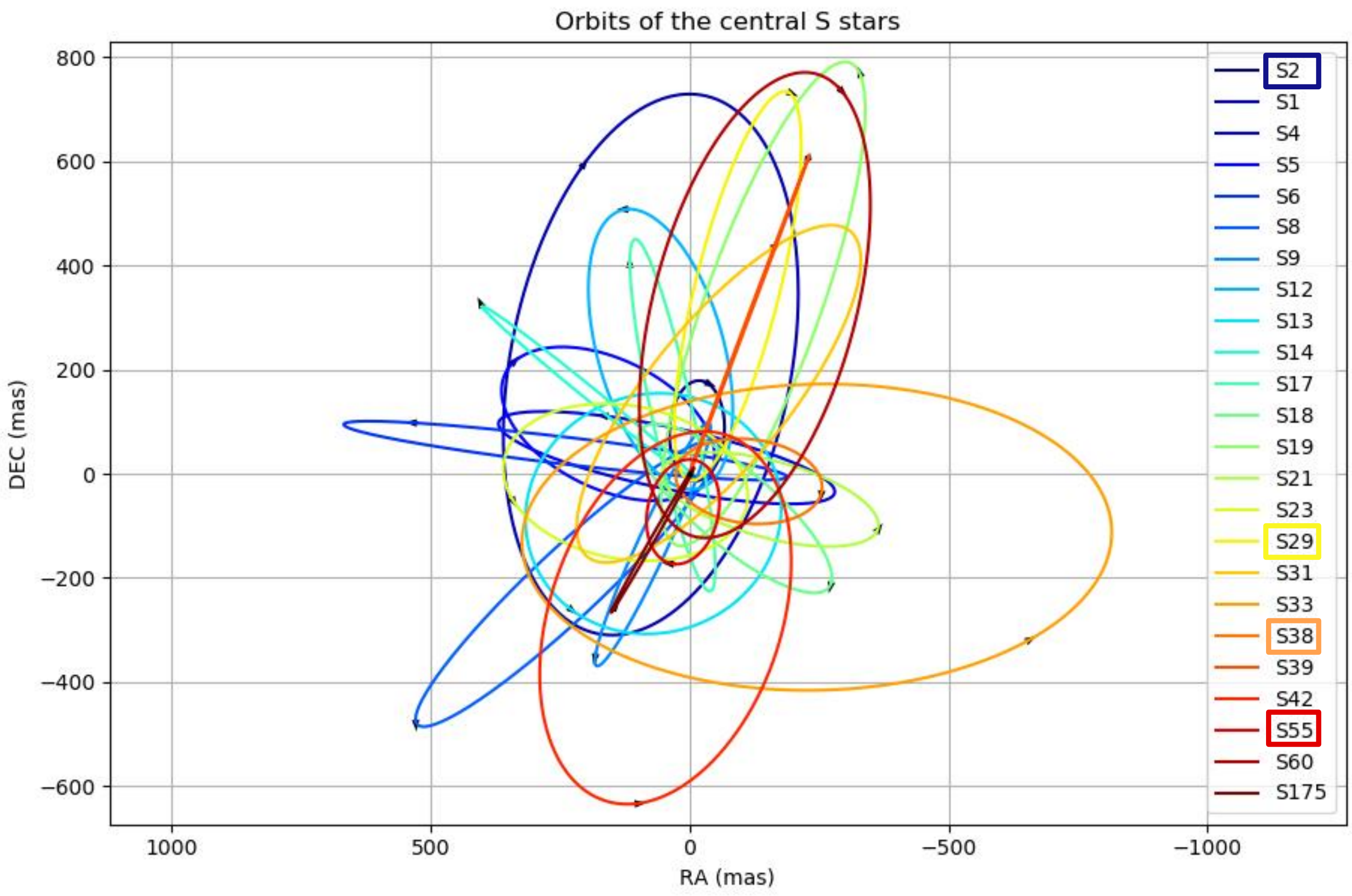
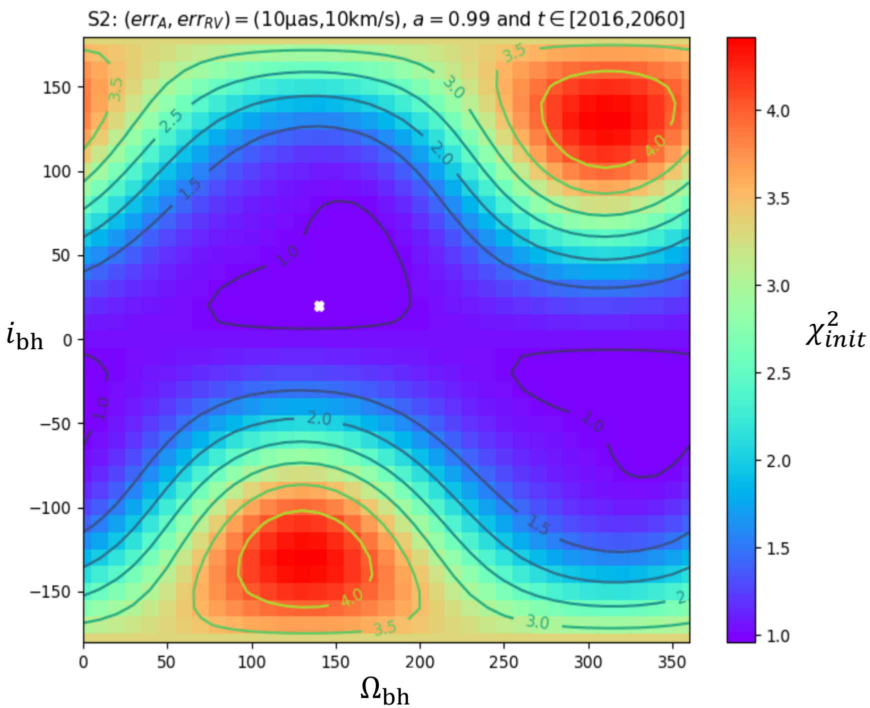


Studying the detectability of the orientation of the spin



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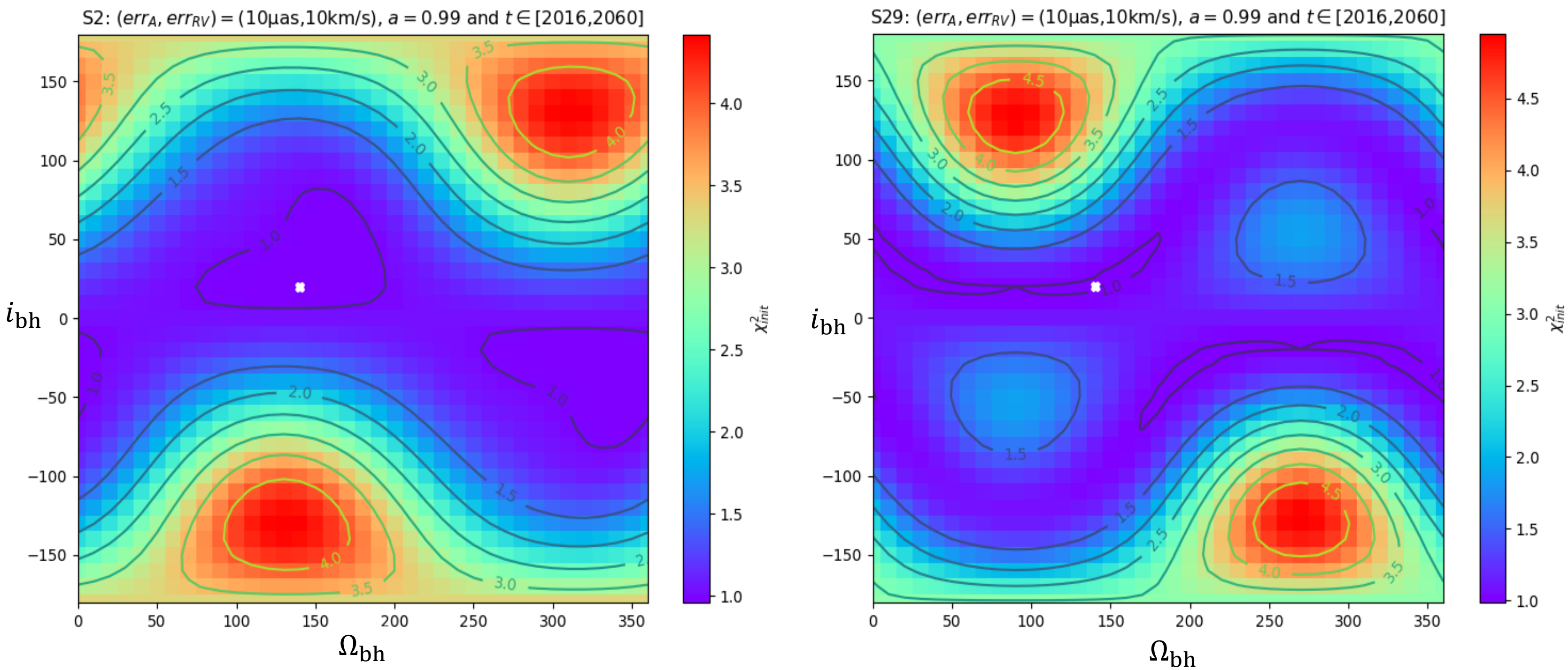
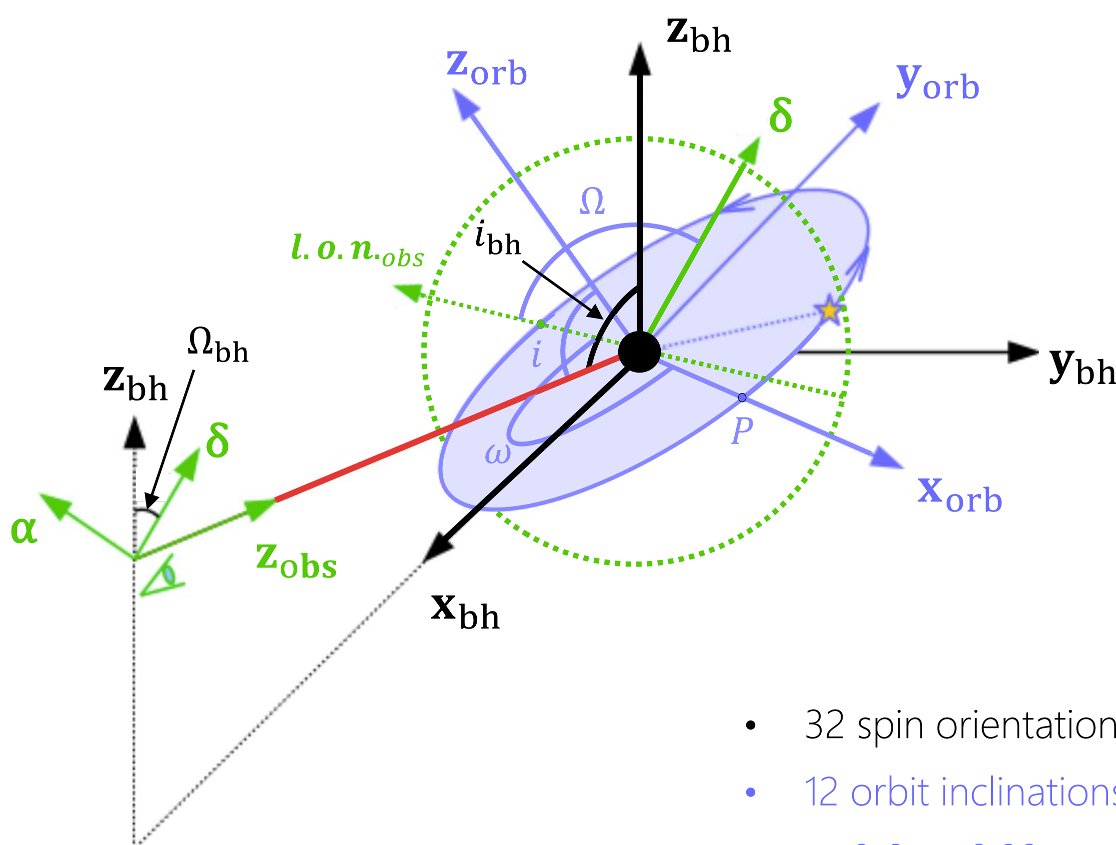


If we fit multiple star orbits at the same time the degeneracy of the spin angles with each other and with the spin magnitude will be reduced !

Studying the impact of the orientation of the spin relative to the orbit and to the observer on the observables



- 32 spin orientations
- 12 orbit inclinations
- e=0 & e=0.99

e=0


Spin effect with different spin and orbit orientations (e=0) t ∈ [2018,2022] using the GR_cd model
max|RA(a=0.99)-RA(a=0)| (μas)
max|DEC(a=0.99)-DEC(a=0)| (μas)

e=0.99


Spin effect with different spin and orbit orientations (e=0.99) t ∈ [2018,2022] using the GR_cd model
max|RA(a=0.99)-RA(a=0)| (mas)
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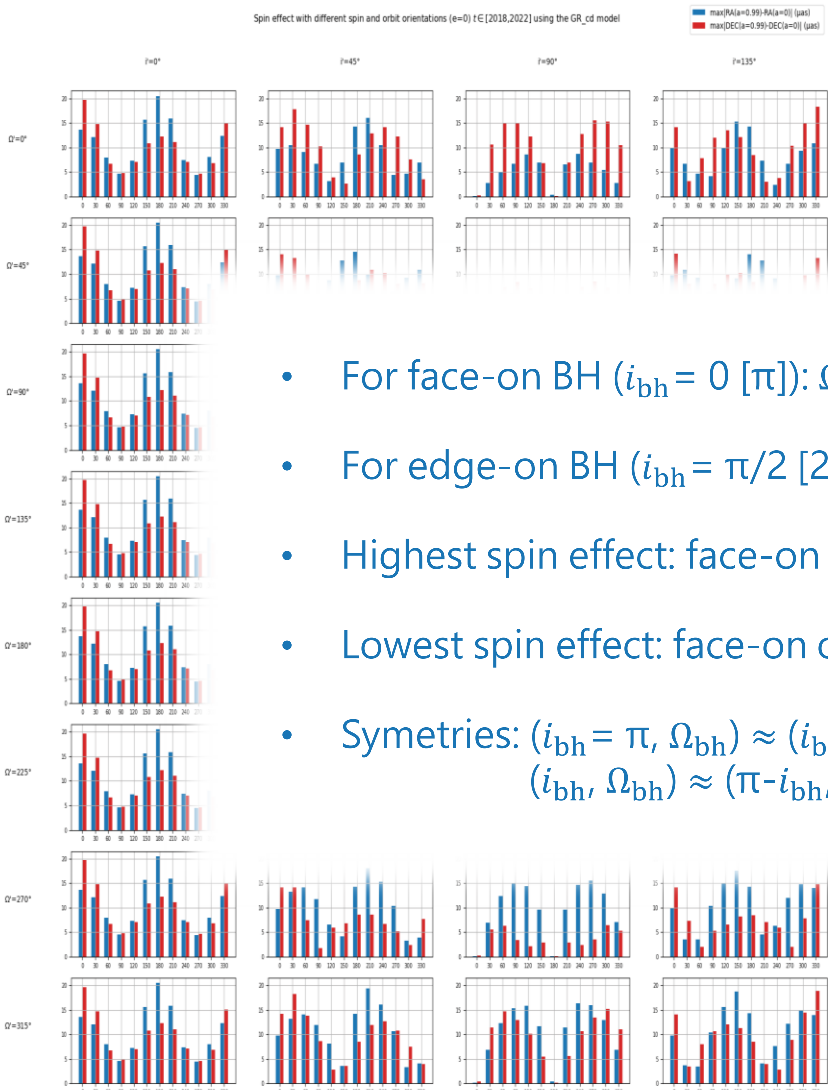
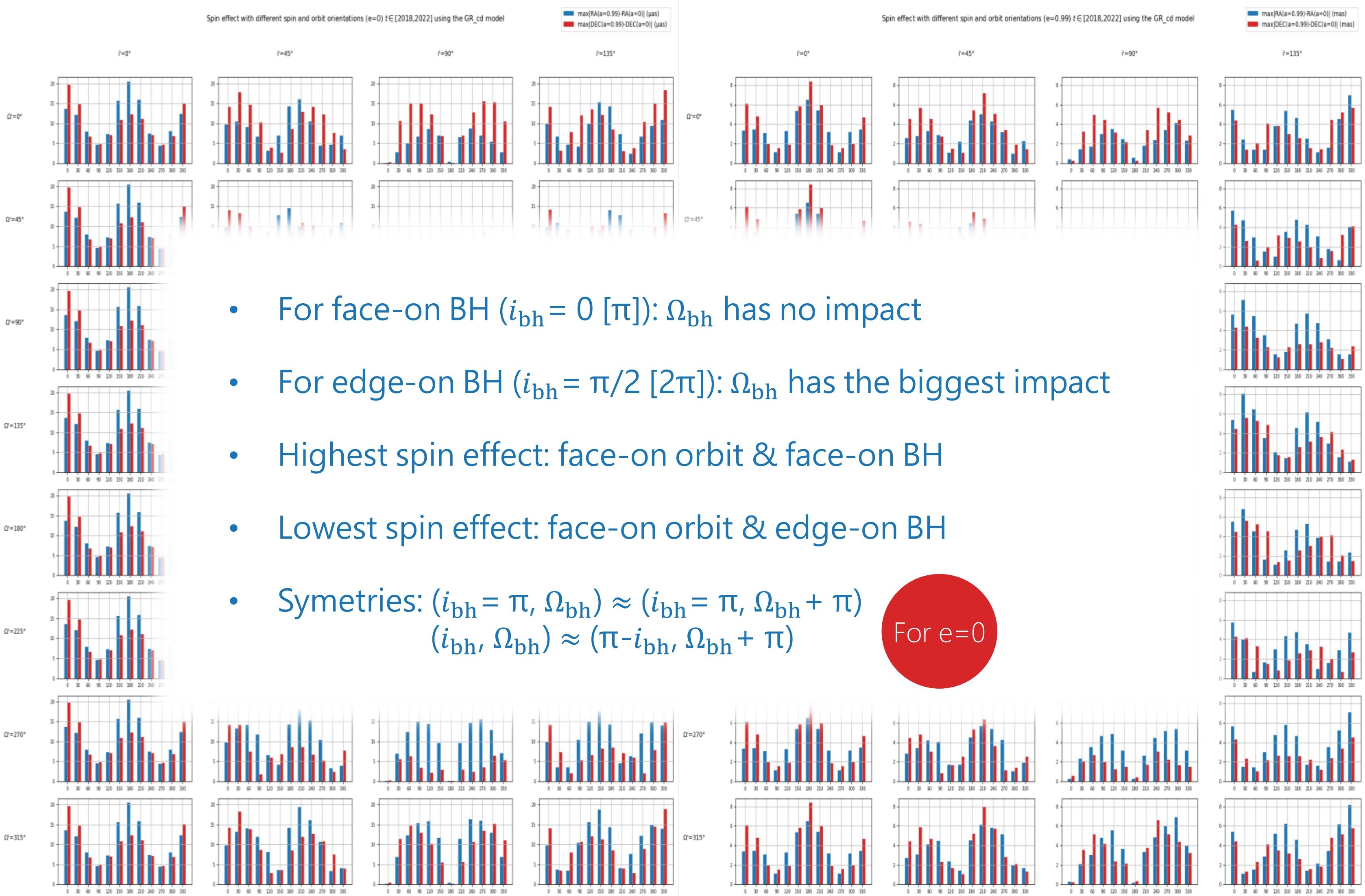
e=0



e=0.99

- For face-on BH ($i_{\mathrm{bh}} = 0\ [\pi]$): Ω_{bh} has no impact
- For edge-on BH ($i_{\mathrm{bh}} = \pi/2\ [2\pi]$): Ω_{bh} has the biggest impact
- Highest spin effect: face-on orbit & face-on BH
- Lowest spin effect: face-on orbit & edge-on BH
- Symetries: $(i_{\mathrm{bh}} = \pi, \Omega_{\mathrm{bh}}) \approx (i_{\mathrm{bh}} = \pi, \Omega_{\mathrm{bh}} + \pi)$
  $(i_{\mathrm{bh}}, \Omega_{\mathrm{bh}}) \approx (\pi\text{-}i_{\mathrm{bh}}, \Omega_{\mathrm{bh}} + \pi)$

e=0 e=0.99



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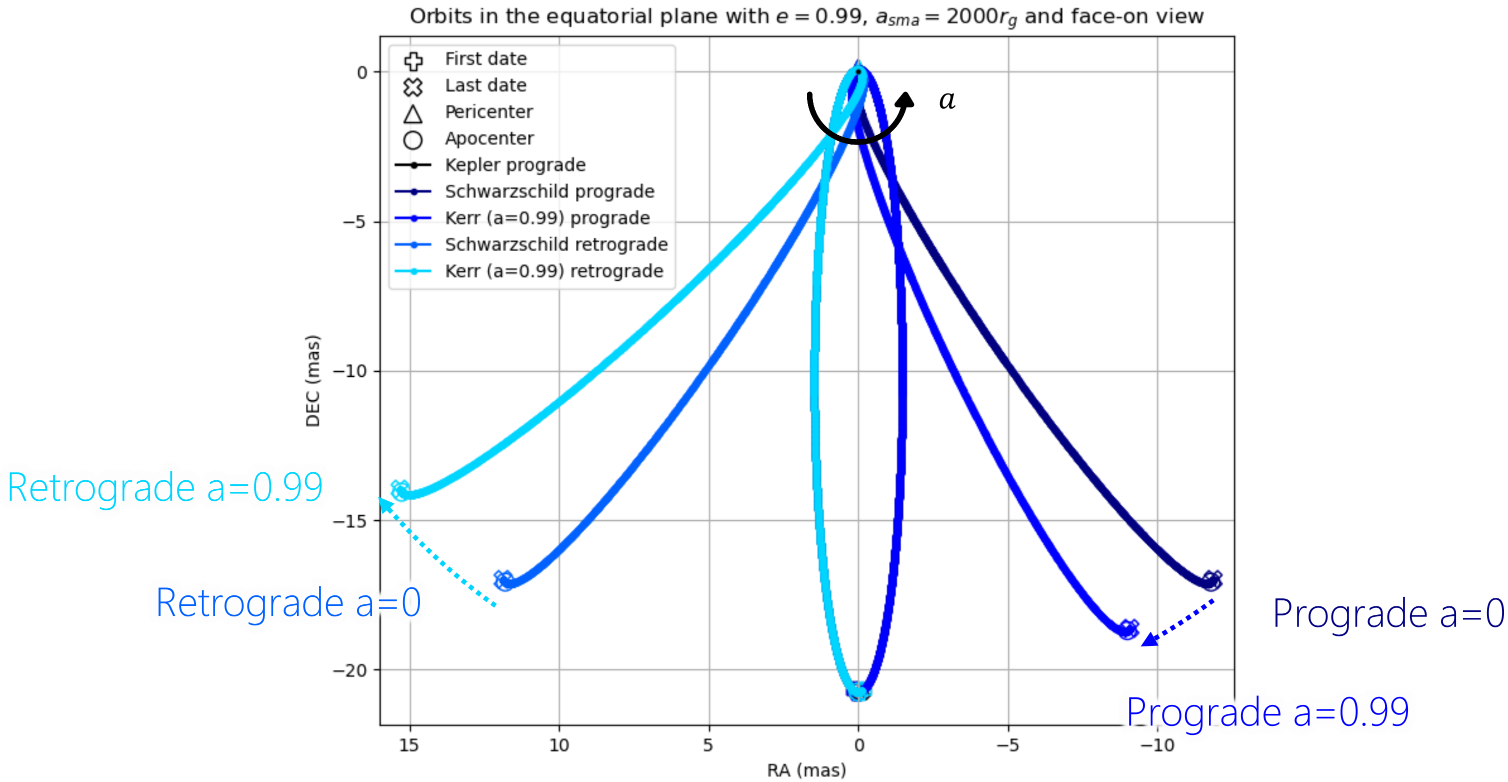
  $(i_{\mathrm{bh}}, \Omega_{\mathrm{bh}}) \approx (\pi - i_{\mathrm{bh}}, \Omega_{\mathrm{bh}} + \pi)$

For e=0

Studying the impact of the orientation of the spin relative to the orbit and to the observer on the observables



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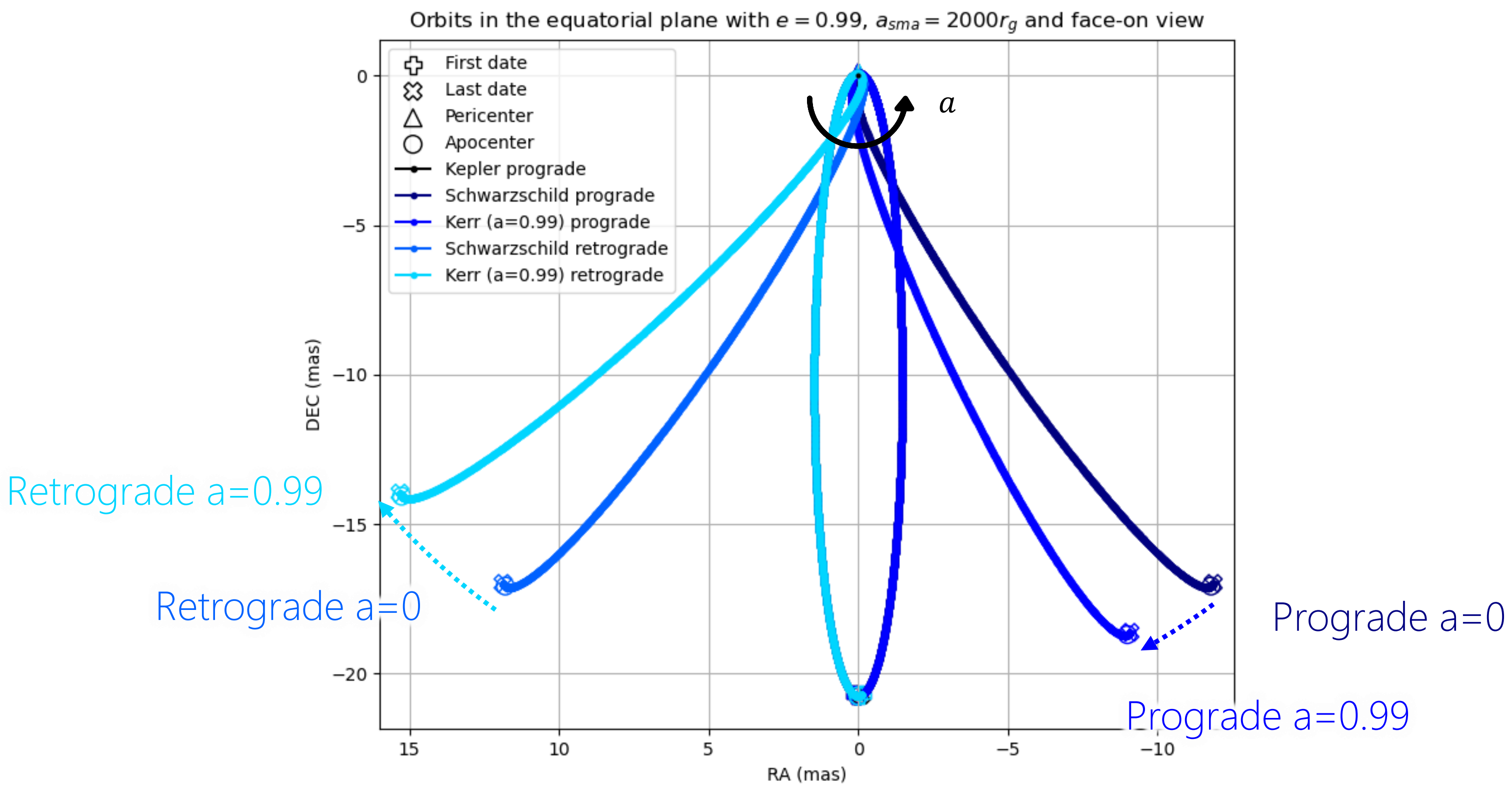
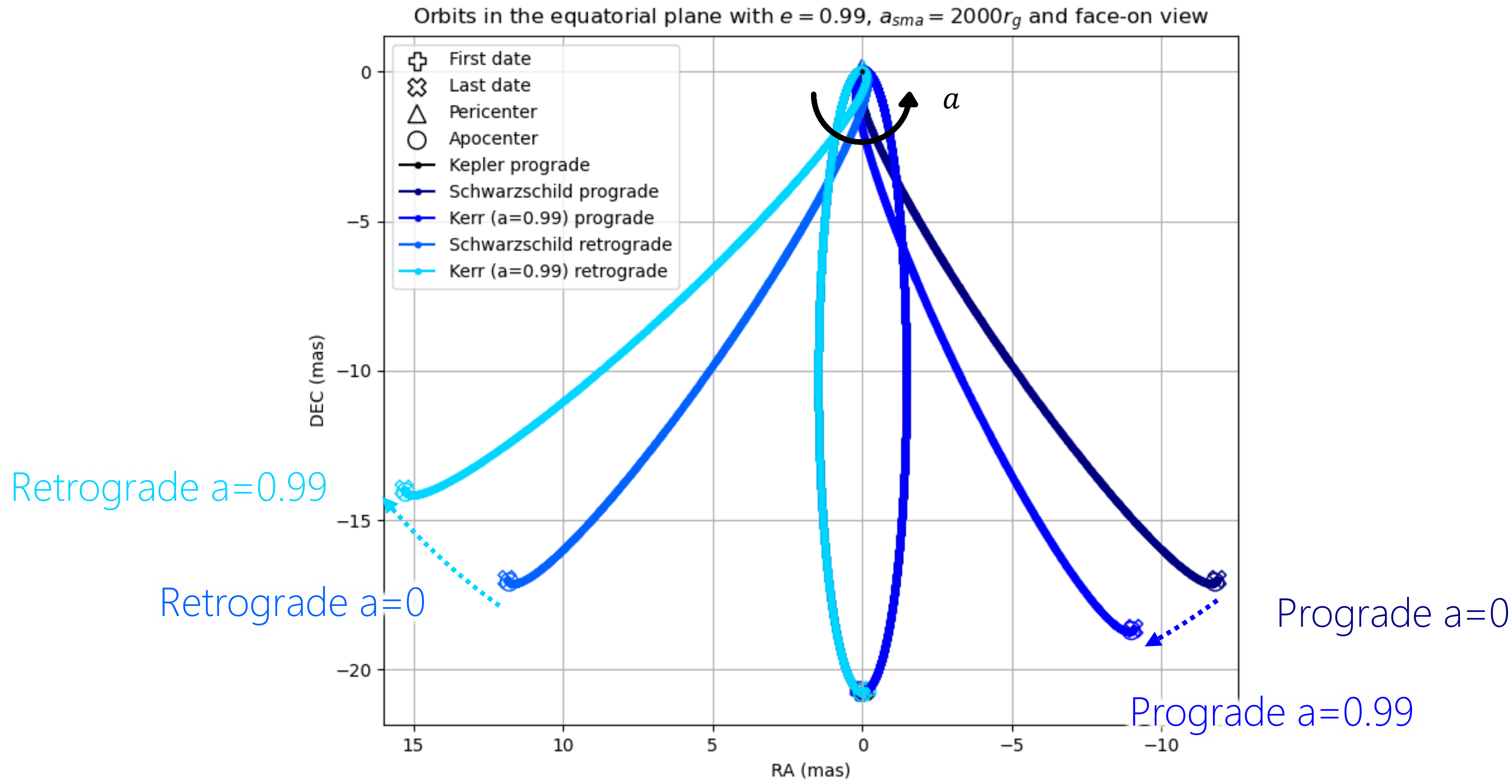


- Noticed that an anticlockwise spin indeed induces a clockwise pericenter advance
- Noticed that the effect of the spin was not the same for a prograde and retrograde orbit!

Studying the impact of the orientation of the spin relative to the orbit and to the observer on the observables



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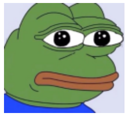


Suspected the quadrupole moment to be responsible → Computed the theoretical expression of the secular shift of orbital parameters induced by the spin and quadrupole moment

$$\chi = J\frac{c}{GM_\bullet^2} \tag{31}$$

$$Q = -\frac{1}{c^2}\frac{J^2}{M_\bullet} = -\frac{G^2M_\bullet^3}{c^4}\chi^2 \tag{32}$$

$$\boldsymbol{v} = v_r\mathbf{n}_{\rm orb} + v_t\mathbf{m}_{\rm orb} \tag{33}$$

$$v_r = \frac{Gm}{p}e\sin f, \quad v_t = \frac{Gm}{p}(1+e\cos f), \quad p = a_{\rm sma}(1-e^2) \tag{34}$$

$$\begin{aligned}
\mathbf{z}_{\rm bh}\cdot\mathbf{n}_{\rm orb} &= \cos i_{\rm bh}\sin i\sin u \\
&+ \sin i_{\rm bh}(\cos u\cos(\Omega+\Omega_{\rm bh}) - \cos i\sin u\sin(\Omega-\Omega_{\rm bh})) \\
&= \sin\theta\cos(\beta-u) = \sin\theta\sin(?+u) \\
\mathbf{z}_{\rm bh}\cdot\mathbf{m}_{\rm orb} &= \cos i_{\rm bh}\sin i\cos u \\
&- \sin i_{\rm bh}(\sin u\cos(\Omega+\Omega_{\rm bh}) + \cos i\cos u\sin(\Omega+\Omega_{\rm bh})) \\
&= \sin\theta\sin(\beta-u) = \sin\theta\cos(?+u) \\
\mathbf{z}_{\rm bh}\cdot\mathbf{z}_{\rm orb} &= \cos i_{\rm bh}\cos i + \sin i_{\rm bh}\sin i\sin(\Omega-\Omega_{\rm bh}) \\
&= \cos\theta
\end{aligned}$$

$$\ddot{\mathbf{r}} = -\frac{Gm}{r^2}\mathbf{n}_{\rm orb} + \mathbf{a}_{\rm PN} \tag{35}$$

$$r = |\mathbf{r}| = \frac{p}{1+e\cos f} \tag{36}$$

$$\mathbf{a}_{\rm PN} \approx \mathbf{a}_{\rm 2PN} = \mathbf{a}_{\rm Sch} + \mathbf{a}_\chi + \mathbf{a}_Q \tag{37}$$

$$\mathbf{a}_{\rm Sch} = \frac{Gm}{c^2r^2}\mathbf{n}_{\rm orb}(4\frac{Gm}{r} - v^2) + 4\frac{Gmv_r}{c^2r^2}\boldsymbol{v} \tag{38}$$

$$\mathbf{a}_\chi = -2\frac{G^2m^2}{c^3r^3}\chi\boldsymbol{v}\times[-\mathbf{z}_{\rm bh} + 3(\mathbf{n}_{\rm orb}\cdot\mathbf{z}_{\rm bh})\mathbf{n}_{\rm orb}] \tag{39}$$

$$= -2\frac{G^2m^2}{c^3r^3}\chi[2\boldsymbol{v}\times\mathbf{z}_{\rm bh} - 3(\mathbf{n}_{\rm orb}\cdot\boldsymbol{v})\mathbf{n}_{\rm orb}\times\mathbf{z}_{\rm bh} - 3\mathbf{n}_{\rm orb}(\mathbf{n}_{\rm orb}\times\boldsymbol{v})\cdot\mathbf{z}_{\rm bh}] \tag{40}$$

$$\mathbf{a}_Q = -\frac{3G^3m^3}{2c^4r^4}\chi^2\Big(5\mathbf{n}_{\rm orb}(\mathbf{n}_{\rm orb}\cdot\mathbf{z}_{\rm bh})^2 - 2(\mathbf{n}_{\rm orb}\cdot\mathbf{z}_{\rm bh})\mathbf{z}_{\rm bh} - \mathbf{n}_{\rm orb}\Big) \tag{41}$$

$$S_{\rm Sch} = \mathbf{a}_{\rm Sch}\cdot\mathbf{n}_{\rm orb} \tag{42}$$

$$S_\chi = \mathbf{a}_\chi\cdot\mathbf{n}_{\rm orb} = 2\frac{(Gm)^{5/2}}{c^3p^{7/2}}(1+e\cos f)^4\cos\theta\,\chi \tag{43}$$

$$S_Q = \mathbf{a}_Q\cdot\mathbf{n}_{\rm orb} = \frac{3G^3m^3}{2c^4p^4}(1+e\cos f)^4(1-3\sin^2\theta\cos^2(\beta-u))\chi^2$$

$$T_{\rm Sch} = \mathbf{a}_{\rm Sch}\cdot\mathbf{m}_{\rm orb} = \tag{45}$$

$$T_\chi = \mathbf{a}_\chi\cdot\mathbf{m}_{\rm orb} = -2\frac{(Gm)^{5/2}}{c^3p^{7/2}}e\sin f(1+e\cos f)^3\cos\theta\,\chi \tag{46}$$

$$T_Q = \mathbf{a}_Q\cdot\mathbf{m}_{\rm orb} = 3\frac{G^3m^3}{c^4p^4}(1+e\cos f)^4\sin^2\theta\cos(\beta-u)\sin(\beta-u)\chi^2 \tag{47}$$

$$W_{\rm Sch} = \mathbf{a}_{\rm Sch}\cdot\mathbf{z}_{\rm orb} = 0$$

$$W_\chi = \mathbf{a}_\chi\cdot\mathbf{z}_{\rm orb} = 2\frac{(Gm)^{5/2}}{c^3p^{7/2}}(1+e\cos f)^3\sin\theta$$
$$\times[2(1+e\cos f)\cos(\beta-u) + e\sin f\sin(\beta-u)]\chi$$

$$W_Q = \mathbf{a}_Q\cdot\mathbf{z}_{\rm orb} = 3\frac{G^3m^3}{c^4p^4}(1+e\cos f)^4\cos\theta\sin\theta\cos(\beta-u)\chi^2 \tag{48}$$

$$\Delta\omega = \Delta\omega_{\rm Sch} + \Delta\omega_\chi + \Delta\omega_Q \tag{49}$$

$$\Delta\Omega = \Delta\Omega_\chi + \Delta\Omega_Q \tag{50}$$

$$\Delta i = \Delta i_\chi + \Delta i_Q \tag{51}$$

$$\Delta\omega_{\rm Sch} = \frac{6\pi Gm}{c^2a_{\rm sma}(1-e^2)} \tag{52}$$

$$\Delta\omega_\chi = -\frac{4\pi}{c^3}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^{3/2}(2\cos\theta + \cot i\sin\theta\sin\beta)\chi \tag{53}$$

$$\Delta\omega_Q = \frac{3\pi}{2c^4}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^2\Big(1 - 3\cos^2\theta - 2\cot i\cos\theta\sin\theta\sin\beta\Big)\chi^2 \tag{54}$$

$$\Delta i_\chi = \frac{4\pi}{c^3}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^{3/2}\sin\theta\cos\beta\;\chi \tag{55}$$

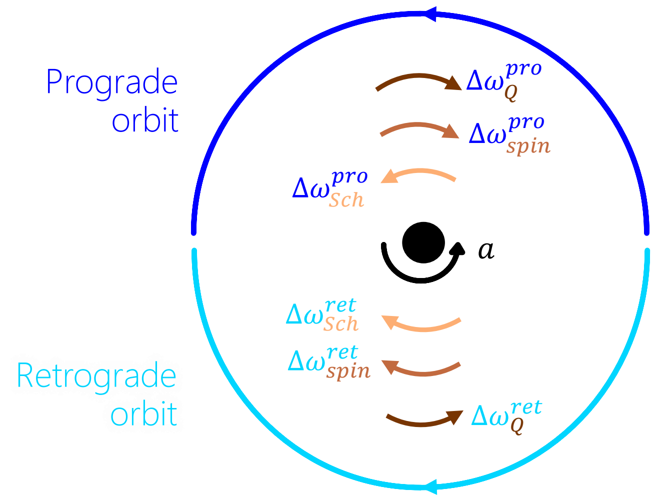
$$\Delta i_Q = \frac{3\pi}{c^4}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^2\cos\theta\sin\theta\cos\beta\;\chi^2 \tag{56}$$

$$\Delta\Omega_\chi = \frac{4\pi}{c^3}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^{3/2}\frac{\sin\theta\sin\beta}{\sin i}\;\chi \tag{57}$$

$$\Delta\Omega_Q = \frac{3\pi}{c^4}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^2\frac{\cos\theta\sin\theta\sin\beta}{\sin i}\;\chi^2 \tag{58}$$

$$\Delta\varpi_\chi = -\frac{8\pi}{c^3}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^{3/2}\cos\theta\,\chi \tag{59}$$

$$\Delta\varpi_Q = \frac{3\pi}{2c^4}\left[\frac{Gm}{a_{\rm sma}(1-e^2)}\right]^2\Big(1 - 3\cos^2\theta\Big)\chi^2 \tag{60}$$



- An Anticlockwise spin induces a clockwise pericenter advance
- Quadrupole moment is responsible for the difference between prograde and retrograde secular shifts
- Prograde shift > Retrograde shift, not the opposite!

Thank you for your attention

Acomplished Work

Spin effects on different S stars

Orbit fitting for spin detectability

Confrontation of the results in KerrKS and KerrBL

Development of the **1.5PN**, $\mathbf{GR_{proj}}$ & $\mathbf{2PN_{Q}}$ models

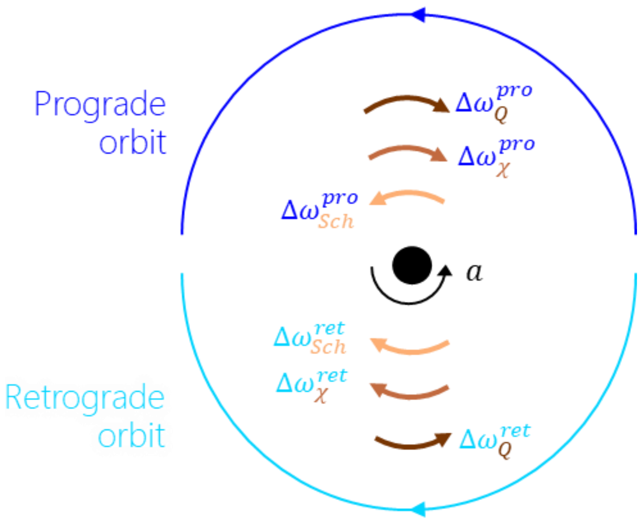
Acquired Knowledge

Galactic center and different effects produced by Sgr A*

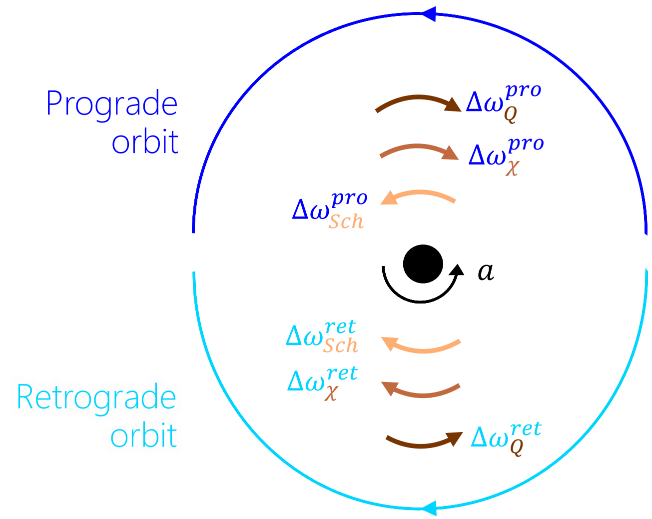
Astrometry and spectroscopy

Fitting mock data with a given model

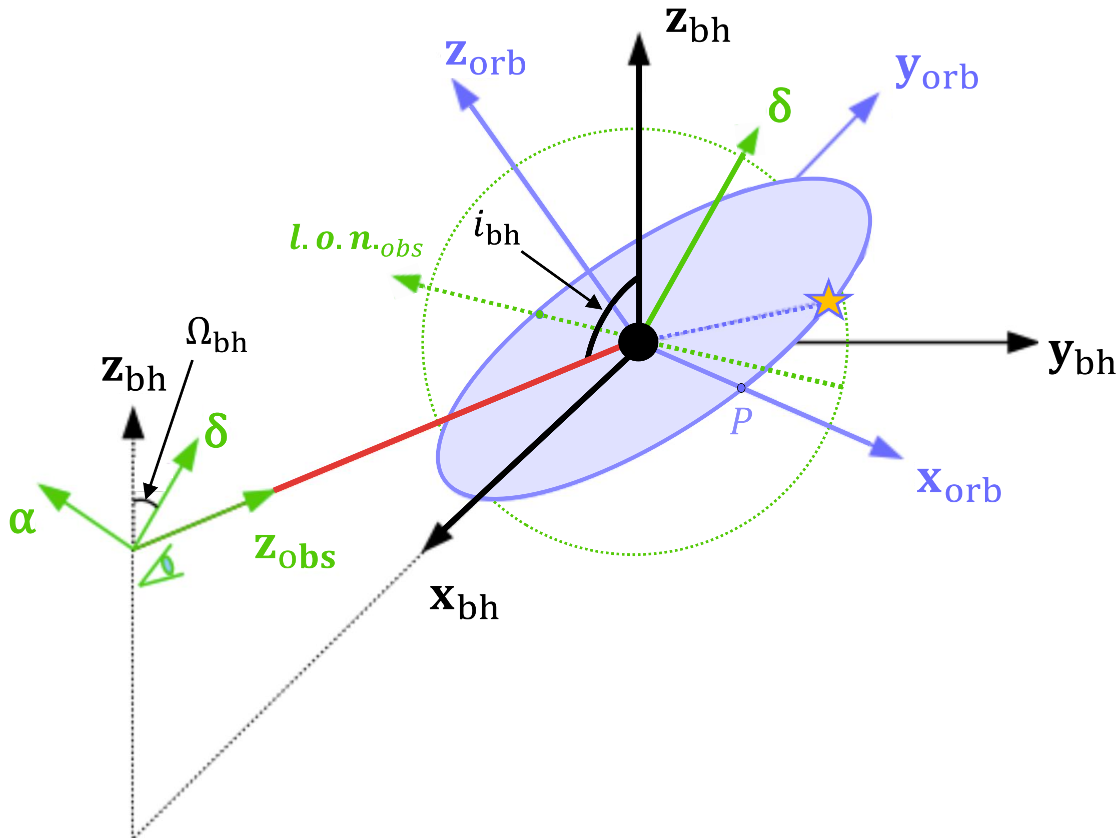
Relativistic dynamics and use of different metrics



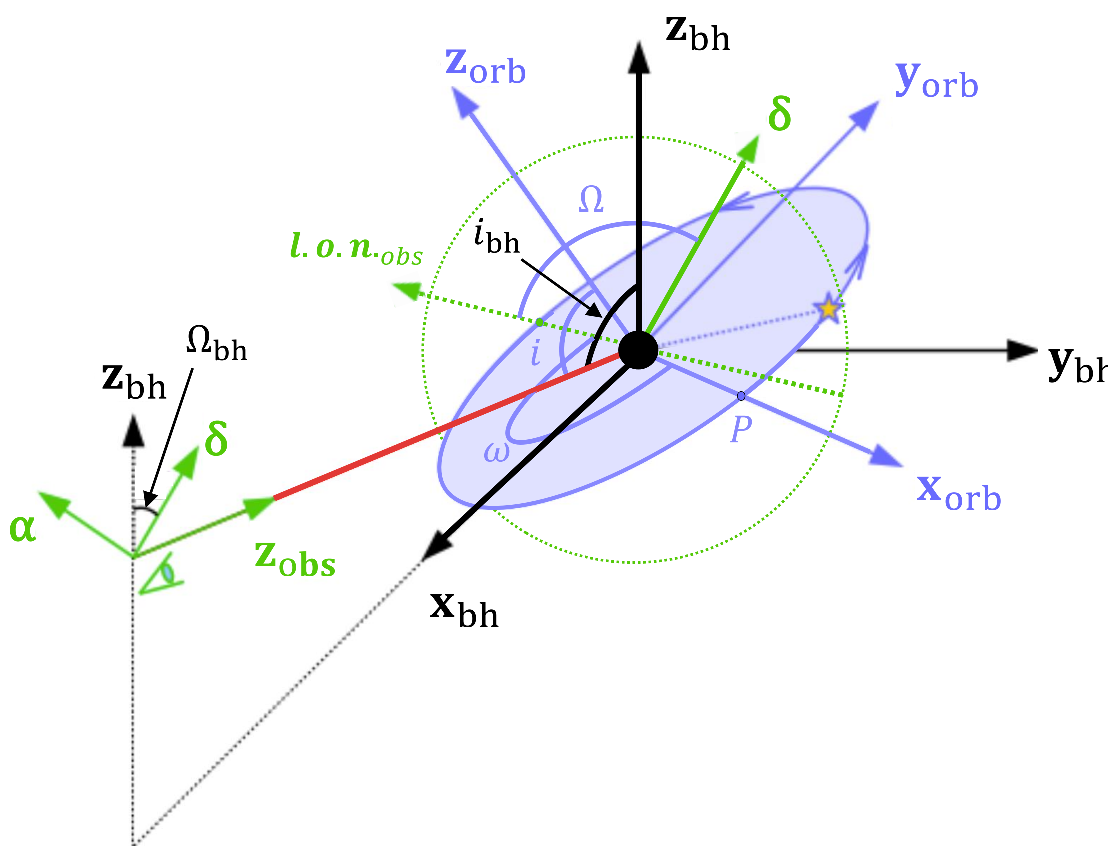
Prograde orbit
$\Delta\omega_{Q}^{pro}$
$\Delta\omega_{\chi}^{pro}$
$\Delta\omega_{Sch}^{pro}$
a
$\Delta\omega_{Sch}^{ret}$
$\Delta\omega_{\chi}^{ret}$
Retrograde orbit
$\Delta\omega_{Q}^{ret}$



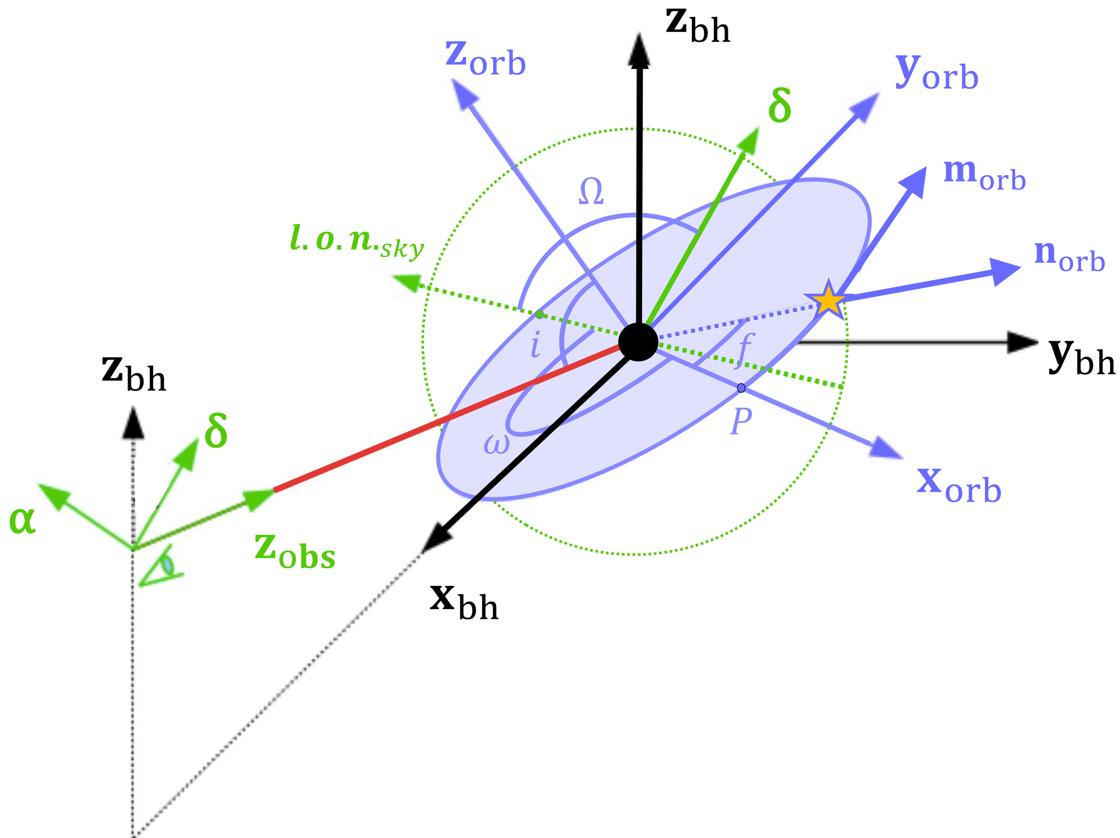
Prograde orbit
$\Delta\omega_{Q}^{pro}$
$\Delta\omega_{\chi}^{pro}$
$\Delta\omega_{Sch}^{pro}$
a
$\Delta\omega_{Sch}^{ret}$
$\Delta\omega_{\chi}^{ret}$
Retrograde orbit
$\Delta\omega_{Q}^{ret}$



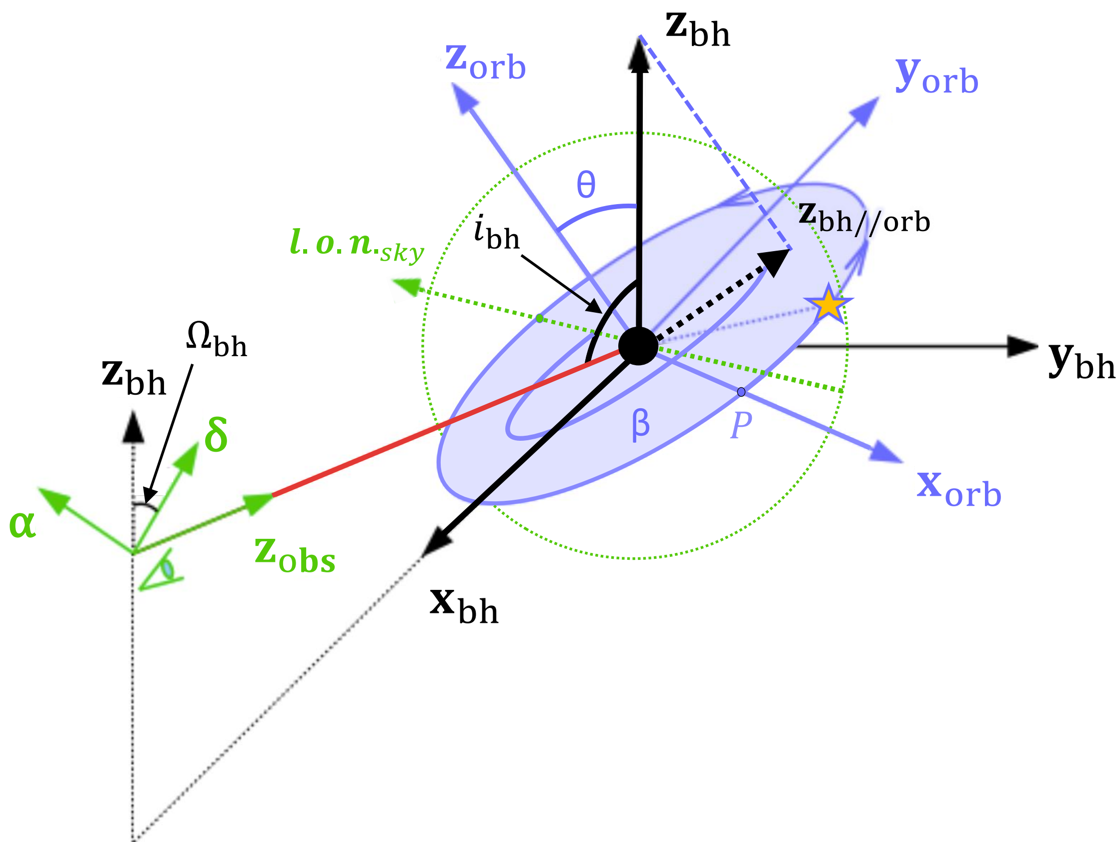
z_bh
z_orb
y_orb
δ
l.o.n._obs
i_bh
Ω_bh
z_bh
δ
α
z_obs
y_bh
P
x_orb
x_bh



$\Delta\omega^{pro}_{Sch}$



$\mathbf{z}_{\rm bh}$
$\mathbf{z}_{\rm orb}$
$\mathbf{y}_{\rm orb}$
δ
$\mathbf{m}_{\rm orb}$
$\mathbf{n}_{\rm orb}$
Ω
$l.o.n._{sky}$
i
f
$\mathbf{y}_{\rm bh}$
$\mathbf{z}_{\rm bh}$
δ
P
ω
$\mathbf{x}_{\rm orb}$
α
$\mathbf{z}_{\rm obs}$
$\mathbf{x}_{\rm bh}$



z_bh
z_orb
y_orb
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z_bh//orb
i_bh
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Ω_bh
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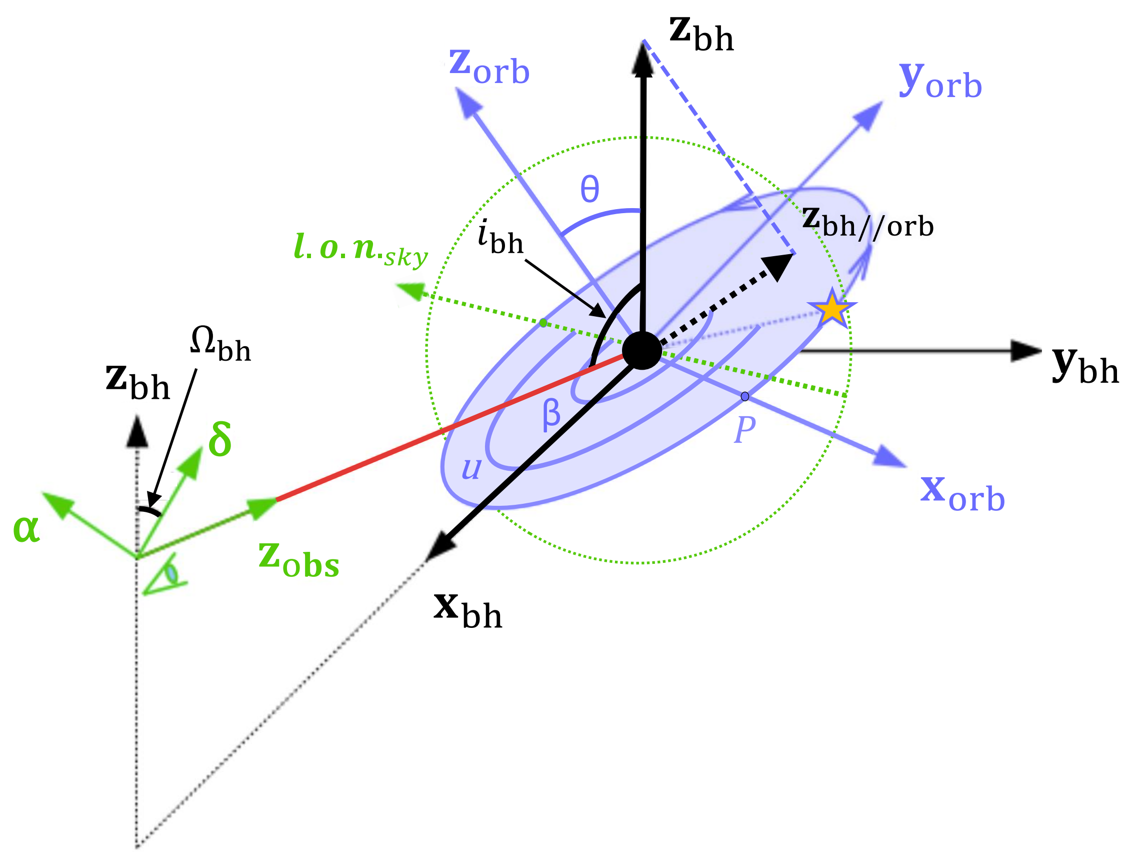
x_bh



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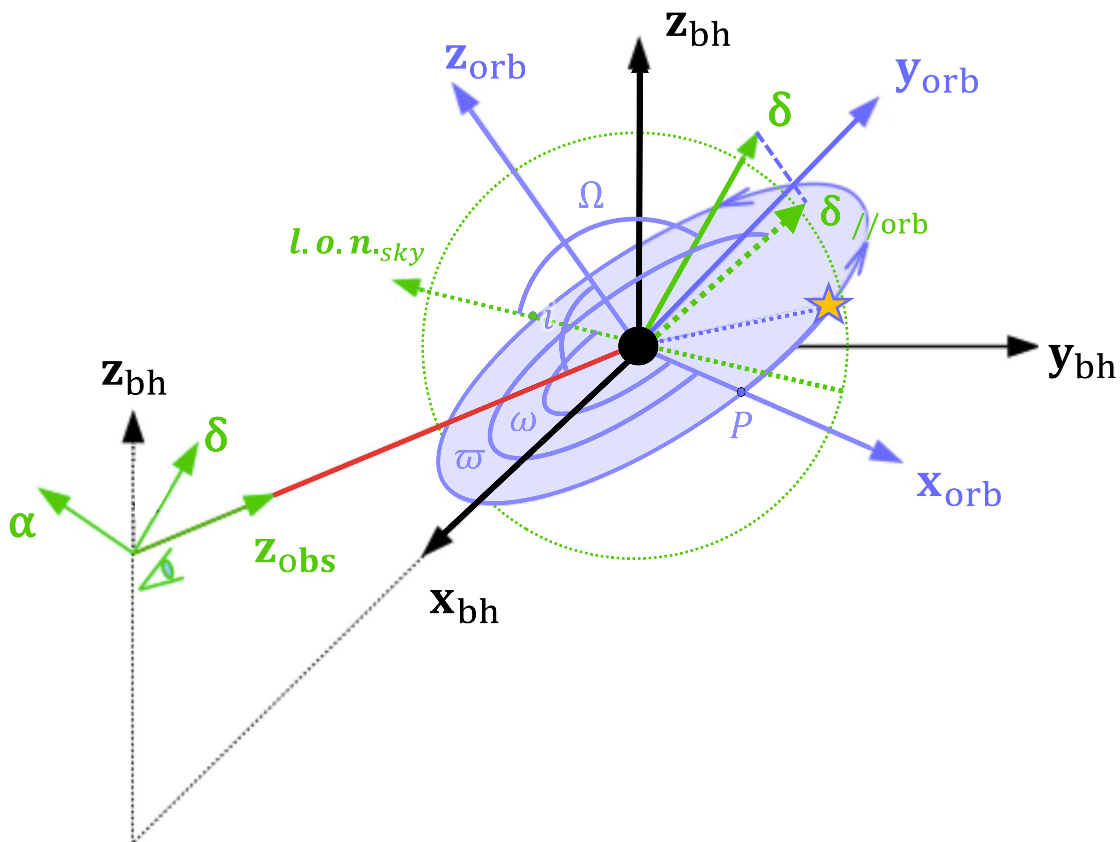

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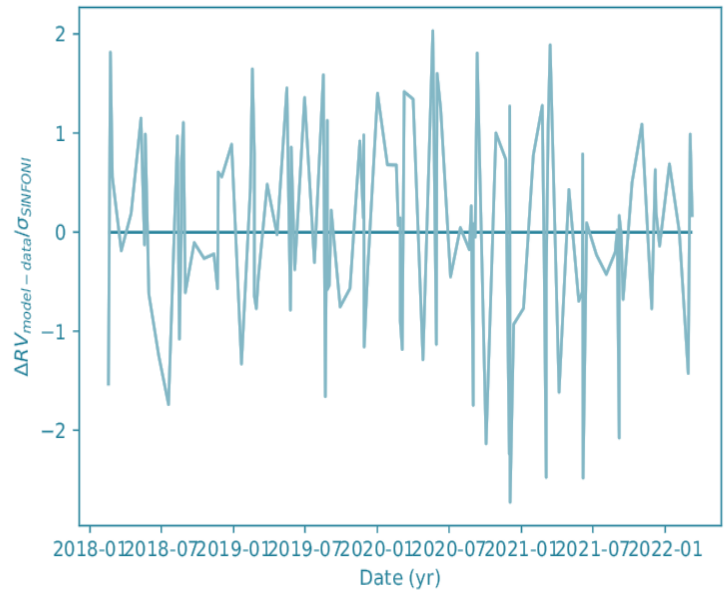
$\mathbf{z}_{obs}$
$\mathbf{x}_{bh}$

Using the faster GR_proj model

ORBIT FITTING OF S2/10 WITH FIXED ZERO SPIN

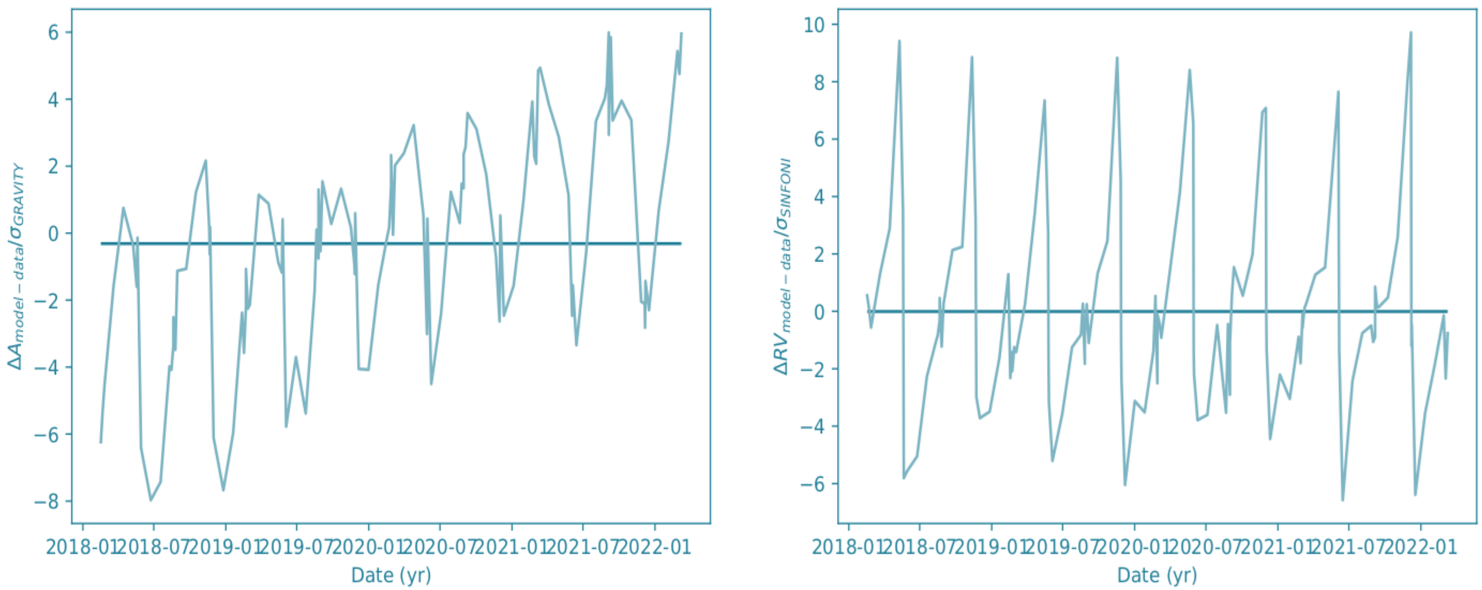
Residual between model and mock data of 8 orbits of S2/10 with a=0 and an orbit fit with the Schwarzschild model:



$\chi_r^2 = 0.99 \pm 0.08$ → Good orbit fitting of the mock data

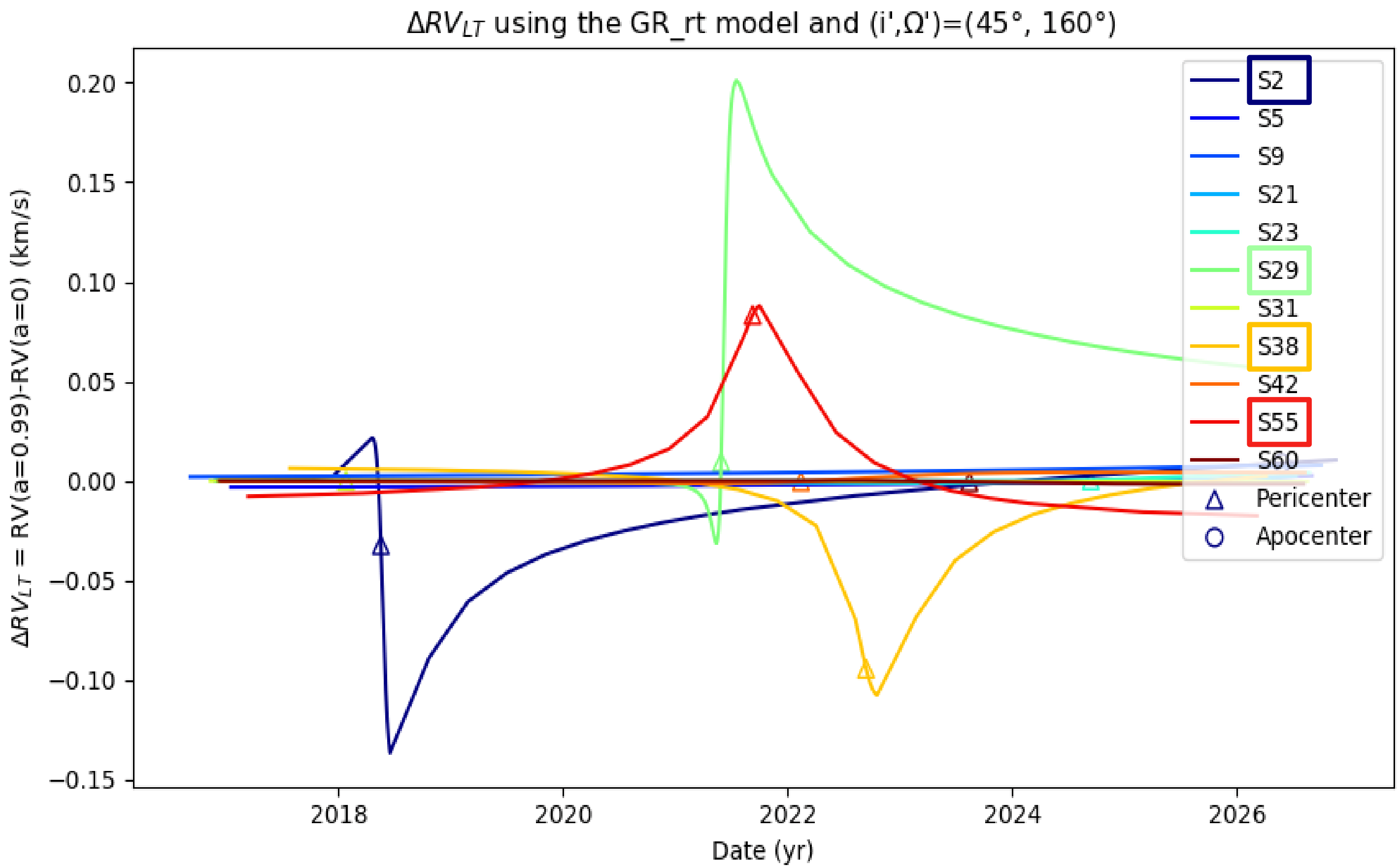
ORBIT FITTING OF S2/10 WITH FIXED ZERO SPIN

Residual between model and mock data of 8 orbits of S2/10 with a=0.99 and an orbit fit with the Schwarzschild model:



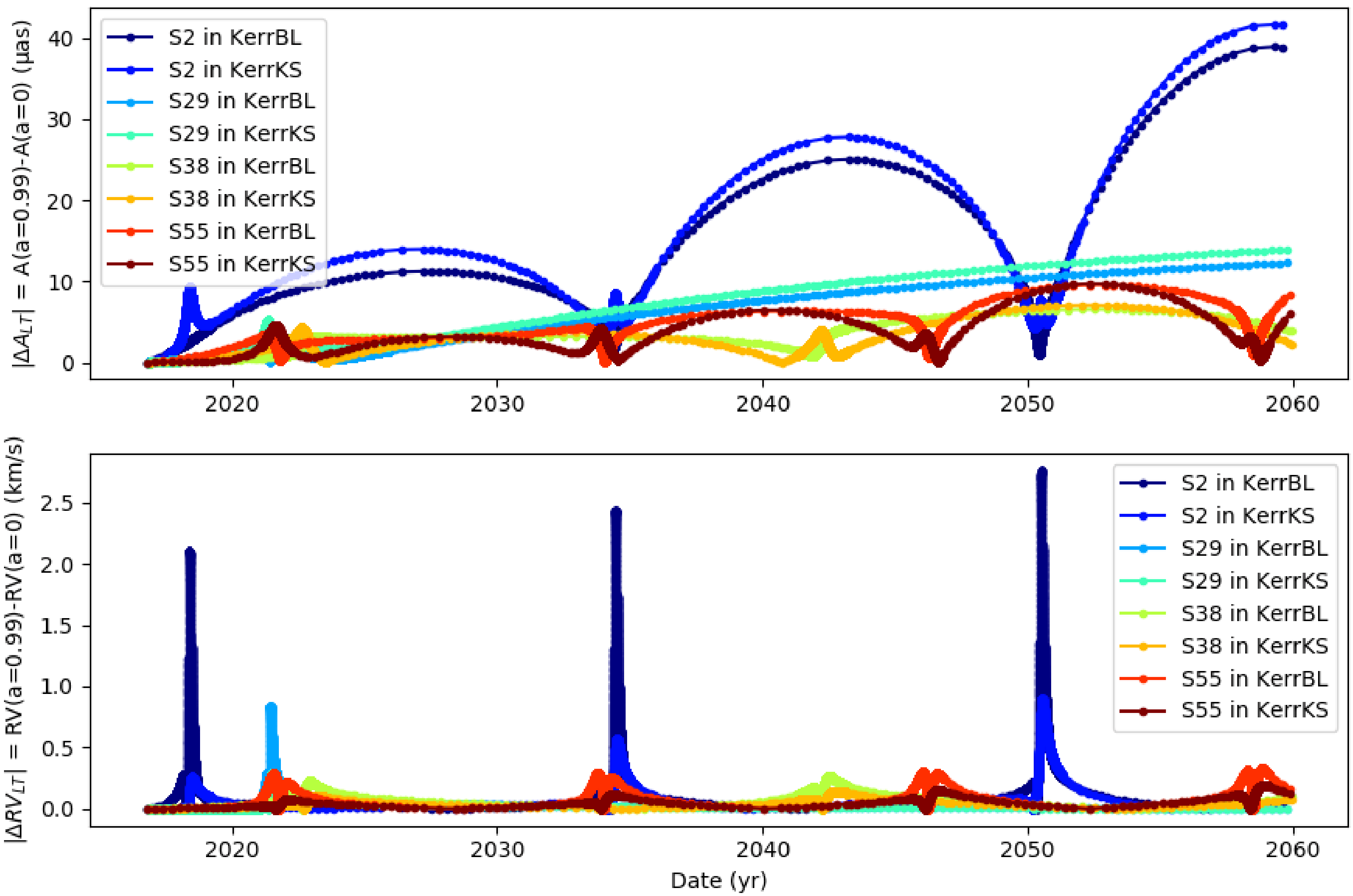
$\chi_r^2 = 13.00 \pm 0.08$ → Very poor orbit fitting of the mock data ✕

Plotting the effects of the spin on radial velocity on our selection of S stars:



The Kerr-Schild (KerrKS) and Boyer Lindquist (KerrBL) coordinate systems give different results for the same set of initial condion

$\Delta A_{LT}/\sigma_A$ and $\Delta RV_{LT}/\sigma_{RV}$ using the GR_cd model and (a,i',Ω')=(45°, 160°)



Before

Set of nominal set of initial conditions

↓

Comparing the spin effect of the KerrKS and KerrBL coordinates (different orbits)

After

Set of nominal set of initial conditions

↓

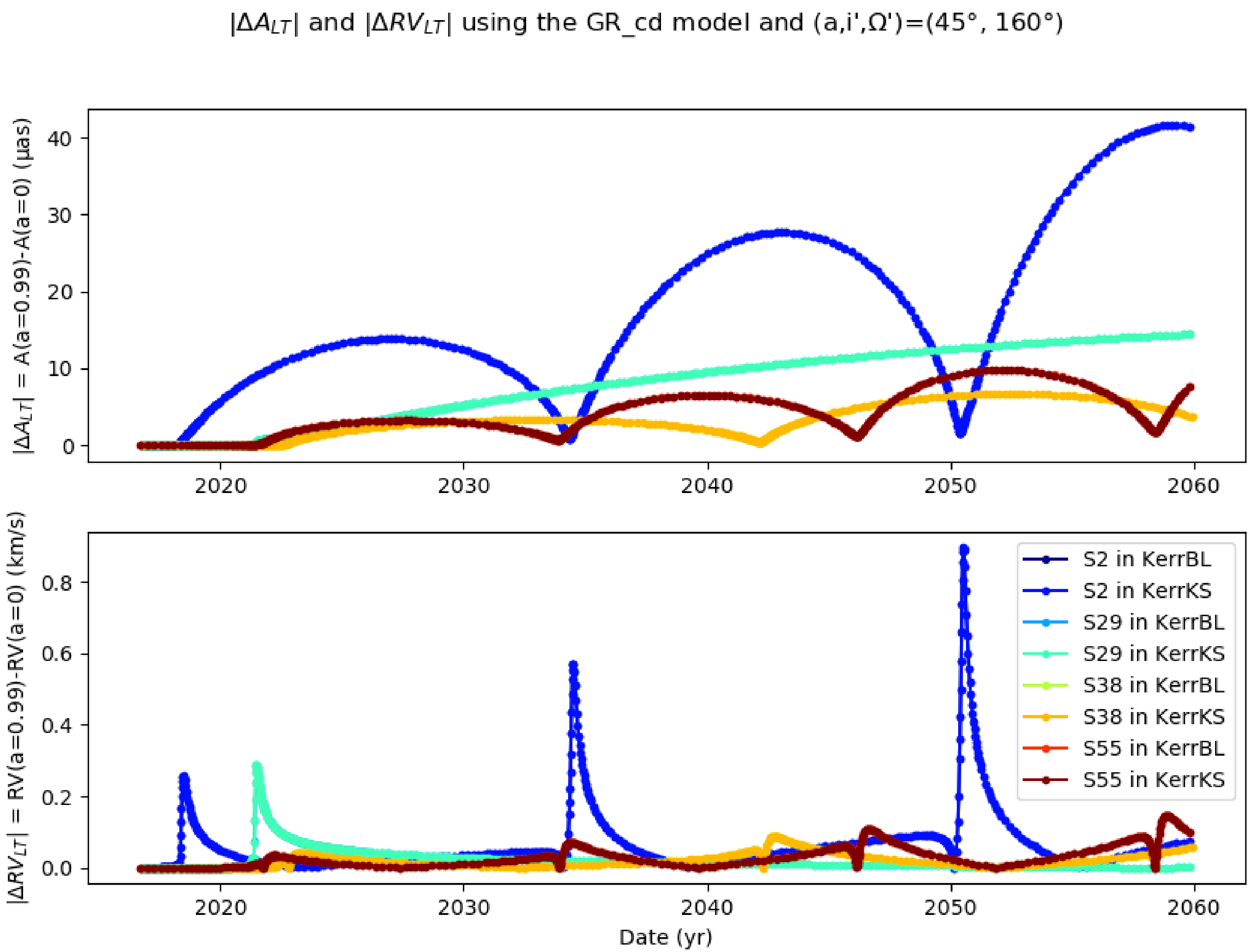
2 curve fits with free orbital parameters: 1 with KerrKS and 1 with KerrBL fitting

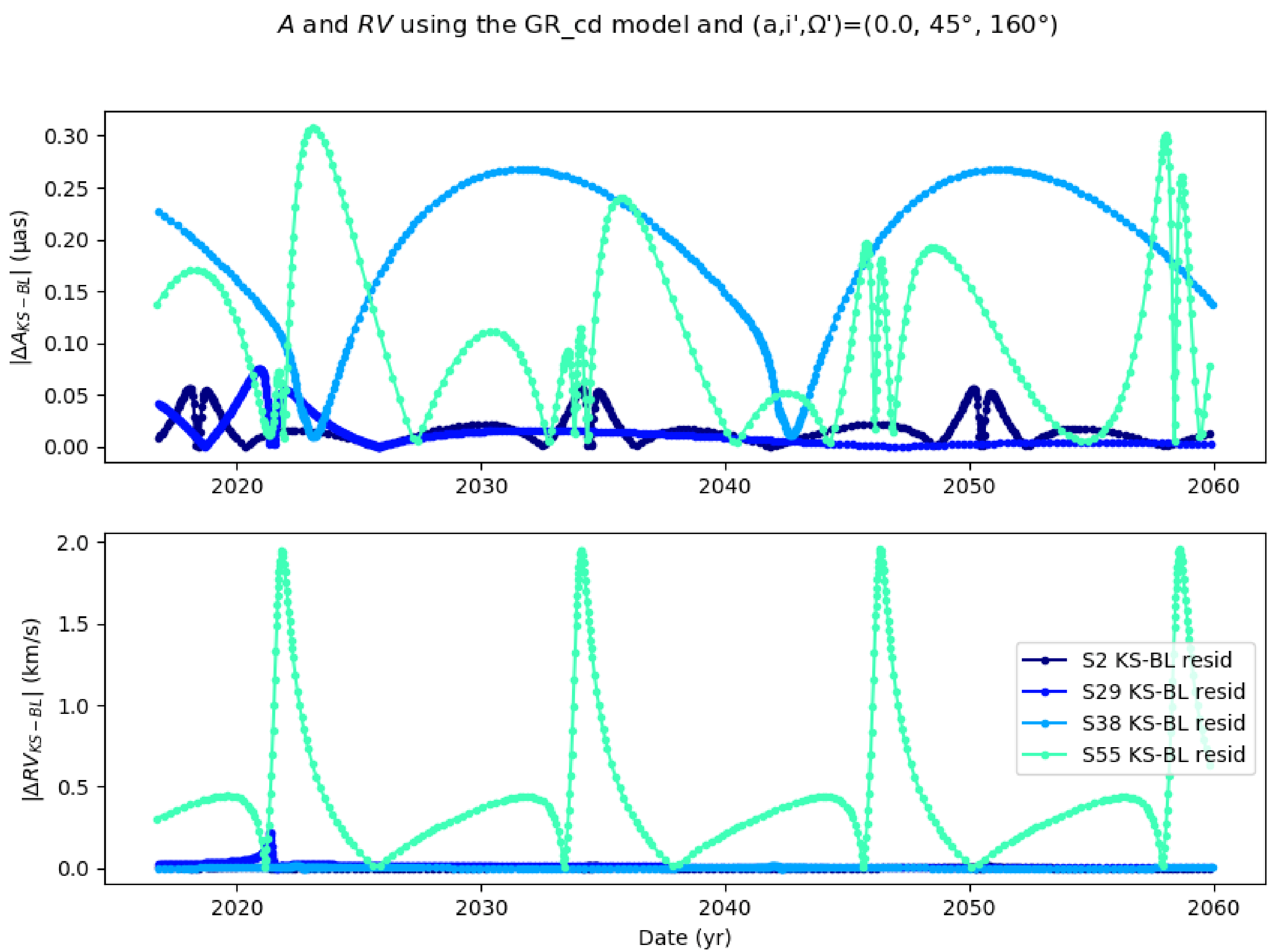
KerrKS orbital parameters forming the **KerrKS orbit**

KerrBL orbital parameters forming the **KerrBL orbit**

+ Removing unnecessary transformations and cleaning the code

Comparing the spin effect of the KerrKS and KerrBL coordinates (approximately same orbits)


|ΔA_{LT}| and |ΔRV_{LT}| using the GR_cd model and (a,i',Ω')=(45°, 160°)
|ΔA_{LT}| = A(a=0.99)-A(a=0) (µas)
40
30
20
10
0
2020
2030
2040
2050
2060
|ΔRV_{LT}| = RV(a=0.99)-RV(a=0) (km/s)
0.8
0.6
0.4
0.2
0.0
Date (yr)
S2 in KerrBL
S2 in KerrKS
S29 in KerrBL
S29 in KerrKS
S38 in KerrBL
S38 in KerrKS
S55 in KerrBL
S55 in KerrKS


A and RV using the GR_cd model and (a,i',Ω')=(0.0, 45°, 160°)
$|\Delta A_{KS-BL}|$ (μas)
$|\Delta RV_{KS-BL}|$ (km/s)
Date (yr)
S2 KS-BL resid
S29 KS-BL resid
S38 KS-BL resid
S55 KS-BL resid

Before

Set of nominal set of initial conditions

↓

Comparing the spin effect of the KerrKS and KerrBL coordinates (different orbits)

After

Set of nominal set of initial conditions

↓

2 curve fits with free orbital parameters: 1 with KerrKS and 1 with KerrBL fitting

↓	↓
KerrKS orbital parameters forming the **KerrKS orbit**	KerrBL orbital parameters forming the **KerrBL orbit**
↓	↓

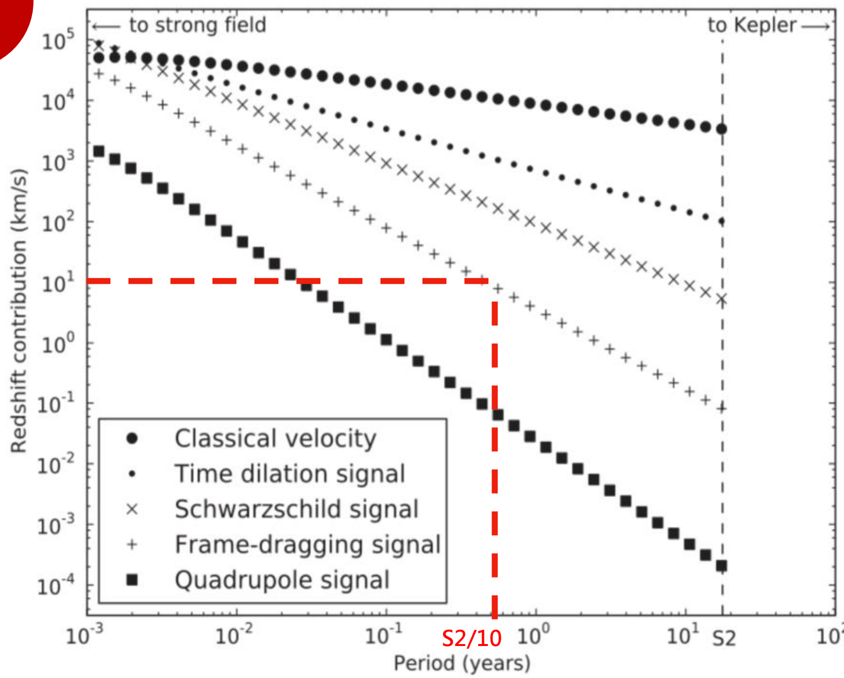
Comparing the spin effect of the KerrKS and KerrBL coordinates (approximately same orbits)

RELATIVISTIC EFFECTS IN THE VICINITY OF SGR A*

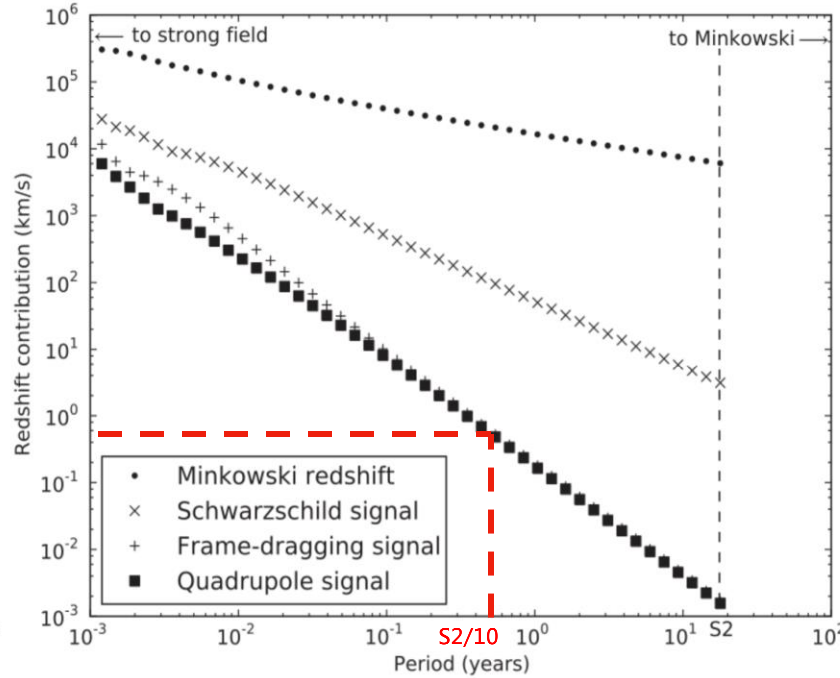
Roemer Effect	Kepler / Minkowski	On the photon trajectory
Schwarzschild Precession	Schwarzschild	On the star's trajectory
Shapiro Effect	Schwarzschild / Kerr	On the photon trajectory
Relativistic Redshifts	Schwarzschild / Kerr	On the photon trajectory
Gravitational Lensing	Schwarzschild / Kerr	On the photon trajectory
Lense-Thirring Effect	Kerr	On the star's trajectory

Angélil et al.

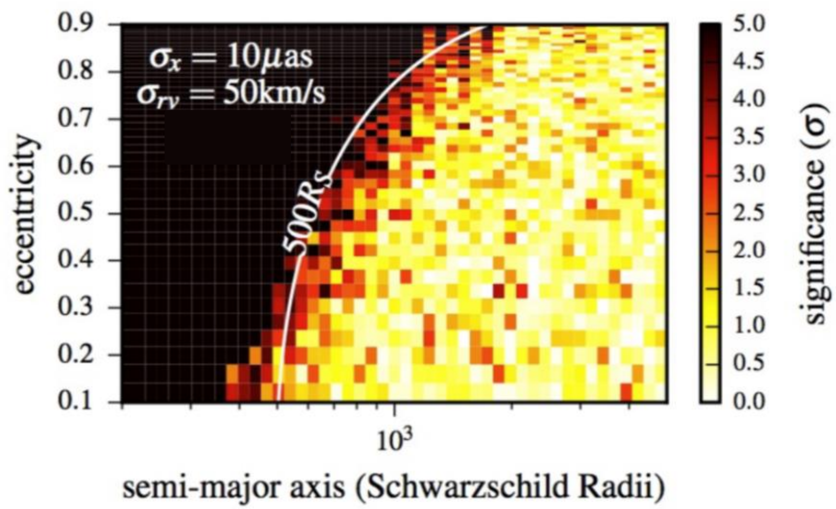
Effects on the orbit of the star



Effects on the photon trajectory



Waisberg et al.

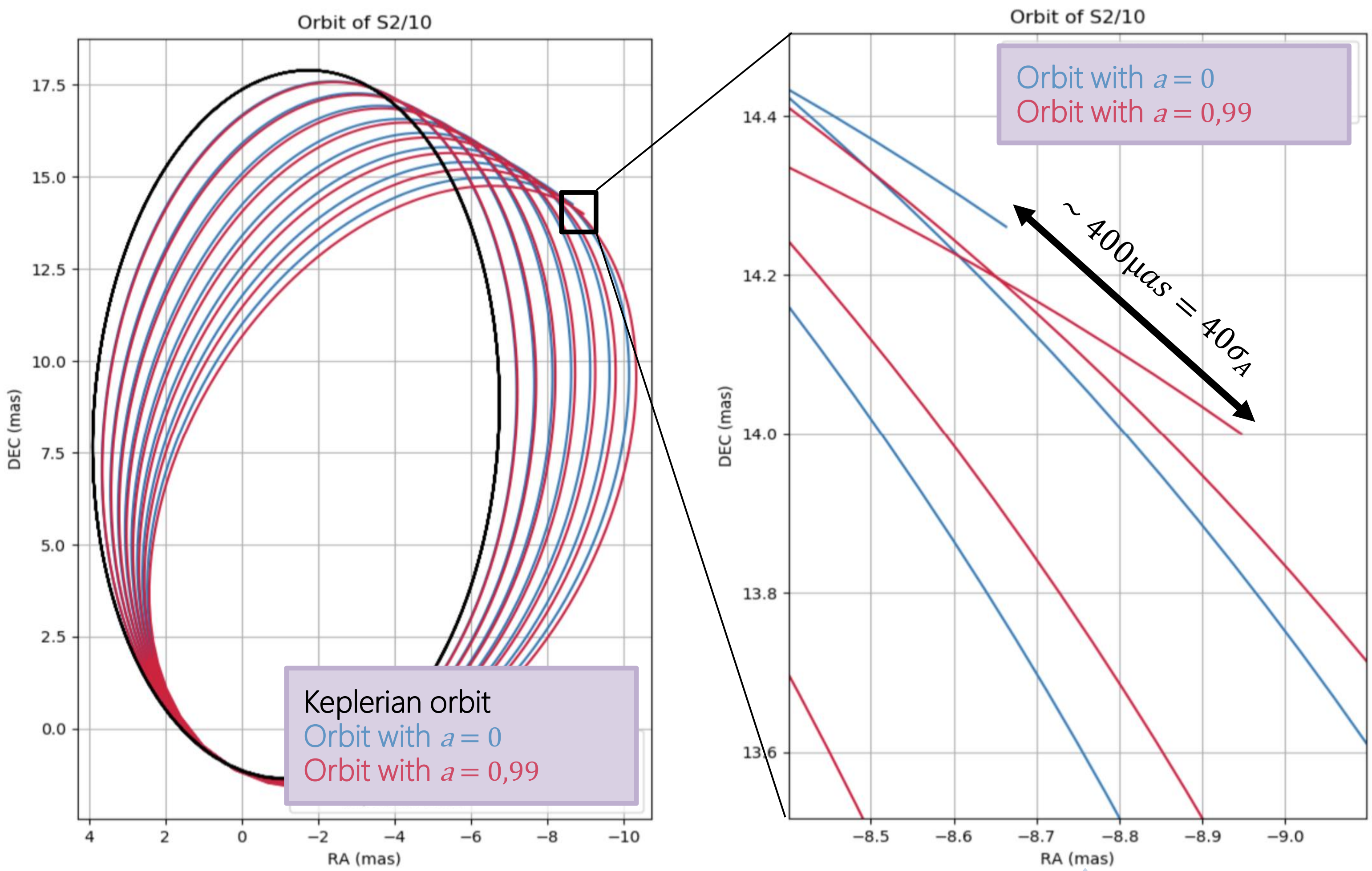


Realistic observation campaign:

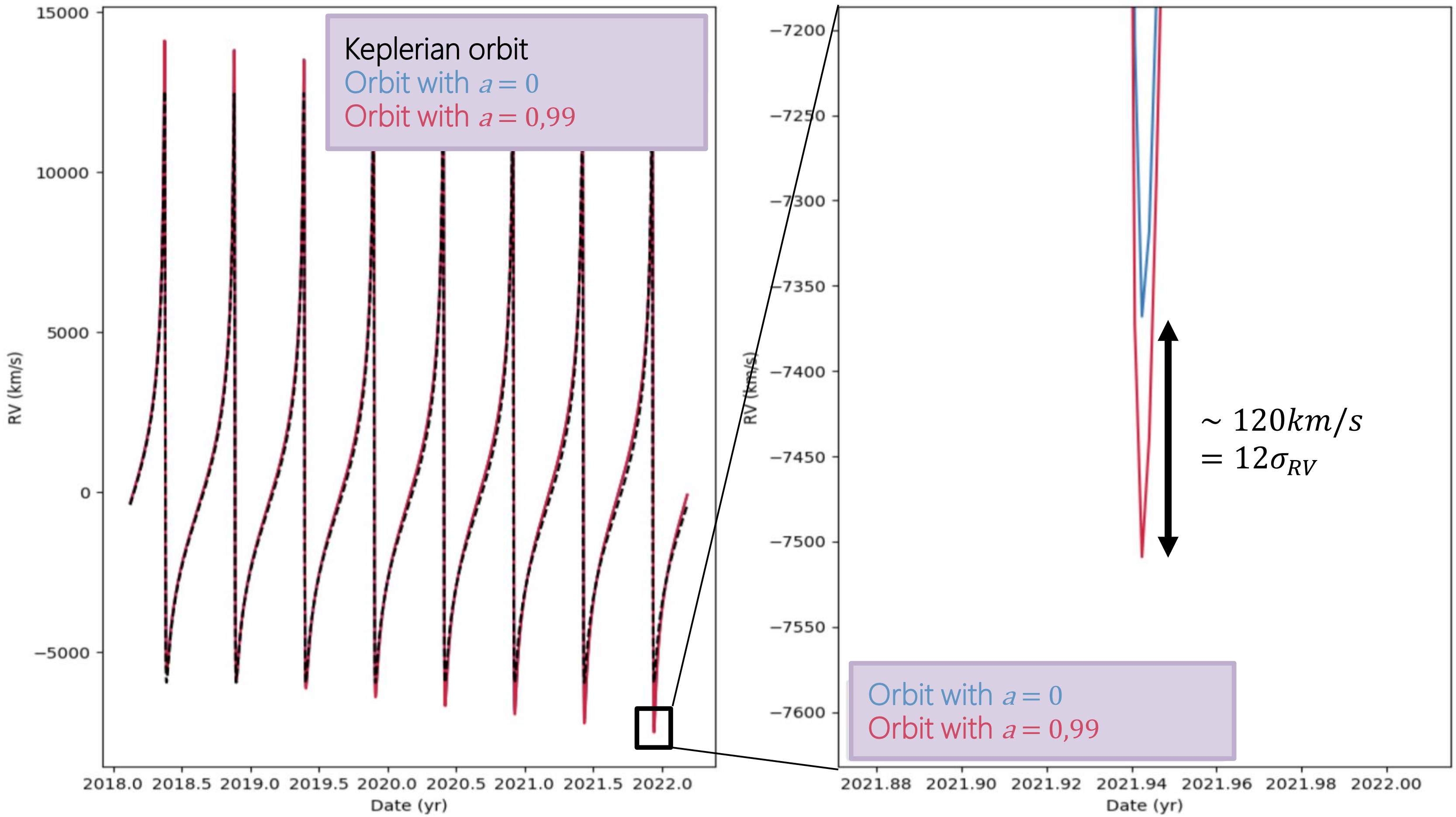
N=120 observations over 4 years

Black area ➡ Spin can be detected

SPIN EFFECTS ON THE ASTROMETRIC MEASUREMENTS OF S2/10

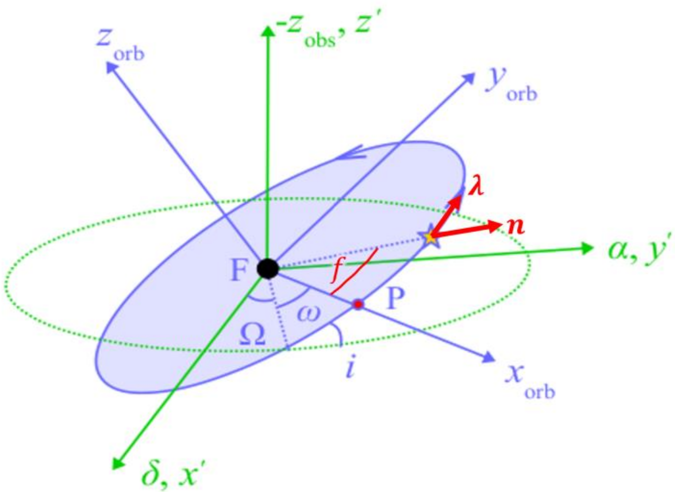


SPIN EFFECTS ON THE SPECTROSCOPIC MEASUREMENTS OF S2/10



Development of the 1.5PN model

Rotating frame: $(\boldsymbol{n}, \boldsymbol{\lambda}, \boldsymbol{z_{orb}})$



With f the true anomaly, let:

$$\xi = \frac{G^2 m^2}{c^3}\sqrt{\frac{Gm}{p}}\left(\frac{1+e\cos f}{p}\right)^3 a$$

We writre the pertubation to the Keplerian problem as*: $\boldsymbol{a}_{p,\,spin} = S\boldsymbol{n} + T\boldsymbol{\lambda} + W\boldsymbol{z_{orb}}$

On $\boldsymbol{n}$ axis: $S = 2\xi\,(1+e\cos f\,)\left[\cos\imath\cos\imath' + \cos\left(\Omega-\Omega'\right)\,\sin\imath\sin\imath'\right]$

On $\boldsymbol{\lambda}$ axis: $T = -\,2\xi e\,\sin f\left[\cos\imath\cos\imath' + \cos\left(\Omega-\Omega'\right)\,\sin\imath\sin\imath'\right]$

On $\boldsymbol{z_{orb}}$ axis:

$$W = \xi\Big[\left(\sin\imath\cos\imath' - \cos\imath\sin\imath'\cos\left(\Omega-\Omega'\right)\right)\left(e\sin\omega + 4\sin(f+\omega) + 3e\sin(2f+\omega)\right)$$
$$-\left(e\cos\omega + 4\cos(f+\omega) + 3e\cos(2f+\omega)\right)\sin\imath'\sin\left(\Omega-\Omega'\right)\Big]$$

* : according to Poisson et Will, *Gravity: Newtonian, Post-Newtonian, Relativistic*, 2014

Development of the $\mathrm{GR_{proj}}$ model

$$g_{tt}u^t_{\mathrm{em}} \gg g_{t\varphi}u^\varphi_{\mathrm{em}}$$

Verified approximation (factor 10^4 at least between the two terms)

$$\mathcal{Z} \approx \frac{1}{2}\epsilon + \frac{\mathcal{V}_\infty^2}{2} - \mathcal{V}_{\mathrm{rad,flat}}$$

With:

$$\epsilon = \frac{2r}{r^2 + a^2\cos^2\theta}$$

Runnig time (s) of 8 orbits	GR_rt	GR_proj
S2/10(a=0)	41.4131805896759	7.1126415729522705
S2/10(a=1)	43.61038303375244	7.116910457611084
S2(a=0)	54.38659143447876	7.016649961471558
S2(a=1)	54.38659143447876	7.016649961471558