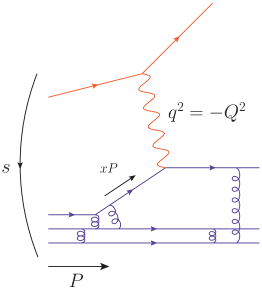
Signatures of saturation involving dissociation

Renaud Boussarie

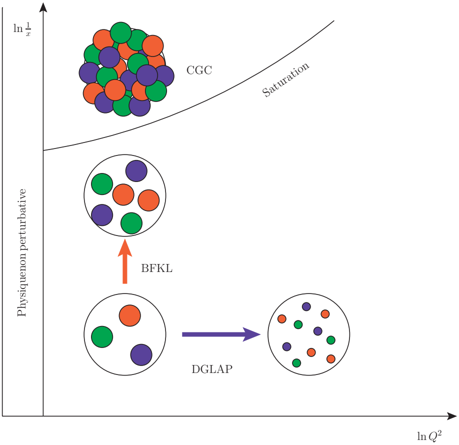
GDR QCD workshop on coherence/incoherence

Overview

Accessing the partonic content of hadrons with an electromagnetic probe

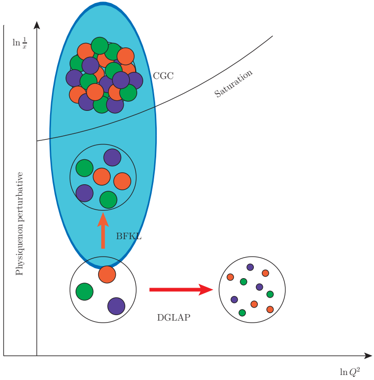


Electron-proton collision (parton model)



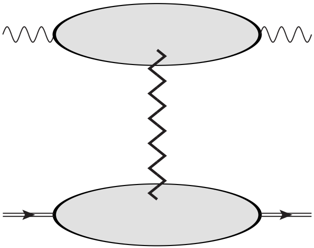
QCD at small $x_B = Q^2/s$

$$Q^2 \ll s$$



The Pomeron

Regge theory: for asymptotic values of s, an effective particle with the quantum numbers of the vacuum is exchanged

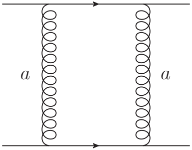


Positive C-parity: Pomeron exchange, negative C-parity: Odderon exchange

- How can we understand the Pomeron and the Odderon in perturbative QCD?

Naive perturbative Pomeron and Odderon

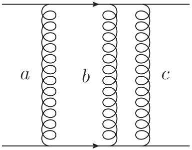
Naive perturbative description of the target hadron



Two gluons on a color singlet state

$$\mathrm{tr}(t^a t^a)$$

Leading Pomeron



Three gluons on a color singlet state

$$\mathrm{tr}(t^a t^b t^c) = \frac{1}{4}(d^{abc} + i f^{abc})$$

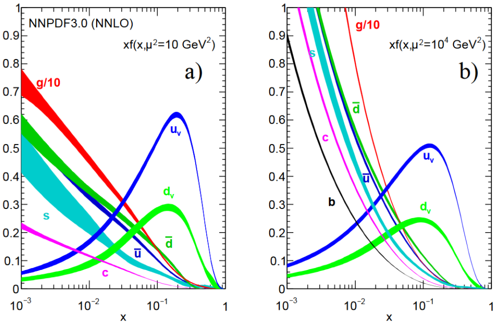
f^{abc}: subleading Pomeron
d^{abc}: leading Odderon

More involved but still for perturbative targets: BFKL, BKP, BLV...

Most general framework: small-x semiclassical effective theory

Parton Distribution Functions

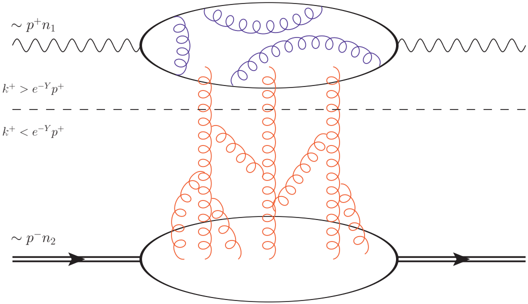
Gluon exchanges dominate at small x



[NNLO NNPDF3.0 global analysis, taken from PDG2018]

Higher chance to see gluons ⇒ enhanced multiple scatterings

Effective semiclassical description of small x QCD

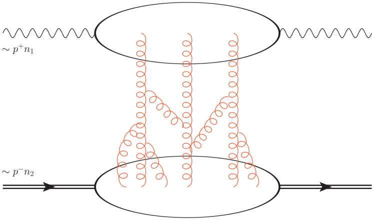


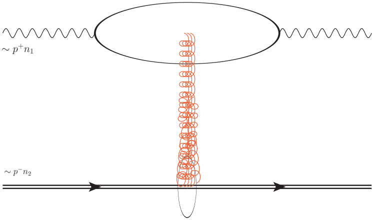
Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A^{\mu a}_{Y_c}(|k^+| > e^{-Y_c} p^+, k^-, \vec{k}) + a^{\mu a}_{Y_c}(|k^+| < e^{-Y_c} p^+, k^-, \vec{k})$$

$e^{-Y_c} \ll 1$

Large longitudinal boost to the projectile frame





$$a^+(x^+, x^-, \vec{x}) \qquad \frac{1}{\Lambda} a^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$a^-(x^+, x^-, \vec{x}) \quad \longrightarrow \quad \Lambda a^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

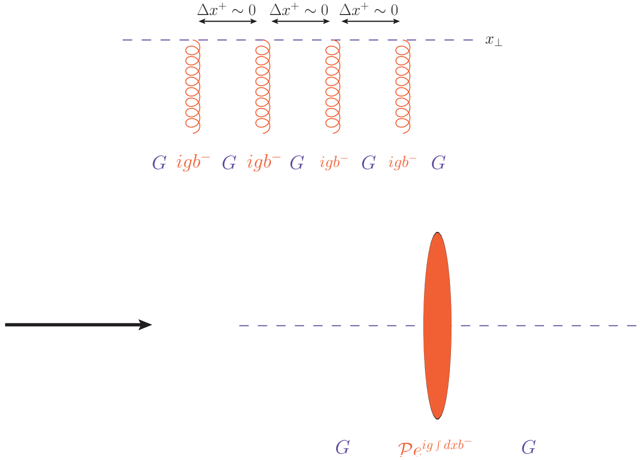
$$a^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad a^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$a^\mu(x) \to a^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

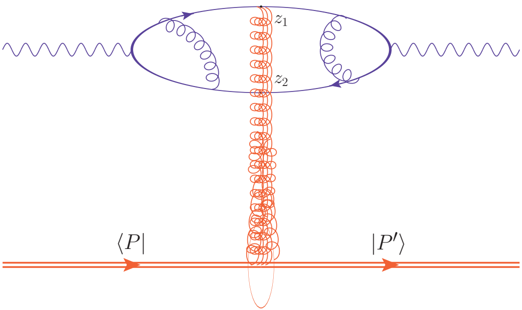
Shock wave approximation

Effective Feynman rules in the slow background field

The interactions with the background field can be exponentiated through resummation



Factorized picture



Factorized amplitude

$$\mathcal{A}^{Y_c} = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^{Y_c}_{\vec{z}_1} U^{Y_c\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

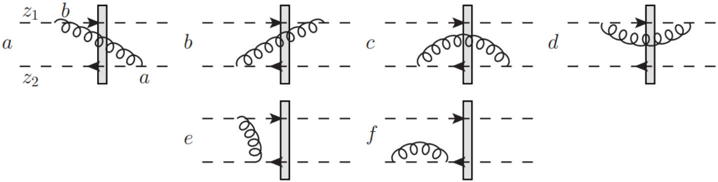
Dipole operator $\mathcal{U}^{Y_c}_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^{Y_c}_{\vec{z}_i} U^{Y_c\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Y_c independence: B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

Evolution for the dipole operator

$$\mathcal{U}_{12}^{Y_c+\delta Y} - \mathcal{U}_{12}^{Y_c}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

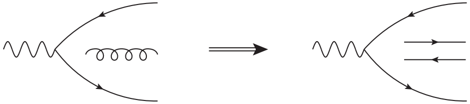
$$\frac{\partial \mathcal{U}_{12}^{Y_c}}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{Y_c} + \mathcal{U}_{32}^{Y_c} - \mathcal{U}_{12}^{Y_c} + \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c}}{\partial Y_c} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \left\langle \mathcal{U}_{12}^{Y_c} \right\rangle}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\left\langle \mathcal{U}_{13}^{Y_c} \right\rangle + \left\langle \mathcal{U}_{32}^{Y_c} \right\rangle - \left\langle \mathcal{U}_{12}^{Y_c} \right\rangle + \left\langle \mathcal{U}_{13}^{Y_c} \right\rangle \left\langle \mathcal{U}_{32}^{Y_c} \right\rangle \right]$$

BFKL/BKP part — Triple pomeron vertex

Non-linear term: recombination effects: saturation

Non-perturbative elements are compatible with CGC-type models

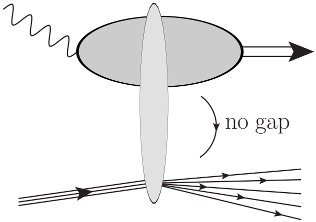
Overview

1. Semi-classical small x physics
2. Dissociation at small x
3. The Color Glass Condensate
4. Dissociation in the CGC
5. Summary

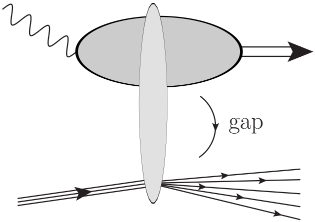
Low x description of diffractive processes

First assumption

Rapidity gap $\Leftrightarrow$ Color singlet exchange

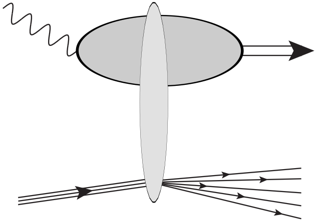


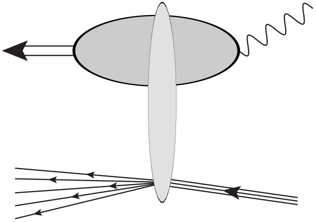
Generic Wilson line operator W



Singlet Wilson line operator $\mathrm{tr}\, W$

Case 1: anything can happen to the target





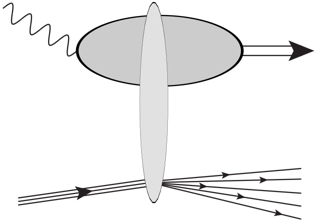
$\langle X|\mathrm{tr}\,W|P\rangle$ $\qquad$ $\langle P|\mathrm{tr}\,W^\dagger|X\rangle$

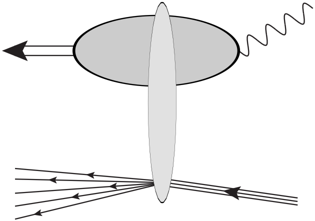
$$\sum_X \int \mathrm{d}\Phi_X \langle P|\mathrm{tr}\,W^\dagger|X\rangle\langle X|\mathrm{tr}\,W|P\rangle = \langle P|\mathrm{tr}\,W^\dagger \mathrm{tr}\,W|P\rangle$$

since $\sum_X \int \mathrm{d}\Phi_X |X\rangle\langle X| = 1$

Case 1b: anything can happen to the target

No "diffraction=singlet" assumption



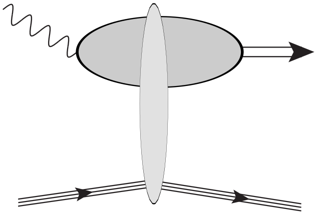


$\langle X \text{ with gap} | W | P \rangle$ $\qquad$ $\langle P | W^\dagger | X \text{ with gap} \rangle$

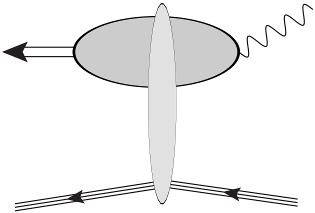
$$\sum_{X \text{ with gap}} \int \mathrm{d}\Phi_X \langle P | W^\dagger | X \rangle \langle X | W | P \rangle \neq \langle P | \mathrm{tr} W^\dagger \mathrm{tr} W | P \rangle$$

since $\sum_{X \text{ with gap}} \int \mathrm{d}\Phi_X |X\rangle\langle X| \neq 1$

Case 2: the target is unbroken



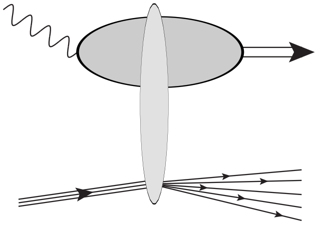
$\langle P' | \mathrm{tr}\, W | P \rangle$



$\langle P | \mathrm{tr}\, W^\dagger | P' \rangle$

$$\langle P | \mathrm{tr}\, W^\dagger | P' \rangle \langle P' | \mathrm{tr}\, W | P \rangle$$

Case 3: dissociative diffraction



$\langle X|\mathrm{tr}W|P\rangle$

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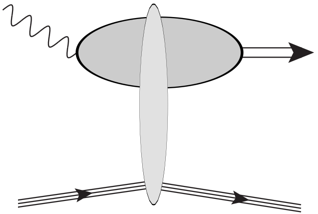
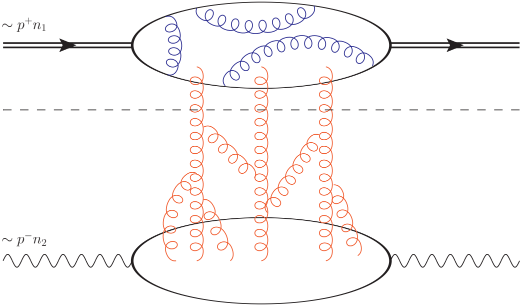


$\langle P'|\mathrm{tr}W|P\rangle$

$$\langle P|\mathrm{tr}W^\dagger \mathrm{tr}W|P\rangle - \langle P|\mathrm{tr}W^\dagger|P'\rangle\langle P'|\mathrm{tr}W|P\rangle$$

Overview

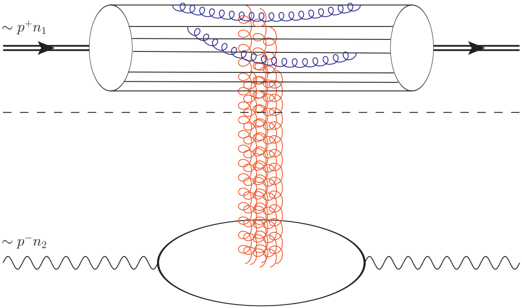
The Color Glass Condensate



fast partons $\leftrightarrow$ valence partons

slow gluons $\leftrightarrow$ wee gluons

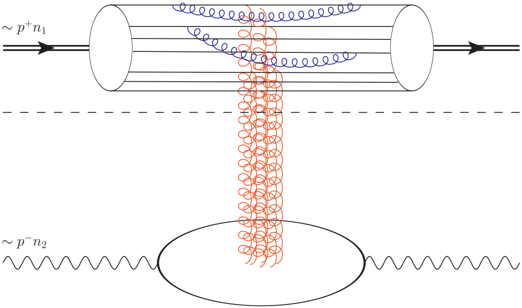
The Color Glass Condensate



Hadron wave function = collection of static color sources

Color sources ρ are classical random variables, treated with a weight function $W_Y[\rho]$

The Color Glass Condensate



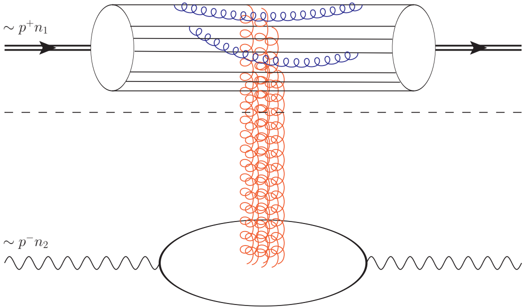
Static source = static current of color charge

$$J_a^{\mu} = \delta^{\mu +} \rho_a(x)$$

Wee gluons: solutions to the classical Yang-Mills equation with the source

$$[D_\nu, F^{\mu\nu}] = \delta^{\mu +} \rho_a(x) T^a$$

The Color Glass Condensate



Target matrix elements → averages over configurations of sources and dynamical fields A^μ

$$\frac{\langle P|\mathcal{O}|P\rangle}{\langle P|P\rangle} \to \langle \mathcal{O} \rangle = \int \mathcal{D}\rho\, \mathcal{D}A^\mu\, W[\rho]\, e^{iS[\rho,A]}\, \mathcal{O}[\rho, A]$$

The MV model

McLerran-Venugopalan model

- Sources $\simeq$ valence quarks $\Rightarrow$ number of sources $\sim N_c A$
- Transverse radius $R_A \sim A^{1/3}\Lambda_{\mathrm{QCD}}^{-1}$
- Transverse resolution of the probe $1/Q^2$
- Number of sources seens by the probe $\Delta N = \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\frac{N_c A^{1/3}}{\pi}$

If $Q^2 \ll \Lambda_{\mathrm{QCD}}^2 A^{1/3}$, a large number of sources is probed

Random distribution of sources, total color charge probed is 0:

$$\langle \mathcal{Q} \rangle = \int_{1/Q^2} d^2\vec{x} \int dx^- \rho(x^-, \vec{x}) = 0$$

The MV model

McLerran-Venugopalan model

- Assume that $\langle \rho_a(x^-, \vec{x}) \rangle = 0$
- Write that
  $\langle \rho_a(x^-, \vec{x}) \rho_b(y^-, \vec{y}) \rangle = g_s^2 \delta_{ab} \delta(x^- - y^-) \delta(\vec{x} - \vec{y}) \lambda(x^-)$
- Assume that higher-point functions vanish

Correlators are generated from a Gaussian weight function

$$\Phi[\rho] \propto \exp\left(-\frac{1}{2}\int d^2\vec{x}\,\frac{\rho_a\,\rho_a}{\mu^2}\right), \quad \mu \propto \int dx^- \lambda(x^-)$$

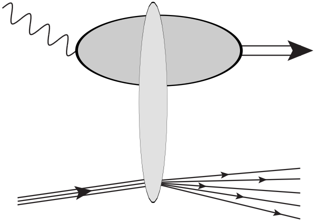
Target matrix elements:

$$\frac{\langle P|\mathcal{O}|P\rangle}{\langle P|P\rangle} \to \langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\rho\,\Phi[\rho]\,\mathcal{O}}{\int \mathcal{D}\rho\,\Phi[\rho]}$$

Overview

Dissociative meson production at small x

Wilson line operator: simple dipole $D(\boldsymbol{b}, \boldsymbol{r})$



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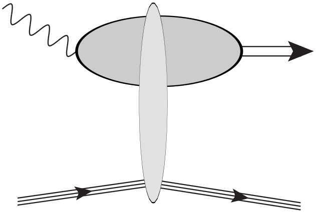


$$\frac{\langle P|\mathrm{tr}D^{\dagger}(\boldsymbol{b}',\boldsymbol{r}')\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})|P\rangle}{\langle P|P\rangle} - \frac{\langle P|\mathrm{tr}D^{\dagger}(\boldsymbol{b}',\boldsymbol{r}')|P'\rangle\langle P'|\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})|P\rangle}{\langle P|P\rangle\langle P|P\rangle}$$

Convoluted with wave function overlaps

$\mathcal{H}(\boldsymbol{b}, \boldsymbol{r}) = \mathrm{e}^{-i(\boldsymbol{\Delta}\cdot\boldsymbol{b})}(\Psi_{\gamma}\Psi^{*}_{\mathrm{meson}})(\boldsymbol{r})$ and $\mathcal{H}^{*}(\boldsymbol{b}', \boldsymbol{r}')$

CGC model for the first term (inclusive diffraction)

$$\int_{\boldsymbol{b}',\boldsymbol{b},\boldsymbol{r}',\boldsymbol{r}} \frac{\langle P|\mathrm{tr}D^\dagger(\boldsymbol{b}',\boldsymbol{r}')\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})|P\rangle}{\langle P|P\rangle}\mathcal{H}^*(\boldsymbol{b}',\boldsymbol{r}')\mathcal{H}(\boldsymbol{b},\boldsymbol{r})$$

$$\to \int_{\boldsymbol{b}',\boldsymbol{b},\boldsymbol{r}',\boldsymbol{r}} \left\langle \mathrm{tr}D^\dagger(\boldsymbol{b}',\boldsymbol{r}')\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})\right\rangle \mathcal{H}^*(\boldsymbol{b}',\boldsymbol{r}')\mathcal{H}(\boldsymbol{b},\boldsymbol{r})$$

$$\to \left\langle \int_{\boldsymbol{b}',\boldsymbol{r}'} \left[\mathrm{tr}D(\boldsymbol{b}',\boldsymbol{r}')\mathcal{H}(\boldsymbol{b}',\boldsymbol{r}')\right]^* \int_{\boldsymbol{b},\boldsymbol{r}} \left[\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})\mathcal{H}(\boldsymbol{b},\boldsymbol{r})\right]\right\rangle$$

This has the form $\langle \mathcal{D}^*\mathcal{D}\rangle = \langle |\mathcal{D}|^2\rangle$

CGC model for the second term (coherent diffraction)

$$\int_{\boldsymbol{b}',\boldsymbol{b},\boldsymbol{r}',\boldsymbol{r}} \frac{\langle P|\mathrm{tr}D^\dagger(\boldsymbol{b}',\boldsymbol{r}')|P'\rangle\langle P'|\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})|P\rangle}{\langle P|P\rangle\langle P|P\rangle}\mathcal{H}^*(\boldsymbol{b}',\boldsymbol{r}')\mathcal{H}(\boldsymbol{b},\boldsymbol{r})$$

$$\to \int_{\boldsymbol{b}',\boldsymbol{b},\boldsymbol{r}',\boldsymbol{r}} \langle\mathrm{tr}D^\dagger(\boldsymbol{b}',\boldsymbol{r}')\rangle\langle\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})\rangle\mathcal{H}^*(\boldsymbol{b}',\boldsymbol{r}')\mathcal{H}(\boldsymbol{b},\boldsymbol{r})$$

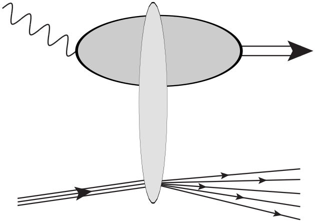
$$\to \left\langle\int_{\boldsymbol{b}',\boldsymbol{r}'}\left[\mathrm{tr}D(\boldsymbol{b}',\boldsymbol{r}')\mathcal{H}(\boldsymbol{b}',\boldsymbol{r}')\right]^*\right\rangle\left\langle\int_{\boldsymbol{b},\boldsymbol{r}}\left[\mathrm{tr}D(\boldsymbol{b},\boldsymbol{r})\mathcal{H}(\boldsymbol{b},\boldsymbol{r})\right]\right\rangle$$

This has the form $\langle\mathcal{D}^*\rangle\,\langle\mathcal{D}\rangle = |\langle\mathcal{D}\rangle|^2$

Bit of cheating involved here since matrix elements are non-diagonal so not simple charge averages

CGC model for dissociative meson production

Wilson line operator: simple dipole $D(\boldsymbol{b}, \boldsymbol{r})$



$\langle|\mathcal{D}|^2\rangle$

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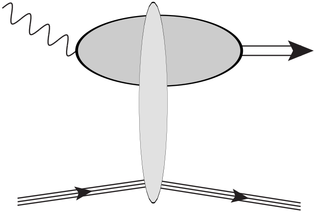

$|\langle\mathcal{D}\rangle|^2$

In a CGC context, this process is sensitive to the variance

$$\langle|\mathcal{D}|^2\rangle - |\langle\mathcal{D}\rangle|^2$$

CGC model for dissociative meson production

This cross section has the form of a variance $\langle|\mathcal{D}|^2\rangle - |\langle\mathcal{D}\rangle|^2$

Randomness: distribution of color sources in the proton.

It is thus sensitive to any fluctuations of proton shapes, hot spots, local color correlations...

In energy/density limits where the mean field approximation is valid, the cross section vanishes.

Particular case: black disk limit $D \sim 1 \rightarrow \mathcal{D} = \mathrm{cste}$

Overview

Summary

- Assuming diffraction $\Leftrightarrow$ singlet exchange
- Assuming large target densities due to high energy or high mass number A
- Assuming we can model non-diagonal matrix elements as color averages
- Then dissociative production is directly sensitive to color fluctuations in the proton.