The impulse approximation

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based on
Chew and Wick, Phys. Rev. 85 (1952)
Askin and Wick, Phys. Rev. 85 (1952)
Chew and Goldberger, Phys. Rev. 87 (1952)

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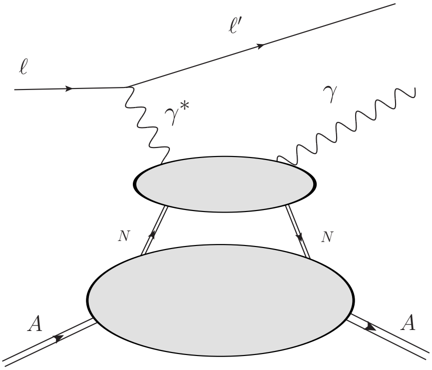
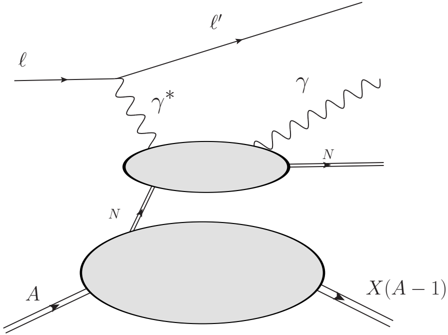
The impulse approximation

This idea is the core of the impulse approximation, which describe such a process as a superposition of two-body interactions

Examples on DVCS on nuclei



Coherent scattering



Incoherent scattering

see *e.g.* S. Fucini *et al.*, Phys.Rev.C 98 (2018) 1, 015203
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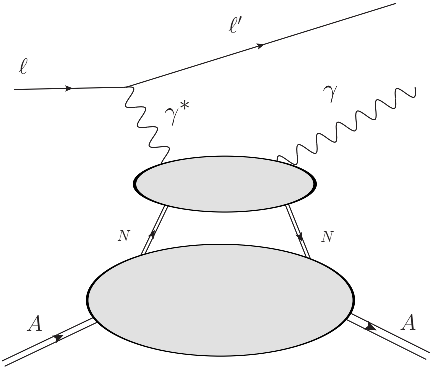

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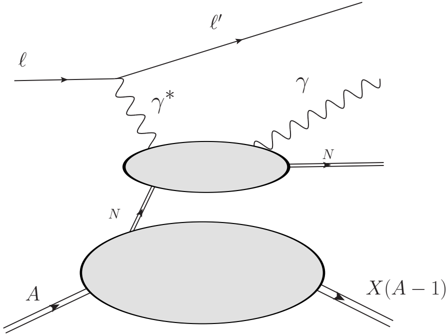

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A similar description can be obtained for J/Psi photoproduction in UPC

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- The discussion was done for quantum but non relativistic systems, which is what I will follow here.
- The discussion rely on hamiltonian formalism, I try to hihglight the important points

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Let's dig a little bit more into these assumptions.

We assume that the collision happens during a time $\tau = t_2 - t_1$ such that:

- before $(t < t_1)$ and after $(t > t_2)$ the collision, the system evolves following the hamiltonian H_0:

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- During the collision, one needs to take into account the full hamiltonian:

  $$H = H_0 + V$$

  where V describe the interaction of the probe with the target.

$$|\psi_i\rangle$$

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Evolution of states

$$T_{fi} = \langle \psi_f | e^{iH_0 t_2} e^{-iH\tau} e^{-iH_0 t_1} | \psi_i \rangle$$

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But is it the end of the story?

We want to compute the first correction in U to our previous estimate to check the consistency of the assumption.

- To do that we write a state during the collision time at t such that $t_1 < t < t_2$:

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- The operator we have to expand is then:

$$e^{i(K+V+U)t_1}e^{-i(K+U)t_1}|\psi_i\rangle$$

The Zassenhaus Formula

Reminder :

$$e^X = \sum_{k=0}^{\infty} \frac{1}{k!} X^k$$

such that

$$e^{X+Y} = \sum_{k=0}^{\infty} \frac{1}{k!} (X+Y)^k \neq e^X e^Y \quad \text{if } [X, Y] \neq 0$$

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In fact the Zassenhaus Fomula yields:

$$e^{t(X+Y)} = e^{tX} e^{tY} e^{-\frac{t^2}{2}[X,Y]} e^{\frac{t^3}{3!}(2[Y,[X,Y]]+[X,[X,Y]])} \dots$$

and highlight the importance of the commutator.

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- Importantly, it appears that strong binding potential are compatible with the impulse approximation, provided that they depend sufficiently weakly of the internal degrees of freedom (nucleons).
- The second order correction has been computed and involve again $[\Omega, U]$, nested in another commutator.

Considerations on assumption II

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- Rescattering effects may also be a challenge, and the rescattered probe wave amplitude should be negligible compare to the incoming one when reaching a second degree of freedom of the system. This is helped by interferences for $\lambda_p << d_{NN}$.
- Unfortunately, there is no general criterium to discuss for assumption I.

Some additional thoughts

Free or effective degrees of freedom?

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- However, nothing prevent these nucleons to be "modified" compared to free ones and the impulse approximation could very well be used for targets made of effective degrees of freedom (providing that the previous assumptions are fulfilled).
- If this is correct, the impulse approximation might not be incompatible with the nuclear effect seen on nuclear PDFs in the diluted region (EMC effect).

Impulse approximation and nuclear PDFs

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- Could it be expected ?

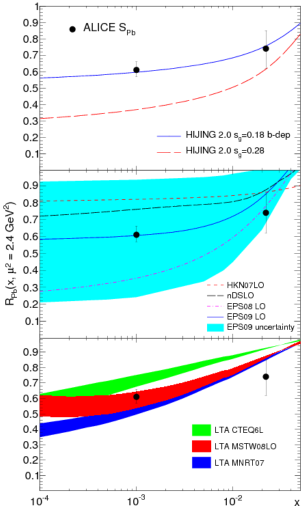
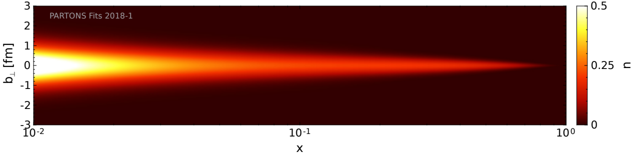


figure from V. Guzey *et al.*, Phys.Lett.B 726 (2013) 290-295

- Deviation ratio R
- Data coming from exclusive J/Psi production in UPC

- First important thing is that the nucleon size and density is x dependent.



H. Moutarde *et al.*, EPJC 78 (2018) 890

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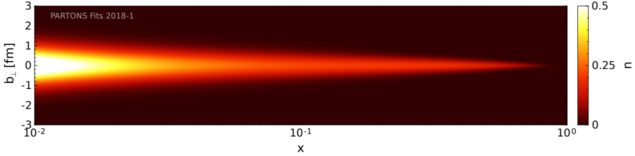

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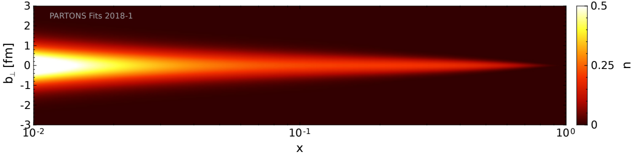

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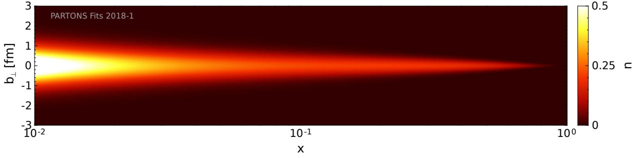

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  - And maybe a higher dependence of U on the nuclear configuration (Assumption III)
- Thus one might expect the influence of rescattering and screening effects to grow.
- There is a competition between the size of the probe and the density of the system.

Conclusion

- The impulse approximation relies on 3 main assumptions
  - The probe interact with one degree of freedom at a time
  - The probe is not affected by the medium
  - The binding forces do not strongly depend on the nucleonic variables
- If these conditions are met, the impulse approximation is a good one, allowing to treat in a much simplified way the collision of a probe with a complex system.

Thank you for your attention

Back up slides