Recent advances from ab initio Self-Consistent Green's function computations of nuclei

EuNPC2025
Sept. 22-26, Caen

Carlo Barbieri
Università degli Studi di Milano & INFN

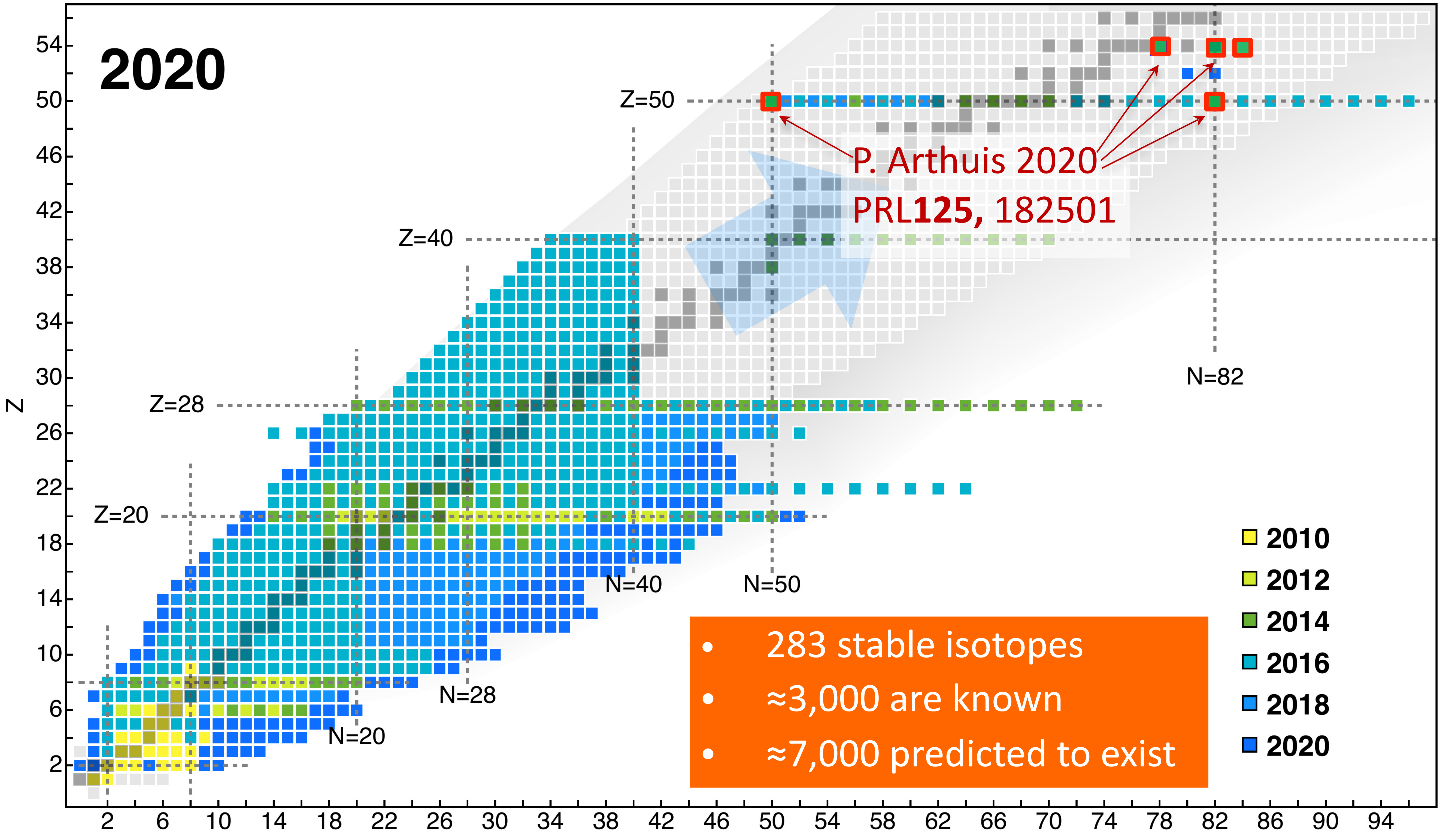
UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

Reach of ab initio methods across the nuclear chart

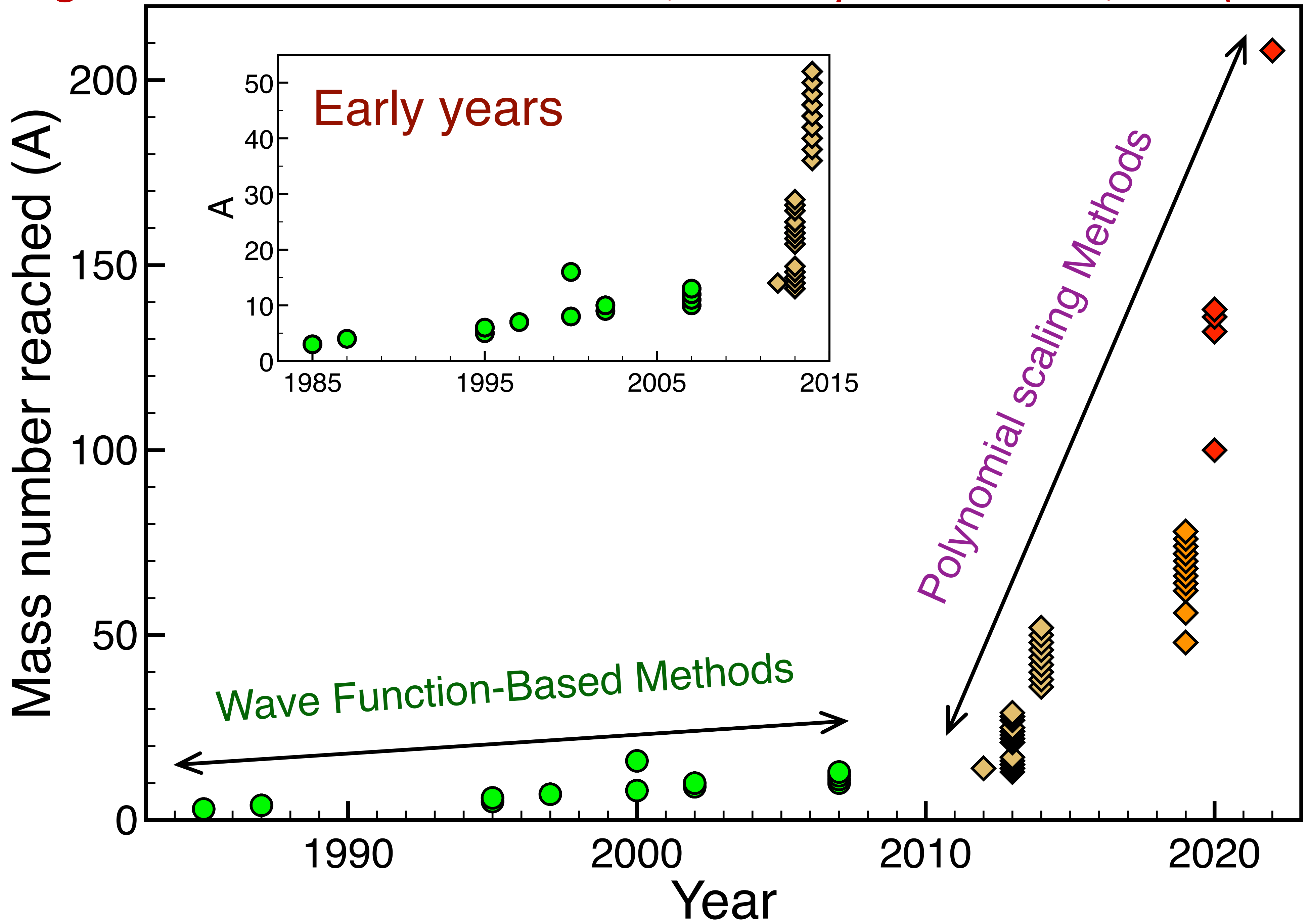
Legnaro Natl' Lab Mid Term Plan; Eur. Phys. J. Plus **138**, 709 (2023)

Extension beyond few-nucleons thanks to:

- Soft (nearly perturbative) effective nuclear forces
- Diagrammatic many-body approaches



H. Hergert, Frontiers in Phys **8**, 379 (2020)
L. Coraggio, S. Pastore, **CB**, Frontiers in Phys **8**, 626976 (2021)



Open challenges:

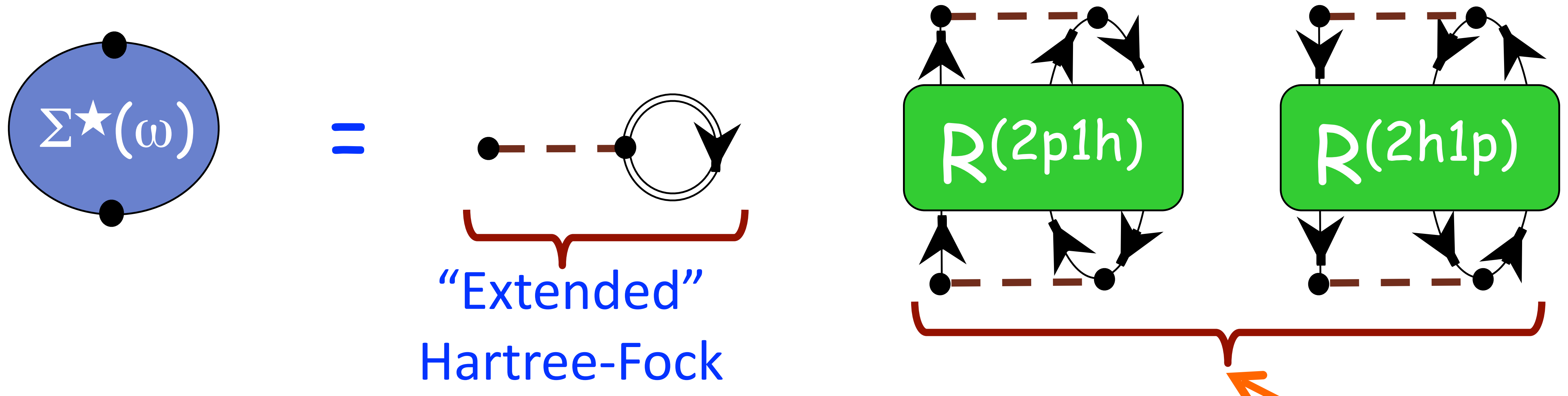
- Accuracy (better theory of nuclear forces)
- Mass number limit (optimised model spaces)
- Precision & scattering (high-order diag. resummations)

The Faddev-RPA and ADC(3) methods in a few words

Compute the nuclear self energy to extract both scattering (optical potential) and spectroscopy.
Both ladders and rings are needed for atomi nuclei:

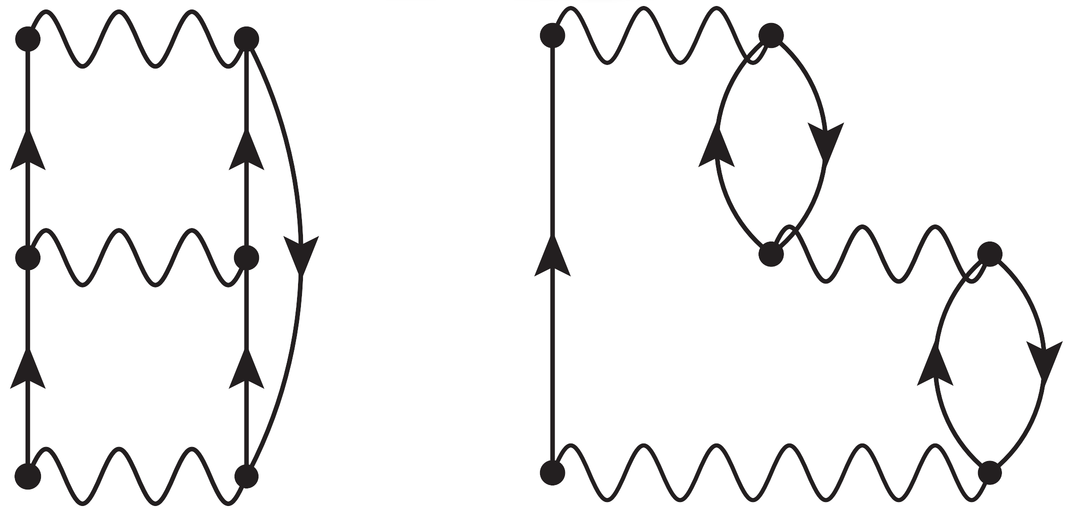
F-RPA:
*Phys. Rev. C**63**, 034313 (2001)*
*Phys. Rev. A**76**, 052503 (2007)*
*Phys. Rev. A**83**, 042517 (2011)*

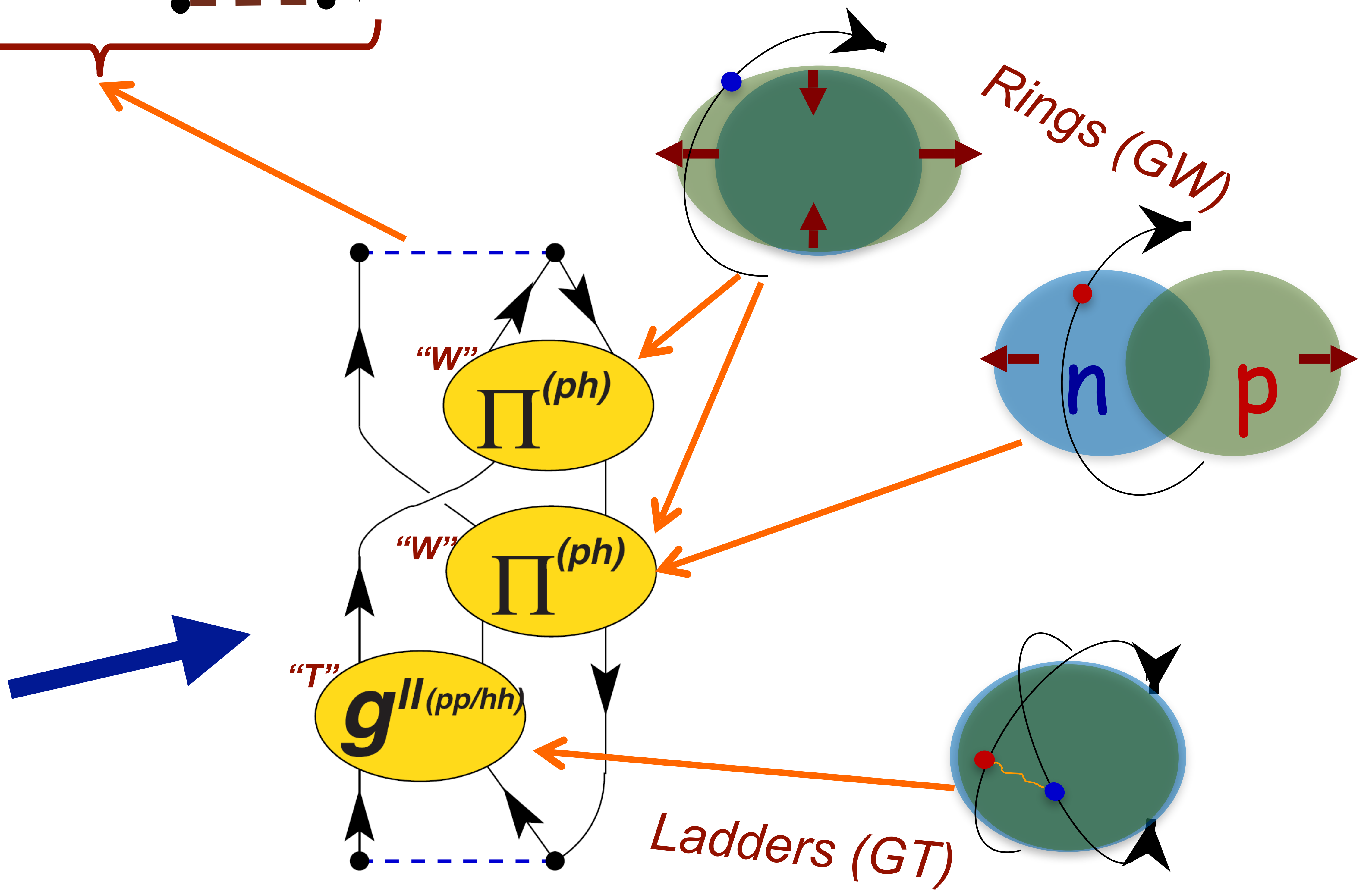
ADC(3):
*Lect. Notes in Phys **936** (2017)-Chapter 11.*



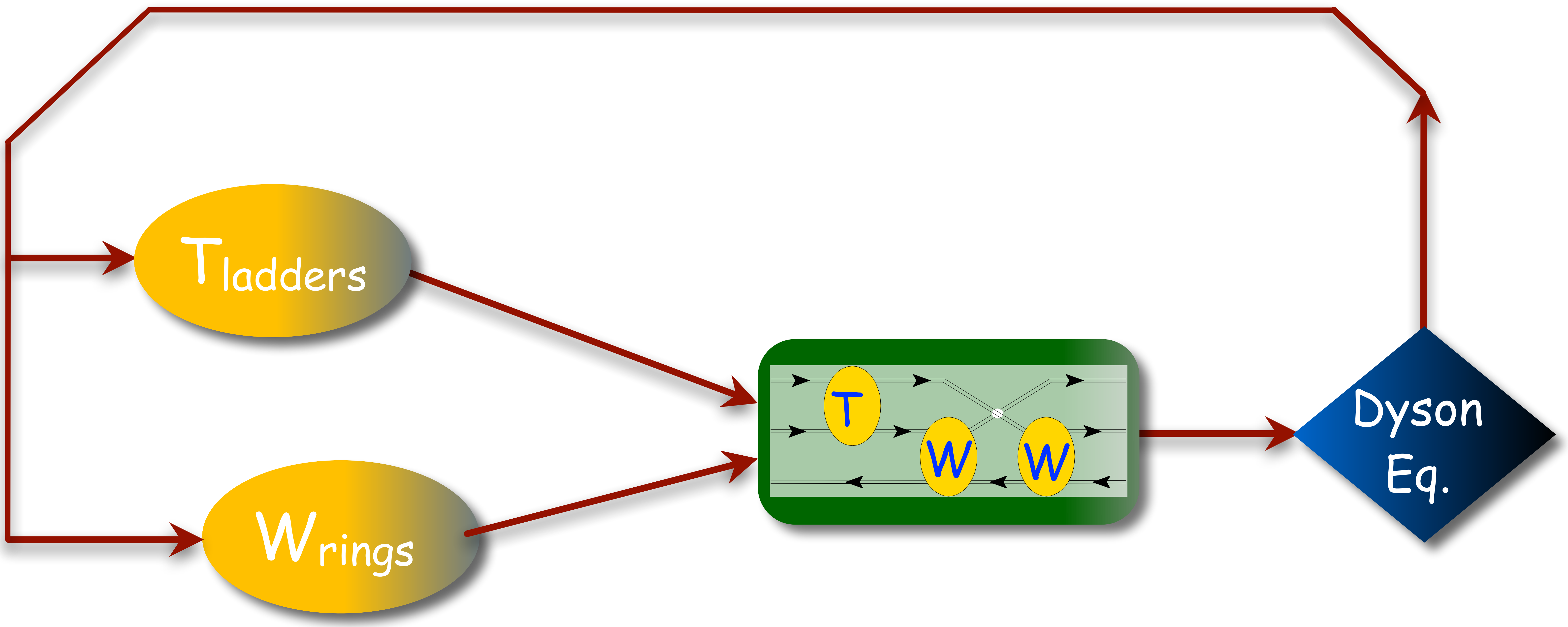
All Ladders (GT) and ring modes (GW) are coupled to all orders. Two approaches:

- Faddev-RPA allows for RPA modes
- ADC(3) Tamn-Dancoff version using 3rd order diagrams as 'seeds':



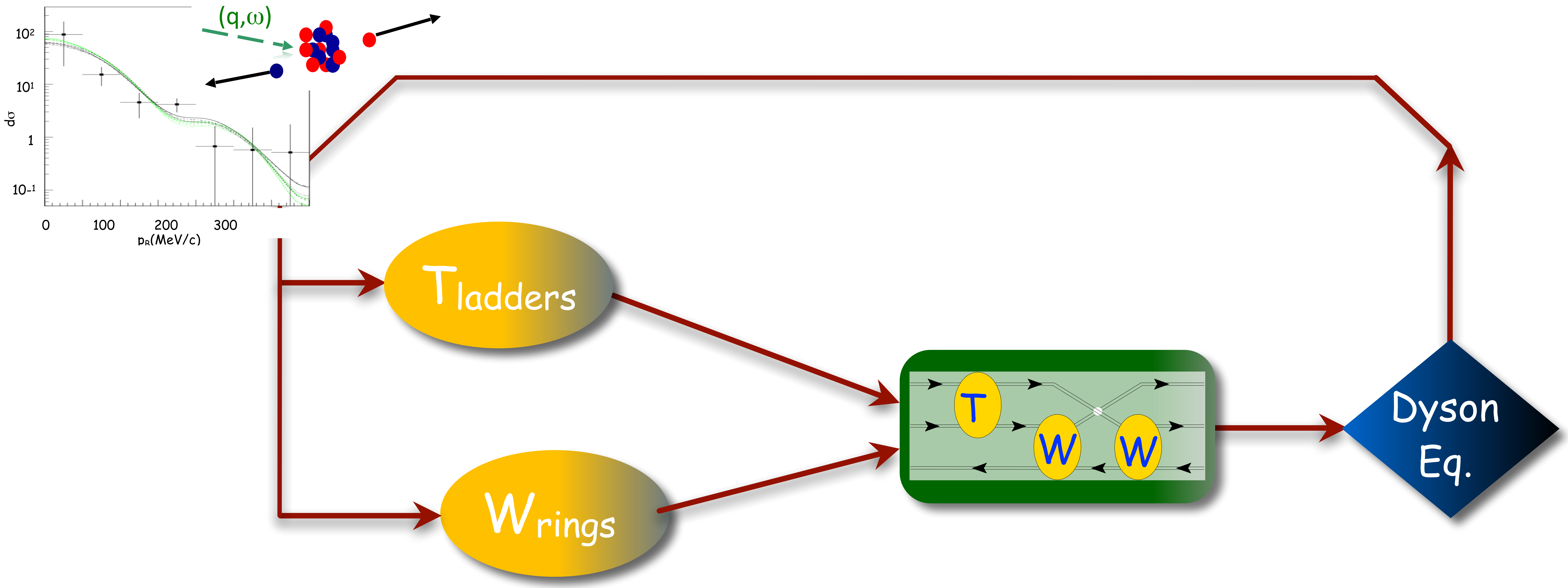


The Self-Consistent Green's Function with Faddev-RPA



The Self-Consistent Green's Function with Faddev-RPA

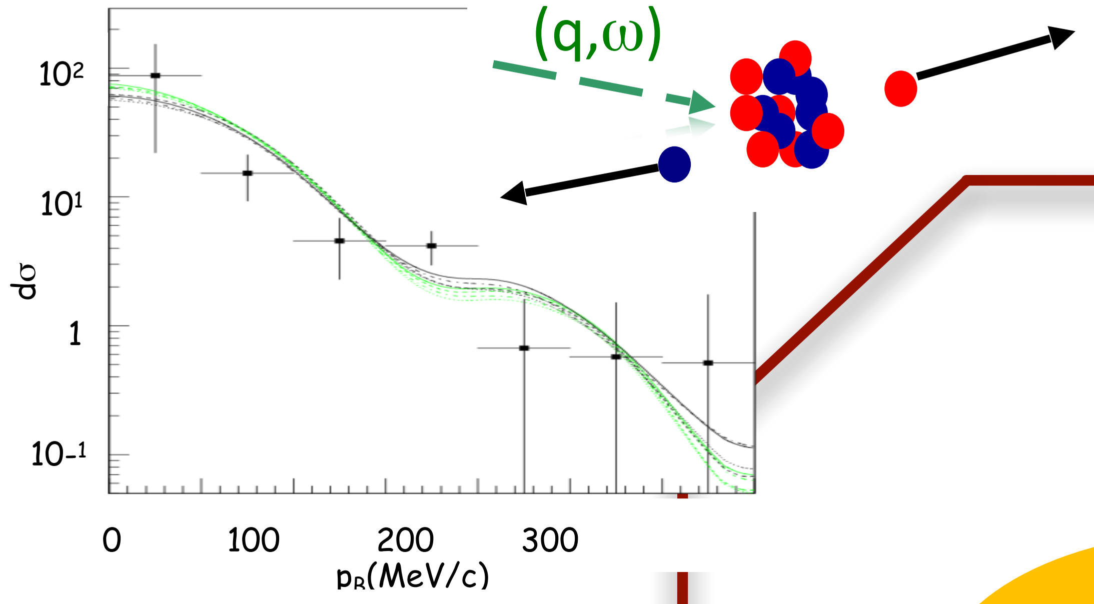
Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$
[Eur. Phys. J. A**43**, 137 (2010)]

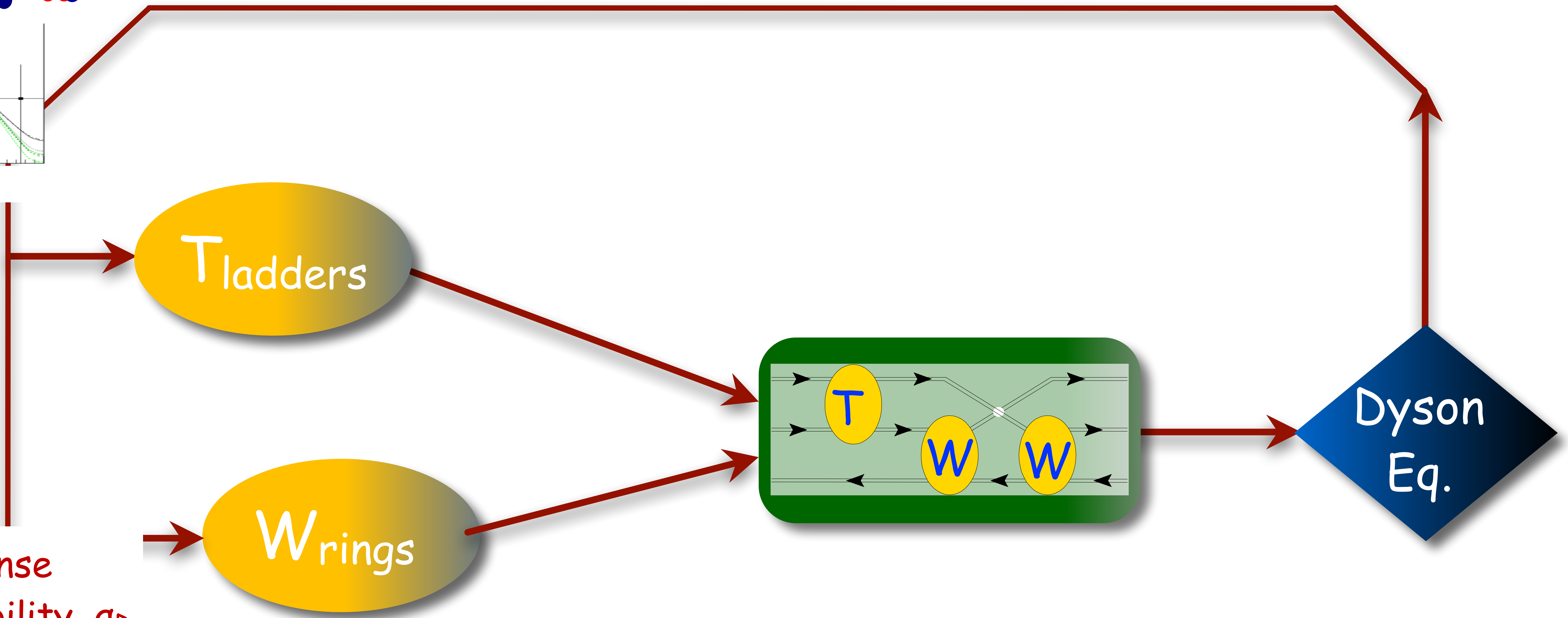


The Self-Consistent Green's Function with Faddev-RPA

Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$

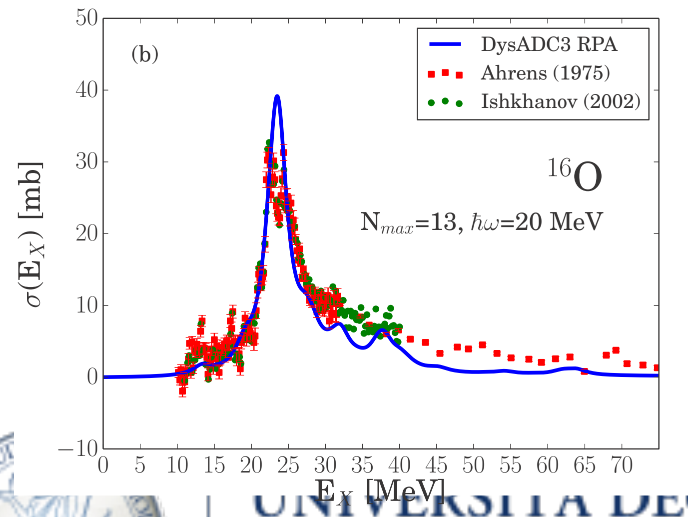
[Eur. Phys. J. A**43**, 137 (2010)]





Nuclear ELM response
and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]



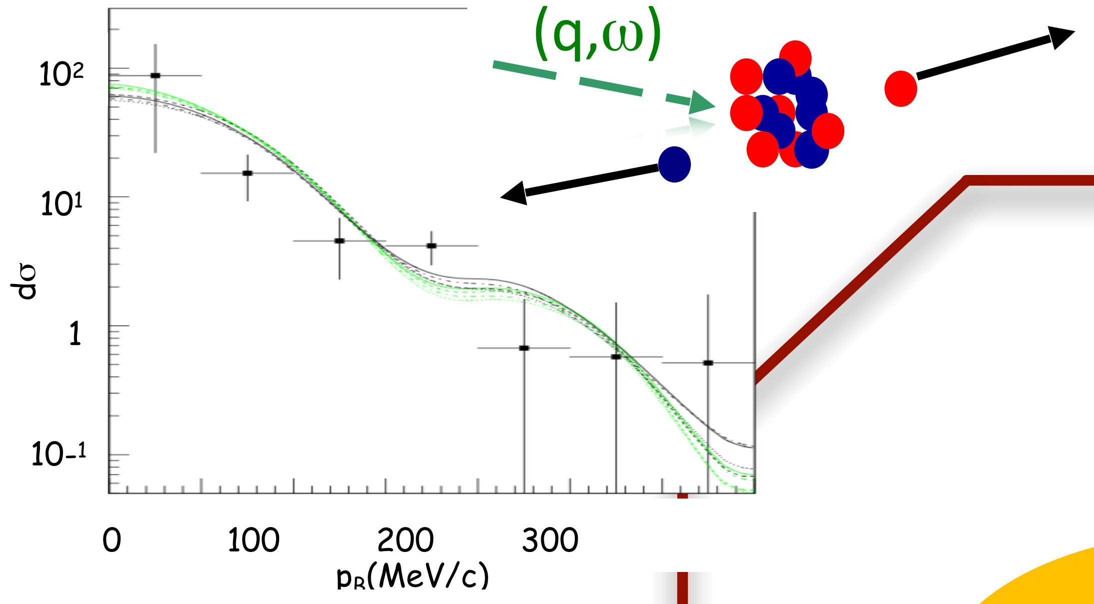
^{68}Ni:

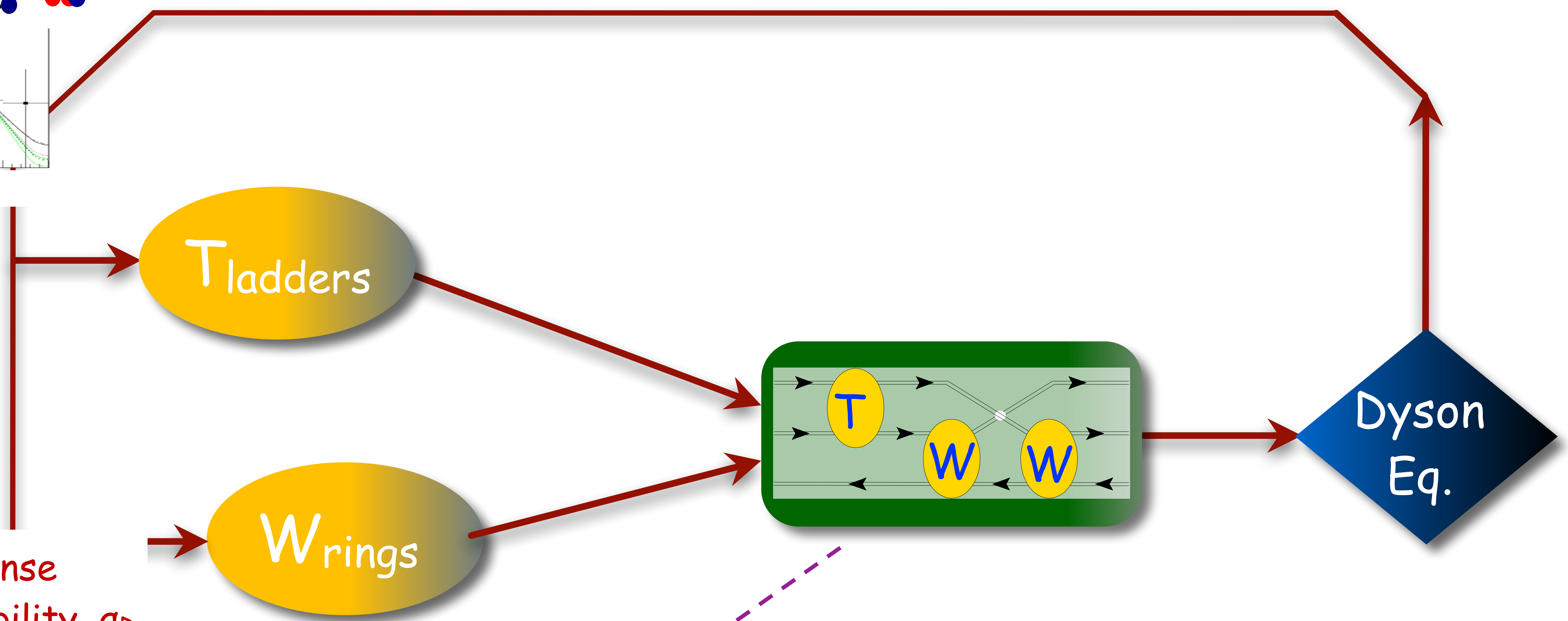
	SCGF	Exp
E_{PDR} (MeV)	10.68 10.92	9.55(17)
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm^3)	3.60	3.40(23) 3.88(31)

The Self-Consistent Green's Function with Faddev-RPA

Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$

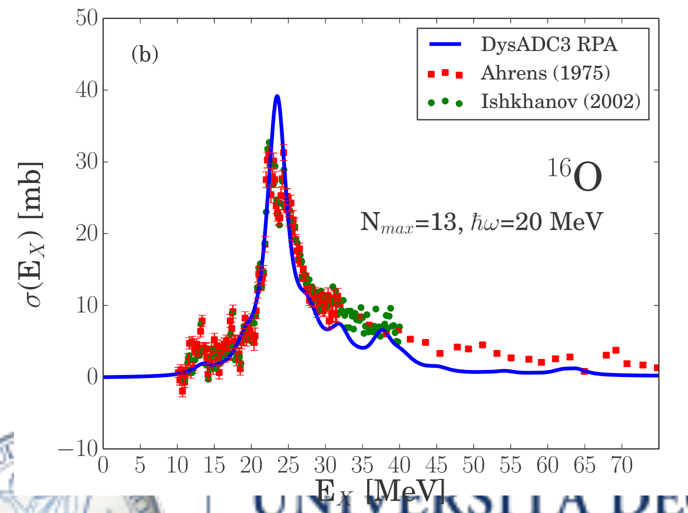
[Eur. Phys. J. A**43**, 137 (2010)]





Nuclear ELM response and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]

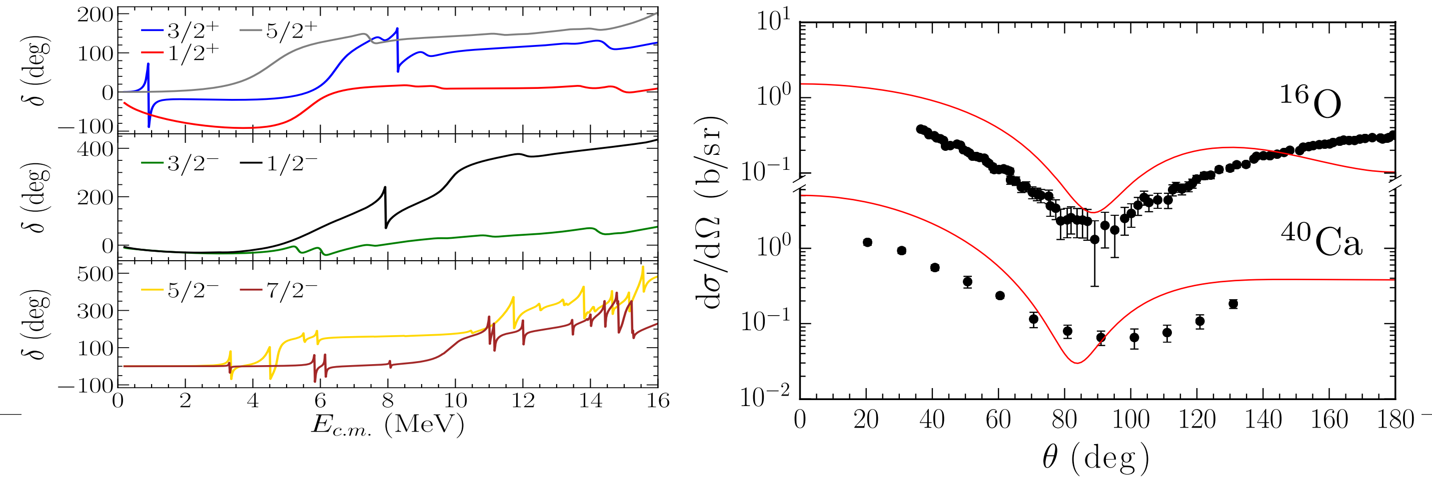


^{68}Ni:

	SCGF	Exp
E_{PDR} (MeV)	10.68 10.92	9.55(17)
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm^3)	3.60	3.40(23) 3.88(31)

Optical potential

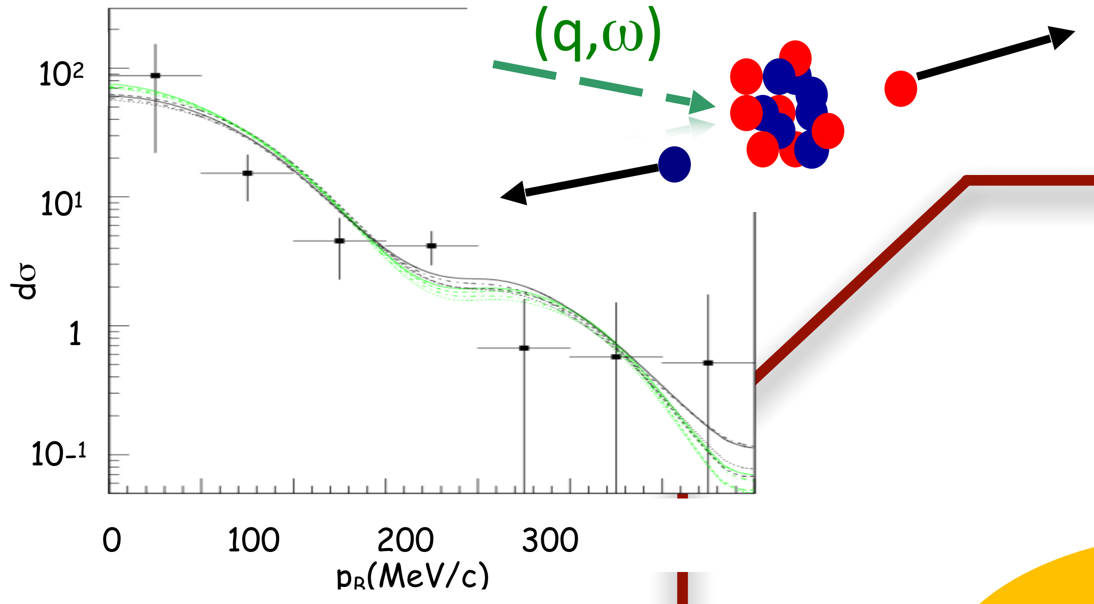
Elastic neutron scattering [Phys Rev. Lett. **123**, 092501 (2013)]

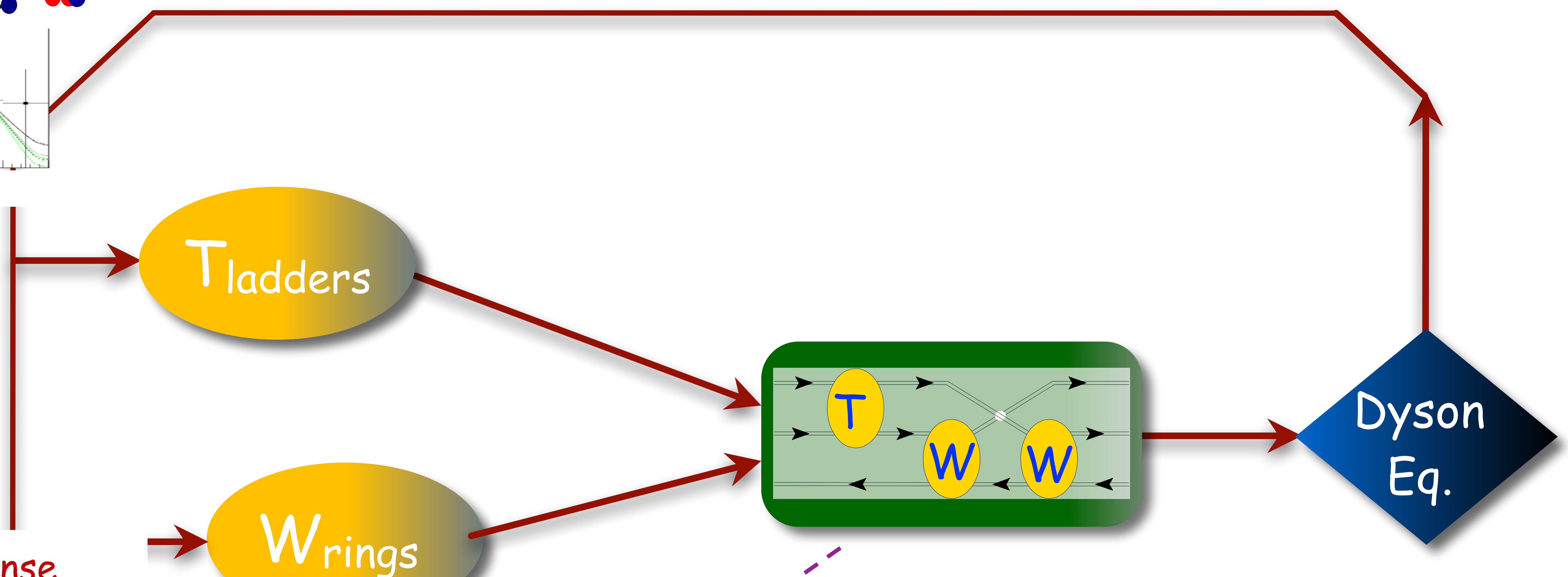


The Self-Consistent Green's Function with Faddev-RPA

Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$

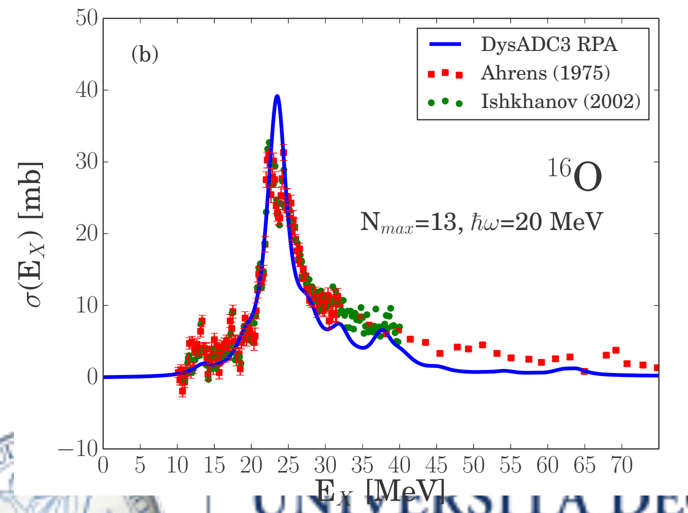
[Eur. Phys. J. A**43**, 137 (2010)]





Nuclear ELM response and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]

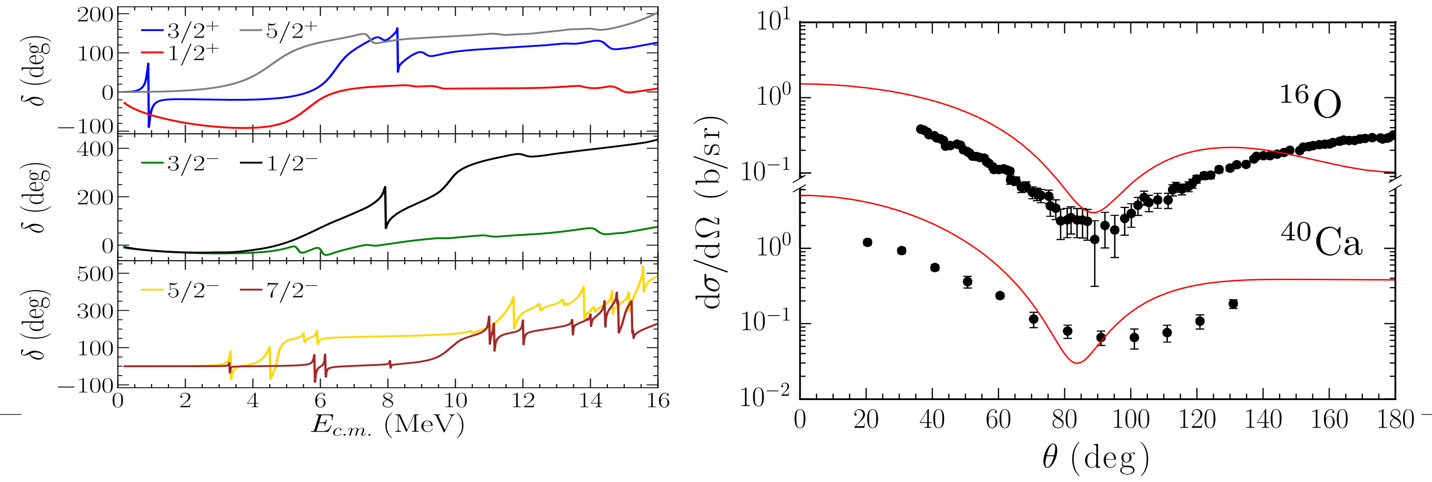


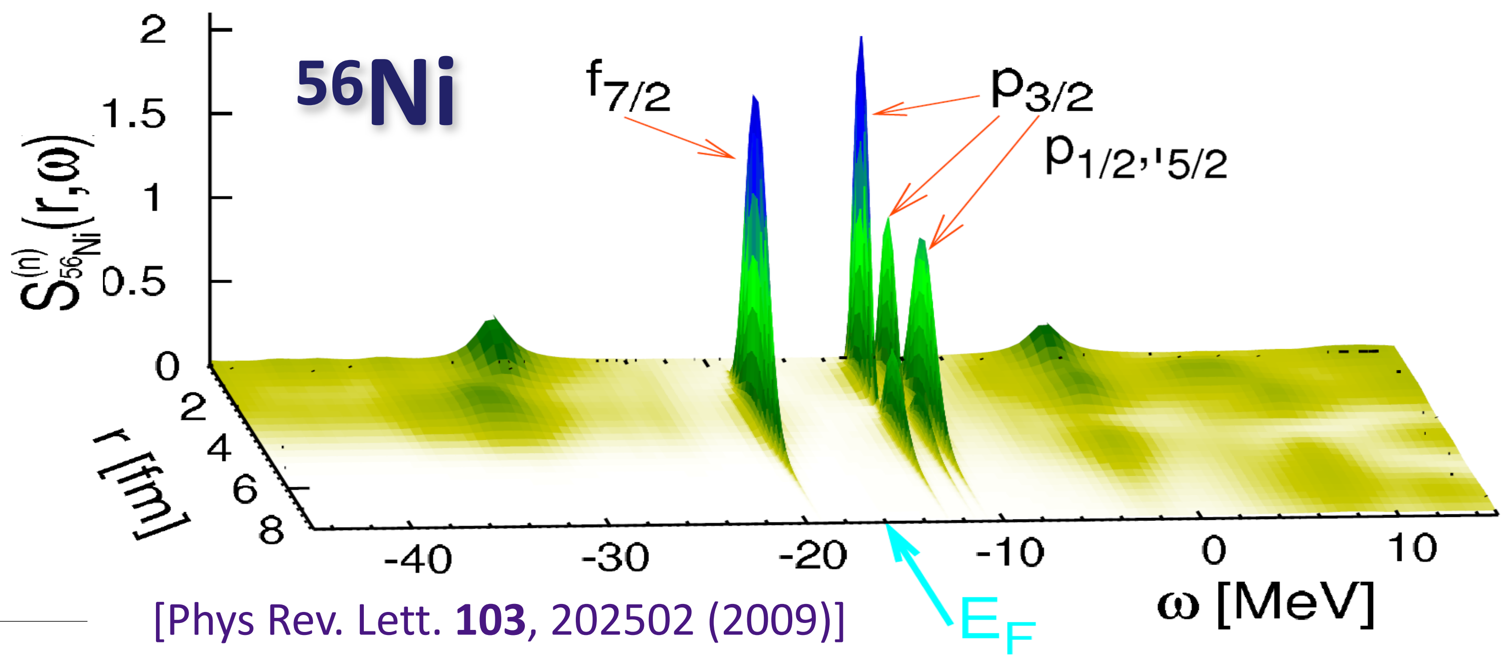
^{68}Ni:

	SCGF	Exp
E_{PDR} (MeV)	10.68 10.92	9.55(17)
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm^3)	3.60	3.40(23) 3.88(31)

Optical potential

Elastic neutron scattering [Phys Rev. Lett. **123**, 092501 (2013)]



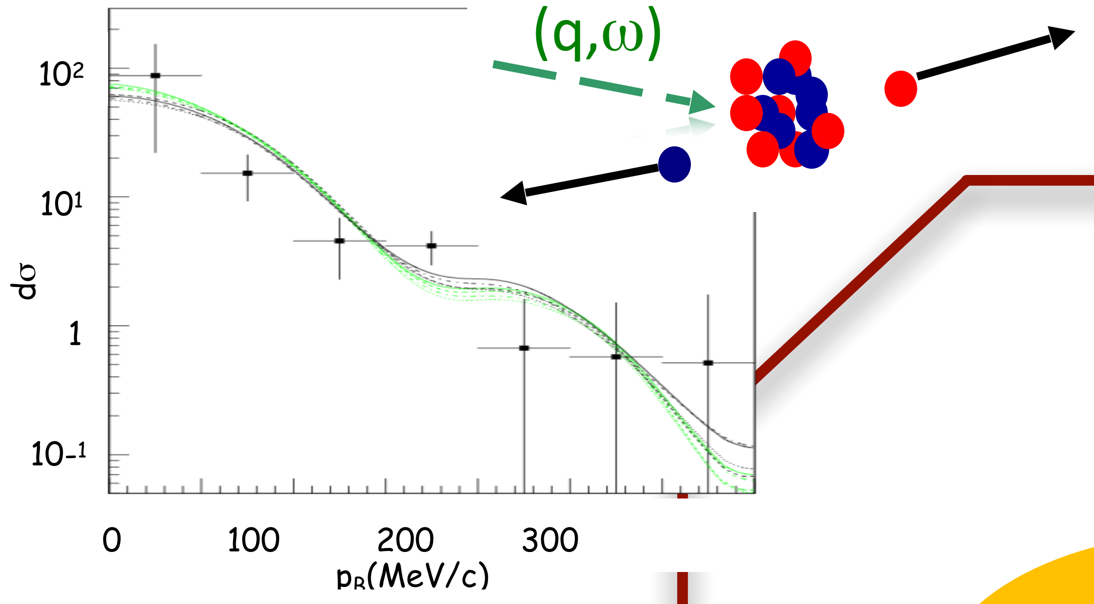


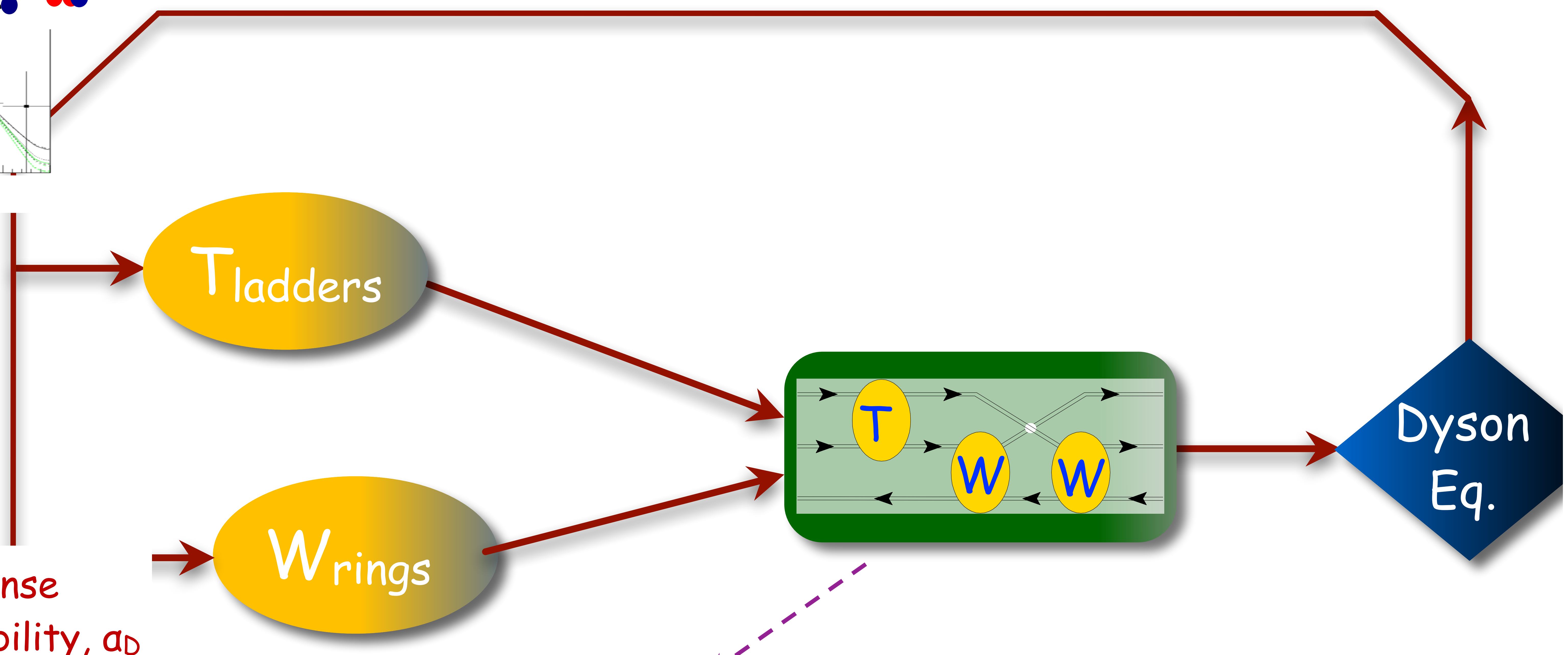
[Phys Rev. Lett. **103**, 202502 (2009)]

The Self-Consistent Green's Function with Faddev-RPA

Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$

[Eur. Phys. J. A**43**, 137 (2010)]





Spectroscopy

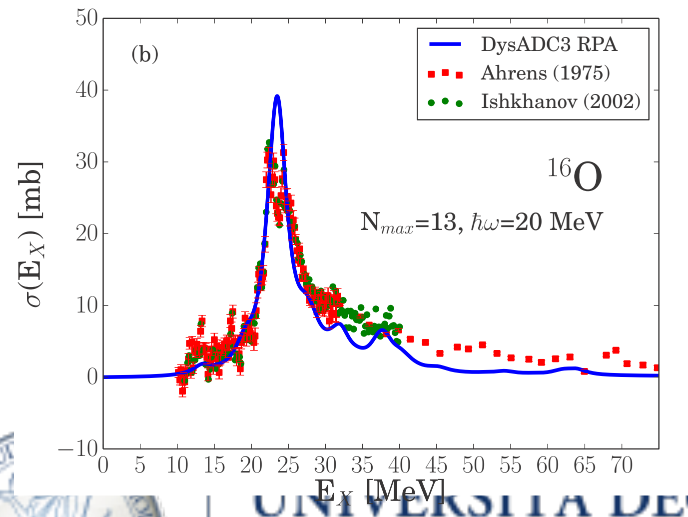
Ionisation energies and affinities for simple atoms and molecules

[Phys Rev. A. **83**, 042517 (2011); **85**, 012501 (2012)]

	Level	ADC(3)	FRPA	FRPA(c)	Expt.
HF					
	1π	16.48	16.05	16.35	16.05
	3σ	20.36	20.03	20.24	20.0
CO					
	5σ	13.94	14.37	13.69	14.01
	1π	16.98	16.95	16.84	16.91
	4σ	20.19	19.46	19.59	19.72
H_2O					
	$1b_1$	12.86	12.62	12.67	12.62
	$3a_1$	15.15	14.91	14.98	14.74
	$1b_2$	19.21	19.06	19.13	18.51
	$\bar{\Delta}$ (eV)	0.30(0.30)	0.25(0.23)	0.31(0.26)	
	Δ_{max} (eV)	0.70(0.70)	0.73(0.73)	0.88(0.62)	

Nuclear ELM response and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]

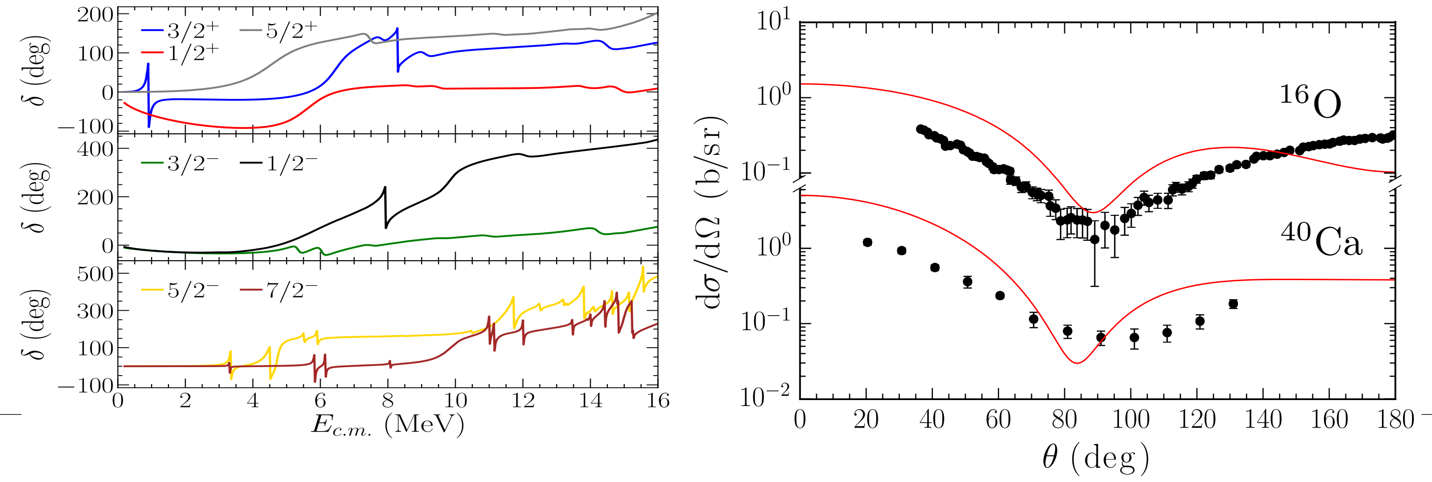


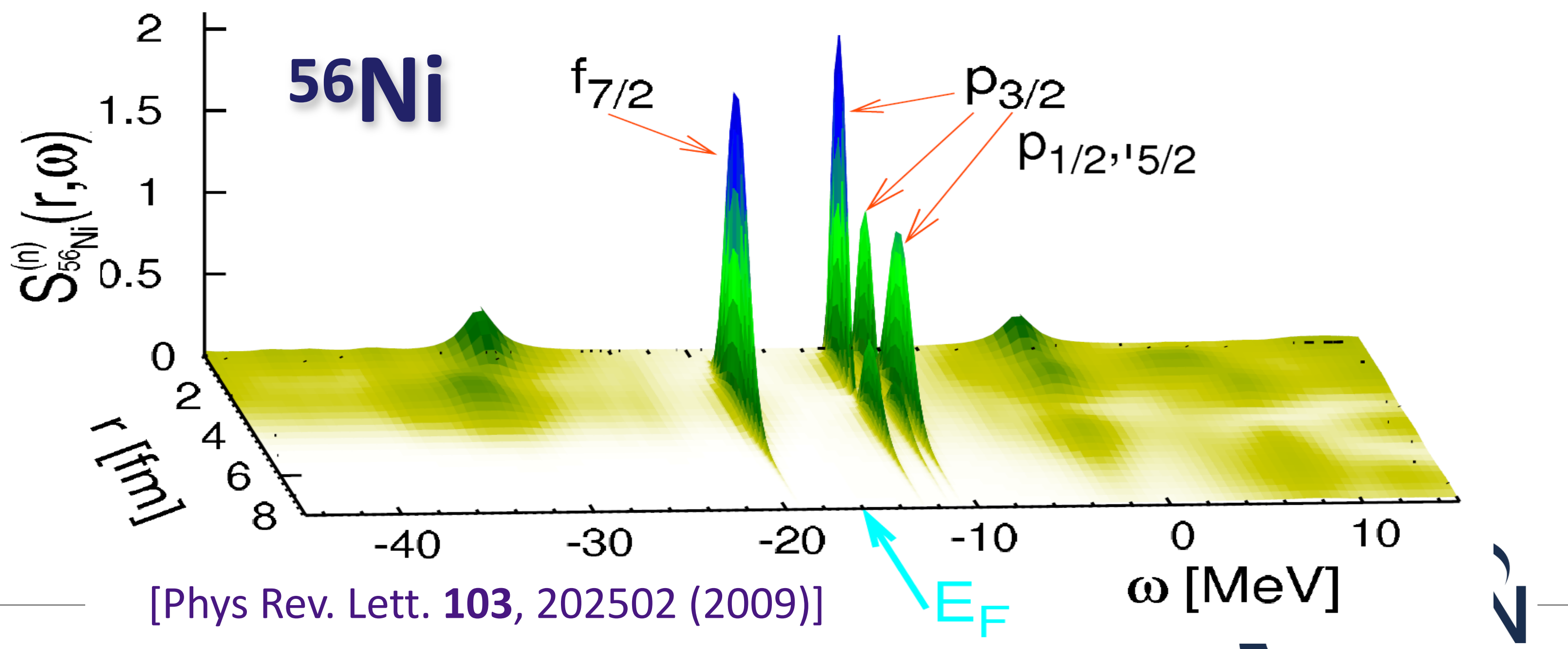
^{68}Ni:

	SCGF	Exp
E_{PDR} (MeV)	10.68 10.92	9.55(17)
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm^3)	3.60	3.40(23) 3.88(31)

Optical potential

Elastic neutron scattering [Phys Rev. Lett. **123**, 092501 (2013)]



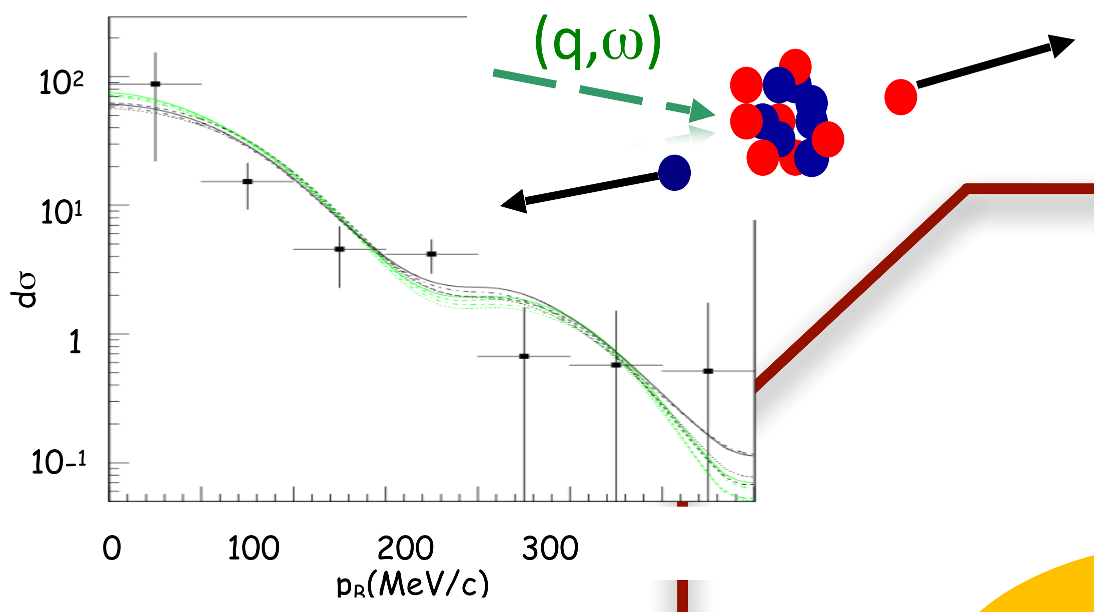


[Phys Rev. Lett. **103**, 202502 (2009)]

The Self-Consistent Green's Function with Faddev-RPA

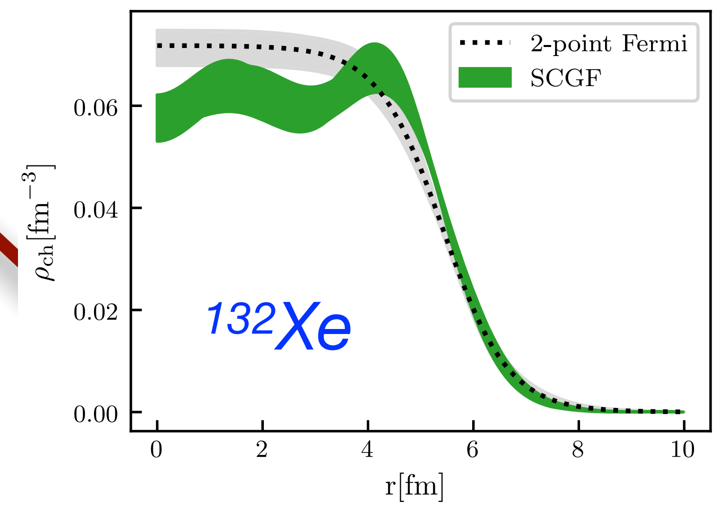
Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$

[Eur. Phys. J. A**43**, 137 (2010)]



Charge & matter distribution

Neutron skins [Phys Rev. Lett. **125**, 182501 (2020)]



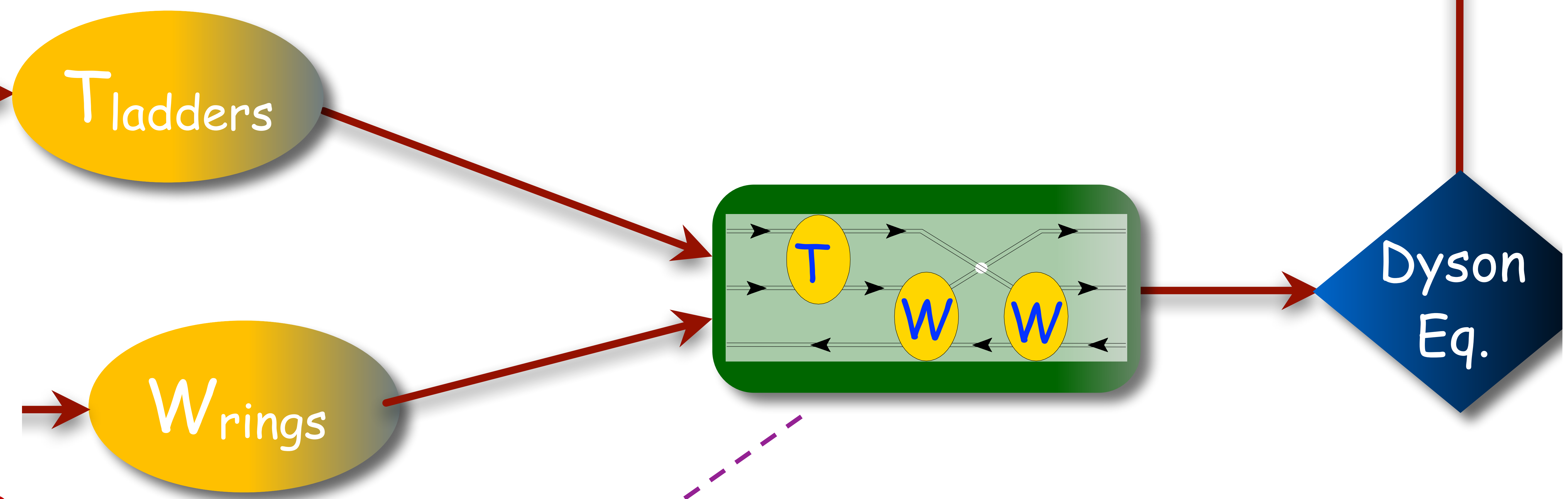
	SCGF	Exp.
^{100}Sn	4.525 – 4.707	
^{132}Sn	4.725 – 4.956	4.7093
^{132}Xe	4.700 – 4.948	4.7859
^{136}Xe	4.715 – 4.928	4.7964
^{138}Xe	4.724 – 4.941	4.8279

Spectroscopy

Ionisation energies and affinities for simple atoms and molecules

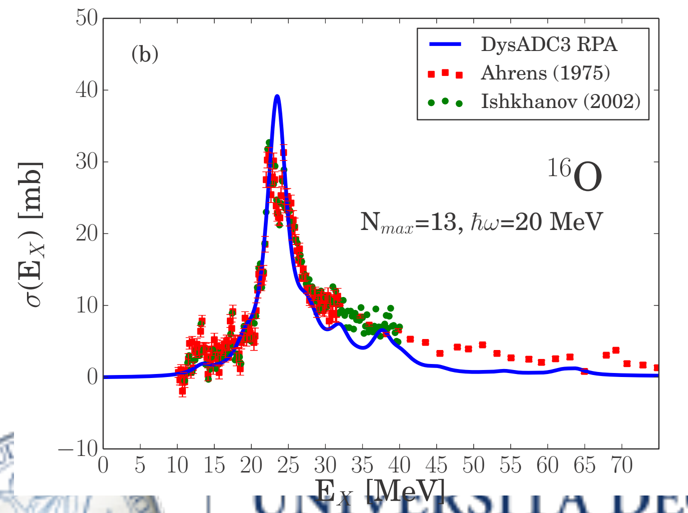
[Phys Rev. A. **83**, 042517 (2011); **85**, 012501 (2012)]

	Level	ADC(3)	FRPA	FRPA(c)	Expt.
HF					
	1π	16.48	16.05	16.35	16.05
	3σ	20.36	20.03	20.24	20.0
CO					
	5σ	13.94	14.37	13.69	14.01
	1π	16.98	16.95	16.84	16.91
	4σ	20.19	19.46	19.59	19.72
H_2O					
	$1b_1$	12.86	12.62	12.67	12.62
	$3a_1$	15.15	14.91	14.98	14.74
	$1b_2$	19.21	19.06	19.13	18.51
	$\bar{\Delta}$ (eV)	0.30(0.30)	0.25(0.23)	0.31(0.26)	
	Δ_{max} (eV)	0.70(0.70)	0.73(0.73)	0.88(0.62)	



Nuclear ELM response and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]

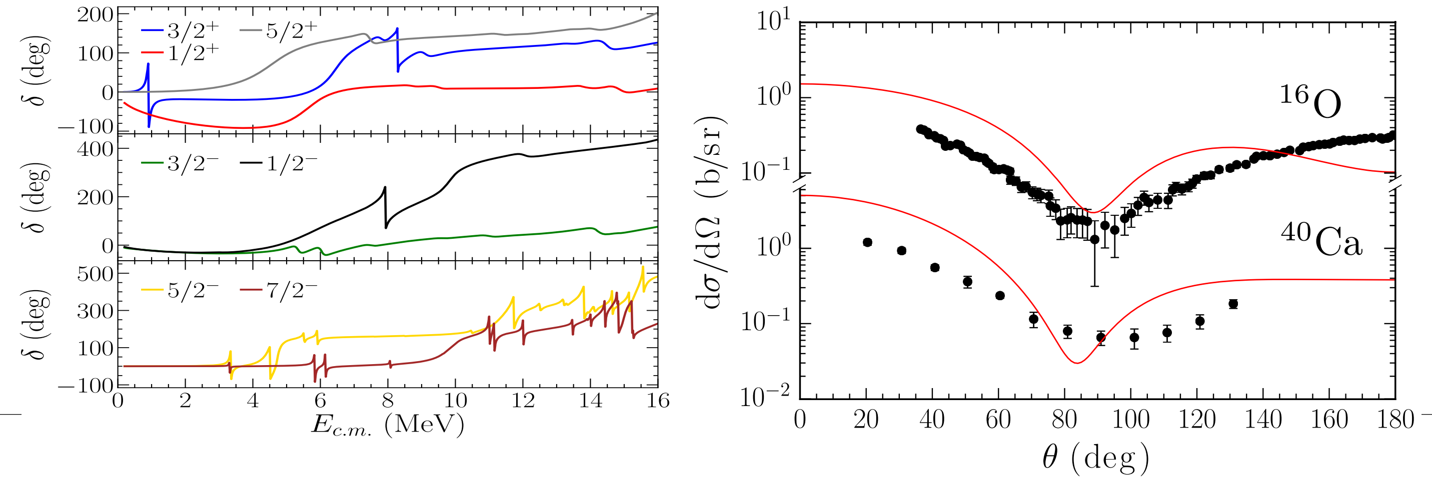


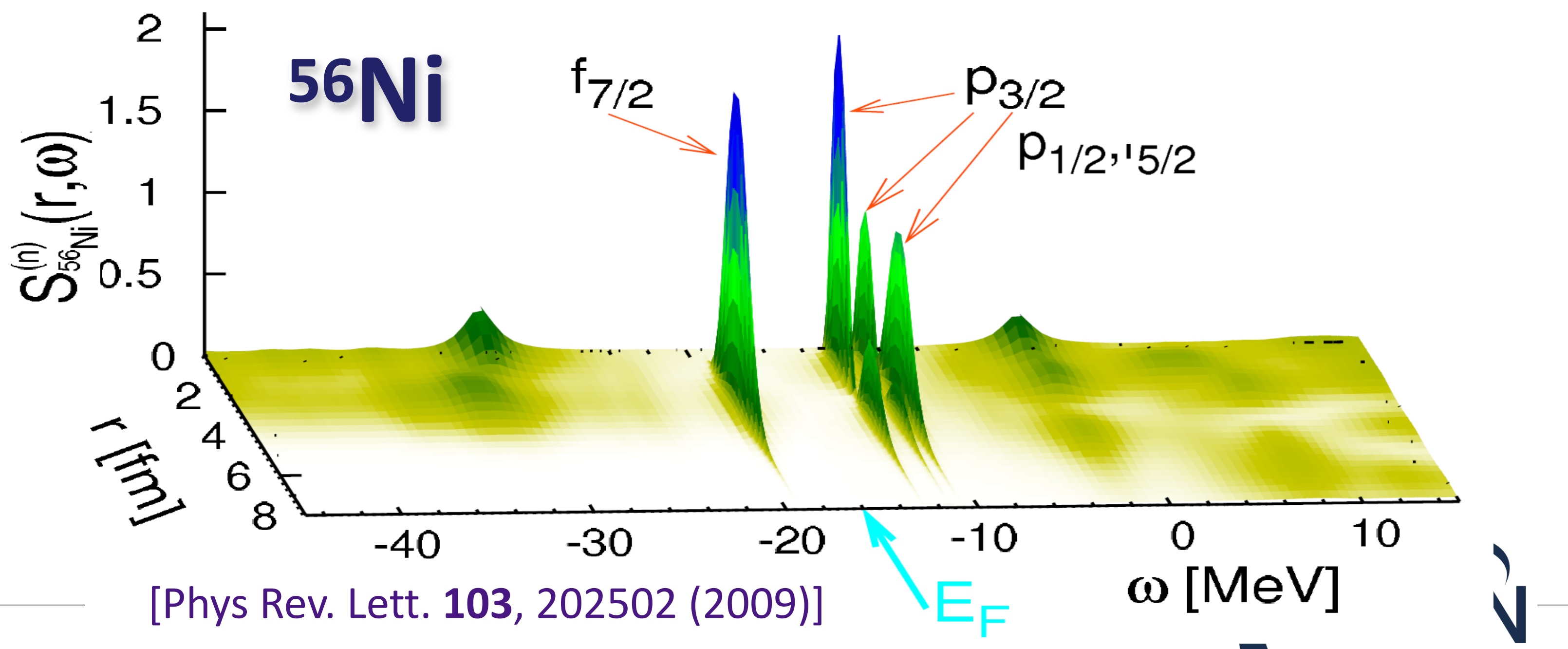
^{68}Ni:

	SCGF	Exp
E_{PDR} (MeV)	10.68 10.92	9.55(17)
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm^3)	3.60	3.40(23) 3.88(31)

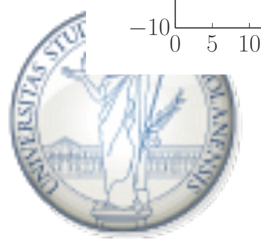
Optical potential

Elastic neutron scattering [Phys Rev. Lett. **123**, 092501 (2013)]





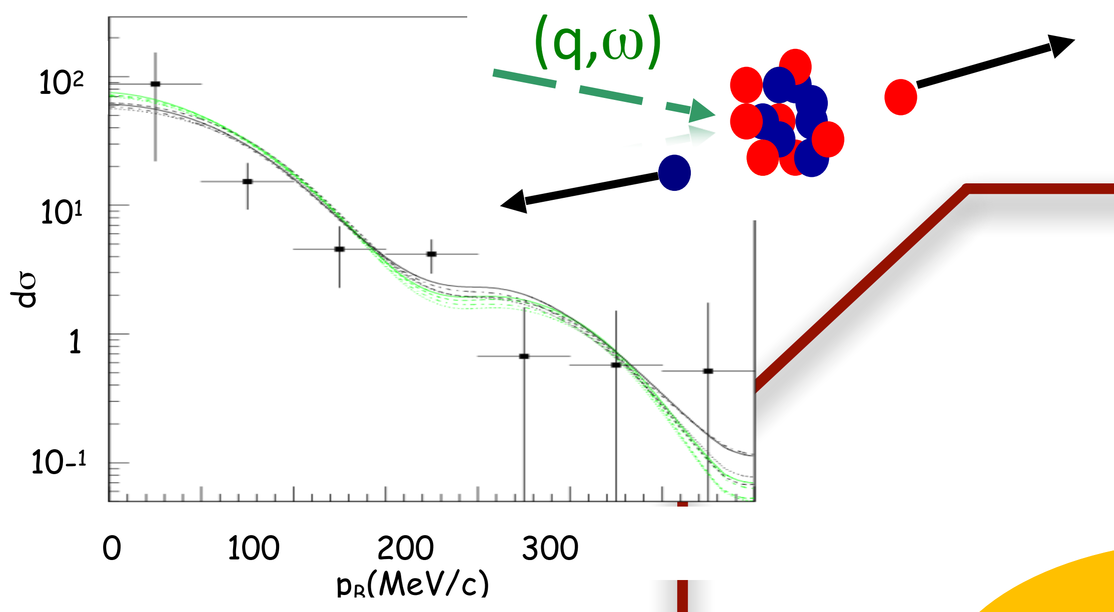
[Phys Rev. Lett. **103**, 202502 (2009)]



The Self-Consistent Green's Function with Faddev-RPA

Two-nucleon emission: $^{16}O(e,e'pn)^{14}N$

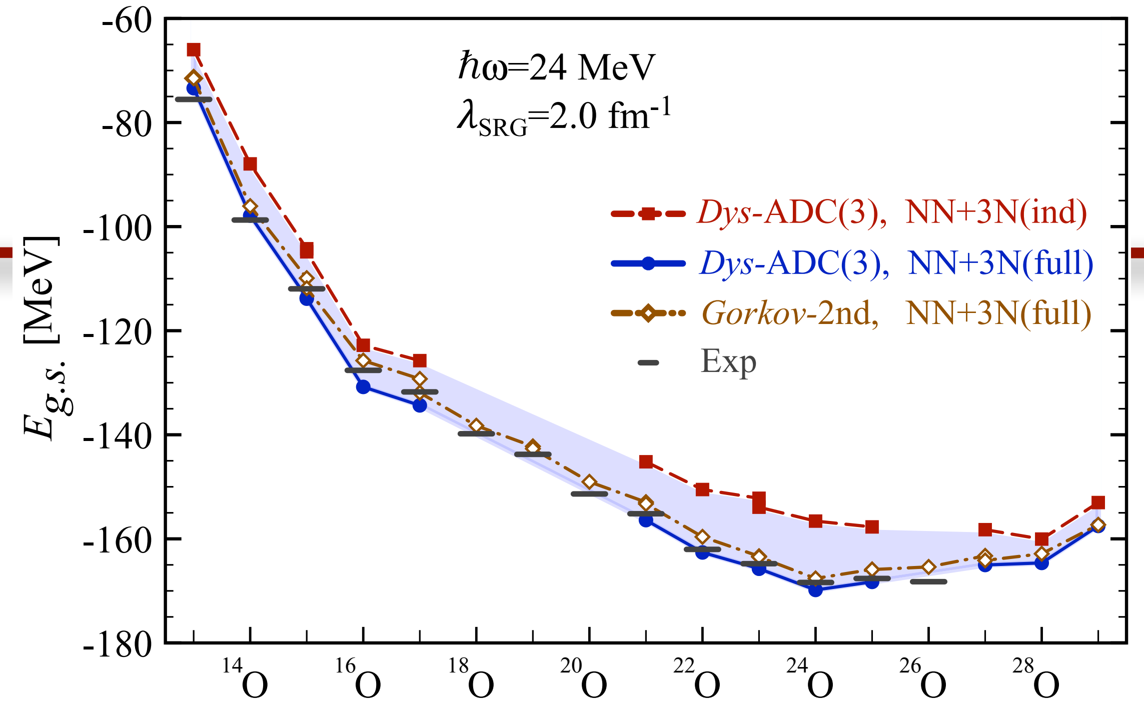
[Eur. Phys. J. A**43**, 137 (2010)]



Binding energies

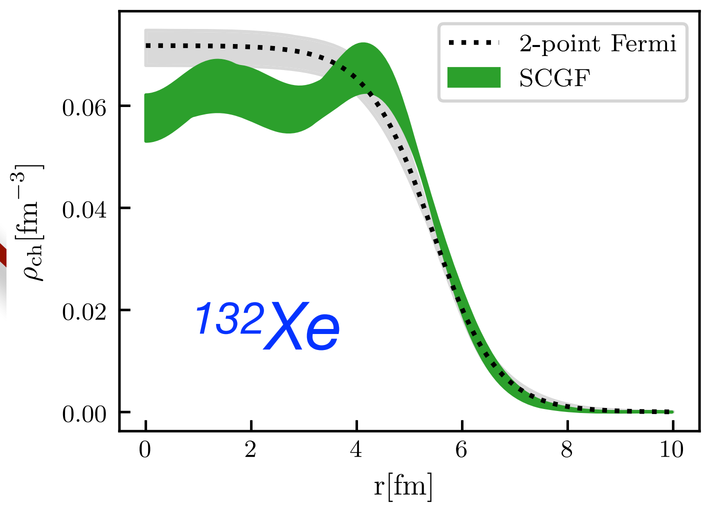
Oxygen drip line

[Phys Rev. Lett. **111**, 062501 (2013)]

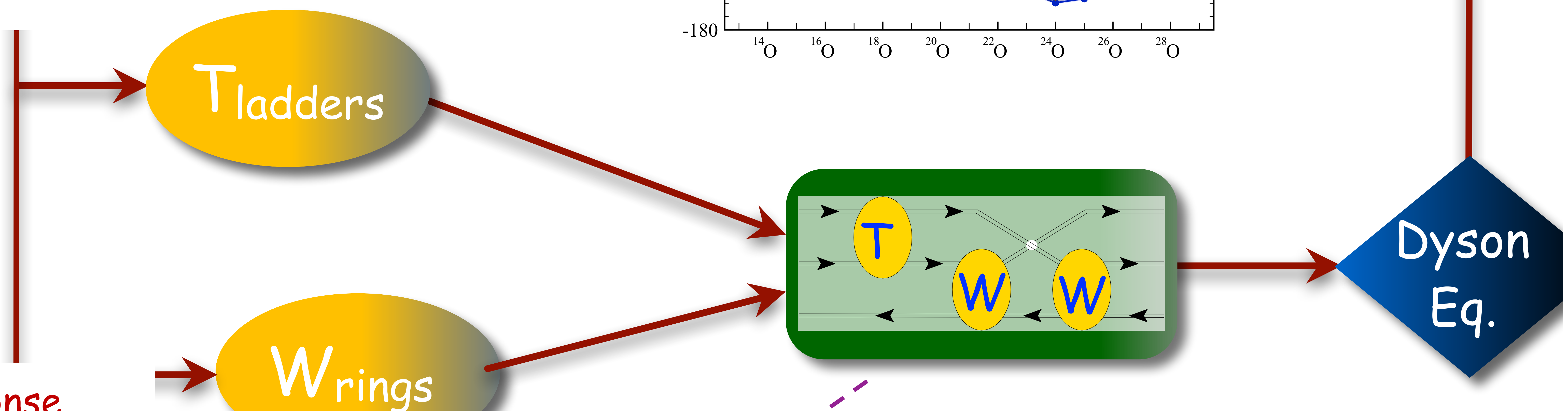


Charge & matter distribution

Neutron skins [Phys Rev. Lett. **125**, 182501 (2020)]



	SCGF	Exp.
^{100}Sn	4.525 – 4.707	
^{132}Sn	4.725 – 4.956	4.7093
^{132}Xe	4.700 – 4.948	4.7859
^{136}Xe	4.715 – 4.928	4.7964
^{138}Xe	4.724 – 4.941	4.8279



Spectroscopy

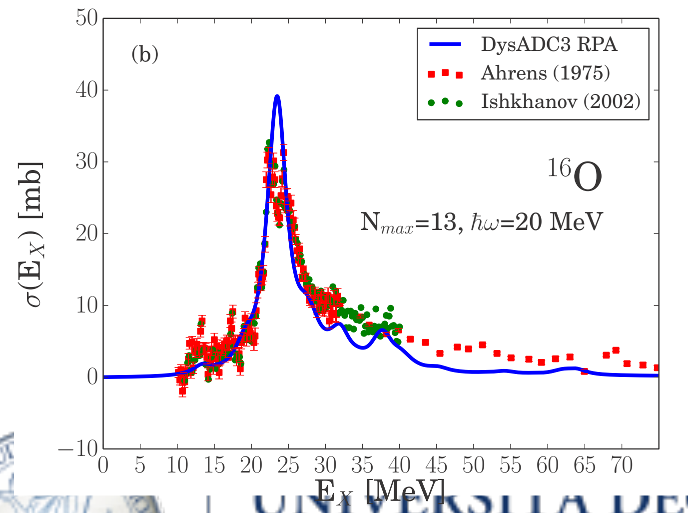
Ionisation energies and affinities for simple atoms and molecules

[Phys Rev. A. **83**, 042517 (2011); **85**, 012501 (2012)]

	Level	ADC(3)	FRPA	FRPA(c)	Expt.
HF					
	1π	16.48	16.05	16.35	16.05
	3σ	20.36	20.03	20.24	20.0
CO					
	5σ	13.94	14.37	13.69	14.01
	1π	16.98	16.95	16.84	16.91
	4σ	20.19	19.46	19.59	19.72
H_2O					
	$1b_1$	12.86	12.62	12.67	12.62
	$3a_1$	15.15	14.91	14.98	14.74
	$1b_2$	19.21	19.06	19.13	18.51
	$\bar{\Delta}$ (eV)	0.30(0.30)	0.25(0.23)	0.31(0.26)	
	Δ_{max} (eV)	0.70(0.70)	0.73(0.73)	0.88(0.62)	

Nuclear ELM response and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]

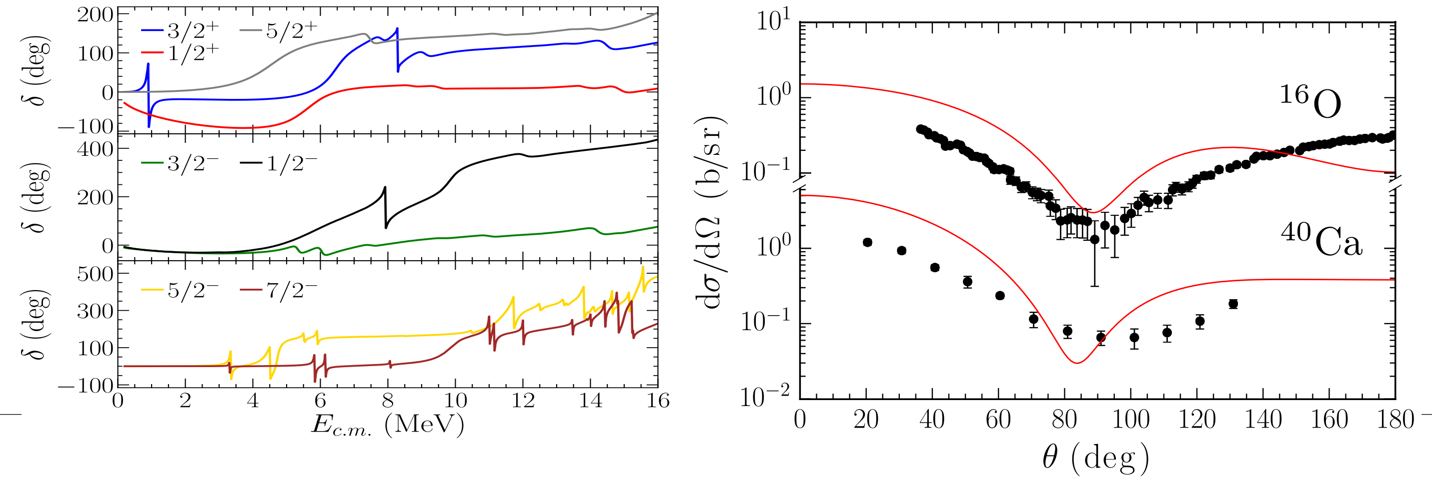


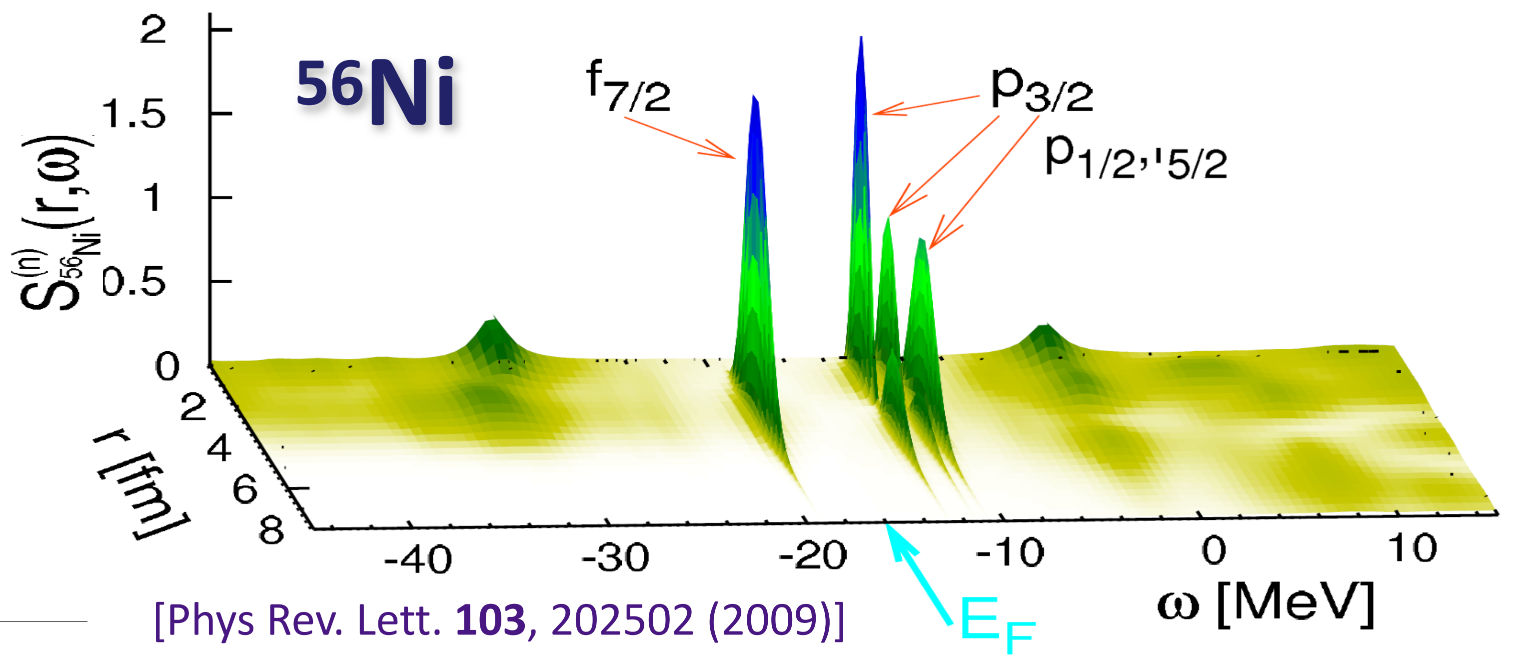
^{68}Ni:

	SCGF	Exp
E_{PDR} (MeV)	10.68 10.92	9.55(17)
E_{GDR} (MeV)	18.1	17.1(2)
α_D (fm^3)	3.60	3.40(23) 3.88(31)

Optical potential

Elastic neutron scattering [Phys Rev. Lett. **123**, 092501 (2013)]





[Phys Rev. Lett. **103**, 202502 (2009)]

The physics contained in the one-boby Green's function

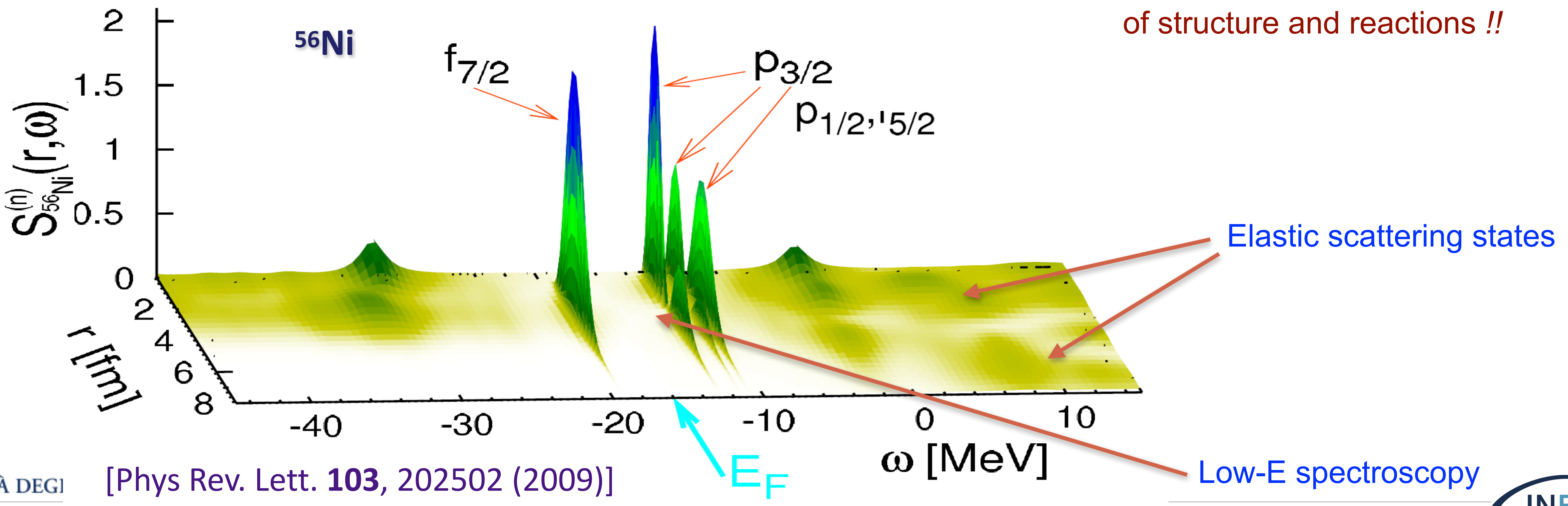
One-body propagator: $g_{\alpha\beta}(t-t') = -i/\hbar \langle \Psi_0^A | T[c_\alpha(t) c_\beta^\dagger(t')] | \Psi_0^A \rangle$

Dyson Equation: $g_{\alpha\beta}(\omega) = g^{(0)}_{\alpha\beta}(\omega) + \sum_{\gamma\delta} g^{(0)}_{\alpha\gamma}(\omega) \Sigma^\star_{\gamma\delta}(\omega) g_{\delta\beta}(\omega)$

(One-body) spectral function: $S(\alpha,\omega) := \pm \frac{1}{2\pi} \Im m\{g_{\alpha\alpha}(\omega)\}$

The self-energy $\Sigma^\star_{\alpha\beta}(\omega)$ 'decides' both ground state observables and elastic scattering.

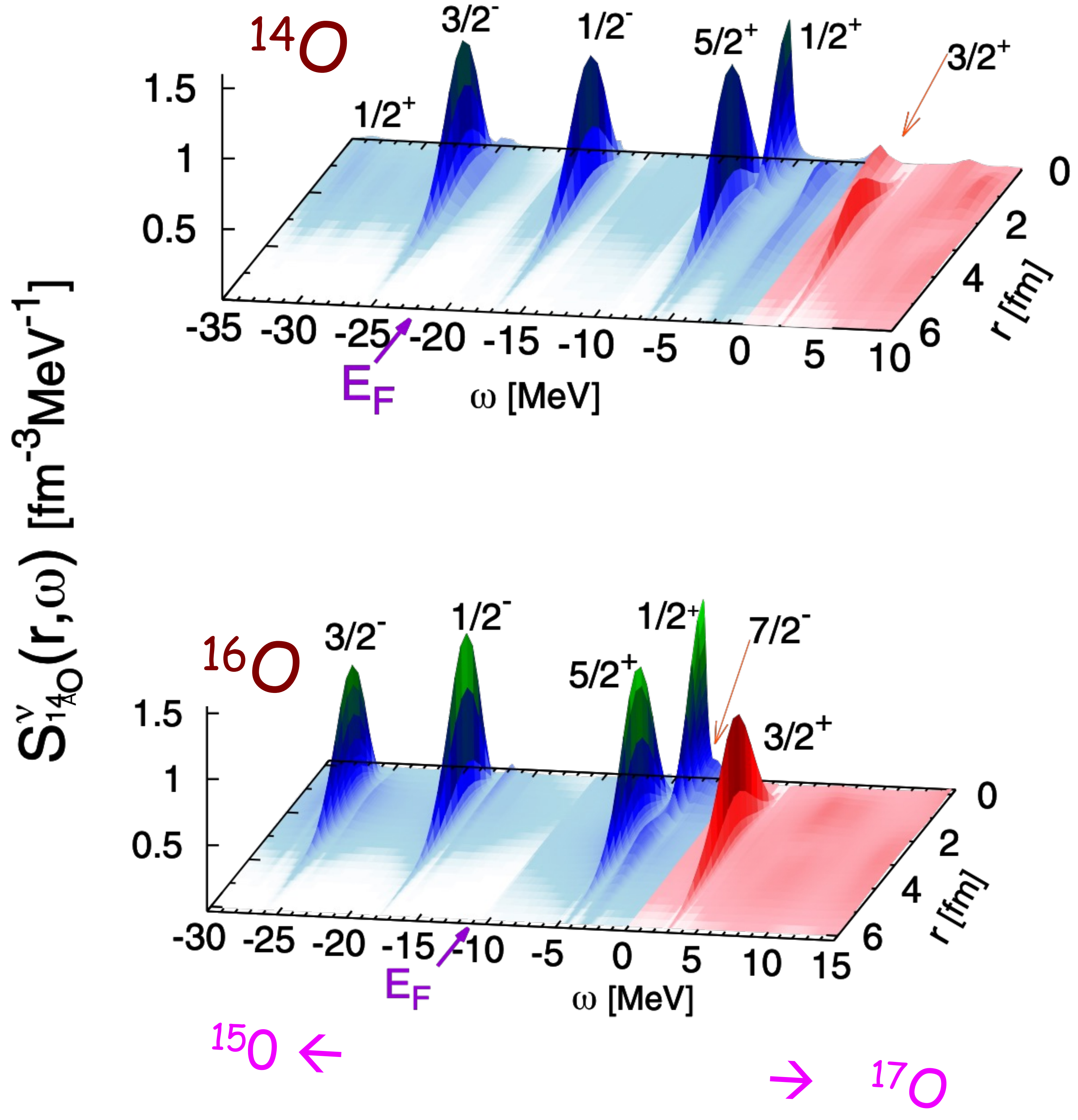
==> Consistent computation of structure and reactions !!

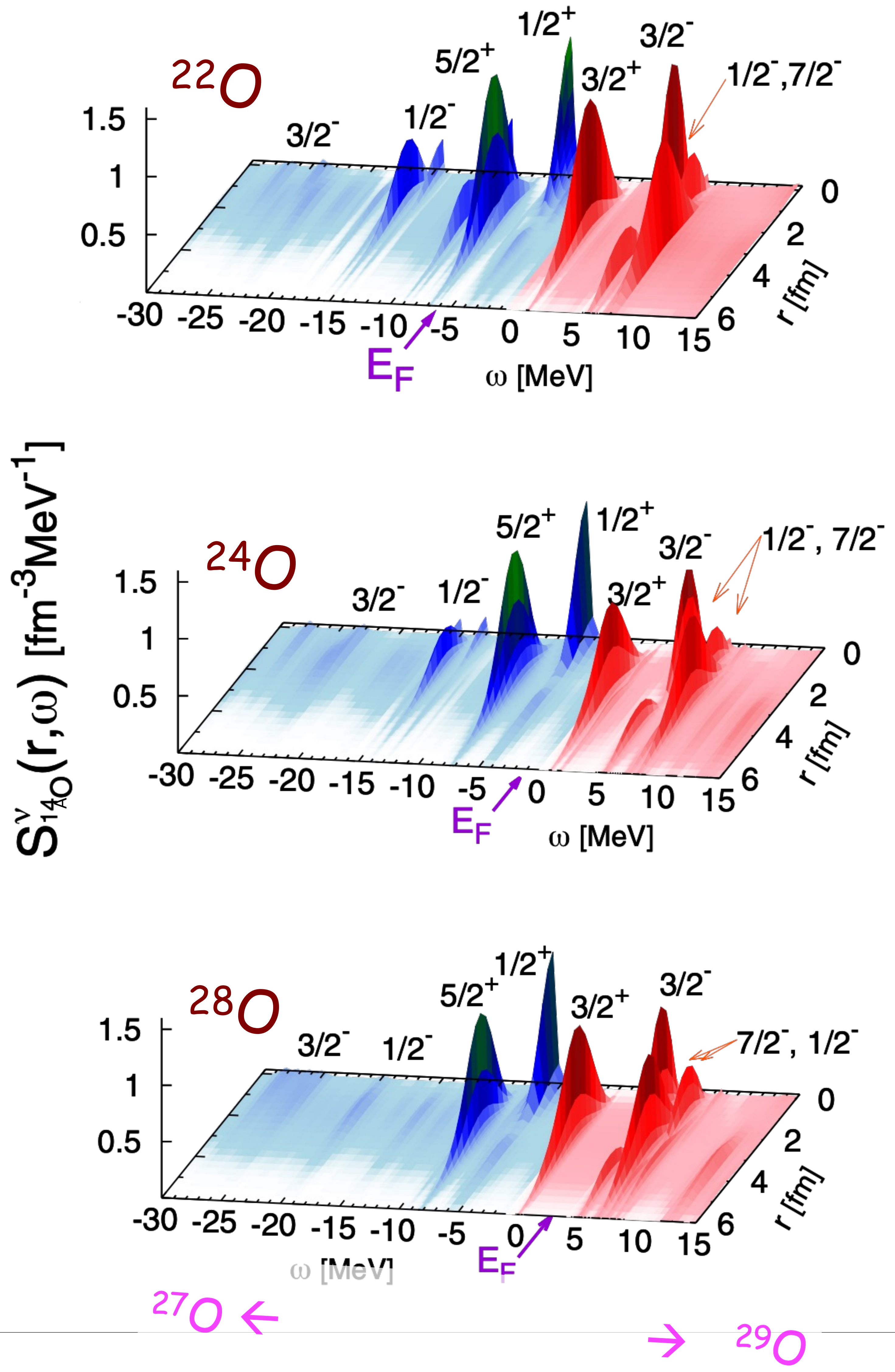


[Phys Rev. Lett. **103**, 202502 (2009)]

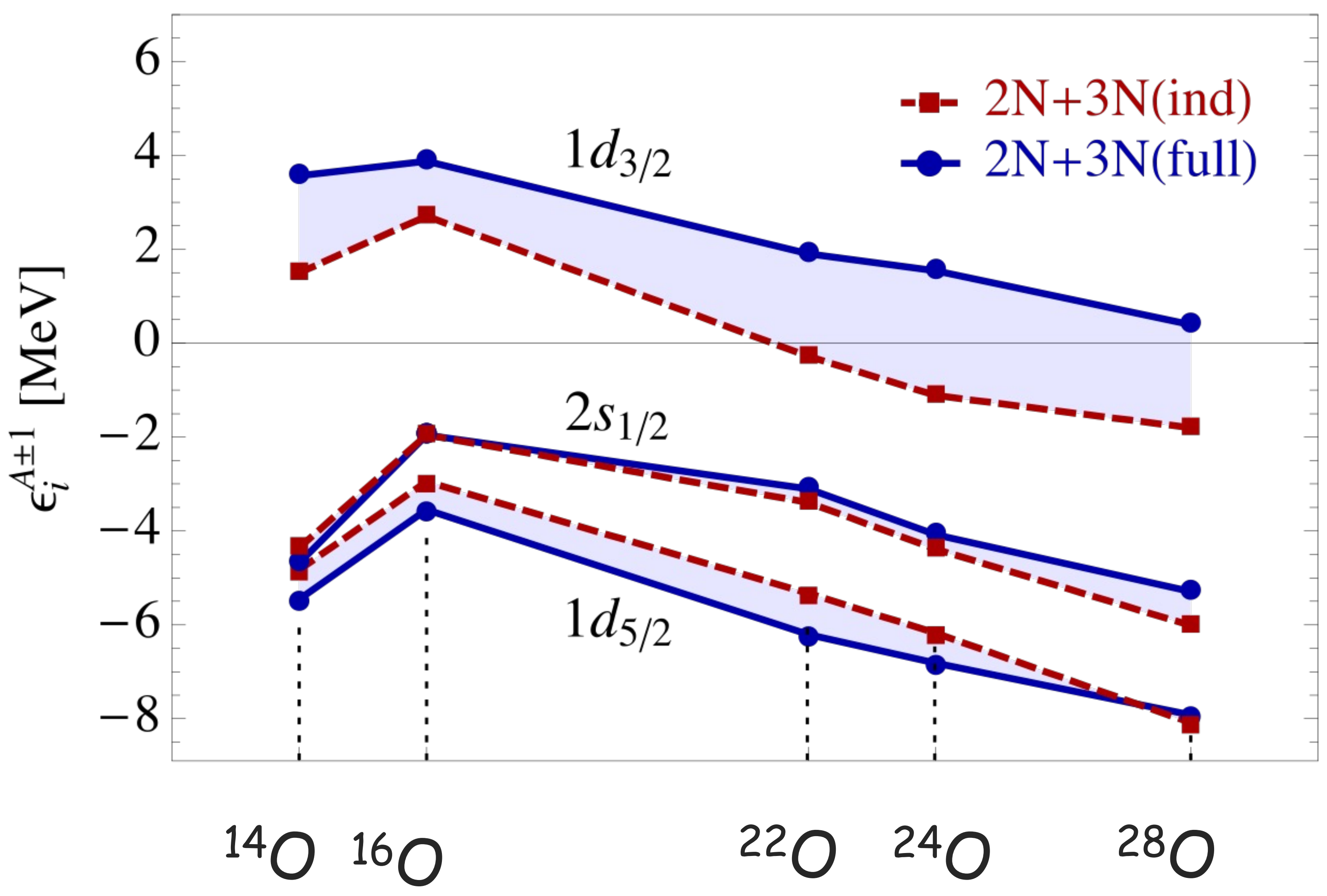
Neutron spectral function for oxygen isotopes

A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* **92**, *014306 (2015)*



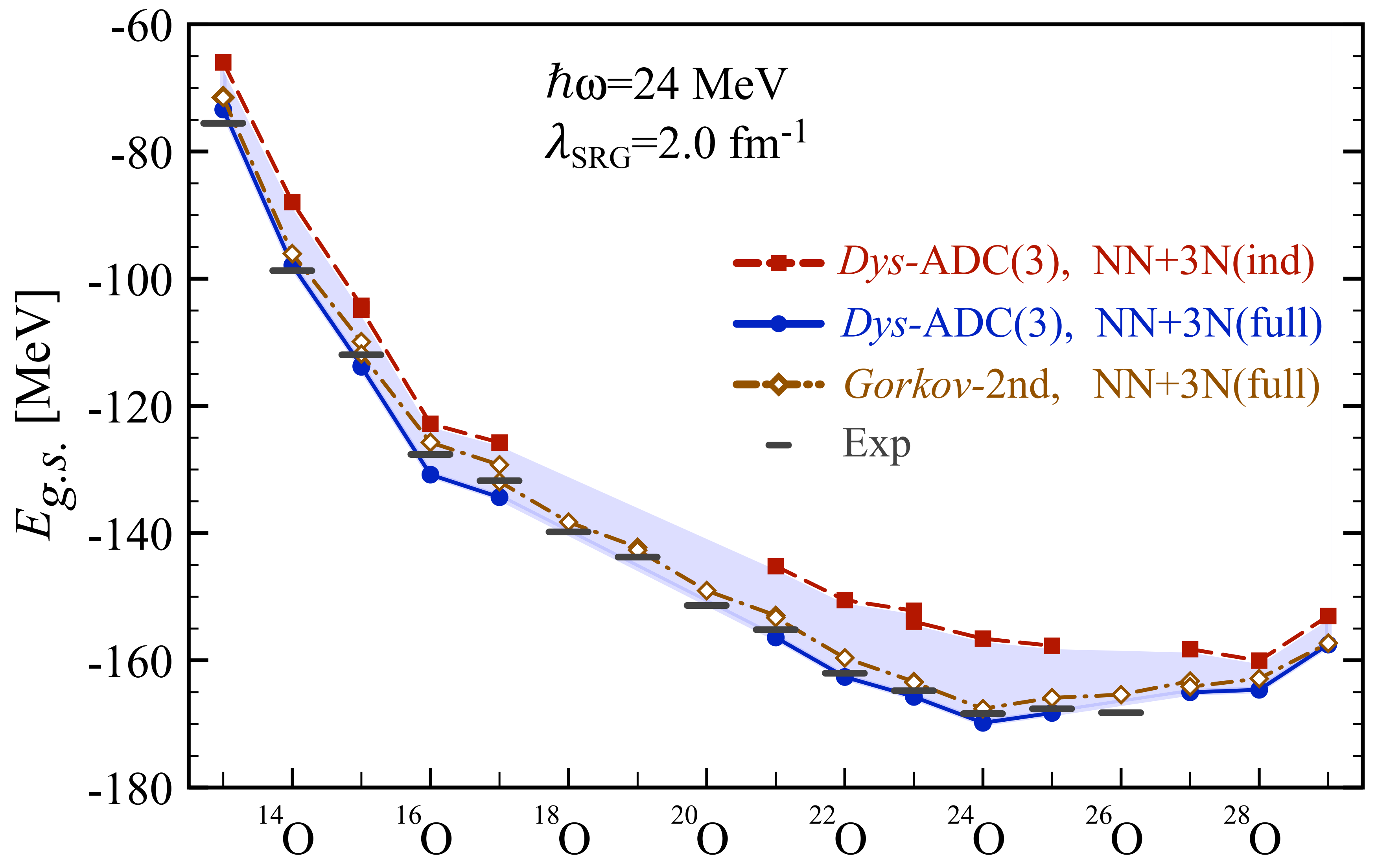


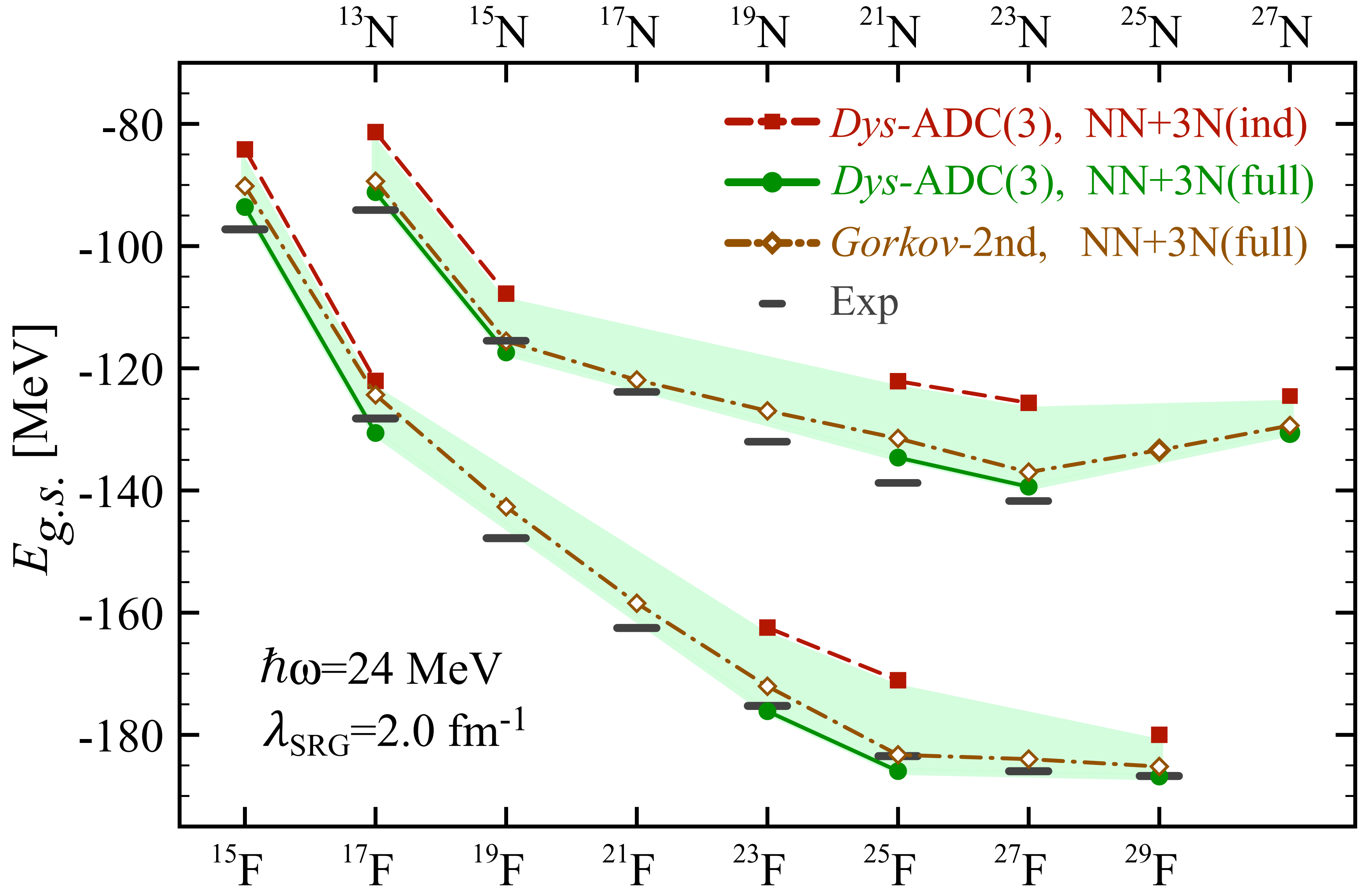
Neutron quasiparticle energies



Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)





→ 3NF crucial for reproducing binding energies and driplines around oxygen

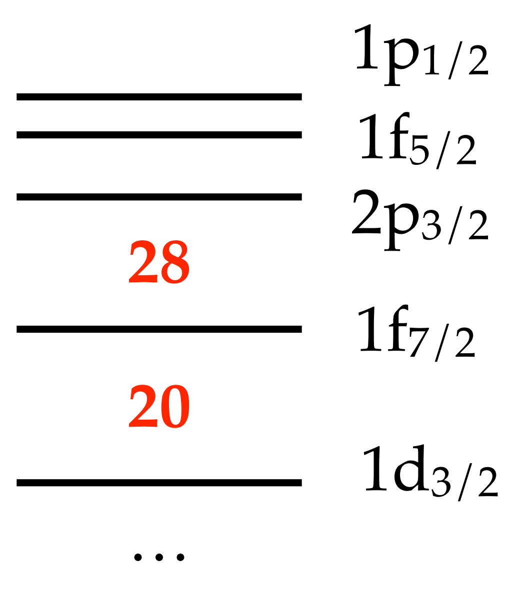
→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

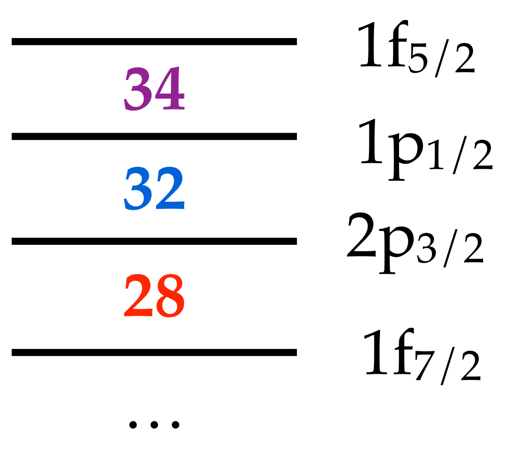
Magic numbers

See talk by V. Soma (Monday)

- Magic numbers: extra-stable combinations of N & Z

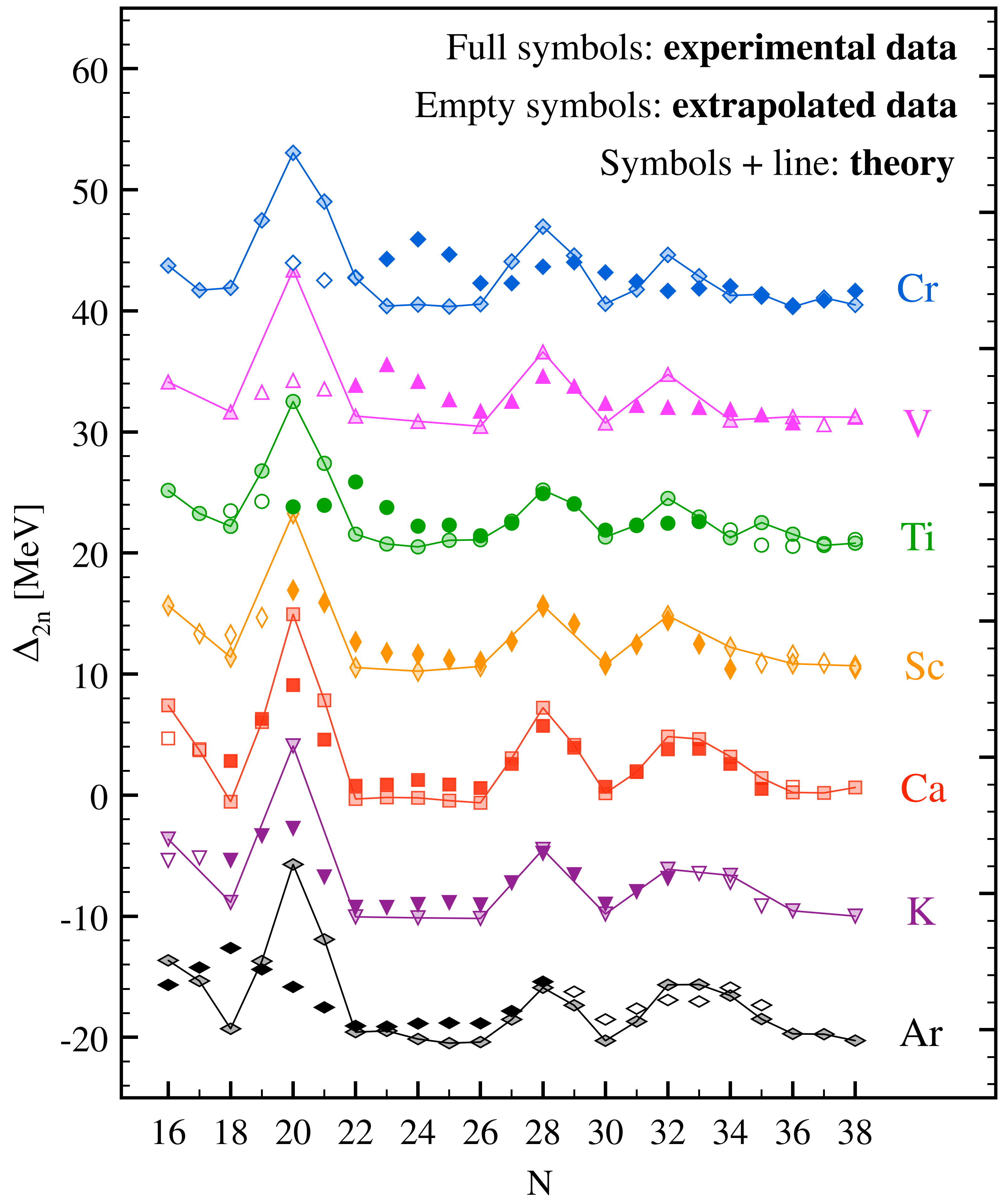


stable

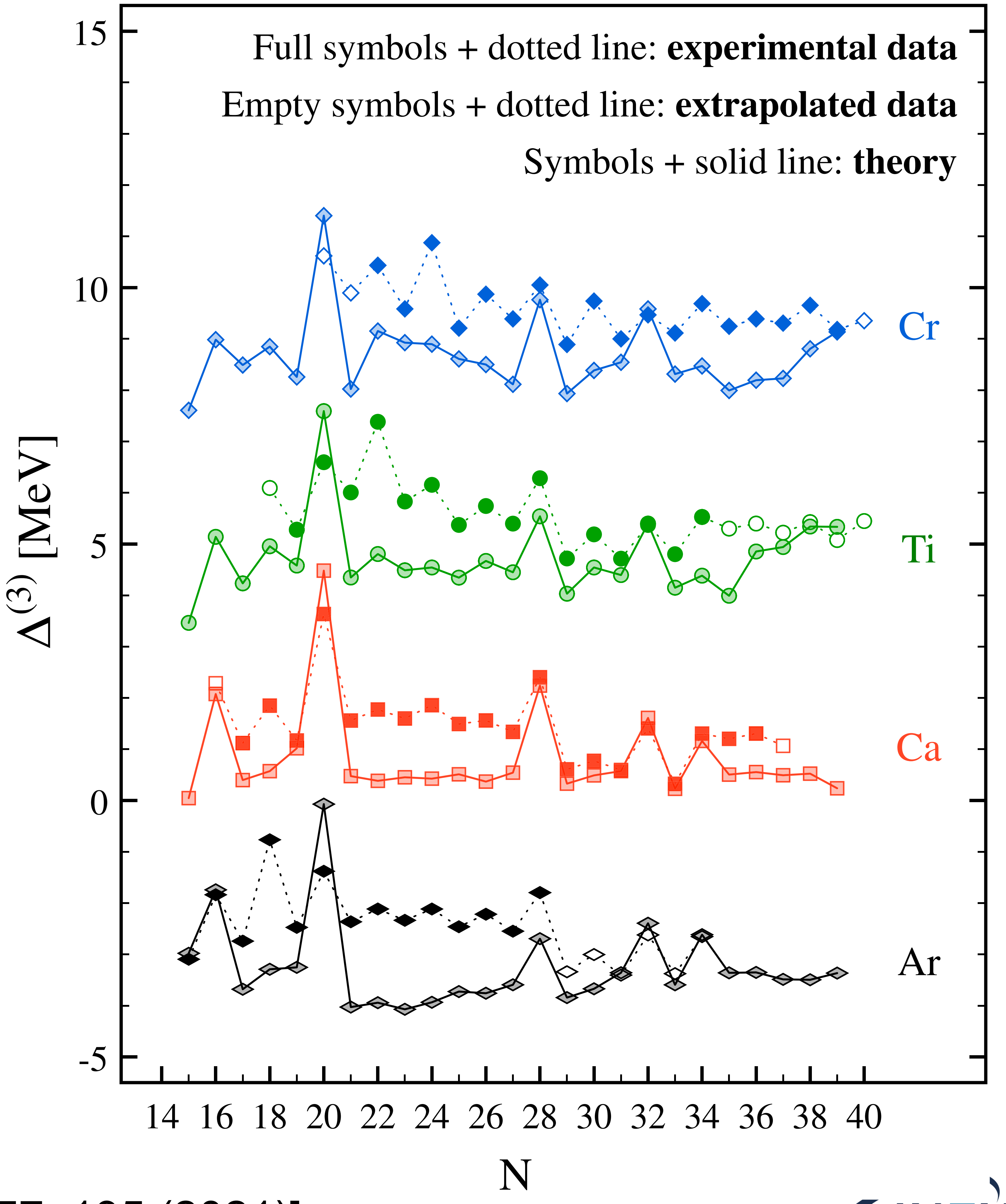


unstable

$$\Delta_{2n}(N,Z) \equiv S_{2n}(N,Z) - S_{2n}(N+2,Z)$$



$$\Delta^{(3)}(N,Z) \equiv \frac{(-1)^N}{2}\left[E(N-1,Z) - 2E(N,Z) + E(N+1,Z)\right]$$

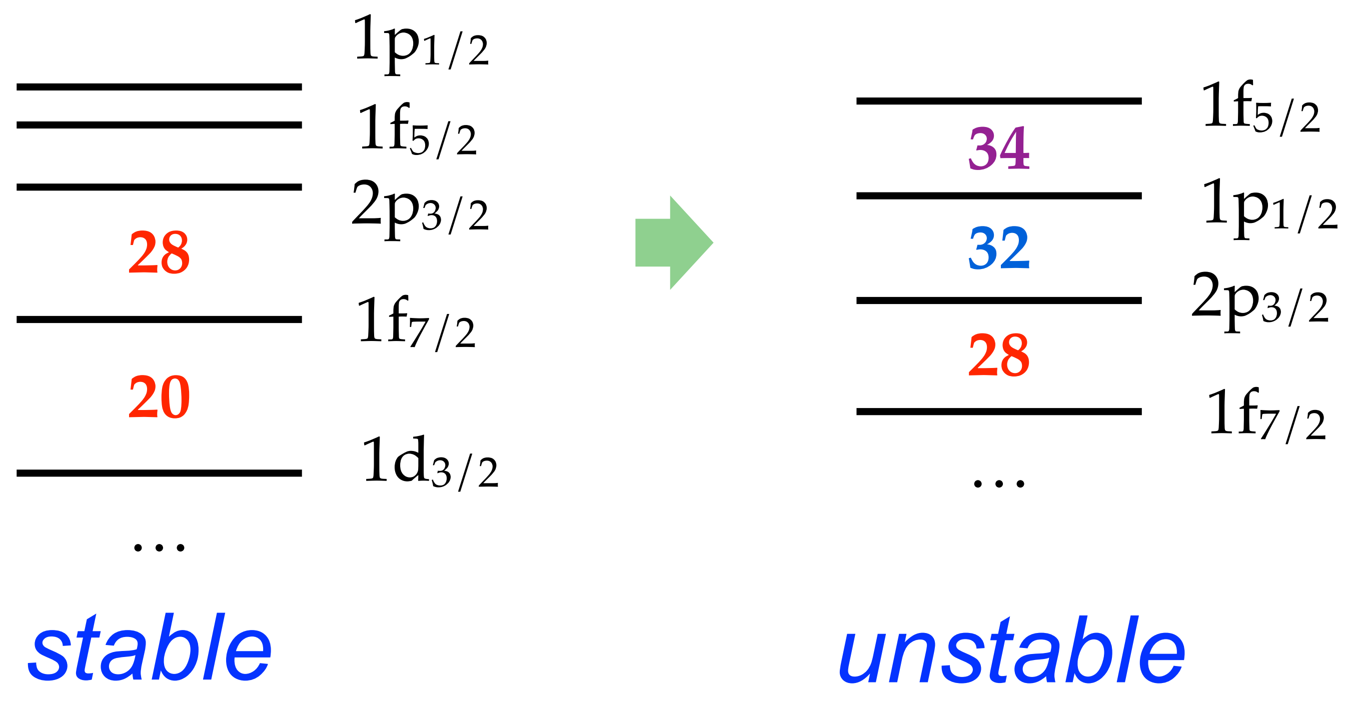


[Somà, CB et al. Eur. Phys. J. A **57**, 135 (2021)]

Magic numbers

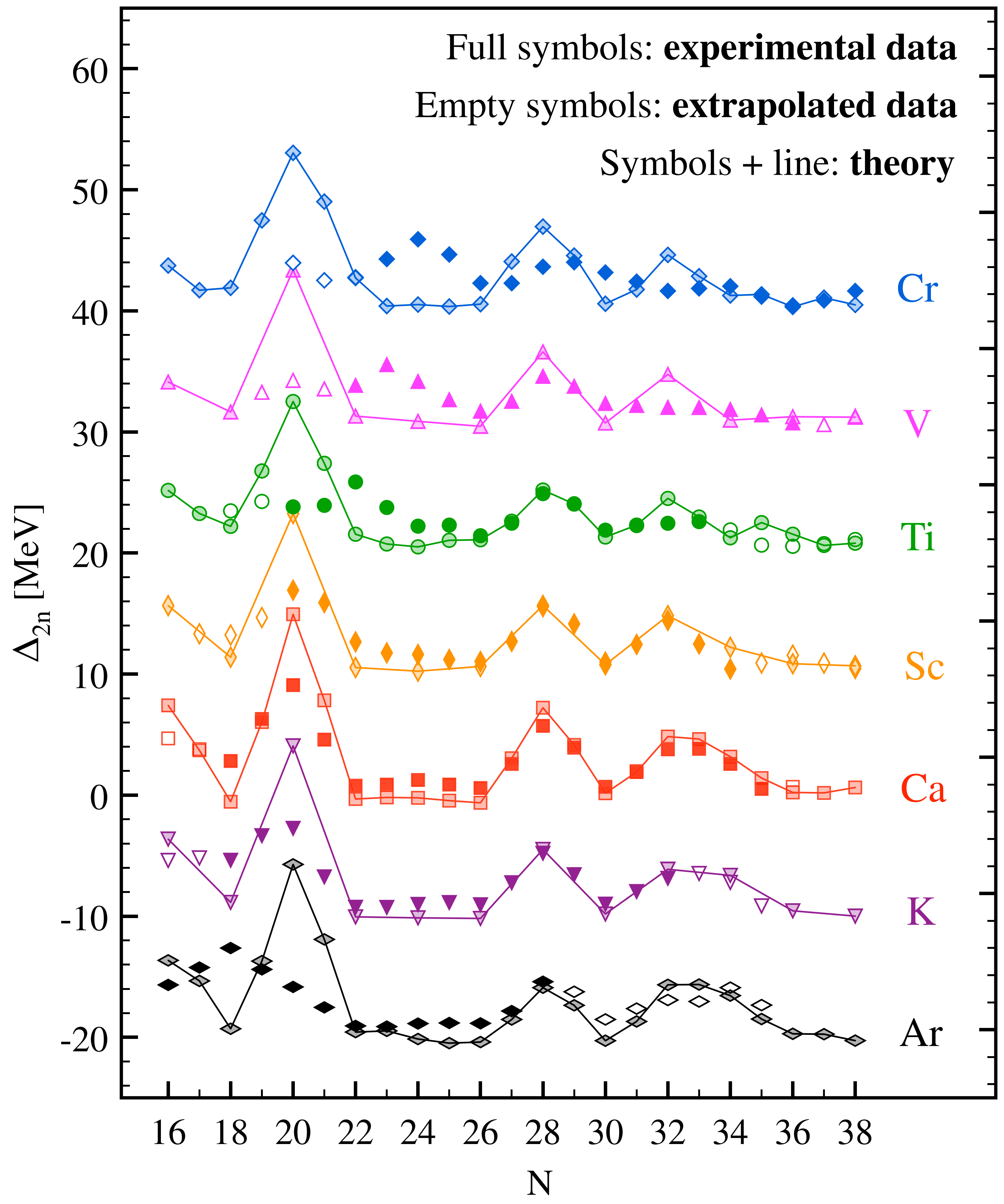
See talk by V. Soma (Monday)

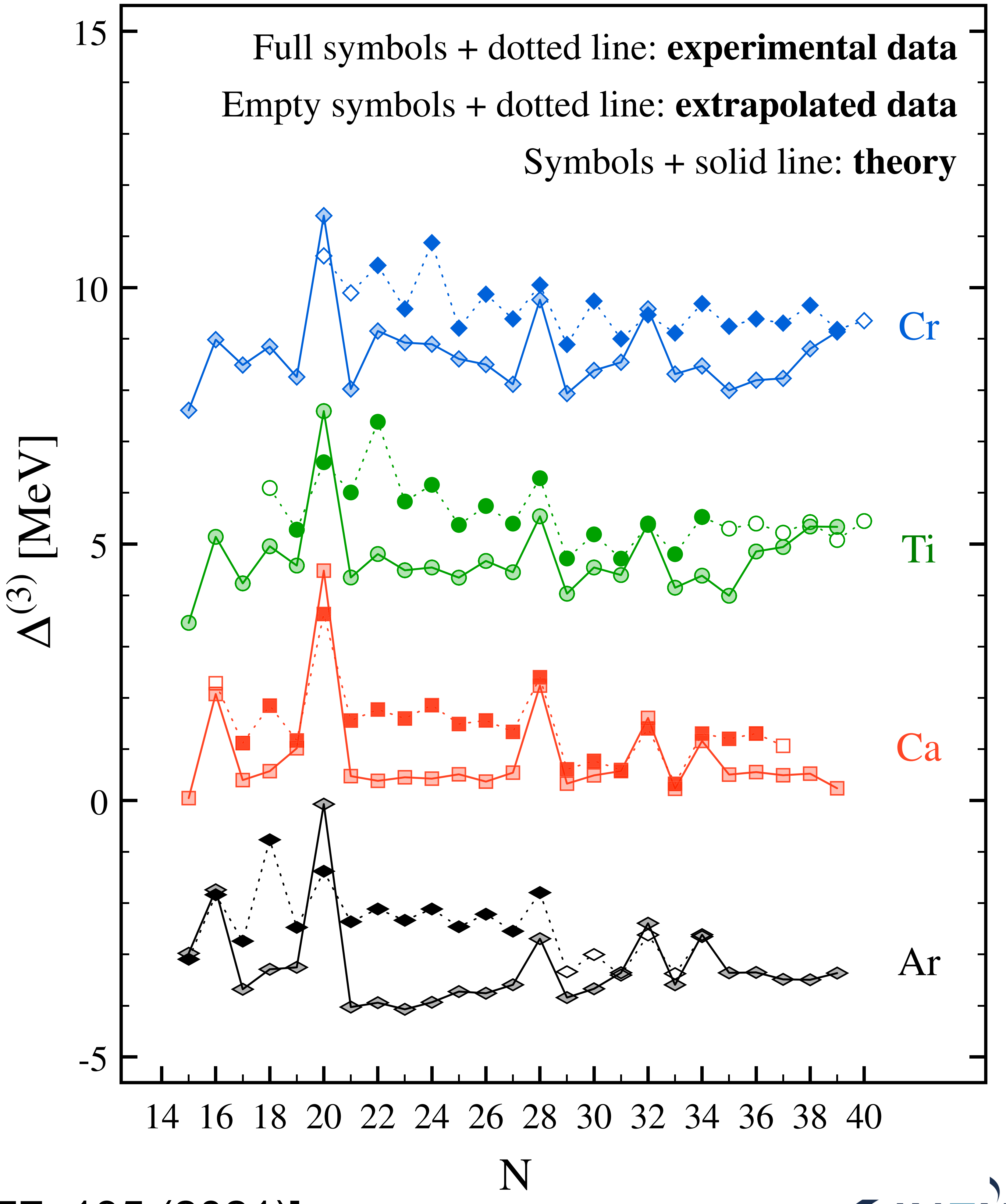
- Magic numbers: extra-stable combinations of N & Z



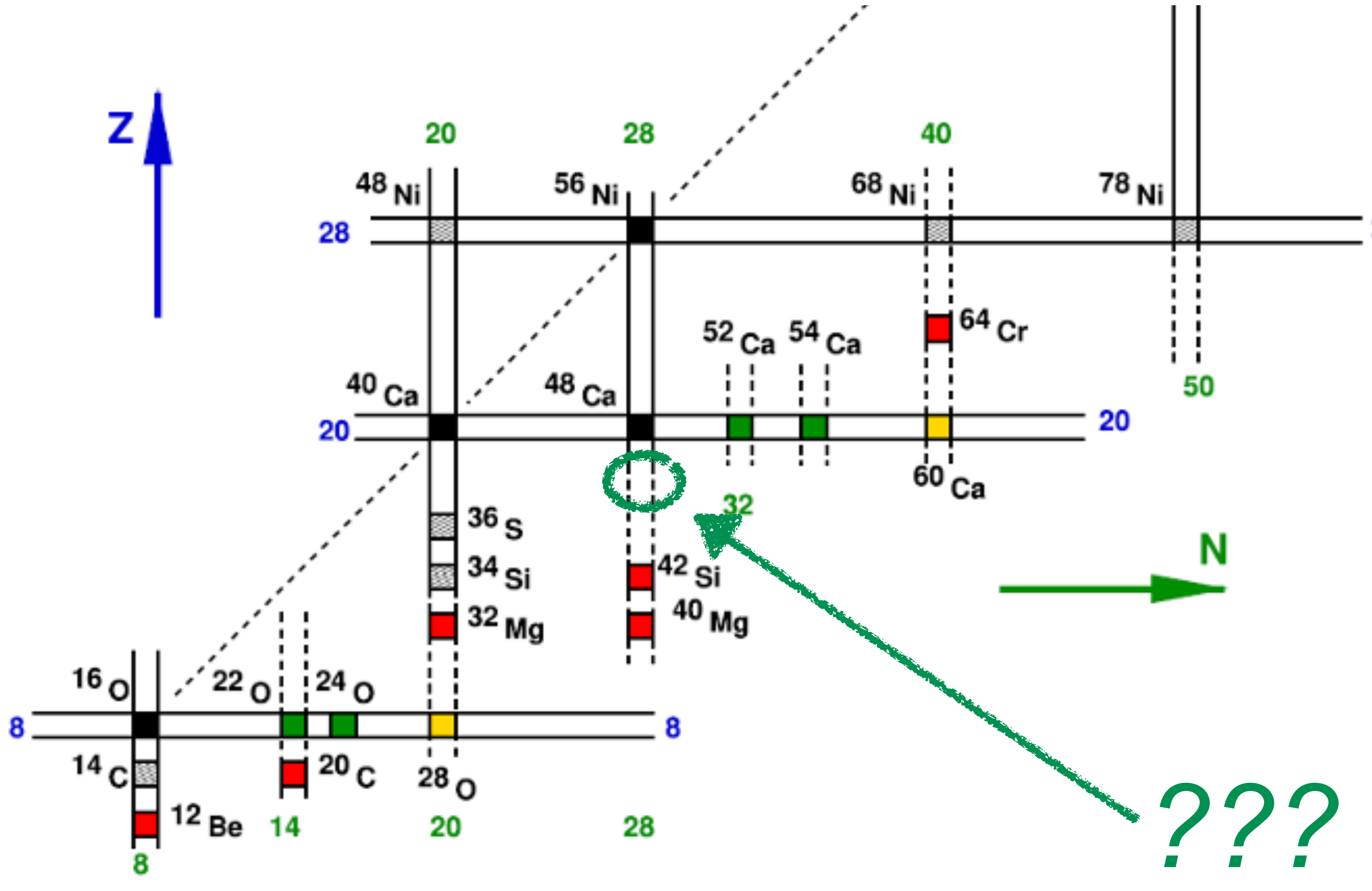
$$\Delta_{2n}(N, Z) \equiv S_{2n}(N, Z) - S_{2n}(N + 2, Z)$$

$$\Delta^{(3)}(N, Z) \equiv \frac{(-1)^N}{2}\left[E(N-1, Z) - 2E(N, Z) + E(N+1, Z)\right]$$





Known magic and semimagic nuclei:



[Nowacki, Obertelli, Poves, PPNP 120, 103866 (2021)]

[Somà, CB et al. Eur. Phys. J. A **57**, 135 (2021)]

The physics contained in the one-boby Green's function

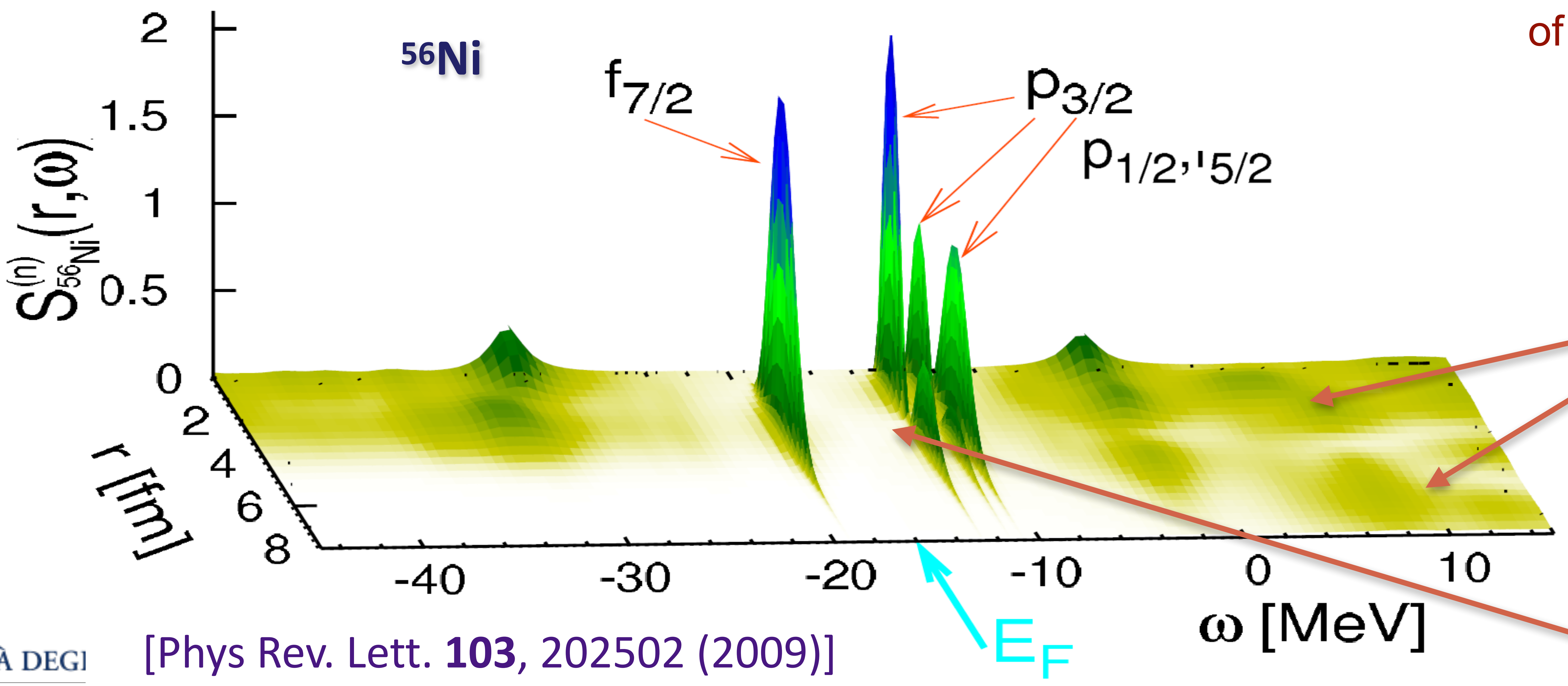
One-body propagator: $g_{\alpha\beta}(t-t') = -i/\hbar \, \langle \Psi_0^A | T[c_\alpha(t) c_\beta^\dagger(t')] | \Psi_0^A \rangle$

Dyson Equation: $g_{\alpha\beta}(\omega) = g_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} g_{\alpha\gamma}^{(0)}(\omega) \Sigma_{\gamma\delta}^{\star}(\omega) g_{\delta\beta}(\omega)$

(One-body) spectral function: $S(\alpha,\omega) := \pm \frac{1}{2\pi} \Im m\{g_{\alpha\alpha}(\omega)\}$

The self-energy $\Sigma_{\alpha\beta}^{\star}(\omega)$ 'decides' both ground state observables and elastic scattering.

==> Consistent computation of structure and reactions !!



Elastic scattering states

Low-E spectroscopy

[Phys Rev. Lett. **103**, 202502 (2009)]

Ab initio optical potentials from propagator theory

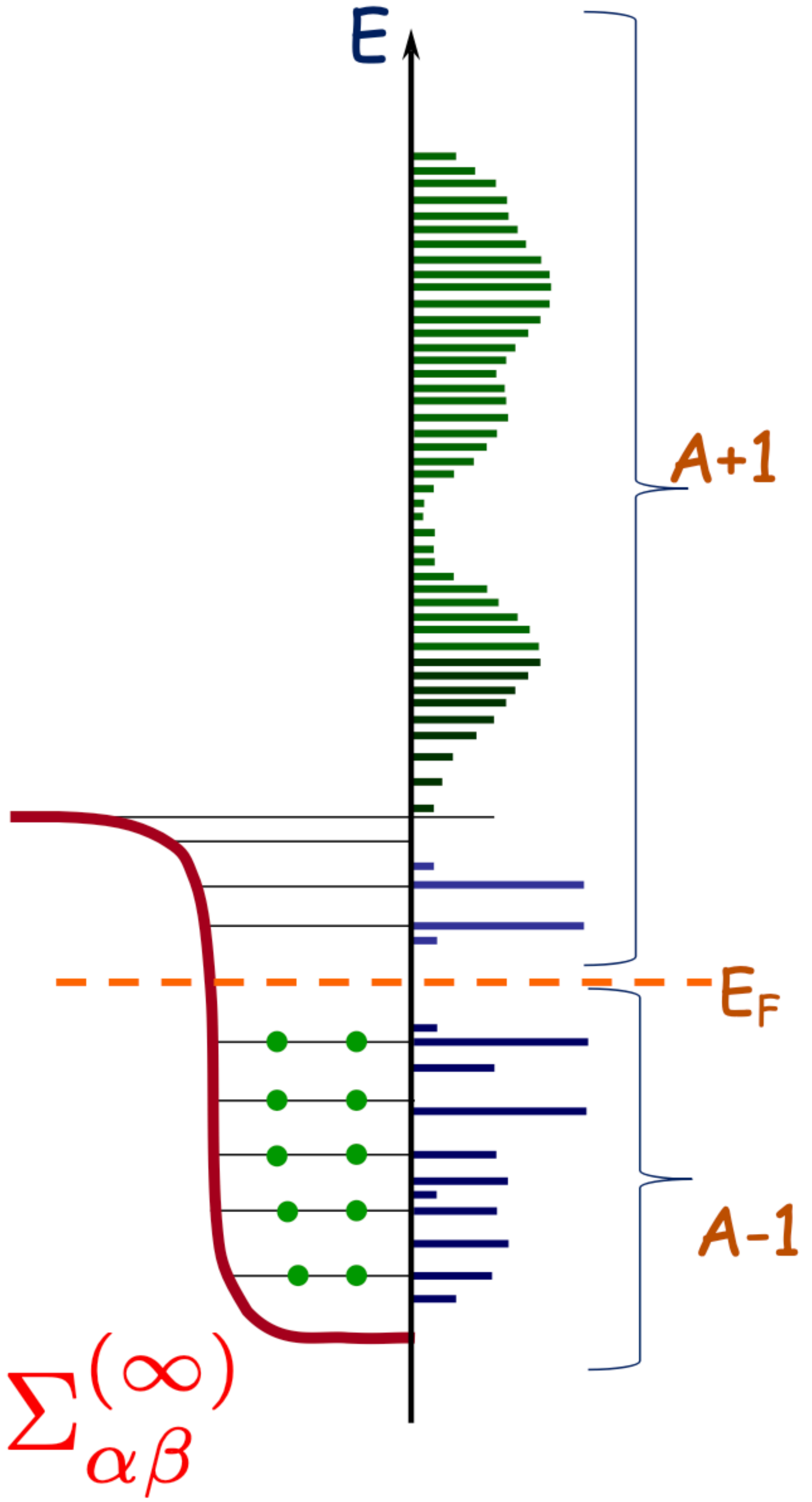
Relation to Fesbach theory:
Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991)
Escher & Jennings Phys. Rev. C**66**, 034313 (2002)

Previous SCGF work:
CB, B. Jennings, Phys. Rev. C**72**, 014613 (2005)
S. Waldecker, CB, W. Dickhoff, Phys. Rev. C**84**, 034616 (2011)
A. Idini, CB, P. Navrátil, Phys. Rv. Lett. **123**, 092501 (2019)
M. Vorabbi, CB, et al., in preparation

State-of-the-art of the field:

C. Hebborn, et al., CB, "*Optical potentials for the rare-isotope beam era*", arXiv:2210.07293 (WP)
Jour. Phys. G (2023), in press.

Microscopic optical potential



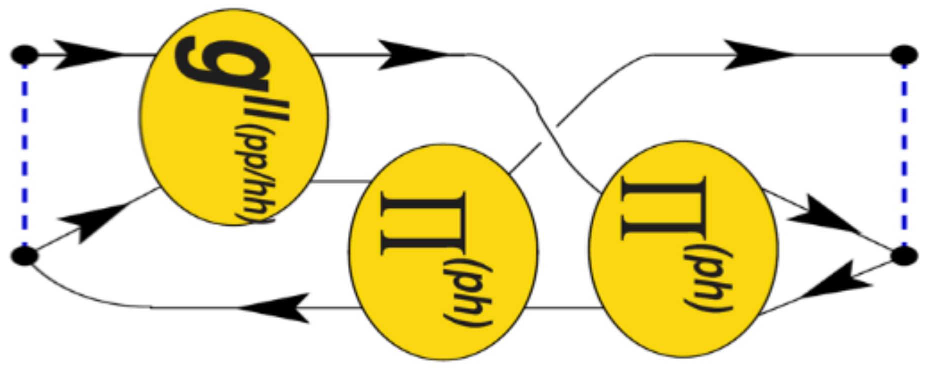
Nuclear self-energy $\Sigma^{\star}(\mathbf{r},\mathbf{r}';\varepsilon)$:

- contains *both particle* and *hole* props.
- it is proven to be a Feshbach opt. pot., in general it is *non-local !*

$$\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \sum_{i,j} \mathbf{M}^{\dagger}_{\alpha,i} \left(\frac{1}{E - (\mathbf{K}^{>} + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^{<} + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}^{\dagger}_{s,\beta}$$

mean-field

Particle-vibration couplings:

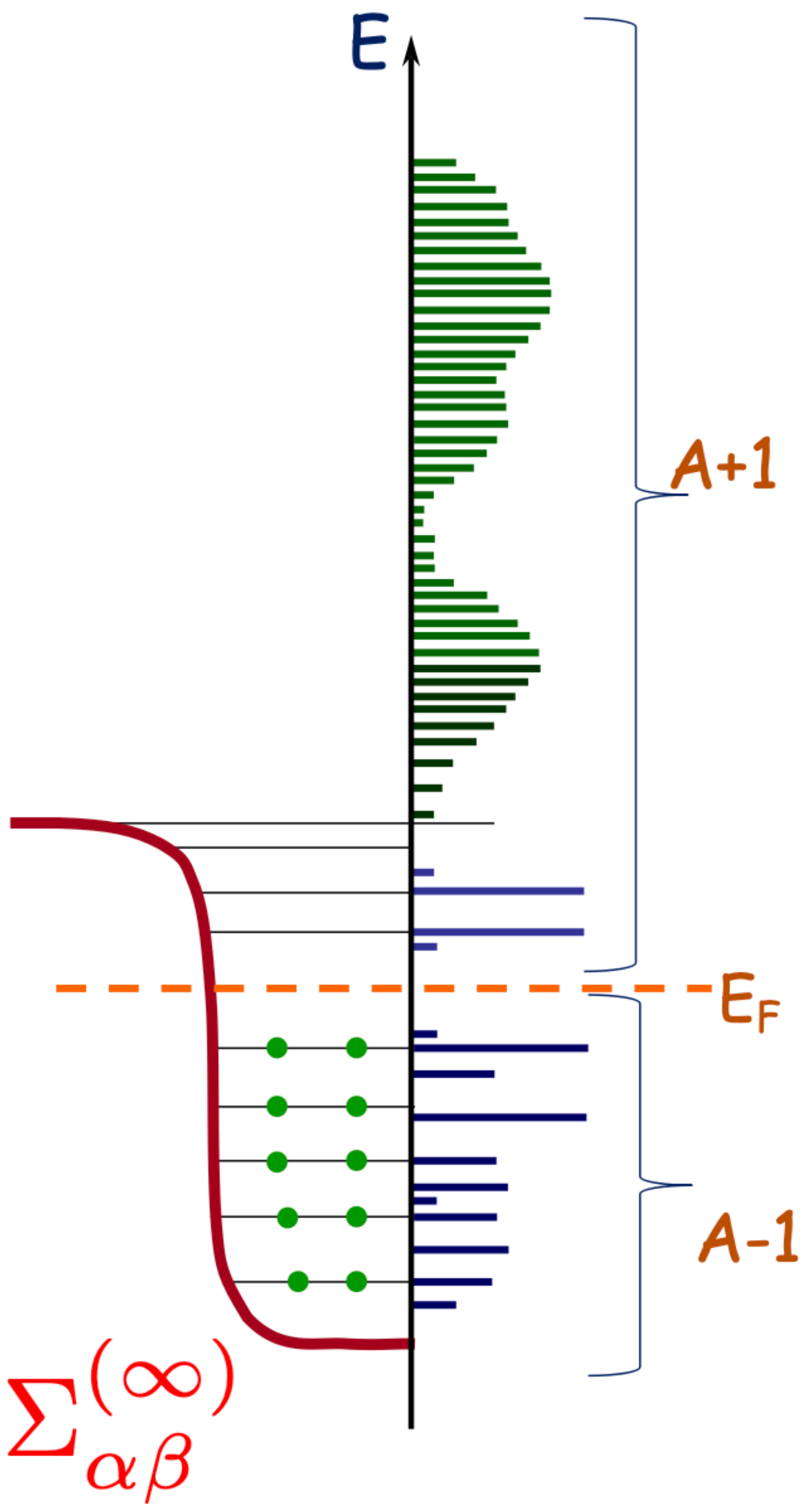


Solve scattering and overlap functions directly in momentum space:

$$\Sigma^{\star\, l,j}(k,k';E) = \sum_{n,\,n'} R_{n\,l}(k)\, \Sigma^{\star\, l,j}_{n,\,n'}\, R_{n\,l}(k')$$

$$\frac{k^2}{2\mu}\psi_{l,j}(k) + \int \mathrm{d}k'\, k'^2\, \Sigma^{\star l,j}(k,k';E_{c.m.})\psi_{l,j}(k') = E_{c.m.}\psi_{l,j}(k)$$

Microscopic optical potential



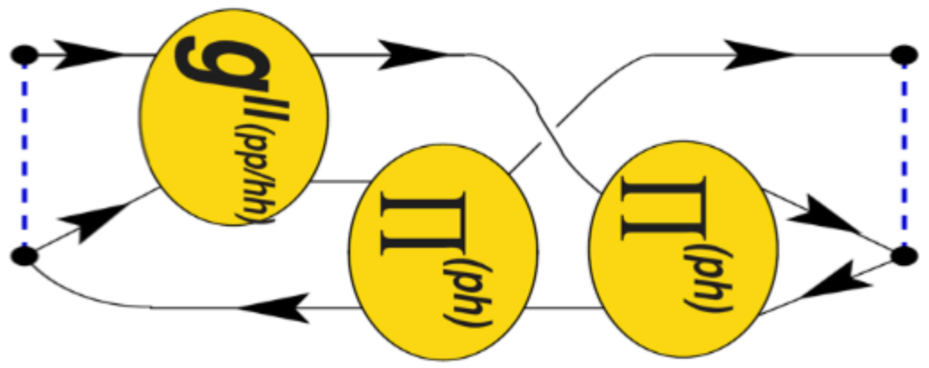
Nuclear self-energy $\Sigma^{\star}(\mathbf{r}, \mathbf{r}'; \varepsilon)$:

- contains *both particle* and *hole* props.
- it is proven to be a Feshbach opt. pot., in general it is *non-local !*

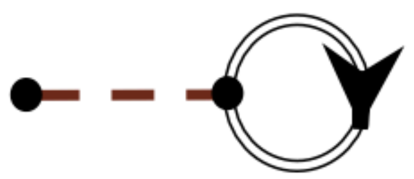
$$\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \sum_{i,j} \mathbf{M}^{\dagger}_{\alpha,i} \left(\frac{1}{E - (\mathbf{K}^{>} + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^{<} + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}^{\dagger}_{s,\beta}$$

mean-field

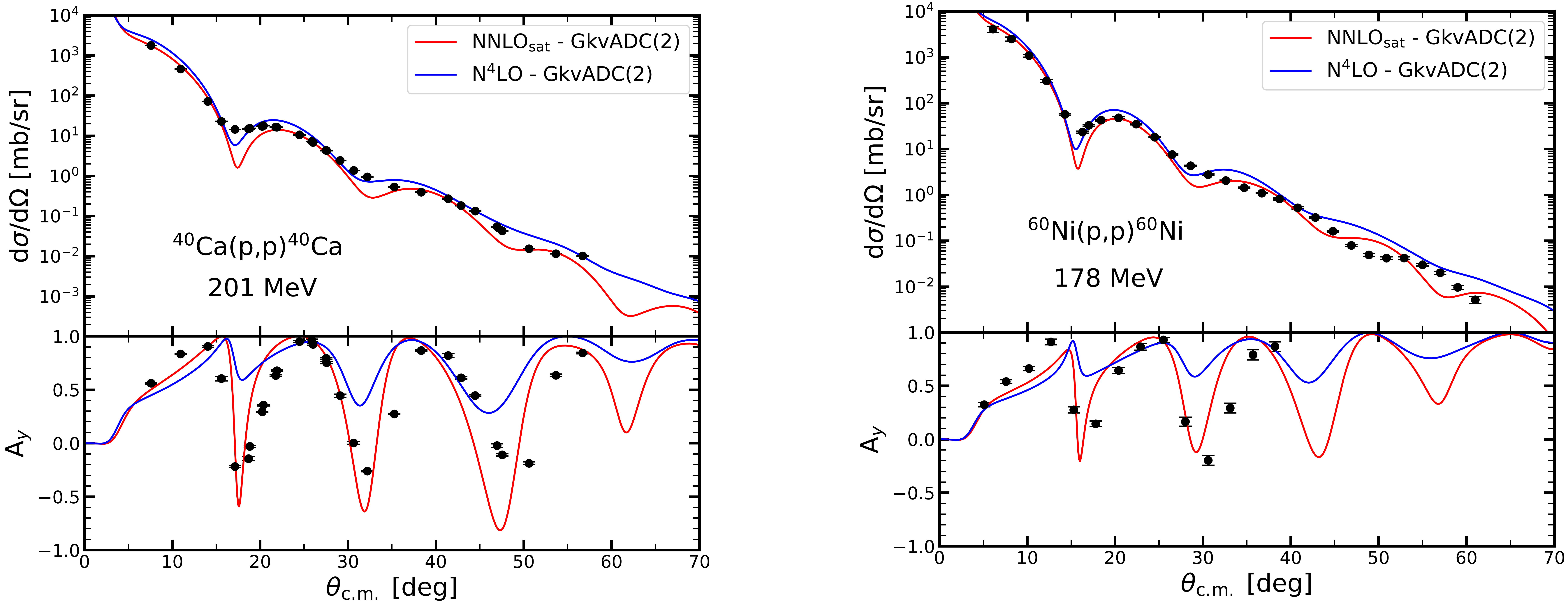
Particle-vibration couplings:



$$\Sigma^{(\infty)}_{\alpha\beta} =$$



Elastic nucleon nucleus scattering



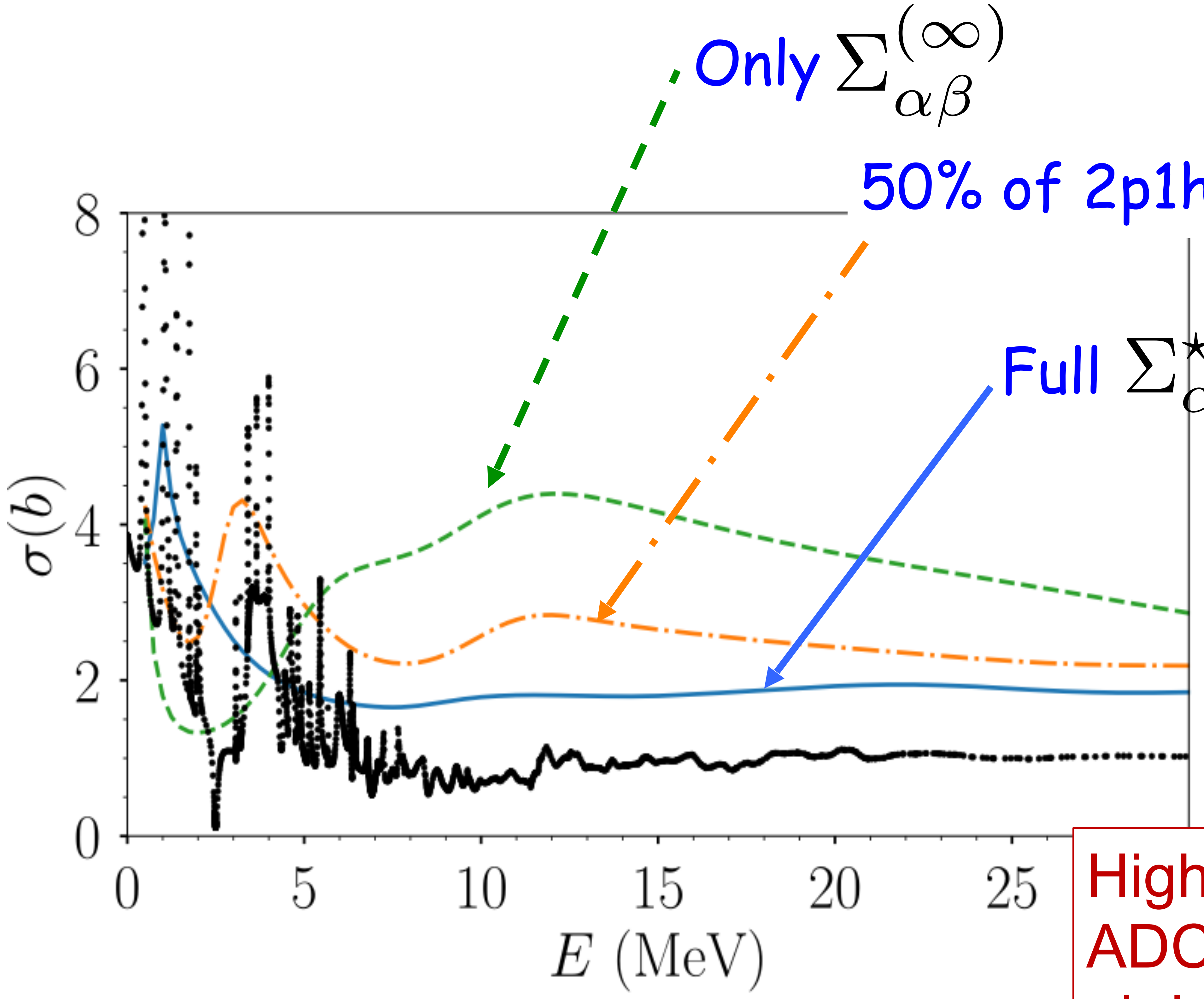
M. Vorabbi
U. of surrey

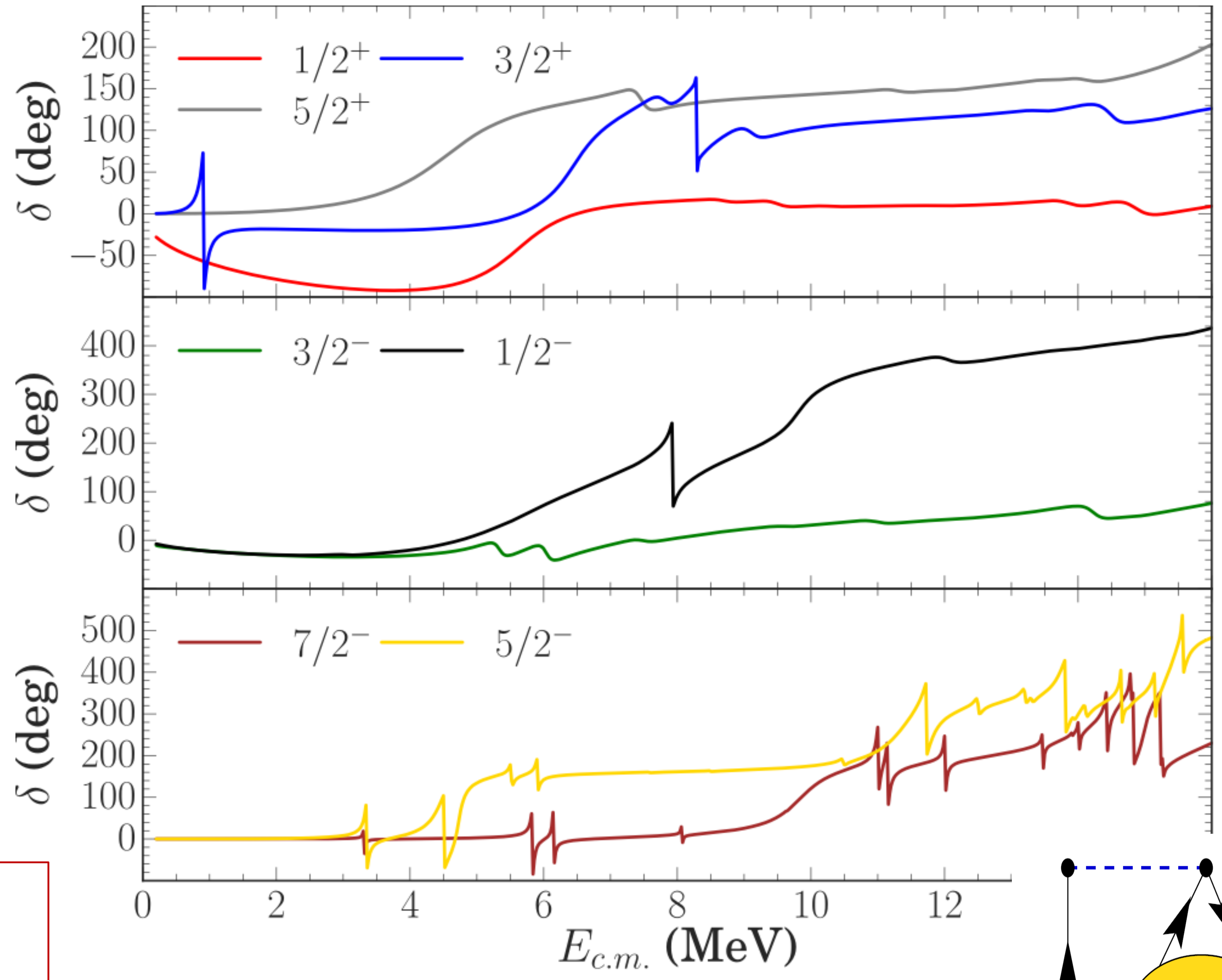
M. Vorabbi, CB et al. *Phys Rev C*, **109**, 034613 (2024)]

Role of intermediate state configurations (ISCs)

n-^{16}O, total elastic cross section

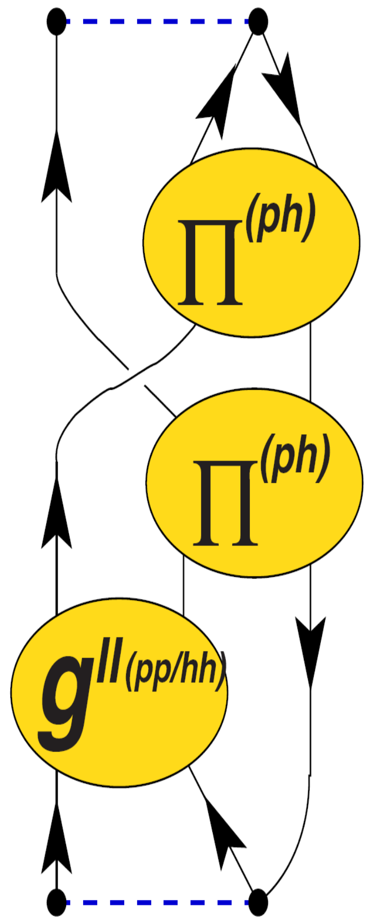
[A. Idini, CB, Navrátil, Phys. Rev. Lett. **123**, 092501 (2019)]





High order configurations, or ADC(n>>3), to be critical for fully ab initio optical potentials

$$\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \sum_{i,j} \mathbf{M}^{\dagger}_{\alpha,i} \underbrace{\left(\frac{1}{E - (\mathbf{K}^{>} + \mathbf{C}) + i\Gamma} \right)_{i,j}}_{\text{2p1h}} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \underbrace{\left(\frac{1}{E - (\mathbf{K}^{<} + \mathbf{D}) - i\Gamma} \right)_{r,s}}_{\text{2h1p}} \mathbf{N}^{\dagger}_{s,\beta}$$



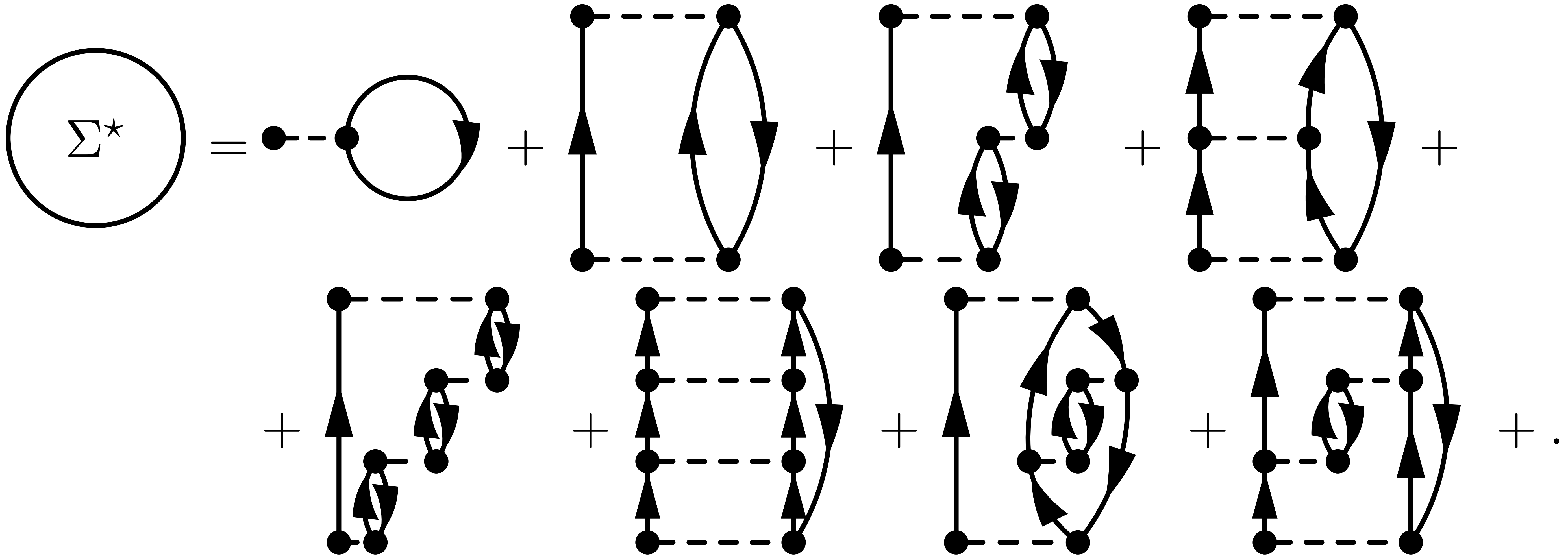
Green's function theory beyond ADC(3)? **DiagMC !!**

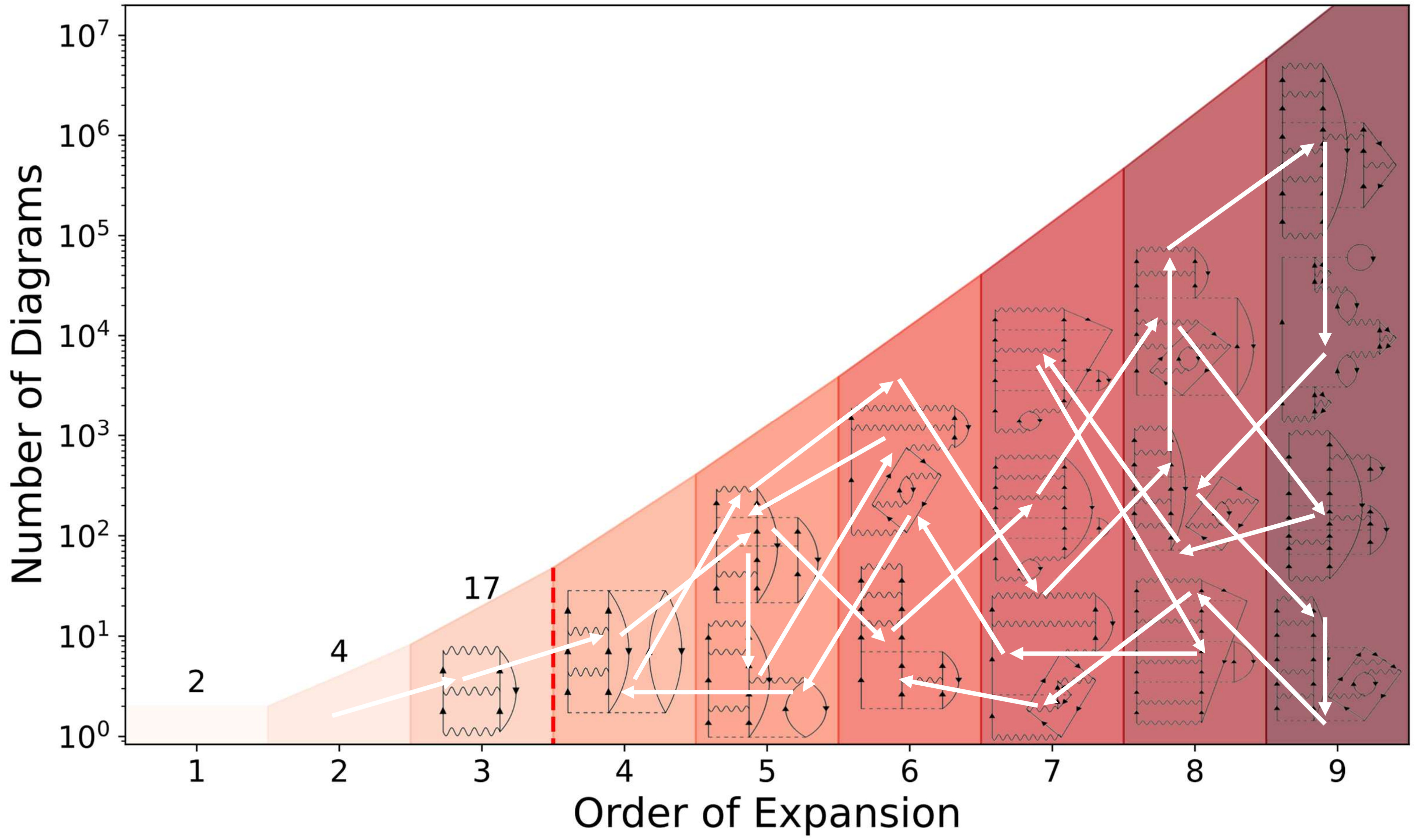
The Green's function is found as the exact solution of the Dyson equation:

$$G_{\alpha\beta}(\omega) = G^{(0)}_{\alpha\beta}(\omega) + \sum_{\gamma\delta} G^{(0)}_{\alpha\gamma}(\omega)\, \Sigma^{\star}_{\gamma\delta}(\omega)\, G_{\delta\beta}(\omega)$$

The number of required diagrams explodes (factorially!) with the order of the approximation…

It requires knowing the self-energy which is the sum of an *infinite series* of Feynman diagrams:





Diagrammatic Monte Carlo (DiagMC) ***samples diagrams in their topological space*** using a Markov chain.

[Brolli, CB, Vigezzi, Phys. Rev. Lett. **134**, 182502 (2025)]

Diagrammatic MC in solid state physics…

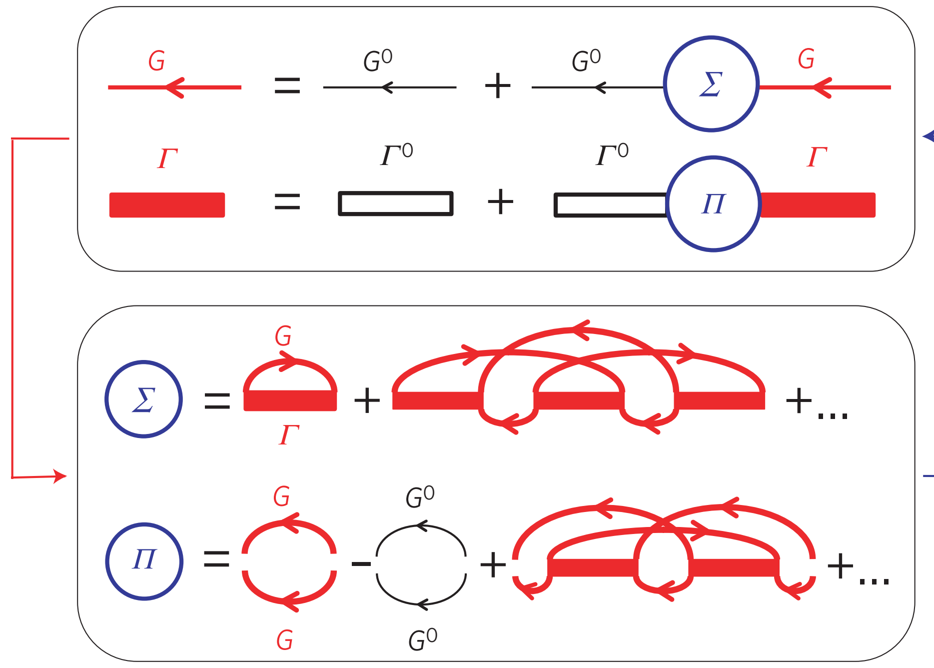


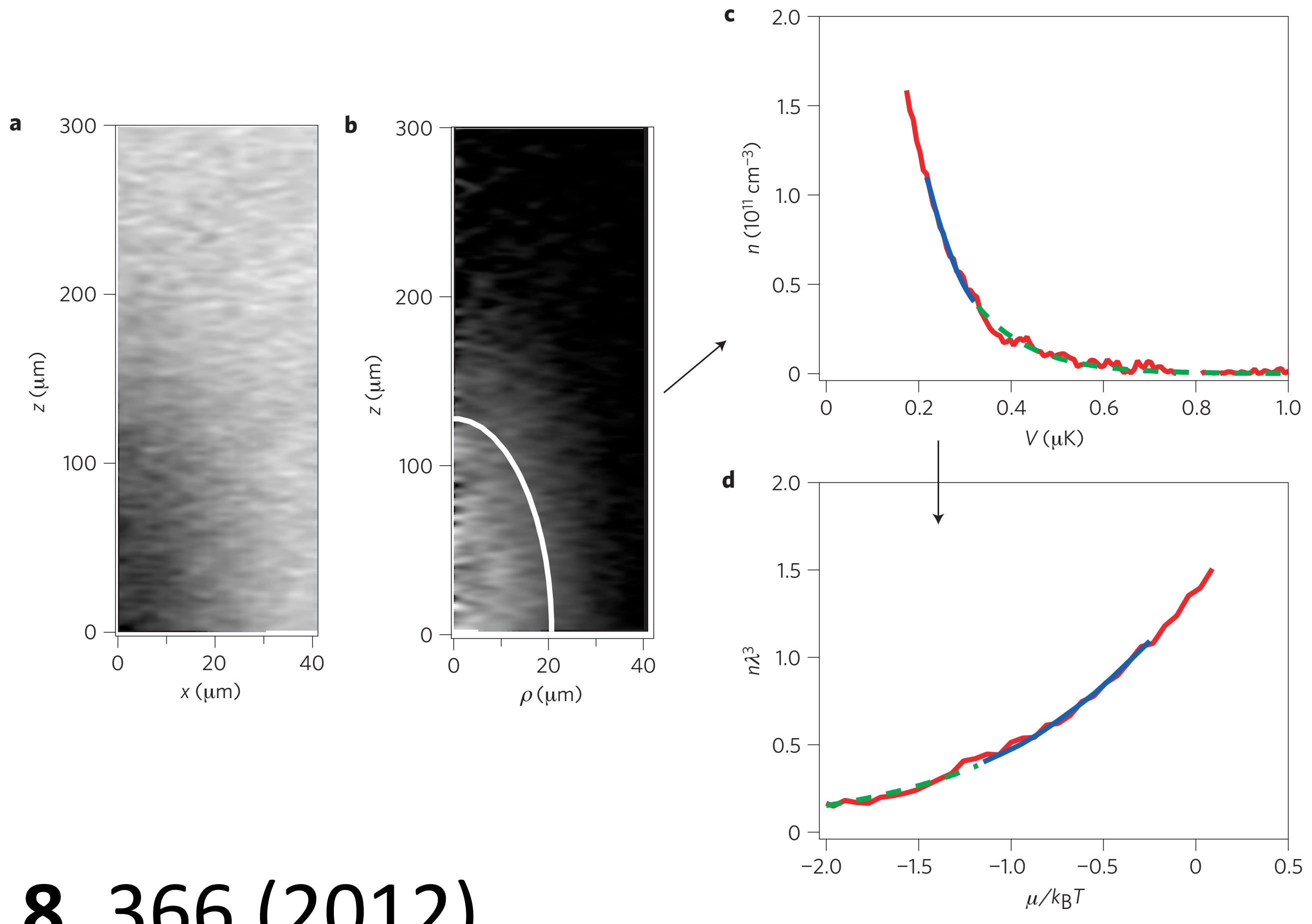
Figure 1 | Bold diagrammatic Monte Carlo The skeleton diagrammatic series for the self-energy Σ and the pair self-energy Π is evaluated

DiagMC:
Phys. Rev. Lett. 81, 2514 (1998)
Phys. Rev. B 99, 035140 (2019)
…

Mostly infinite matter at T≠0…

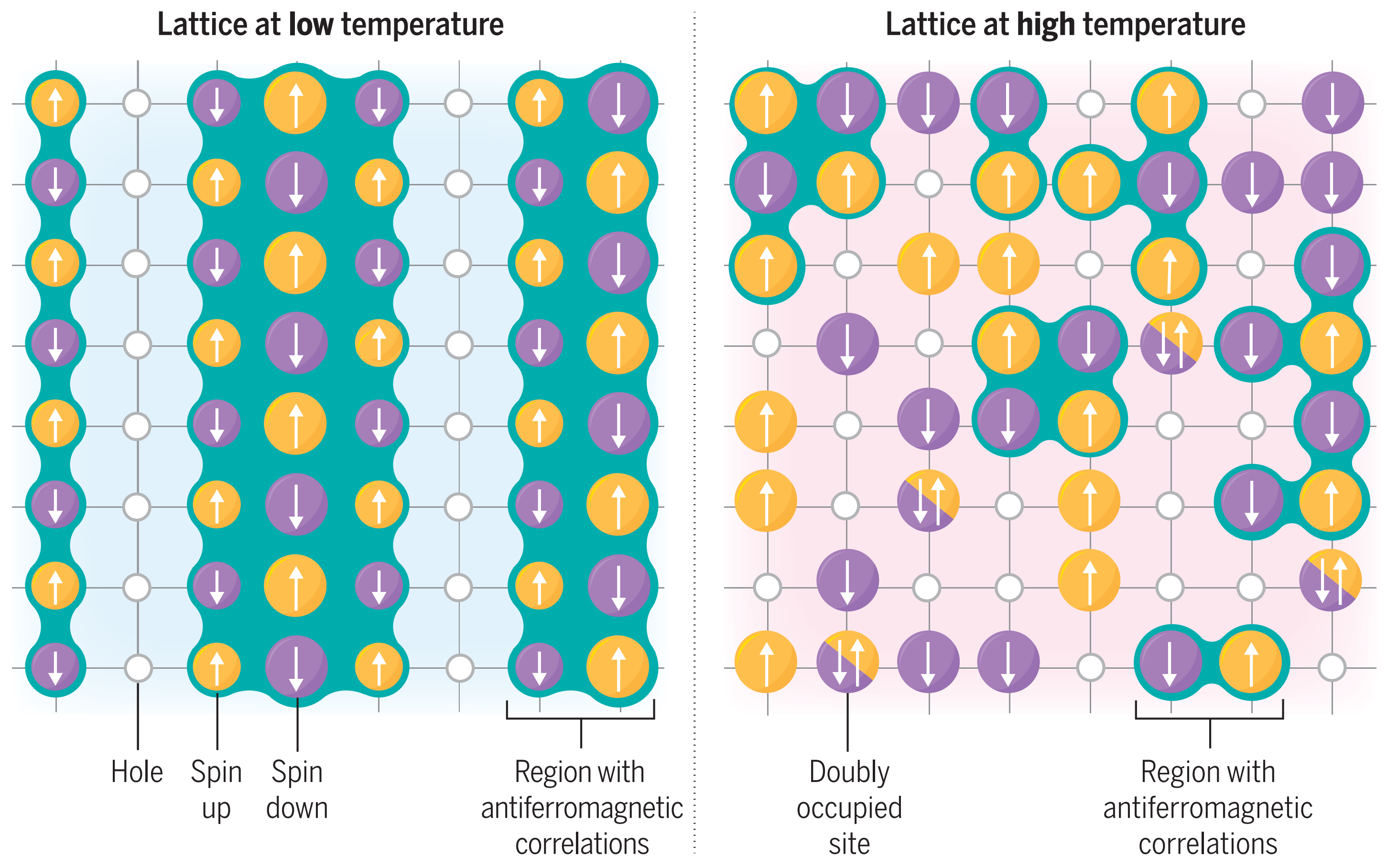
…what about finite nuclei??

EOS unitary fermi gas…



Nature Phys. **8**, 366 (2012)

Spins in the Hubbard model…



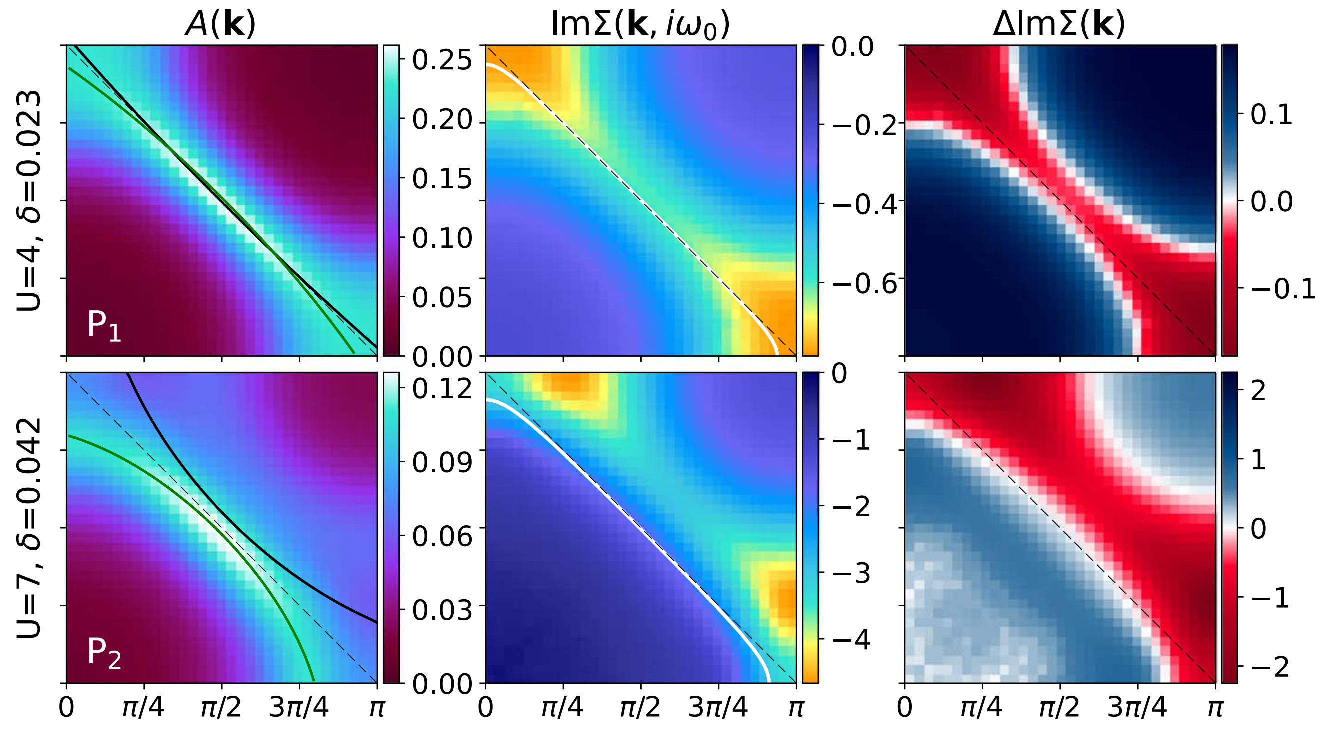


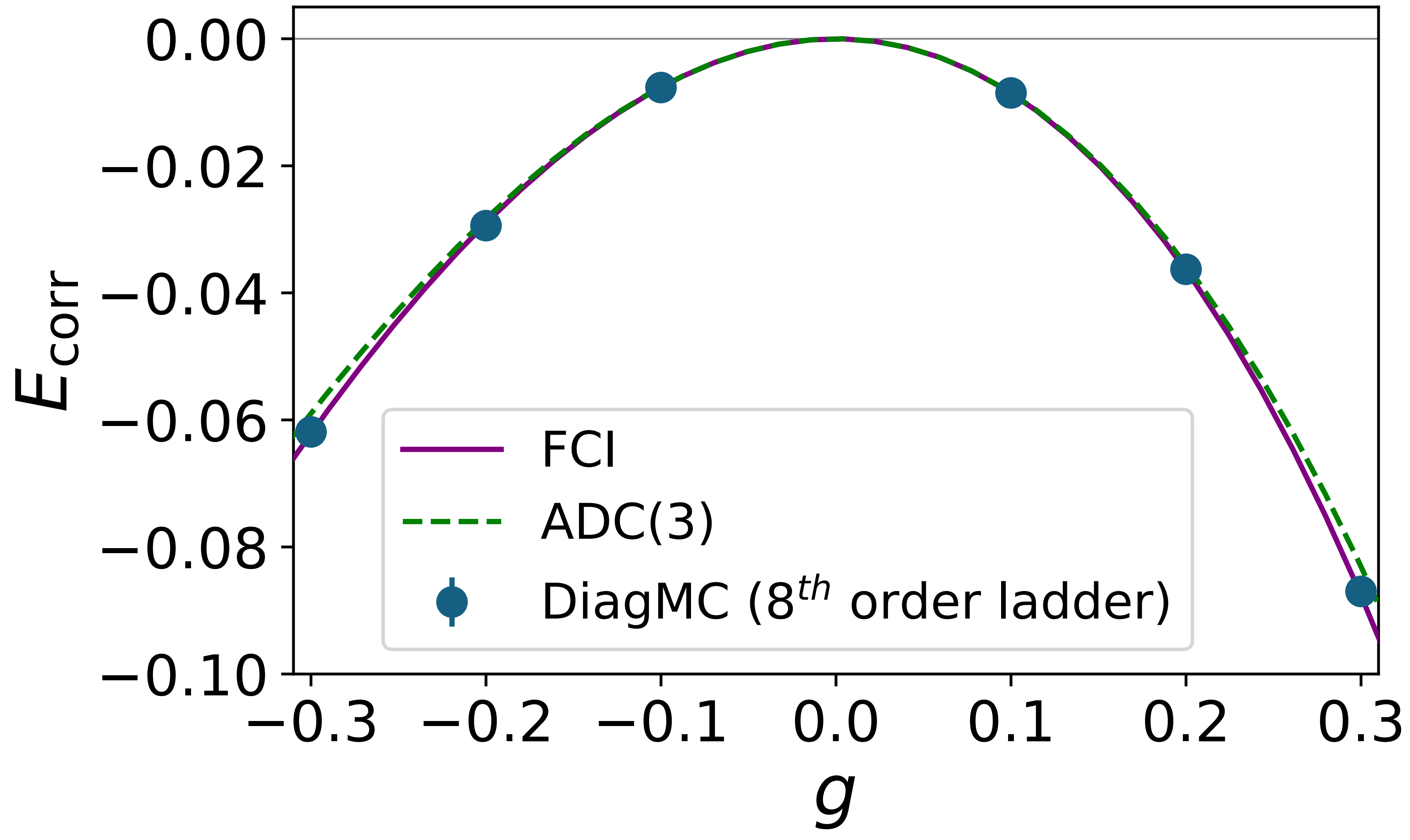
Fig. 2. Fingerprints of the different regimes. The momentum-resolved spectral function, A(**k**) (left); the imaginary part of the self-energy, Im Σ(**k**,iω₀) (middle); and the difference between the imaginary part of the self-

Science **385**, eade9194 (2024)

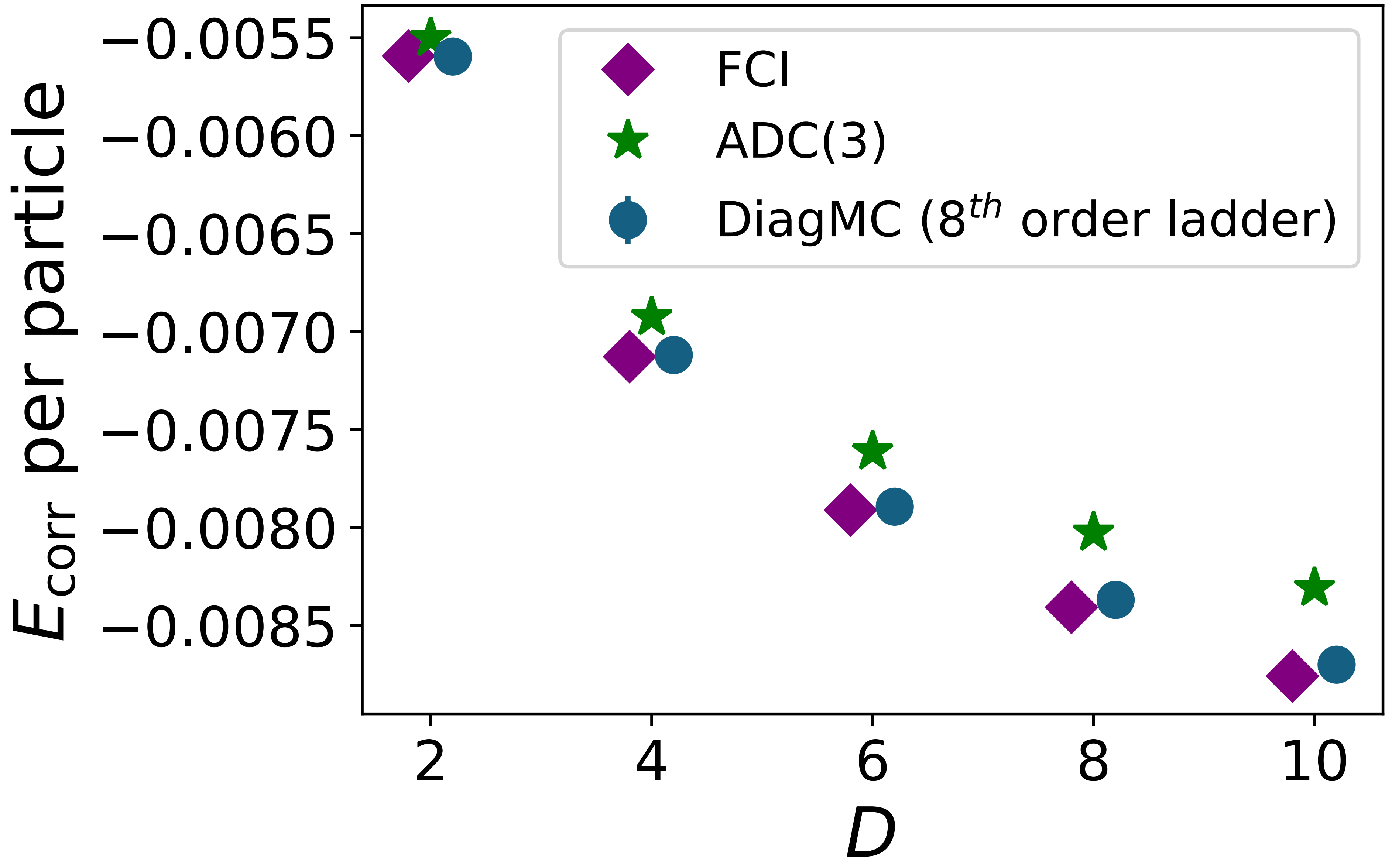
Results of the simulation for D=2-10 levels

See talk by S. Brolli (Monday)

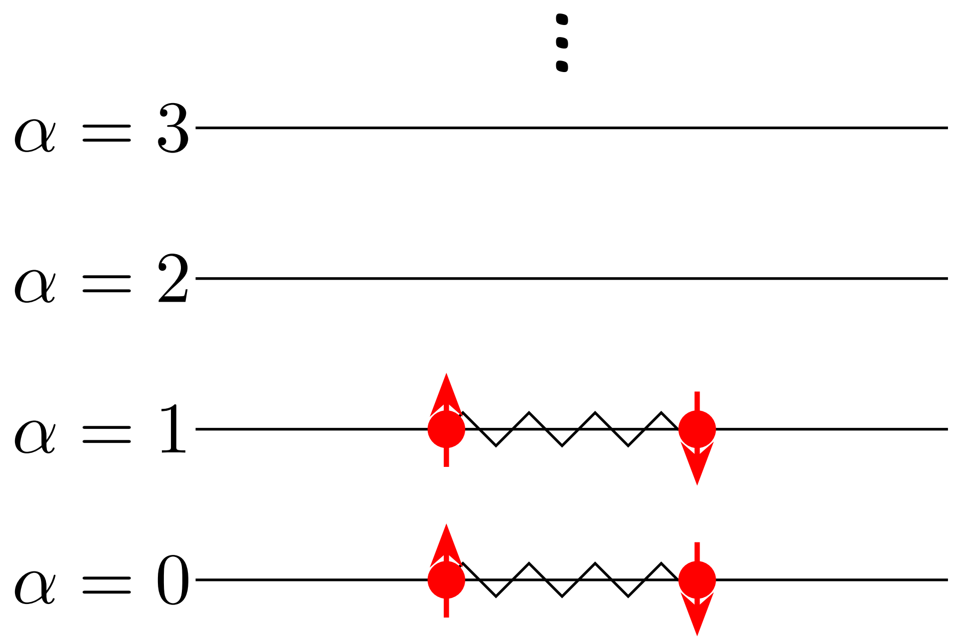
Correlation energy $\Delta E = E - E_{HF}$ as a function of interaction strength (*g*):



Accuracy for different model spaces model spaces (*D=2-10* levels):



Spectroscopic function for *D=10* levels :



$$H = \xi \sum_{\alpha=0}^{D-1} \sum_{\sigma=+,-} \alpha c^\dagger_{\alpha\sigma} c_{\alpha\sigma} - \frac{g}{2} \sum_{\alpha,\beta=0}^{D-1} c^\dagger_{\alpha+} c^\dagger_{\alpha-} c_{\beta-} c_{\beta+}$$

[Brolli, CB, Vigezzi, Phys. Rev. Lett. **134**, 182502 (2025)]

Applying DiagMC to nuclei…

See talk by
S. Brolli (Monday)

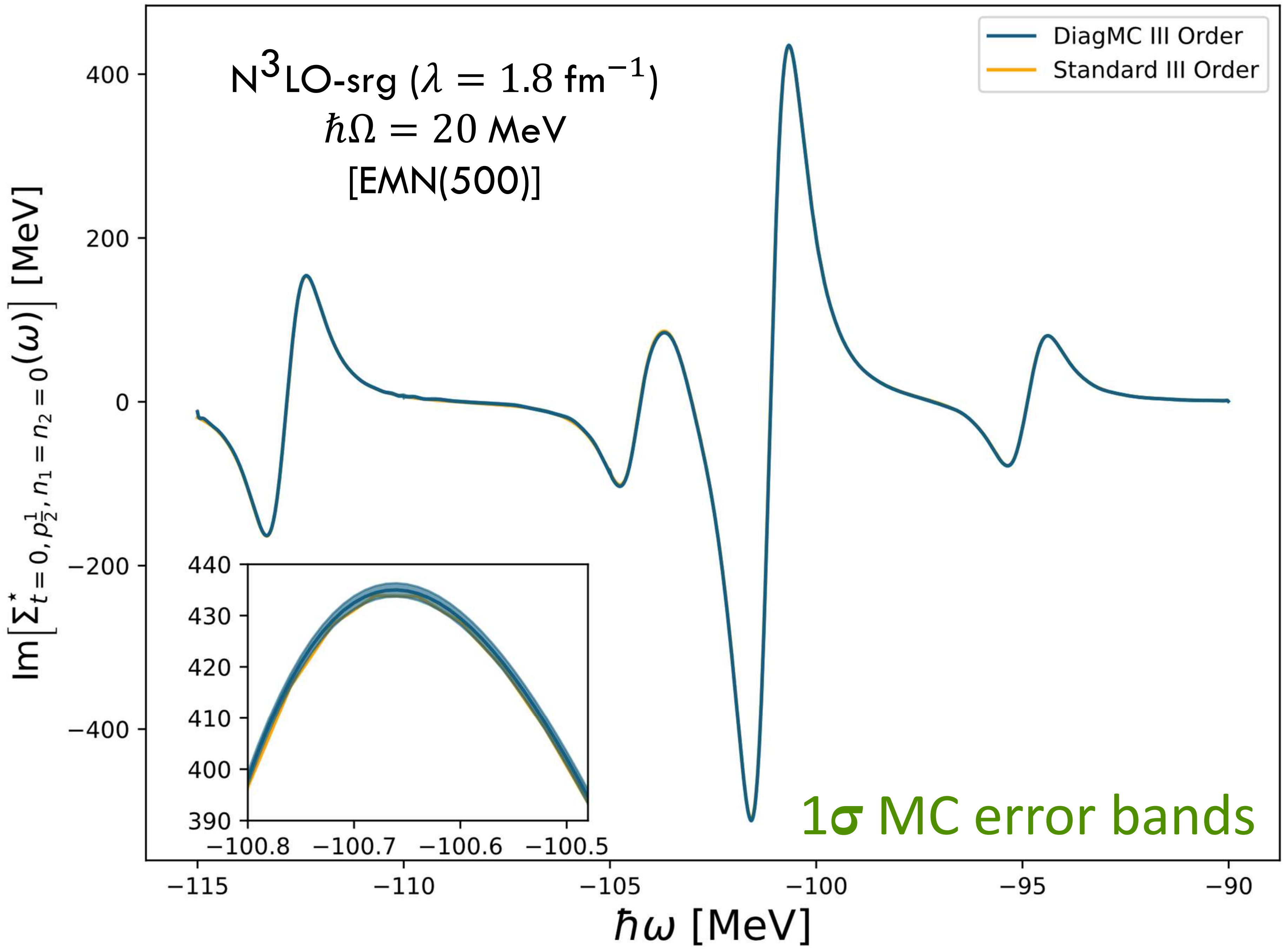
DiagMC is being extended to treat realistic microscopic nuclear Hamiltonians

^{16}O in harmonic oscillator space (N_{max} = 2, for now)

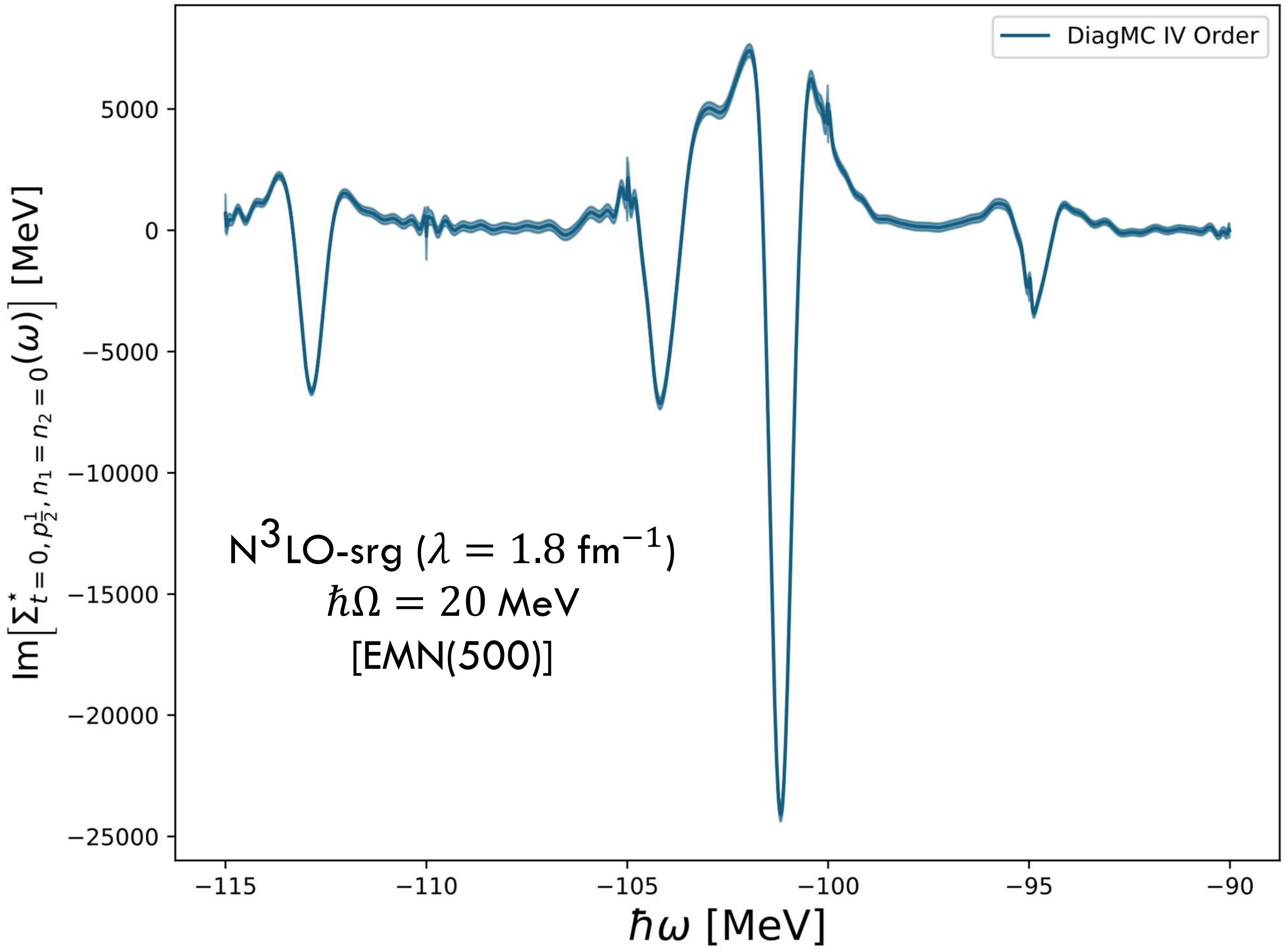
Example of neutron $p_{1/2}$ self-energy up to 3rd order

S. Brolli & CB – preliminary !!!

THIRD ORDER RESULTS

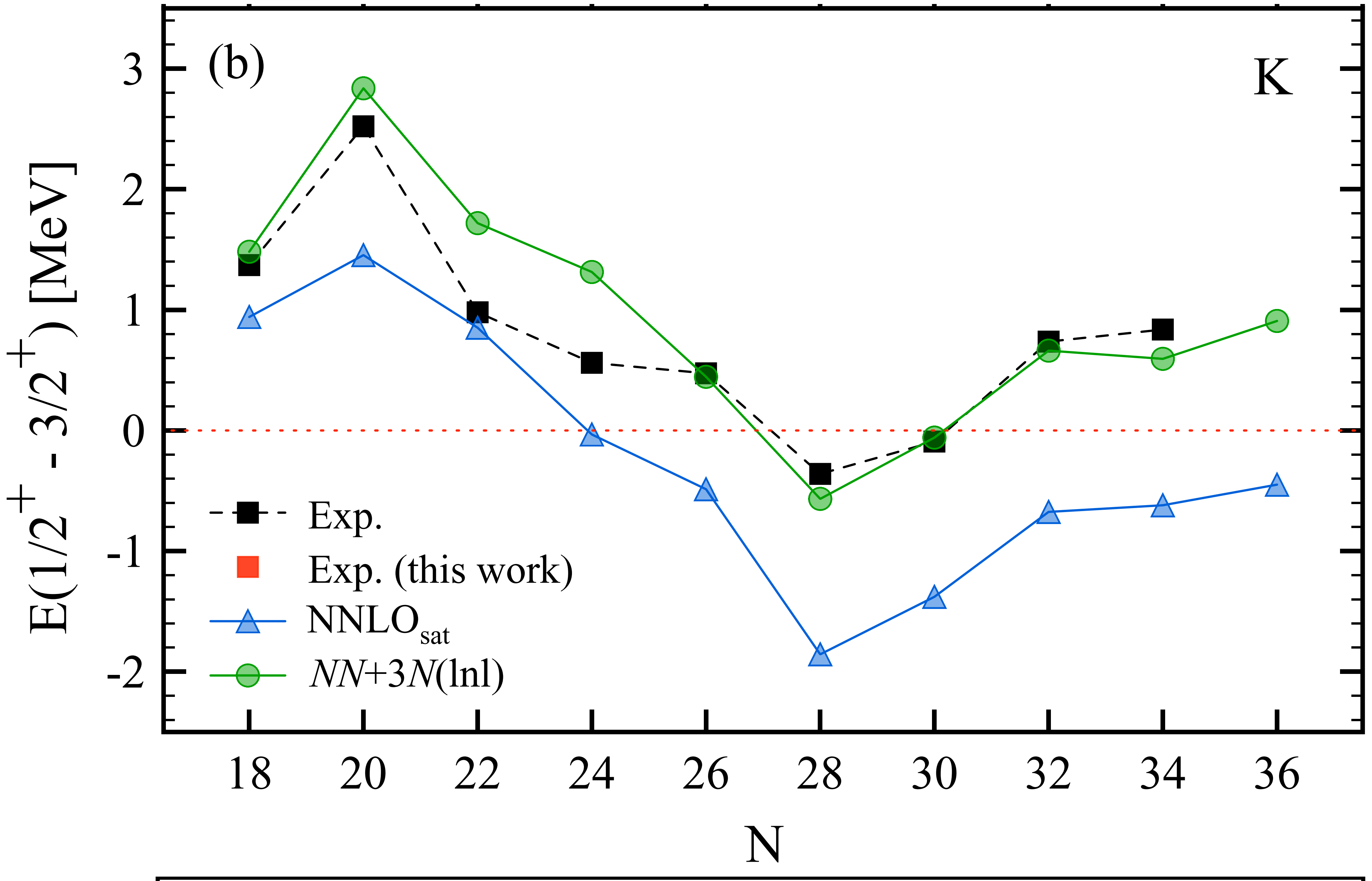


FOURTH ORDER RESULTS



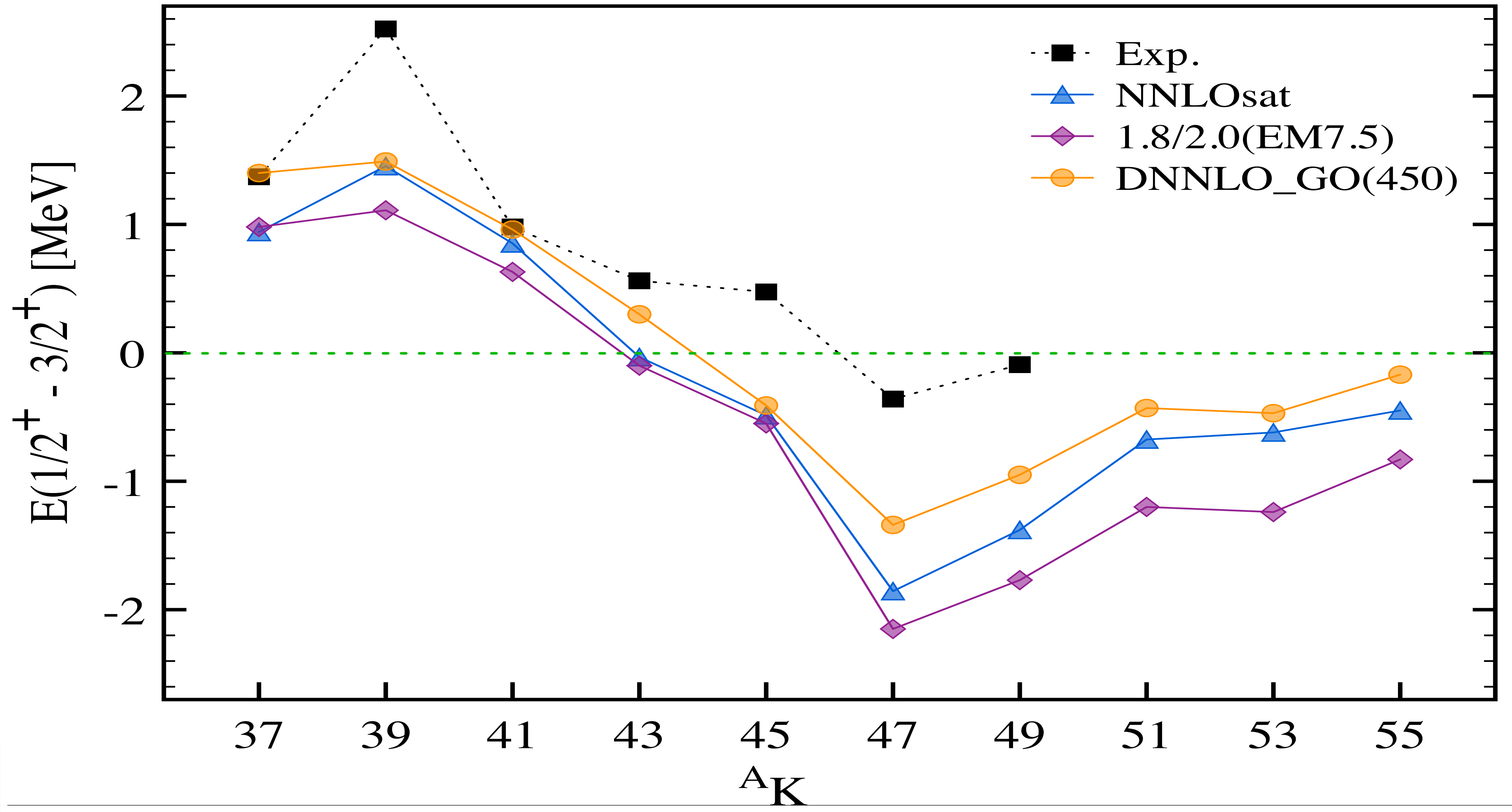
Imaginary part of the neutron p $_{1/2}$ hole self-energy in ^{16}O.

$d_{3/2}$ — $s_{1/2}$ inversion of protons at N=28



V. Somà, P. Navrátil, F. Raimondi, CB, T. Duguet, PRC**104**, 024315 (2021)

Both states populated by *proton removal form ^{A}Ca*

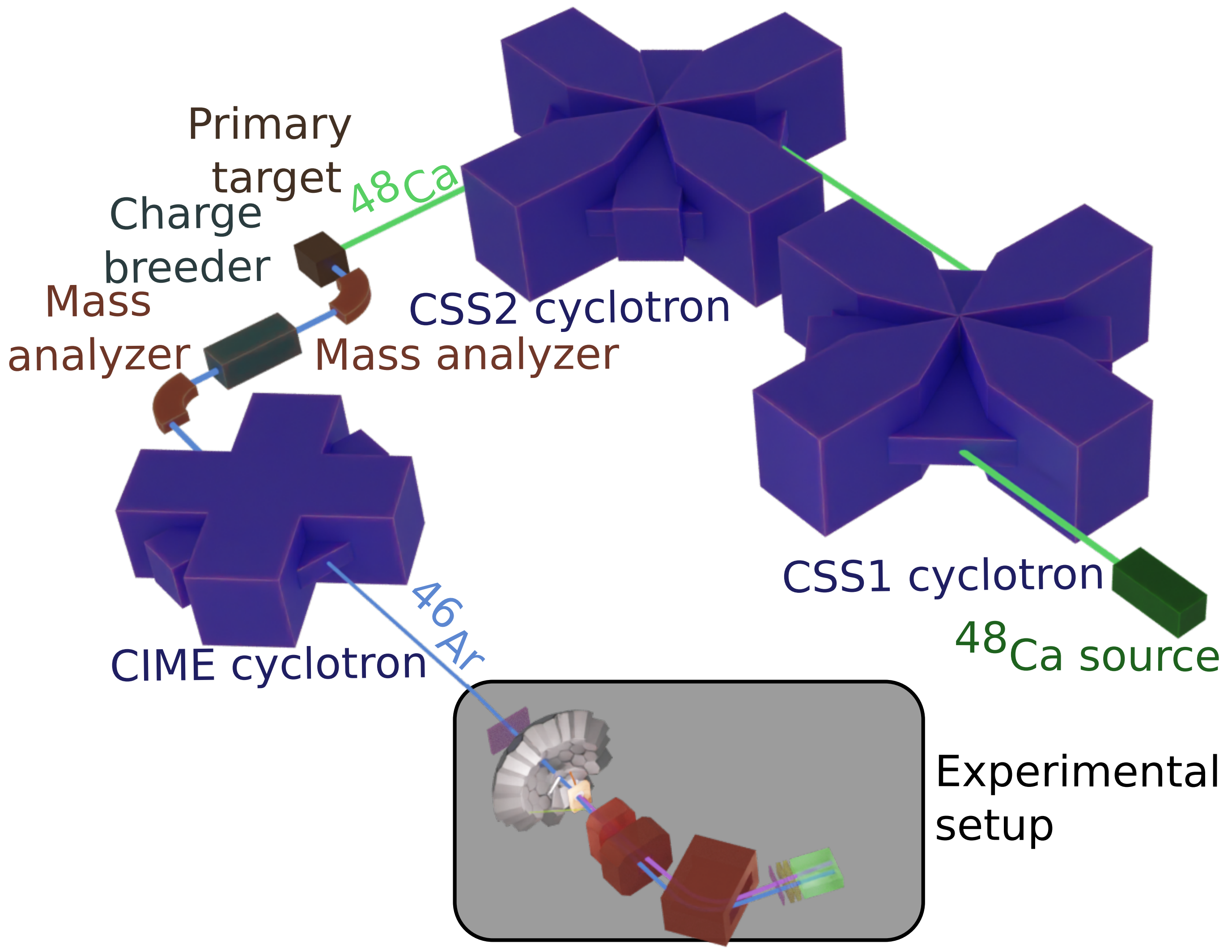


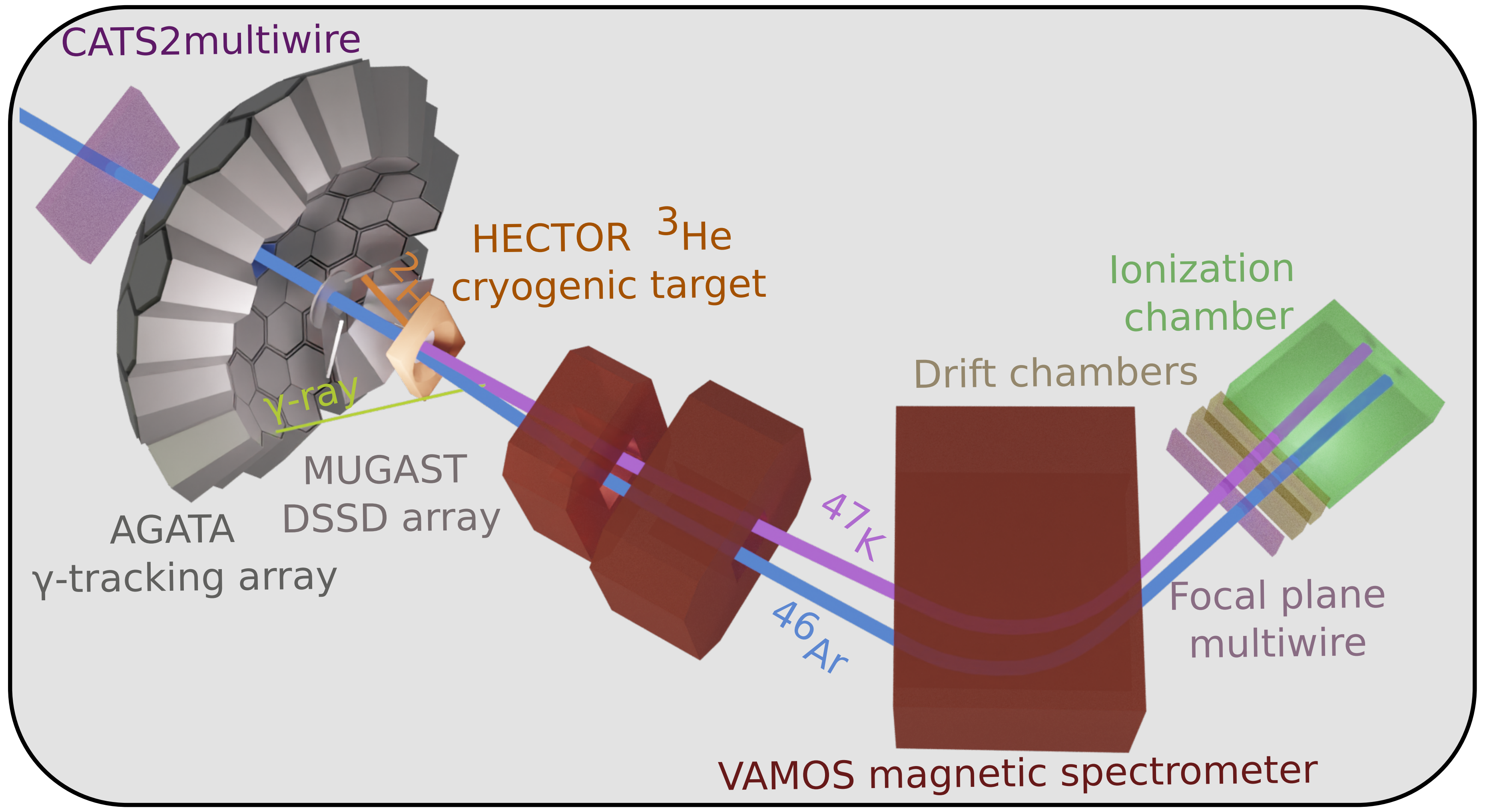
Papuga et al., PRL**110**, 172503 (2013); PRC**90**, 034321 (2014)

RIKEN, SEASTAR coll., Phys. Lett. **B802** 135215 (2020)

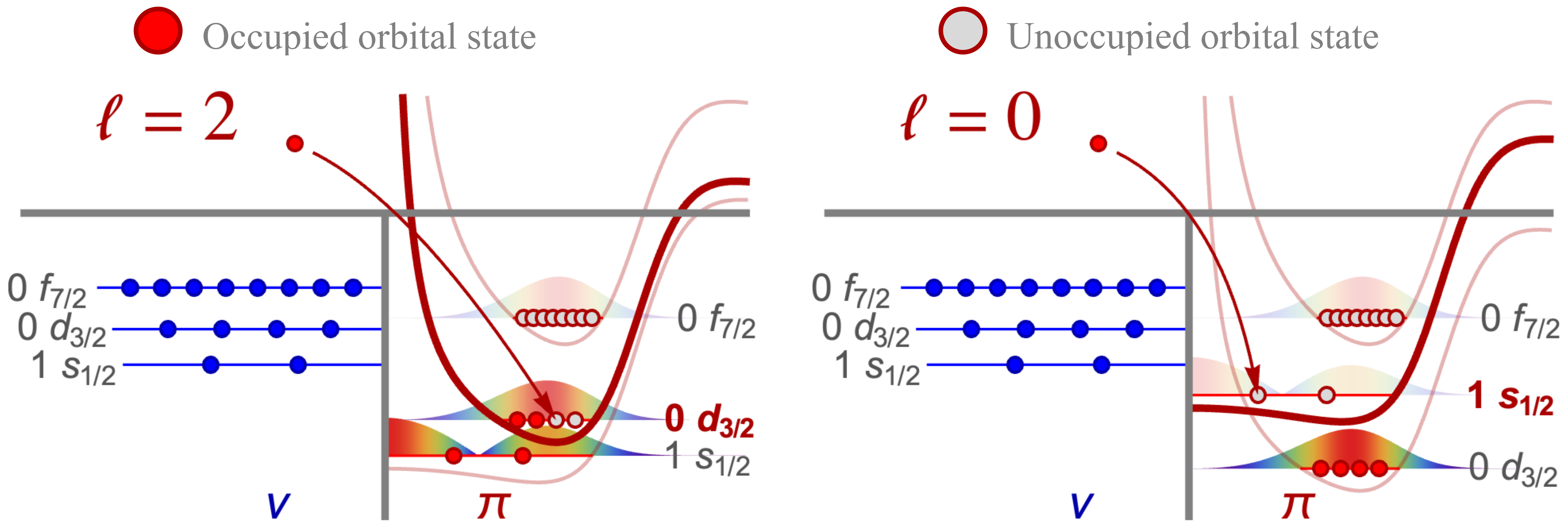
Linh, Gillibert, et al., PRC**104**, 044331 (2021)

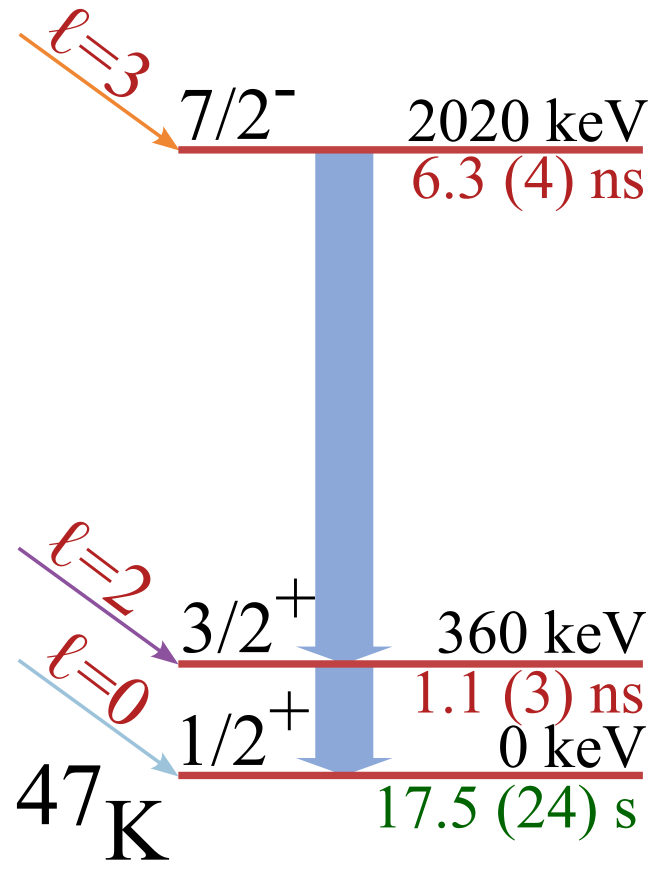
$^{46}Ar(^{3}He,d)^{47}K$ at GANIL





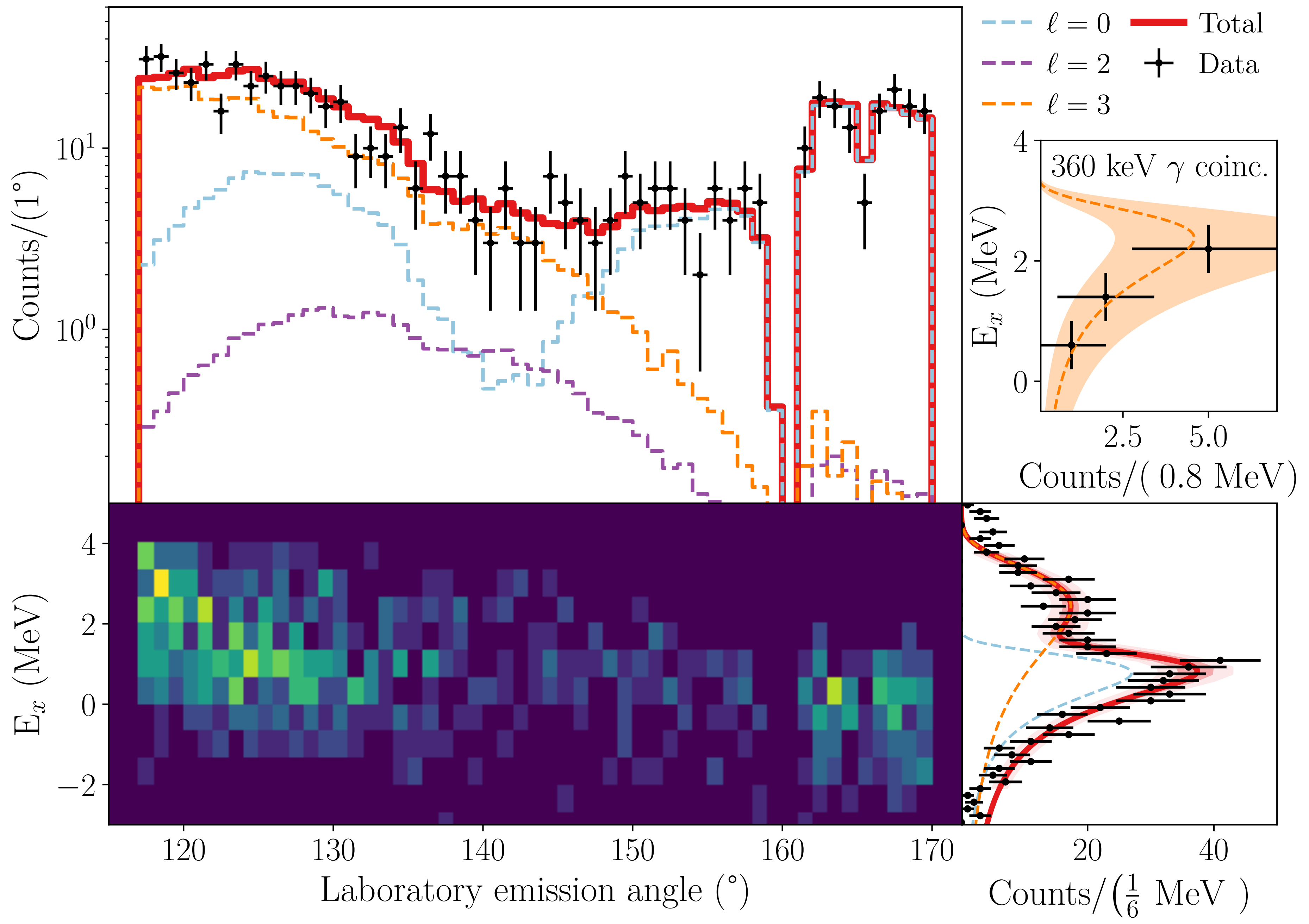
d3/2 - s1/2 inversion revisited from adding protons to ^{46}Ar

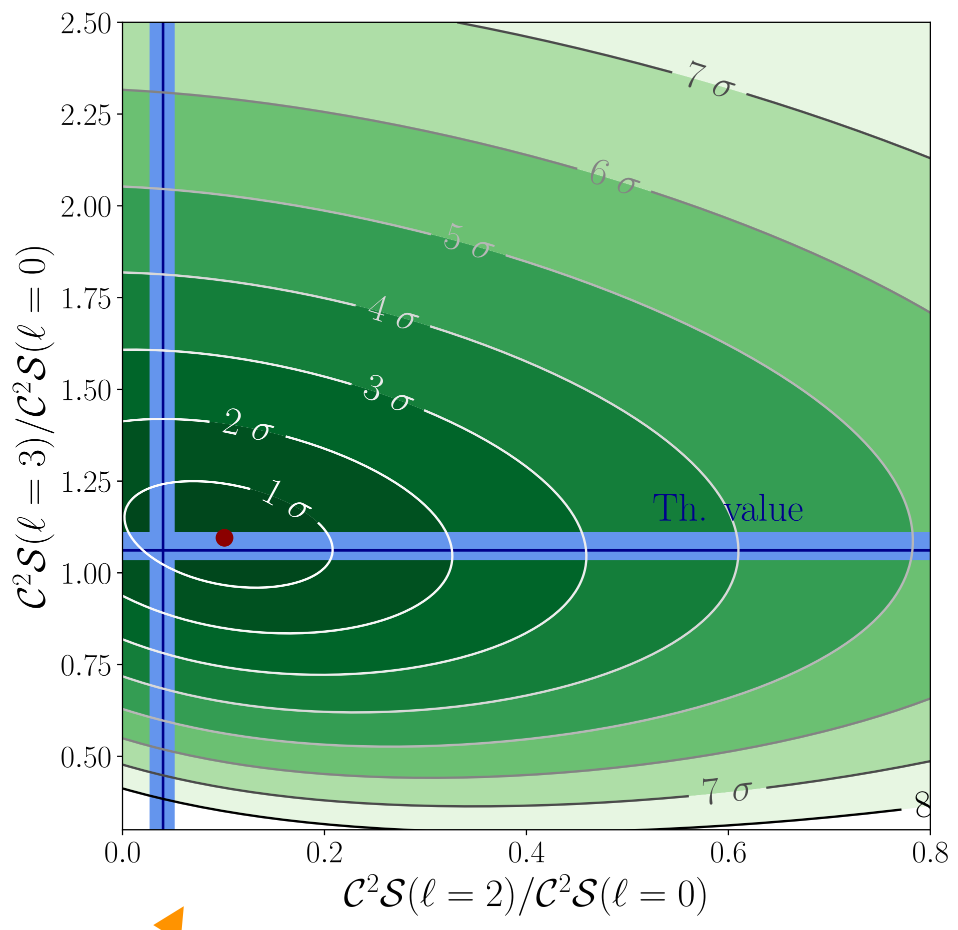




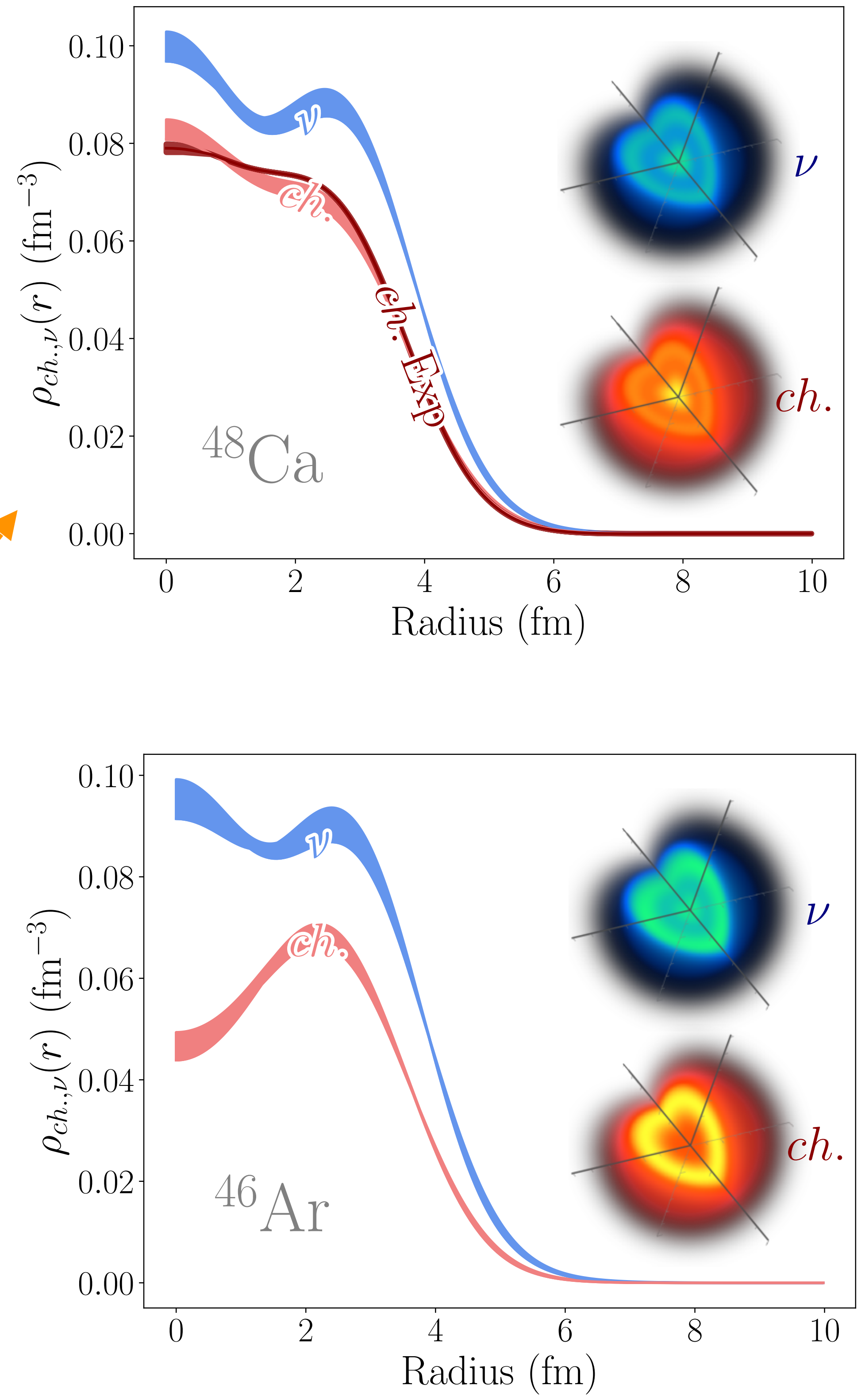
D. Brugnara, A. Gottardo, CB et al…

^{46}Ar(^{3}He,d)^{47}K at GANIL : New charge bubble in ^{46}Ar





$$\frac{d\sigma}{d\Omega} = \sum_k g_k \, \mathcal{C}^2\mathcal{S}_k \, \frac{d\sigma_k^{SP}}{d\Omega}$$



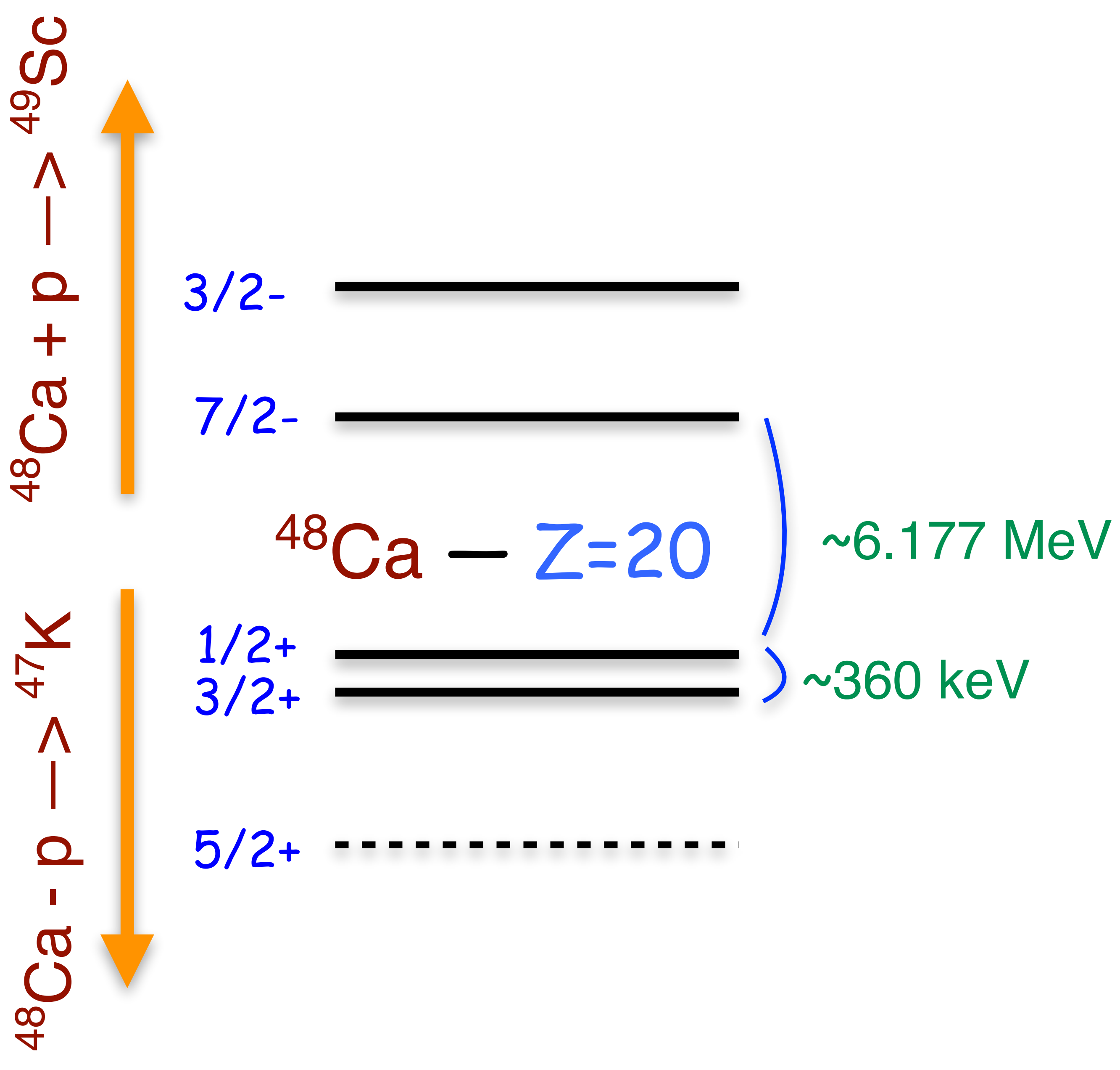
d3/2 - s1/2 inversion revisited from adding protons to ^{46}Ar

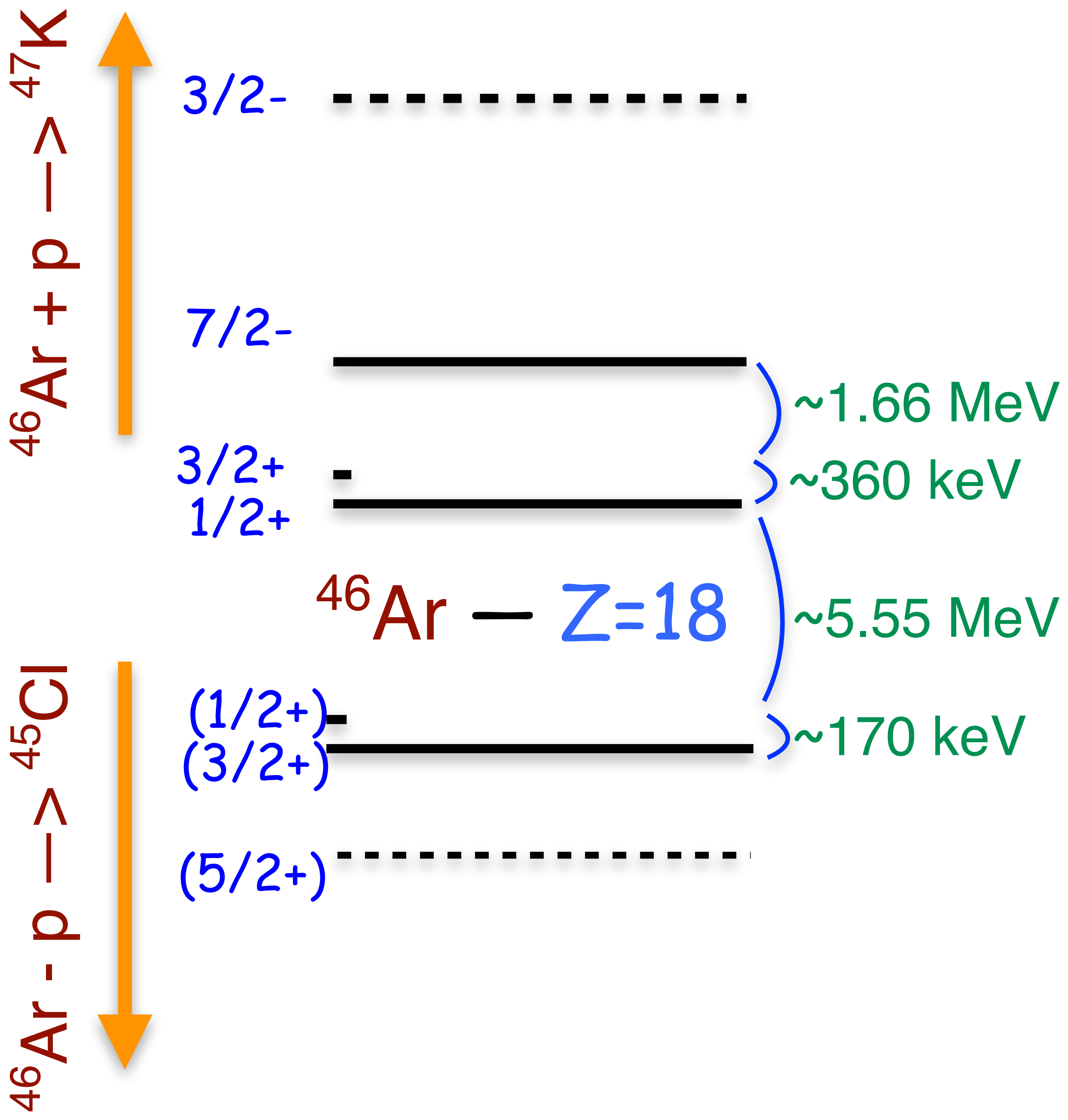
Theory & experiment for relative SFs agree within 1 sigma and confirms charge depletion in ^{46}Ar

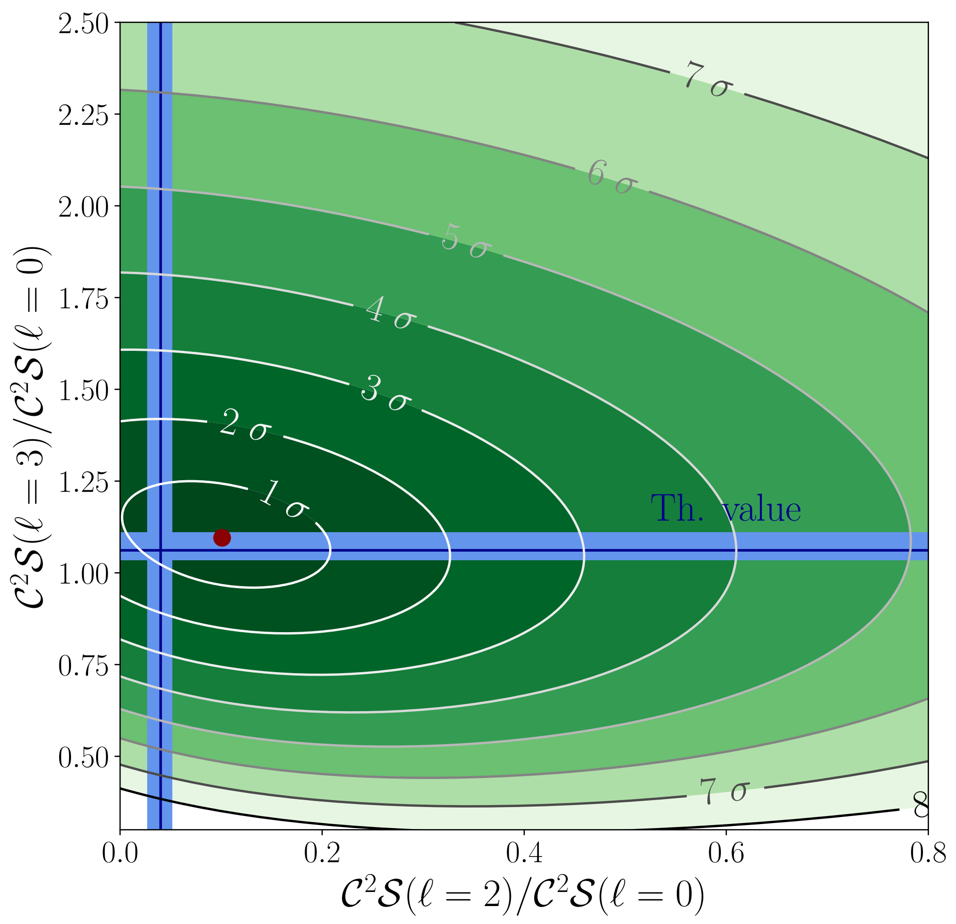
Theory bands are a combination of NNLOsat, Δ-full DN^{2}LO(394) and DN^{2}LO(450), as well as Darmstadt's saturating "magic force" 1.8/2.0 (7.5) —all of these have constrained LECs using the ^{16}O radius and few other mid-mass nuclei data.

D. Brugnara, A. Gottardo, CB et al… to be publishsed

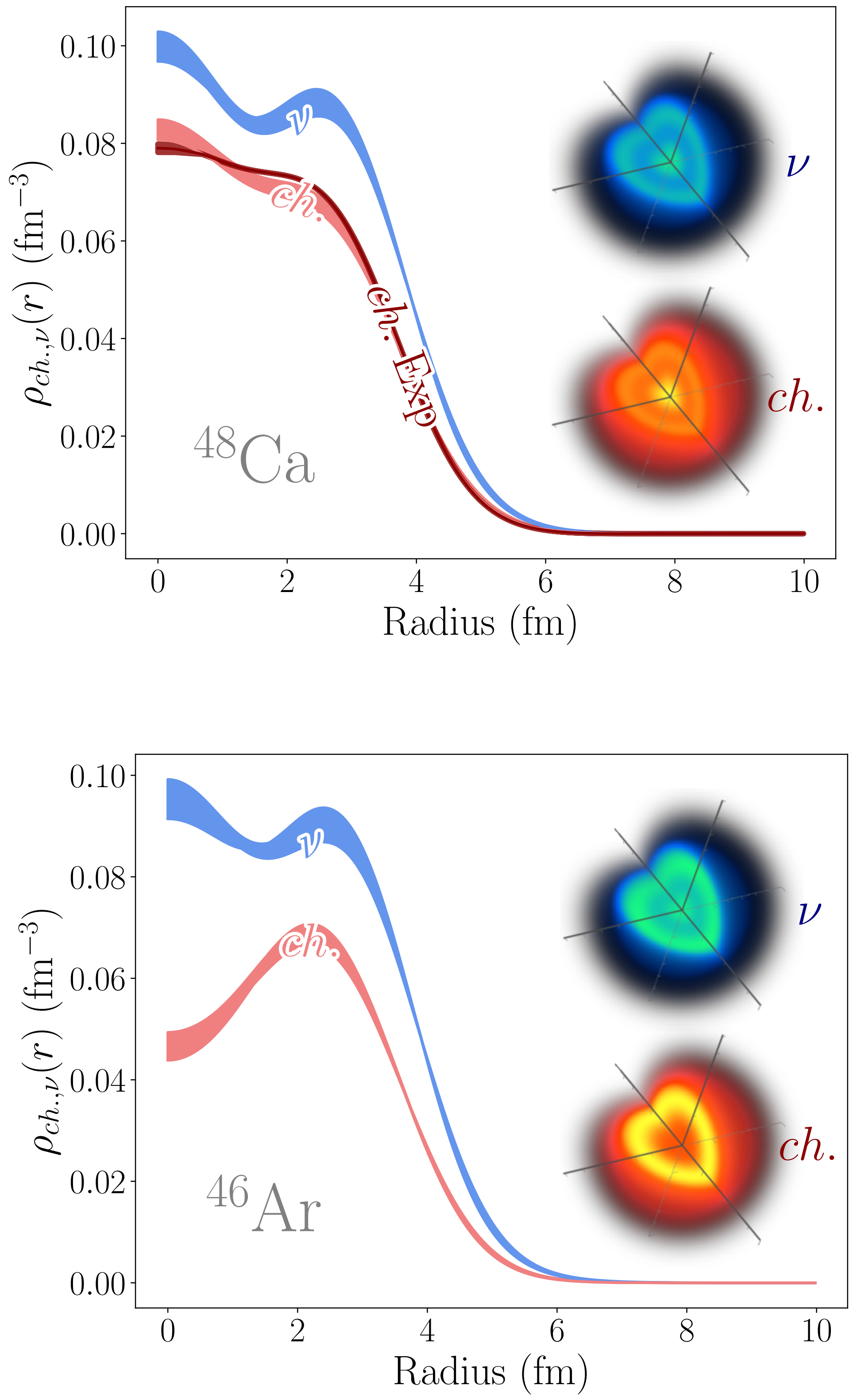
^{46}Ar(^{3}He,d)^{47}K at GANIL : New charge bubble in ^{46}Ar







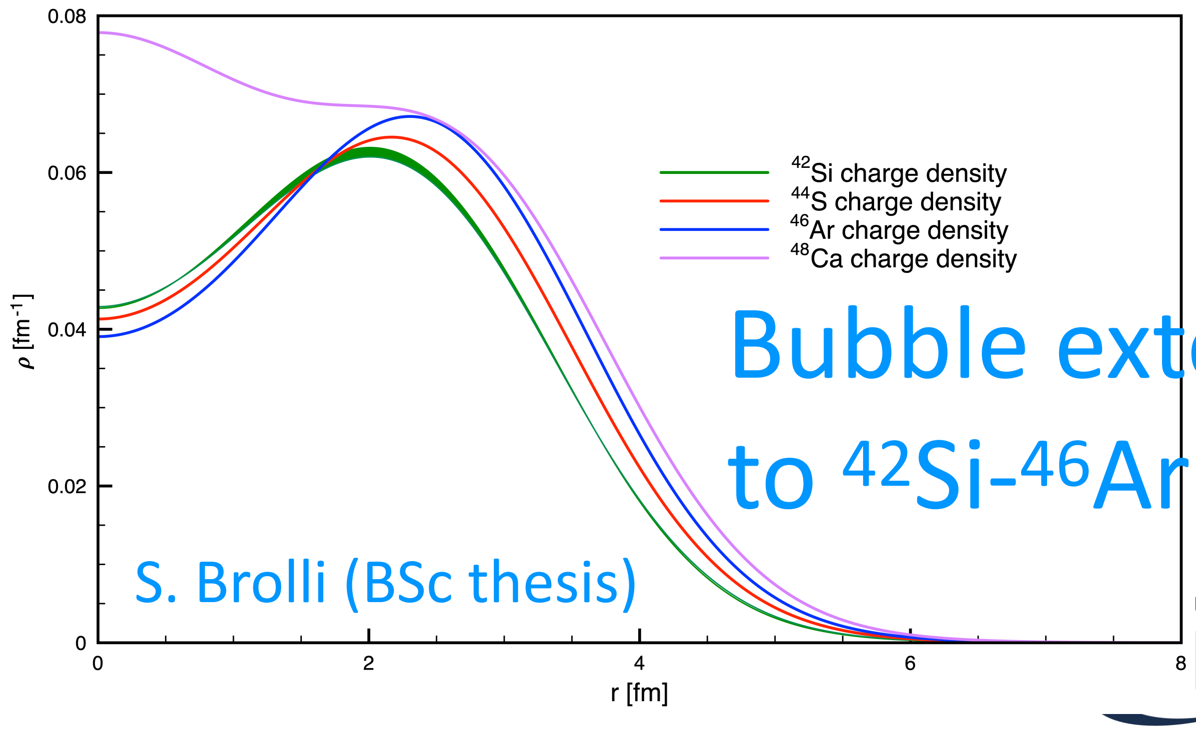
$$\frac{d\sigma}{d\Omega} = \sum_k g_k\, \mathcal{C}^2\mathcal{S}_k \frac{d\sigma_k^{SP}}{d\Omega}$$



ΔE$_{1/2+ - 3/2+}$ (exp) = 360 keV

ΔE$_{1/2+ - 3/2+}$ (exp) = 5,55 MeV
(theo) = 2,59 MeV

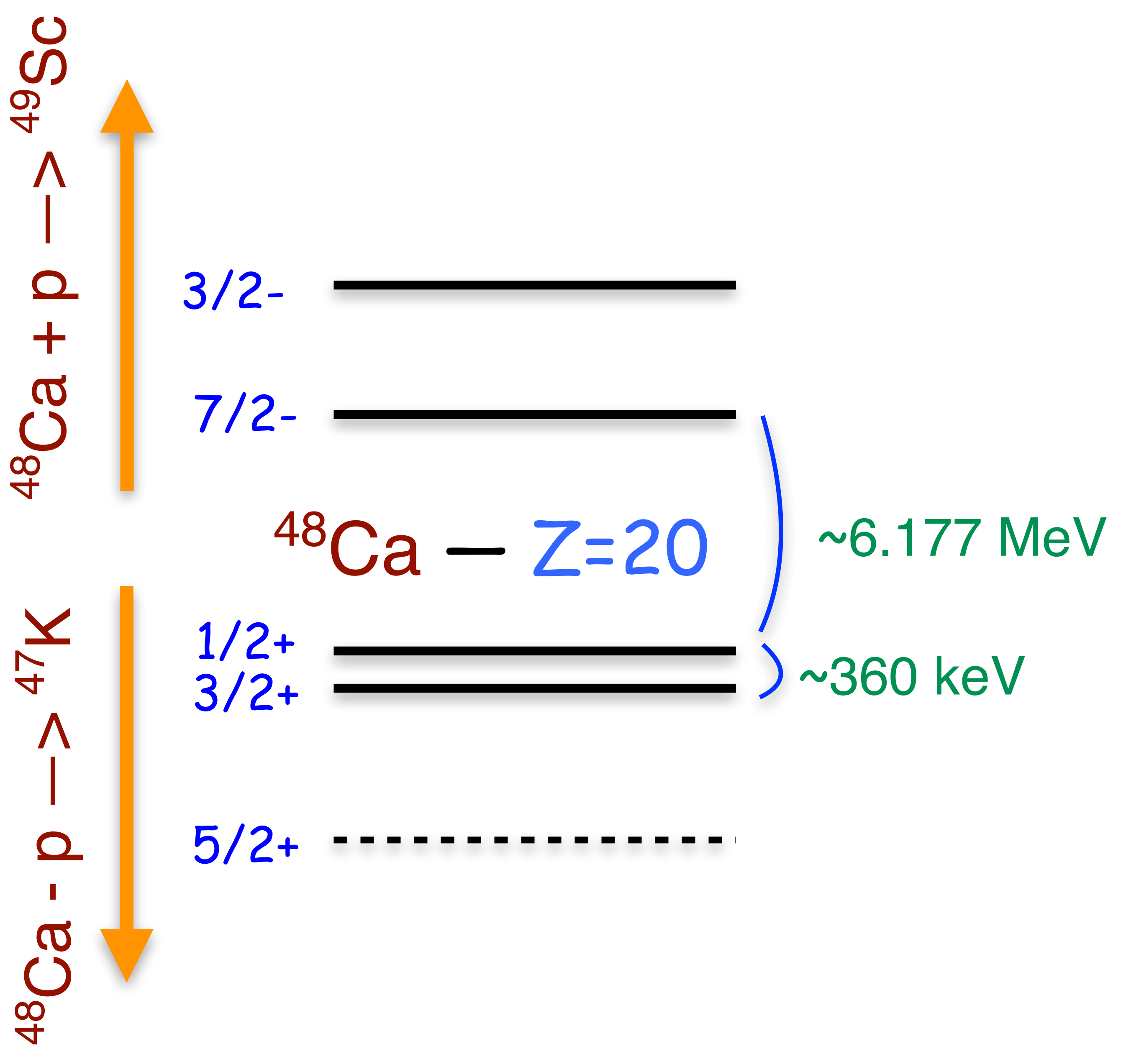
The d3/2 - s1/2 gap opening and new shell closure at Z=18 !!

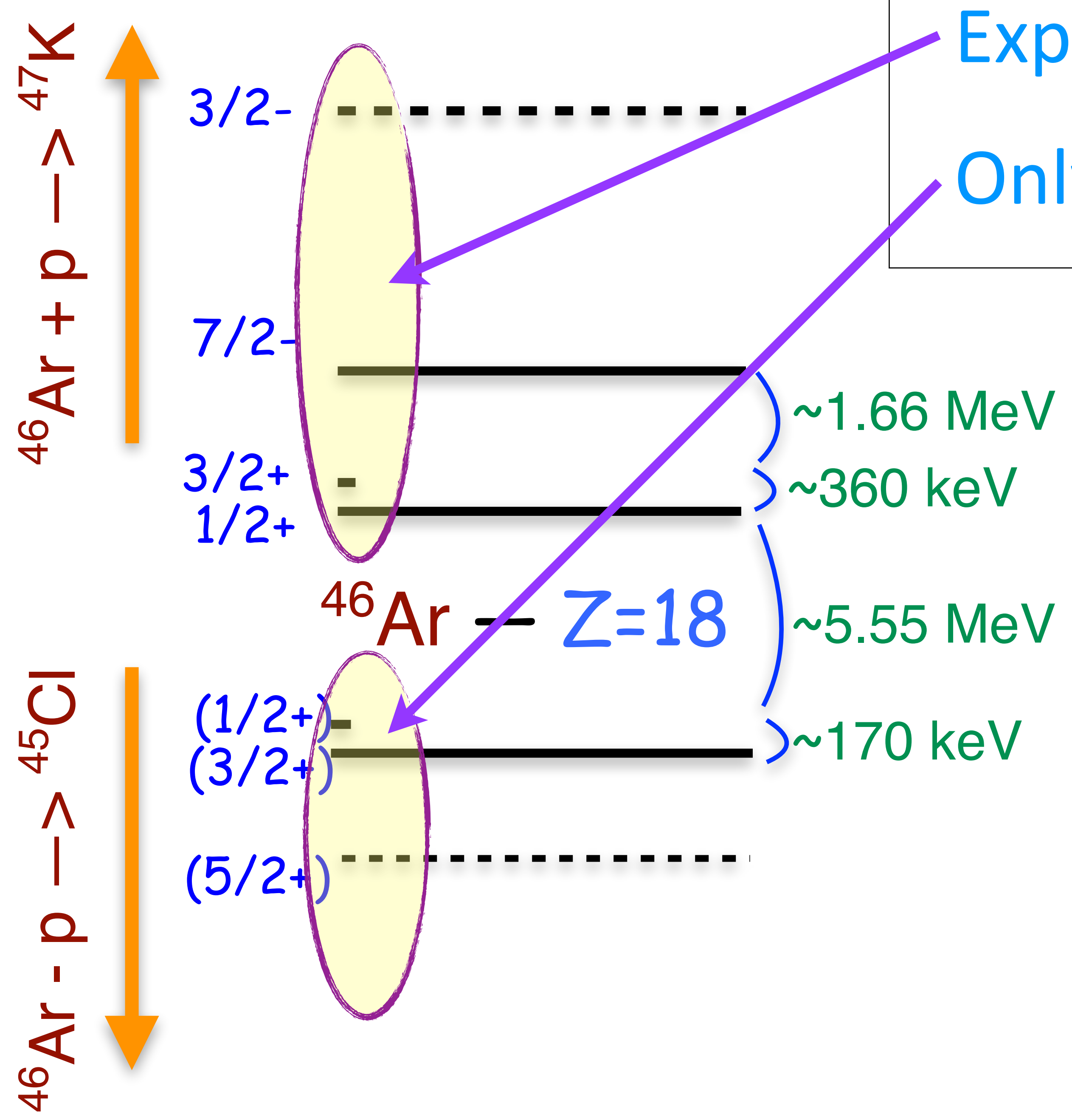


Bubble extends to ^{42}Si-^{46}Ar ??

D. Brugnara, A. Gottardo, CB et al... to be publishsed

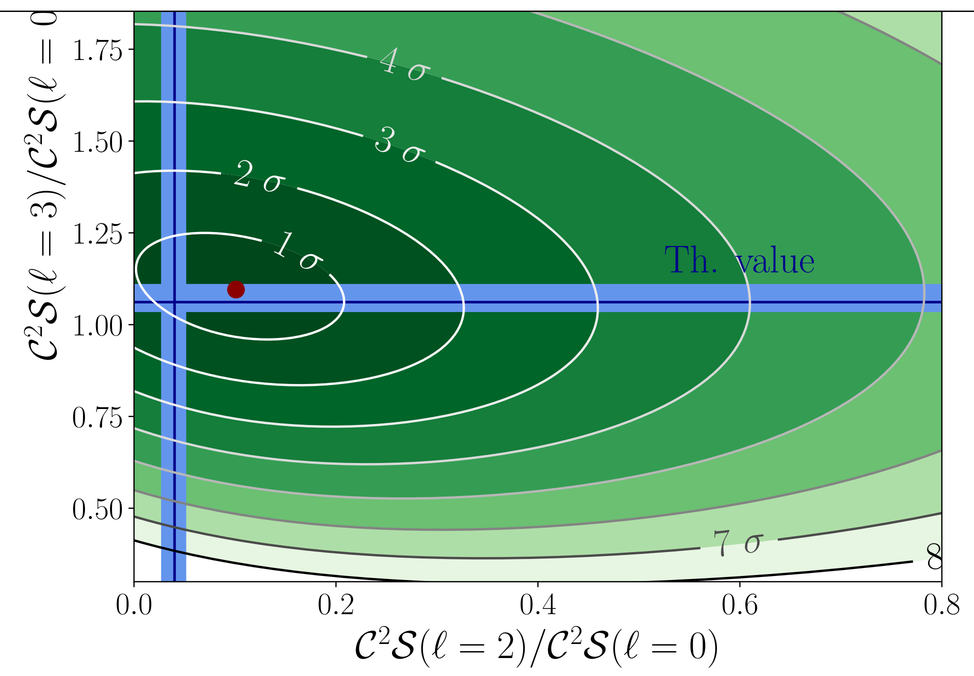
$^{46}Ar(^{3}He,d)^{47}K$ at GANIL : New charge bubble in ^{46}Ar



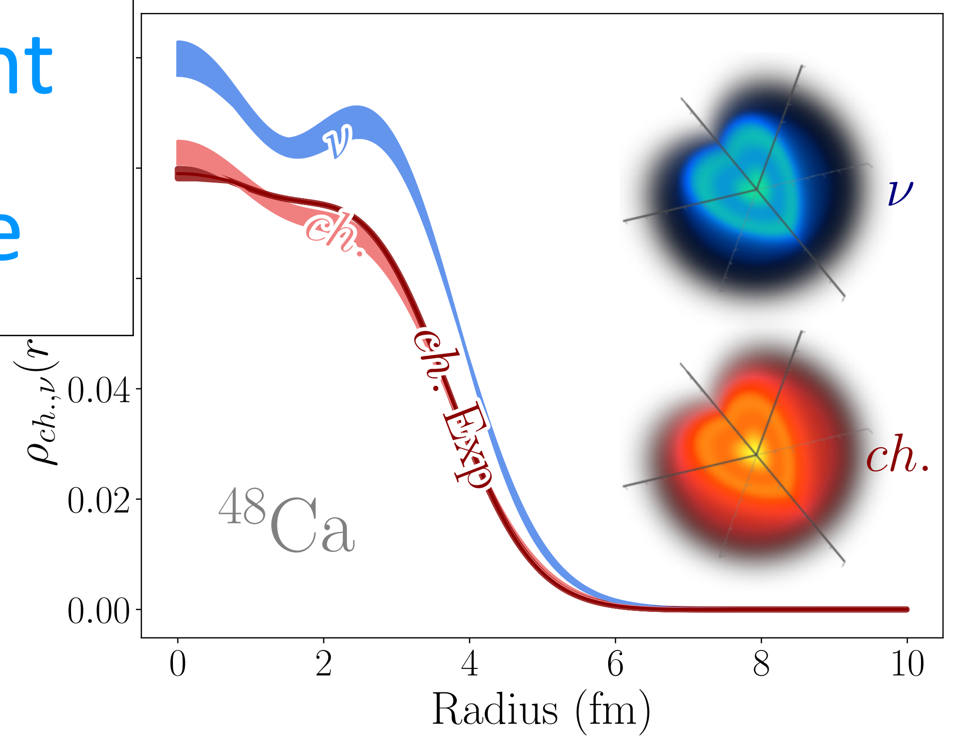


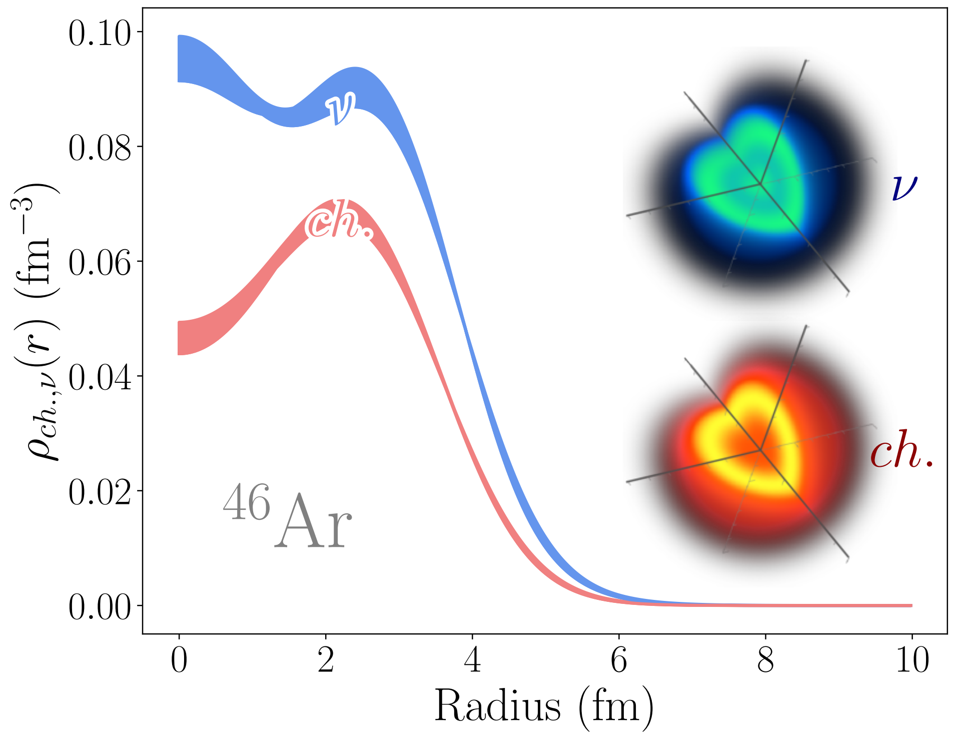
Experiment & theory in agreement

Only theory prediction is available



$$\frac{d\sigma}{d\Omega} = \sum_k g_k \, C^2 S_k \, \frac{d\sigma_k^{SP}}{d\Omega}$$

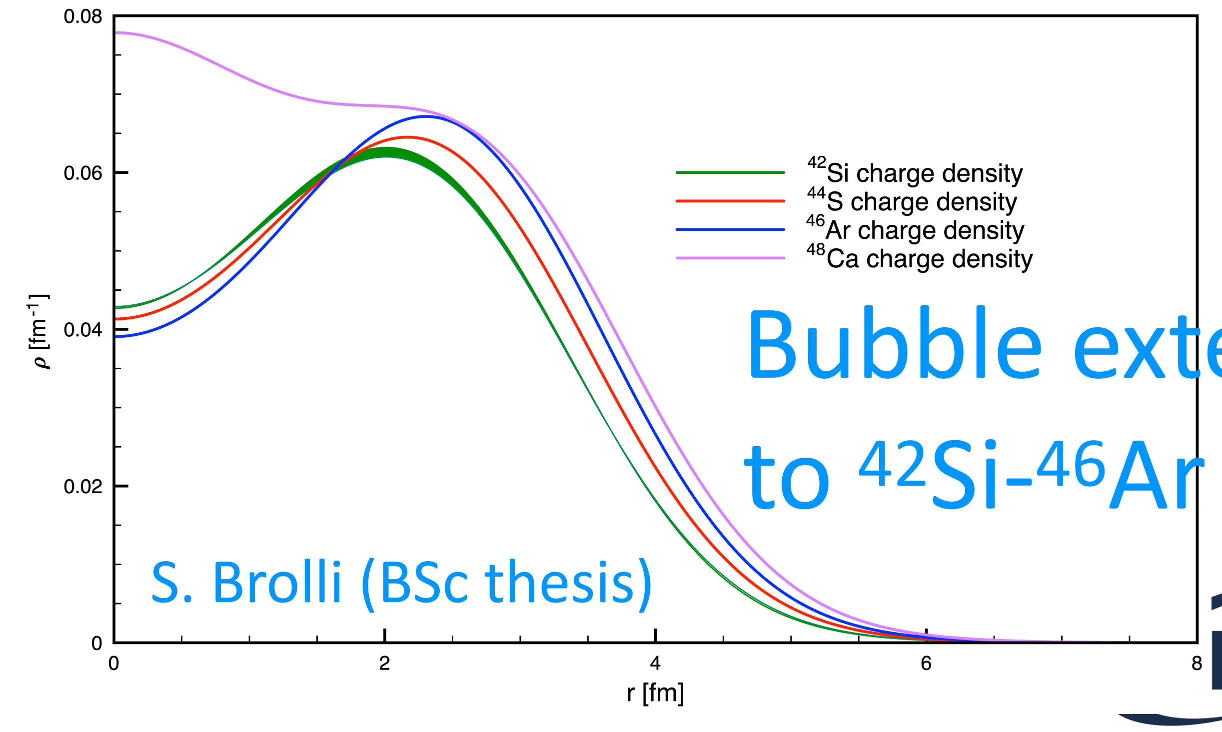




$\Delta E_{1/2+ - 3/2+}$ (exp) = 360 keV

$\Delta E_{1/2+ - 3/2+}$ (exp) = 5,55 MeV
(theo) = 2,59 MeV

The d3/2 - s1/2 gap opening and new shell closure at Z=18 !!



Bubble extends to ^{42}Si-^{46}Ar ??

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

D. Brugnara, A. Gottardo, CB et al… to be publishsed

INFN MILANO

Nuclear Density Functional from Ab Initio Theory

PHYSICAL REVIEW C **104**, 024315 (2021)

Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino,[1,2,*] C. Barbieri,[1,2] A. Carbone,[3] G. Colò,[1,2] A. Lovato,[4,5] F. Pederiva,[6,5] X. Roca-Maza,[1,2] and E. Vigezzi[2]

[1]*Dipartimento di Fisica "Aldo Pontremoli," Università degli Studi di Milano, 20133 Milano, Italy*

[2]*Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy*

[3]*Istituto Nazionale di Fisica Nucleare–CNAF, Viale Carlo Berti Pichat 6/2, 40127 Bologna, Italy*

See talk by F. Marino on Friday (EPS PhD thesis prize)

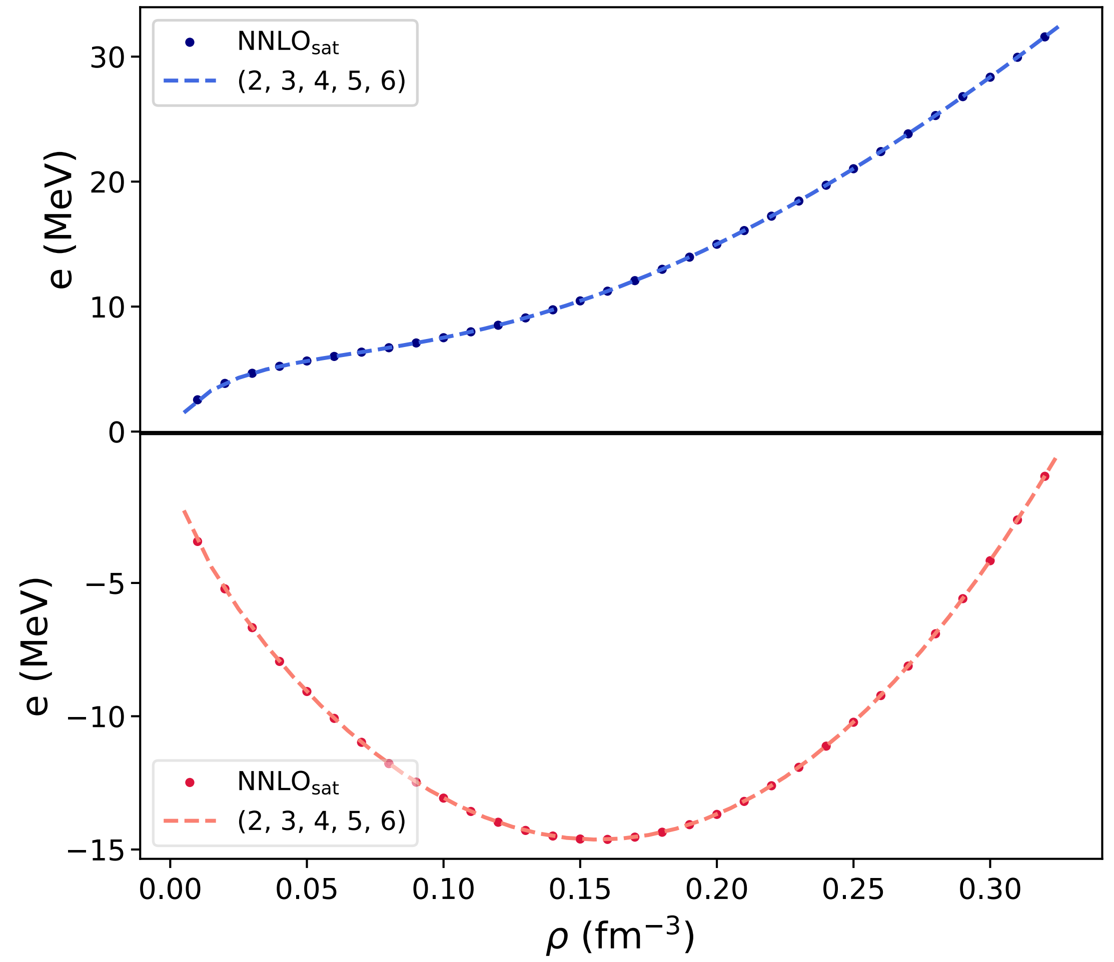
DFT is in principle exact – but the energy density functional (EDF) is not known

For nuclear physics this is even more demanding: need to link the EDF to theories rooted in QCD!

Machine-learn DFT functional on the nuclear equation of state

Jacob's ladder

+ approximate GA



Benchmark in finite systems

$$E = \int d\mathbf{r}\, \mathcal{E}(\mathbf{r}) = E_{\text{kin}} + E_{\text{pot}} + E_{\text{Coul}}$$

$$E_{\text{GA}} = E_{\text{LDA}} + E_{\text{surf}}$$

$$E_{\text{surf}} = \int d\mathbf{r} \left[\sum_{t=0,1} C_t^{\Delta} \rho_t \Delta \rho_t - \frac{W_0}{2} \left(\rho \nabla \cdot \mathbf{J} + \sum_q \rho_q \nabla \cdot \mathbf{J}_q \right) \right]$$

Algebraic diagrammatic construction [ADC(3)] for infinite matter

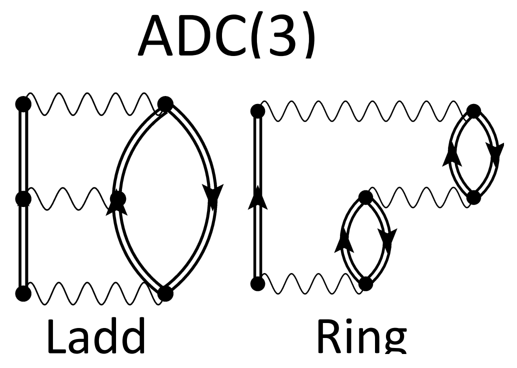
Finite size box (of length L) with periodic Boundary conditions:

$$\rho = \frac{A}{L} \qquad p_F = \sqrt[3]{\frac{6\pi^2\rho}{\nu_d}} \qquad \mathbf{k} = \frac{1}{L}\left(2\pi\mathbf{n} + \boldsymbol{\theta}\right), \qquad |\mathbf{k}|^2 = k_x^2 + k_y^2 + k_z^2 \leq \frac{4\pi^2}{L^2} N_{max}$$

$$\Sigma^{11}(\omega) = \Sigma^{11(\infty)} + \tilde{\Sigma}(\omega)$$

ADC(3) self energy:

$$\Sigma^{(\star)}_{\alpha\beta}(\omega) = -U_{\alpha\beta} + \Sigma^{(\infty)}_{\alpha\beta} + M^{\dagger}_{\alpha,r}\left[\frac{1}{\omega - [E^{>} + C]_{r,r'} + i\eta}\right]_{r,r'} M_{r',\beta} + N_{\alpha,s}\left[\frac{1}{\omega - (E^{<} + D) - i\eta}\right]_{s,s'} N^{\dagger}_{s',\beta}$$



Gorkov superfluid formulation (at 1st order)

$$\mathbf{g}(\omega) = \mathbf{g}^{(0)}(\omega) + \mathbf{g}^{(0)}(\omega)\mathbf{\Sigma}^{\star}(\omega)\mathbf{g}(\omega)$$

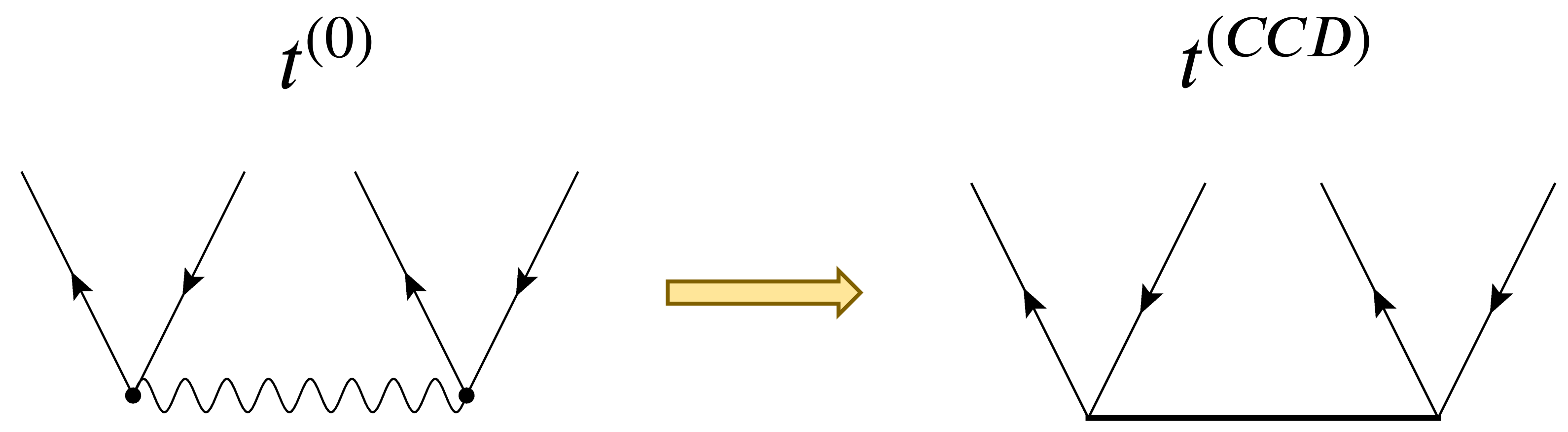
Normal self-energy — Pairing field

$$\Sigma^{\star}(\omega) = \begin{pmatrix} \Sigma^{11}(\omega) & \Delta \\ \Delta^{*} & \Sigma^{22}(\omega) \end{pmatrix}$$

Extension of ADC(3) ⟹ **ADC(3)-D**

In $\tilde{\Sigma}(\omega)$, replace $t^{(0)}$ with **converged T_2 amplitudes**

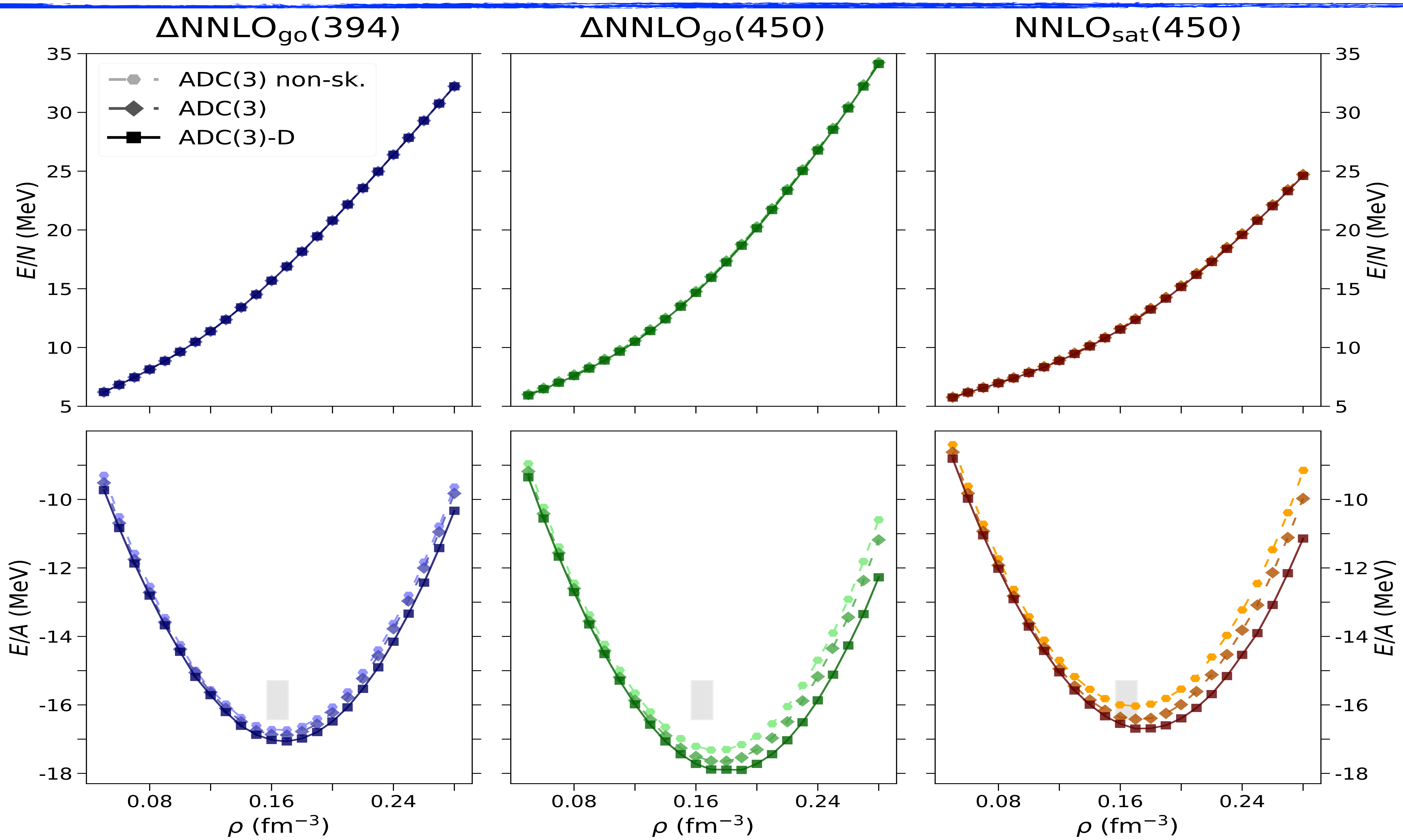
$$(t^{(0)})^{ab}_{ij} = \frac{\langle ab|v|ij\rangle_A}{\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b}$$



Somà et al., Phys. Rev. C **84**, 064317 (2011)
Raimondi et al., Phys Rev C **97**, 054308 (2018)
Barbieri et al., Lect. Notes Phys. **936**, 571 (2017) - Ch 11
Barbieri et al., Phys. Rev. C **105**, 044330 (2022)

F. Marino, CB, and G. Colò, arXiv:2510.xxx, to be submitted

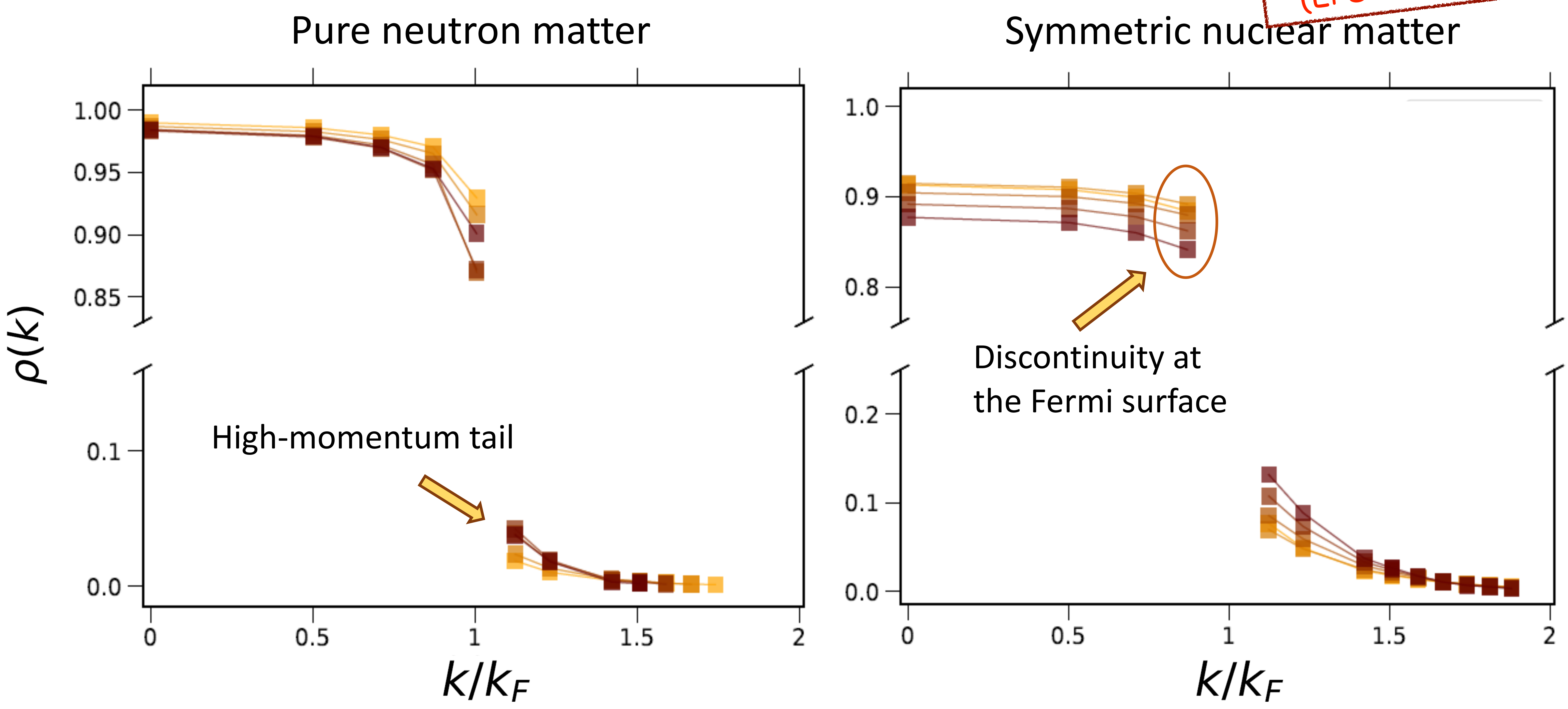
Equations of state (T=0)



F. Marino, CB, and G. Colò, arXiv:2510.xxx, to be submitted

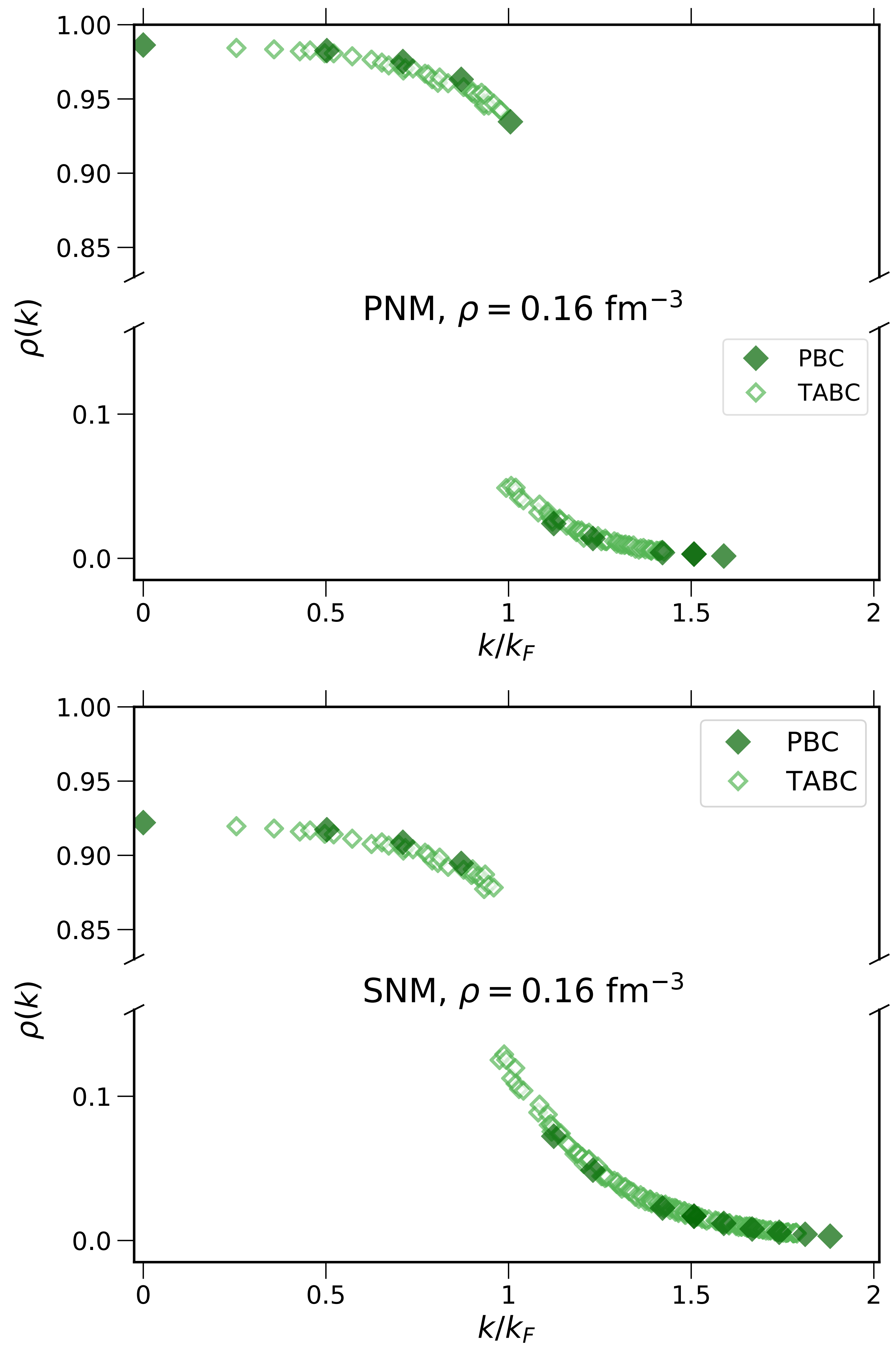
Momentum distributions

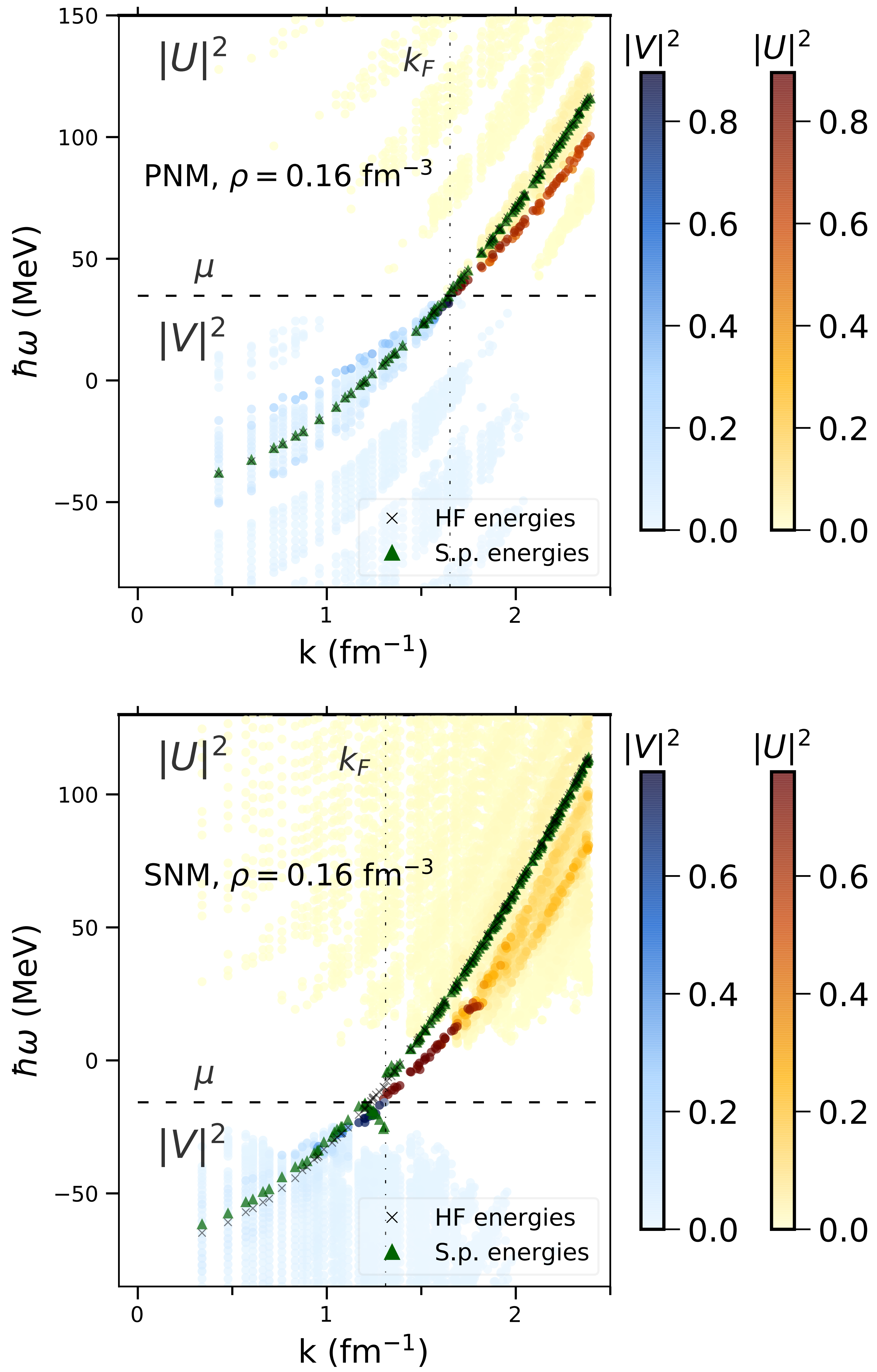
See talk by F. Marino on Friday (EPS PhD thesis prize)



F. Marino, CB, et al, PoS QNP2024, 180 (2025)
F. Marino, CB, and G. Colò, arXiv:2510.xxx, to be

Removing finite size effects - Twisted Angle BC





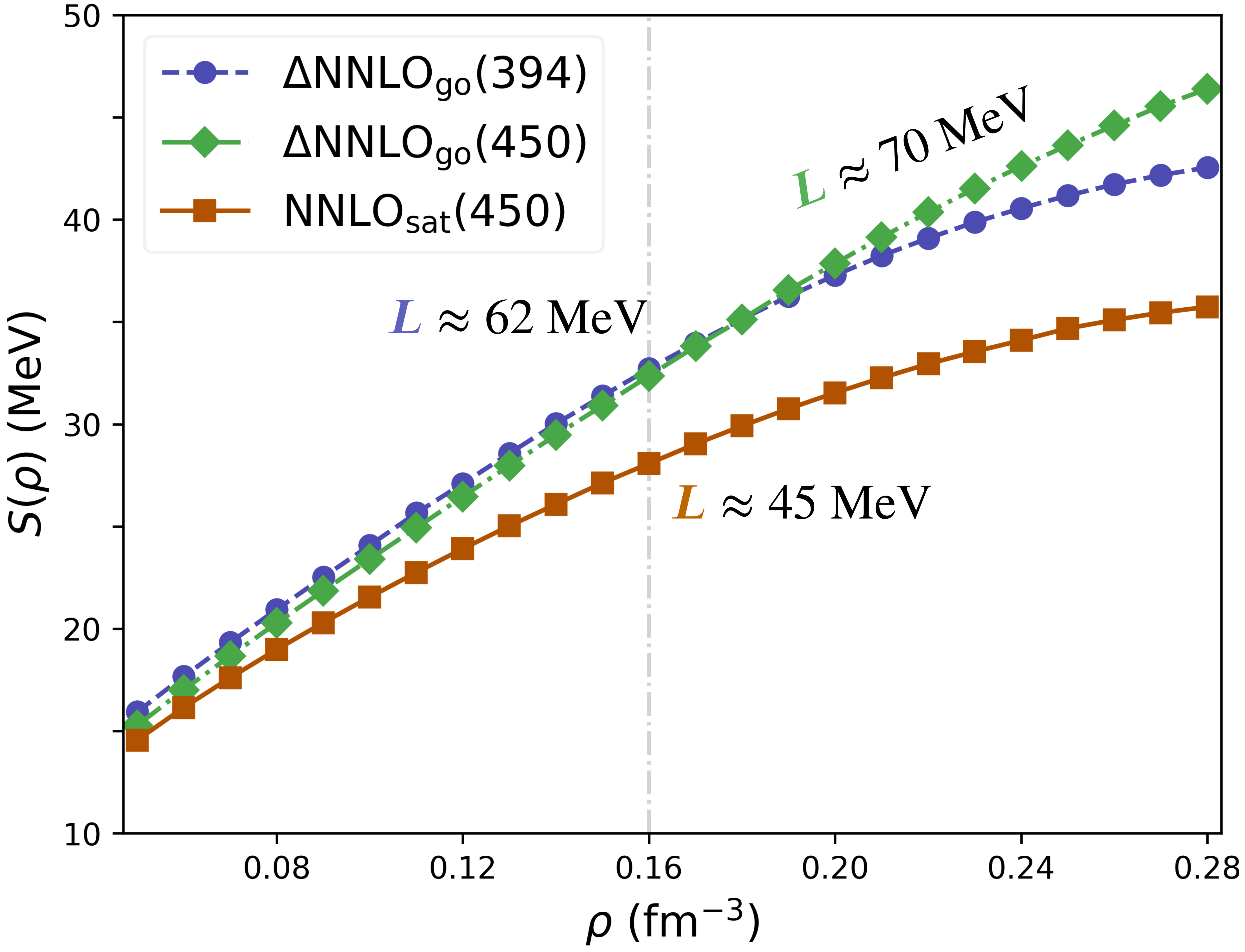
$\Delta NNLO_{go}(450)$

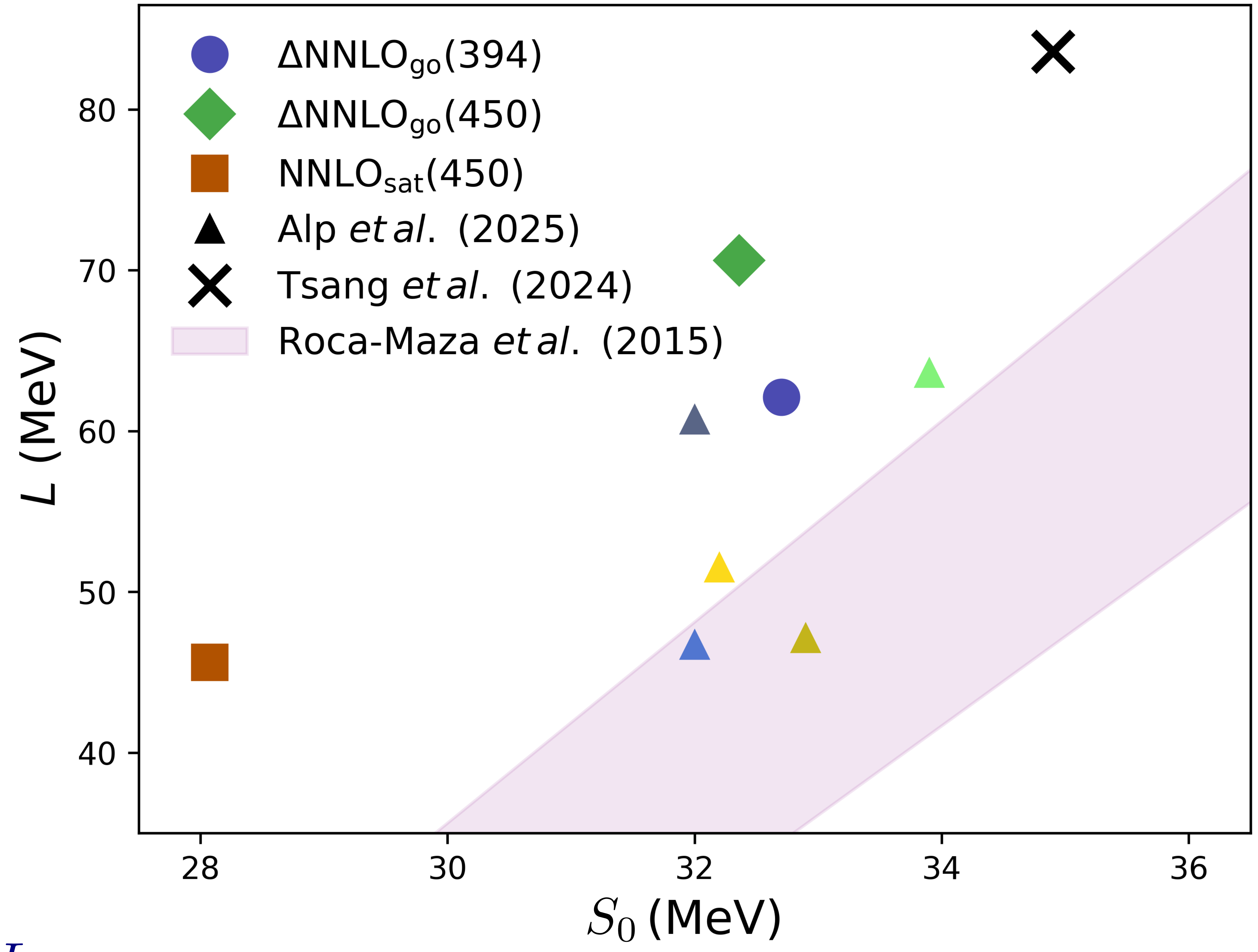
See talk by F. Marino on Friday (EPS PhD thesis prize)

F. Marino, CB, and G. Colò, arXiv:2510.xxx, to be submitted

Symmetry energy

See talk by
F. Marino on Friday
(EPS PhD thesis prize)





$$S(\rho) = S(\rho_0) + \frac{L}{3\rho_0}(\rho - \rho_0) + \cdots$$

F. Marino, CB, and G. Colò, arXiv:2510.xxx, to be submitted.

Alp et al., arXiv:2504.18259
Burgio et al., Front. Astron. Space Sci. **11**, 1505560 (2024)
Lynch and Tsang, Phys. Lett. B **830**, 137098 (2022)
Roca-Maza et al., Phys. Rev. C **92**, 064304 (2015)

UNIVERSITÀ DEGLI STUDI DI MILANO

RPMBT23

Upcoming 23rd conference on "Recent Progress in MAny-Body Theory" RPMBT-XXIII

Milan, Italy: 14-18 Sept 2026 (main event)

9-11 Sept. (satellite school on QC)

Duomo

RPMBT23

Porta Nuova district

UNIVERSITÀ DEGLI STUDI DI MILANO

RPMBT23

Upcoming 23rd conference on "Recent Progress in MAny-Body Theory" RPMBT-XXIII

Milan, Italy: 14-18 Sept 2026 (main event)

9-11 Sept. (satellite school on QC)

Duomo

RPMBT23

Thank you for your attention !!!

Porta Nuova district

Thank you for your attention !!!

Removing finite size effects - Twisted Tngle BC