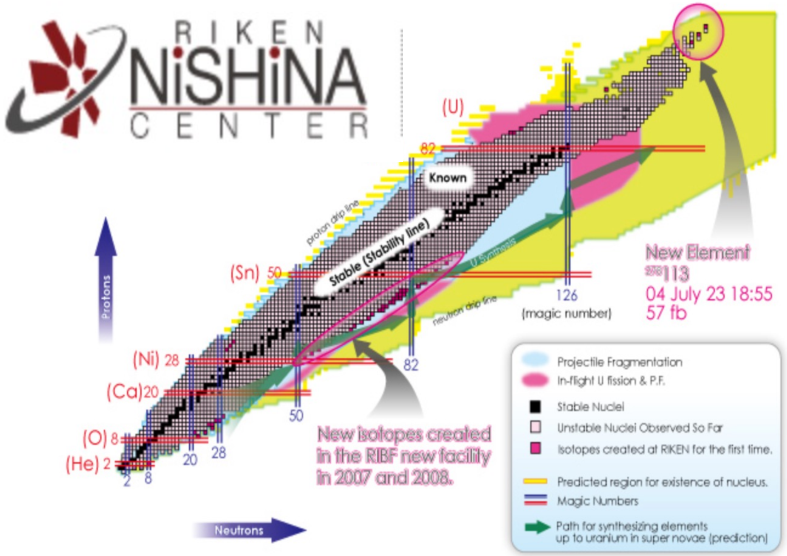
Predicting reaction observables at intermediate energies: selected recent developments

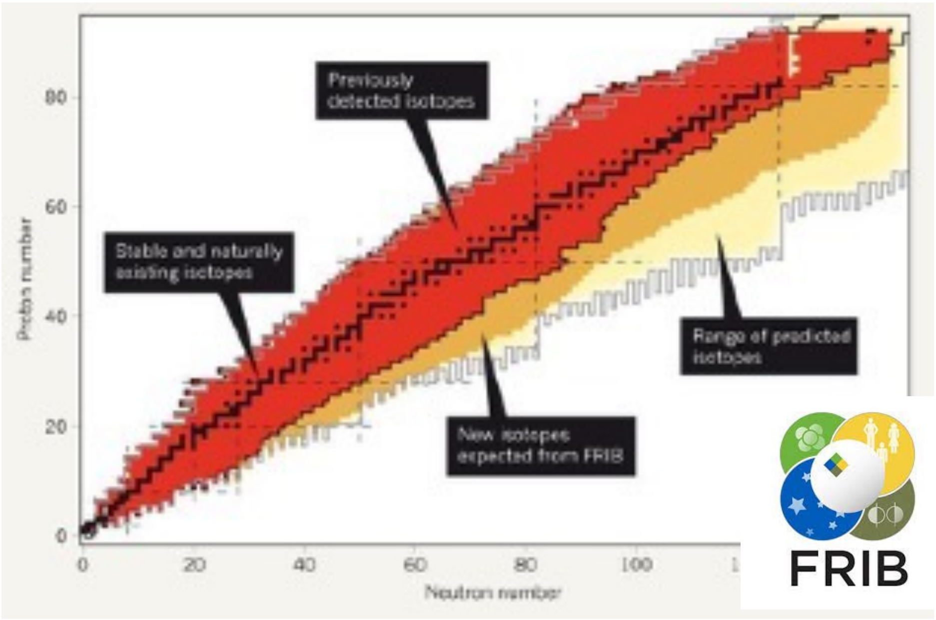
Chloë Hebborn (IJClab, Orsay, France)

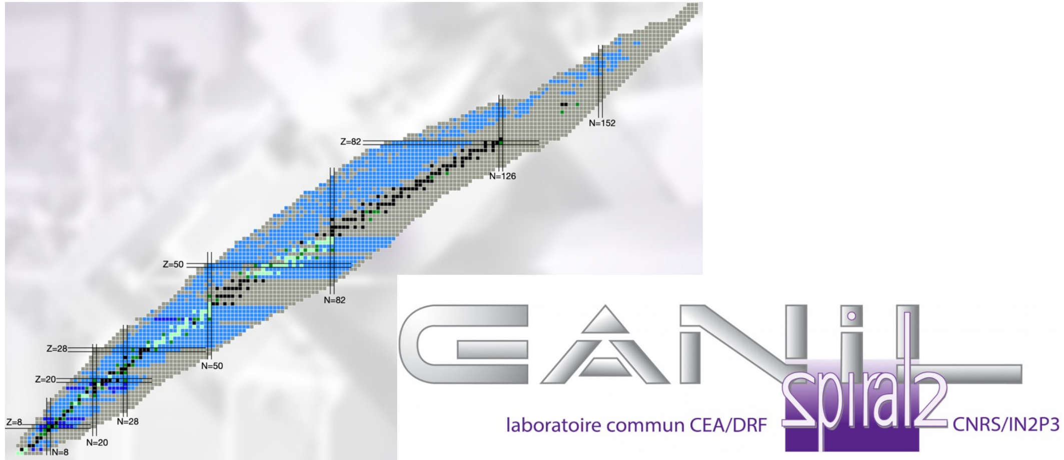
More info …

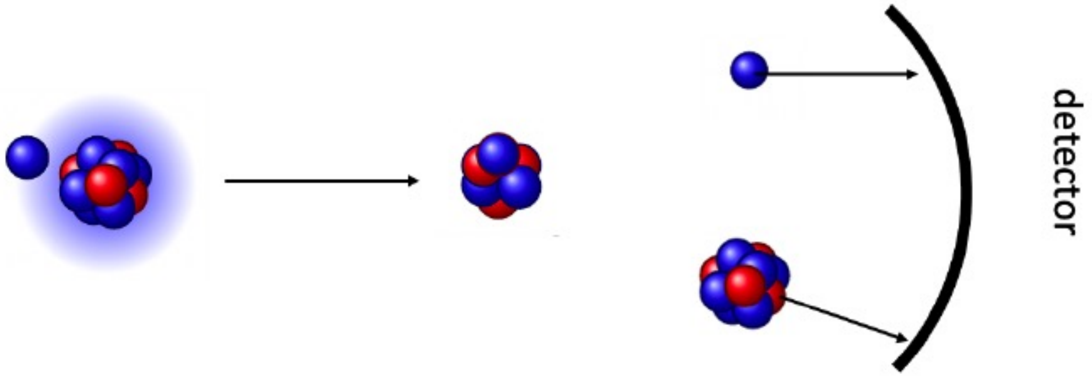
Hebborn, Nunes, Lovell, PRL **131**, 212503 (2023). Erratum: PRL **132**, 139901 (2024)
Hebborn, Nunes, Frontiers **13** (2025).
Shiu, Hebborn, Nunes, arXiv:2507.13291
Hebborn, Potel, PRC **107** 014607 (2023).
Smith, Hebborn, Nunes, Zegers PRC **110** 034602 (2024).

Exciting time to be a nuclear physicists as RIB facilities enable the study of many unstable nuclei!

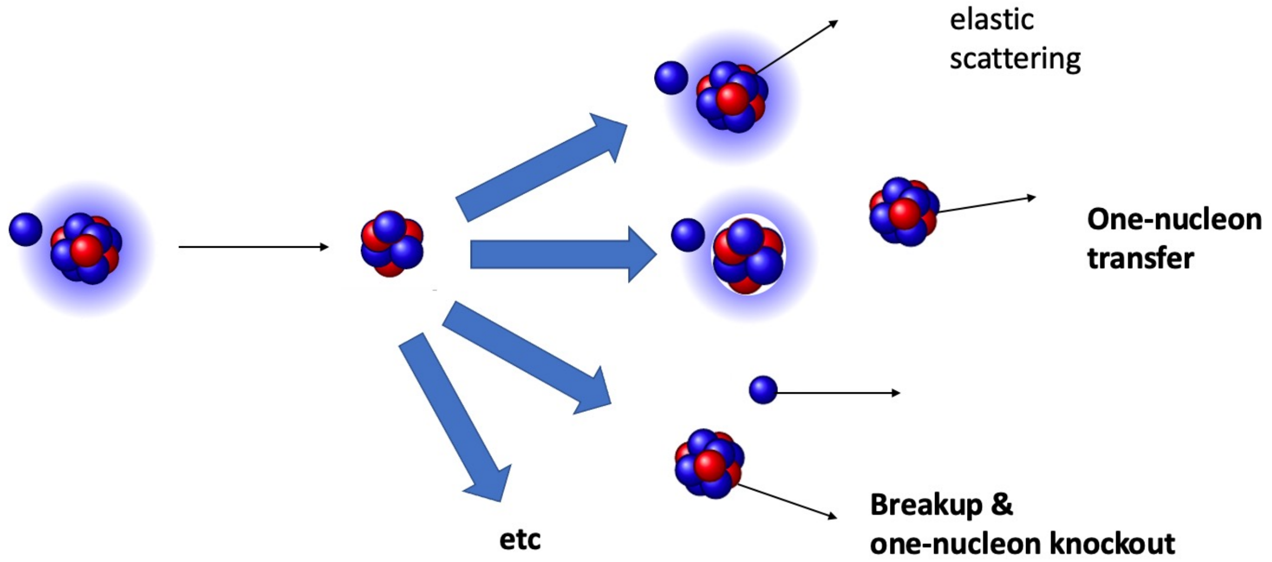




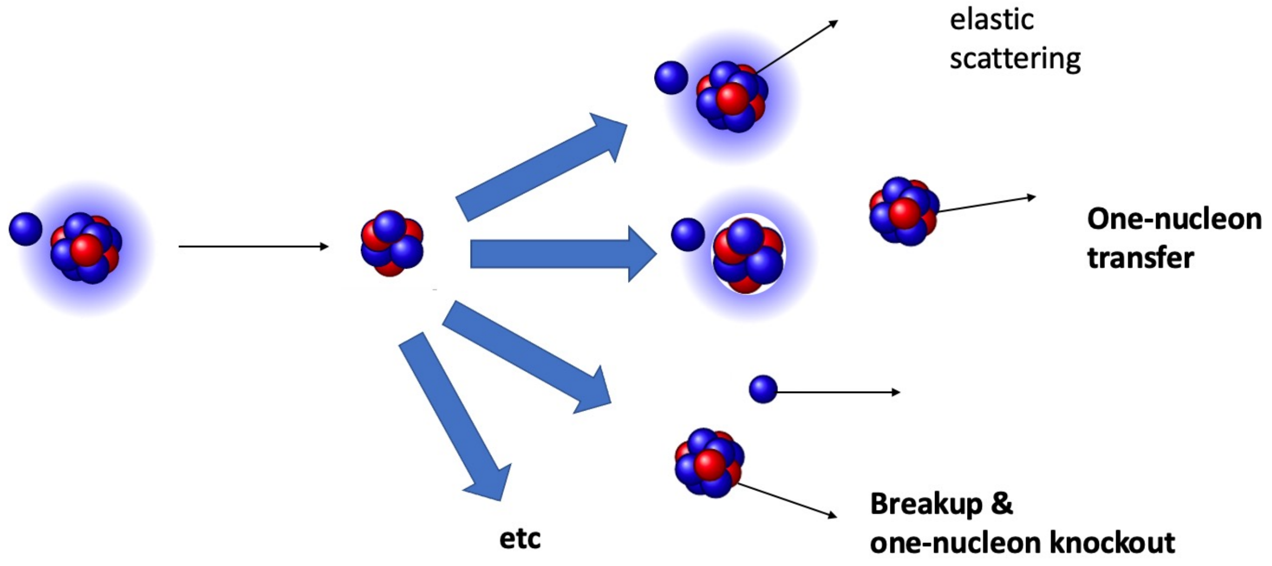




One-nucleon removal reactions give insights on long-and short-range correlations in nuclei



One-nucleon removal reactions give insights on long-and short-range correlations in nuclei

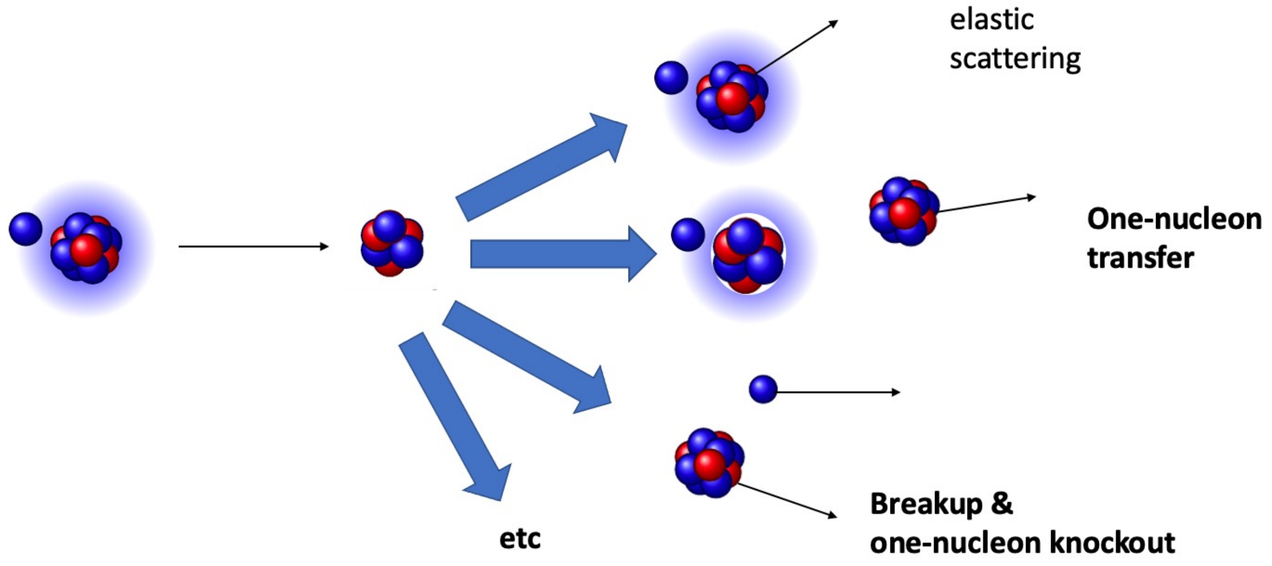


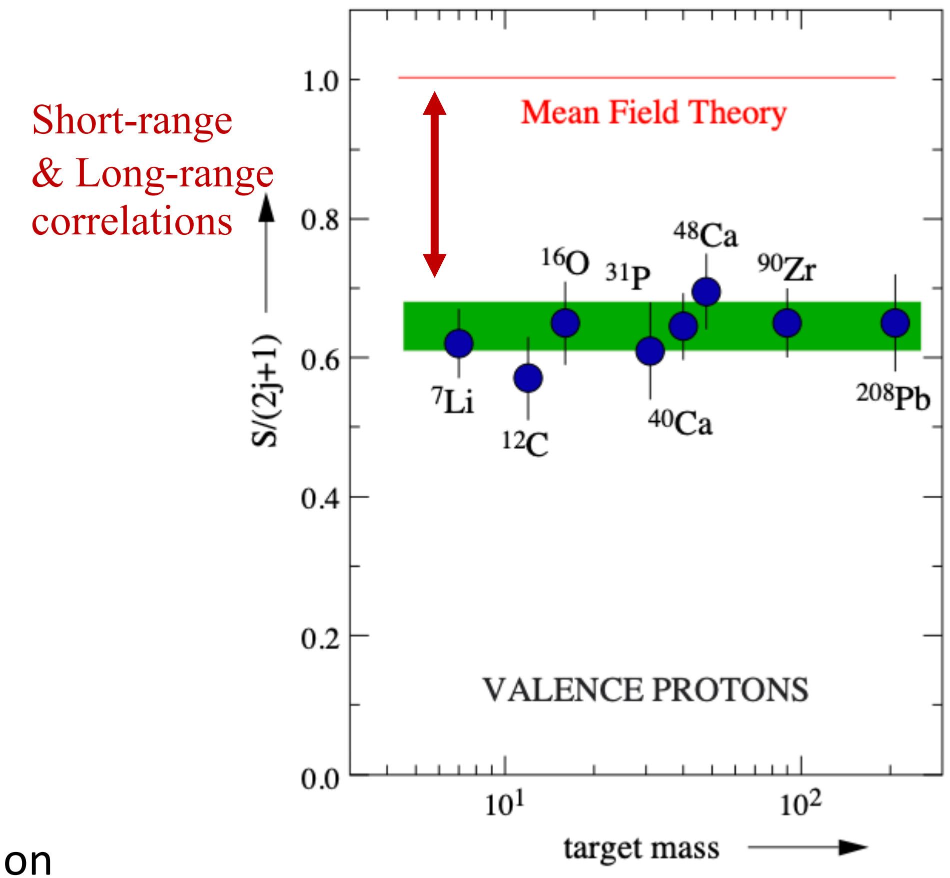
One can extract spectroscopic factors (SFs): $\boldsymbol{\sigma_{exp} = SF^{exp} \;\; \sigma_{th}^{sp}}$

→Model dependent quantity: run with the resolution scale of the interaction

T. Duguet *et al.* PRC **92** 15 (2015) & A. J. Tropiano *et al.* PRC **104** 16 (2021)

One-nucleon removal reactions give insights on long-and short-range correlations in nuclei





One can extract spectroscopic factors (SFs): $\boldsymbol{\sigma_{exp} = SF^{exp} \;\; \sigma_{th}^{sp}}$

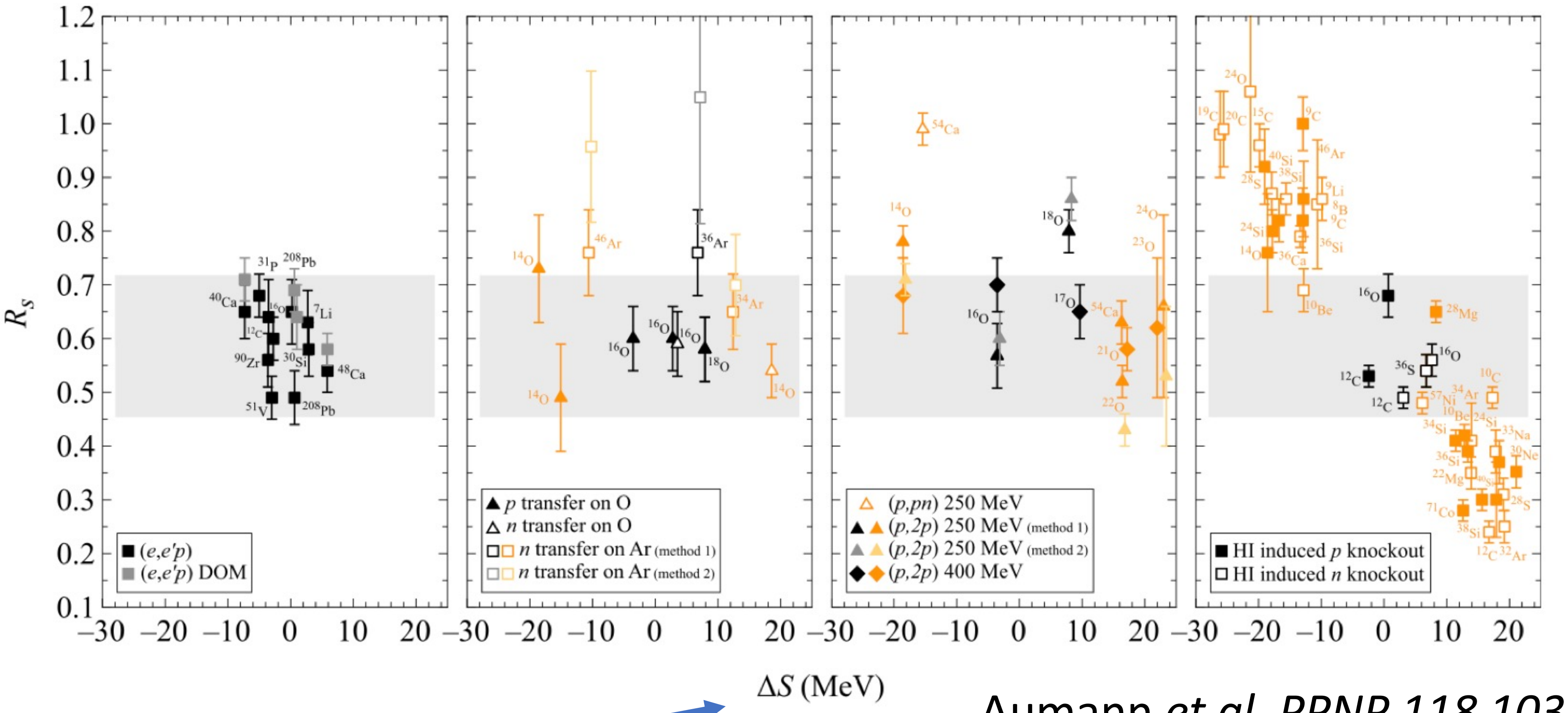
→Model dependent quantity: run with the resolution scale of the interaction

T. Duguet *et al.* PRC **92** 15 (2015) & A. J. Tropiano *et al.* PRC **104** 16 (2021)

Dickhoff and Barbieri PPNP **52** 377 (2004)

Different SFs for different probes at various beam energies → What could cause this?

$$R_s = \frac{\sigma^{exp}}{SF^{th}\sigma_{th}^{sp}}$$



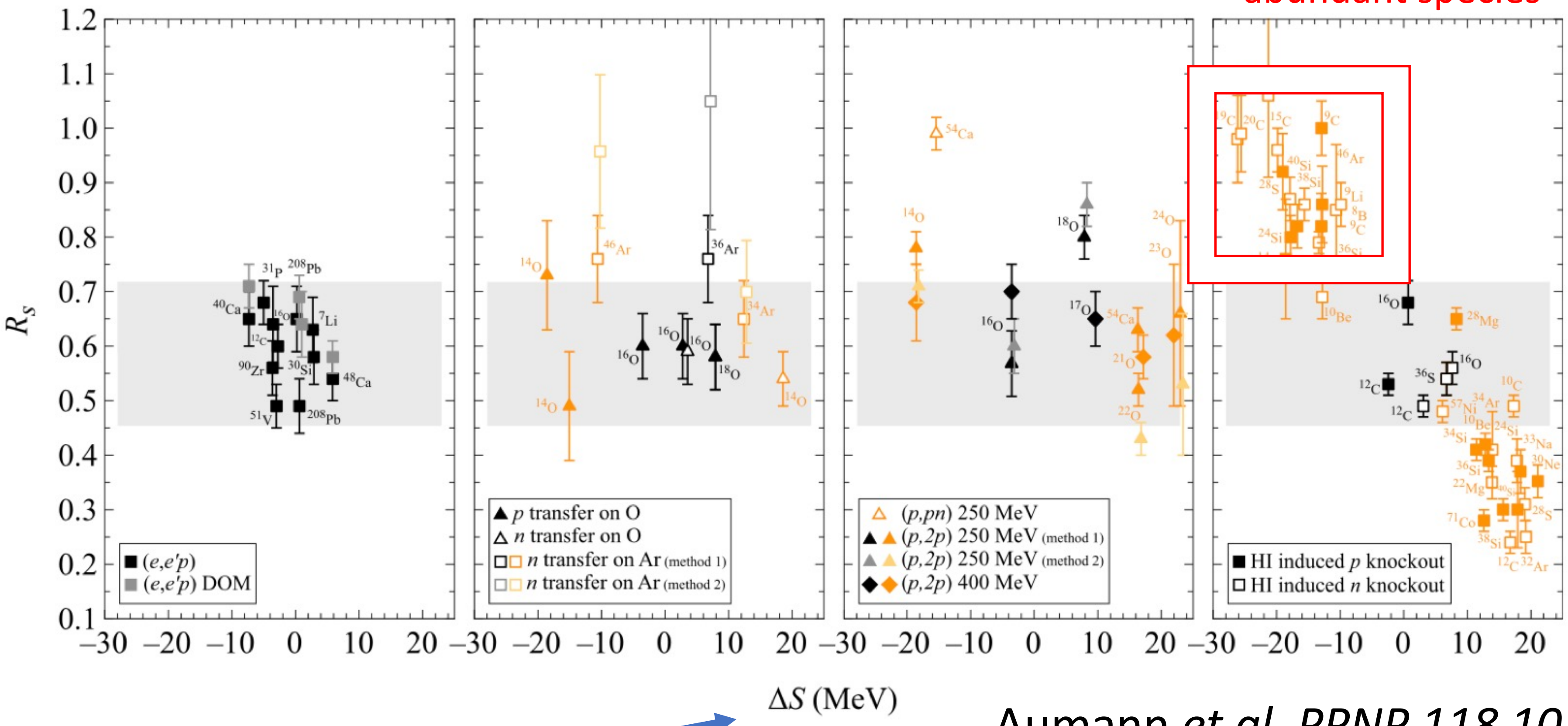
Aumann *et al. PPNP 118 103847 (2021)*

$\Delta S = S_n - S_p$ or $\Delta S = S_p - S_n$ quantifies the n/p asymmetry of the nucleus

Different SFs for different probes at various beam energies
→ What could cause this?

$$R_s = \frac{\sigma^{exp}}{SF^{th}\sigma_{th}^{sp}}$$

Removal of the abundant species

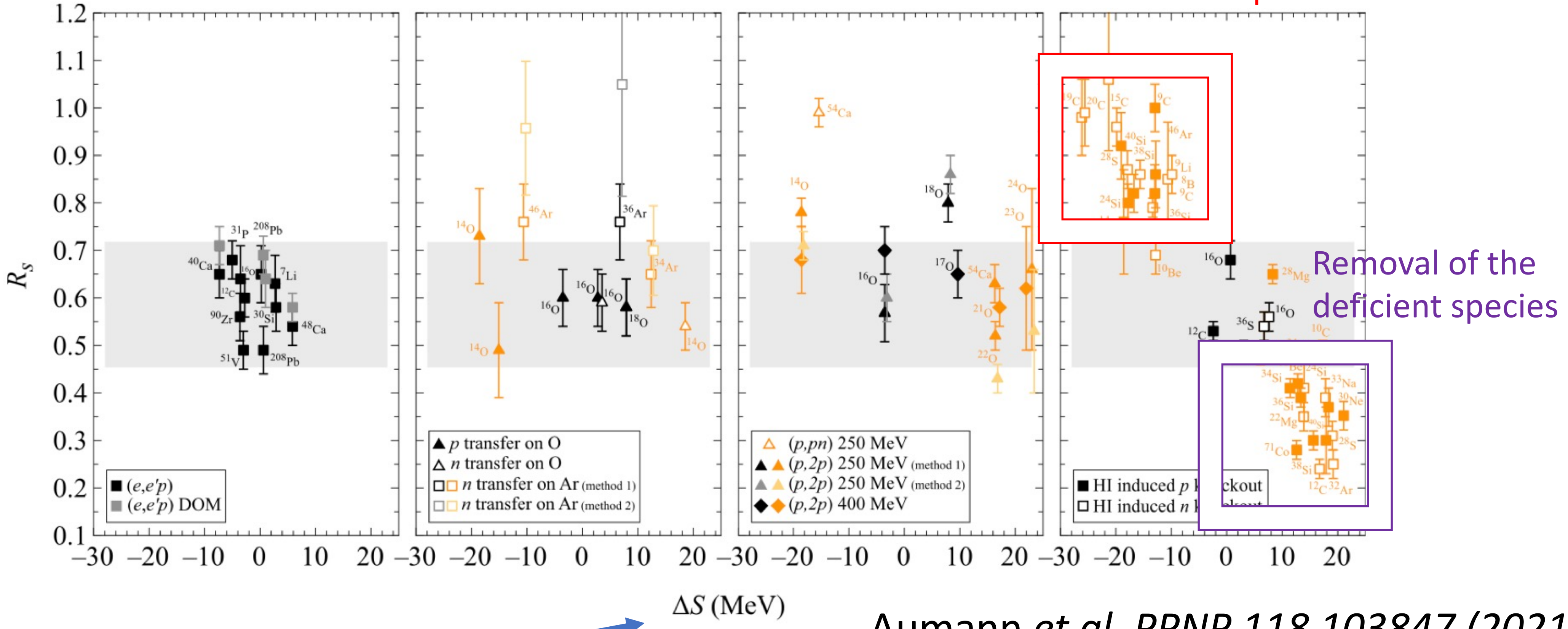


Aumann *et al. PPNP 118 103847 (2021)*

$\Delta S = S_n - S_p$ or $\Delta S = S_p - S_n$ quantifies the n/p asymmetry of the nucleus

Different SFs for different probes at various beam energies → What could cause this?

$$R_s = \frac{\sigma^{exp}}{SF^{th}\sigma^{sp}_{th}}$$



Aumann *et al. PPNP 118 103847 (2021)*

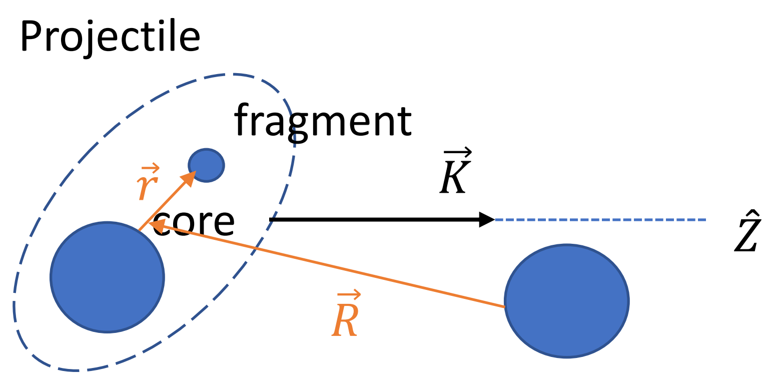
$\Delta S = S_n - S_p$ or $\Delta S = S_p - S_n$ quantifies the n/p asymmetry of the nucleus

How do we infer these SFs from experimental data?

Ratio dependent on theoretical predictions $R_s = \frac{\sigma^{exp}}{SF^{th}\sigma_{th}^{sp}}$

How do we infer these SFs from experimental data?

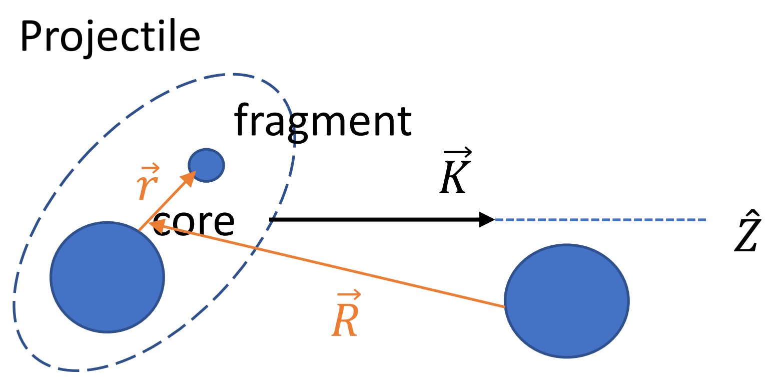
Ratio dependent on theoretical predictions $R_s = \frac{\sigma^{exp}}{SF^{th}\sigma^{sp}_{th}}$



$$\left[T_{\vec{R}} + h_{cf} + V_{cT}(R_{cT}) + V_{fT}(R_{fT})\right]\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r})$$

How do we infer these SFs from experimental data?

Ratio dependent on theoretical predictions $R_s = \frac{\sigma^{exp}}{SF^{th}\sigma_{th}^{sp}}$



To compute observables, one needs

$$\left[T_{\vec{R}} + h_{cf} + V_{cT}(R_{cT}) + V_{fT}(R_{fT})\right]\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r})$$

1. A few-body reaction model

    e.g. DWBA (1-step description) vs Eikonal (semiclassical model)

2. Single-particle description of the projectile nucleu

    Woods-Saxon fit to Sn,

3. Optical potentials

To make a meaningful comparison of SFs extracted from different probes, one needs to be consistent…

Inconsistencies in SF^{th} : some use SF^{SM} and some use $(2j+1)$

$$R_s = \frac{\sigma^{exp}}{SF^{th}\sigma_{th}^{sp}}$$

In the computation of σ_{th}^{sp} several inputs

1. A few body model

   DWIA (e,e'p), Eikonal Knockout, DWBA/ADWA transfer

2. A single-particle description of the projectile nucleus

   a real Woods-Saxon fit to S_n

Often not consistent in the analysis of the different probes

3. Optical potentials for nucleon- and nucleus-target interactions

UQ not quantified

We carried out the first consistent analysis of (p,d) and knockout data with UQ

(p,d) at 30A MeV and knockout data at 60 A MeV

on ^{32,34,36,46}Ar which span $\Delta S \sim -10 \ -20$ MeV

→Measurements done using the same setup!

We carried out the first consistent analysis of (p,d) and knockout data with UQ

(p,d) at 30A MeV and knockout data at 60 A MeV

on ^{32,34,36,46}Ar which span $\Delta S \sim -10\ -20$ MeV

→Measurements done using the same setup!

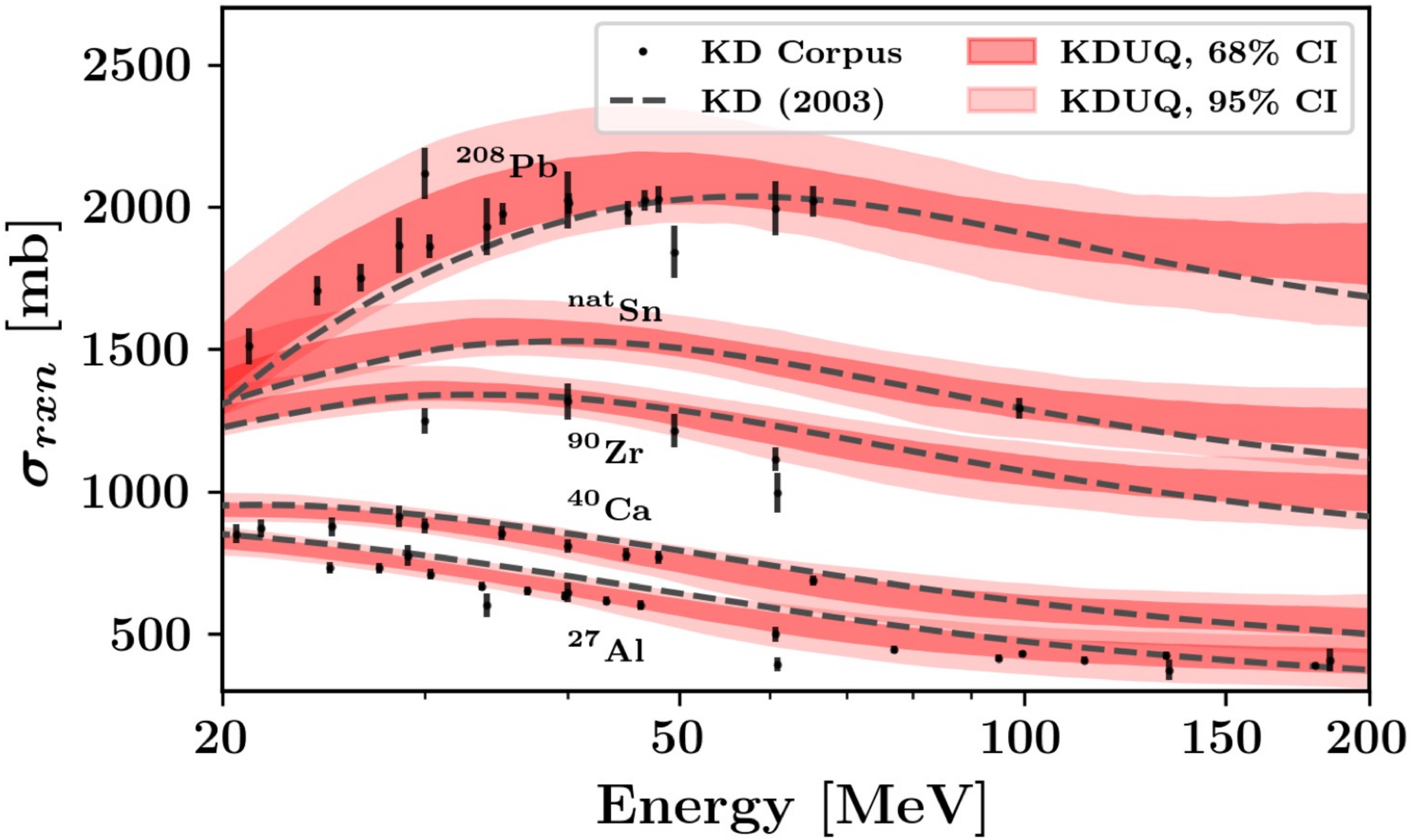
1. Transfer: ADWA calculations & Knockout eikonal

2. Consistent single-particle description: $r_0 = 1.25$ fm and $a = 0.65$ fm

3. UQ due to the nucleon-nucleus optical potential: optimal tool is…

Uncertainty quantified phenomenological global nucleon-nucleus optical potential KDUQ

Bayesian analysis using the same experimental protocol as in the original KD2003 parameterizations

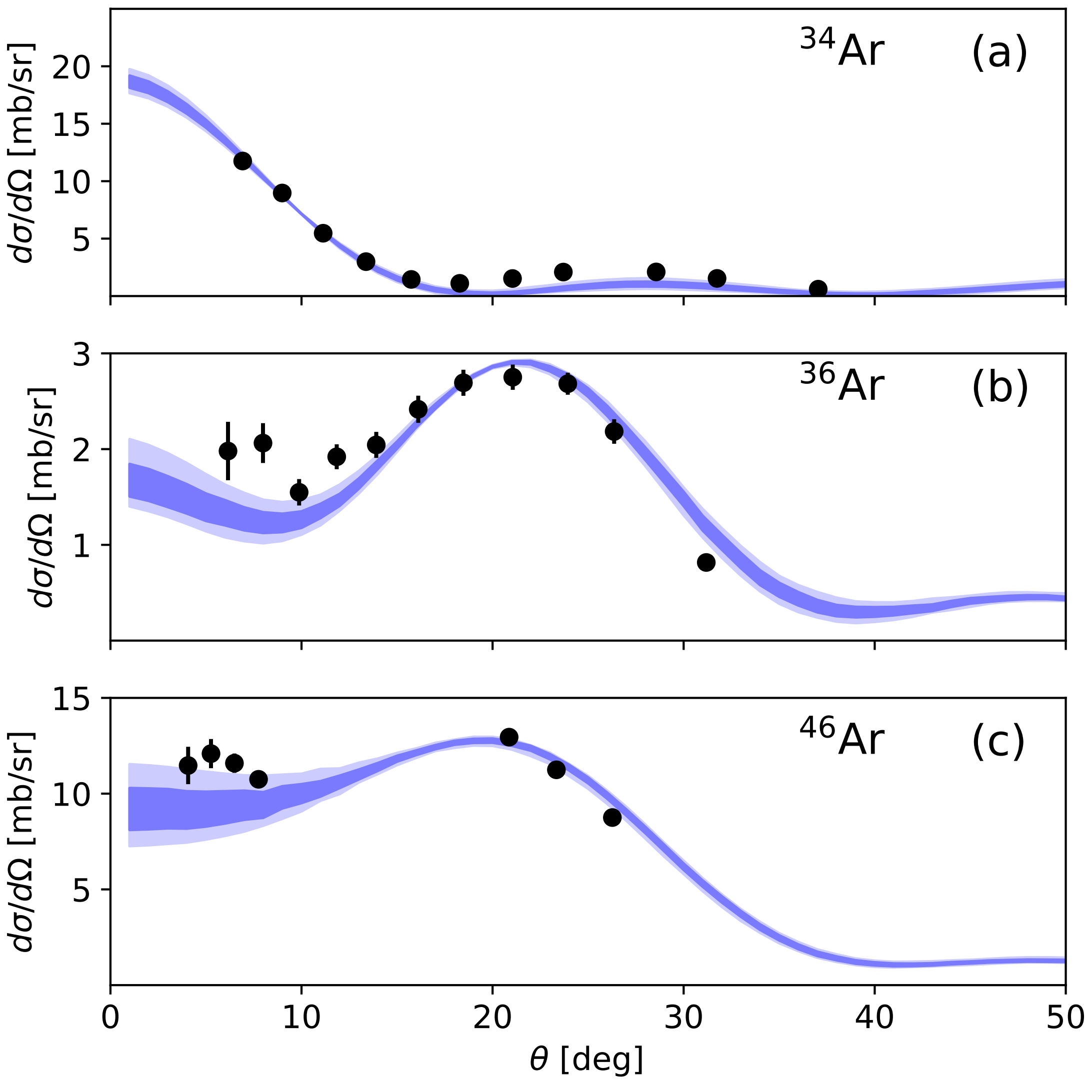


[Pruitt et al. PRC 107, 014602 (2023) including python scripts !]

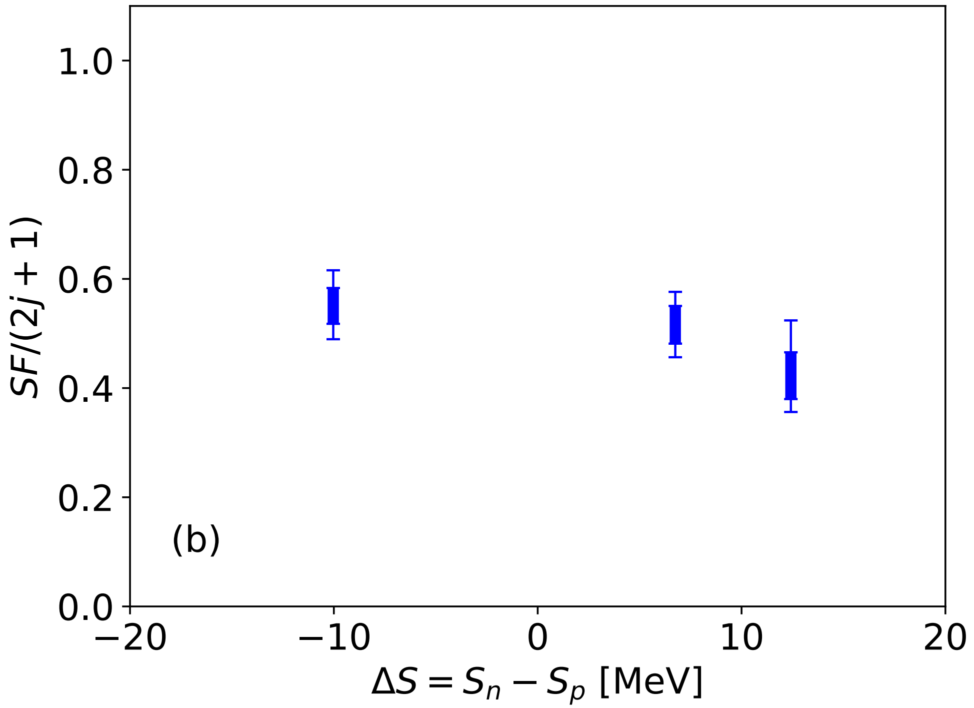
Transfer results

- ADWA reproduce well the shape for all nuclei

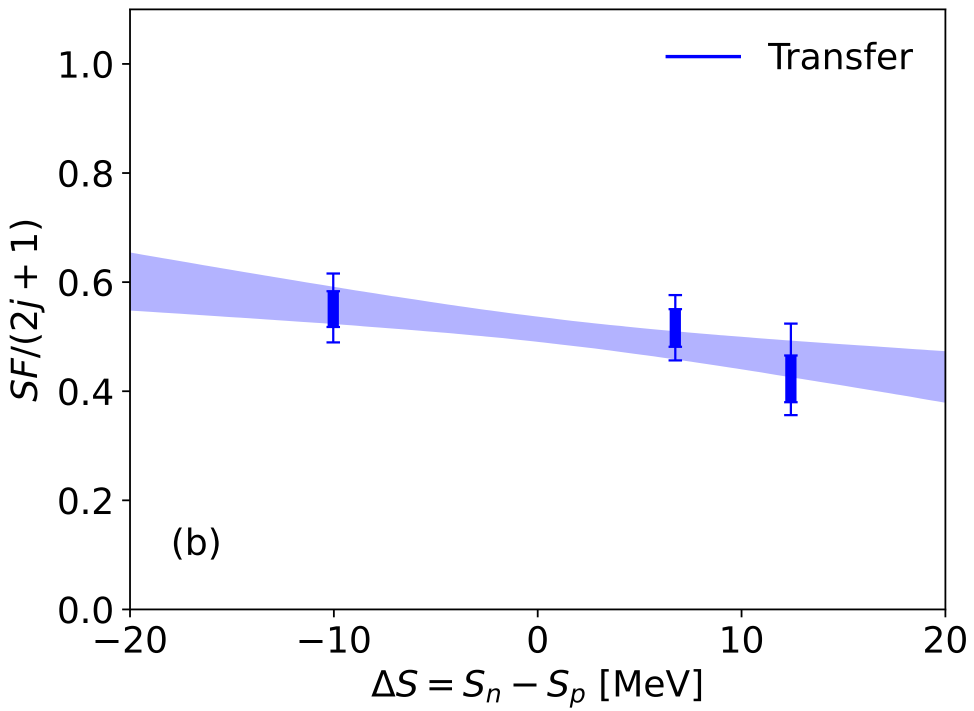
- SFs extracted by adjusting to data where σ_{tran} is maximal



Uncertainties on SFs extracted from transfer data are 10-15%!



Uncertainties on SFs extracted from transfer data are 10-15%!



$$\frac{SF}{2j+1} = -0.0044(\pm 0.0022)\,\Delta S + 0.51\,(\pm\,0.02)$$

SFs extracted from transfer show only a mild dependence on ΔS consistent with previous transfer studies

Kay *et al.* PRL **111** 042502 (2013)

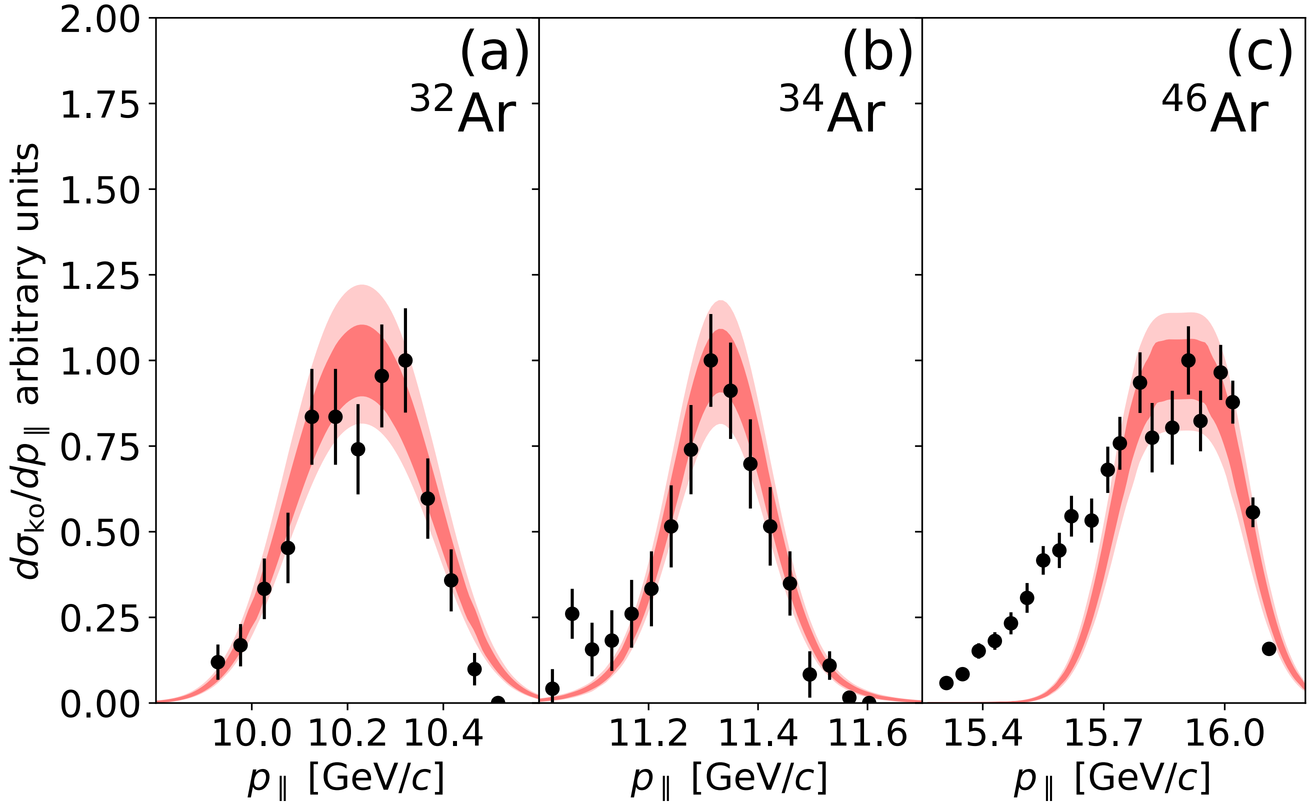
Knockout results

- Eikonal misses the asymmetry for some nuclei

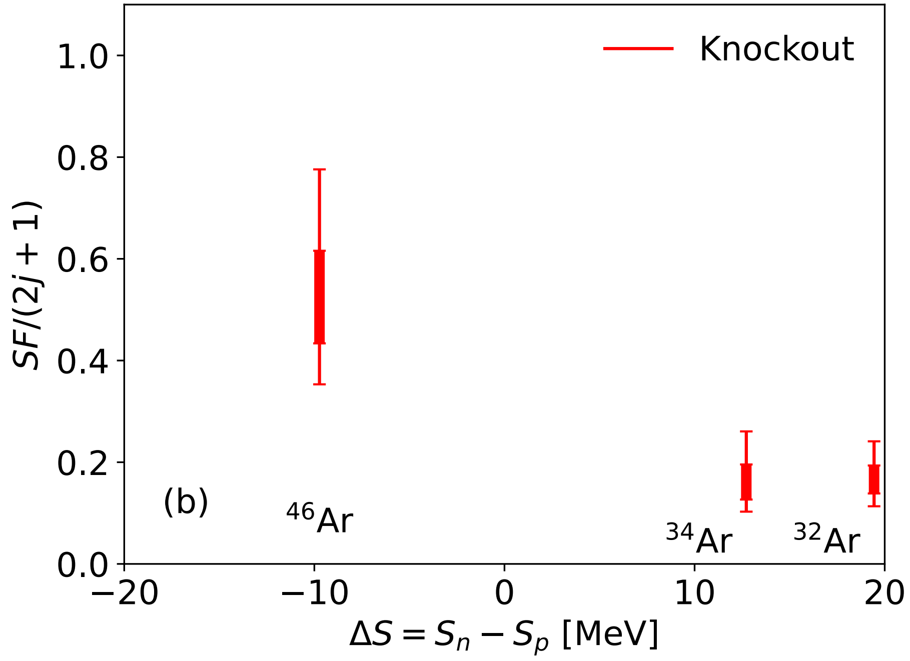
  → due to the adiabatic approx.

- SFs extracted by taking

$$SF_{exp} = \frac{\sigma_{exp}}{\sigma_{th}^{sp}}$$



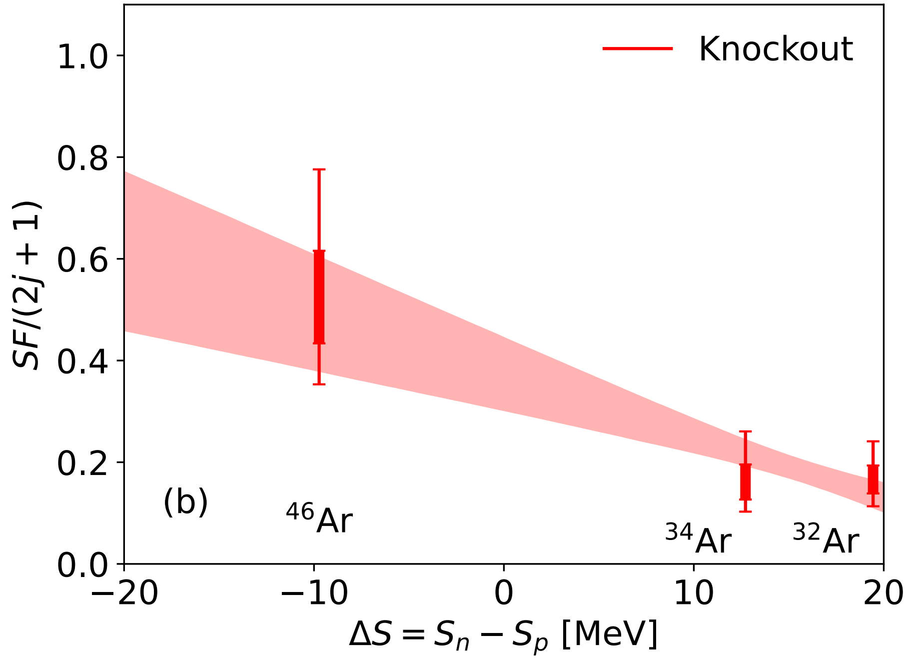
Uncertainties on SFs extracted from knockout data larger than previously estimated (~30%)!



SFs extracted from knockout consistent with previous studies

Tostevin and Gade, PRC **103** 054610 (2021)

Uncertainties on SFs extracted from knockout data larger than previously estimated (~30%)!

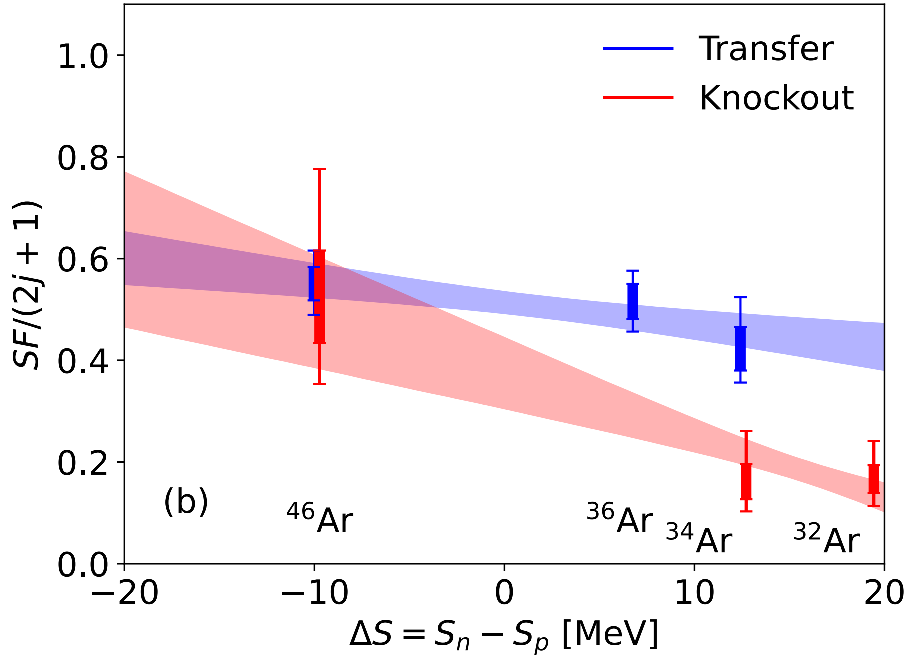


$$\frac{SF}{2j+1} = -0.0122\ (\pm\ 0.0043)\ \Delta S + 0.37\ (\pm\ 0.07)$$

SFs extracted from knockout consistent with previous studies

Tostevin and Gade, PRC **103** 054610 (2021)

Uncertainties on SFs extracted from knockout data larger than previously estimated (~30%)!



$$\frac{SF}{2j+1} = -0.0122\ (\pm\ 0.0043)\ \Delta S + 0.37\ (\pm\ 0.07)$$

$$\frac{SF}{2j+1} = -0.0044(\pm 0.0022)\ \Delta S + 0.51\ (\pm\ 0.02)$$

SFs extracted from knockout consistent with previous studies

Tostevin and Gade, PRC **103** 054610 (2021)

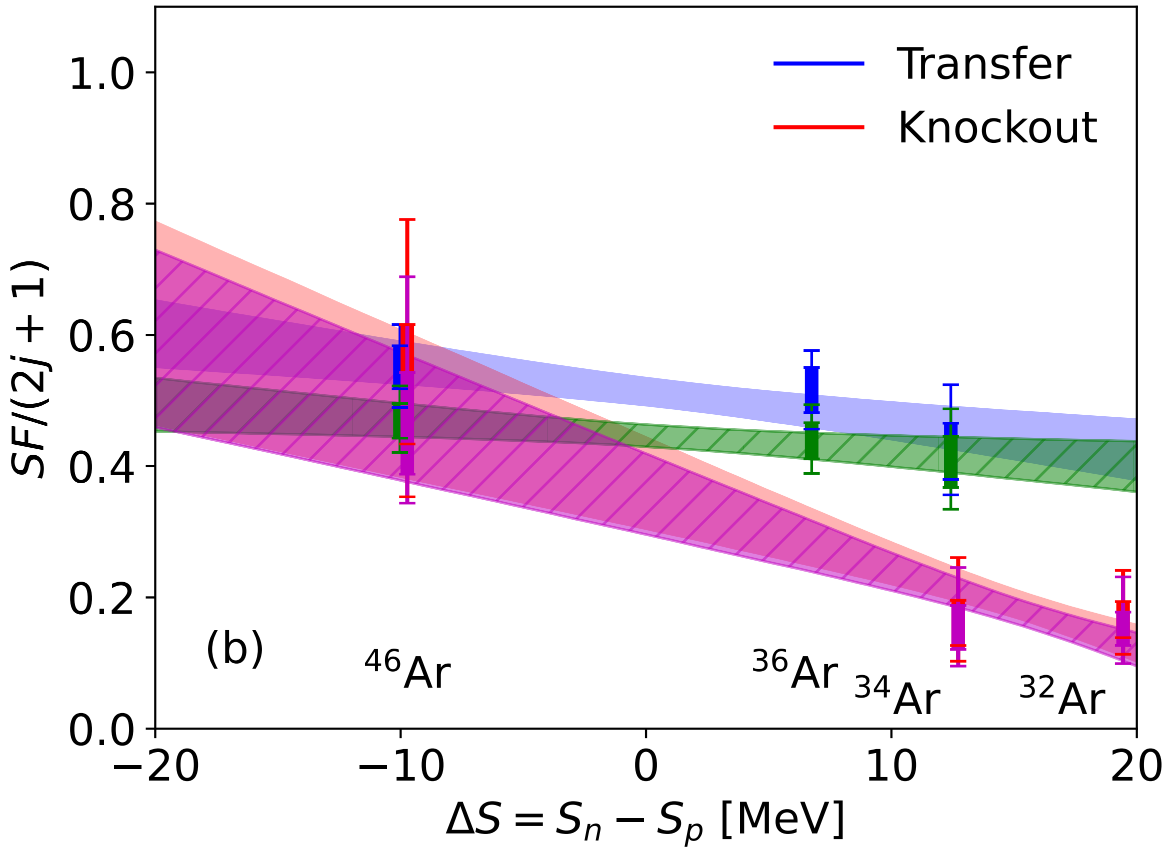
Agreement for ΔS <0 but 2σ disagreement for $\Delta S \geq 0$ (quantitative)

Hebborn, Nunes, Lovell, PRL **131**, 212503 (2023). Erratum: PRL **132**, 139901 (2024)

This study leads to more questions….

- What happens if one consider another single-particle potential?

Considering another single-particle potential shift the trend up and down … but the conclusions still hold!



Indicate that :

- One needs to be consistent across an isotopic chain

- Importance of radii..

This study leads to more questions….

- What happens if one consider another single-particle potential?
- Why transfer and knockout observables have so different parametric uncertainties?

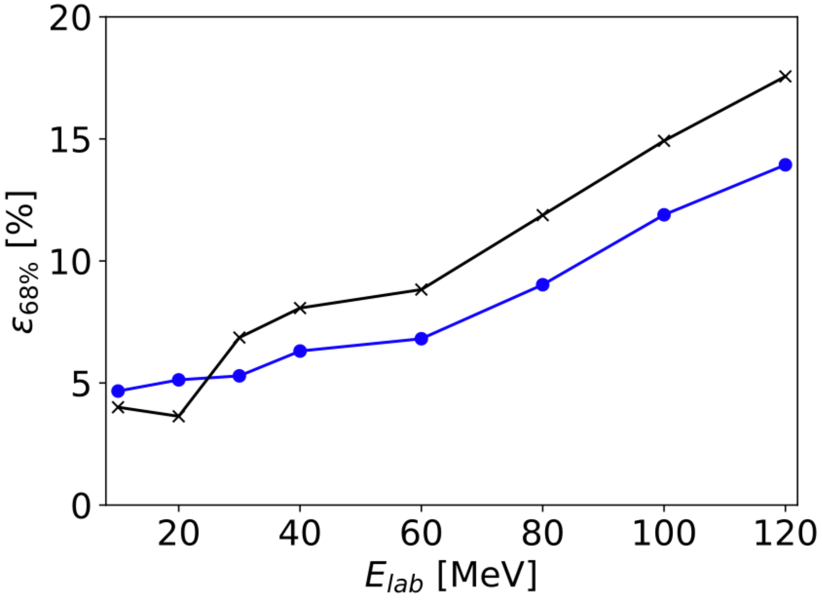
Why transfer and knockout observables have so different parametric uncertainties?

1. Is it the beam energy?

Why transfer and knockout observables have so different parametric uncertainties?

1. Is it the beam energy? Yes, higher beam energy leads to larger uncertainties!

Relative uncertainty at the peak of transfer angular distribution depending on E_{lab} for (d,p)

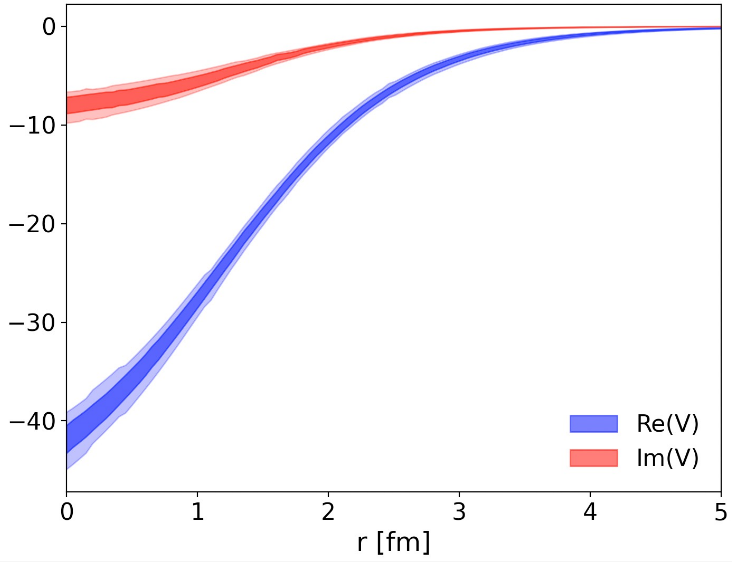


Extensive sensitivity study for transfer: Hebborn, Nunes, Frontiers **13** (2025)

Why transfer and knockout observables have so different parametric uncertainties?

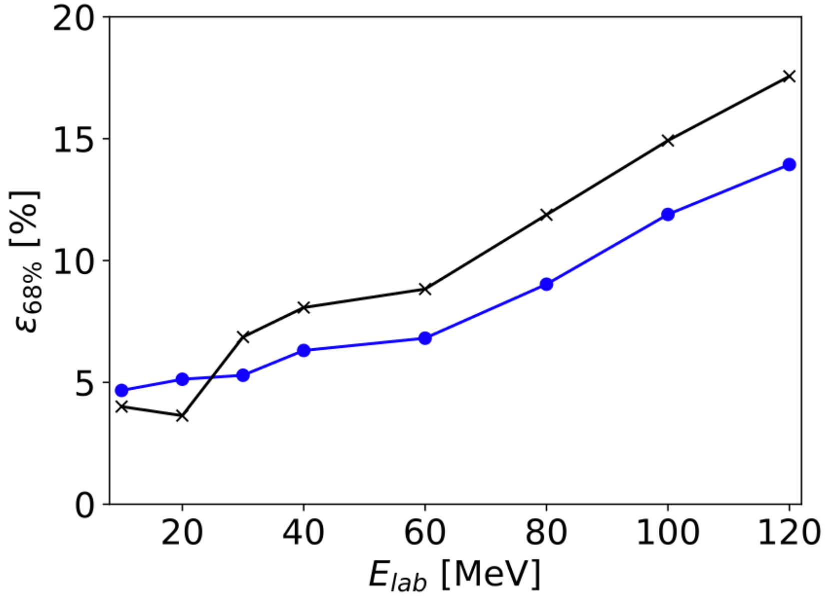
1. Is it the beam energy? Yes, higher beam energy leads to larger uncertainties!

e.g. proton-49Ca at 26 MeV



Imaginary part less constrained (relatively)… Knockout cross sections very senstive to the imaginary part !

Relative uncertainty at the peak of transfer angular distribution depending on E_{lab} for (d,p)



Extensive sensitivity study for transfer: Hebborn, Nunes, Frontiers **13** (2025)

Why transfer and knockout observables have so different parametric uncertainties?

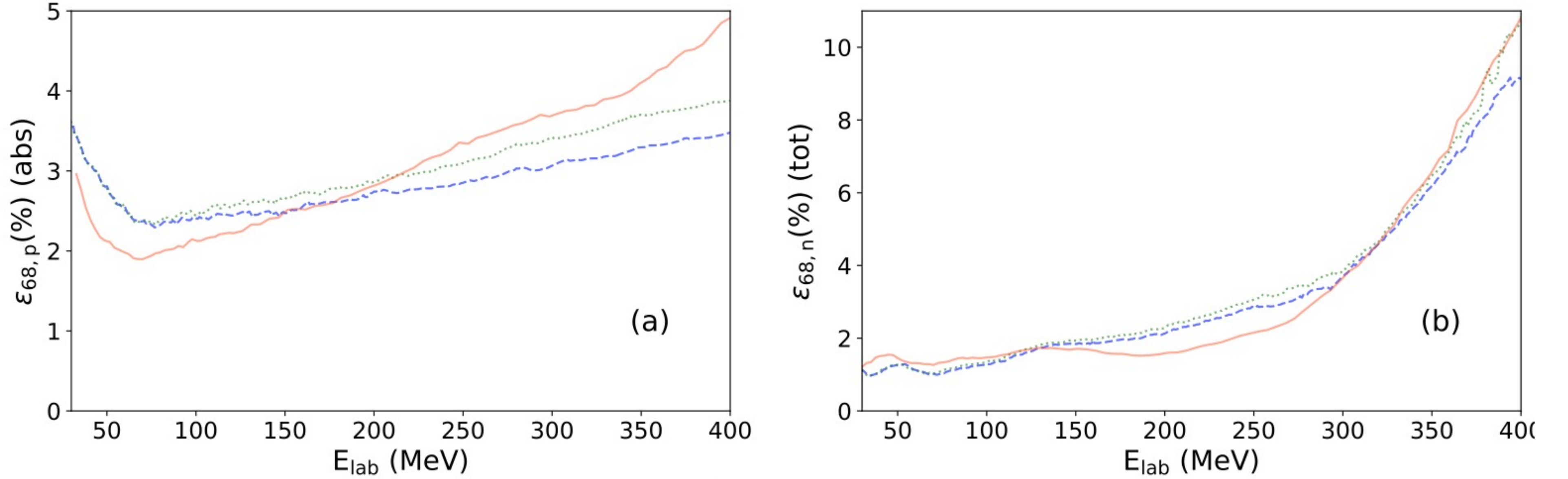
2. Do uncertainties propagate differently in the eikonal model and DWBA/ADWA?
Is it due to the eikonal approximation of the scattering waves?

Why transfer and knockout observables have so different parametric uncertainties?

2. Do uncertainties propagate differently in the eikonal model and DWBA/ADWA?
Is it due to the eikonal approximation of the scattering waves?

1 sigma uncertainties obtained for the eikonal model vs exact solution for 2-body problems

p+^{208}Pb n+^{208}Pb



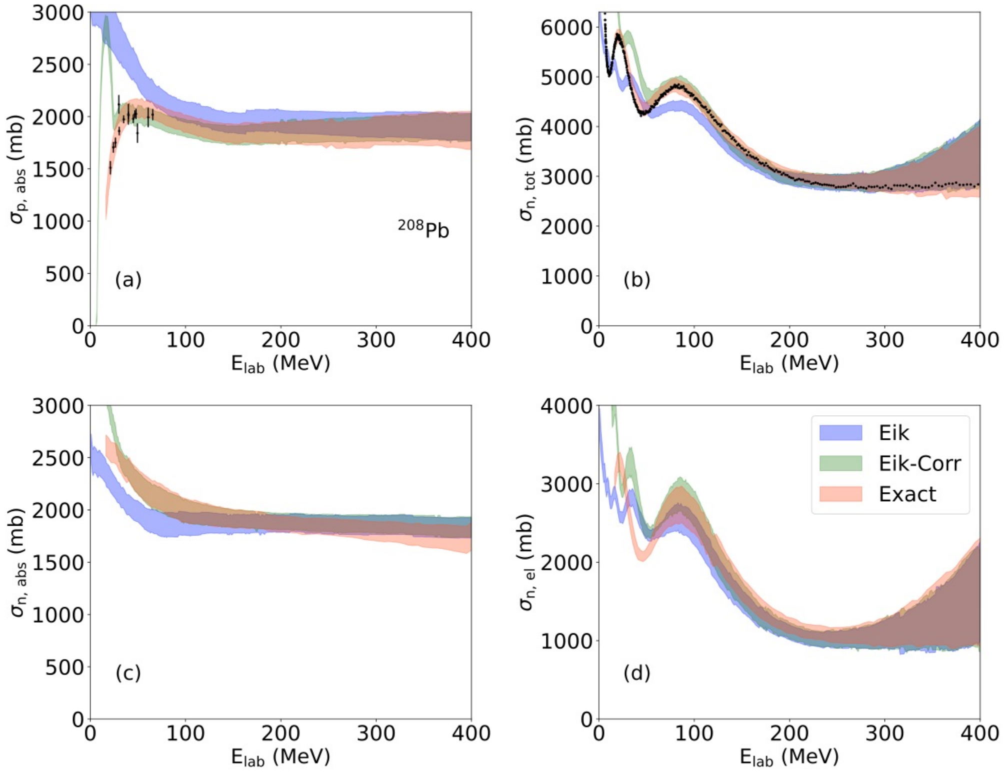
Similar uncertainties for both models! [**Shiu,** Hebborn, Nunes, arXiv:2507.13291]

This study leads to more questions….

- What happens if one consider another single-particle potential?
- Why transfer and knockout observables have so different parametric uncertainties?
- Model uncertainty: is the eikonal approximation valid at these energies?

Model uncertainty: is the eikonal approximation valid at these energies?

Comparison of the eikonal model, a correction vs exact solution for 2-body problems



		E^{eik}_{min} (MeV)	$E^{eik-corr}_{min}$ (MeV)
	^{208}Pb	60	25
	^{90}Zr	35	30
$\sigma_{p,abs}$	^{40}Ca	25	25
	^{27}Al	25	25
	^{208}Pb	110	50
	^{90}Zr	160	40
$\sigma_{n,tot}$	^{40}Ca	150	50
	^{27}Al	120	40

Inaccuracies at energy <100 MeV, need for correction!

Shiu, Hebborn, Nunes, arXiv:2507.13291

Other uncertainties in knockout that we expect to be important

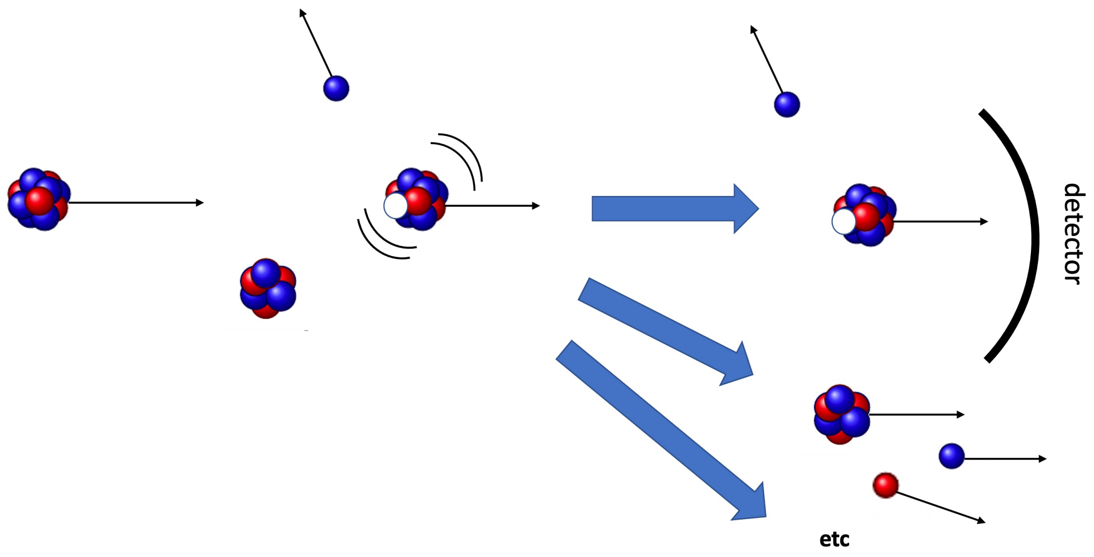
- Eikonal model for 3-body reaction:
  violates energy conservation & treat the core as a spectator

Other uncertainties in knockout that we expect to be important

- Eikonal model for 3-body reaction:
  violates energy conservation & treat the core as a spectator

- Dissipative processes could be important

Gomez Ramos *et al.* Phys. Lett B 847, 138284 (2023)
Bertulani Phys. Lett B 846, 138250 (2023)



Simulates the hole propagation in the many-body systems with DOM potentials

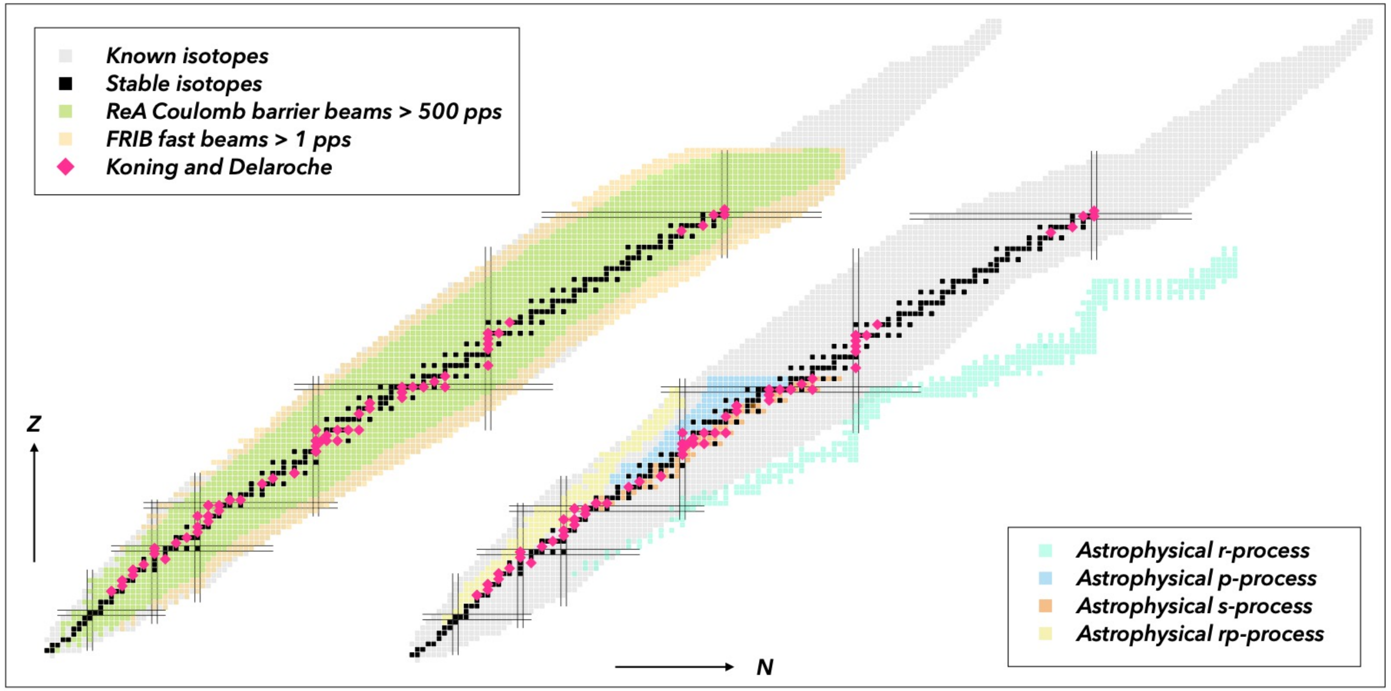
→ GFK with Potel, Pruitt & Escher (LLNL)

PRC **107** 014607 (2023)

This study leads to more questions….

- What happens if one consider another single-particle potential?
- Why transfer and knockout observables have so different parametric uncertainties?
- Model uncertainty: is the eikonal approximation valid at these energies?
- Are optical potentials accurate when dealing with unstable systems?

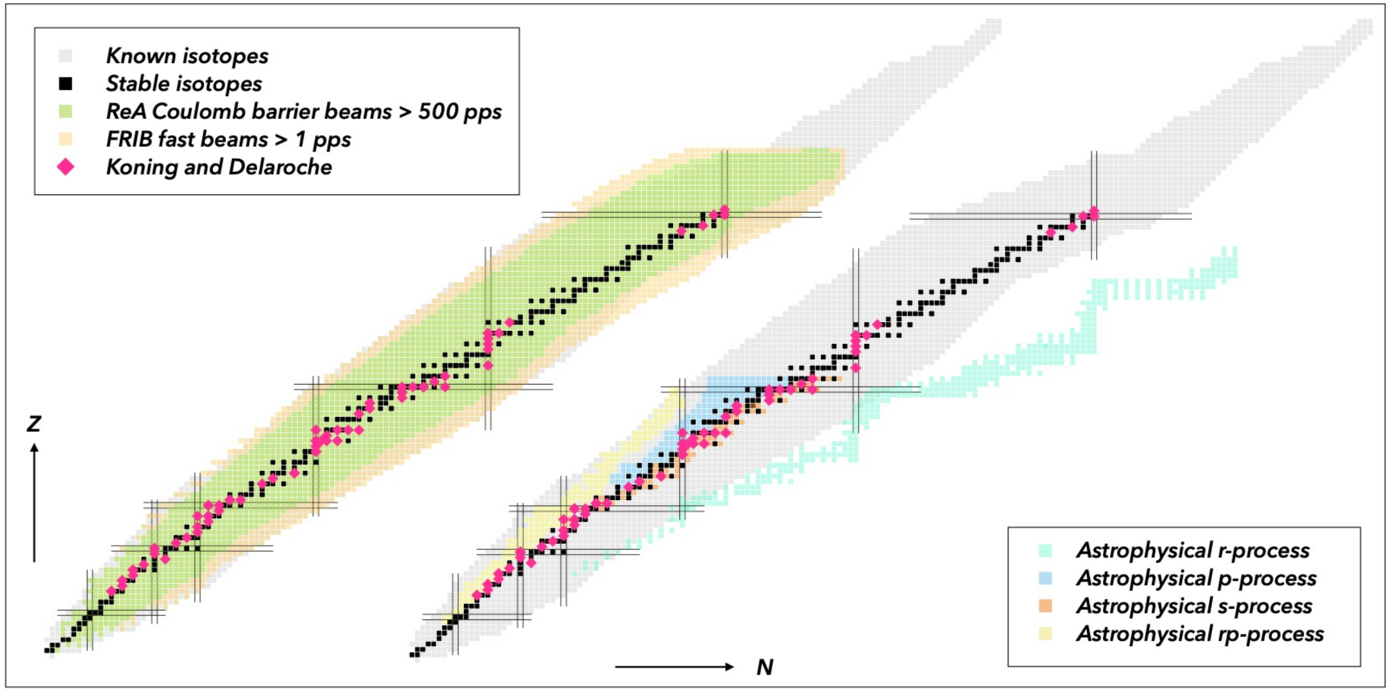
Are optical potentials accurate when dealing with unstable systems?



Hebborn *et al.* J. Phys. G **50**, 060501 (2023)

Unstable nuclei characterize with extreme N/Z ratio… but optical potentials fitted next to stability

Are optical potentials accurate when dealing with unstable systems?



Hebborn *et al.* J. Phys. G **50**, 060501 (2023)

Unstable nuclei characterize with extreme N/Z ratio… but optical potentials fitted next to stability

Lane consistent form

$$U_{\mathrm{n,p}} = U_0 \pm \frac{N-Z}{A} U_1$$

Extrapolation driven by U_1, how constrained is U_1?

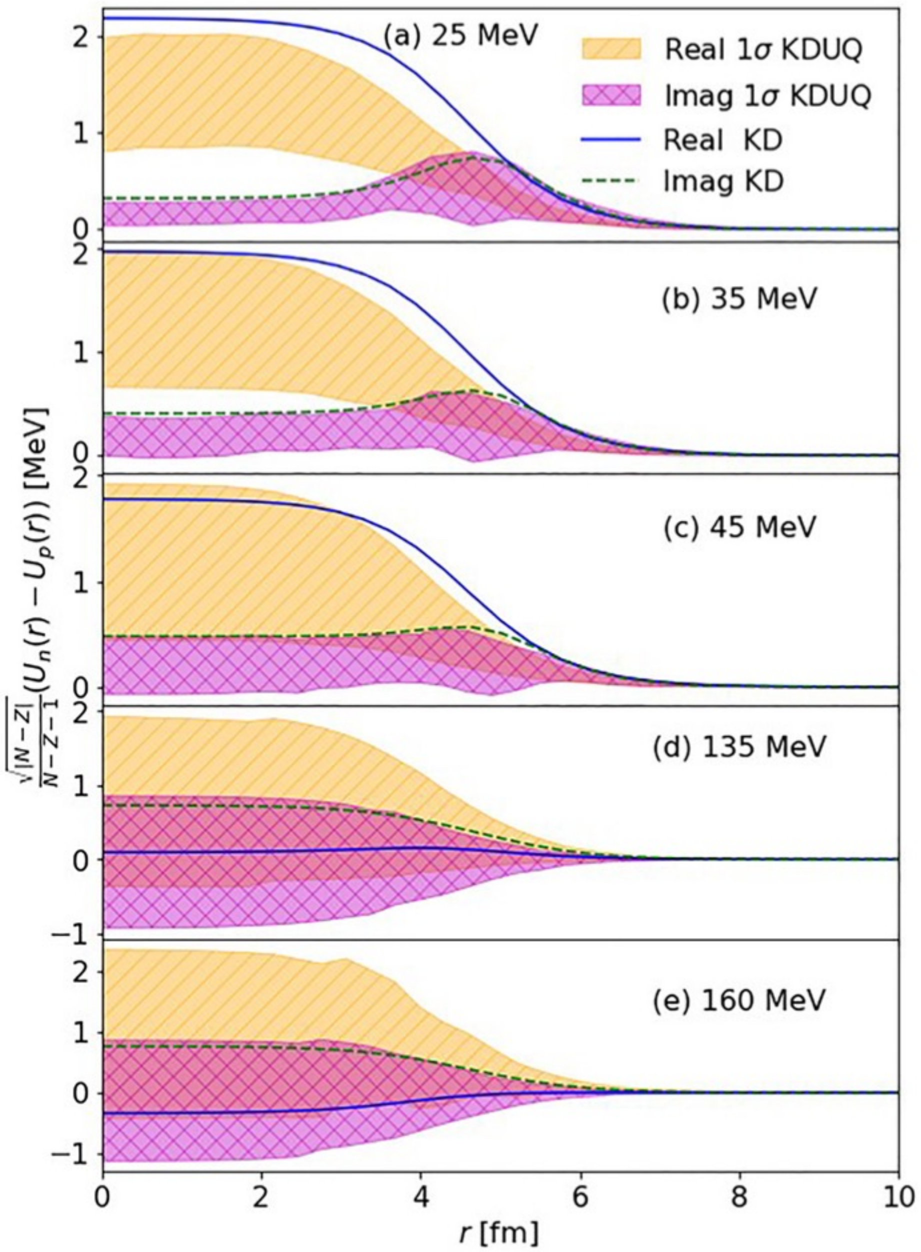
Are optical potentials accurate when dealing with unstable systems?

U_1,poorly constrained (50% to 100% uncertainties)

→ Increased uncertainties for unstable systems…

→ Need to further constrain optical potentials

Smith, Hebborn, Nunes, Zegers PRC **110** 034602 (2024).



Conclusions, open questions and prospects

- **First consistent analysis of transfer and knockout data with UQ:**

  Quenching of SF consistent between knockout and transfer for $\Delta S < 0$ MeV & 2σ disagreement for $\Delta S \geq 0$

  → verify for other isotopic chain?

Conclusions, open questions and prospects

- **First consistent analysis of transfer and knockout data with UQ:**

  Quenching of SF consistent between knockout and transfer for $\Delta S < 0$ MeV & 2σ disagreement for $\Delta S \geq 0$

  → verify for other isotopic chain?

- **Parametric uncertainties**: larger for knockout because imaginary part less constrained & performed at higher beam energies

Conclusions, open questions and prospects

- **First consistent analysis of transfer and knockout data with UQ:**

  Quenching of SF consistent between knockout and transfer for $\Delta S < 0$ MeV & 2σ disagreement for $\Delta S \geq 0$

  → verify for other isotopic chain?

- **Parametric uncertainties**: larger for knockout because imaginary part less constrained & performed at higher beam energies

- **Model uncertaintes:** likely larger for knockout as inaccuracies for <100 AMeV and core-spectator assumption

  → addressing these inaccuracies: ongoing work

Conclusions, open questions and prospects

- **First consistent analysis of transfer and knockout data with UQ:**

  Quenching of SF consistent between knockout and transfer for $\Delta S < 0$ MeV & 2σ disagreement for $\Delta S \geq 0$

  → verify for other isotopic chain?

- **Parametric uncertainties**: larger for knockout because imaginary part less constrained & performed at higher beam energies

- **Model uncertaintes:** likely larger for knockout as inaccuracies for <100 AMeV and core-spectator assumption
  → addressing these inaccuracies: ongoing work

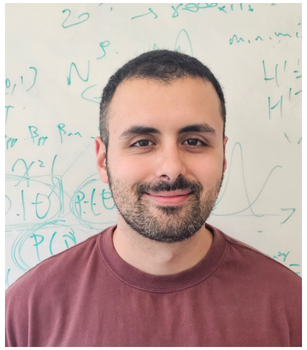
- **Reactions with unstable systems**: likely large uncertainties as U_1 poorly constrained… → Future: dispersive optical model, microscopic?

[whitepaper JPG 50 060501 (2023)]

Thanks to the few-body group at MSU & all my collaborators…


Filomena Nunes


Chloë Hebborn


Ibrahim Abdurrahman

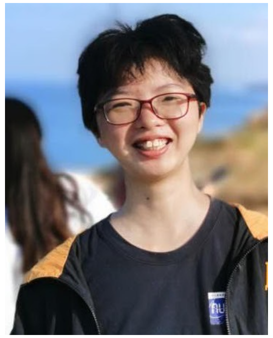

Kyle Beyer


Patrick McGlynn


Cate Beckman


Manuel Catacora Rios


Andy Smith


Zetian Ma


Daniel Shiu


Pablo Giuliani


Grigor Sargsyan

MICHIGAN STATE UNIVERSITY — R. T. Zegers

Los Alamos NATIONAL LABORATORY — A. E Lovell

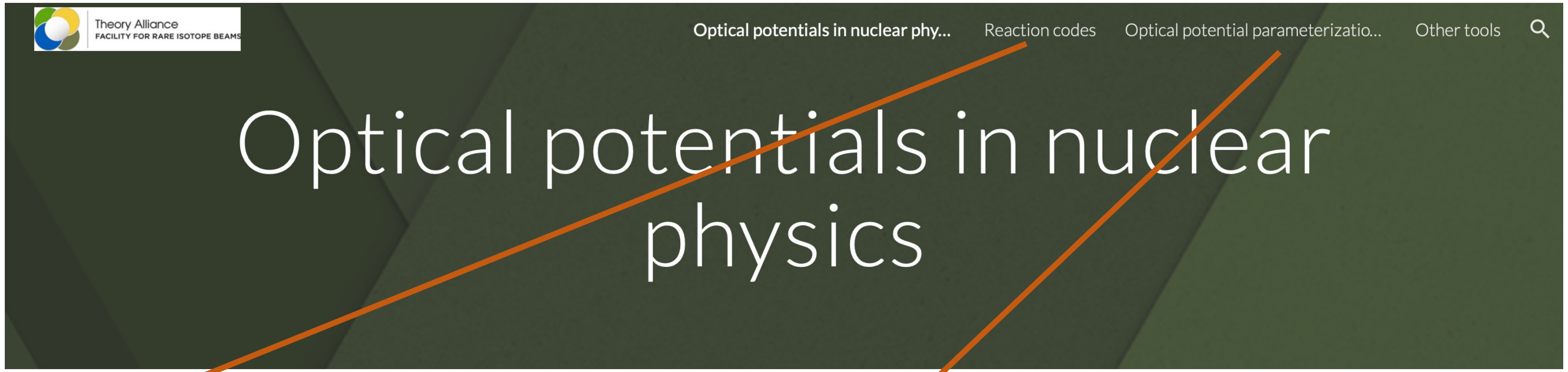
UNIVERSIDAD DE SEVILLA — G. Potel

Lawrence Livermore National Laboratory — J. Escher, C. D. Pruitt

and thank you for your attention ☺ !!

Useful resources for those who do reaction calculations

https://sites.google.com/view/opticalpotentials/



Links to reaction codes publicly available (direct & compound)

Links to optical potentials parametrization

SCAN ME