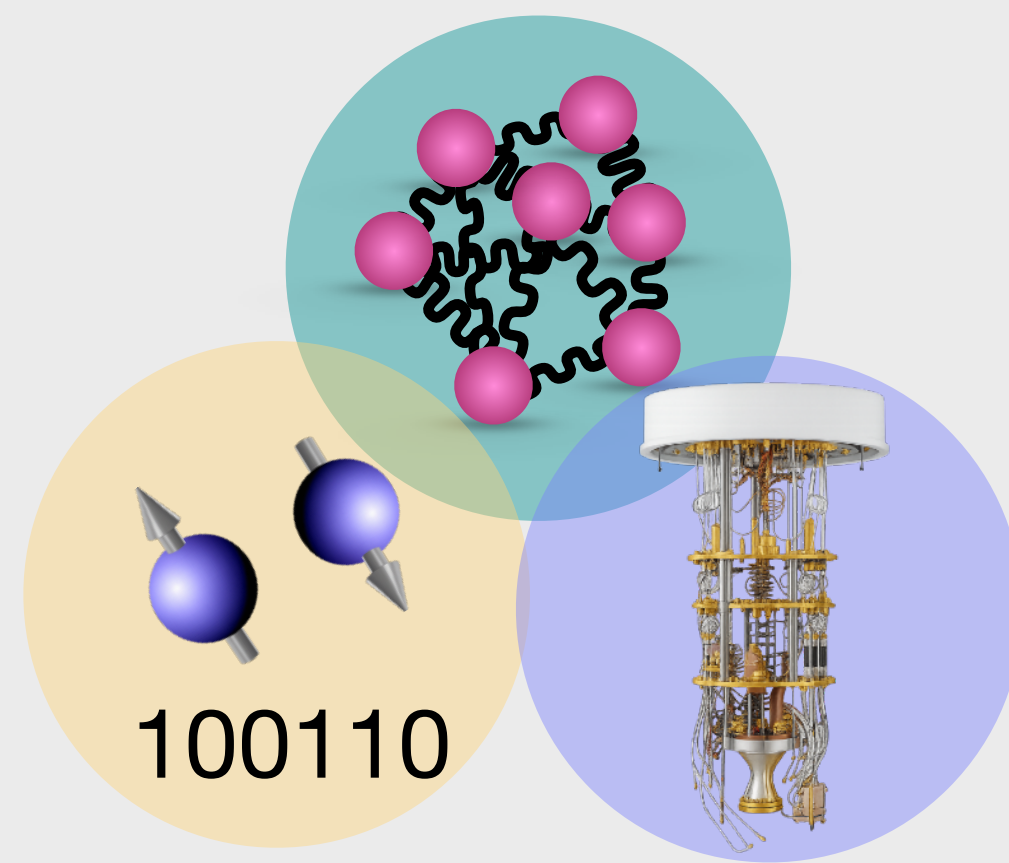


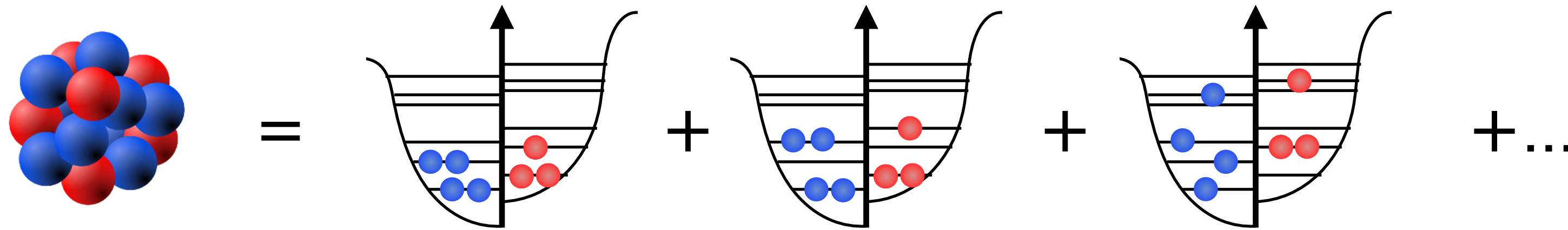


European Nuclear Physics Conference 2025

ENTANGLEMENT, COMPLEXITY AND QUANTUM COMPUTATIONS OF NUCLEAR MANY-BODY SYSTEMS

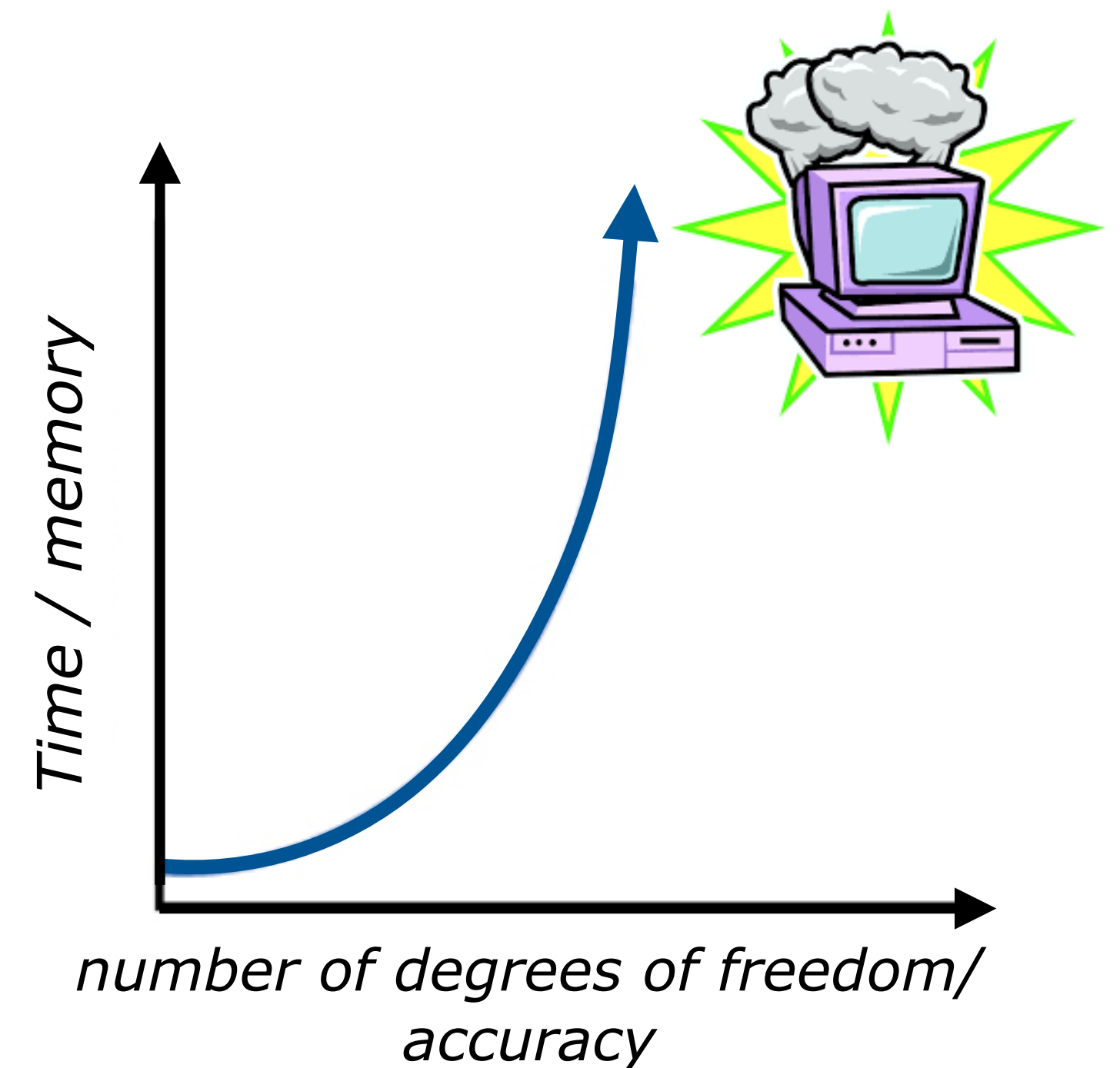
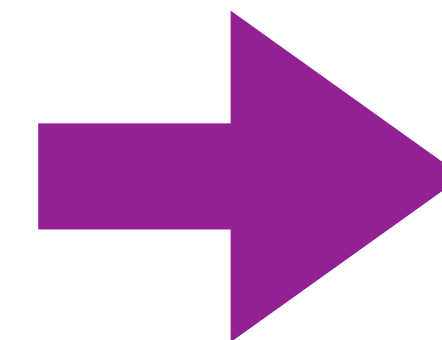
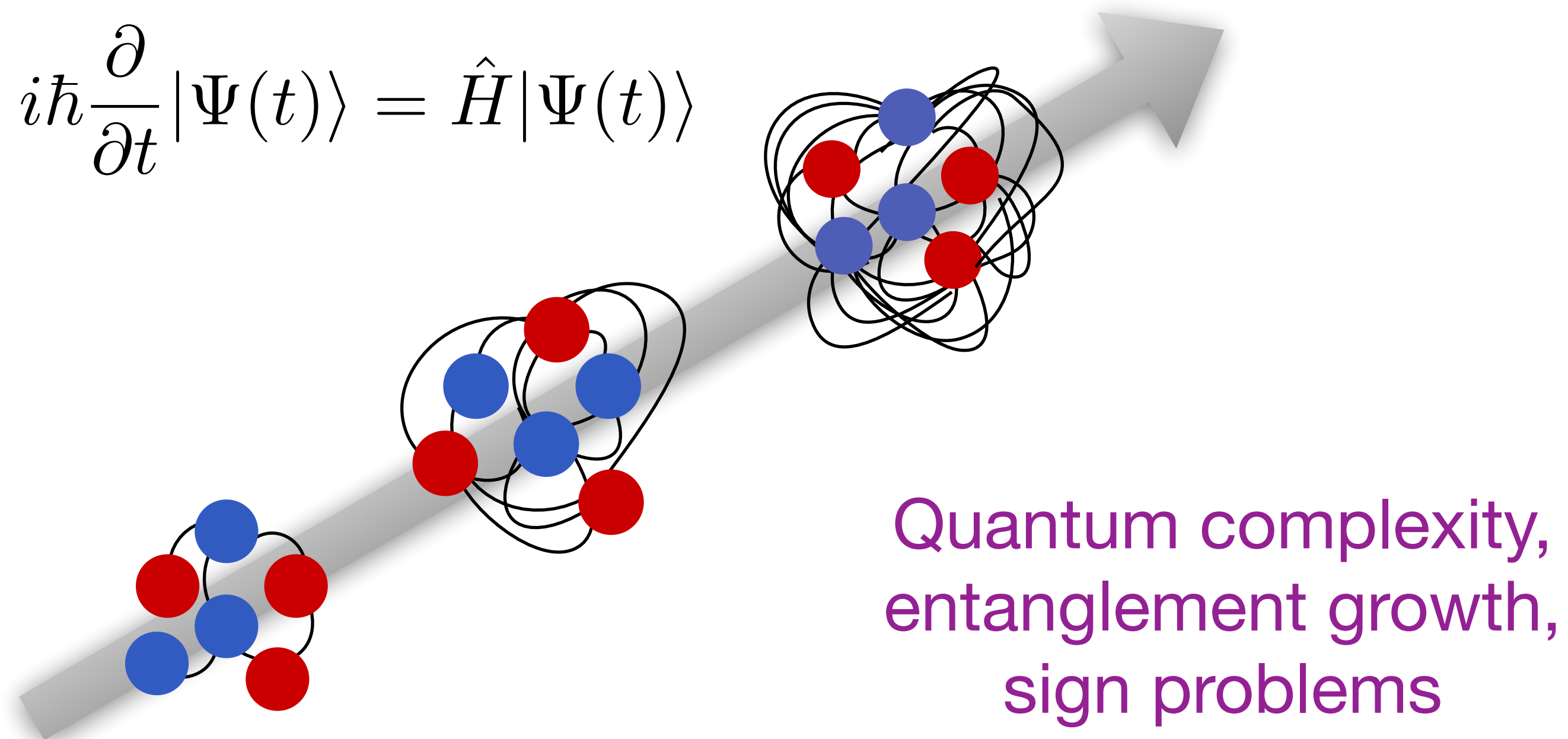
CAROLINE ROBIN

(Nuclear) Quantum Many-Body Problems

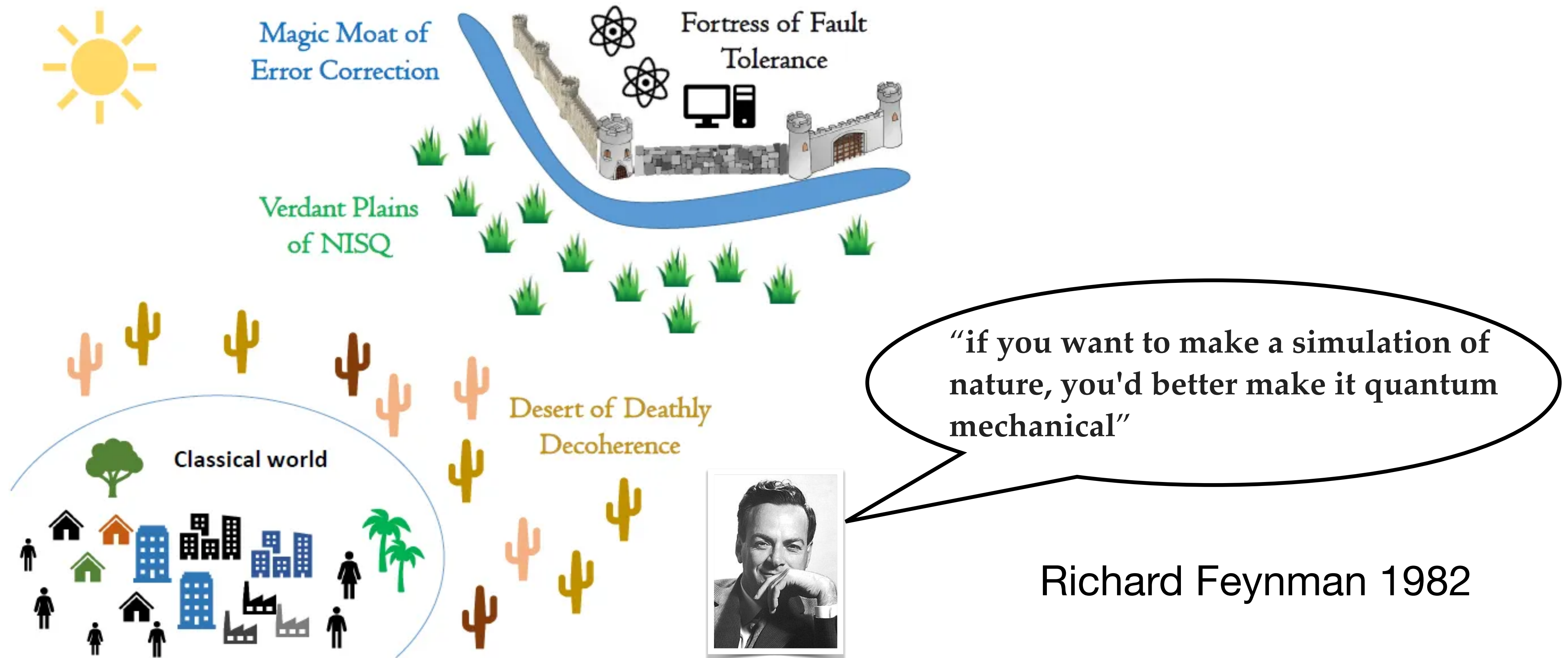


$$|\Psi\rangle = \sum_i^{exp(N)} C_i |\Phi_i\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$

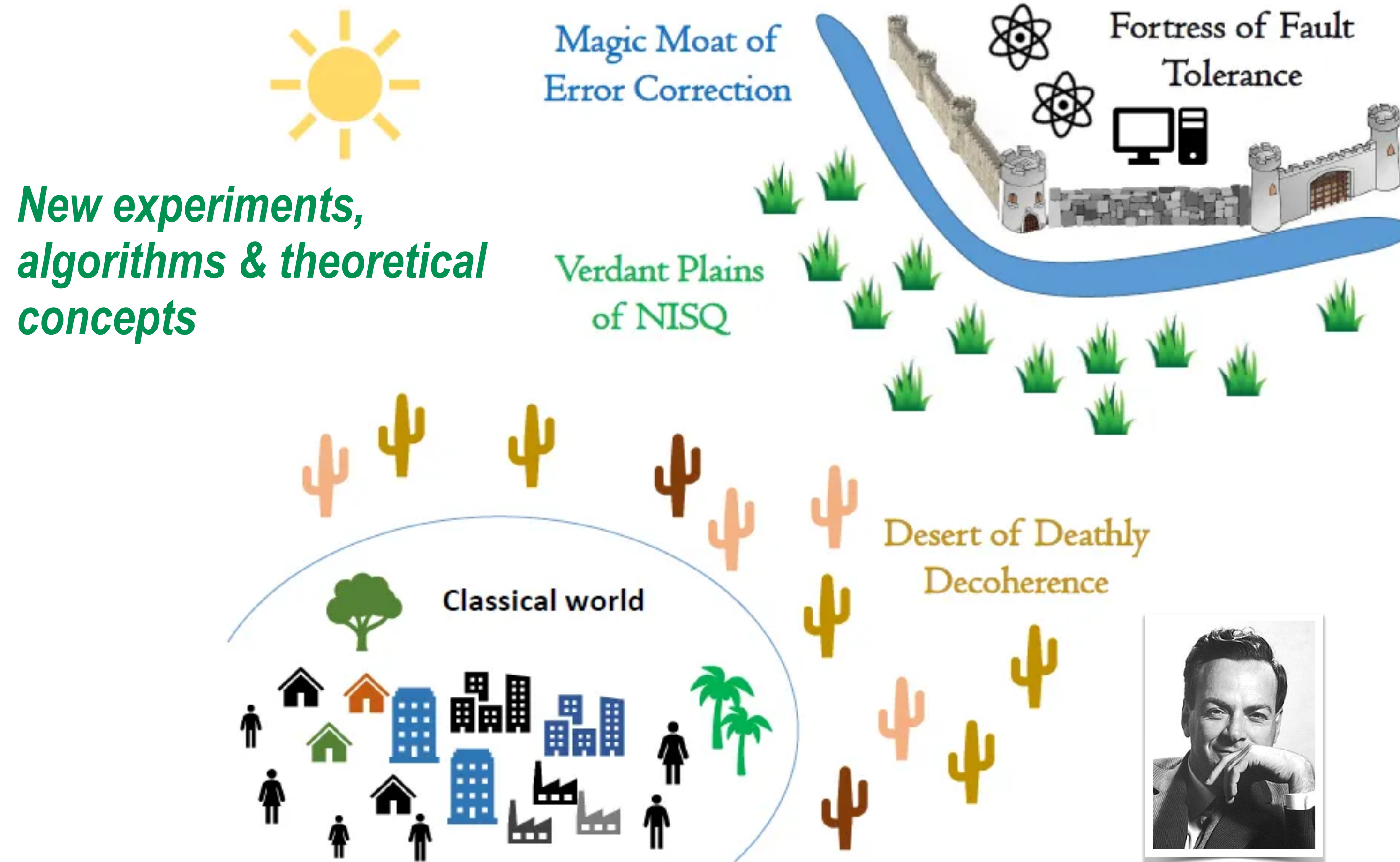


Progress in Quantum Computation

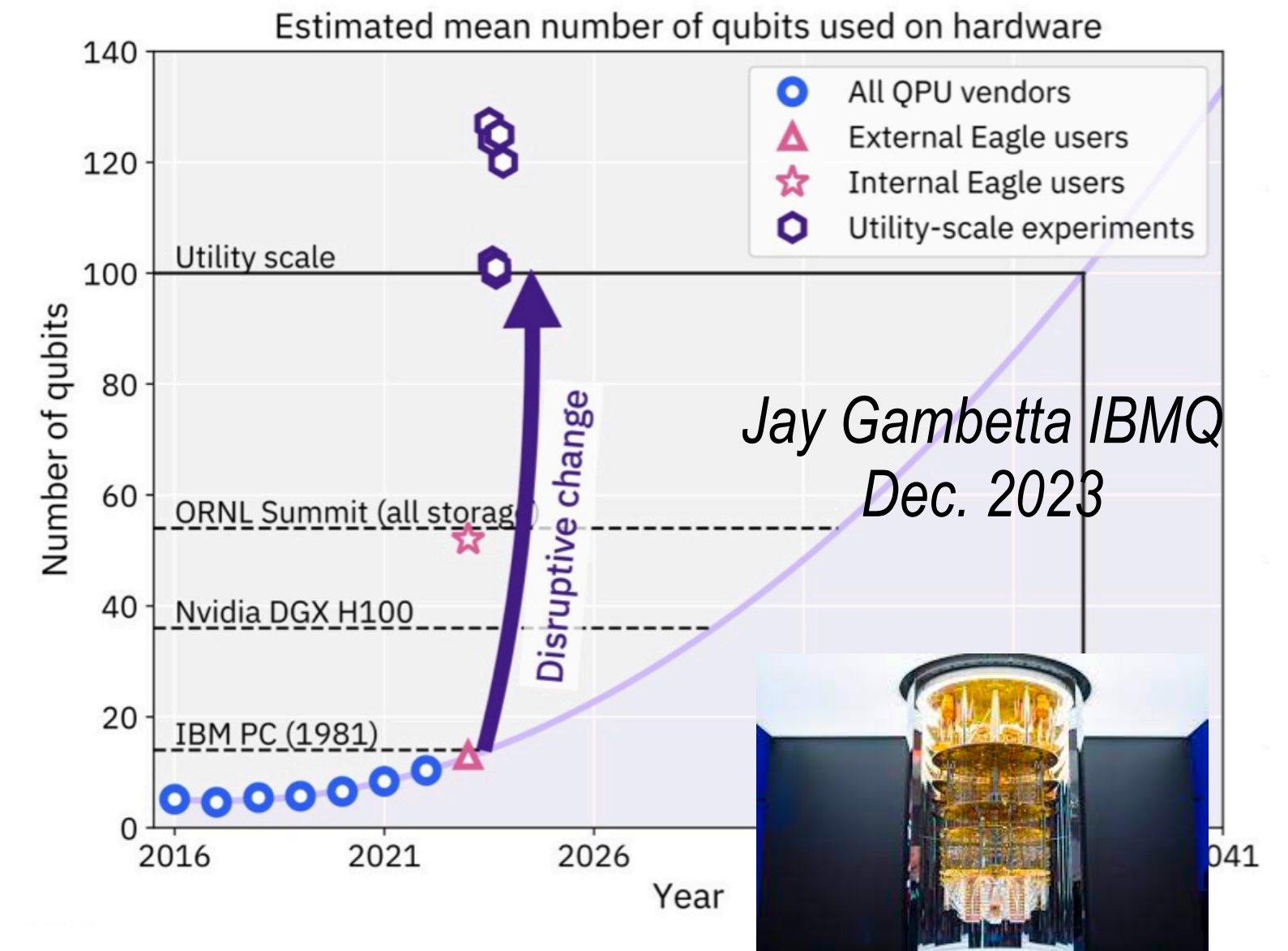


Adapted from Ewan Munro, Co-Founder of Entropica Labs. Landscape of quantum computing from an error correction perspective. Inspired by a figure by Daniel Gottesman.

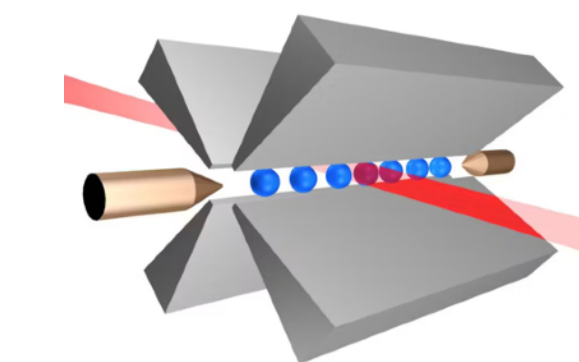
Progress in Quantum Computation



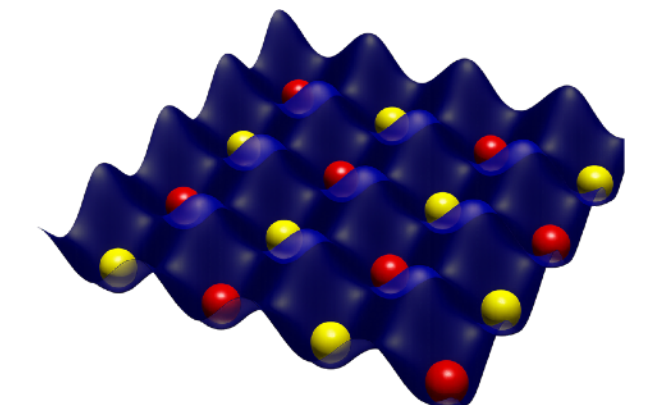
New experiments,
algorithms & theoretical
concepts



Superconductors



Trapped ions

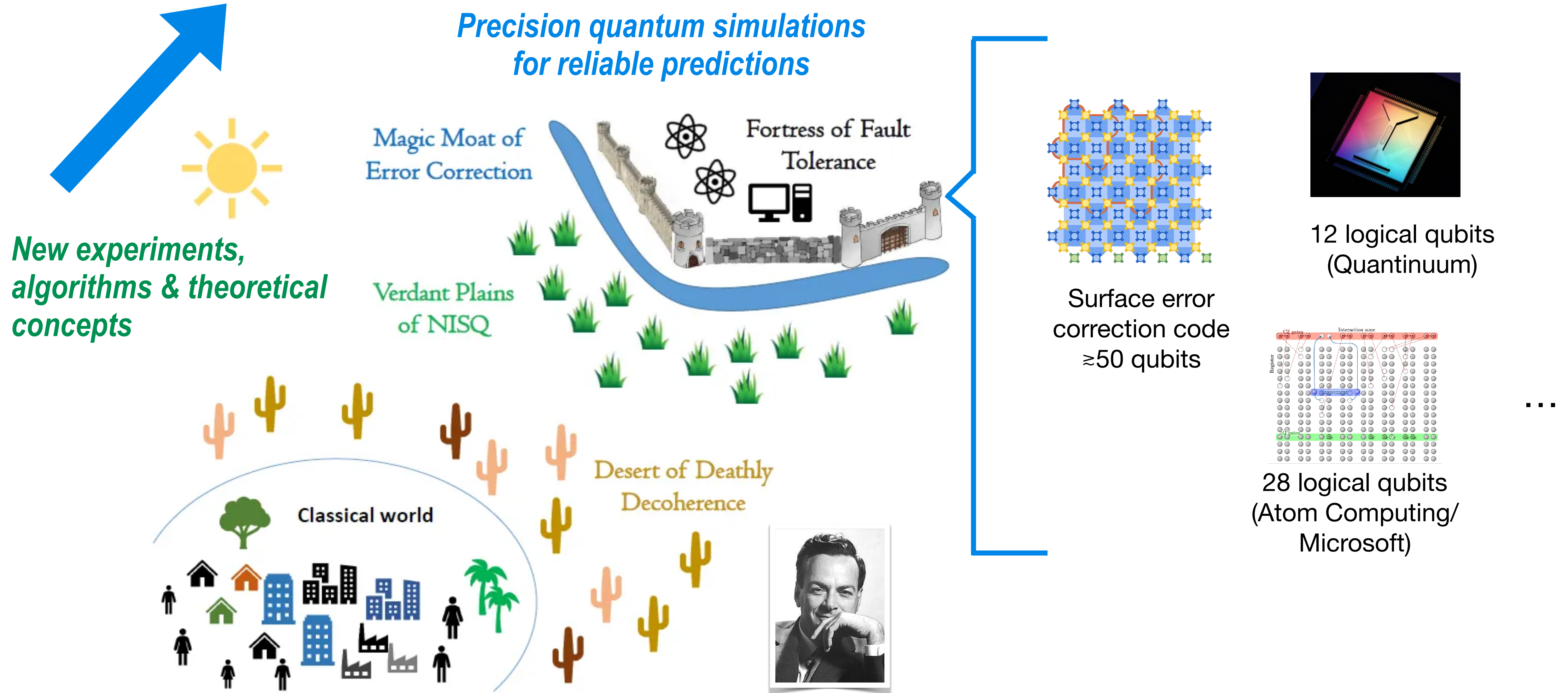


Cold atoms

...

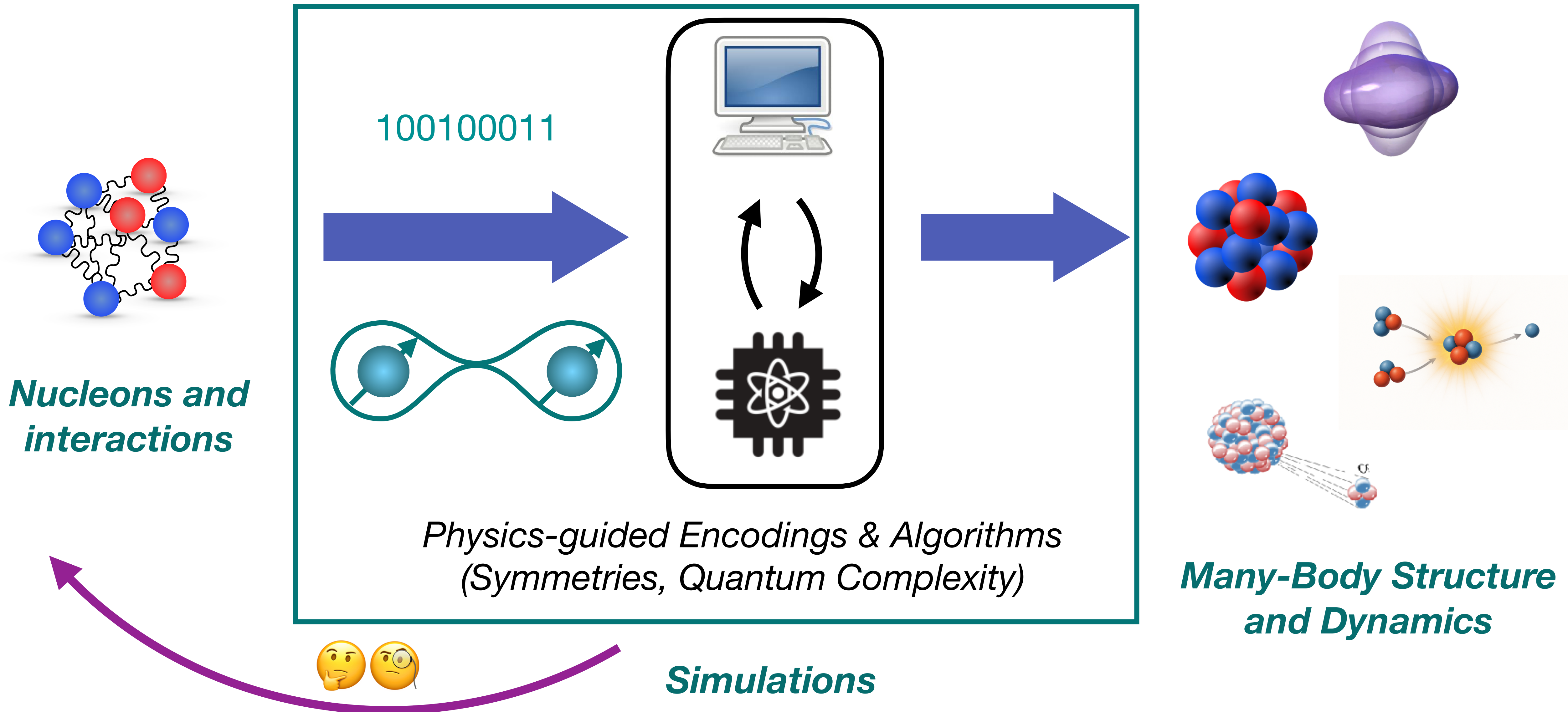
Adapted from Ewan Munro, Co-Founder of Entropica Labs. Landscape of quantum computing from an error correction perspective. Inspired by a figure by Daniel Gottesman.

Progress in Quantum Computation



Adapted from Ewan Munro, Co-Founder of Entropica Labs. Landscape of quantum computing from an error correction perspective. Inspired by a figure by Daniel Gottesman.

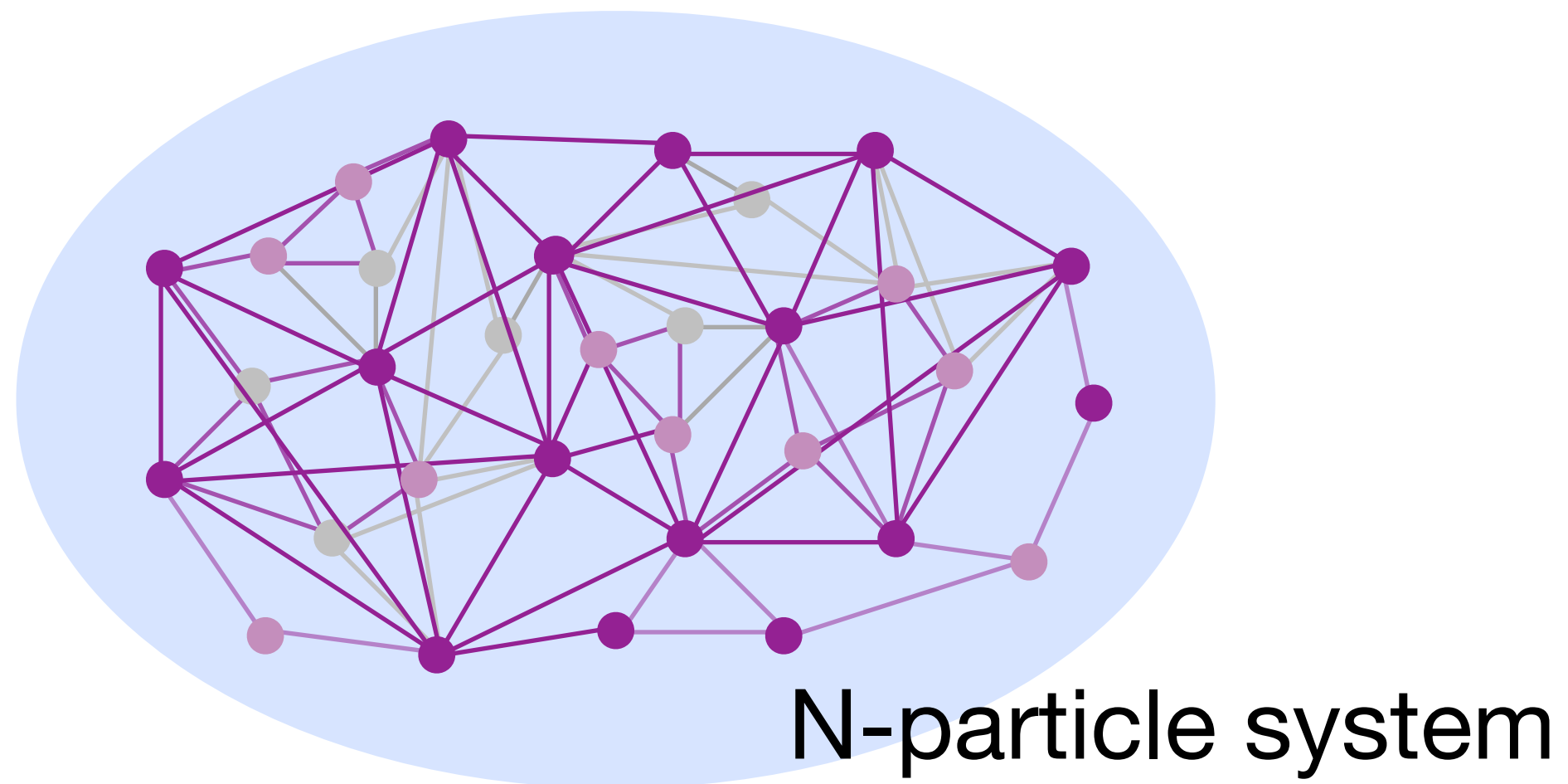
Towards Quantum Computations of (Nuclear) Many-Body Systems



Complexity of Physical Systems — when is a Quantum Computer required?

Information theory: the complexity of a system $\sim$ amount of information needed to describe it

(I) Quantum Entanglement

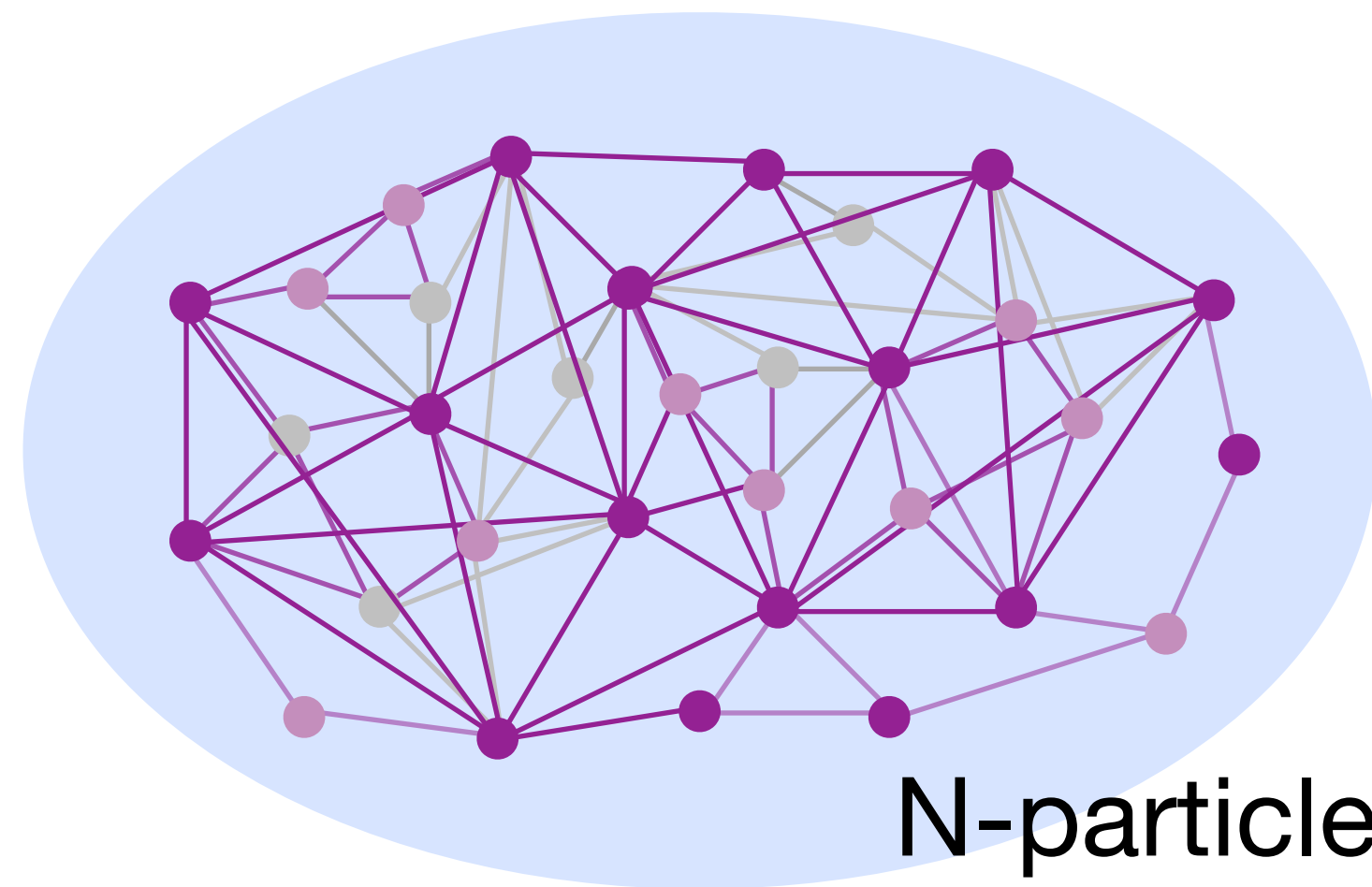


Arbitrary entangled state:
 $\sim \exp(N)$ bits

Complexity of Physical Systems — when is a Quantum Computer required?

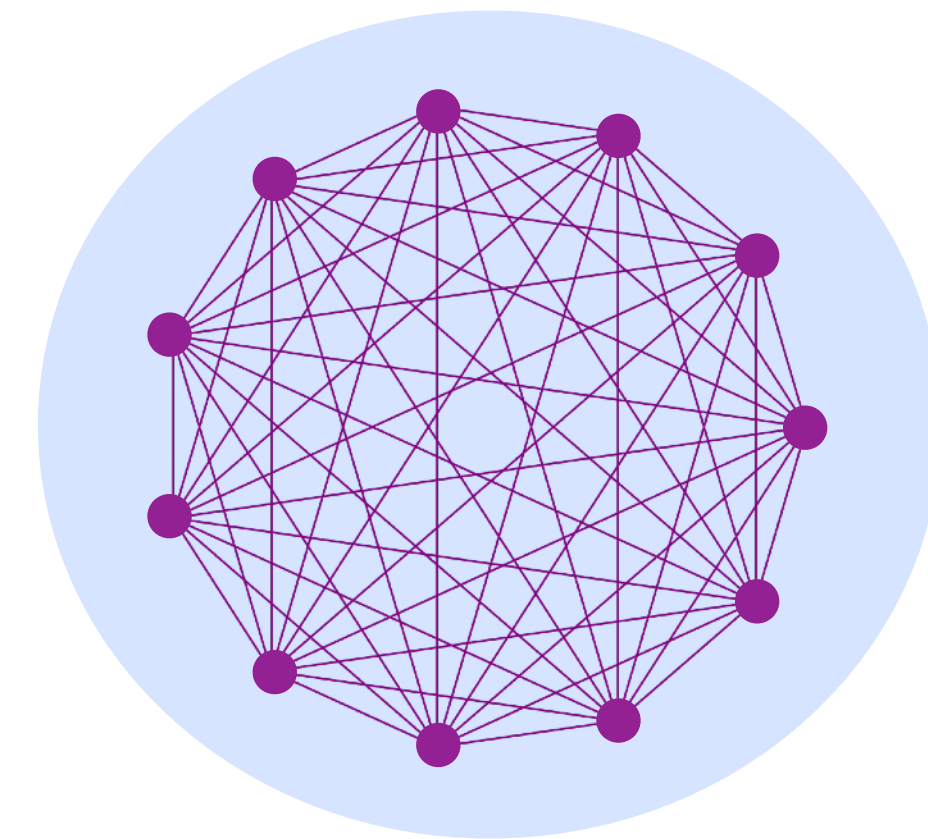
Information theory: the complexity of a system $\sim$ amount of information needed to describe it

(I) Quantum Entanglement



Arbitrary entangled state:
 $\sim \exp(N)$ bits

But... **entanglement can be simple**



→ highly-entangled “stabilizer states”
 $\sim$ graph states $\sim N^2$ bits

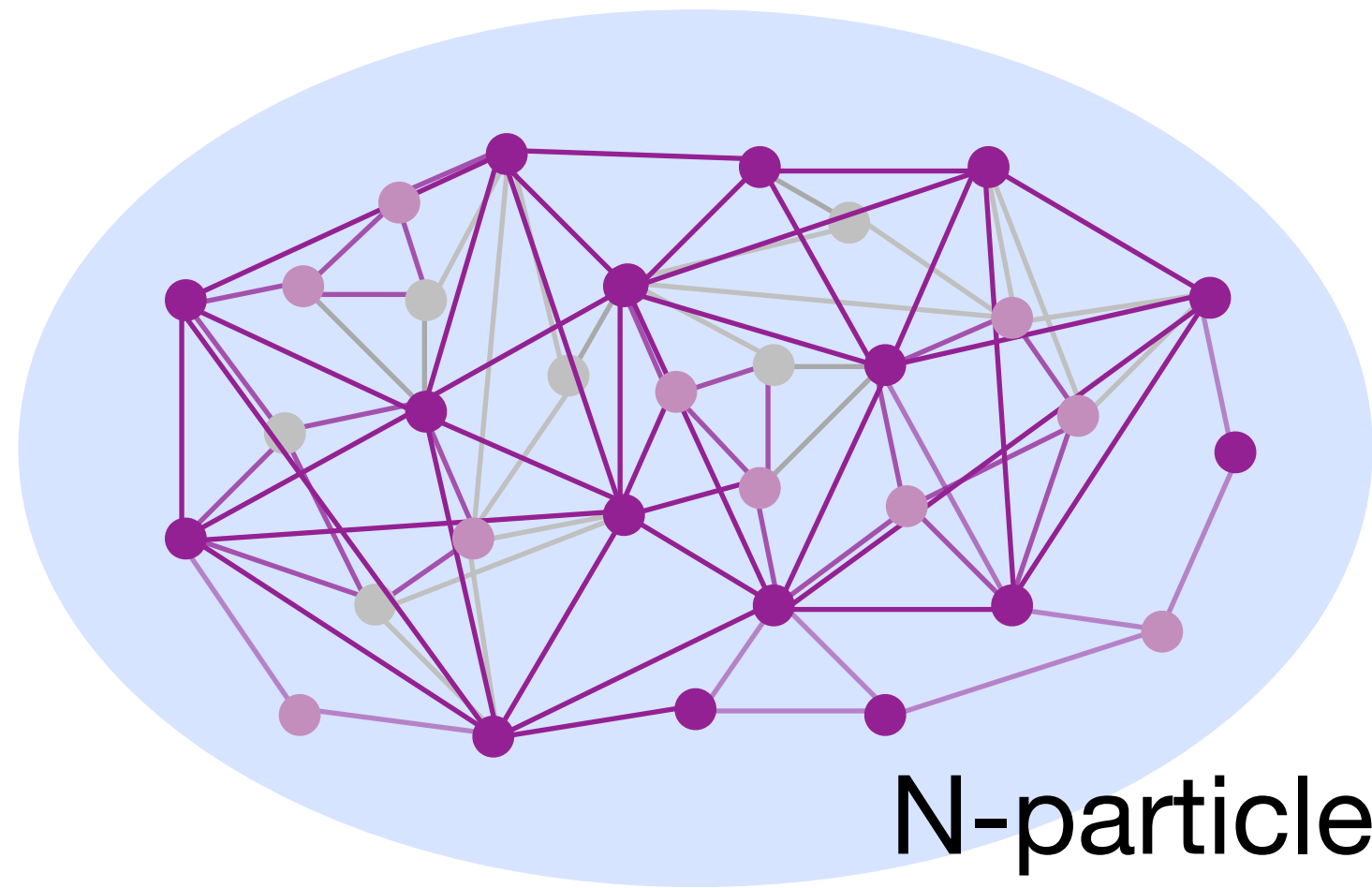
⇒ *No need for quantum computers*

*Gottesman & Knill (1998),
Aaronson & Gottesman (2004)*

Complexity of Physical Systems — when is a Quantum Computer required?

Information theory: the complexity of a system $\sim$ amount of information needed to describe it

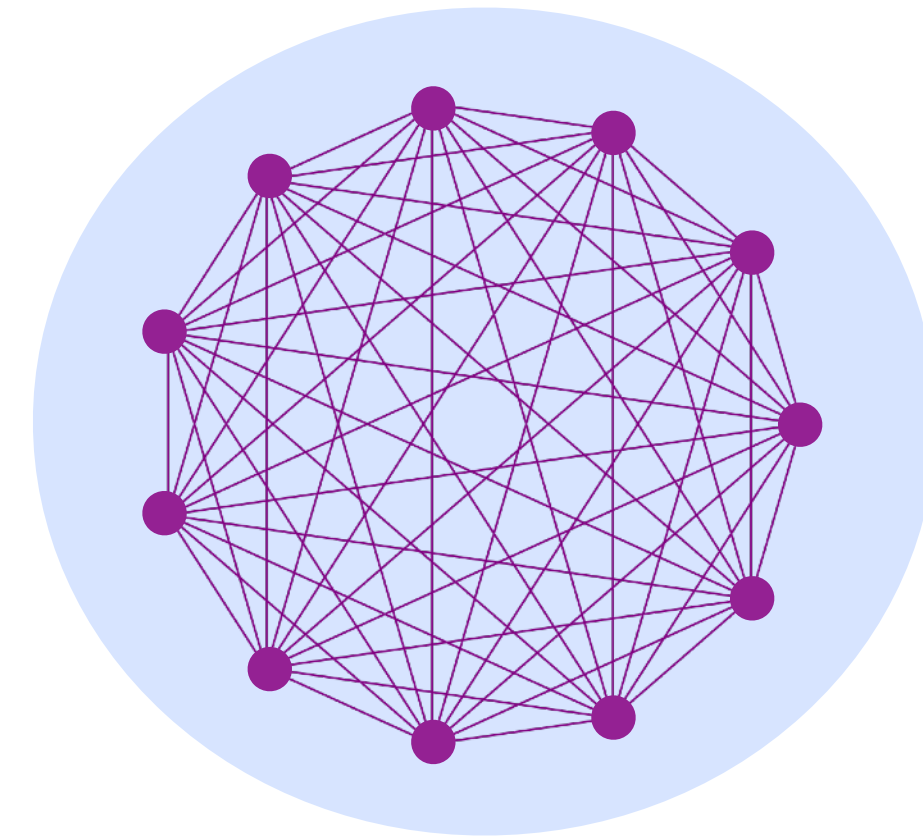
(I) *Quantum Entanglement*
+
(II) *Non-Stabilizerness (Quantum “Magic”)*



N-particle system

Arbitrary entangled state:
 $\sim \exp(N)$ bits

But... **entanglement can be simple**



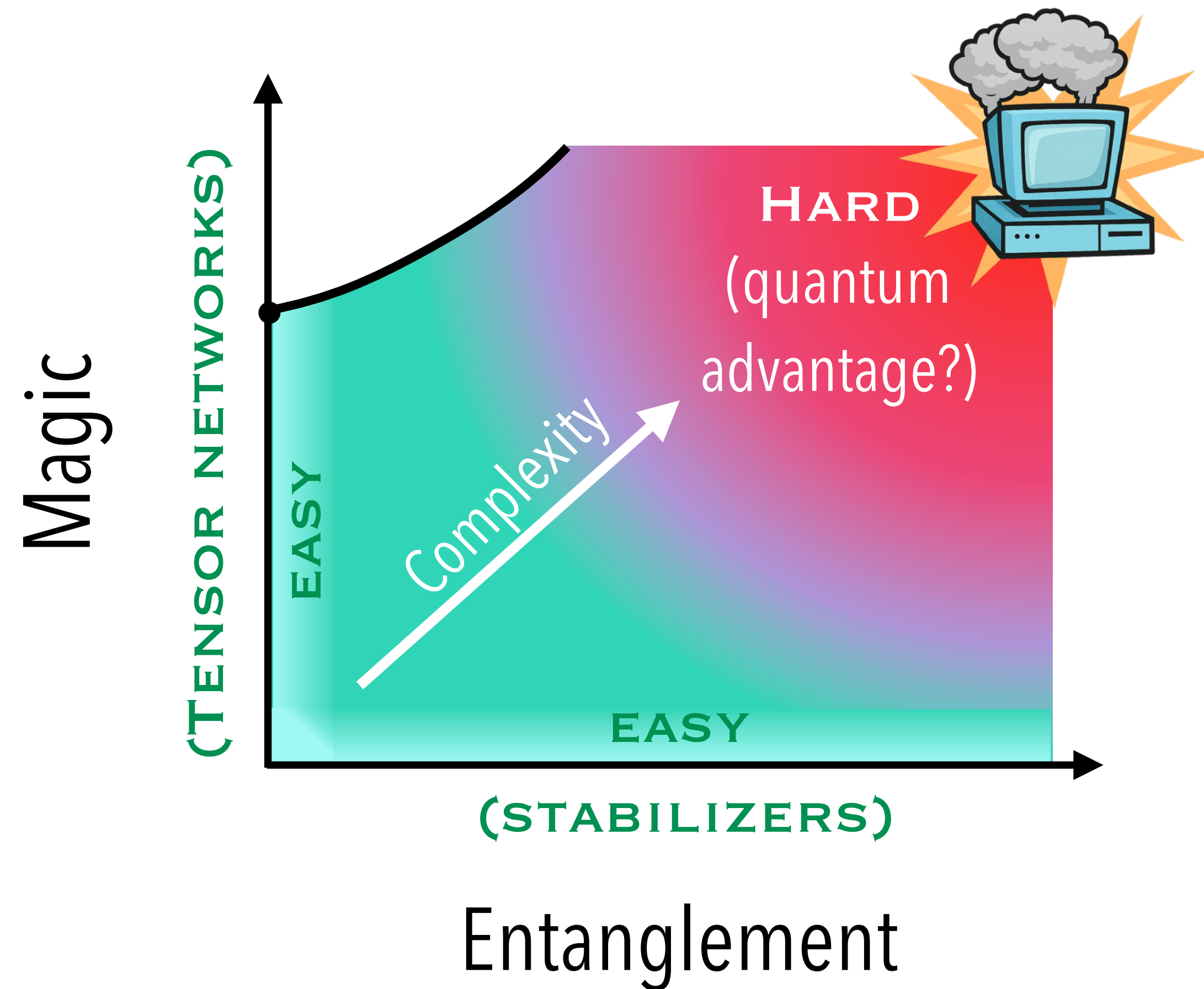
→ highly-entangled “stabilizer states”
 $\sim$ graph states $\sim N^2$ bits

⇒ *No need for quantum computers*

*Gottesman & Knill (1998),
Aaronson & Gottesman (2004)*

Complexity of Physical Systems — when is a Quantum Computer required?

Complexity phase diagram, hardness of classical simulations



*Inspired by Alioscia Hamma, Qmeets seminar,
https://www.youtube.com/watch?v=UrOCCGpL_go*

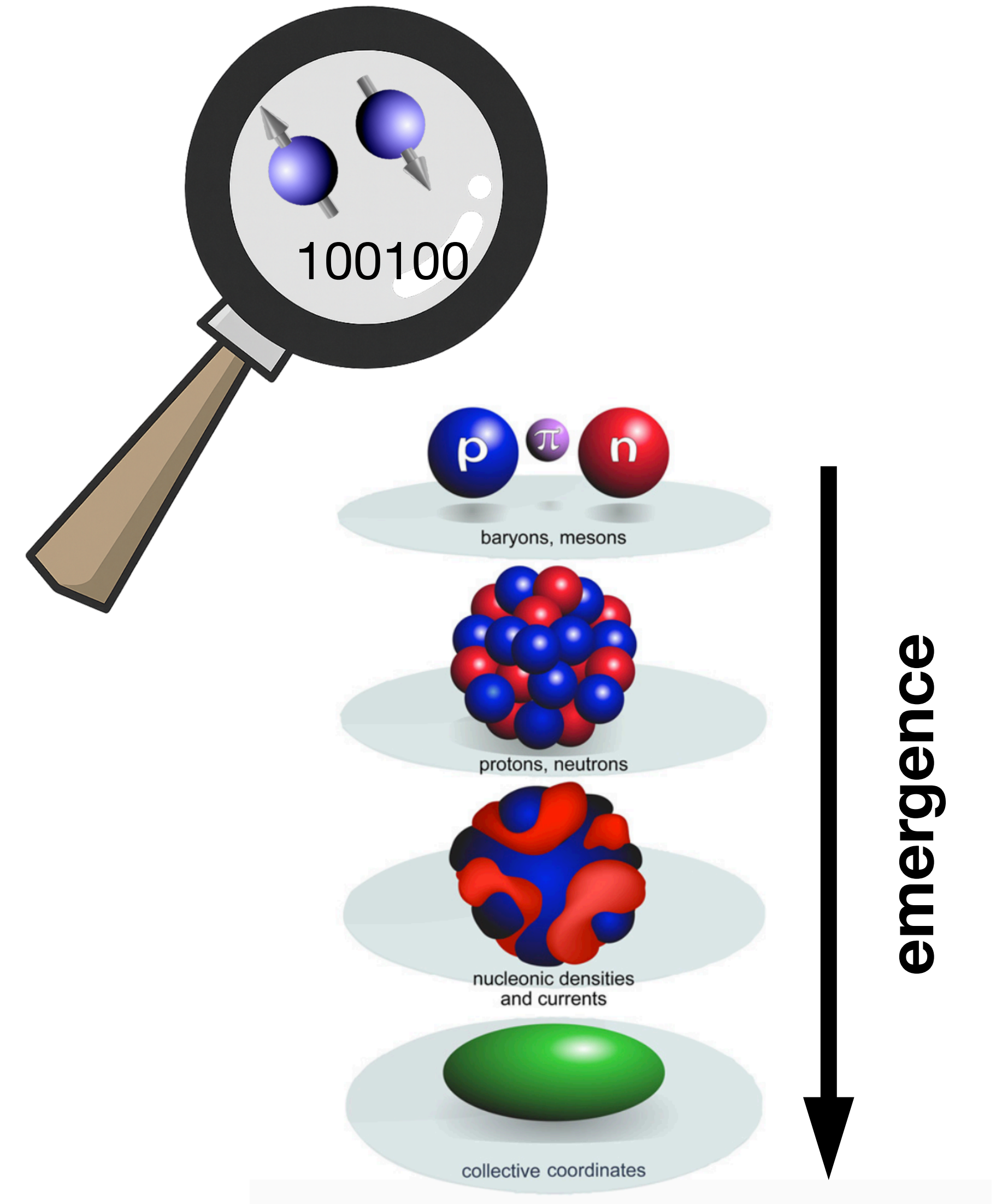
Motivational Questions & Outline

- Where do nuclear systems stand in the quantum complexity phase diagram? How are nuclear structure and dynamics rooted in quantum information?

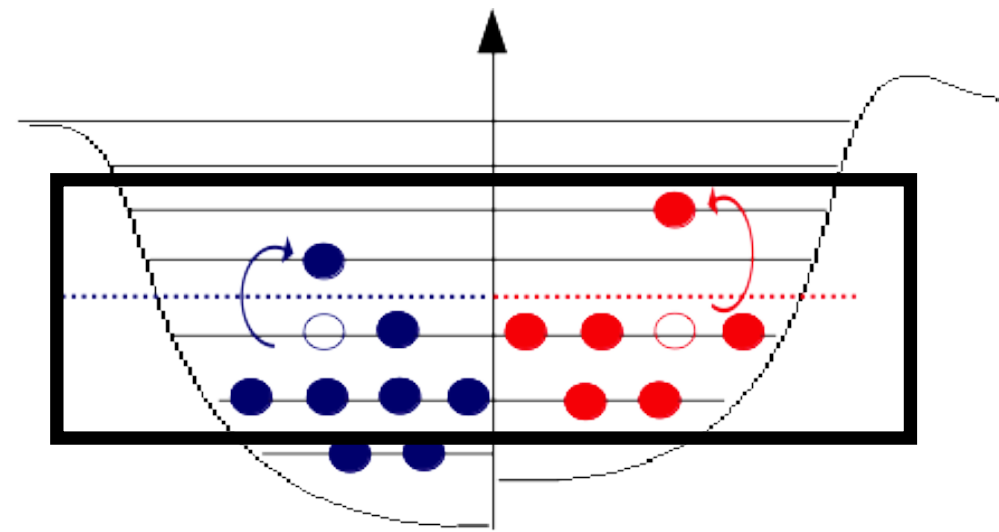
→ *Entanglement, magic and deformation in nuclear systems*

- Can QIS concepts help reformulate QMB problems and improve classical/quantum computation algorithms?

→ *Demonstrations with the Lipkin-Meshkov-Glick model*



Entanglement and Magic in Shell-Model Nuclei



Orbital-to-qubit
Jordan-Wigner mapping

$$a_i^\dagger \rightarrow \left(\prod_{j < i} \hat{\sigma}_z^{(j)} \right) (\hat{\sigma}_x^{(i)} - i \hat{\sigma}_y^{(i)}) / 2$$

Magic via Stabilizer Rényi Entropy:

Leone, Oliviero, Hamma, PRL 128, 050402 (2022)

$$\mathcal{M}_\alpha(|\Psi\rangle) = -\log(d) + \frac{1}{1-\alpha} \log \left(\sum_P \frac{\langle \Psi | \hat{P} | \Psi \rangle^{2\alpha}}{d^\alpha} \right)$$

$\Rightarrow \sim$ distance to the nearest stabilizer state

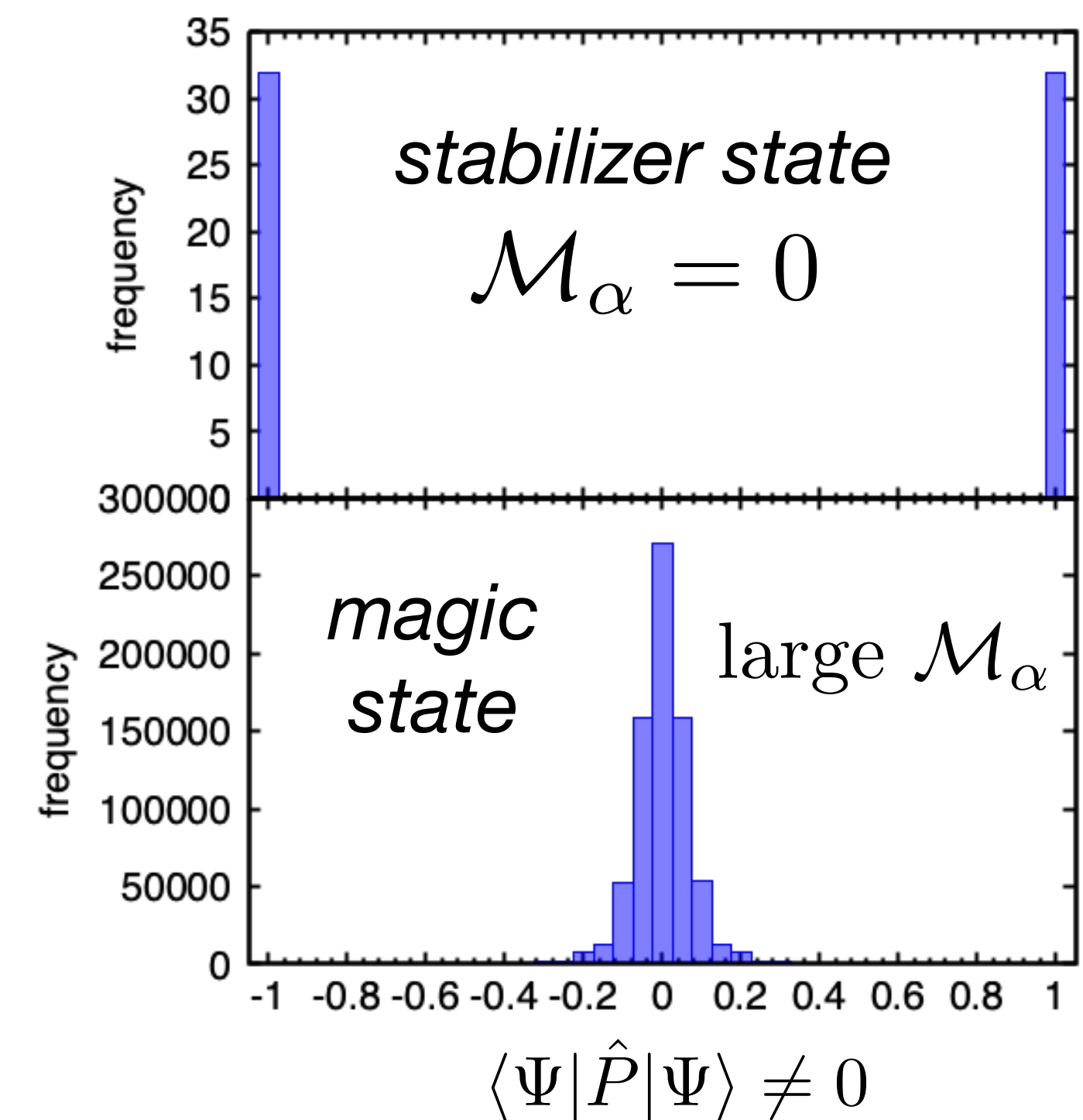
$$d = 2^{n_{\text{qubits}}} \quad \hat{P} = \text{Tensor product of Pauli operators}$$

Multi-Partite entanglement via n -tangles

Wong, Christensen, PRA 63, 044301 (2001)

$$\tau_{(i_1 \dots i_n)}^{(n)} = \left| \langle \Psi | \hat{\sigma}_y^{(i_1)} \otimes \dots \otimes \hat{\sigma}_y^{(i_n)} | \Psi^* \rangle \right|^2$$

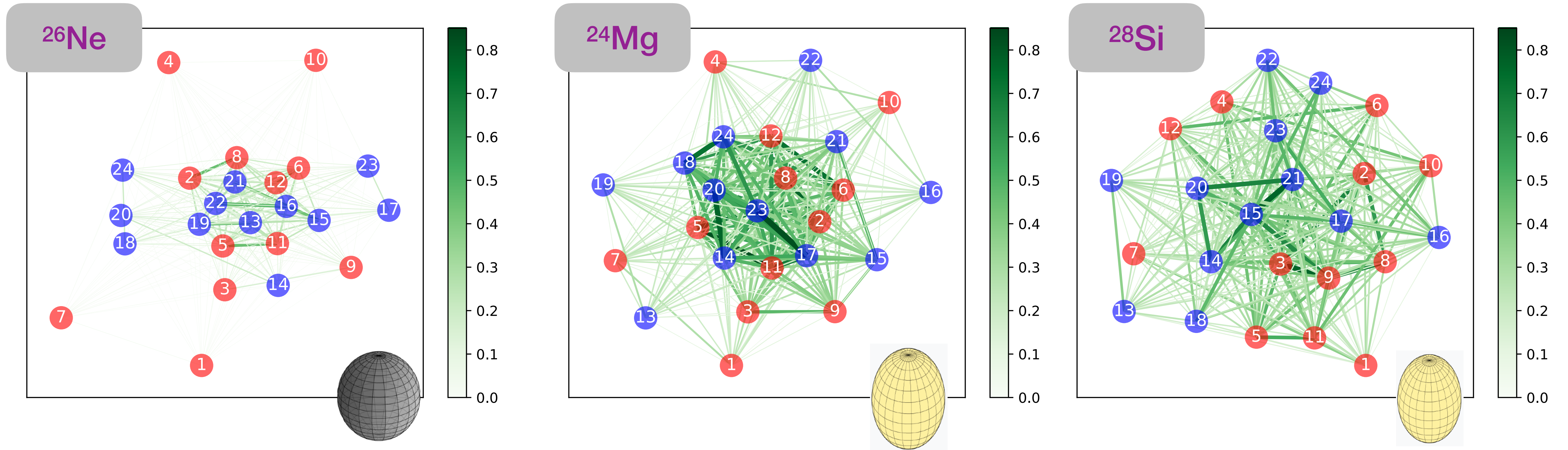
$\Rightarrow$ n -tangles are related to $n/2$ -body entanglement



Multi-Partite Entanglement in sd-shell Nuclei

Brökemeier, CR+, PRC 111, 034317 (2025)

Collective entanglement networks (8-tangles)

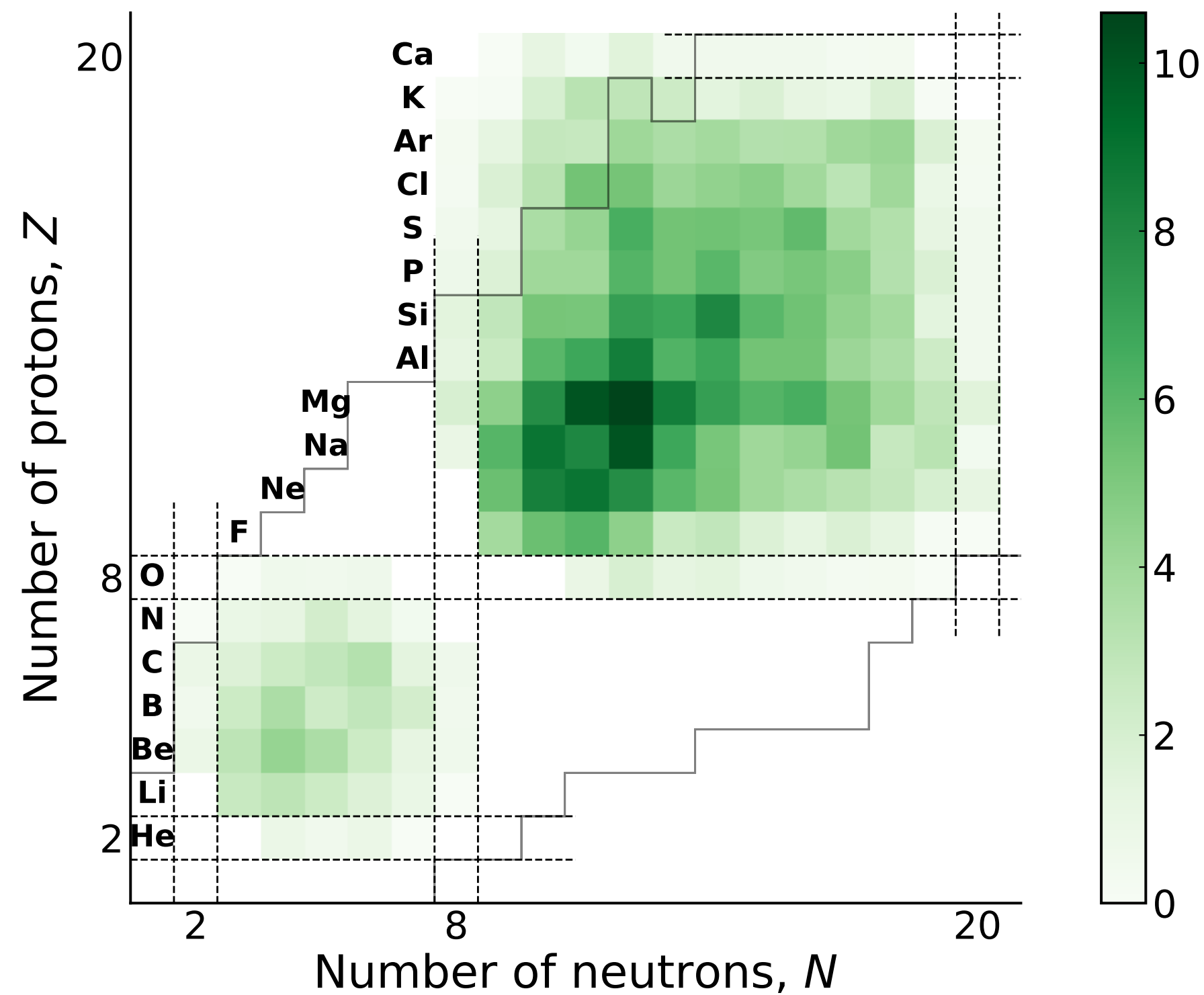


Edge values

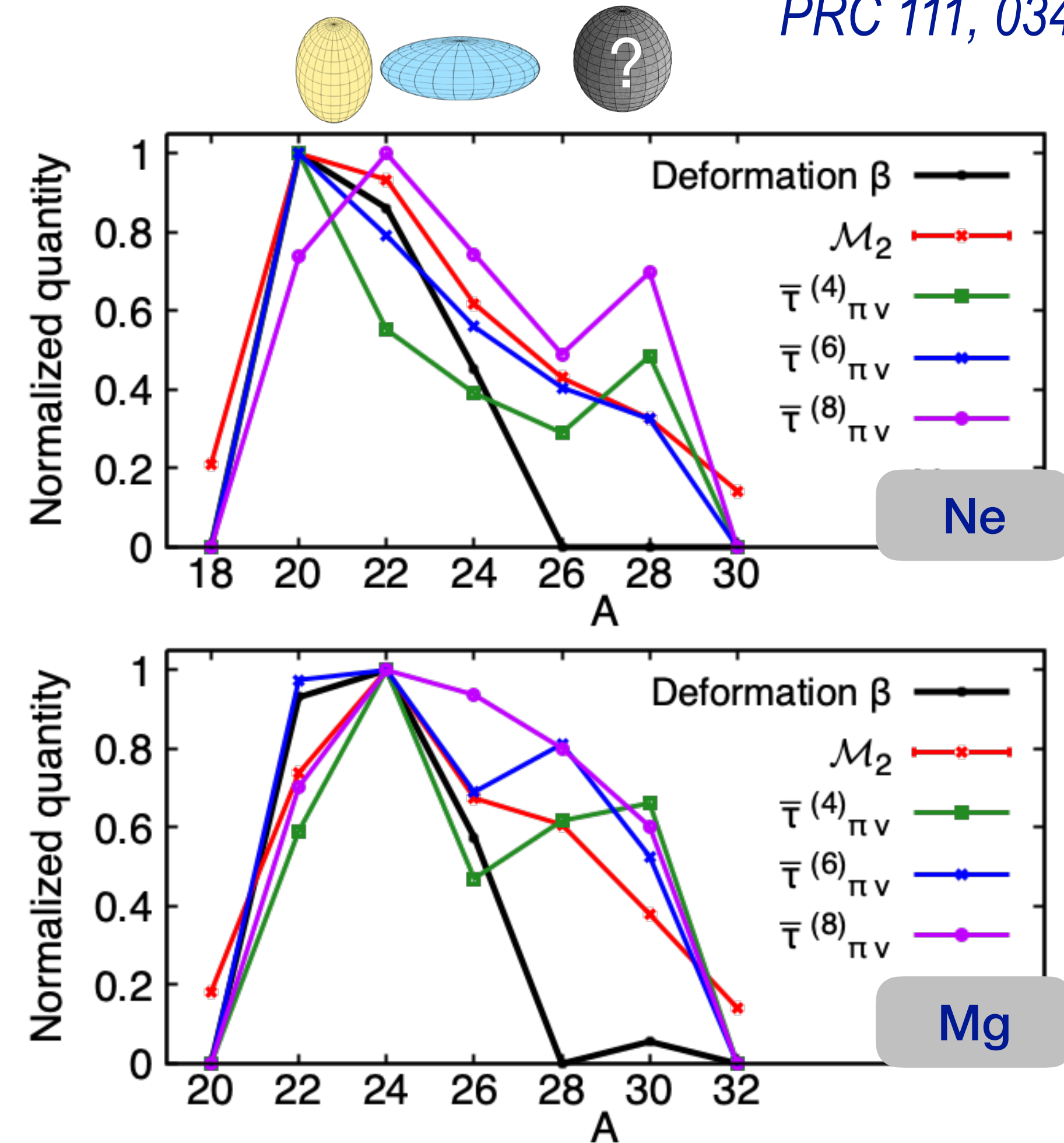
$$e_{i_1 i_2}^{(8)} = \sum_{i_3 < i_4 < i_5 < i_6 < i_7 < i_8} \tau_{(i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8)}^{(8)}$$

Signature of Shape Collectivity in Magic and Multi-pn Entanglement

Stabilizer 2-Rényi Entropy $\mathcal{M}_2$ Exact and MCMC computations



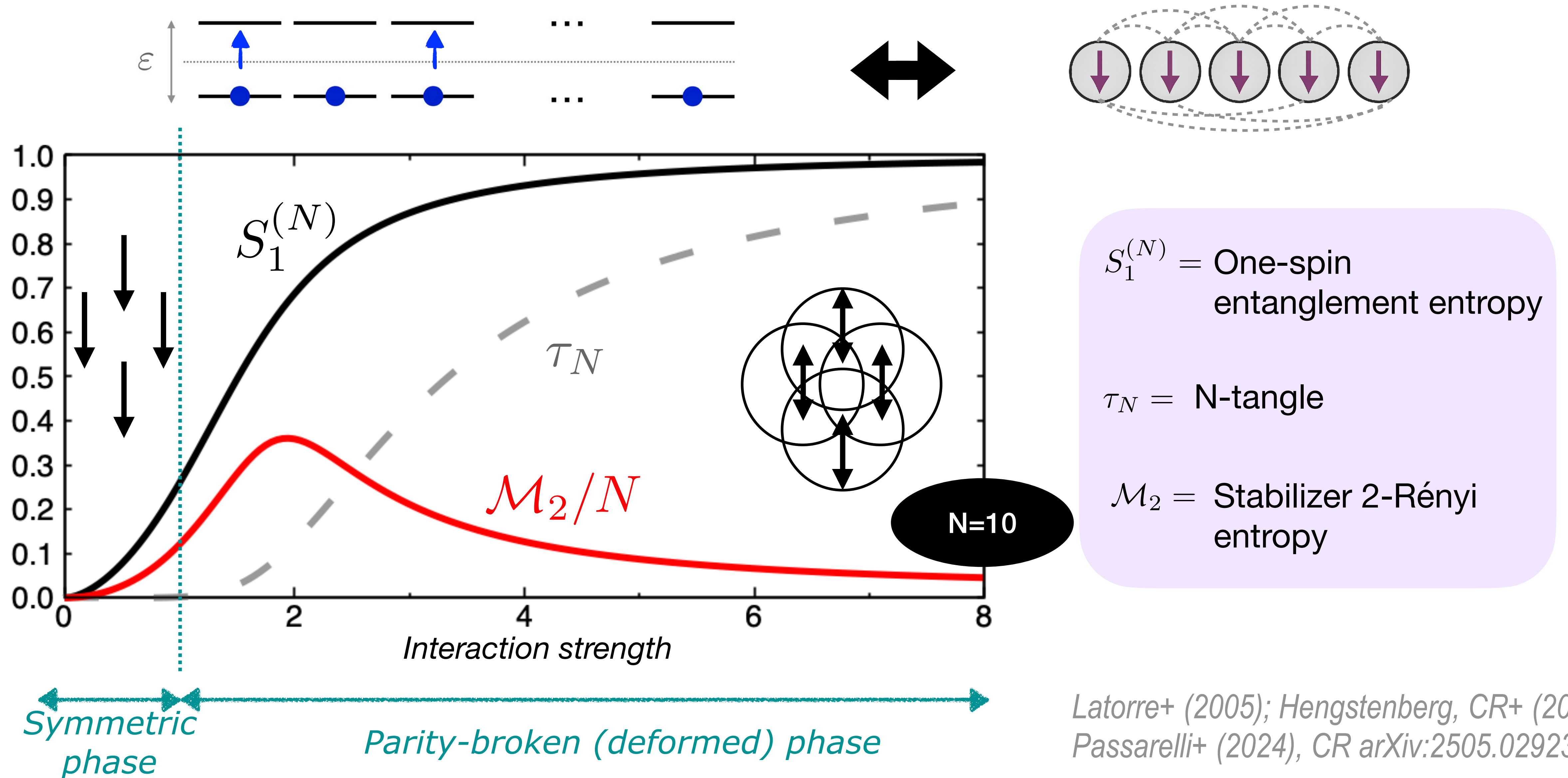
PRC 111, 034317 (2025)



- Maximal magic and proton-neutron tangles coincide with maximal deformation in nuclei
- Magic and tangles also persist in the region where axial deformation vanishes (shape co-existence, other correlations?)

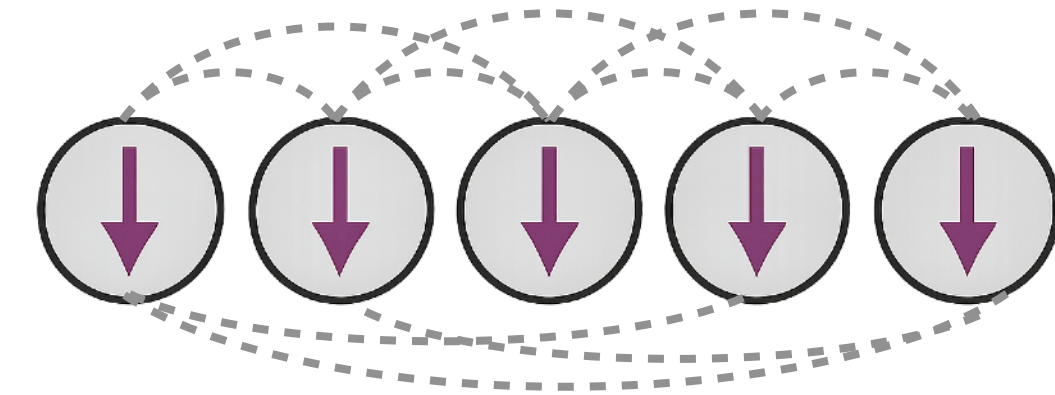
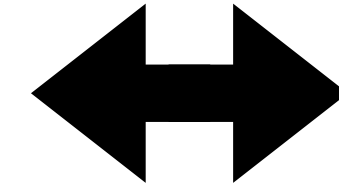
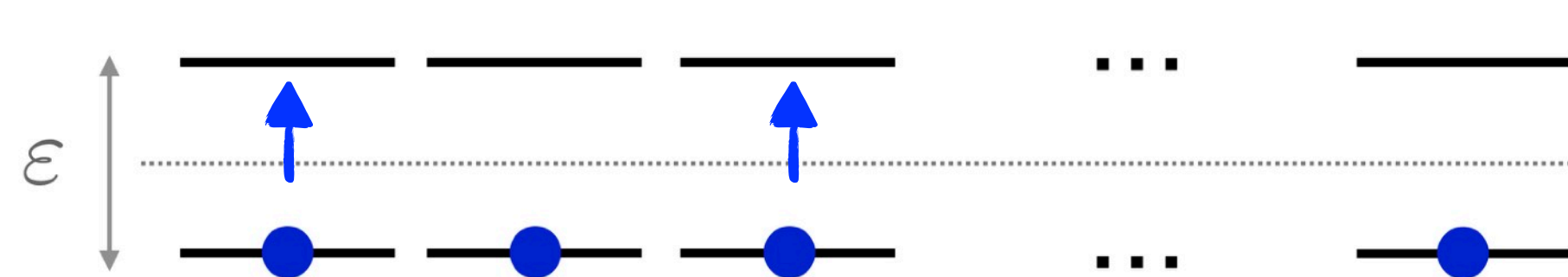
Singling out Deformation: the Lipkin-Meshkov-Glick model

Relevance for nuclear physics, condensed matter, trapped-ion quantum computing, quantum sensing/metrology



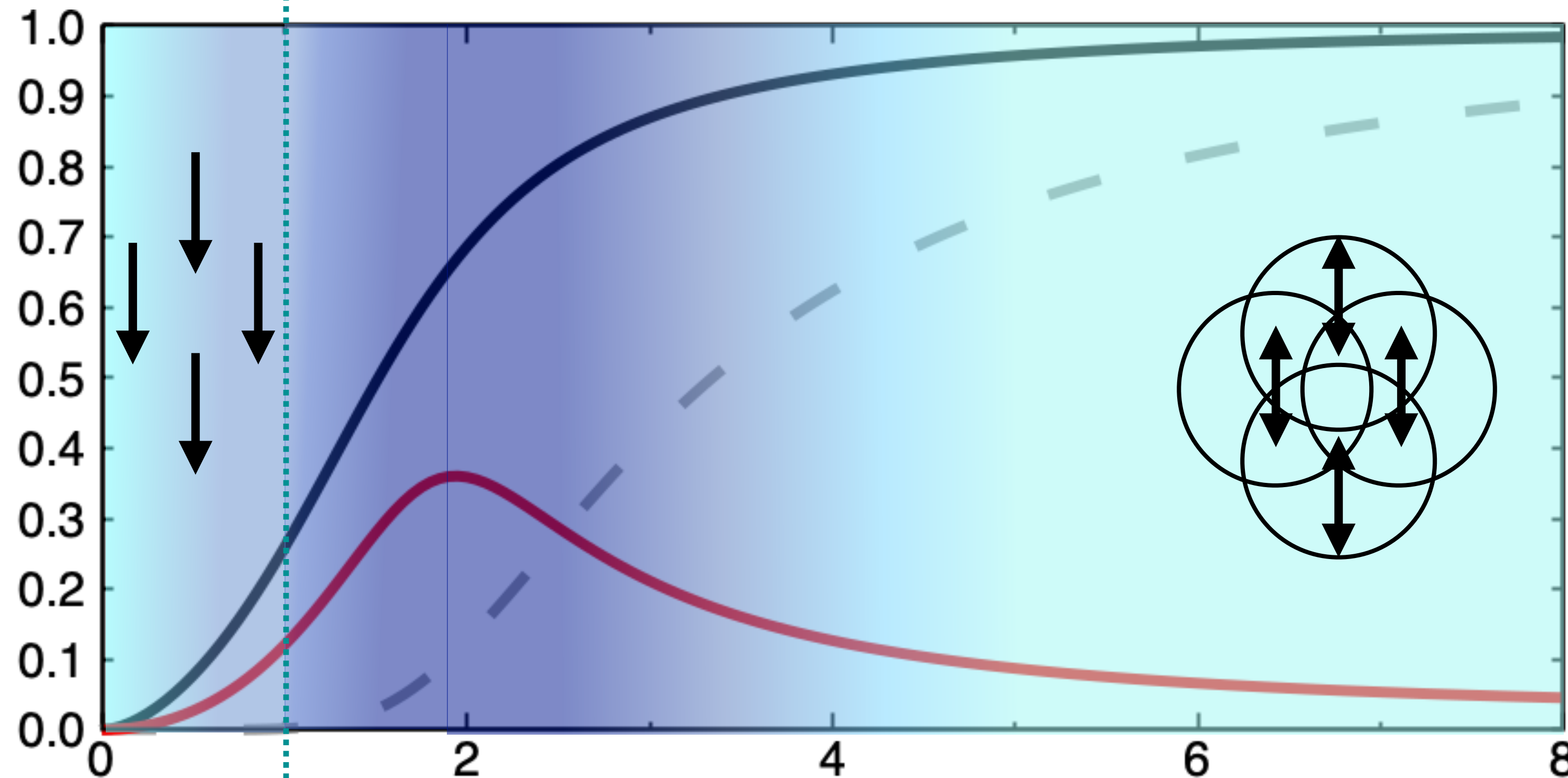
Singling out Deformation: the Lipkin-Meshkov-Glick model

Relevance for nuclear physics, condensed matter, trapped-ion quantum computing, quantum sensing/metrology



Can stabilizer states capture deformation in an efficient & symmetry-preserving way?

CR arXiv:2505.02923



Simple (trivial)

Complex

Simple (entangled)



Stabilizer Ground-State Method

👉 **Strategy:**

CR arXiv:2505.02923

$$\hat{H} = \underbrace{\sum_{P \in \mathcal{S}} a_P \hat{P}}_{\hat{H}_{stab}} + \underbrace{\sum_{P \in \mathcal{G}_N(H) \setminus \mathcal{S}} a_P \hat{P}}_{\hat{W}},$$

Stabilizer part

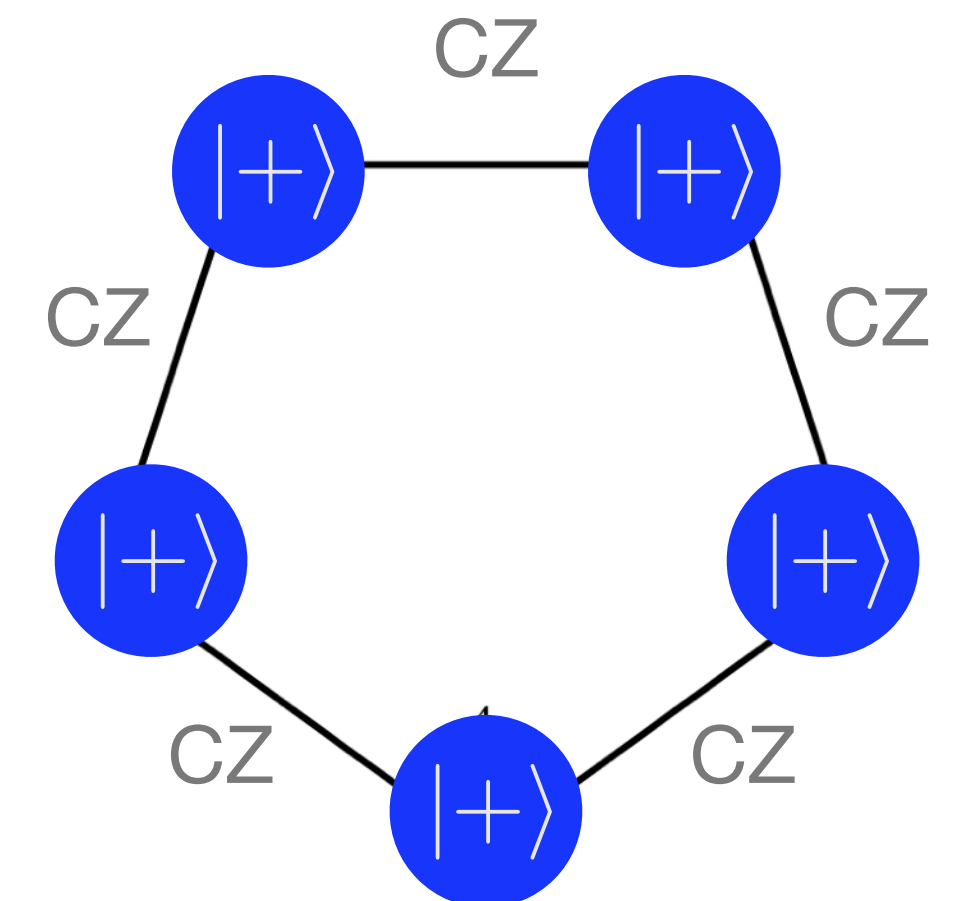
Magic-inducing part

See also Sun+ (2024), Gu+ (2024), Weaving+ (2022)...

$\mathcal{S} = \langle g_1, g_2, \dots, g_N \rangle$
Stabilizer group

- Explore all partitionings, and pick the energy-minimizing solution
- The stabilizer ground state can be prepared using graph states

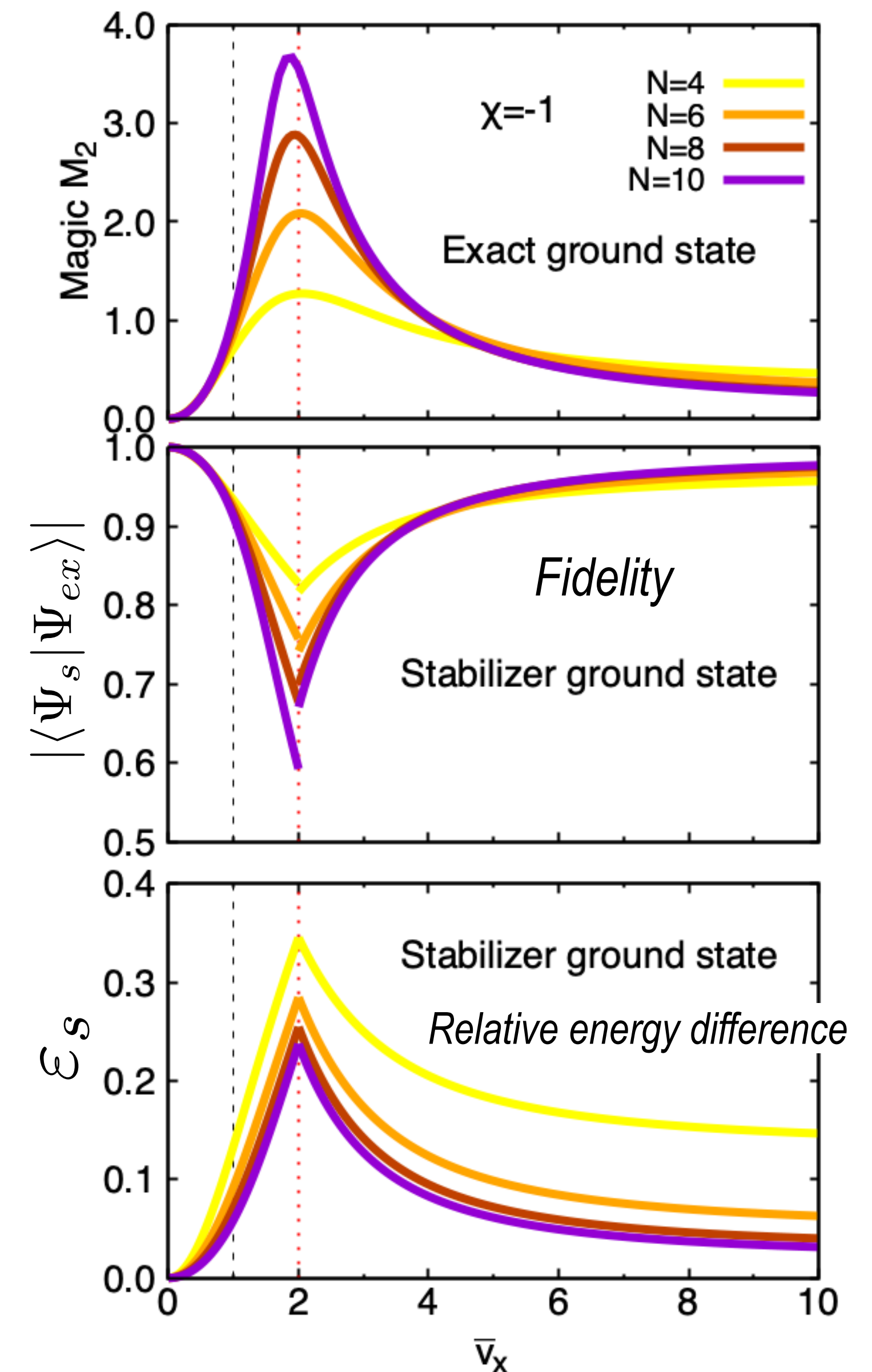
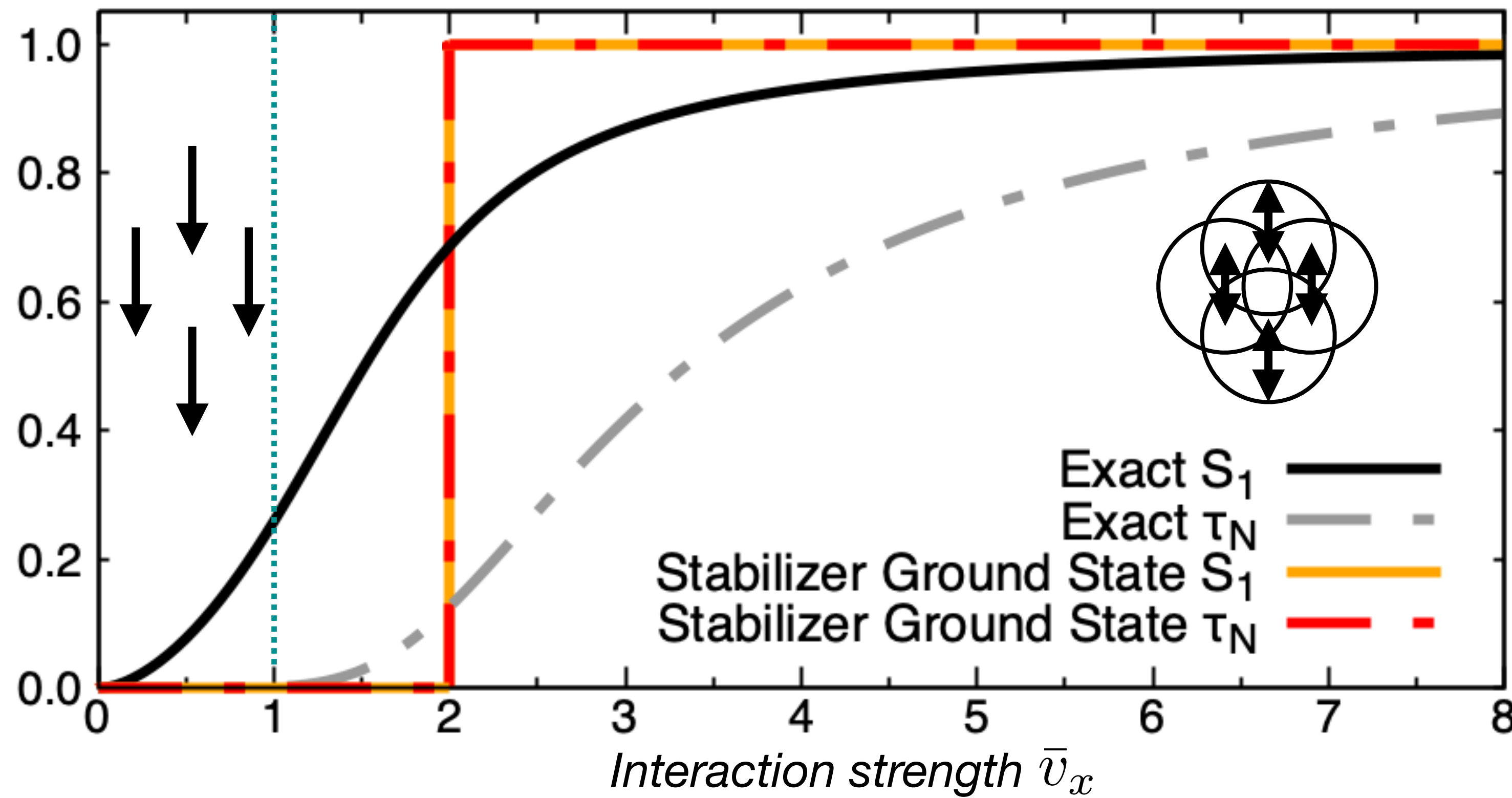
$$|\Psi_s\rangle = \prod_{i=1}^N C_i |G\rangle \quad \text{where} \quad |G\rangle = \left(\prod_{e \in E} CZ_e \right) |+\rangle^{\otimes N}$$



The 1-qubit Clifford C_i 's can be determined efficiently, using stabilizer tableaus

see e.g. Van den Nest+ PRA 69, 022316 (2004).

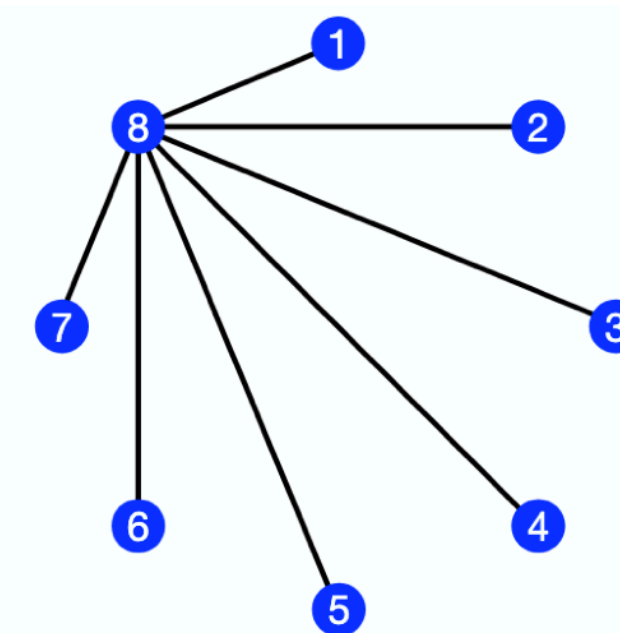
Stabilizer Ground State in the LMG model



$$|\Psi_s\rangle = |\downarrow\downarrow \dots \downarrow\rangle$$

$$|\Psi_s\rangle = (X_1)^{N \bmod 2} H_N |G\rangle$$

$$|G\rangle =$$



Beyond stabilizer ground state?
(Q)ITE, ADAPT-VQE...

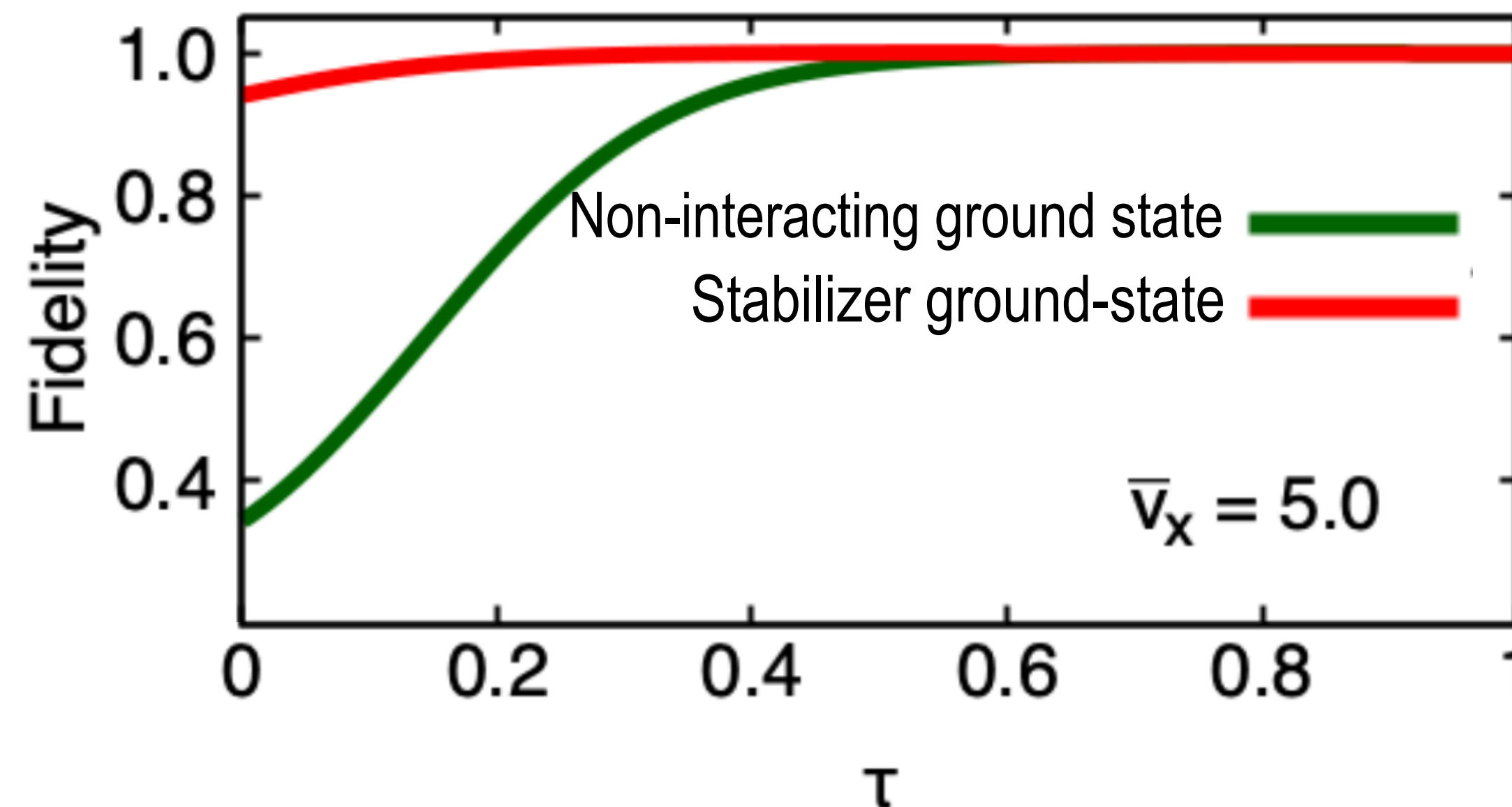
CR arXiv:2505.02923

Beyond Stabilizer Ground State with Magic Injection

★ Example: (Quantum) Imaginary Time Evolutions

$$e^{-(\hat{H}-\bar{E}_0)\tau} |\eta\rangle = \sum_n e^{-(E_n-\bar{E}_0)\tau} \langle \Phi_n | \eta \rangle |\Phi_n\rangle \xrightarrow{\tau \rightarrow \infty} \propto |\Phi_0\rangle$$

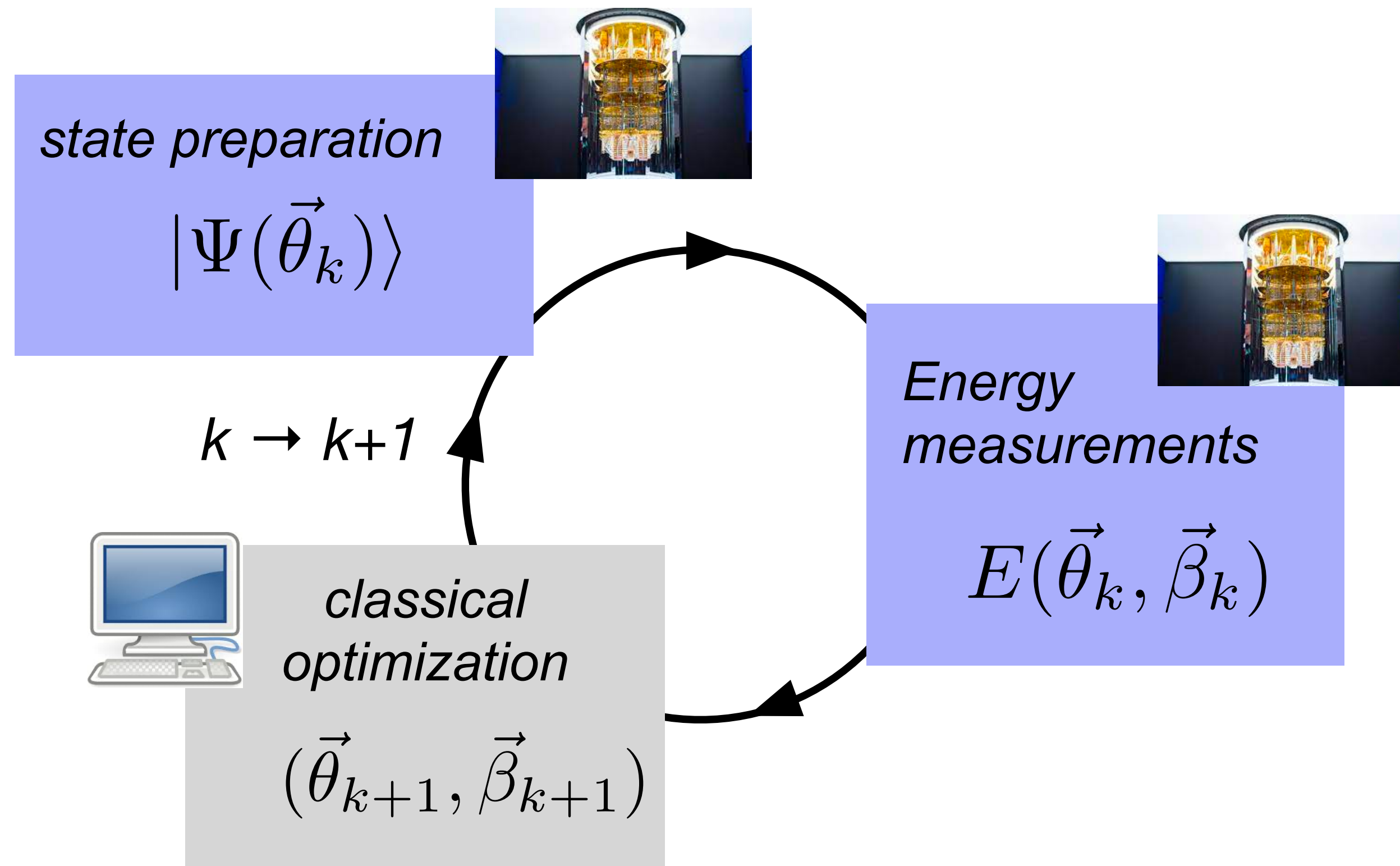
Convergence with different initial states:



Information Rearrangement in Classical and Quantum Computations

★ *Hamiltonian-Learning-VQE Algorithm for ground-state finding:*

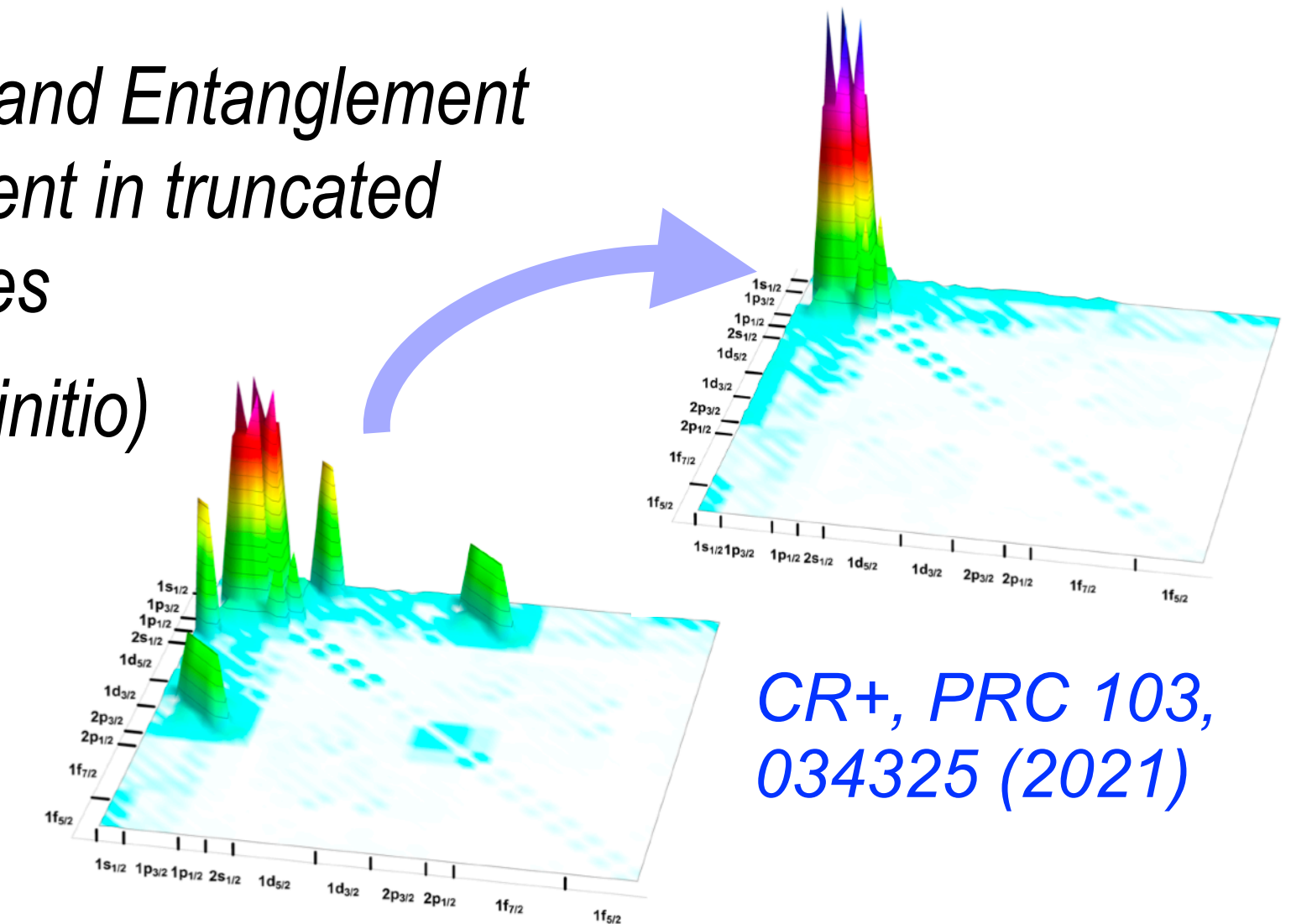
CR, Savage PRC 108, 024313 (2023)



$\Rightarrow$ learns the effective Hamiltonian and identifies the associated ground state simultaneously

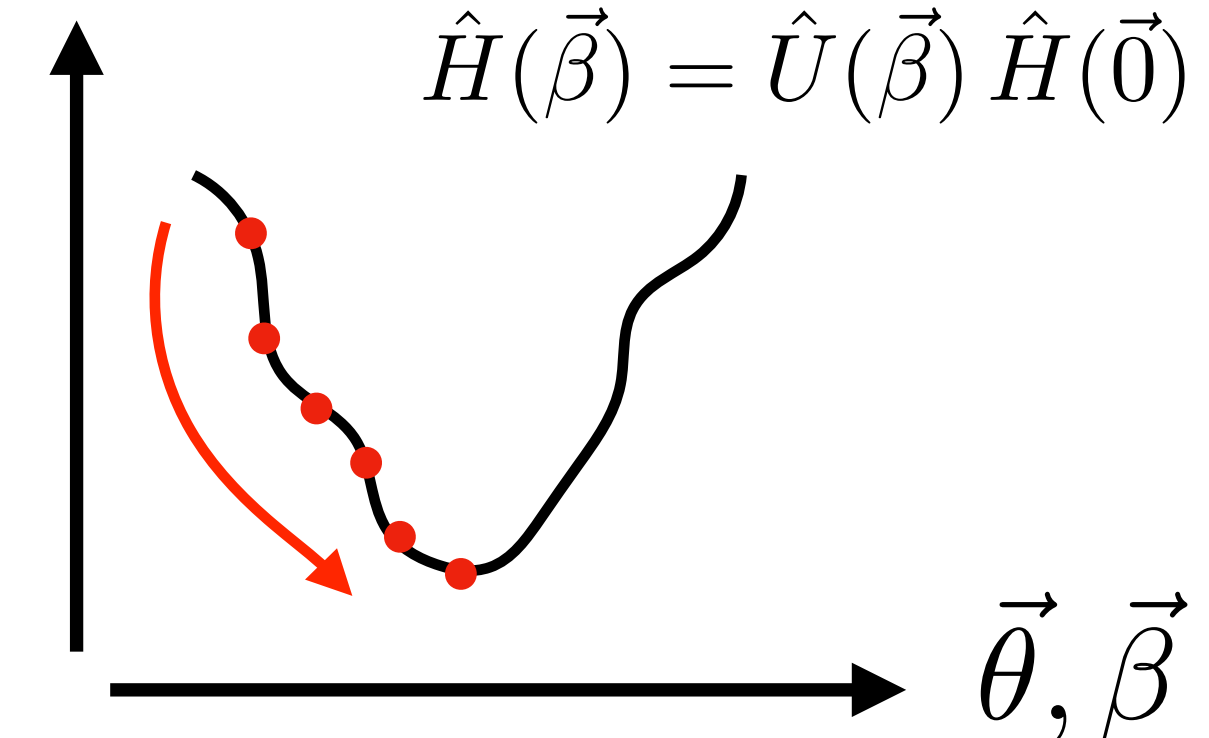
Information and Entanglement rearrangement in truncated model spaces

(^6He , ab-initio)



CR+, PRC 103, 034325 (2021)

$$E(\vec{\theta}, \vec{\beta}) = \langle \Psi(\vec{\theta}) | \hat{H}(\vec{\beta}) | \Psi(\vec{\theta}) \rangle$$
$$\hat{H}(\vec{\beta}) = \hat{U}(\vec{\beta}) \hat{H}(\vec{0}) \hat{U}^\dagger(\vec{\beta})$$

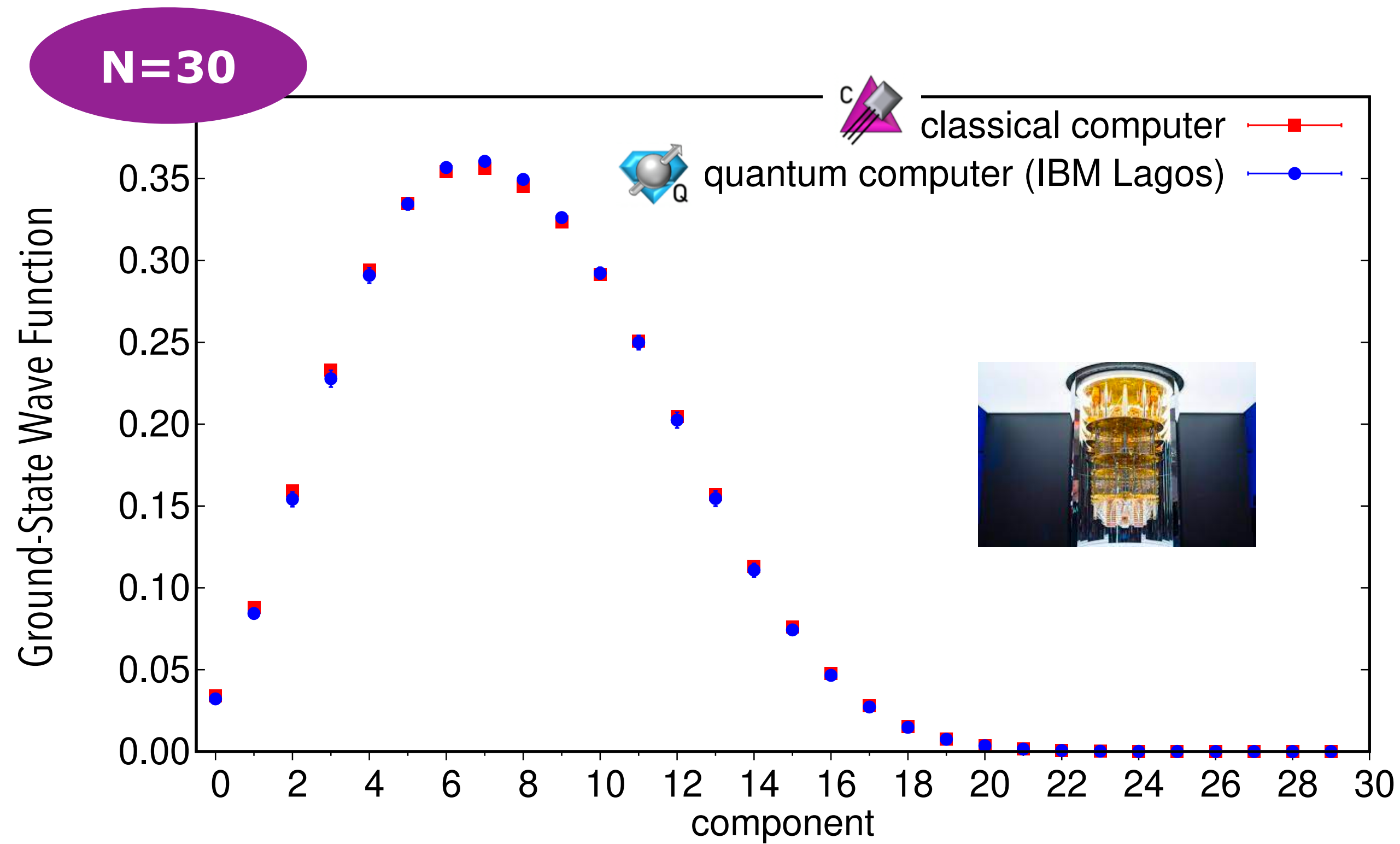


Information Rearrangement in Classical and Quantum Computations

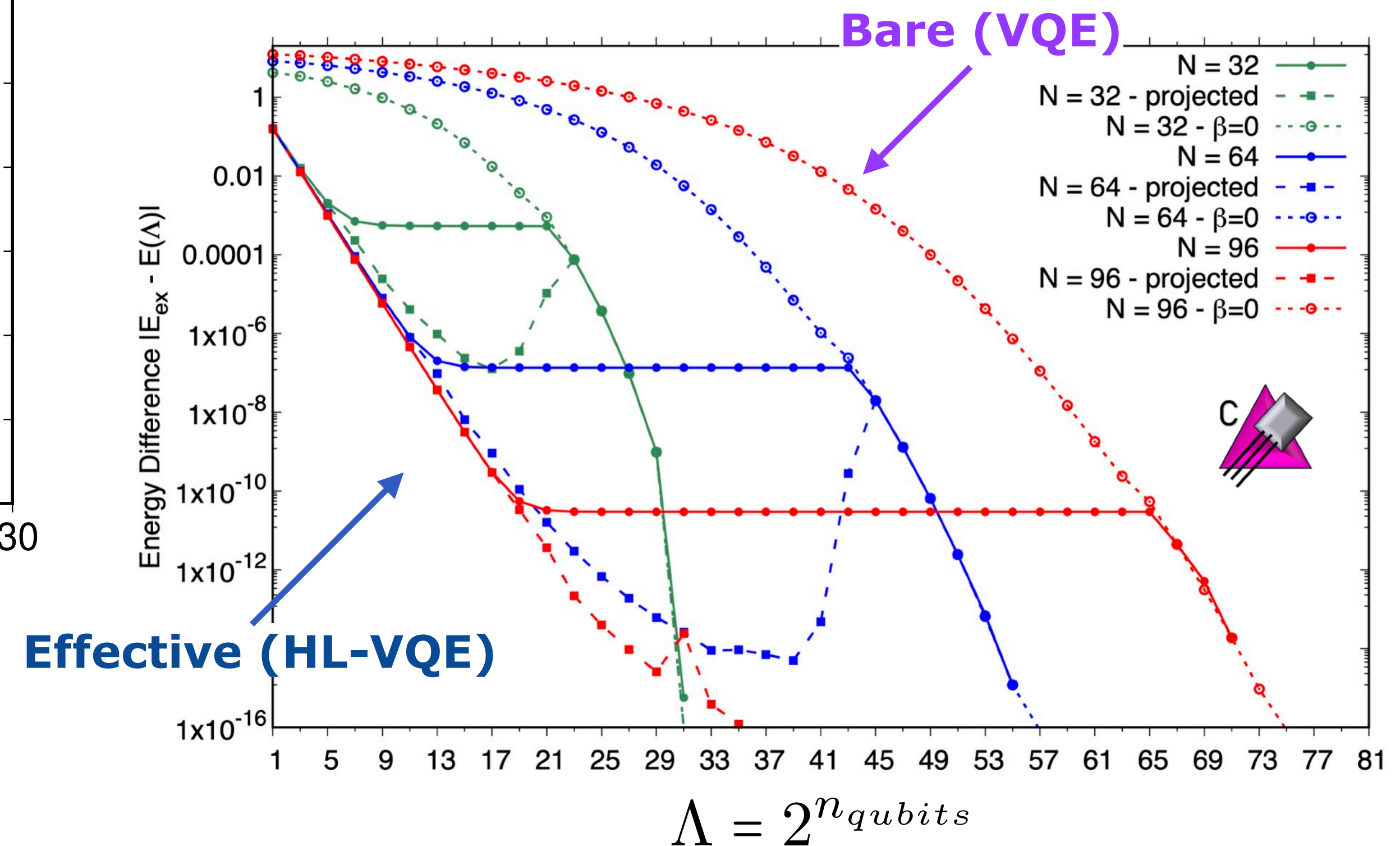
→ **Application to the LMG model:**

CR, Savage PRC 108, 024313 (2023)

Wave function extracted from IBM quantum computer



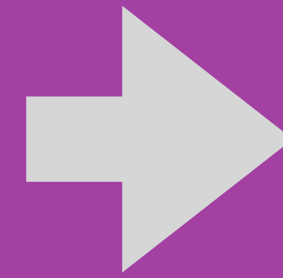
Exponential Acceleration in the expected convergence:



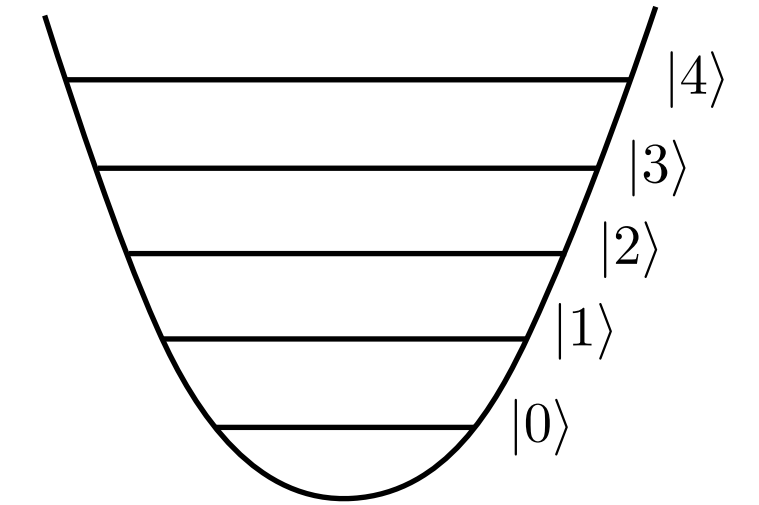
Symmetry-guided Quantum Simulations with Qudits

Quantum Computers with qudits ($d > 2$ -level quantum systems) are becoming available and allow for naturally mapping symmetries

Agassi model*: deformation + pairing
with underlying $so(5)$ algebra

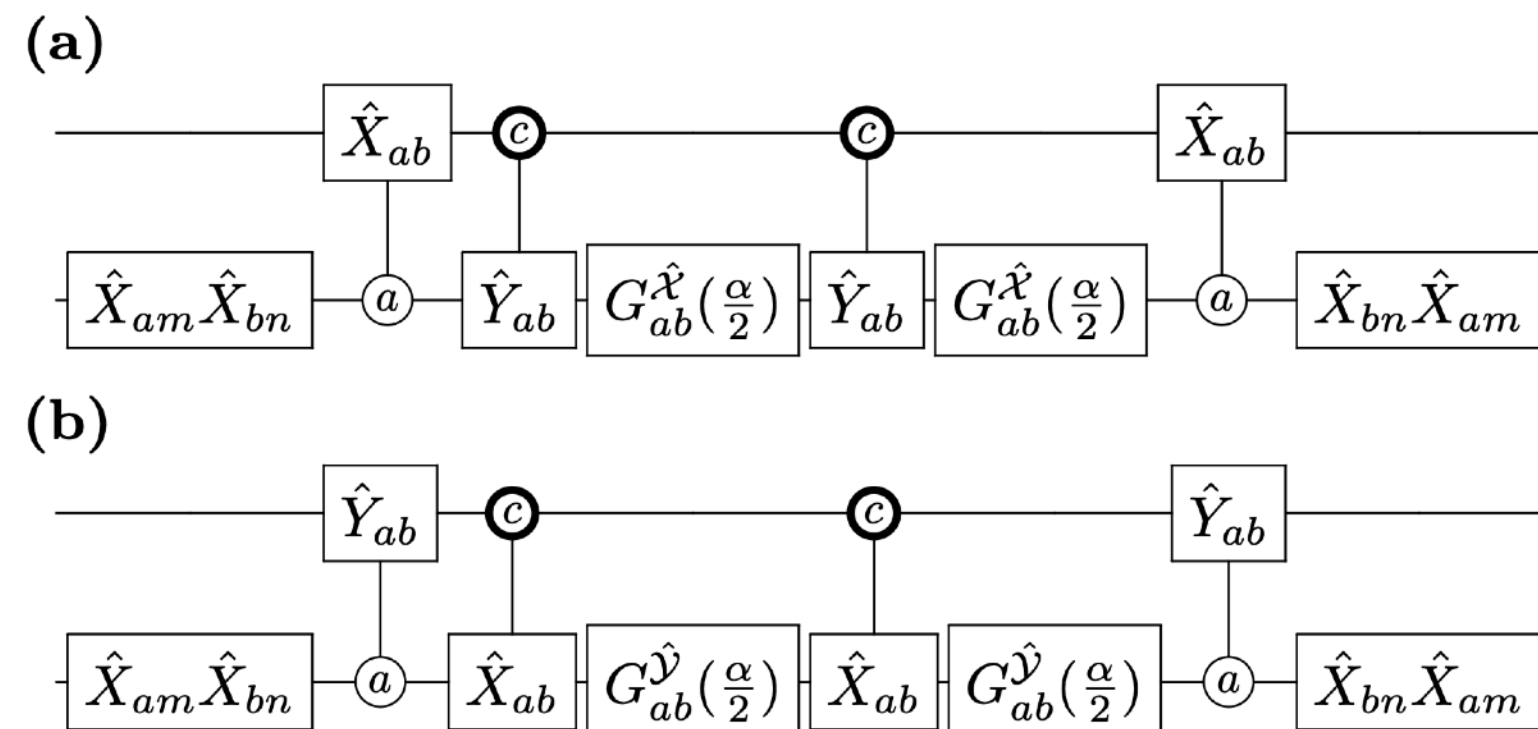


Naturally maps onto "qu5its"
[qudits with $d=5$]

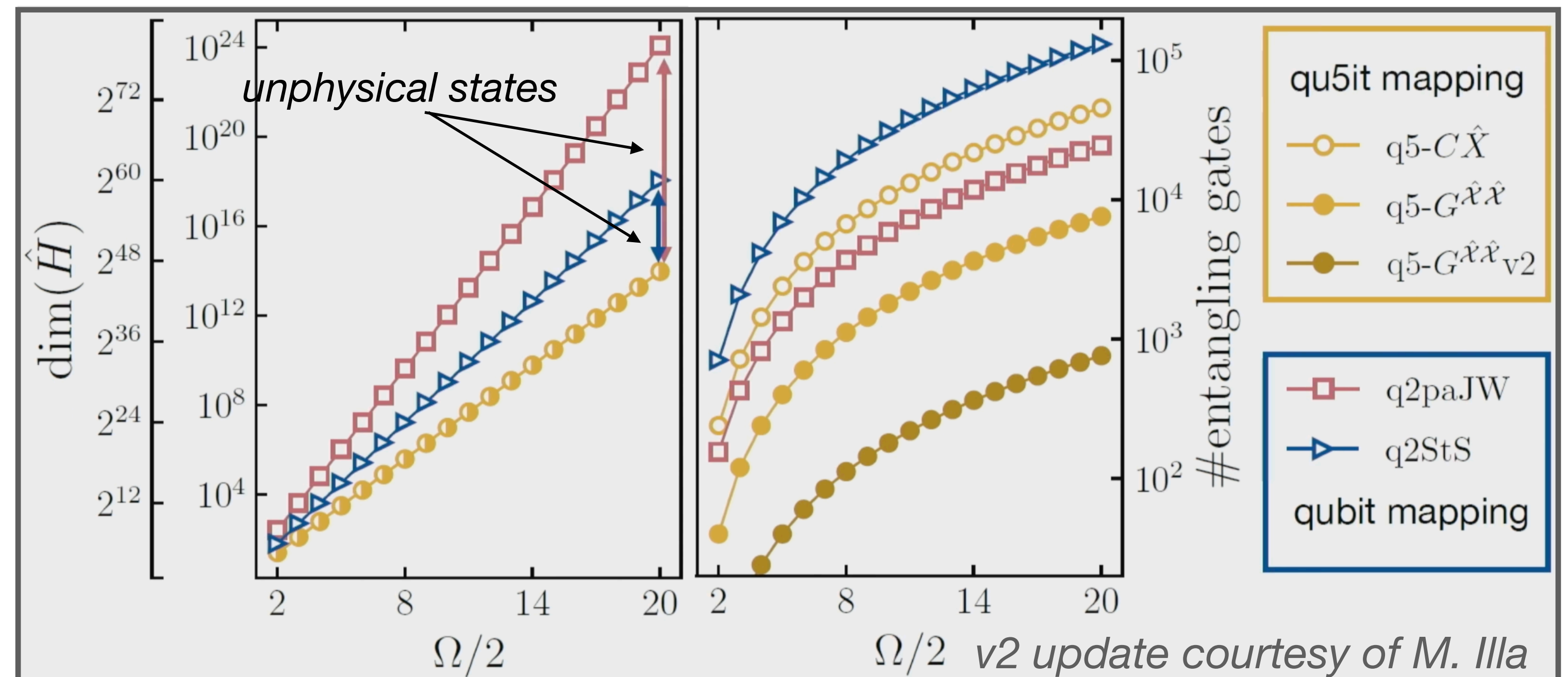


Developed a qudit-system
simulator for unitary
dynamics using Google's *cirq*

[Illa, CR, Savage PRC 108,
064306 \(2023\)](#)



Errors & Resource Requirements in Simulating Real-Time Dynamics:



See also *1+1D Lattice QCD with qu8its*, *PRD* 110, 014507 (2024)

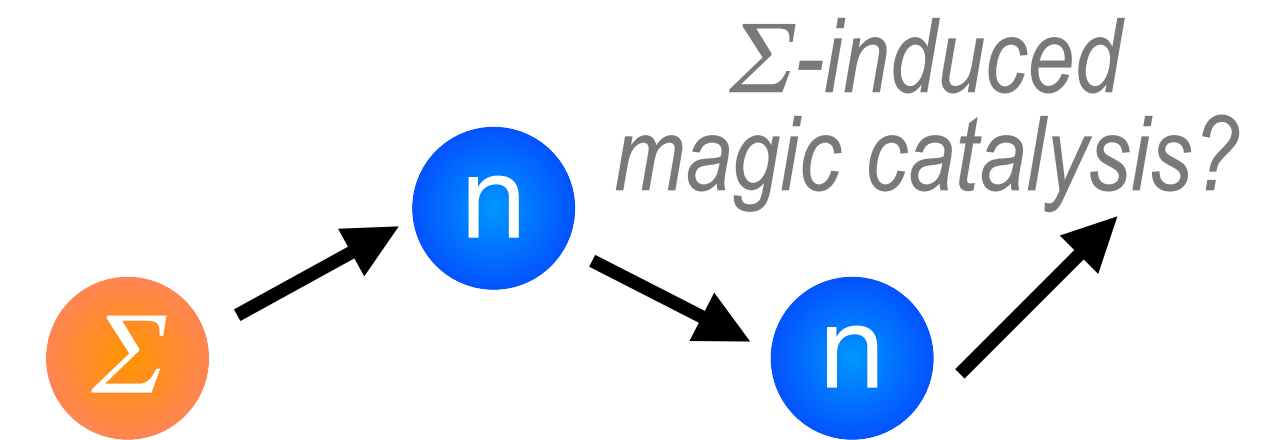
*D. Agassi, *Nucl. Phys. A* 116, 49 (1968)

Conclusions

★ Concepts of QIS bring new insights into many-body phenomena and underlying forces

→ *Better understand the underlying mechanisms governing the emergence of collective behaviours (superfluidity ongoing)*

See also NN & YN scattering CR & Savage arXiv:2405.10268,
3-flavour neutrino dynamics Chernyshev, CR, Savage PRR 7, 023228 (2025)

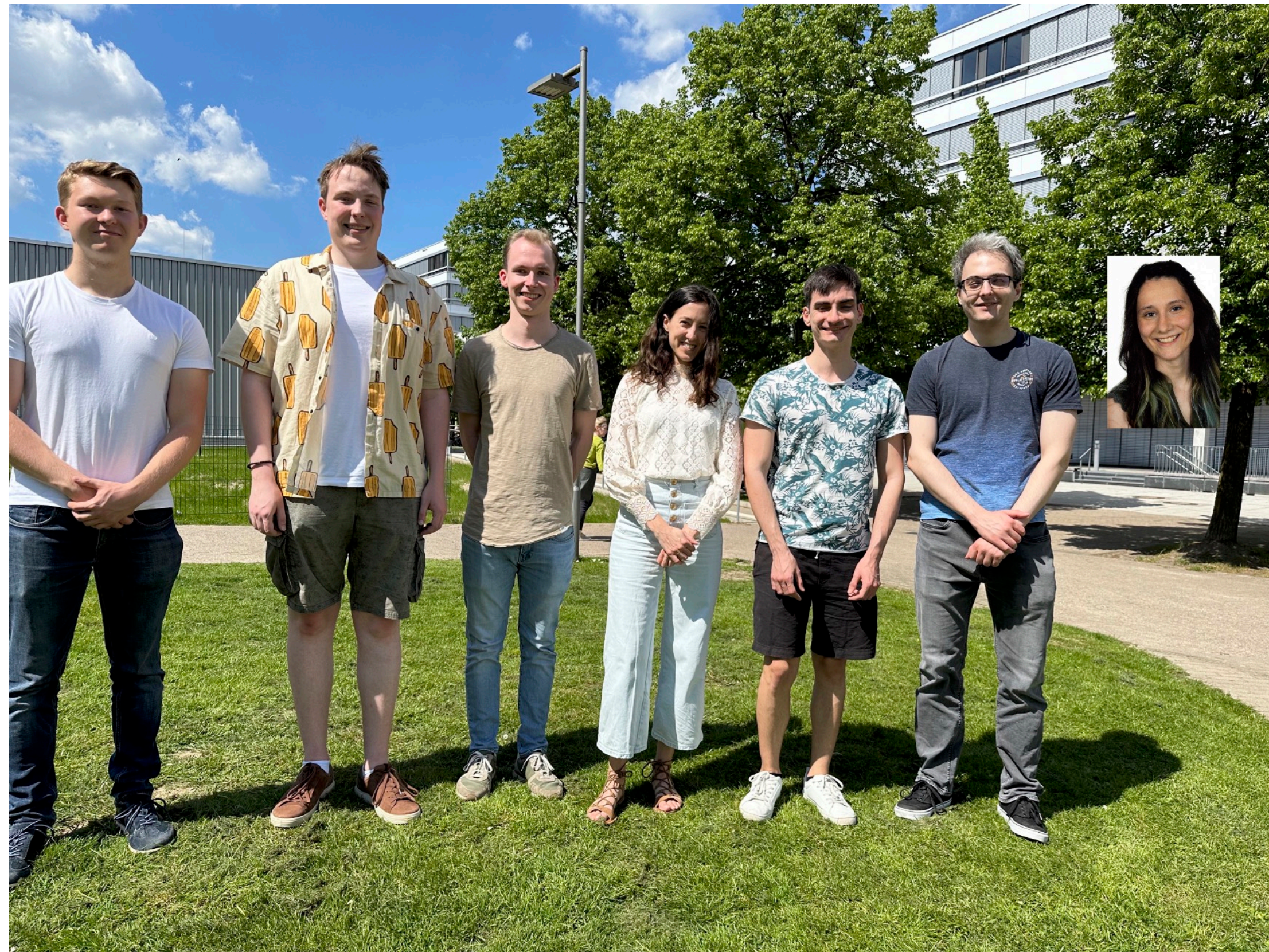


★ Entanglement, Magic and Symmetries are key ingredients for designing resource efficient quantum computations of many-body structure and real-time dynamics

→ *Combine these aspects to develop efficient algorithms for present (NISQ) and future fault-tolerant quantum computers (magic states are often the most resource intensive in FTQC)*

THANKS TO COLLABORATORS!

Uni Bielefeld group



From left to right:
Erik Müller, Florian Brökemeier, Momme Hengstenberg, CR,
Federico Rocco, James Keeble, Elisabeth Hahm

IQuS @ UW Seattle



Martin Savage



Marc Illa
(Now at PNNL)

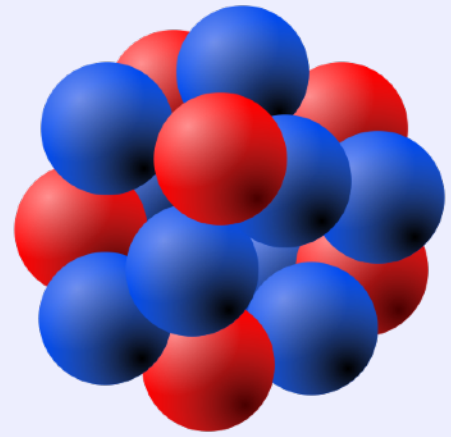


Ivan Chernyshev
(Now at LANL)

THANK You!

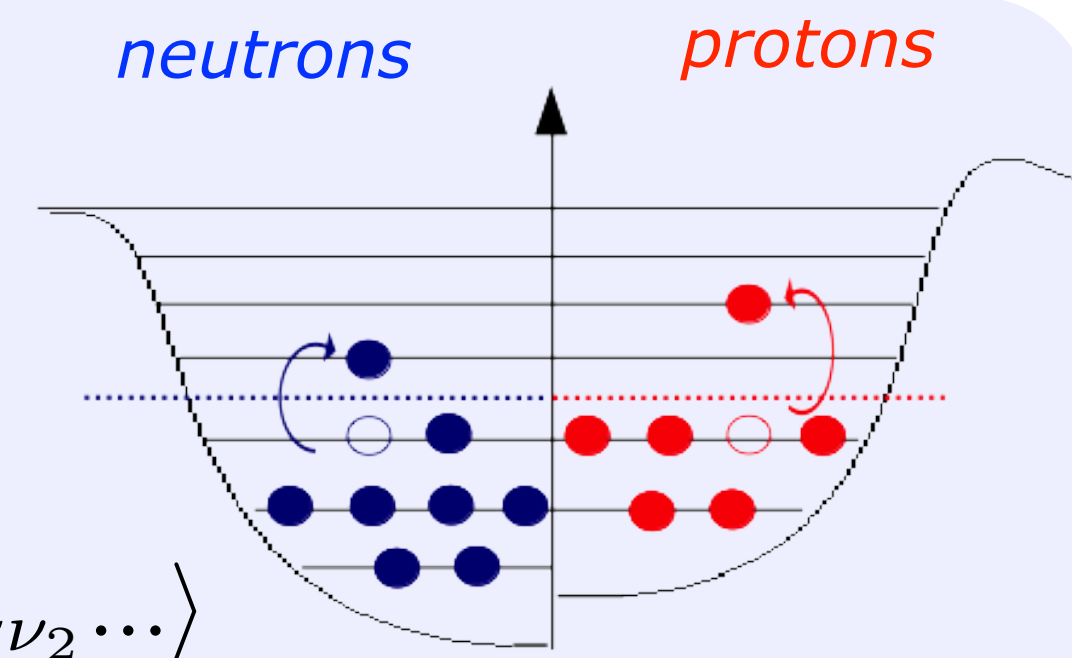
BACKUP

Entanglement in Nuclei



$$|\Psi\rangle = \sum_{\pi\nu} C_{\pi\nu} |\phi_\pi\rangle \otimes |\phi_\nu\rangle$$

$$= \sum_{n_{\pi_1} n_{\pi_2} \dots n_{\nu_1} n_{\nu_2} \dots} C_{n_{\pi_1} n_{\pi_2} \dots n_{\nu_1} n_{\nu_2} \dots} |n_{\pi_1} n_{\pi_2} \dots n_{\nu_1} n_{\nu_2} \dots\rangle$$



occupation numbers $n_i = 0$ or 1

*Entanglement between proton and neutron subsystems

See e.g. Papenbrock & Dean (2003), Gorton & Johnson (2023)

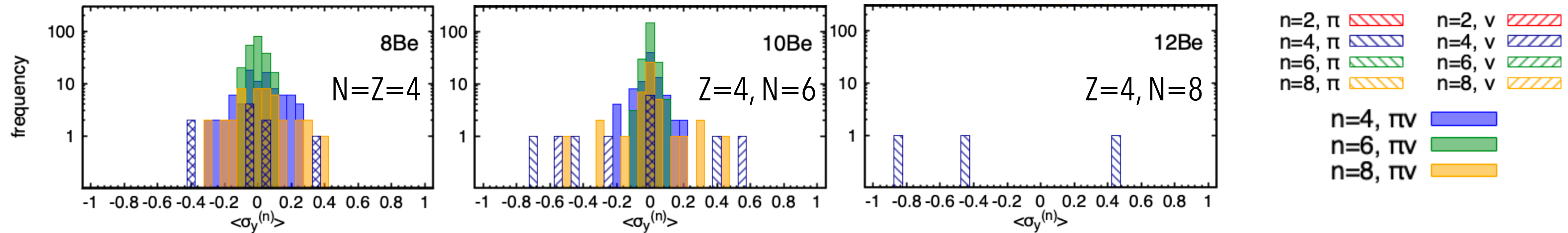
*Entanglement of modes (single-particle orbitals)

See e.g. Legeza+ (2015), CR+ (2020), Tichai+ (2022), Pérez-Obiol+ (2023) ...

→ One-Orbital Von Neumann Entropy; Two-Orbital Mutual Information, Negativity

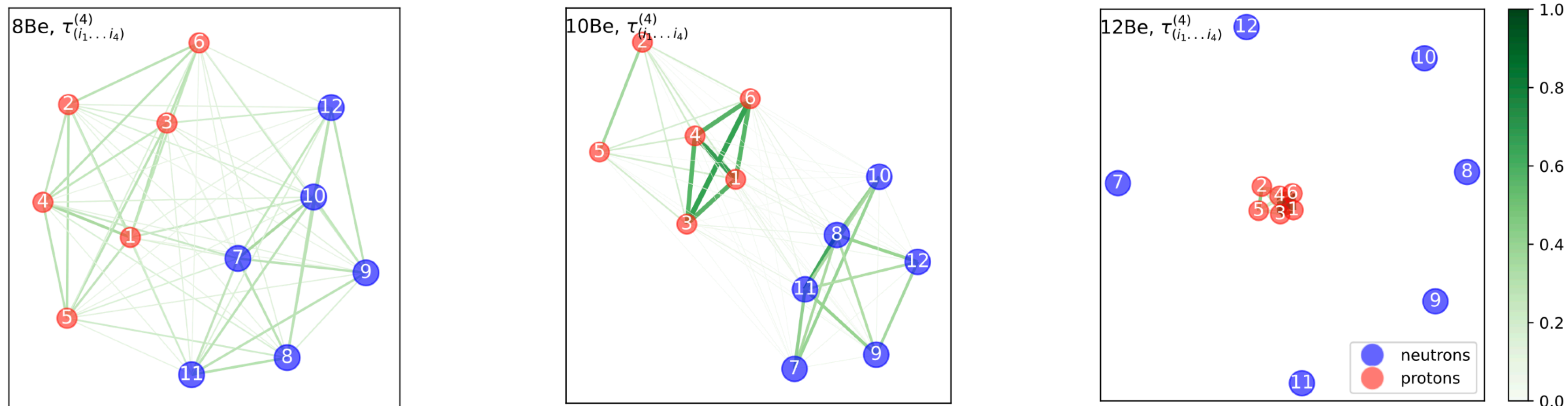
Multi-Partite Entanglement in p -shell (12-qubit) Nuclei

*Distribution of the $\sigma_y^{\otimes n}$ Pauli strings expectation values



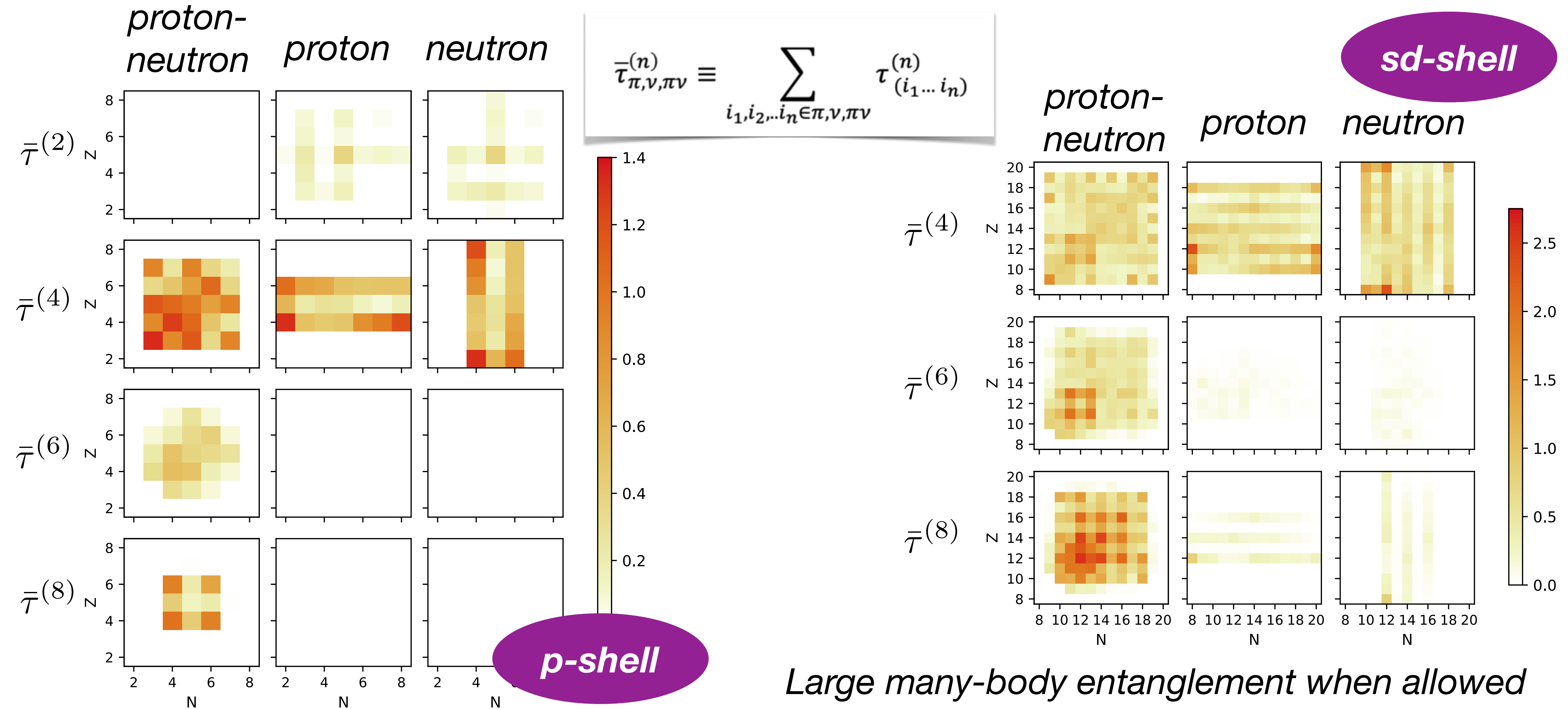
- proton-neutron entanglement is more collective than pure proton or neutron entanglement
- large proton-neutron 8-tangles $\rightarrow$ hint of alpha correlations?

*Network plots

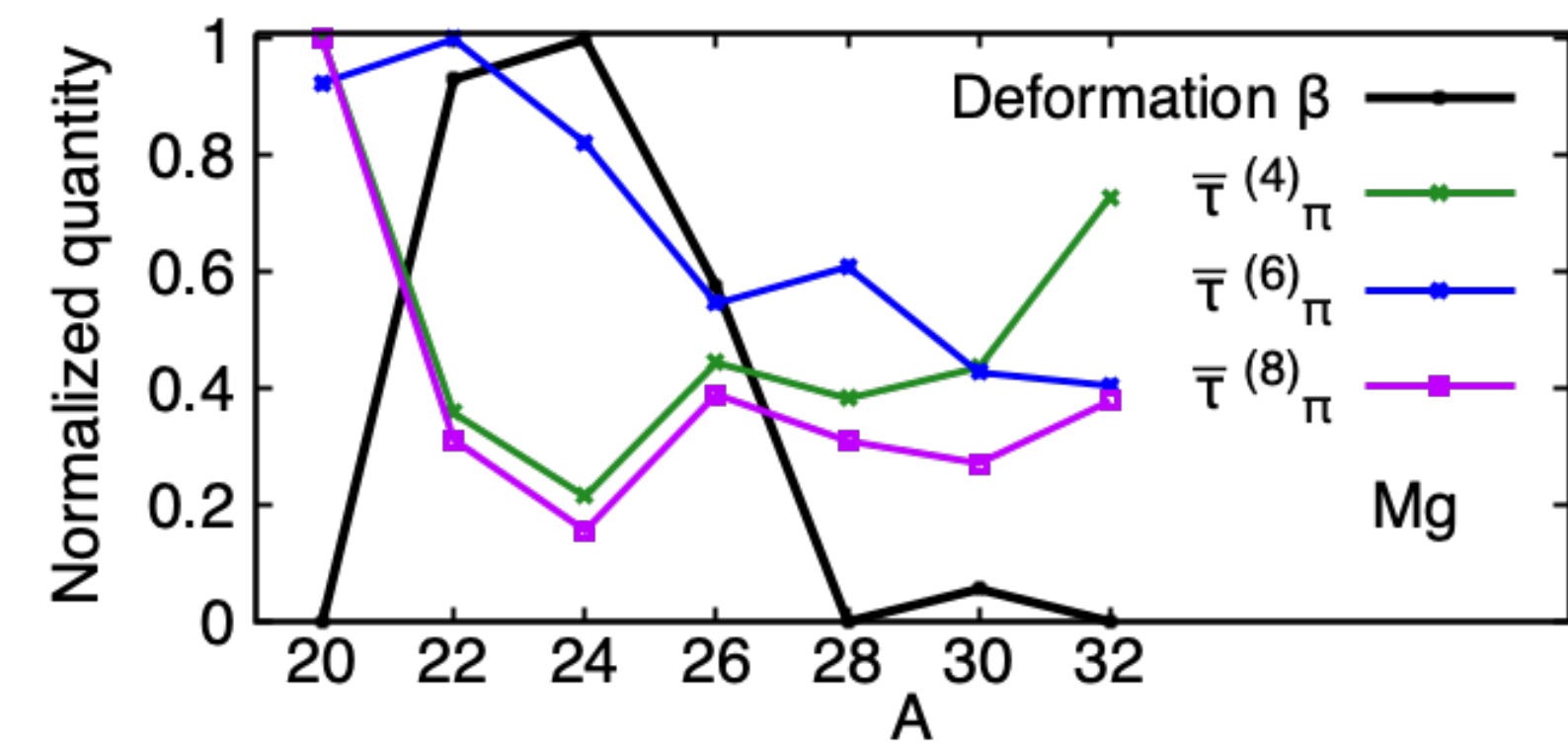
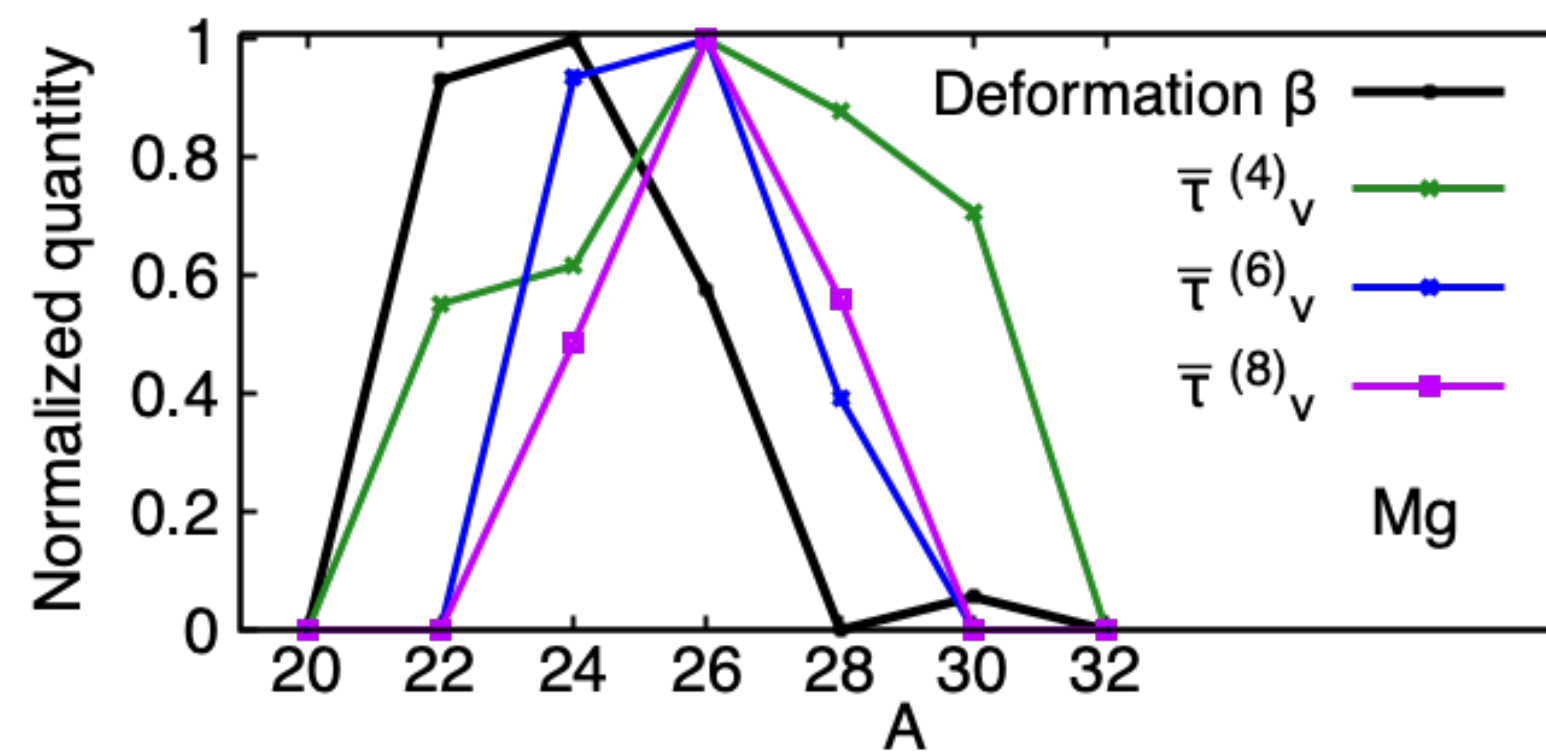
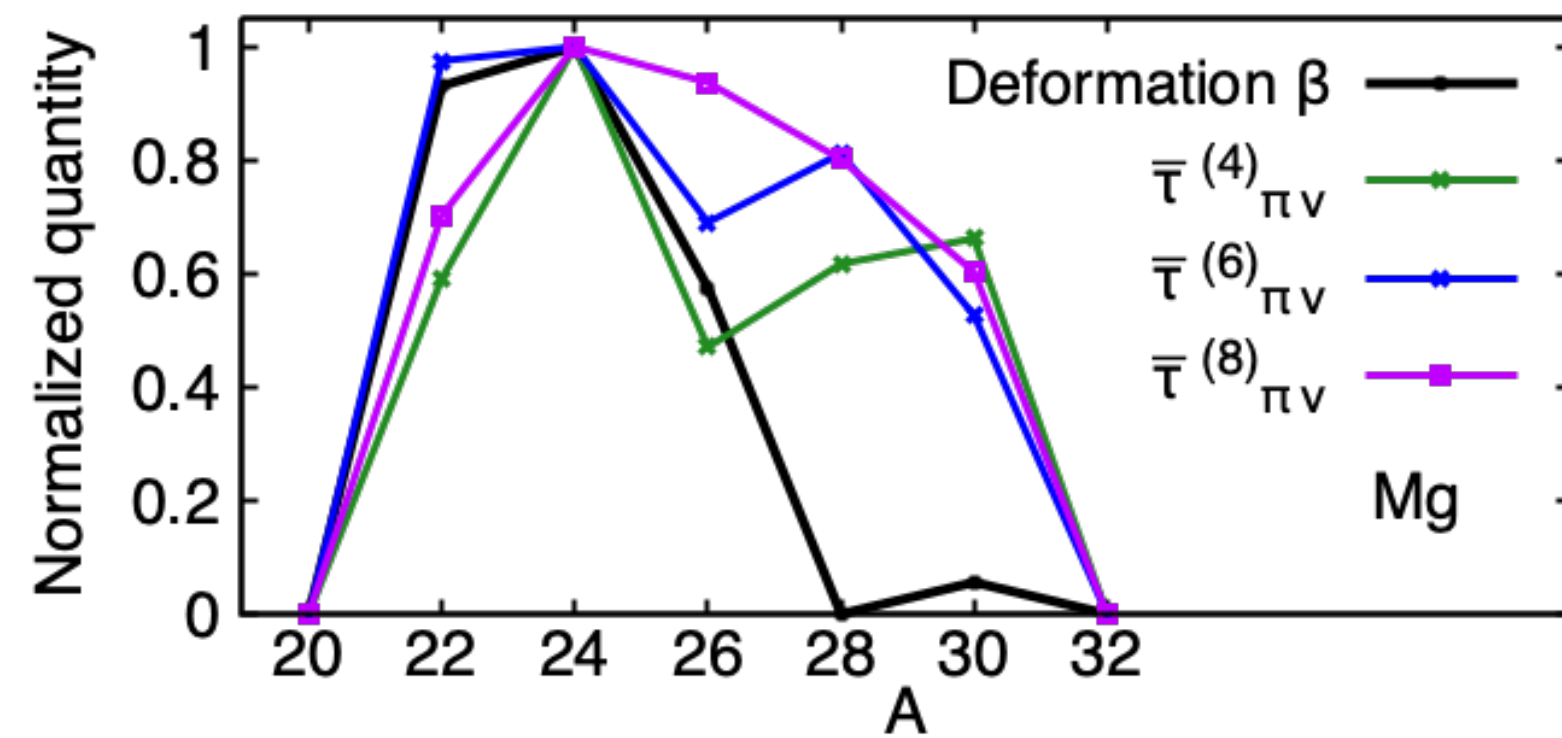
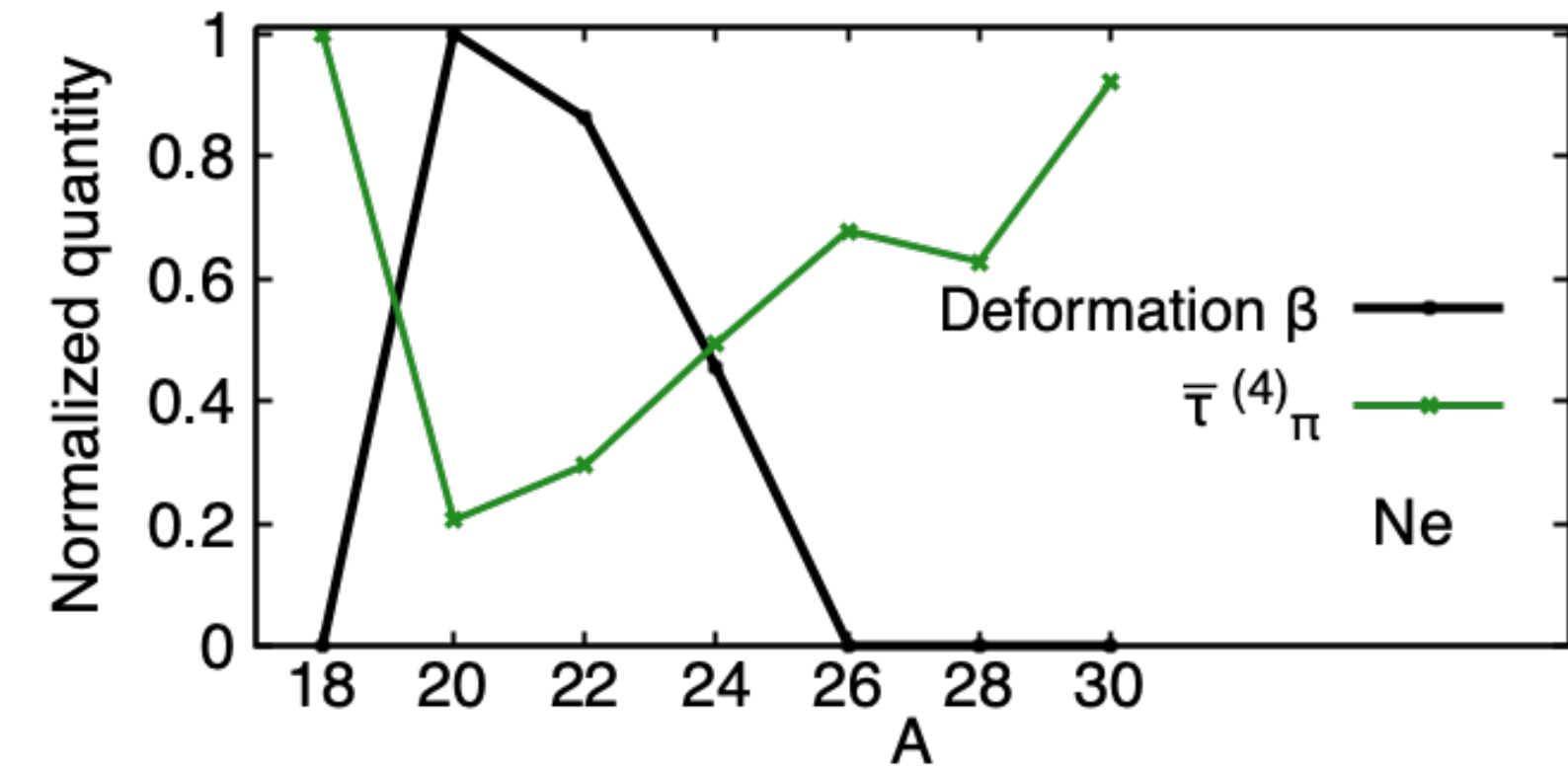
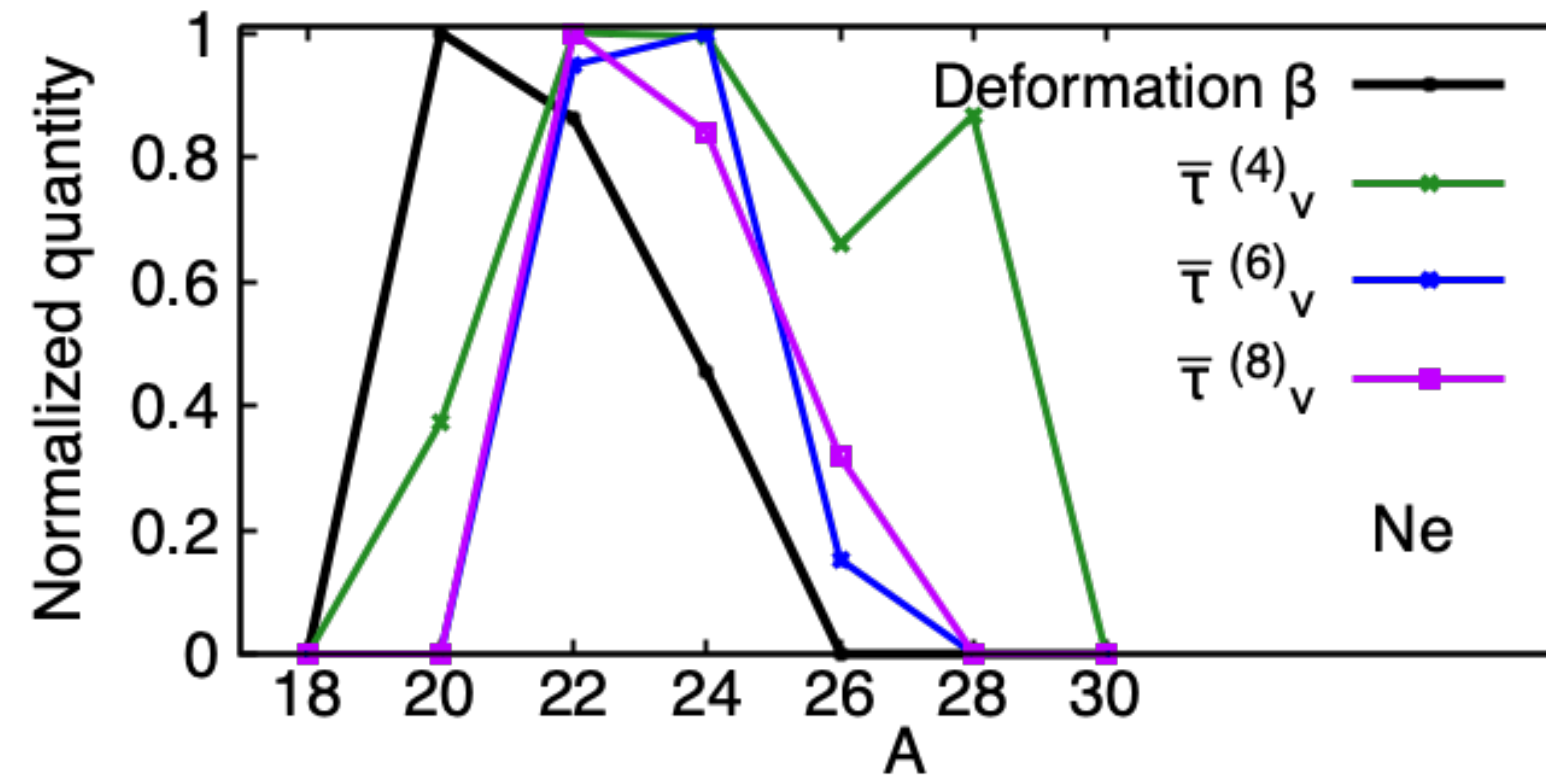
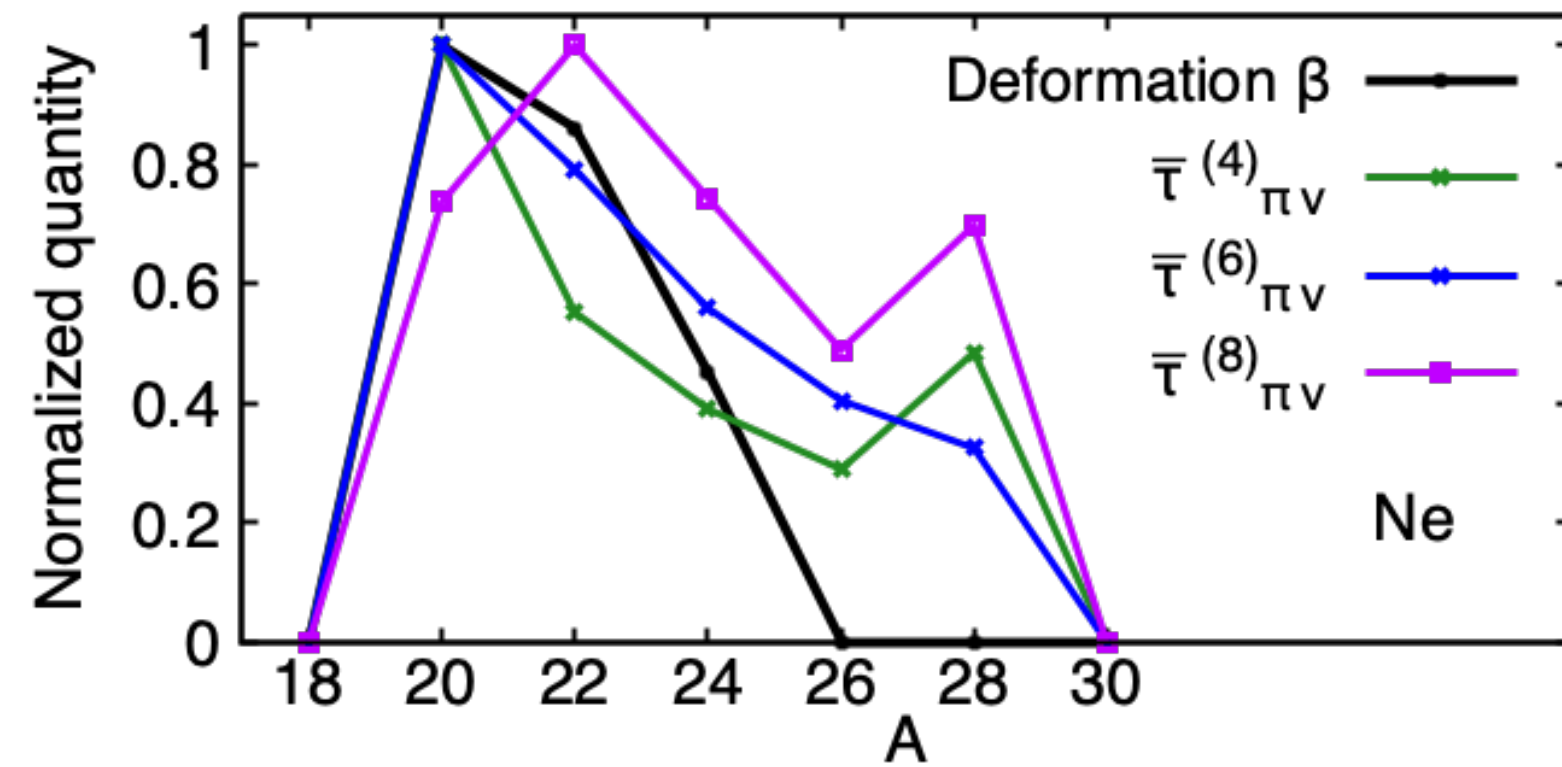


$$e_{i_1 i_2}^{(4)} = \sum_{i_3 < i_4} \tau_{(i_1, i_2, i_3, i_4)}^{(4)}$$

Multi-Partite Entanglement in Shell-Model Nuclei



Multi-Partite Entanglement in Shell-Model Nuclei

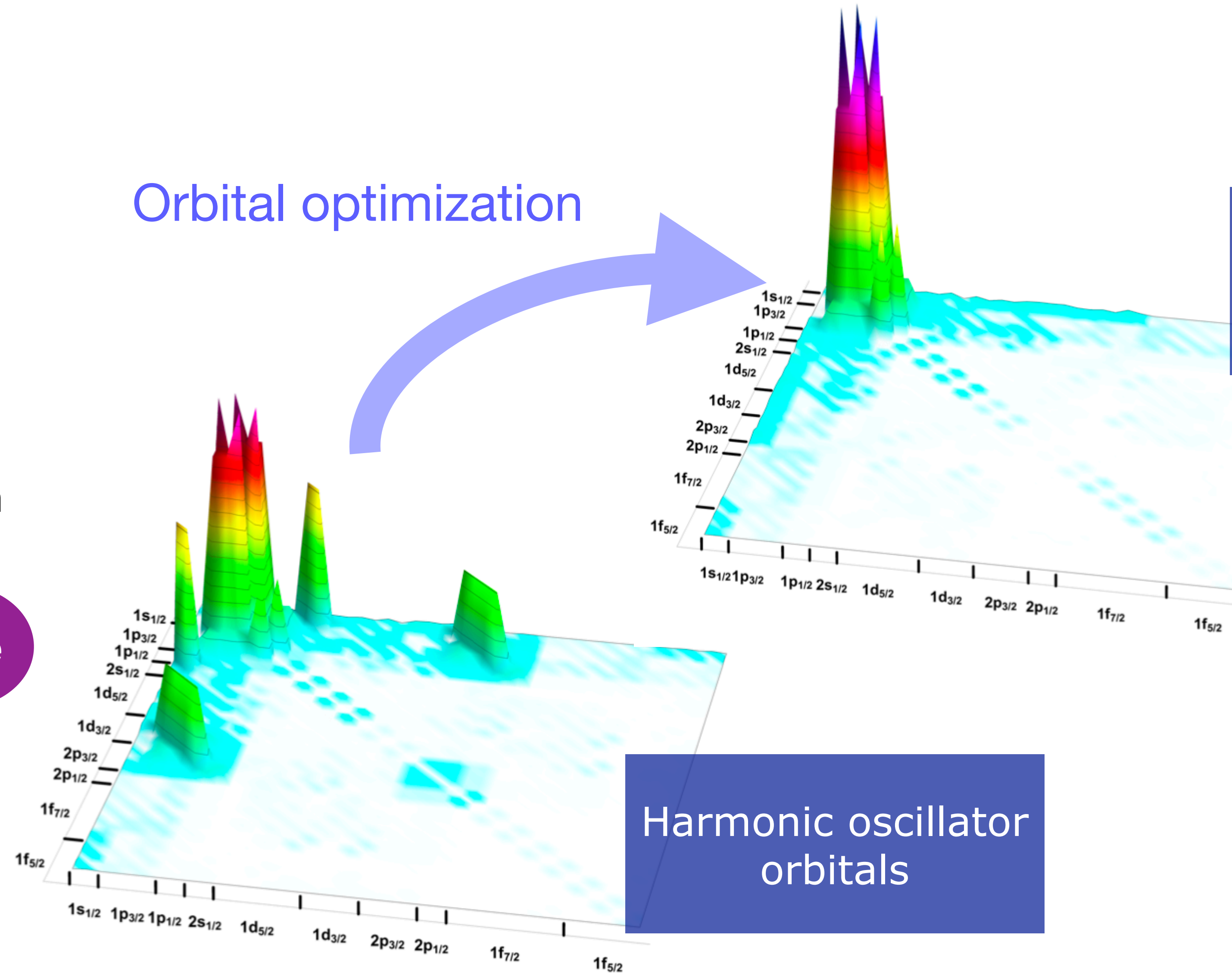


Entanglement Rearrangement In Nuclei

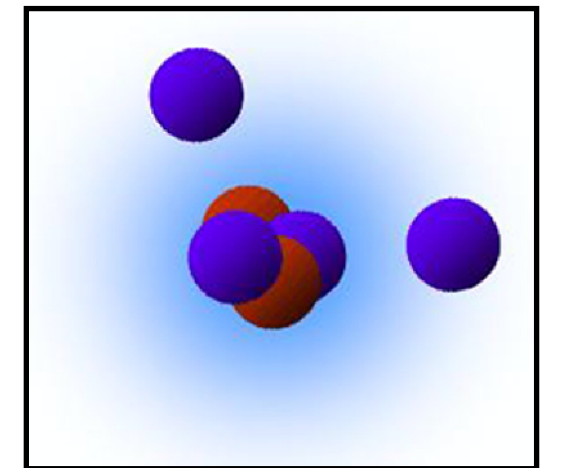
Neutron-neutron
mutual information

${}^6\text{He}$

Orbital optimization



Effective
(variational natural)
orbitals



⇒ *emergence of ${}^4\text{He}$ -core
+ nn-valence structure (?)*

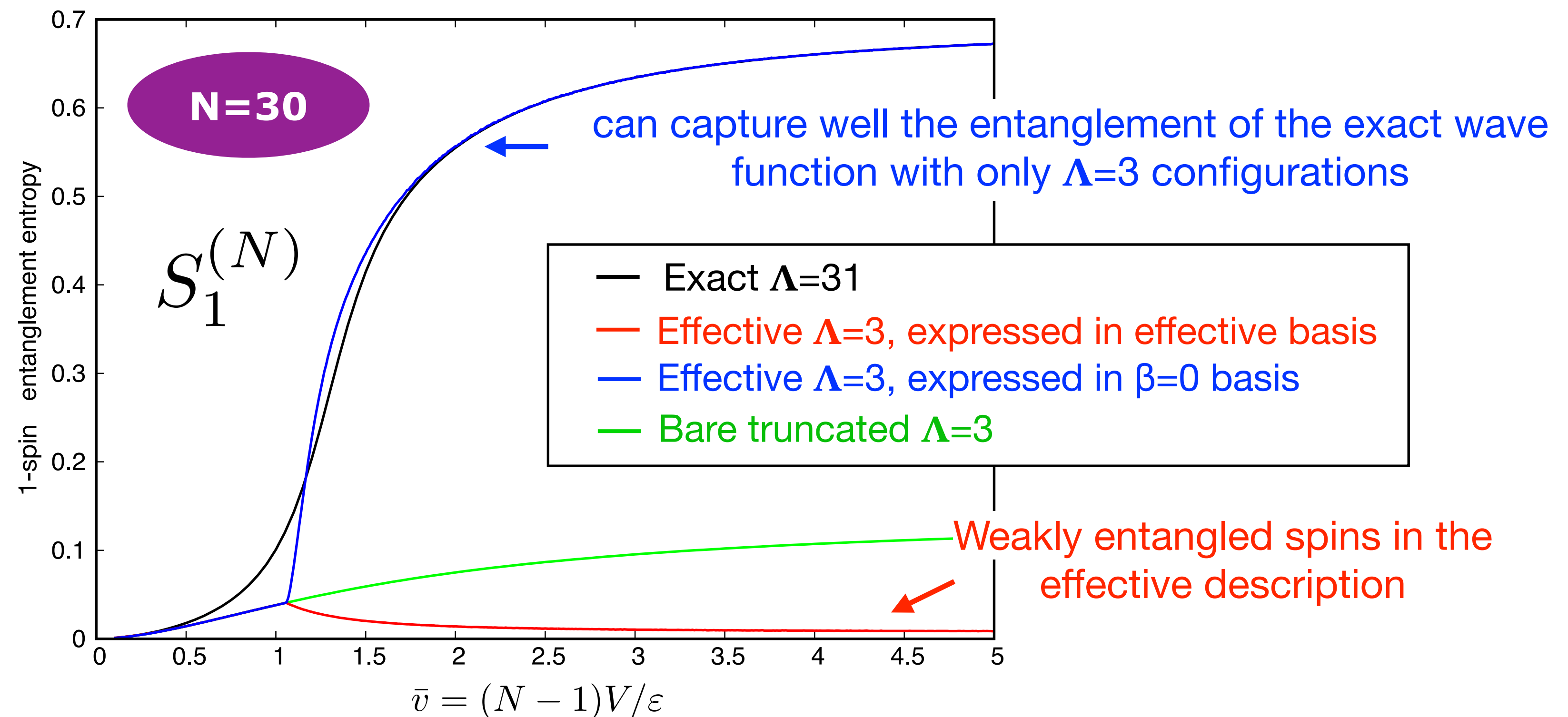
Entanglement Rearrangement in the LMG Model

Rotation of the spins
as “disentangler”

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} = \cos(\beta/2) \begin{array}{c} \text{---} \\ \text{---} \end{array} + \sin(\beta/2) \begin{array}{c} \bullet \\ \text{---} \end{array}$$

➡ Effective Hamiltonian: $H(\beta) = U(\beta)^\dagger H U(\beta)$ with $U(\beta) = e^{-iJ_y\beta}$
determined variationally

$$|\Psi\rangle^\Lambda = \sum_{n=0}^{\Lambda-1} A_n |n\rangle$$



Beyond stabilizer Ground State with Magic Injection

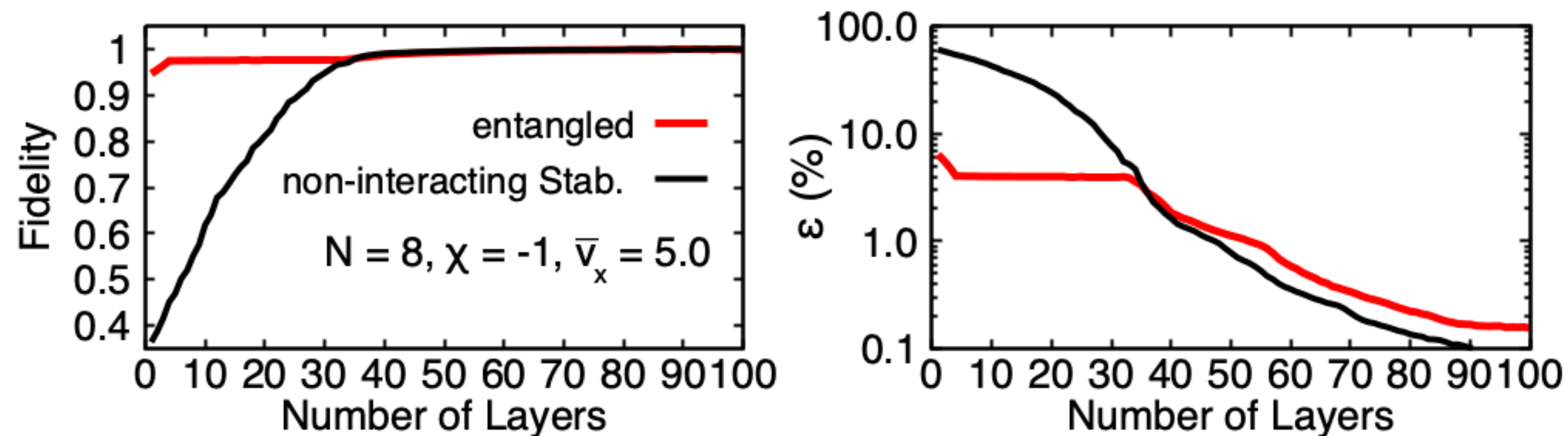
* Grimsley+ Nature Comm. 10, 3007 (2019)

★ ADAPT-VQE *

$$|\Phi(\theta_1, \dots, \theta_L)\rangle = \prod_{l=1}^L e^{i\theta_l \hat{T}_l} |\phi\rangle \quad \frac{\partial E}{\partial \theta_l} \Big|_{\theta_l=0} = -i \langle \Phi_{l-1} | [\hat{T}_l, \hat{H}] | \Phi_{l-1} \rangle$$

For the LMG model: $\hat{T}_{ij}^{\pm} = X_i Y_j \pm Y_i X_j$

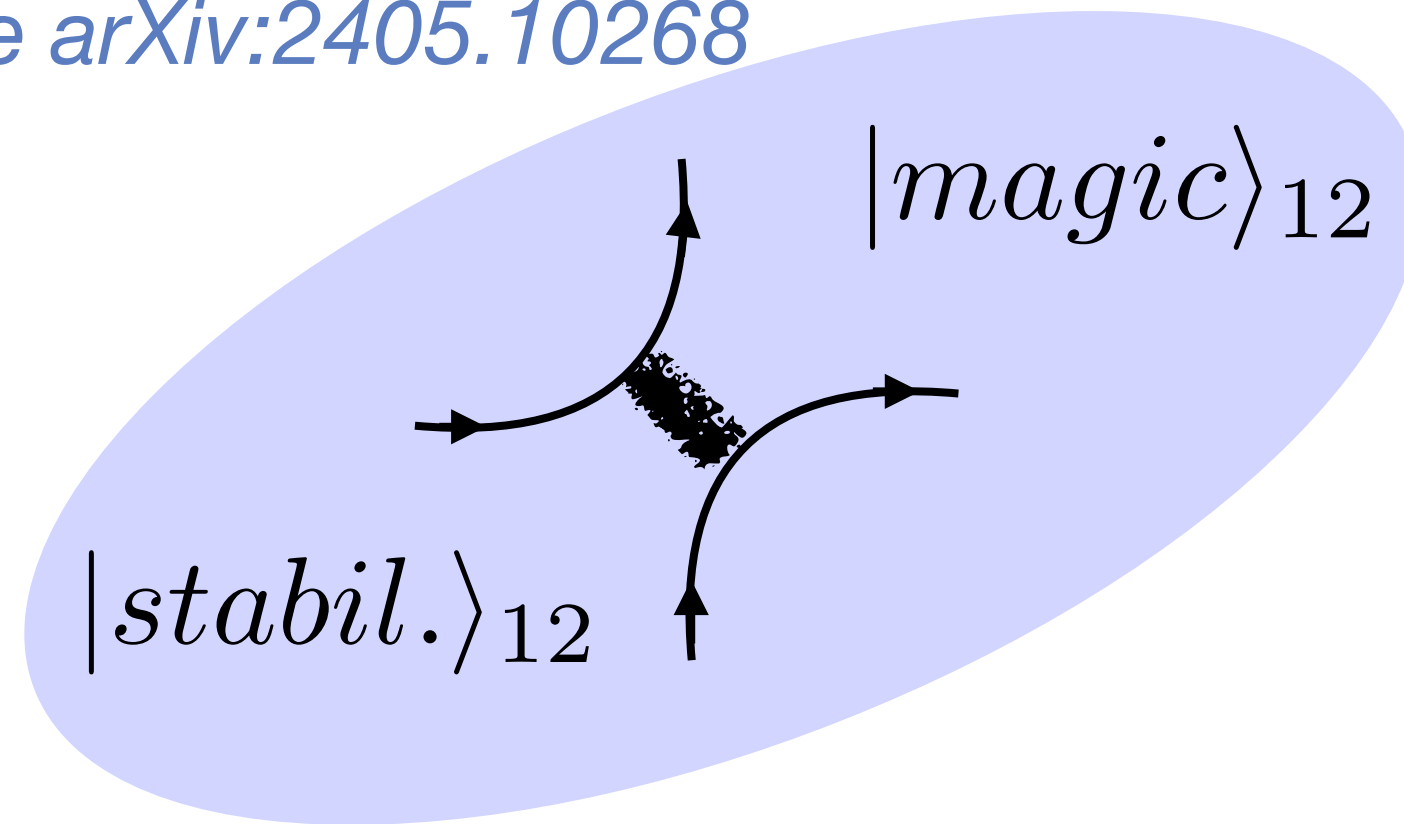
CR arXiv:2505.02923



- Long plateaux due to very small energy gradients for many iterations
- Destroys the coherence, and rebuilds it

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268



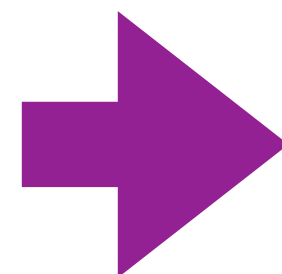
Magic power of the S-matrix:

$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}} \sum_{i=1}^{\mathcal{N}_{ss}} \mathcal{M}(\hat{\mathbf{S}} |\Psi_i\rangle)$$

Average fluctuations in magic induced by the S-matrix

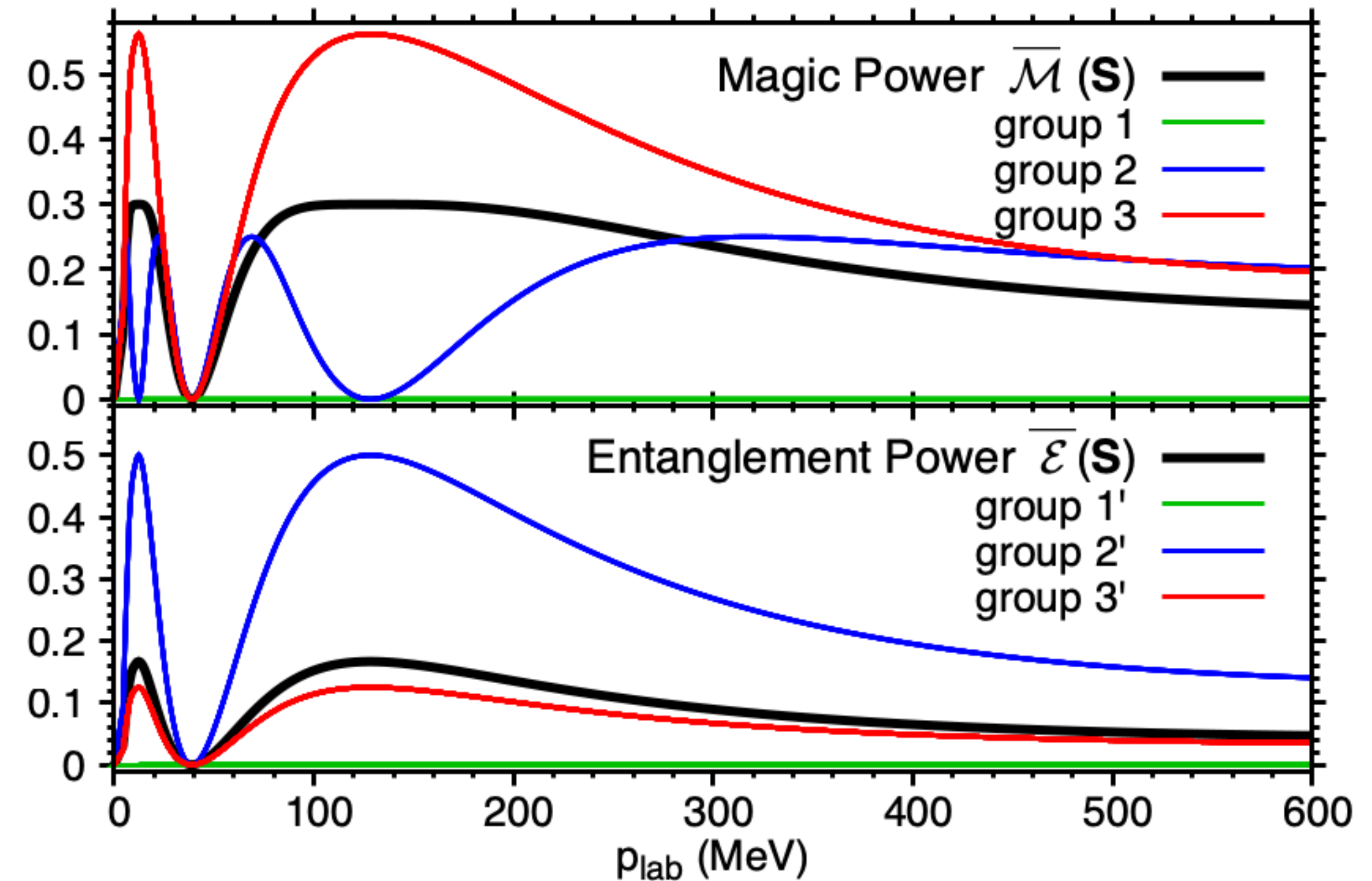
Entanglement power of the S-matrix

$$\overline{\mathcal{E}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}^{TP}} \sum_{i=1}^{\mathcal{N}_{ss}^{TP}} \mathcal{E}(\rho_i^{(1)}(\hat{\mathbf{S}}))$$



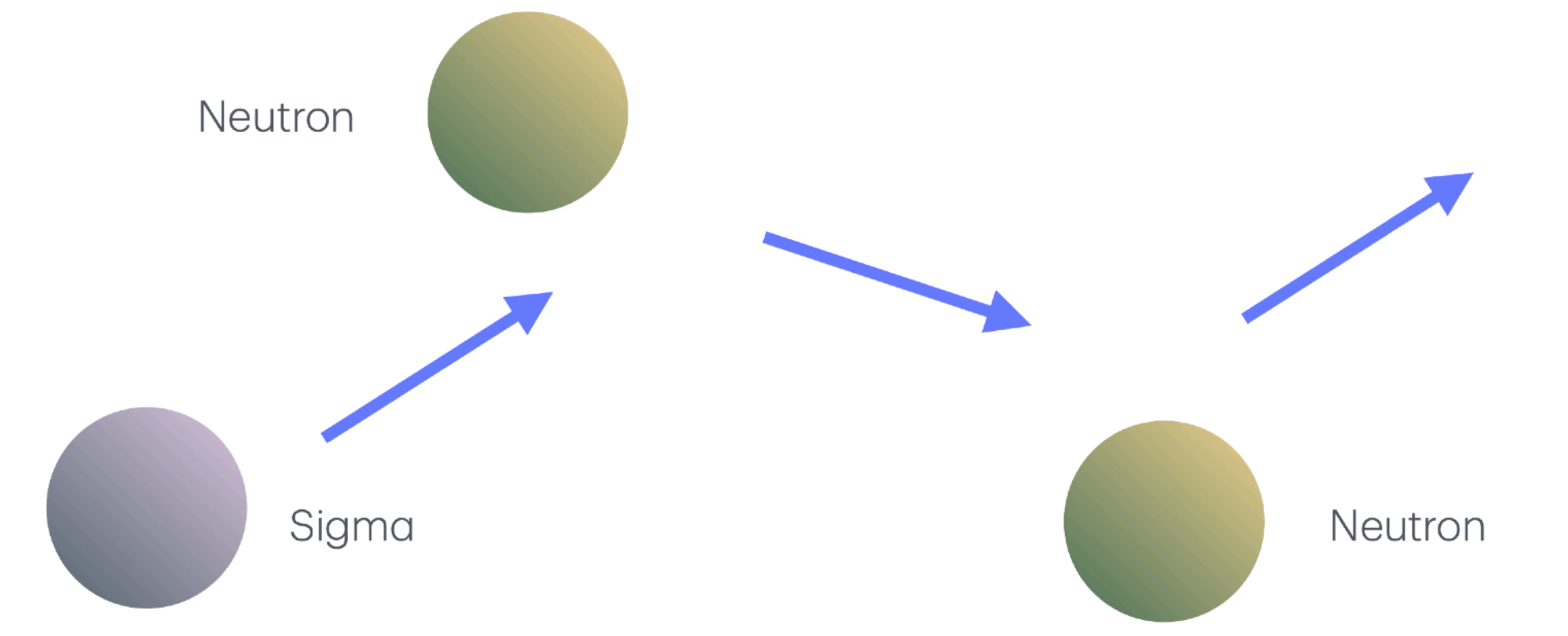
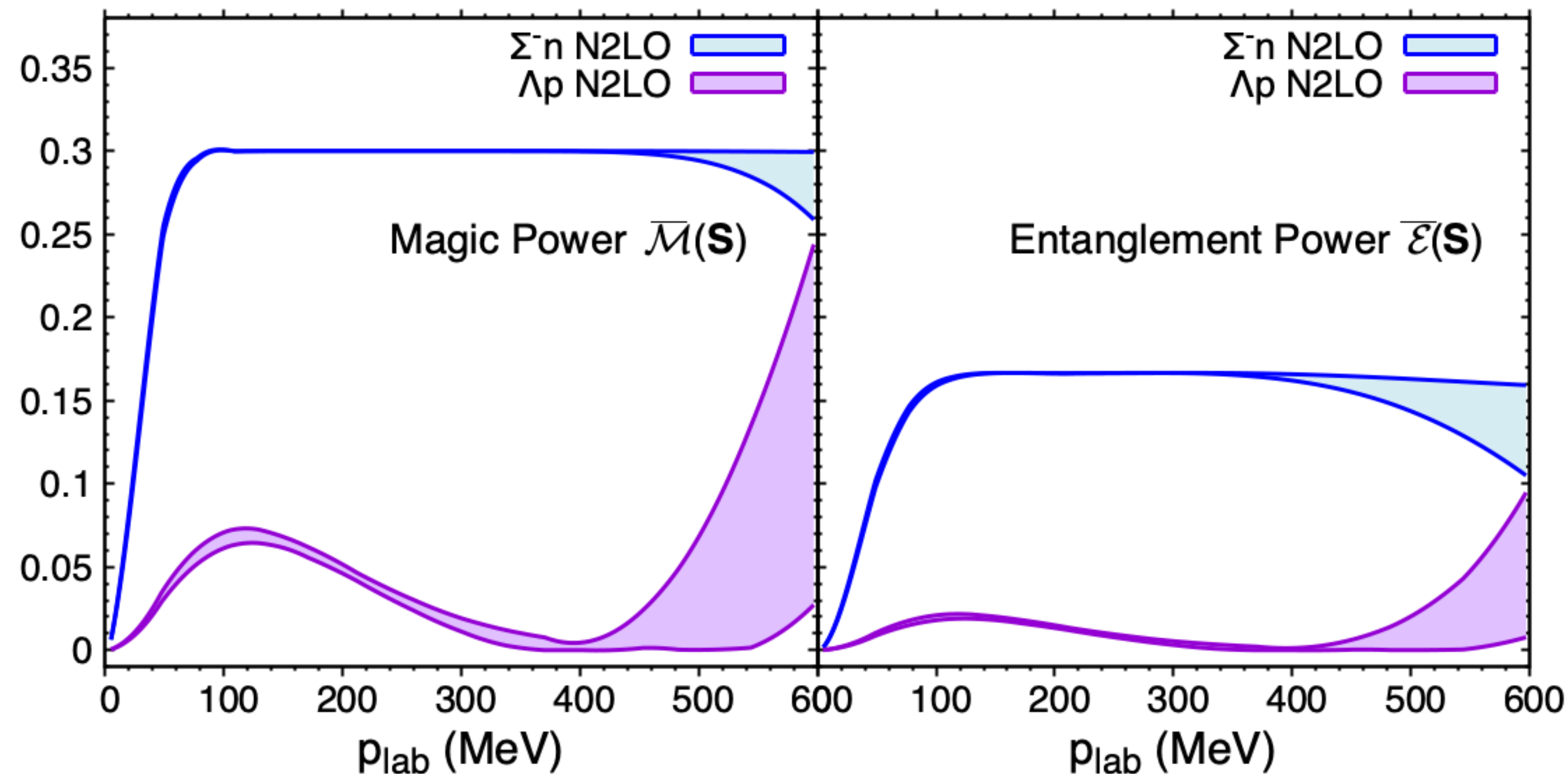
Same results as in Beane+ PRL 122, 102001 (2019) with continuous integration over spin orientations of initial tensor-product states

$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) = \frac{3}{20} \left(3 + \cos(4 \Delta\delta) \right) \sin^2(2 \Delta\delta) \quad \overline{\mathcal{E}}(\hat{\mathbf{S}}) = \frac{1}{6} \sin^2(2 \Delta\delta)$$

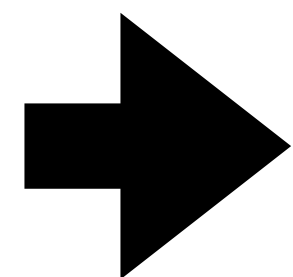


The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268



Magic between neutrons is induced by successive Σ -n scatterings (assumed no decoherence)

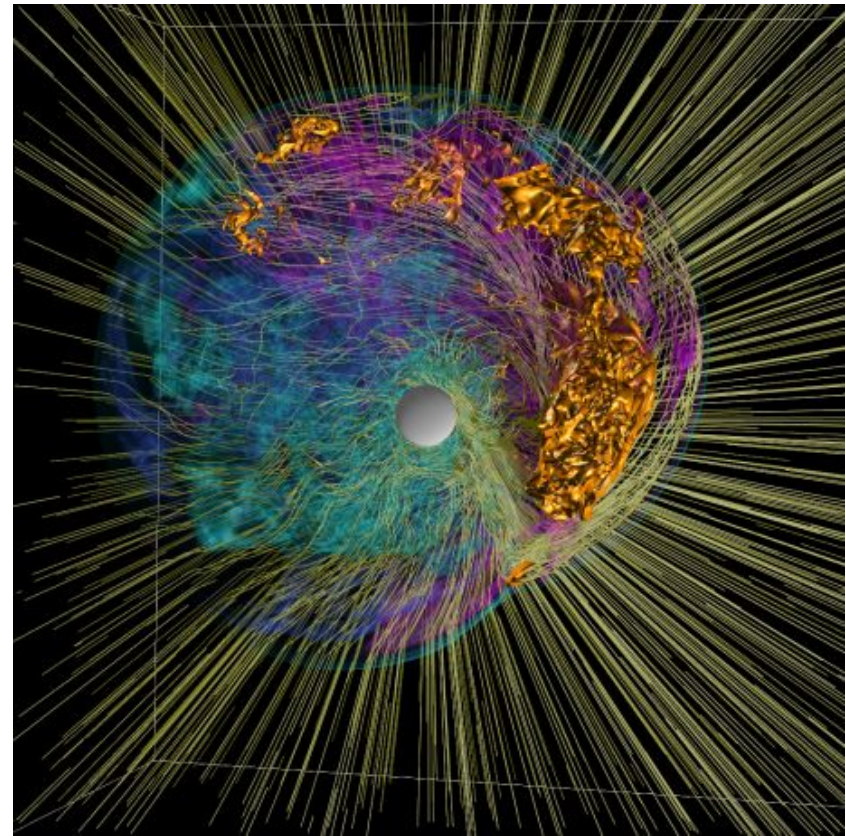


Σ -baryon is identified as a potential candidate catalyst for enhanced spreading of magic and entanglement in dense matter

Entanglement power also in Beane+ PRL 122, 102001 (2019); Liu+ PLB 856, 138899 (2024)

Magic and Entanglement in Neutrino Dynamics

Chernyshev, CR, Savage PRR 7 023228 (2025)



Neutrinos from core-collapse supernovae

Wikimedia commons

1-flavour

$$|\nu_e \nu_e \nu_e \nu_e \nu_e\rangle$$

VS

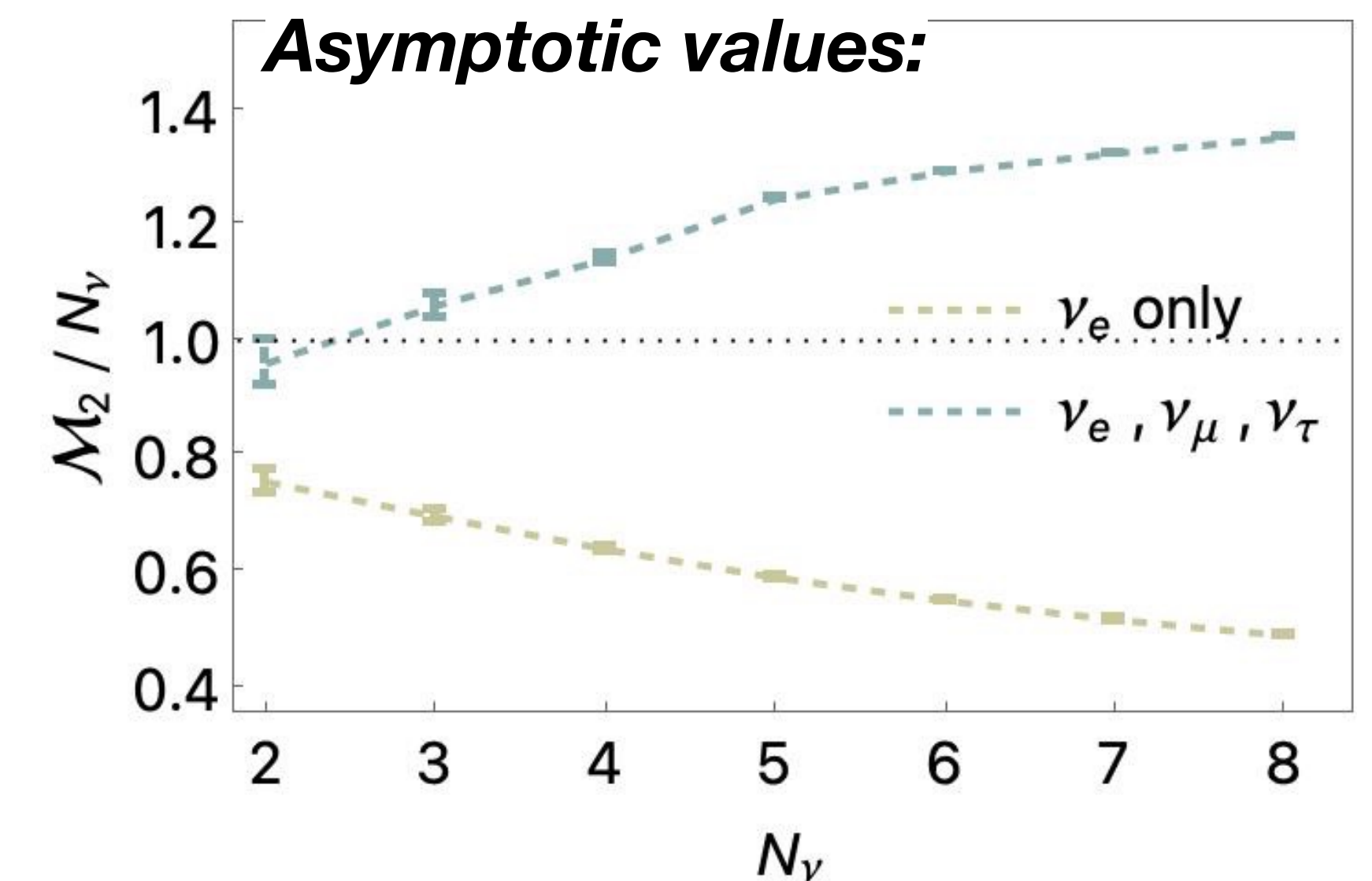
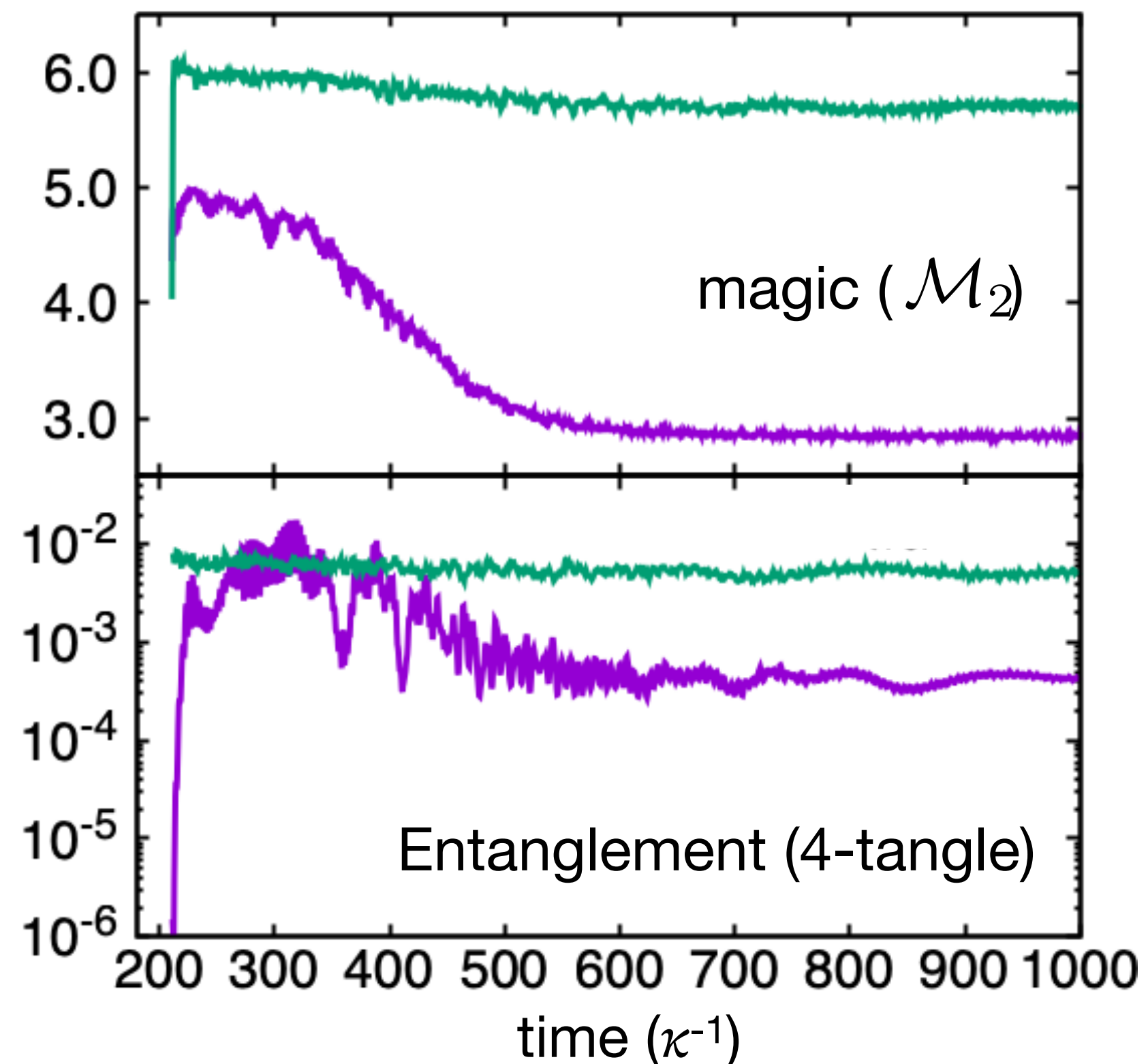
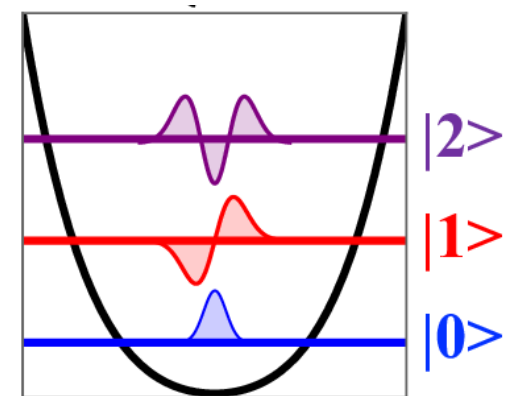
3-flavour

$$|\nu_e \nu_e \nu_\mu \nu_\mu \nu_\tau\rangle$$

initial state

Neutrinos = weakly-interacting, strongly-correlated particles

3 flavours of neutrinos $|\nu\rangle_e, |\nu\rangle_\mu, |\nu\rangle_\tau$ mapped onto qutrits:

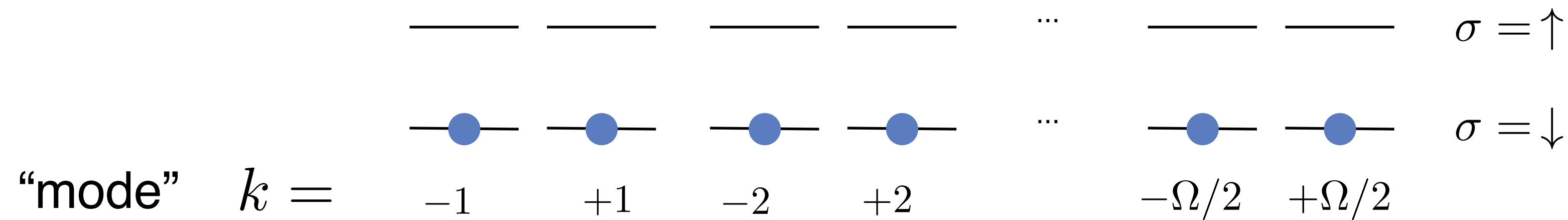


👉 Flavour mixing enhances complexity during the evolution

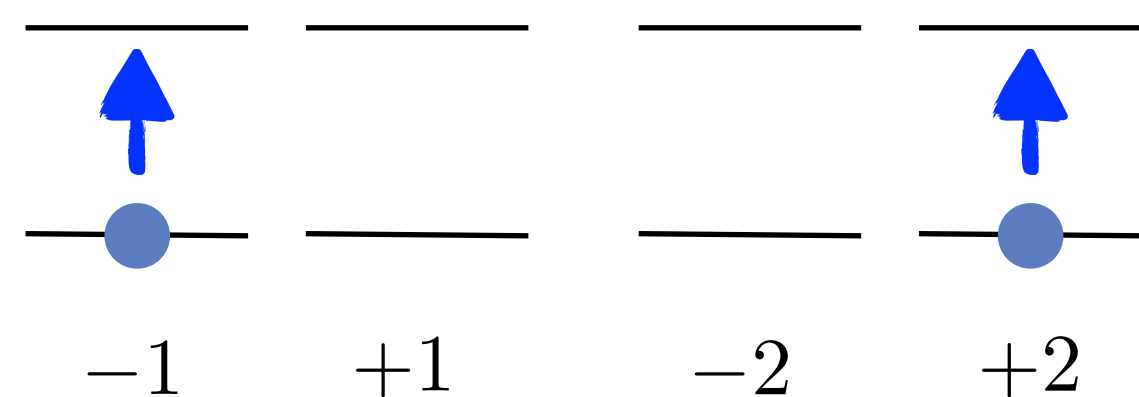
The Agassi model as demonstration of symmetry-guided mapping

★The Agassi model

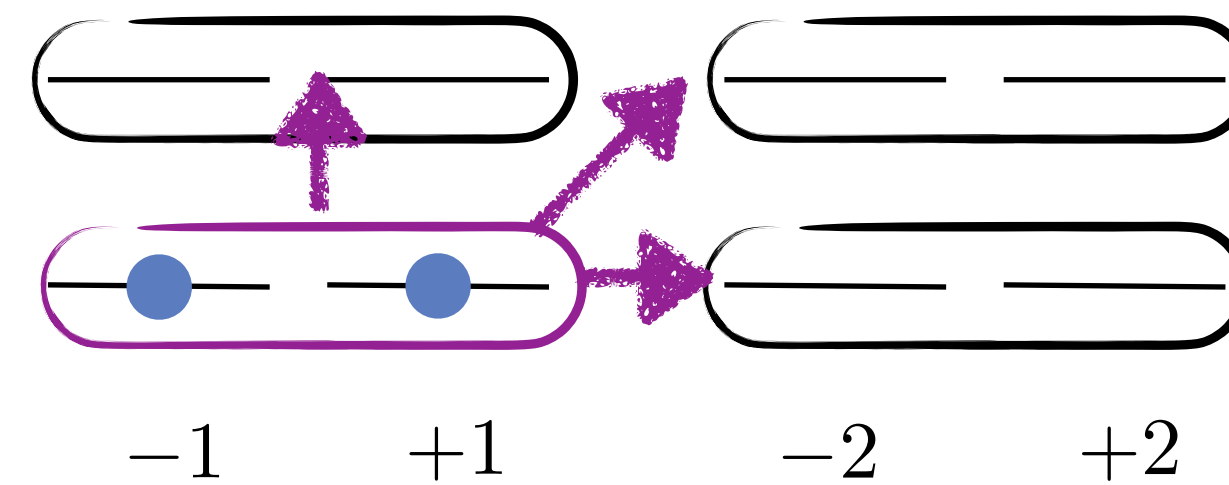
= extension of the LMG model with superfluid pairing



$$\hat{H} = \varepsilon \hat{J}_z - \frac{V}{2} \left(\hat{J}_+^2 + \hat{J}_-^2 \right) - g \sum_{\sigma\sigma'} \hat{B}_\sigma^\dagger \hat{B}_{\sigma'}$$

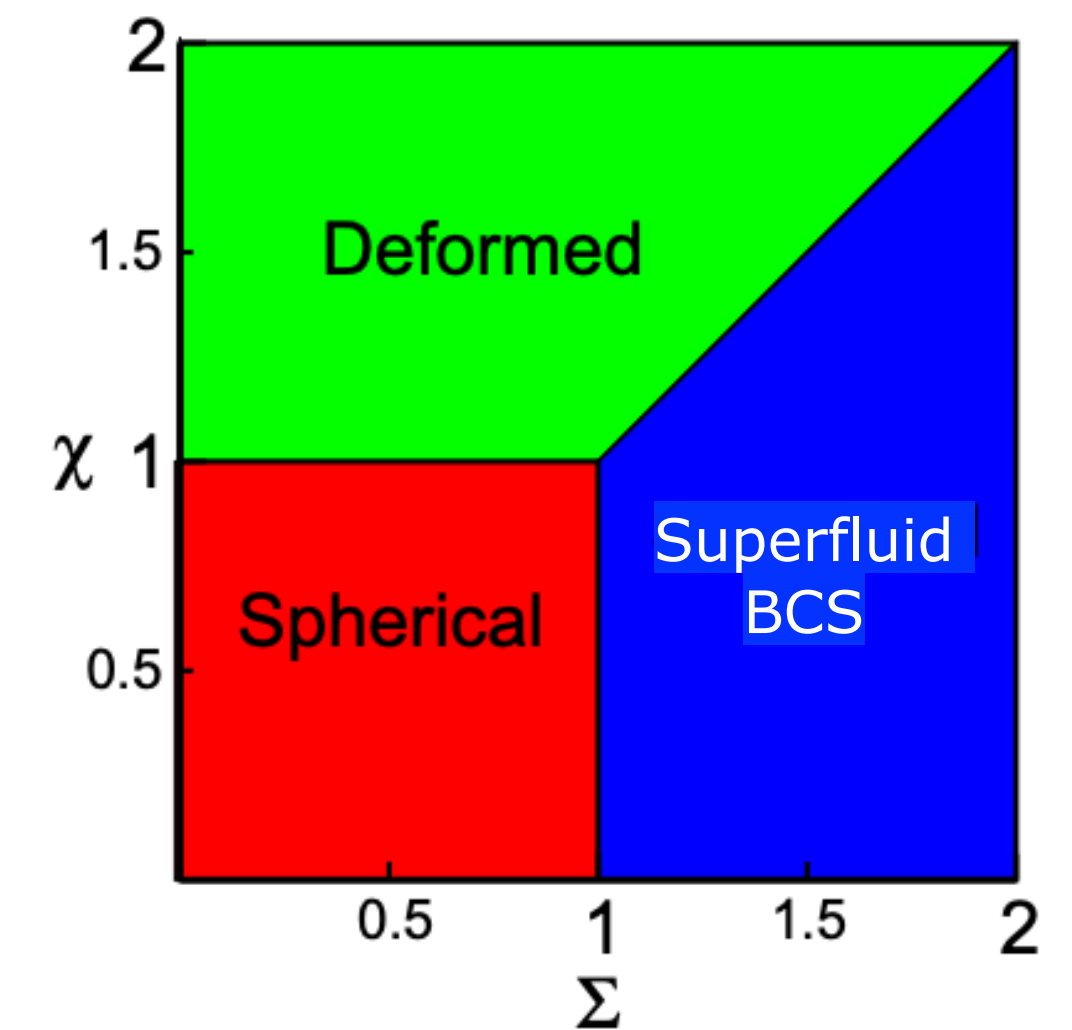


particle-hole interaction V



pairing interaction g

*D. Agassi, Nucl. Phys. A 116, 49 (1968)

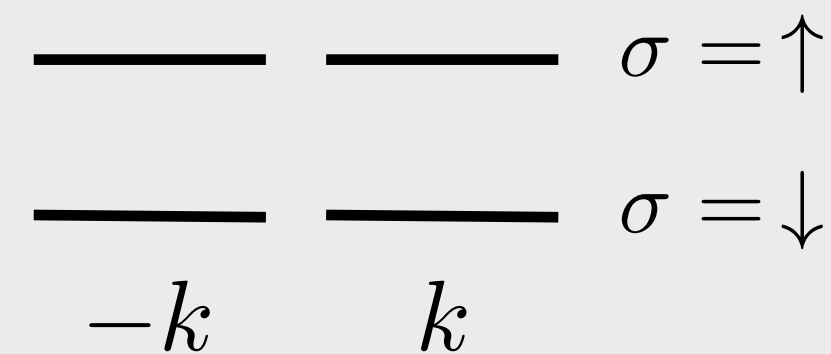


[Pérez-Fernández+ PLB 829 137133 (2022)]

Symmetry-guided mapping of the Agassi model onto qudit systems

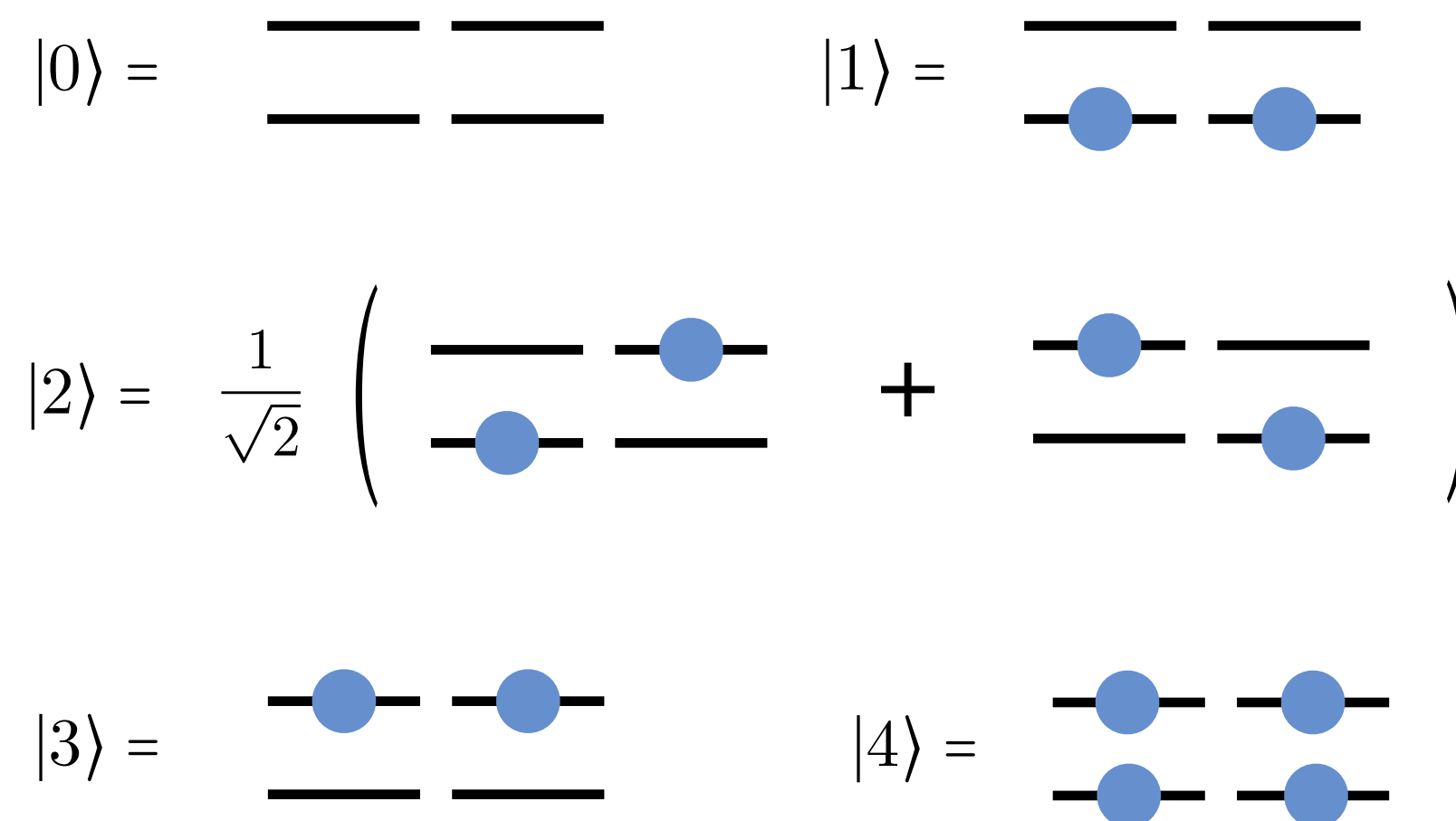
* Make use of the $SO(5)$ symmetry:

Degrees of freedom = pairs of modes

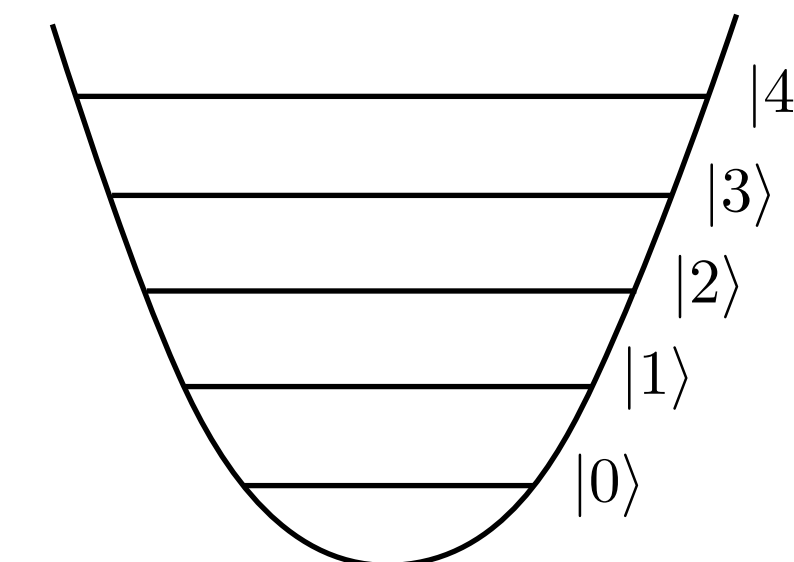


$J_z, J_{\pm}, B_{\uparrow,\downarrow}, B_{\uparrow,\downarrow}^{\dagger}$
= generators of $SO(5)$

$\Rightarrow$ 5 states:



Naturally maps onto “qu5its”
[qudits with $d=5$]



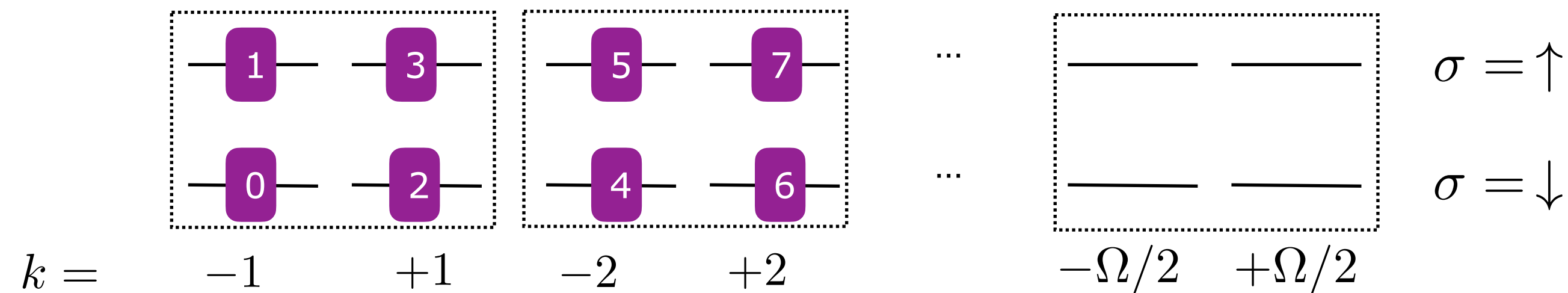
Qu5it resource requirements and comparison with qubit mappings

★ Resource requirements and comparison with mappings onto qubits

A) "Physics-Aware" Jordan-Wigner (paJW) mapping

— $\equiv |0\rangle$ —●— $\equiv |1\rangle$

Organization in terms of mode-pairs:



**Bosonization
made explicit**

→ minimizes the number of phase operators Z

→ 4 qubits per mode pair

B) State-to-state (StS) qubit-qu5it mapping

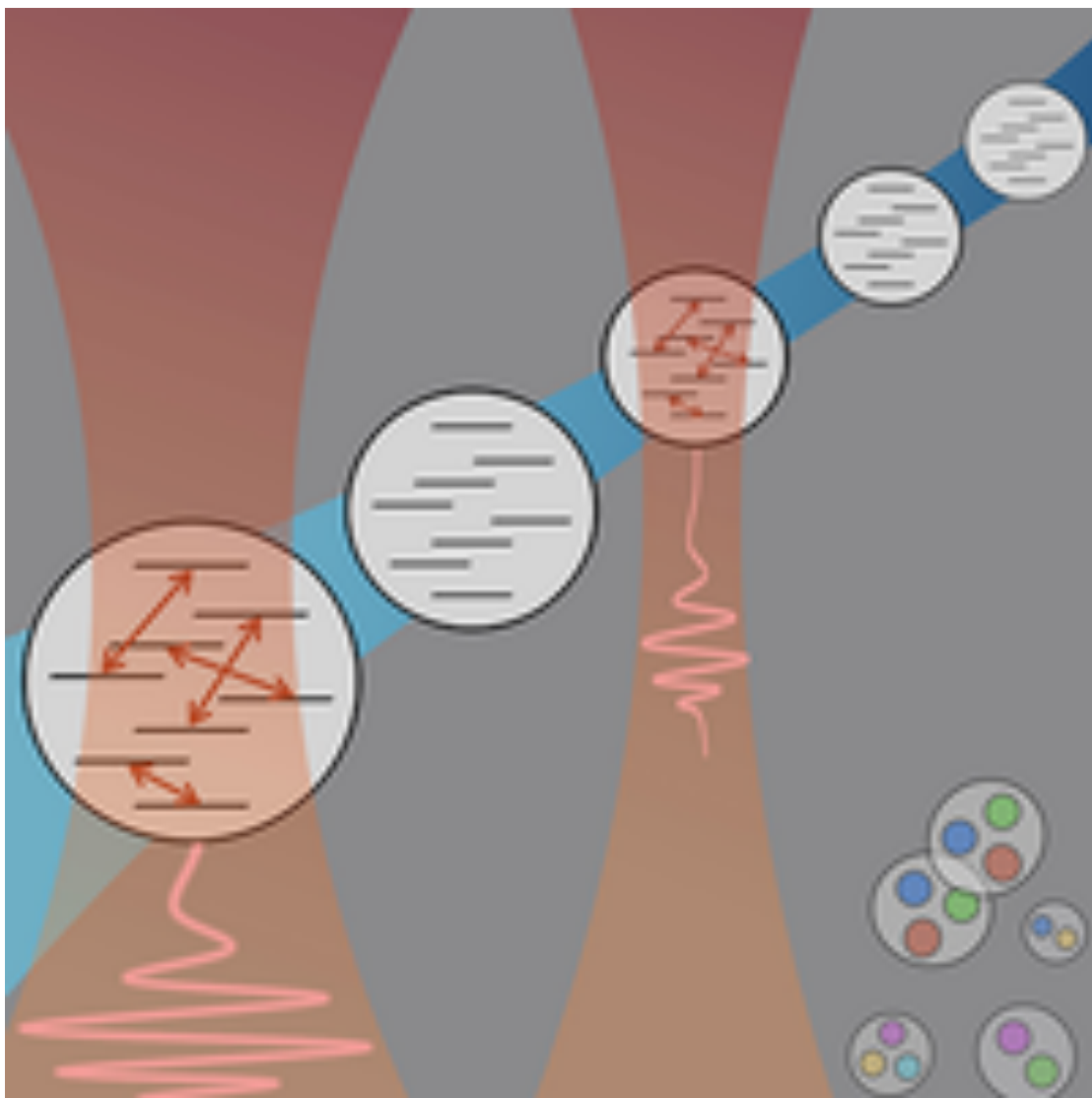
→ 3 qubits are used to map the 5 states of one mode pair

$$|0\rangle = |000\rangle, \quad |1\rangle = |001\rangle, \quad |2\rangle = |010\rangle, \quad |3\rangle = |011\rangle, \quad |4\rangle = |100\rangle$$

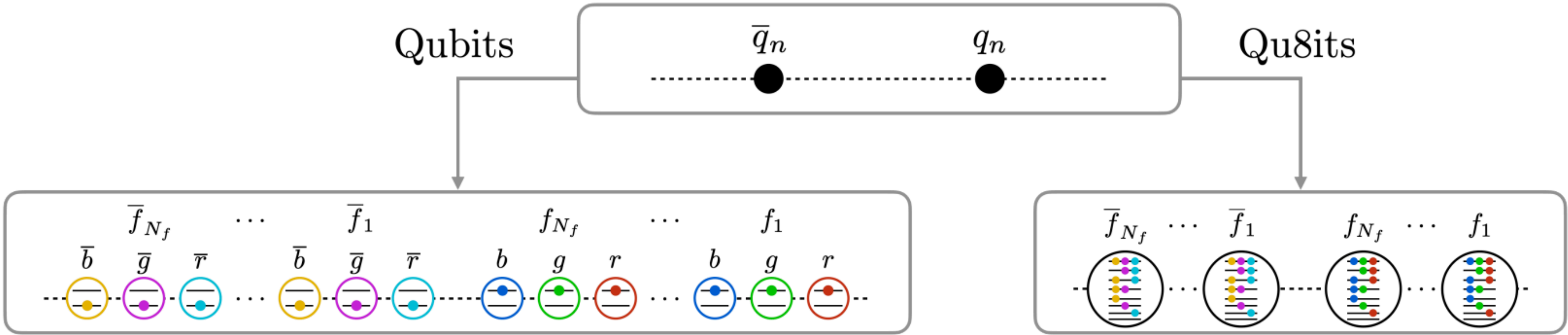
Qu8its for Quantum Simulations of 1+1D Lattice QCD

Illa, CR, Savage, PRD 110, 014507 (2024)

Editor's suggestion



$$H = \sum_f \left[\frac{1}{2} \sum_{n=0}^{2L-2} \left(\phi_n^{(f)\dagger} \phi_{n+1}^{(f)} + \text{h.c.} \right) + m_f \sum_{n=0}^{2L-1} (-1)^n \phi_n^{(f)\dagger} \phi_n^{(f)} \right] + \frac{g^2}{2} \sum_{n=0}^{2L-2} \sum_{a=1}^8 \left(\sum_{m \leq n} Q_m^{(a)} \right)^2$$



Resource for time evolution (single Trotter step):

Qudits	Number of qudits	U_{kin} ent. gates	U_{el} ent. gates
Qubit ($d = 2$)	$6N_f L$	$6N_f(8L - 3) - 4$	$N_f(2L - 1)[23N_f(2L - 1) - 17]$
Qu8it ($d = 8$)	$2N_f L$	$6N_f(2L - 1)$	$4N_f(2L - 1)[N_f(2L - 1) - 1]$
Reduction in resources ($L \rightarrow \infty$)	3	4	5.75