Field-theoretical treatment of the deuteron breakup in the reaction $e + d \to e' + p + n$ on $e'p$-coincidence

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1 Introduction

Though our consideration is related to the exclusive $d(e,e'p)n$ reaction many results may be a departure point in studying other reactions, e.g., induced by other leptons: neutrino, muons, etc.

2 Basic definitions and relations

Following our previous studies [1] (see Appendix A therein), the S-matrix for the process of the deuteron breakup by electrons ($e + d \to e' + n + p$) to the one-photon-exchange approximation (OPEA) Fig. 1, can be written as

$$\langle f|S|i\rangle = \langle e'np; out|ed; in\rangle = 2\pi i \frac{em_e}{\sqrt{EE'}} \bar{u}_e(k')\gamma_\mu u_e(k) \times \delta(p'+k'-p-k)\langle np; out|J^\mu(0)|d\rangle/q^2, \quad (2.1)$$

where m_e is the mass of the physical electron, q is the 4-momentum transferred and the electron's Dirac spinor $u_e(k)$ is normalized as in Sec. 3 of Ref. [2].

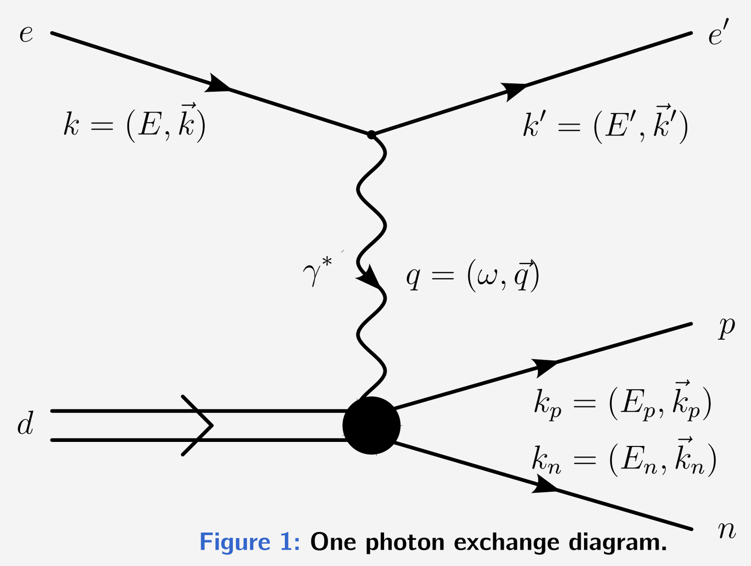

Figure 1: One photon exchange diagram.

2.1 Angular distributions and polarization of protons in the reaction $d(e,e'p)n$.

The cross section of interest in the lab. system

$$\sigma_0 \equiv \frac{d^3\sigma}{dE' d\Omega_e d\Omega_p} = l^{\mu\nu} W_{\mu\nu} R, \quad (2.1.1)$$

can be written as the convolution of the leptonic tensor $l_{\mu\nu} = (k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k'k)/2E'E$ and hadronic tensor

$$W_{\mu\nu} = \frac{1}{3} \mathrm{Sp}\left[\mathcal{F}_\mu(\vec{p}_0,\vec{q})\mathcal{F}^+_\nu(\vec{p}_0,\vec{q})\right]. \quad (2.1.2)$$

Henceforth, the metric tensor is given by $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. The kinematic factor R is equal to

$$R = E_p|\vec{k}_p|\left[1 - \frac{E_p}{E_n}\left(\frac{|\vec{q}|}{|\vec{k}_p|}\cos(\widehat{\vec{q}\vec{k}_p}) - 1\right)\right]^{-1}. \quad (2.1.3)$$

The energies and momenta involved are shown in Fig. 1.
Here

$$\mathcal{F}^\mu(\vec{p}_0,\vec{q}) = \frac{2\alpha E'}{q^2}\langle \Psi^{(-)}_{\vec{q},\vec{p}_0 S M_S}|J^\mu(0)|\Psi_{M_d}\rangle \quad (2.1.4)$$

is the matrix elements of the electromagnetic (e.m.) Noether current density operator $J_\mu(x)$ taken at the space-time point $x = (t,\vec{x}) = 0$ sandwiched between the initial deuteron state $|\Psi_{M_d}\rangle$ at rest and final np-pair state $|\Psi^{(-)}_{\vec{q},\vec{p}_0 S M_S}\rangle$ with spin S, its projection M_S, total momentum $\vec{q}$ and relative momentum $\vec{p}_0$. These states are the $P = (H,\vec{P})$ eigenvectors, viz.,

$$P|\Psi_{M_d}\rangle = (m_d,\vec{0})|\Psi_{M_d}\rangle, \quad P|\Psi^{(-)}_{\vec{q},\vec{p}_0 S M_S}\rangle = (E_{\frac{\vec{q}}{2}+\vec{p}_0} + E_{\frac{\vec{q}}{2}-\vec{p}_0}, \vec{q})|\Psi^{(-)}_{\vec{q},\vec{p}_0 S M_S}\rangle.$$

Unlike

$$F_\mu(\vec{p}_0,\vec{q}) = \frac{2\alpha E'}{q^2}\langle \Psi^{(-)}_{\vec{p}_0 S M_S}|J_\mu(\vec{q})|\Psi_{M_d}\rangle, \quad (2.1.5)$$

from Eq. (3) of Ref. [3] we handle $\mathcal{F}^\mu(\vec{p}_0,\vec{q})$ defined by Eq. (2.1.4), where, in general, the momenta $\vec{p}_0$ and $\vec{q}$ can not be separated.
Let us consider the coplanar case and introduce the orthonormal basis,

$$\hat{\vec{n}}_z = \vec{q}/q, \quad \hat{\vec{n}}_y = \vec{k}\times\vec{k}'/|\vec{k}\times\vec{k}'|, \quad \text{and} \quad \hat{\vec{n}}_x = \hat{\vec{n}}_y \times \hat{\vec{n}}_z, \quad (2.1.6)$$

assuming that $\hat{\vec{n}}_x$ and $\hat{\vec{n}}_z$ form the reaction plane (Fig. 2).

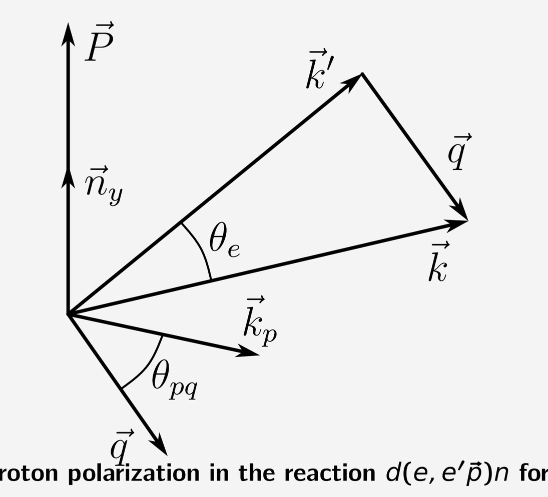

Figure 2: Proton polarization in the reaction $d(e,e'\vec{p})n$ for coplanar geometry.

According to Ref. [4], one has

$$l^{\mu\nu}W_{\mu\nu} = \sigma_M\left\{\xi^2 W_C + (\tfrac{1}{2}\xi + \eta)W_T + (\xi+\eta)W_S + \xi\sqrt{\xi+\eta}W_I\right\}, \quad \xi = q^2/\vec{q}^2, \quad \eta = \tan^2(\theta_e/2), \quad (2.1.7)$$

where σ_M is the Mott cross section, θ_e is the electron scattering angle. The structure functions (SFs): the Coulomb SF $W_C = W_{00}$ is determined by the longitudinal current component, while $W_T = 2W_{yy}$ and $W_S = W_{xx} - W_{yy}$ depend on the transverse current components. The function $W_I = -W_{x0} - W_{0x}$ is the interference term. To eliminate the matrix elements with longitudinal current component we used the continuity equation

$$q_\mu \mathcal{F}^\mu(\vec{p}_0,\vec{q}) = 0 \quad (2.1.8)$$

In the coplanar case the proton polarization in the reaction $d(e,e'\vec{p})n$ with unpolarized electrons and deuteron is orthogonal to the reaction plane

$$\vec{P} = P\vec{n}_y, \quad \vec{P} = \mathrm{Sp}\{\vec{\sigma}(1)\mathcal{F}\mathcal{F}^\dagger\}/\mathrm{Sp}\{\mathcal{F}\mathcal{F}^\dagger\}. \quad (2.1.9)$$

By using condition (2.1.8) one can show that

$$\sigma_0 P = l_{\mu\nu}\Sigma^{\mu\nu}_y R = \sigma_M\left\{\xi^2\Sigma_C + \left(\tfrac{1}{2}\xi+\eta\right)\Sigma_T + (\xi+\eta)\Sigma_S + \xi\sqrt{\xi+\eta}\Sigma_I\right\}R \quad (2.1.10)$$

where the polarization SFs $\vec{\Sigma}_i$ ($i = C,T,S,I$) are connected to the components of the polarization hadronic tensor

$$\vec{\Sigma}_{\mu\nu} = \tfrac{1}{3}\mathrm{Sp}\{\vec{\sigma}(1)\mathcal{F}_\mu(\vec{p}_0,\vec{q})\mathcal{F}^\dagger_\nu(\vec{p}_0,\vec{q})\} \quad (2.1.11)$$

via the same relations as those connecting W_i with $W_{\mu\nu}$.

2.2. Reaction $d(\vec{e},e'\vec{p})n$ with polarized electrons. Transfer polarization.

In the reaction with linearly polarized electrons and unpolarized deuterons the proton polarization $\vec{P}_\lambda$ has a component $\vec{P}'$ due to the electron helicity λ

$$\vec{P}_\lambda = \vec{P} + \lambda\vec{P}', \quad (2.2.1)$$

where $\vec{P}$ is the polarization vector (2.1.9) in case of unpolarized electrons. For the coplanar kinematics (2.1.6) the transfer polarization vector lies in the plane and its components are given by

$$\sigma_0 P'_{x,z} = \sigma_M R\sqrt{\eta}\left\{\xi\Sigma'^{x,z}_I + \sqrt{\eta+\xi}\Sigma'^{x,z}_T\right\}, \quad P'_y = 0, \quad (2.2.2)$$

where the interference $\vec{\Sigma}'_I$ and the transverse $\vec{\Sigma}'_T$ SFs are given by

$$\vec{\Sigma}'_I = i\left[\vec{\Sigma}_{0y} - \vec{\Sigma}_{y0}\right], \quad \vec{\Sigma}'_T = i\left[\vec{\Sigma}_{yx} - \vec{\Sigma}_{xy}\right]. \quad (2.2.3)$$

Thus, the transfer polarization $\vec{P}'$ depends on reaction amplitudes that do not appear in the so-called induced polarization $\vec{P}$. In contrast to the latter, the transferred polarization does not vanish when the final state interaction (FSI) is neglected. This circumstance can be used to extract additional information on the electromagnetic properties of a bound nucleon (e.g., the neutron) under conditions where FSI and many-body current effects can be neglected.

3 Underlying formalism

We start from the instant form of relativistic quantum dynamics [5] for a system of interacting particles, where amongst the ten generators of the Poincaré group Π only the Hamiltonian H and the boost operator $\vec{B}$ carry interactions. Following the conception of clothed particles, put forward by Greenberg and Schweber [6, 7] and developed in Refs. [8–10], we have constructed [11] the interaction operators for the clothed nucleons and mesons (π, ρ, δ, η, ω, σ). Doing so, the one-clothed-particle state $|\vec{p}; cloth\rangle$ of the particle with physical mass m and momentum $\vec{p}$, becomes the total H eigenvector

$$H|\vec{p}; cloth\rangle = E_{\vec{p}}|\vec{p}; cloth\rangle, \quad E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}, \quad (3.1)$$

e.g., see Ref. [12]. The so-called bare particle states do not meet this property. Using the same clothing procedure we have obtained [1, 13] many-body current operators starting with nucleon and meson Noether currents in the bare particle representation.

3.1 Links between clothed particle representation and in(out) formalism.

As well-known, when evaluating the S-matrix in the Heisenberg picture,

$$S_{if} = \langle f; out \mid i; in\rangle \quad (3.1.1)$$

one has to deal with the in(out) states (see, e.g., [14]), in particular, one-particle state

$$|\vec{p};\, in\,(out)\rangle = a^\dagger_{in\,(out)}(\vec{p})\,|\Omega\rangle\,, \quad (3.1.2)$$

where $|\Omega\rangle$ is the physical vacuum. The creation (destruction) in(out) operators $a^\dagger_{in(out)}$ ($a_{in(out)}$) meet canonical commutation relations for bosons and fermions. By definition, these states are the H eigenstates

$$H|\vec{p}; in(out)\rangle = E_{\vec{p}}|\vec{p}; in(out)\rangle. \quad (3.1.3)$$

Omitting important details [13, 15], one can prove the relations between states in the clothed particle representation and in(out) formalism for the one particle

$$|\vec{p}; in(out)\rangle \equiv a^\dagger_{in(out)}(\vec{p})|\Omega\rangle = a^\dagger_c(\vec{p})|\Omega\rangle, \quad (3.1.4)$$

and two particle states

$$|\vec{p}_1\vec{p}_2; in\rangle \equiv a^\dagger_{in}(\vec{p}_1)a^\dagger_{in}(\vec{p}_2)|\Omega\rangle = \Omega^{(+)}_c a^\dagger_c(\vec{p}_1)a^\dagger_c(\vec{p}_2)|\Omega\rangle,$$
$$|\vec{p}_1\vec{p}_2; out\rangle \equiv a^\dagger_{out}(\vec{p}_1)a^\dagger_{out}(\vec{p}_2)|\Omega\rangle = \Omega^{(-)}_c a^\dagger_c(\vec{p}_1)a^\dagger_c(\vec{p}_2)|\Omega\rangle \quad (3.1.5)$$

with the Møller operators $\Omega^{(\pm)}_c \equiv \lim_{t\to\mp\infty} \exp(iHt)\exp(-iK_F t)$, that hold under the condition

$$\lim_{t\to\pm\infty} W_D(t) = 1. \quad (3.1.6)$$

To some extent, relation (3.1.4) does not seem unexpected, since both one-particle clothed states and in(out) states, being equally normalized, are H eigenvectors. Of course, it does not mean that $a_{in(out)}(\vec{p}) = a_c(\vec{p})$!
One of the useful results of these links is that they make it possible to use equation (2.1) for the S-matrix in conjunction with the UCT method for current operator matrix elements.

3.2 The construction of the initial and final states.

Handling the two-nucleon sector of the Fock space the initial and final states in the center of mass system (c.m.s.) can be represented as

$$|\Psi_{M_d}\rangle = \sum_{M'_S}\int d\vec{p}'\,\Psi_{M_d}(\vec{p}'1M'_S)|\vec{p}'1M'_S\rangle, \quad (3.2.1)$$

$$|\Psi^{(-)}_{\vec{0},\vec{p}SM_S}\rangle \equiv |\Psi^{(-)}_{\vec{p}SM_S}\rangle = \sum_{S'M'_S}\int d\vec{p}'\,\Psi^{(-)}_{\vec{p}SM_S}\left(\vec{p}'S'M'_S\right)|\vec{p}'S'M'_S\rangle, \quad (3.2.2)$$

where $$|\vec{p}SM_S\rangle = \sum_{\mu_1\mu_2}(\tfrac{1}{2}\mu_1\,\tfrac{1}{2}\mu_2|S\,M_S)b^\dagger_c(\vec{p}\mu_1)b^\dagger_c(-\vec{p}\mu_2)|\Omega\rangle. \quad (3.2.3)$$

The $\mathbb{C}$-number coefficient $\Psi_{M_d}(\vec{p}1M_S)$ can be expressed through the S- and D-components of the deuteron wave function (WF) in the momentum space

$$\Psi_{M_d}(\vec{p}1M_S) = \sum_{l=0,2}(lm_l1M_S|1M_d)u_l(|\vec{p}|)Y_{lm_l}(\hat{\vec{p}}). \quad (3.2.4)$$

They have been calculated in Refs. [16, 17] using the so-called Kharkiv potential [11].
For this consideration we confine ourselves to the approximation with separable variables

$$|\Psi^{(-)}_{\vec{q},\vec{p}SM_S}\rangle \approx \int d\vec{p}'\,\Psi^{(-)}_{\vec{p}SM_S}\left(\vec{p}'\mu'_1\mu'_2\right)b^\dagger_c\left(\tfrac{\vec{q}}{2}+\vec{p}'\mu'_1\right)b^\dagger_c\left(\tfrac{\vec{q}}{2}-\vec{p}'\mu'_2\right)|\Omega\rangle,$$

that allows us to get the matrix element of interest as a vacuum expectation value

$$\mathcal{F}^\mu(\vec{p}_0,\vec{q}) = \frac{2\alpha E'}{q^2}\sum_{\mu'_1\mu'_2\mu_1\mu_2}\int d\vec{p}'d\vec{p}[\Psi^{(-)}_{\vec{p}_0SM_S}(\vec{p}'\mu'_1\mu'_2)]^*\Psi_{M_d}(\vec{p}\mu_1\mu_2) \times \langle\Omega|b_c\left(\tfrac{\vec{q}}{2}-\vec{p}'\mu'_2\right)b_c\left(\tfrac{\vec{q}}{2}+\vec{p}'\mu'_1\right)J^\mu(0)\,b^\dagger_c(\vec{p}\mu_1)b^\dagger_c(-\vec{p}\mu_2)|\Omega\rangle. \quad (3.2.5)$$

In order to calculate the np-pair states in the rest frame we address the partial wave decomposition (see details in Ref. [3]).

3.3 Current density operator.

The Noether current operator $J(0)$ being expressed through the clothed operators takes the form [1, 9]

$$J^\mu(0) = e^R J^\mu_c(0)e^{-R} = J^\mu_c(0) + [R, J^\mu_c(0)] + \tfrac{1}{2}[R,[R,J^\mu_c(0)]] + \ldots, \quad (3.3.1)$$

where $J^\mu_c(0)$ is the primary Noether current in which "bare" operators $\{\alpha\}$ are replaced by the clothed ones $\{\alpha_c\}$ and R is the generator of the clothing transformation $W = \exp(R)$. Decomposition (3.3.1) involves one-body, two-body and more complicated interaction currents that often call the meson exchange currents (MEC), if one uses the terminology addopted in the theory of e.m. interactions with nuclei.
In Eq.(3.2.5) the current density operator $J^\mu(0)$ is sandwiched between clothed two-nucleon states. As shown in Ref. [1] for such expectation values the current $J^\mu(0)$ will contribute only with its one-body $J_{[1]}$ and two-body $J_{[2]}$ parts

$$\langle\vec{p}'_1\mu'_1,\vec{p}'_2\mu'_2|J^\mu(0)|\vec{p}_1\mu_1,\vec{p}_2\mu_2\rangle = \langle\vec{p}'_1\mu'_1,\vec{p}'_2\mu'_2|(J^\mu_{[1]} + J^\mu_{[2]})|\vec{p}_1\mu_1,\vec{p}_2\mu_2\rangle,$$

where the one-body and two-body currents are given by

$$J^\mu_{[1]} = \sum_{\mu'\mu}\int d\vec{p}'d\vec{p}\,F^\mu(\vec{p}'\mu',\vec{p}\mu)b^\dagger_c(\vec{p}'\mu')b_c(\vec{p}\mu), \quad (3.3.2)$$

$$F^\mu(\vec{p}'\mu',\vec{p}\mu) = \frac{em}{\sqrt{E_{\vec{p}'}E_{\vec{p}}}}\bar{u}(\vec{p}'\mu')\Big[F_1[(p'-p)^2]\gamma^\mu + i\sigma^{\mu\nu}\frac{(p'-p)_\nu}{2m}F_2[(p'-p)^2]\Big]u(\vec{p}\mu) \quad (3.3.3)$$

$$J^\mu_{[2]} = \int d1'd2'd1d2\,F^\mu_{MEC}(1',2',1,2)b^\dagger_c(1')b^\dagger_c(2')b_c(1)b_c(2). \quad (3.3.4)$$

In Refs. [1, 13], considering the first non-vanishing contribution to the two-nucleon current from the series (3.3.1) that stem from the commutator $\frac{1}{2}[R,[R,J^\mu_c(0)]]$ we have found F^μ_{MEC} for the model of interacting clothed nucleons and π, ρ, ω, η, δ, σ mesons. A unique feature of this new family of MECs is that they are derived just from the given conserved nucleon and meson Noether currents.
The evaluation of the matrix elements (3.2.5) with the one-body current reduces to the three-dimensional integrals

$$S_{[1]}(p_0,\vec{q}) = \int \psi^{JST}_{p_0 ll'}(p')Y^*_{l'm'}\left(\widehat{\vec{p}'}\right)F^\mu\left(\vec{p}'+\tfrac{\vec{q}}{2}\mu',\vec{p}'-\tfrac{\vec{q}}{2}\mu\right) \times u_L\left(\left|\vec{p}'-\tfrac{\vec{q}}{2}\right|\right)Y_{LM}\left(\widehat{\vec{p}'-\tfrac{\vec{q}}{2}}\right)d\vec{p}'. \quad (3.3.5)$$

In turn, the angular integrations involved can be simplified applying the Wigner-Eckart theorem and using explicit expressions for the S and D deuteron wave function components (cf. Eqs. (A.1)–(A.7) from [3]).

4 Results and discussion

In Fig. 3–5, both the direct mechanism of the proton knockout – plane-wave impulse approximation (dotted curves) and the recoil mechanism due to the e.m. interaction with the spectator neutron – Born approximation (solid curves) were considered. In Fig. 3, we also have inclusion of the FSI effects – distorted-wave Born approximation (dash-dotted curves).

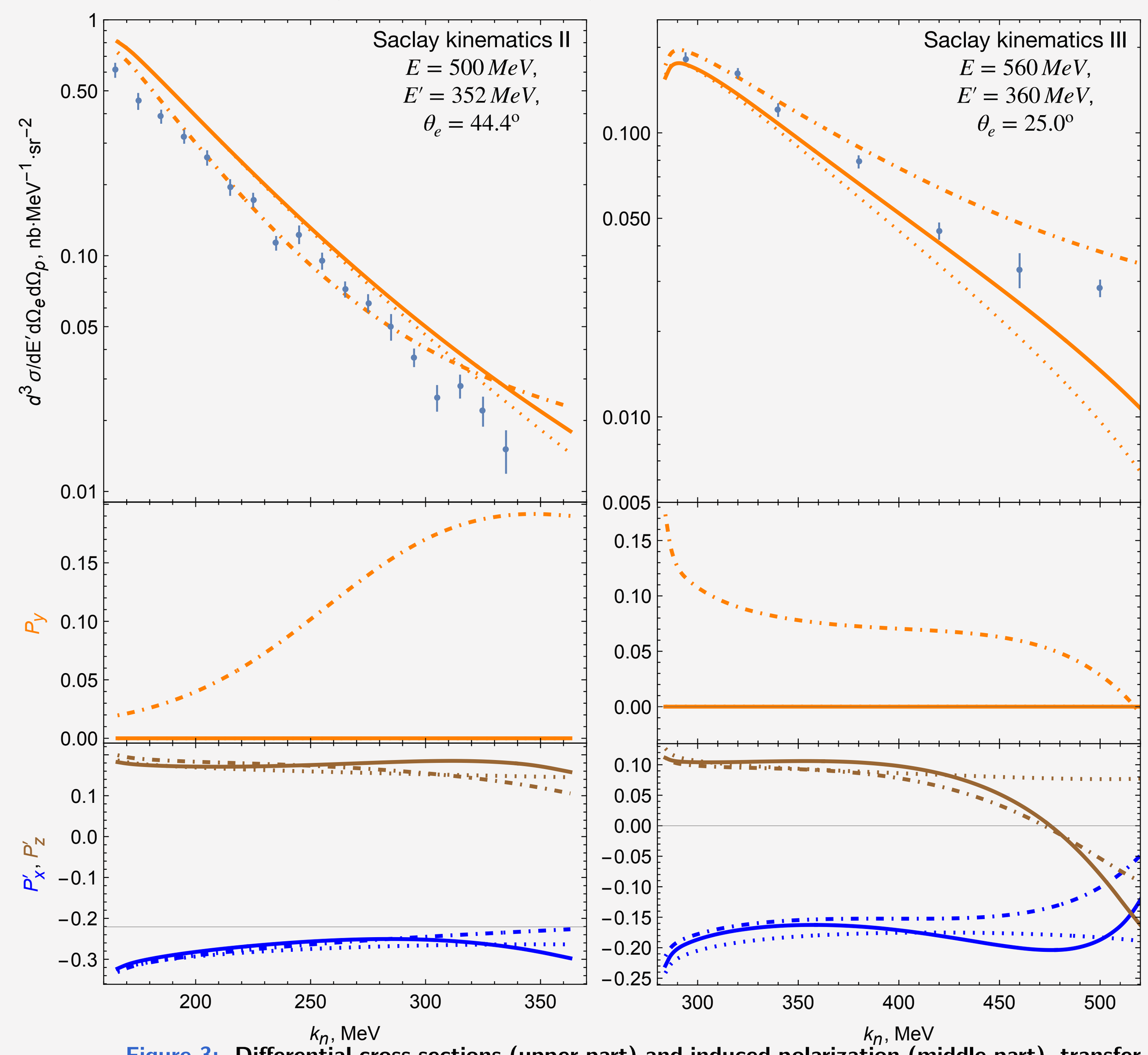

Figure 3: Differential cross sections (upper part) and induced polarization (middle part), transfer polarization (lower part) of knocked-out protons versus the spectator momentum k_n for the Saclay kinematics [18, 19].

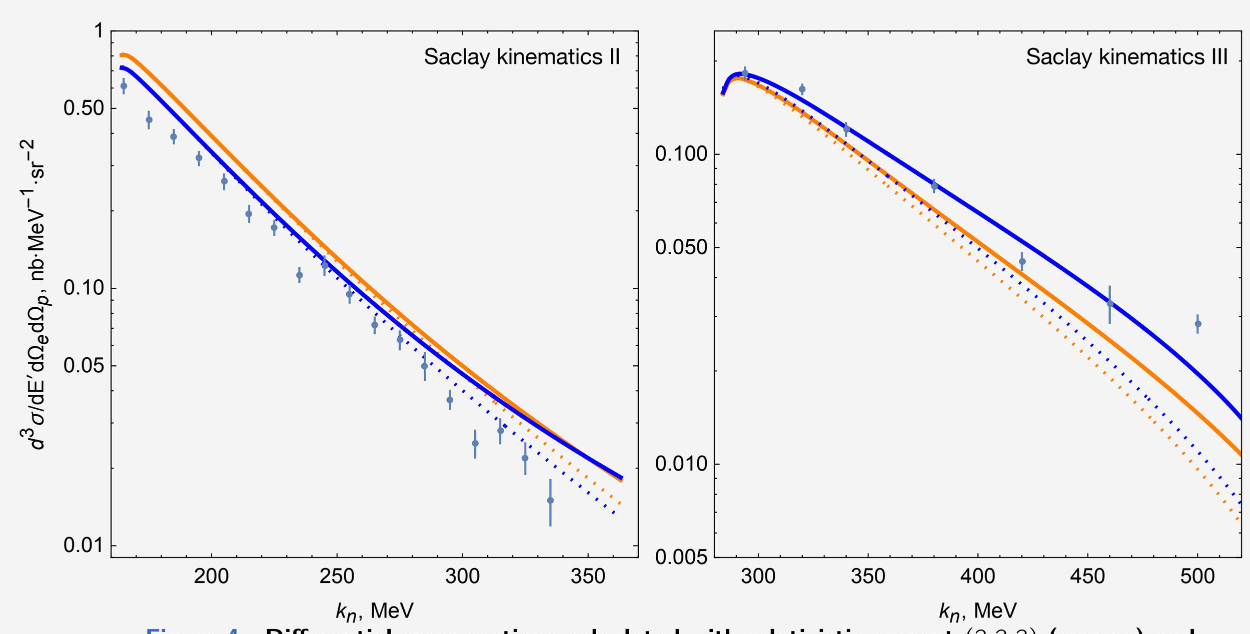

Figure 4: Differential cross sections calculated with relativistic current (3.3.3) (orange) and current from Ref. [20] (blue).

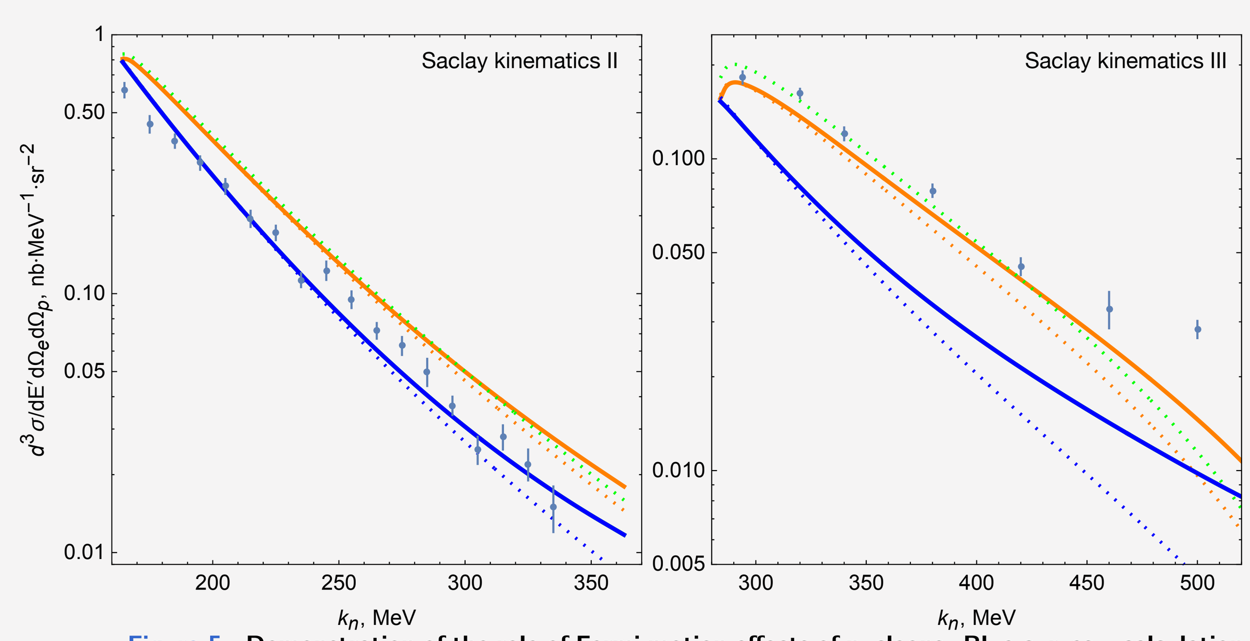

Figure 5: Demonstration of the role of Fermi motion effects of nucleons. Blue curves – calculation with "frozen" current, i.e., in formula (3.3.3), momenta are fixed $\vec{p} = 0$, $\vec{p}' = \vec{q}$; Green curves – arguments of the form factors $(p'+p)^2$ are replaced with q^2; Orange curves – the same as in Fig. 4. The solid Green and Blue curves (BA) overlap.

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