Fluctuation measurements as a probe of hot QCD matter

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(Yale University)

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Wright Laboratory

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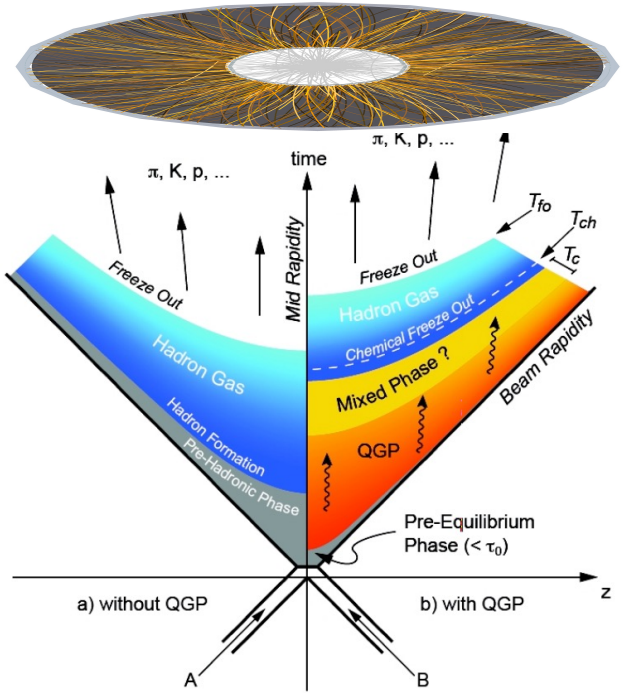
Outline

- Event-by-event fluctuations to study QCD phase diagram
  - Theory vs Experiment
- What have we learned so far from low to high energies?
  - Take away messages (TA)
  - Room for improvement (RI)
- What do we expect from the future?

Evolution of QGP



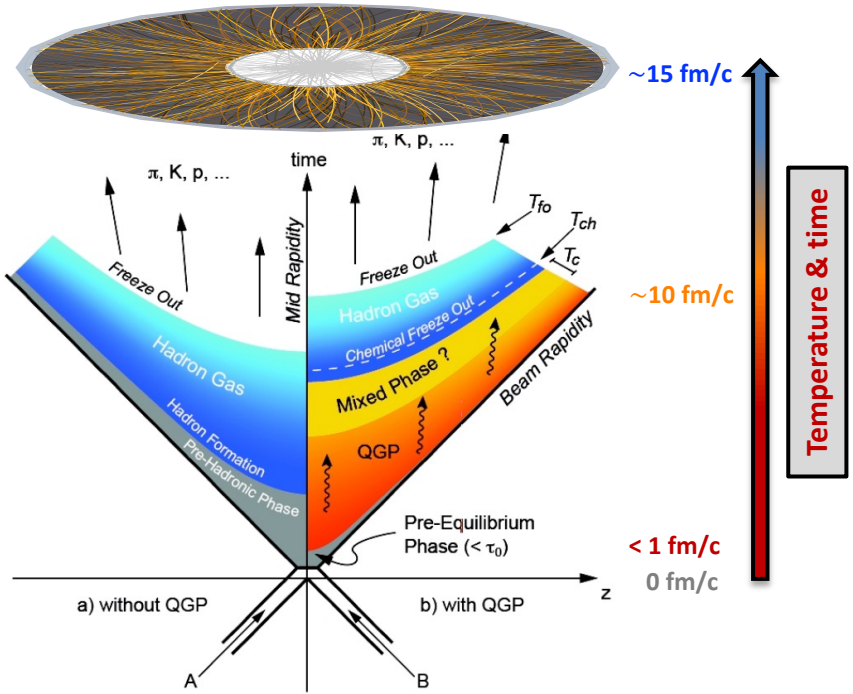
~15 fm/c

~10 fm/c

< 1 fm/c

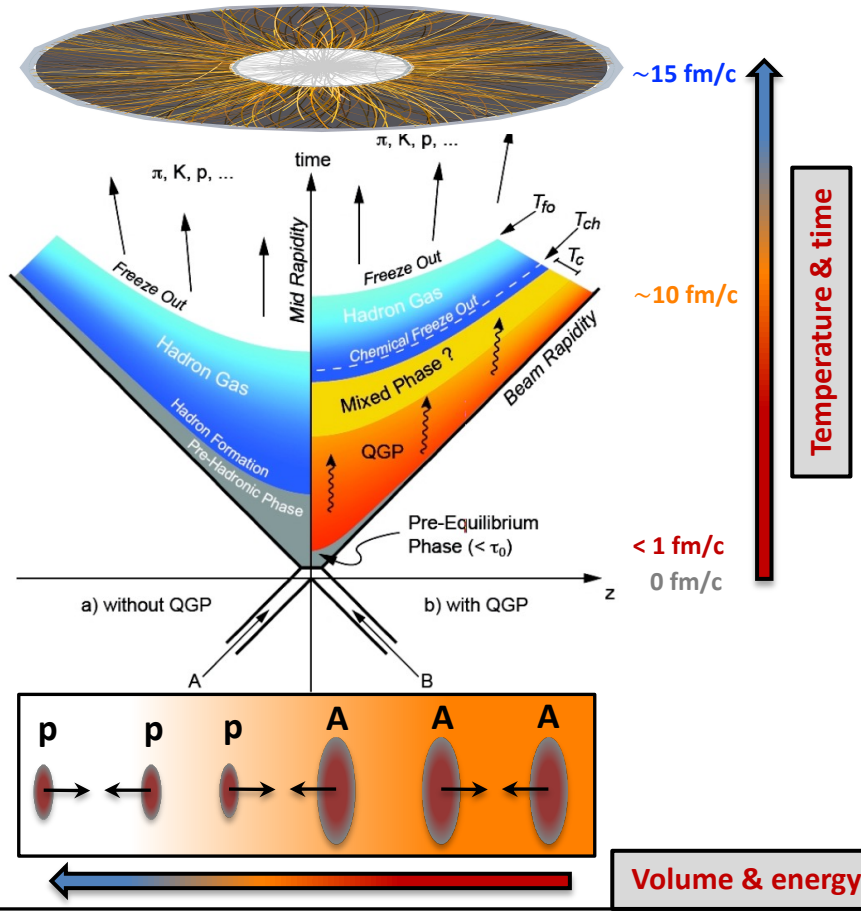
0 fm/c

Evolution of QGP



- **Increased fluctuations** → Statistical vs critical
- **Long/short range correlations** → Early vs late production

Evolution of QGP



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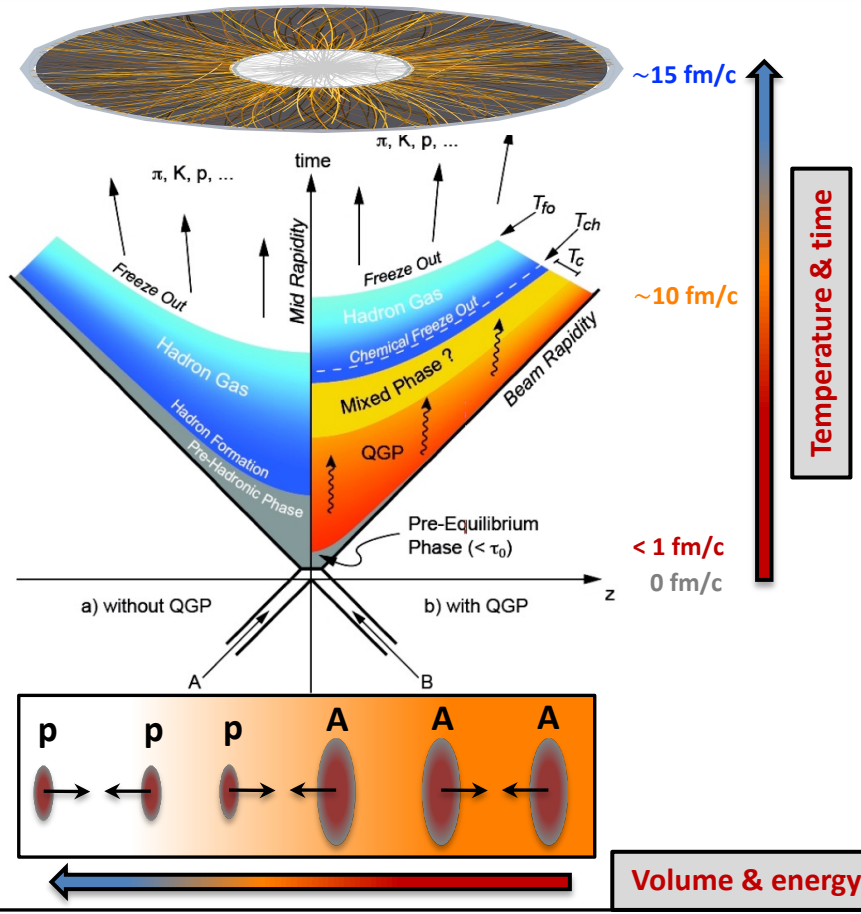

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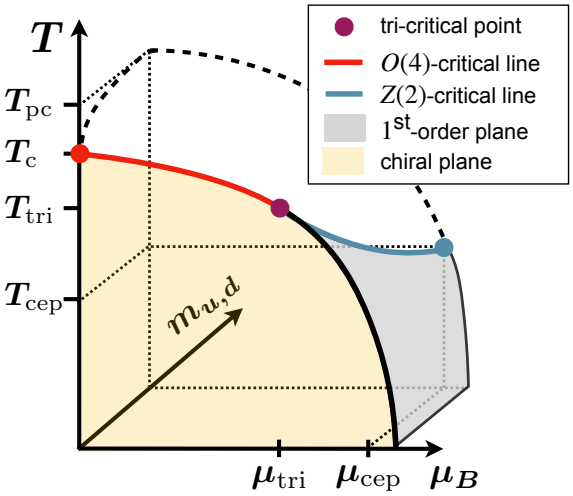
How much of the full history survives until freezout?
Is all experimental evidence consistent?

QGP as a thermodynamic system

Theory



H.-T. Ding et. al, Phys.Rev.D 109 (2024) 114516

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \vec{\mu})$$

Chiral condensate, susceptibilities …

QGP as a thermodynamic system

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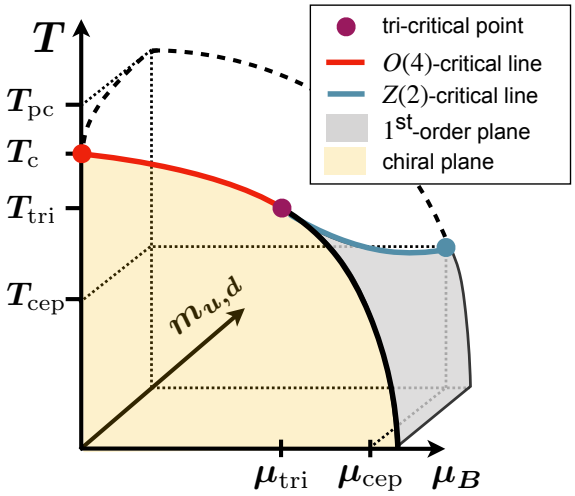

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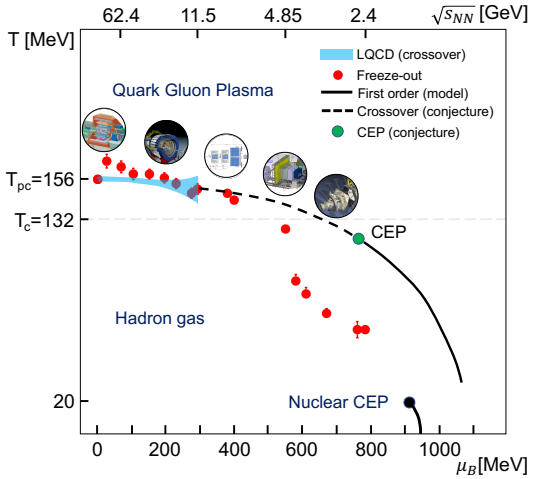

P. B.-Munzinger, A. Rustamov, J. Stachel EPJC 83 (2023) 1125

$$\langle N \rangle = \sum_j N_j p_j = T \frac{\partial \ln Z_{GCE}}{\partial \mu}\bigg|_V$$

Moments, (factorial) cumulants …

QGP as a thermodynamic system

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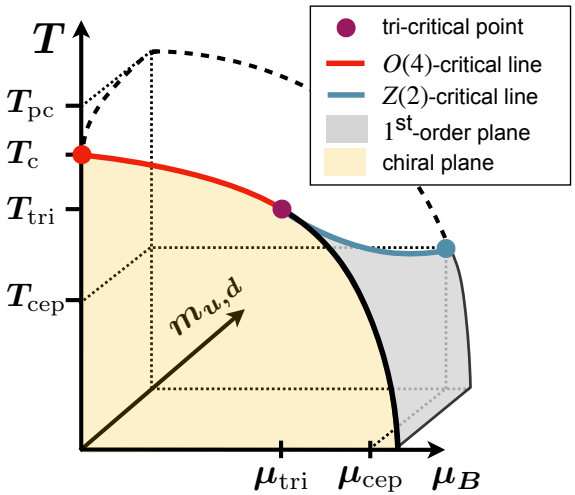
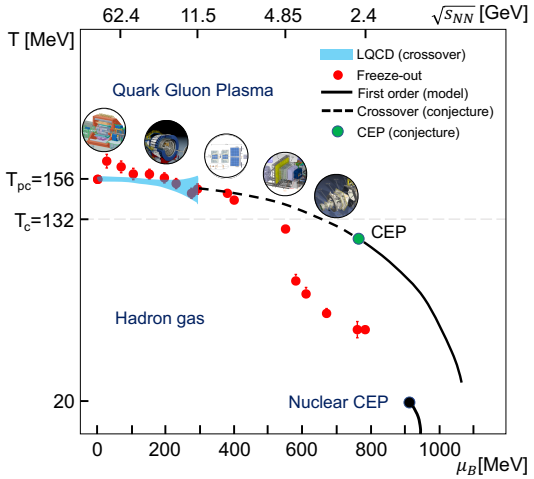


H.-T. Ding et. al, Phys.Rev.D 109 (2024) 114516

Static	Dynamic
Coordinate space	Momentum space
Net-baryon	Net-proton
Fixed V	Fluctuating V
…	…

Experiment



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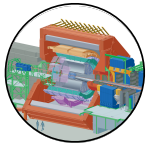
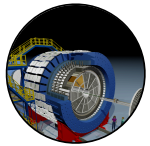
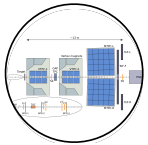
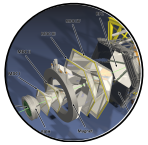
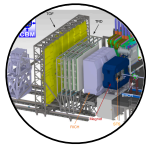
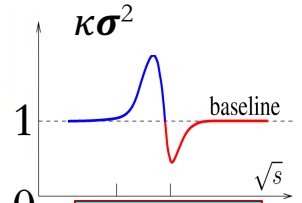
Event-by-event fluctuations: Big picture

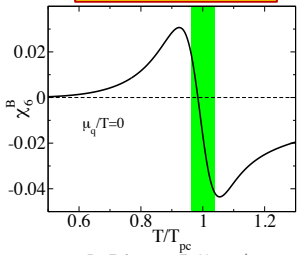






M. Stephanov, PRL107, 052301(2011)



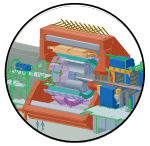
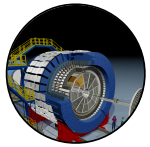
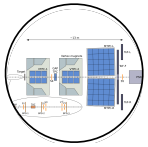
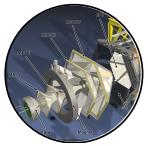
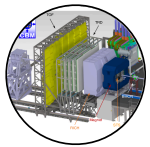
Qualitative

Quantitative



B. Friman, F. Karsch, K. Redlich, V. Skokov EPJC (2011) 71 1694

Event-by-event fluctuations: Big picture

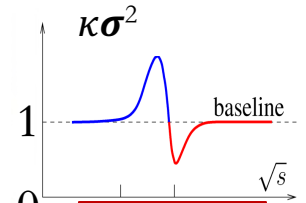
How to interpret: "Establishing a non-critical baseline"

- ❖ **Critical fluctuations & Critical End Point (CEP)**
- ➢ Global/local charge conservation
- ➢ Different hadronisation mechanisms
- ➢ Annihilation, resonances, hydrodynamic evolution …

Experimental challenges:

- ➢ Detection efficiency correction
- ➢ Event pileup
- ➢ Particle identification
- ➢ **Volume fluctuations**
- ➢ …

M. Stephanov, PRL107, 052301(2011)



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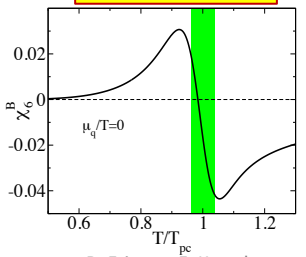
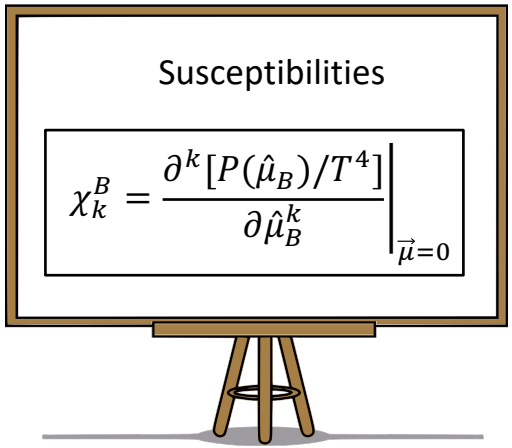


B. Friman, F. Karsch, K. Redlich, V. Skokov EPJC (2011) 71 1694

How to link experiment to theory
&
interpret the results?

How to design an observable?

LQCD



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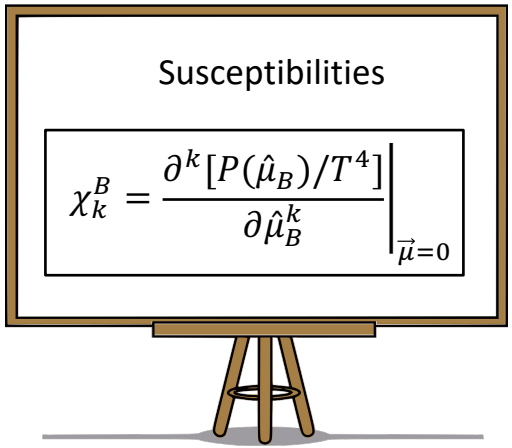
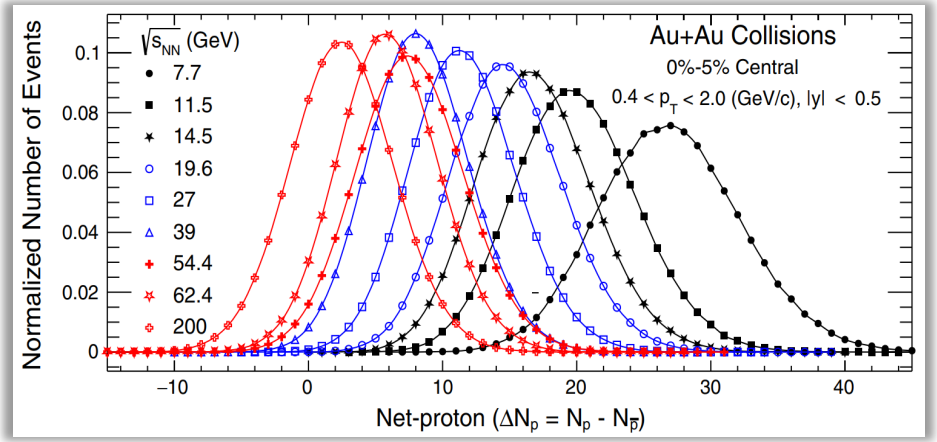


Experiment



STAR Coll., Phys. Rev. Lett. 126, 092301 (2021)

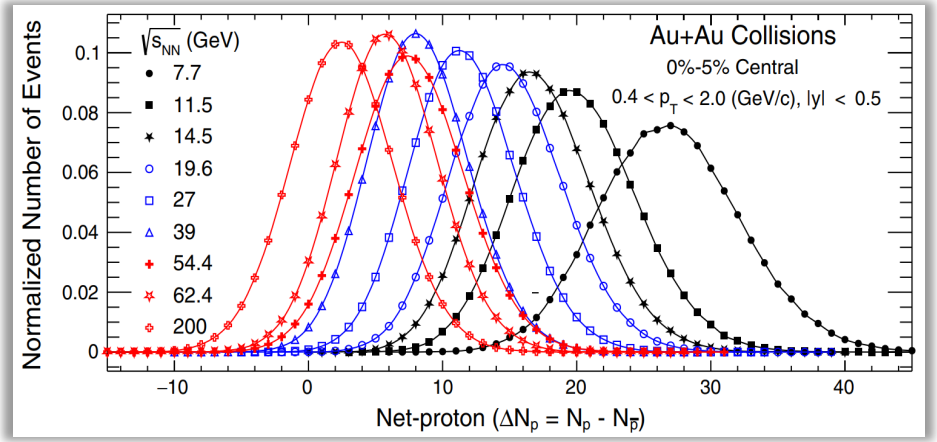
How to design an observable?

LQCD

Susceptibilities

$$\chi_k^B = \frac{\partial^k [P(\hat{\mu}_B)/T^4]}{\partial \hat{\mu}_B^k}\bigg|_{\vec{\mu}=0}$$

Experiment



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Cumulants

$$\kappa_1 = \langle N \rangle$$
$$\kappa_2 = \langle N^2 \rangle - \langle N \rangle^2$$
$$\kappa_3 = \langle N^3 \rangle + 3\langle N^2 \rangle \langle N \rangle + 2\langle N \rangle^3$$
$$\kappa_4 = \langle N^4 \rangle - 4\langle N^3 \rangle \langle N \rangle - 3\langle N^2 \rangle^2 + 12\langle N^2 \rangle \langle N \rangle^2 - 6\langle N \rangle^4$$

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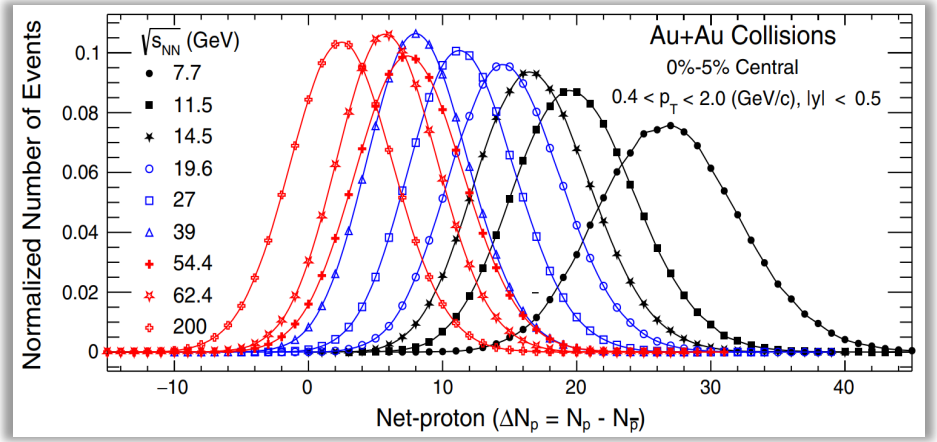

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Factorial cumulants

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How to design an observable?

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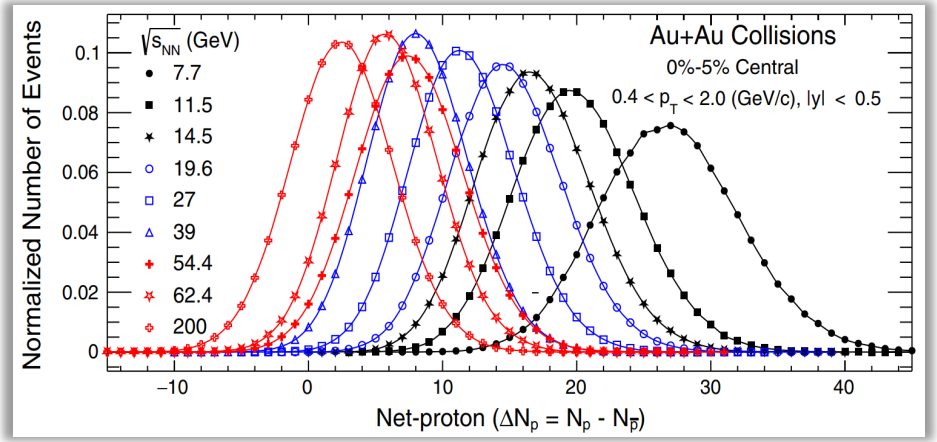

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Link theory to experiment

$$\chi_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3} \Rightarrow \frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\chi_4^B}{\chi_2^B}$$

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_B \rangle$$

κ_n/κ_2 is 0 (n odd) or 1 (n even)

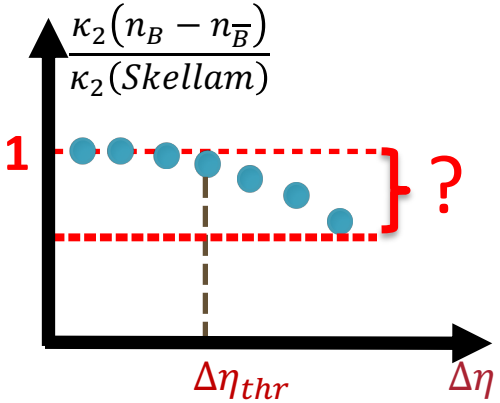
Caution

STAR
Cumulants (C)
Factorial cumulants (κ)

Others
Cumulants (κ)
Factorial cumulants (C)

Acceptance & baseline

What is the source of deviation from baseline?



- Baryon number conservation
- Volume fluctuations
- Resonance decays
- Initial-state fluctuations
- Effect of initial magnetic field
- …

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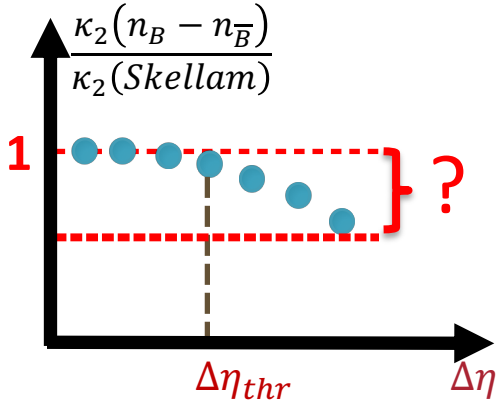

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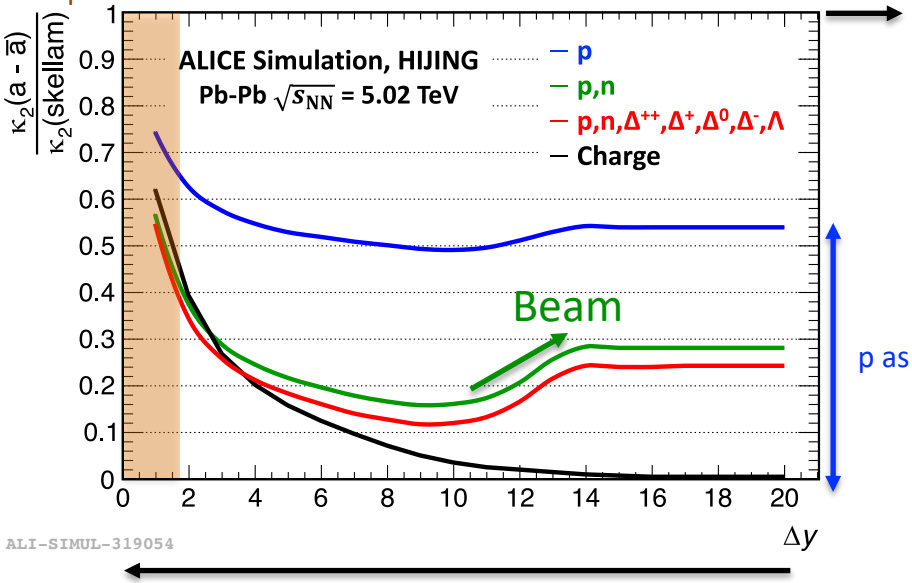
Event generator (HIJING)

Experimental acceptance at the LHC



Poissonian baseline & LQCD prediction

p as proxy to B

Towards Poissonian limit

Acceptance & baseline

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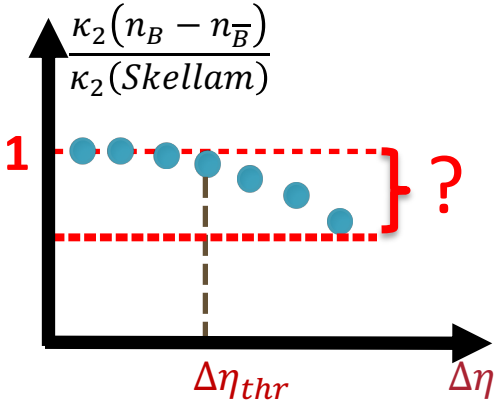

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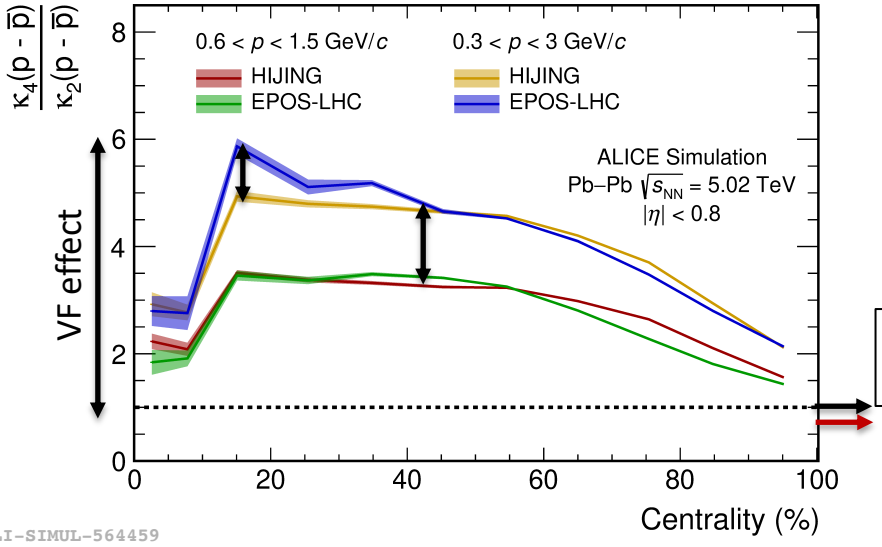
Volume Fluctuations (VF)



Poissonian baseline: 1

LQCD: ≈0.75

How to overcome volume fluctuations?

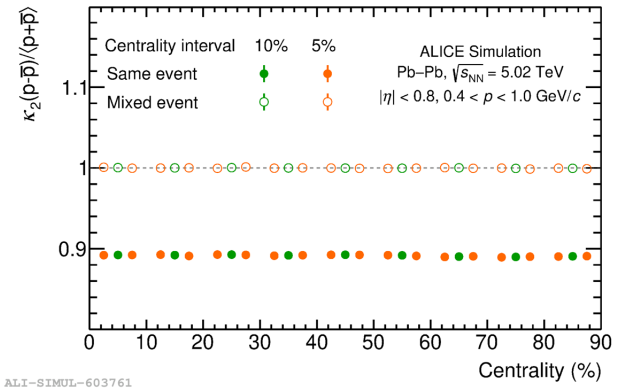
- **Idea:** Mix events in a given category wrt event shape and centrality

A. Rustamov, R. Holzmann, J. Stroth, NPA 1034 (2023) 122641, V.Koch, R. Holzmann, A. Rustamov, J. Stroth, NPA 1050 (2024) 122924

How to overcome volume fluctuations?

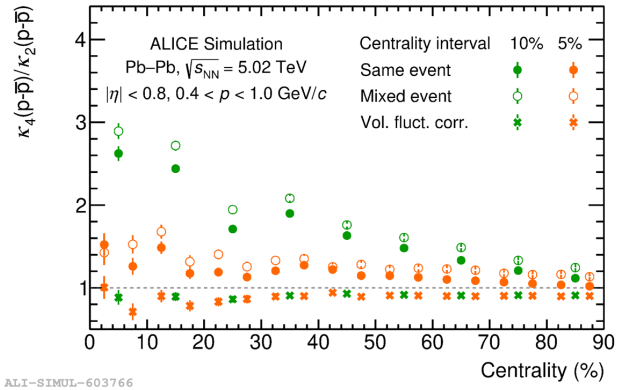
➢ **Idea:** Mix events in a given category wrt event shape and centrality

2nd order

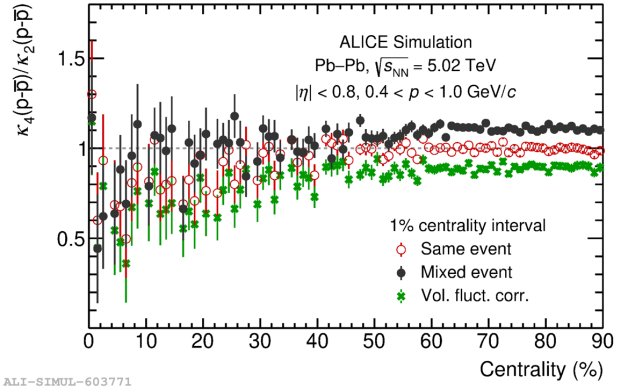


It kills all correlations

4th order



Volume fluctuations remain



Very narrow CBW do not help

A. Rustamov, R. Holzmann, J. Stroth, NPA 1034 (2023) 122641, V.Koch, R. Holzmann, A. Rustamov, J. Stroth, NPA 1050 (2024) 122924

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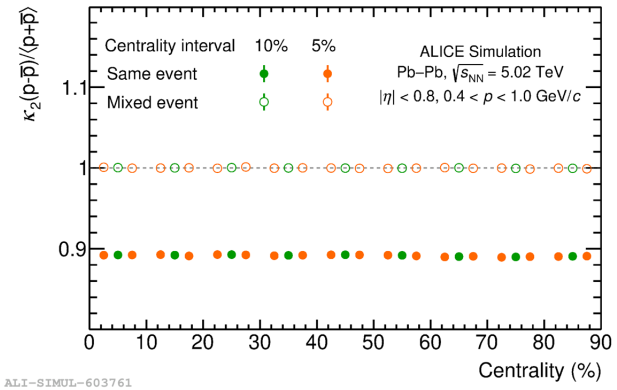

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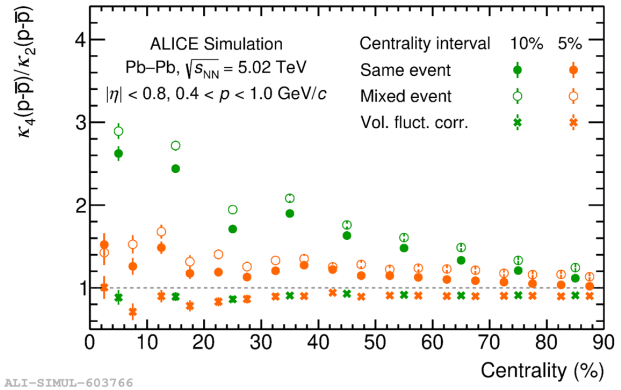
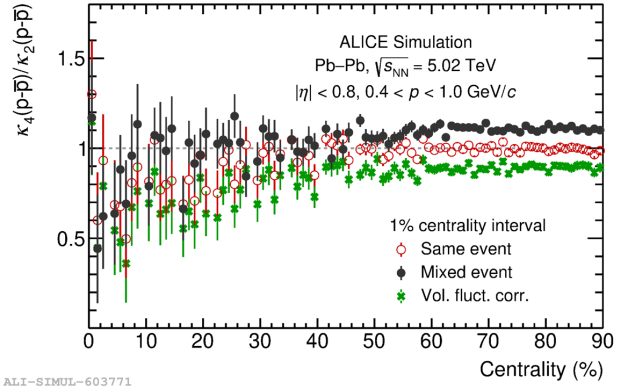


Volume fluctuations remain



Very narrow CBW do not help

➢ **TA:** CBWC does not completely eliminate VF, but mixed event technique does

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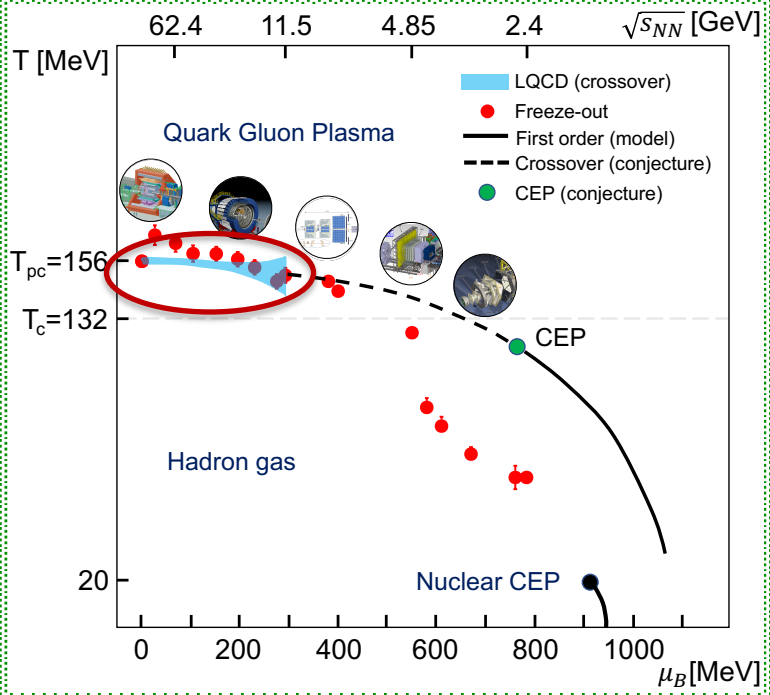
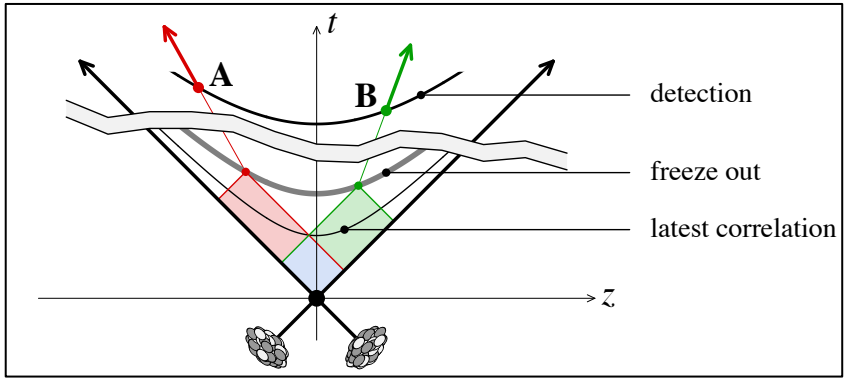
News from crossover



Can we test different hadronization scenarios?

Early correlations → longer correlation length



$$\tau \leq \tau_{\text{freeze out}}\, e^{-\frac{1}{2}|y_A - y_B|}$$

A. Dumitru, F. Gelis, L. McLerran, and R. Venugopalan, *Nucl. Phys. A* 810 (2008) 91

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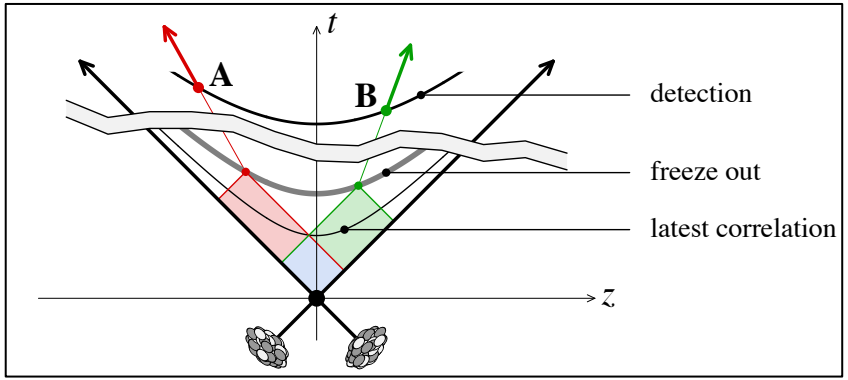

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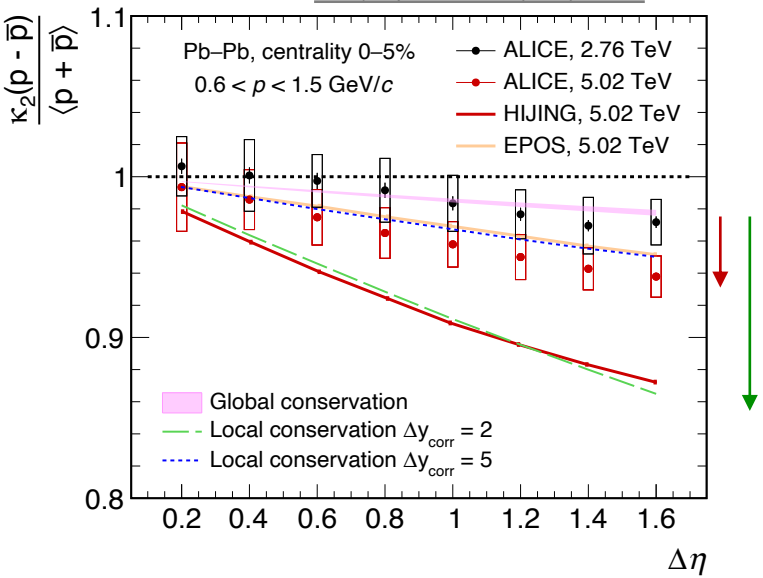
ALICE, Phys. Lett. B 844 (2023) 137545



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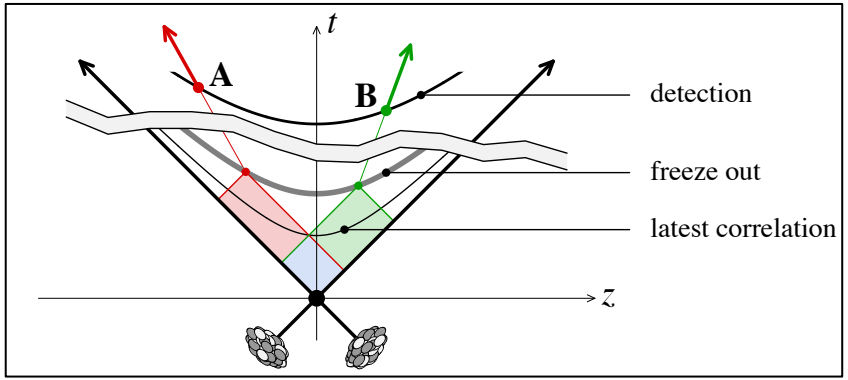

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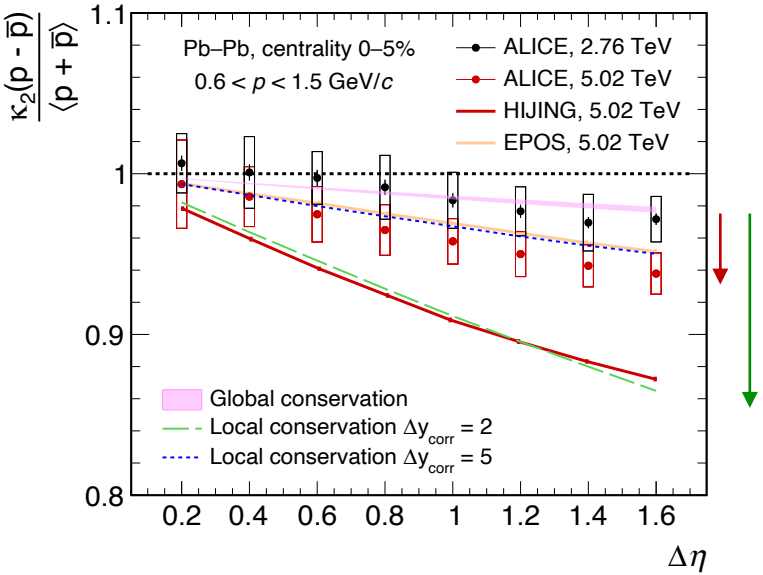
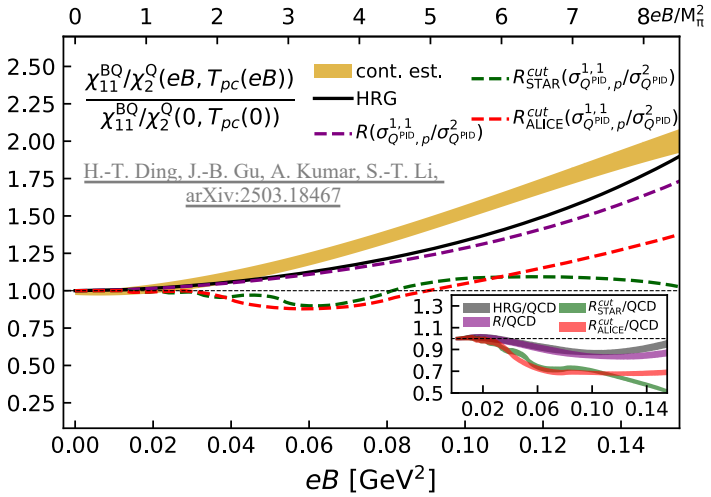
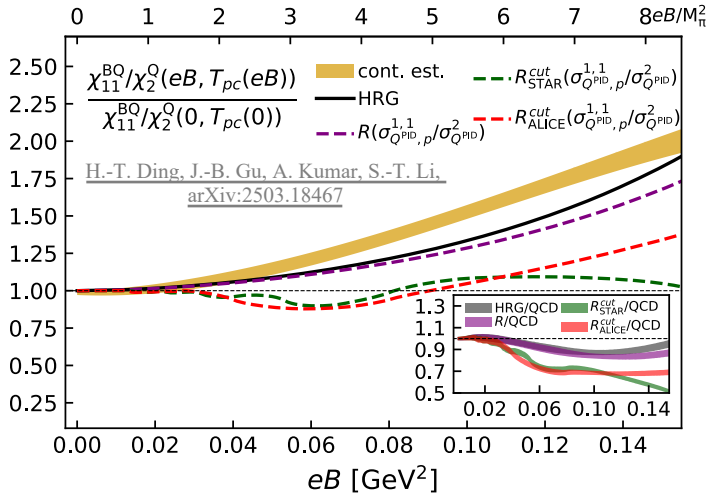


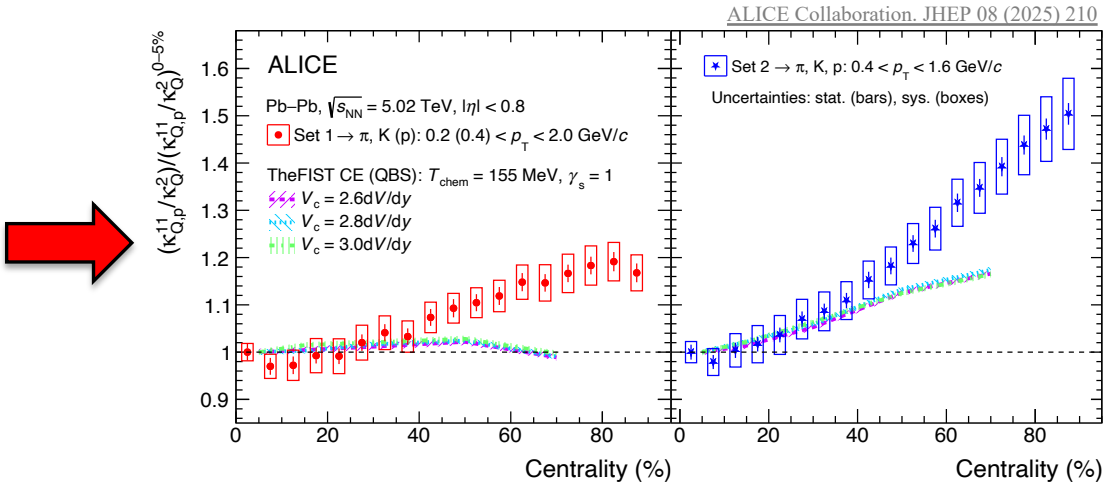
- **TA:** String fragmentation picture fails to describe data on the second order. Data suggests much earlier correlations
- **RI:** *"Enhancement of p/π in jets"* → Can this be the reason for increasing locality at 5 TeV? Implications for the higher orders?

Probing magnetic field with cumulants?



Probing magnetic field with cumulants?





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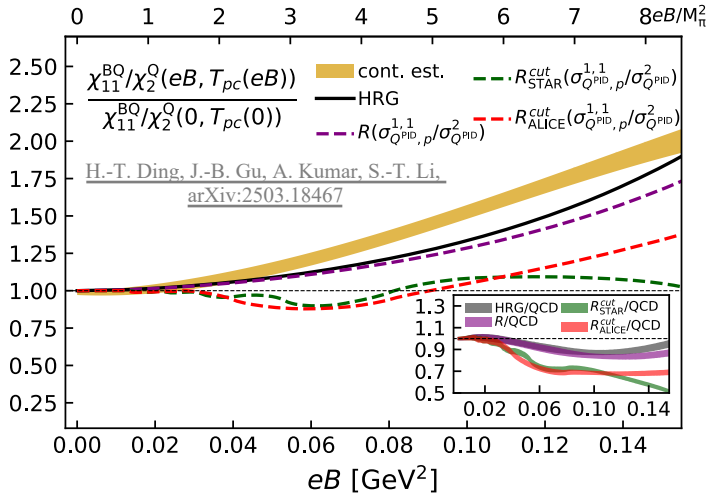
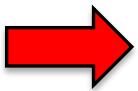
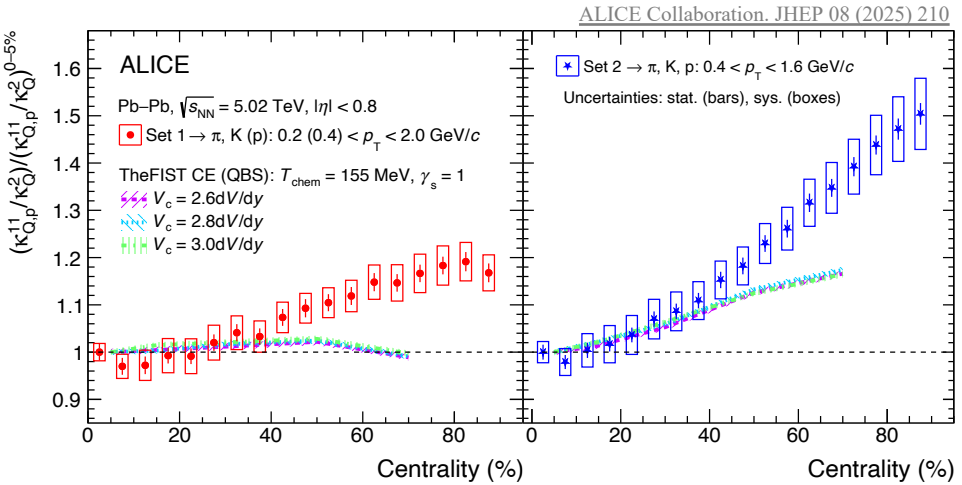
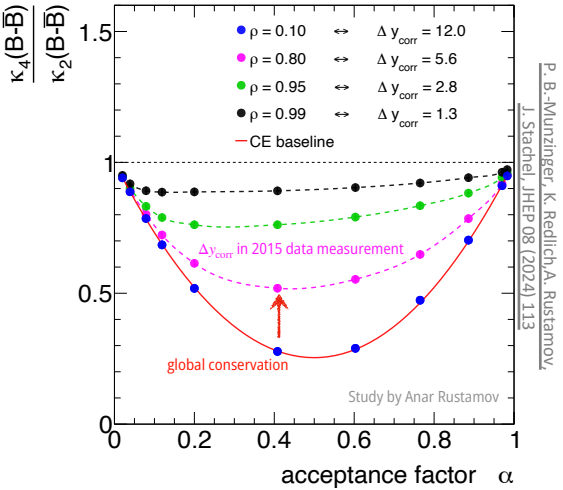






- **TA:** Very promising signal
- **RI**: Does momentum range make a difference? Higher orders?

Any light at the 4th order in A-A collisions?



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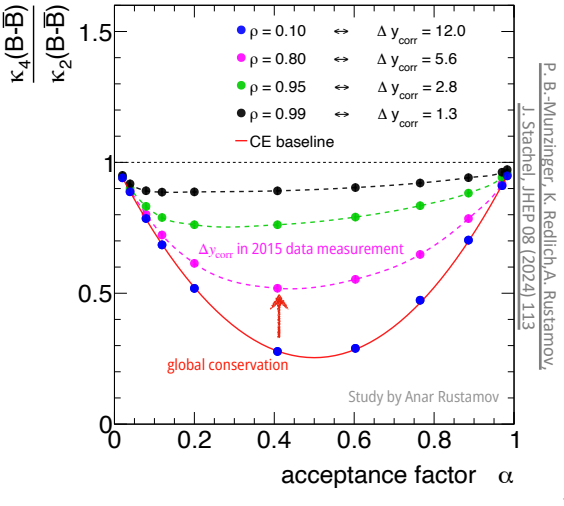
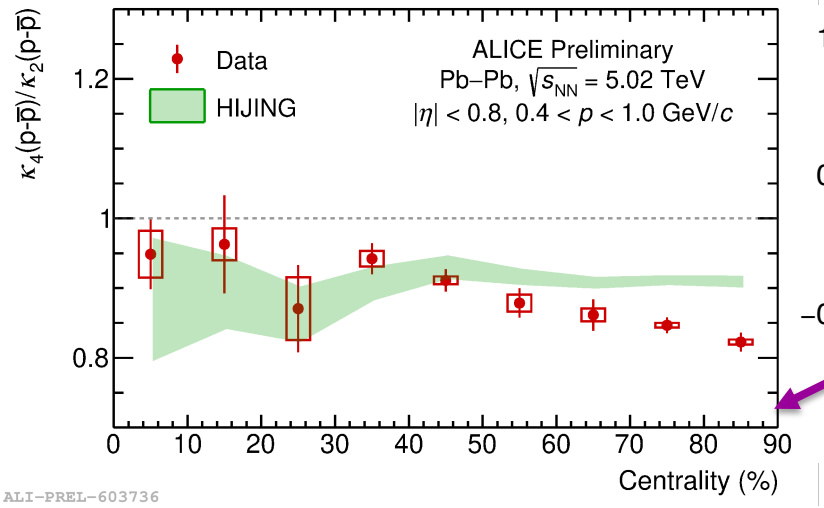
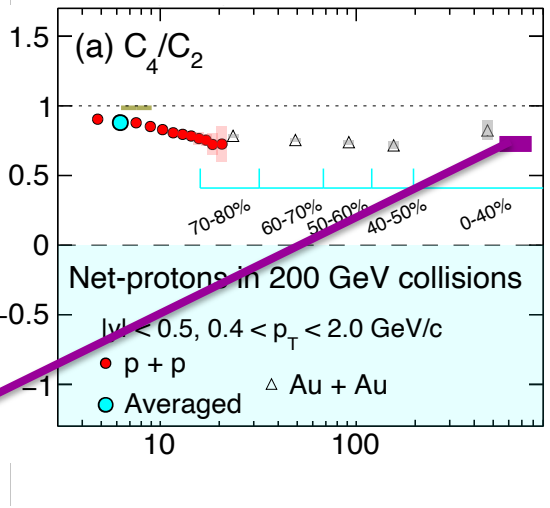
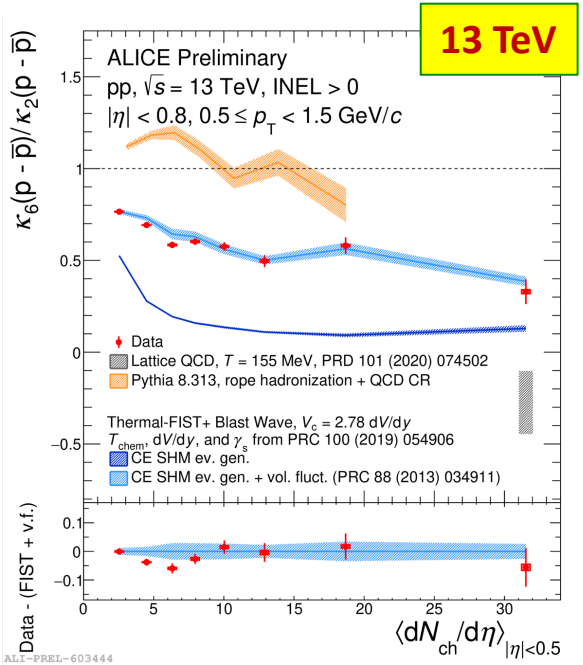


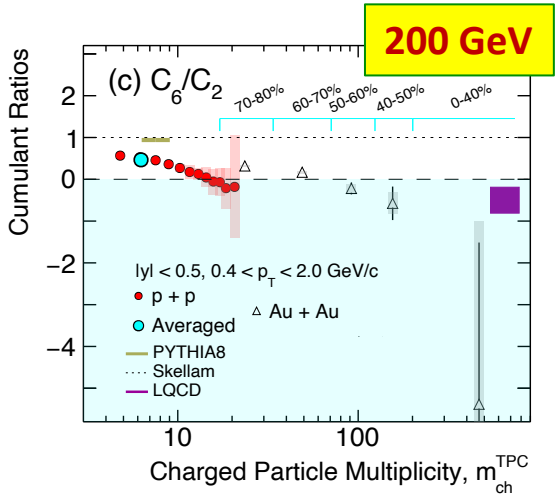




- **TA:** Hint of deviation from Poissonian baseline
- **RI:** CE baseline for the peripheral collisions + larger acceptance and statistics + mixed event correction for the VF at low energies

How about critical fluctuations in pp collisions on the 6th order?





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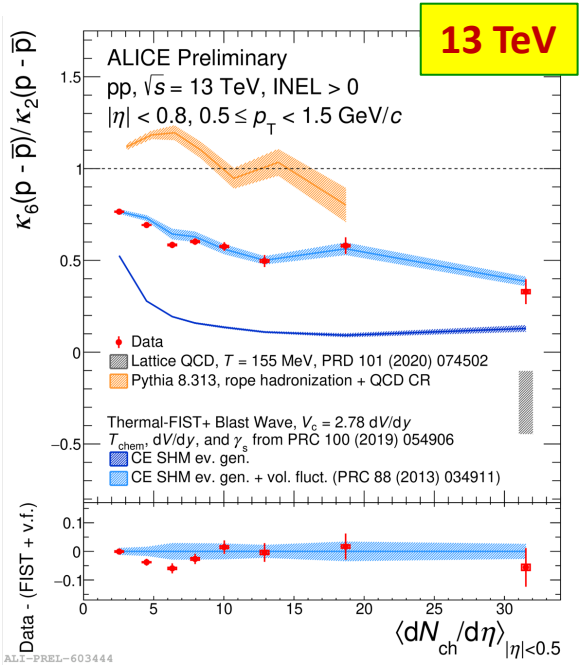
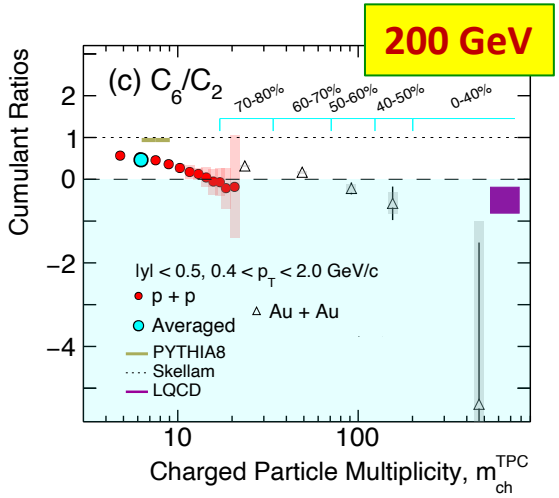
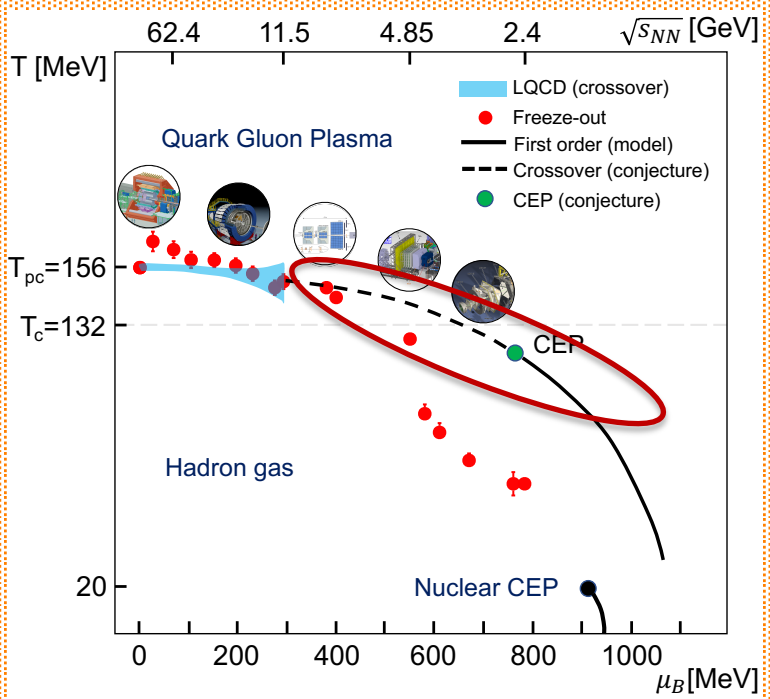


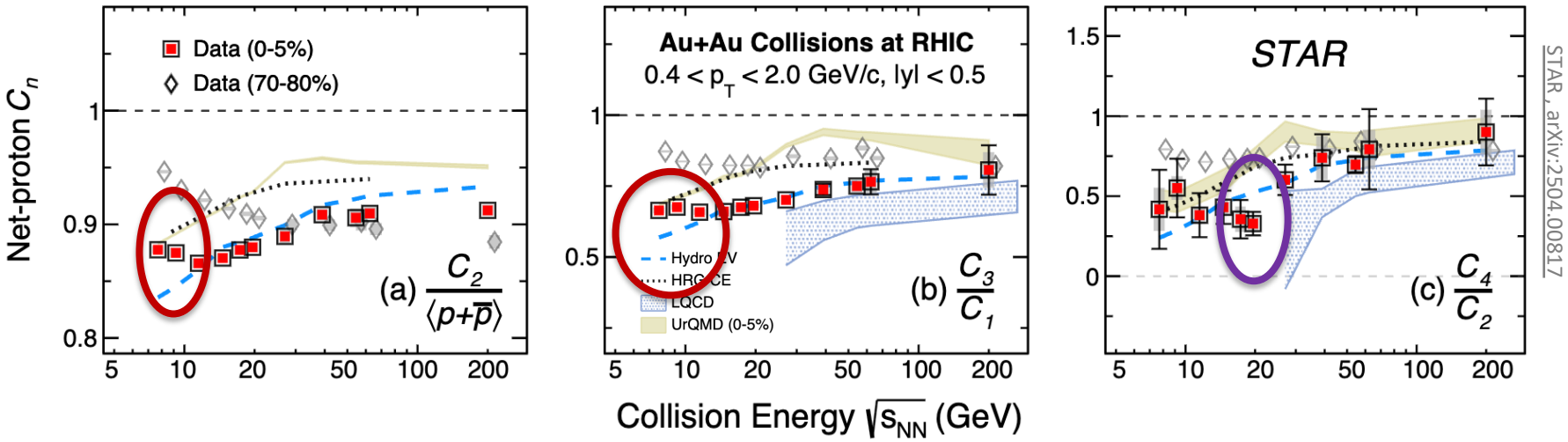


- **TA:** Looks like no sign of criticality
- **RI:** VFC using mixed event & analysis wrt event shape & higher multiplicities & non-critical baseline

Towards CEP


62.4
11.5
4.85
2.4
$\sqrt{s_{NN}}$ [GeV]
T [MeV]
LQCD (crossover)
Freeze-out
First order (model)
Crossover (conjecture)
CEP (conjecture)
Quark Gluon Plasma
T_{pc}=156
T_c=132
CEP
Hadron gas
20
Nuclear CEP
0
200
400
600
800
1000
μ_B[MeV]

Nonmonotonic behavior as a function of energy?



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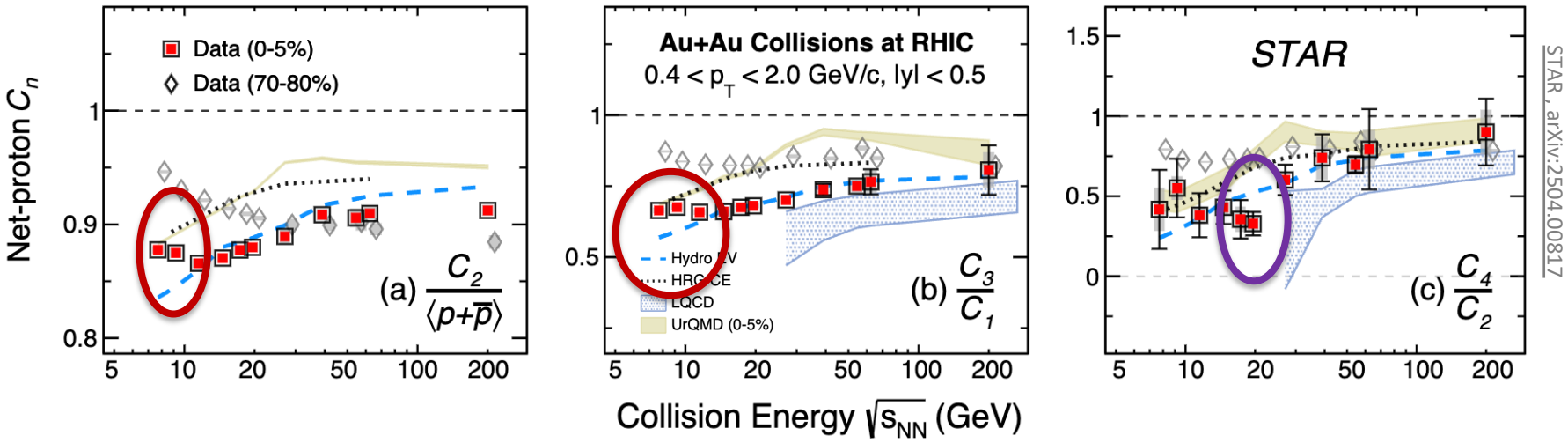
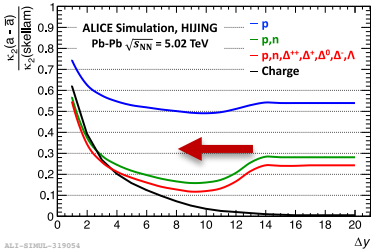


- **TA:** Promising signal at 20 GeV (2-3 σ deviation from "hydro EV" baseline) and below 10 GeV for both C and κ
- **RI**: Modelling of stopped protons and volume fluct. correction with mixed event

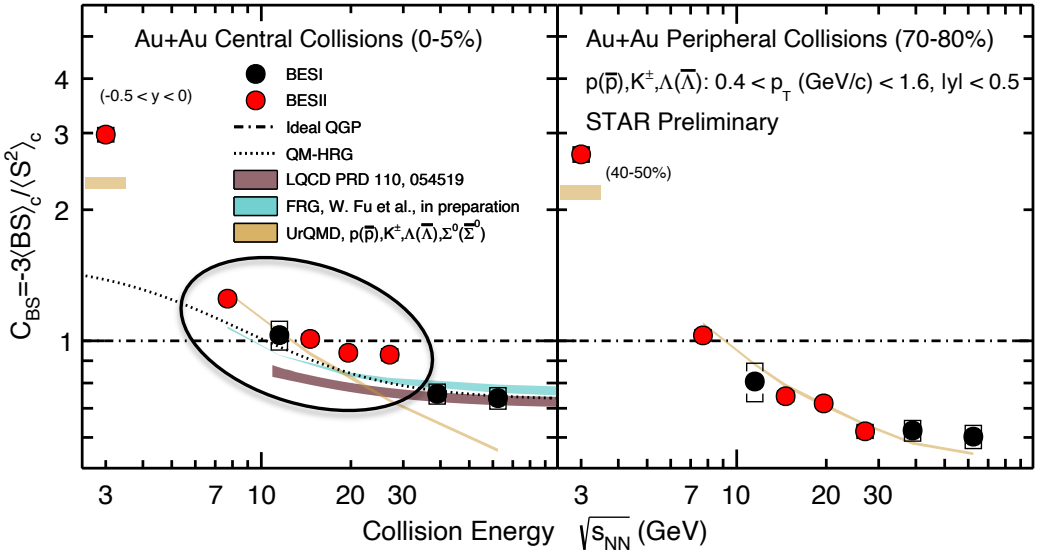


How about cross cumulants as a function of energy?

$$C_{BS} = -3\frac{\langle BS\rangle_c}{\langle S^2\rangle_c} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2}$$

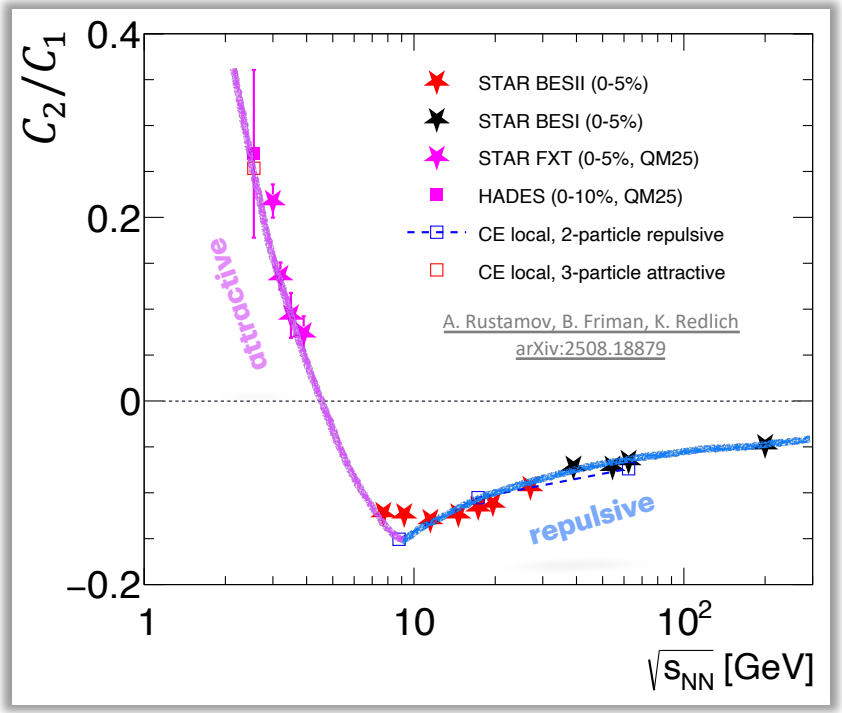
net-B : $\delta p + \delta\Lambda(+\delta\Xi^-)$

net-S : $\delta K^+ - \delta\Lambda(-2\delta\Xi^-)$



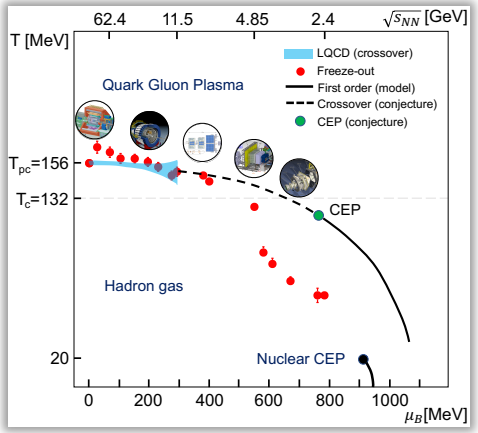
- **TA:** Calculations can describe the data below 30 GeV data
- **RI:** Stopped protons + VFC + non-critical baseline

Critical point or first order phase transition?



- **TA:** There is a striking jump from repulsive to attractive interaction! Sign of a first order phase transition?
- **RI:** Different acceptances and VFC & Modelling of stopped protons & confirmation by another observable; intermittency?

FUTURE?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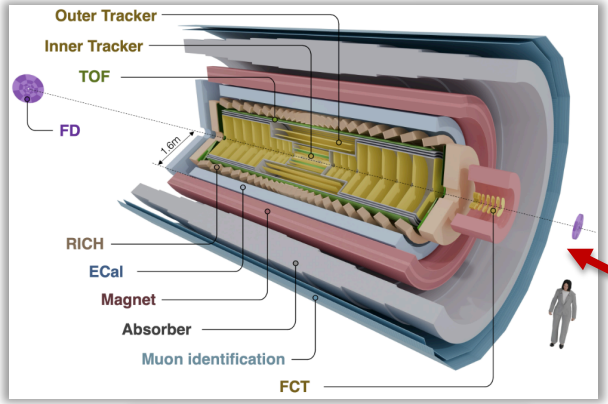
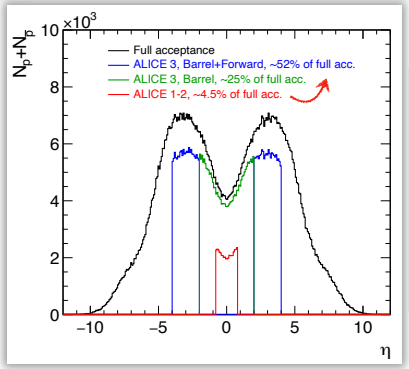
FUTURE?

ALICE 3

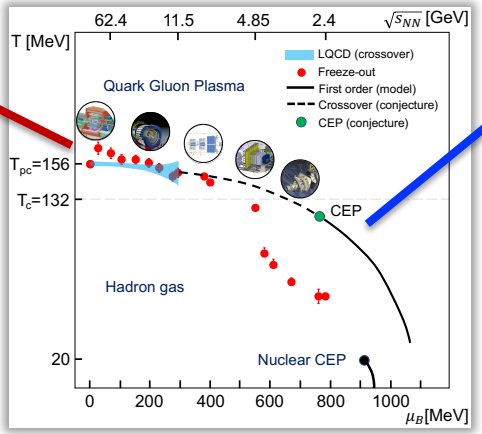




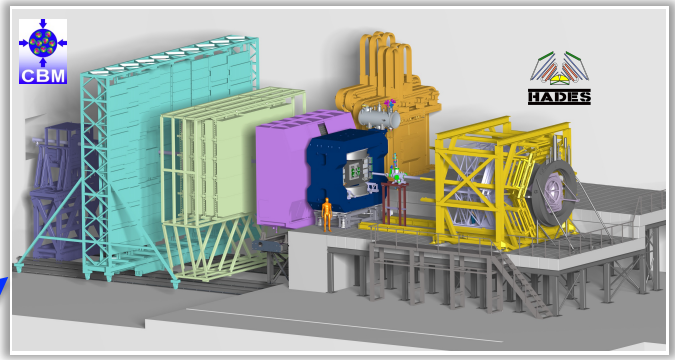
ALICE, CERN-LHCC-2022-009

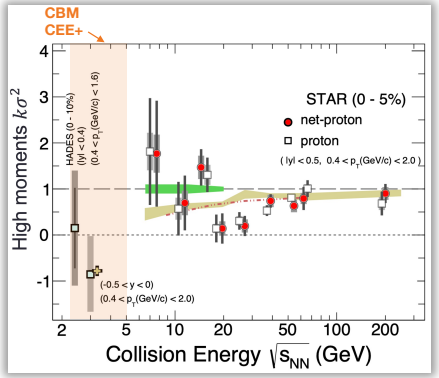
Factor >10 larger acceptance!

Very bright!



CBM





STAR, PRL 128 (2022) 20, 202303
HADES, PRC 102 (2020) 2, 024914

High precision at < 5GeV

Summary

1) **Experiment vs Theory:** “Almost all” experimental challenges are resolved
2) **A-A:** Exciting signals at high (crossover) and low (first order line) energies
3) **pp:** No sign of critical behavior (?)

Summary

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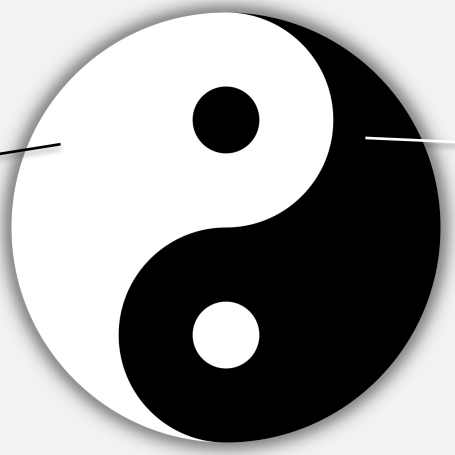
Experiment

High precision in larger acceptance

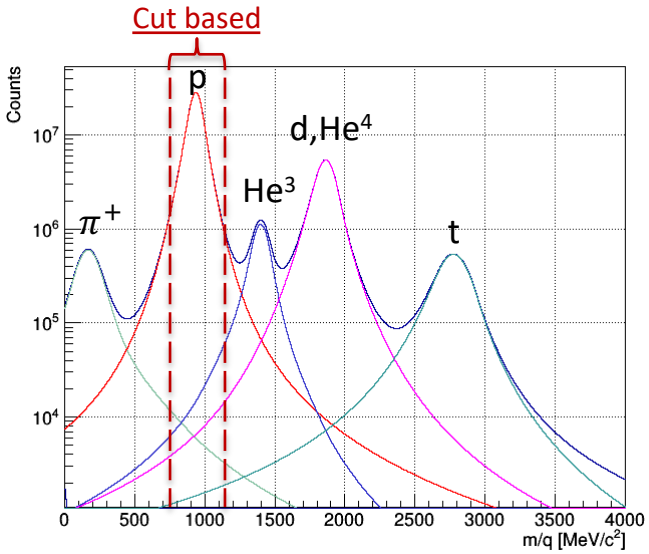


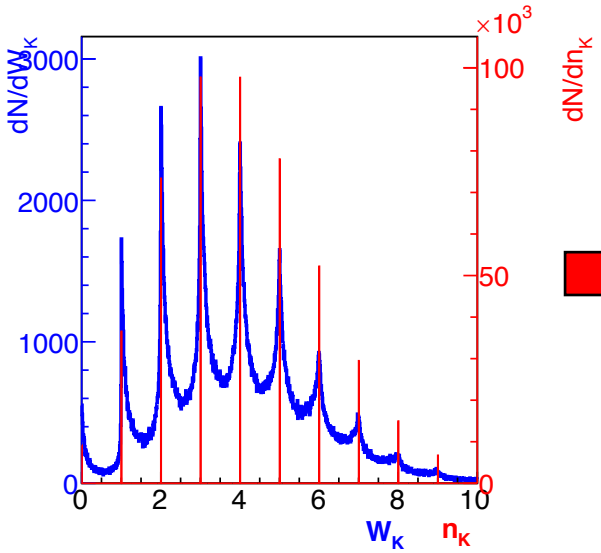
Theory

Isolate critical fluct. from the ”Baseline”

BACKUP

Solution for the misidentification





$$\langle N_j^n \rangle = \mathrm{A}^{-1} \langle W_j^n \rangle$$

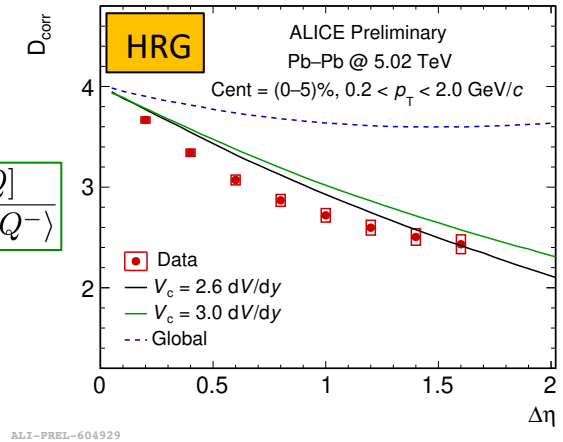
A. Rustamov, Phys.Rev.C 110 (2024) 6, 064910
A. Rustamov, M. Gazdzicki, M. I. Gorenstein, PRC 86, 044906 (2012), PRC 84, 024902 (2011)
A. Rustamov, M. Arslandok, Nucl. Instrum. A946 (2019) 162622}

2nd order net-Q: Subvolume vs Correlation length

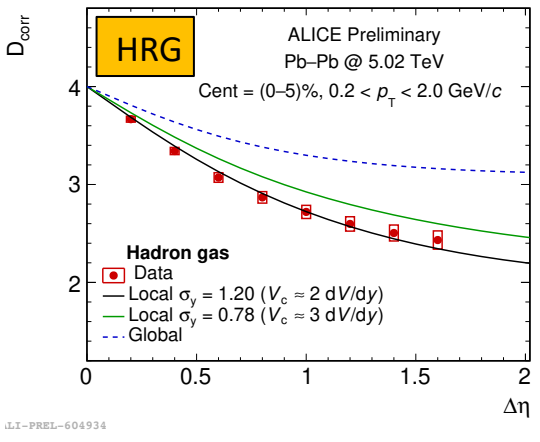
➢ **Question:** What is the right modeling of charge conservation?

$$D = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^- \rangle}$$

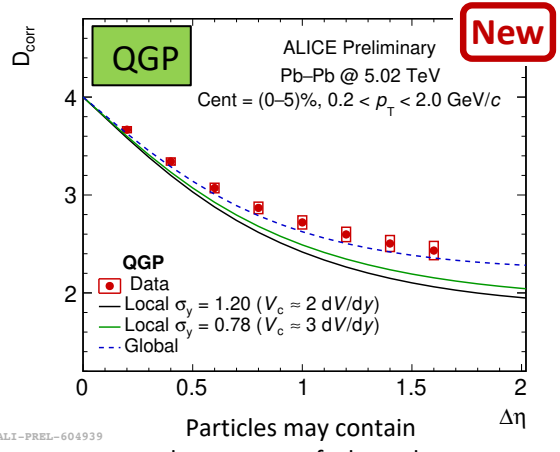
Subvolume approach



Gaussian correlation



Gaussian correlation



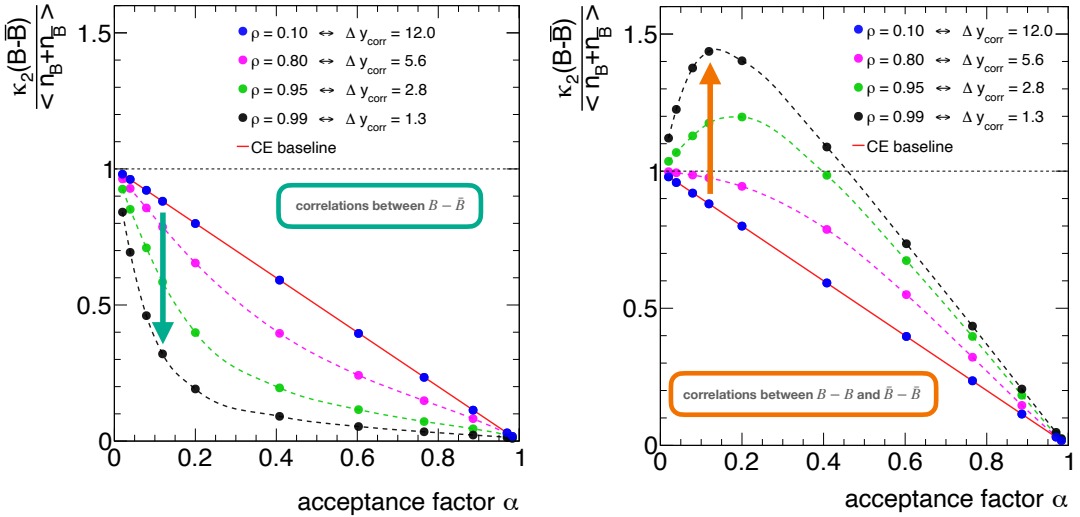
Particles may contain the memory of where the Q-fluctuations freeze out

➢ **TA:** Gaussian correlation is the right answer, possibility to probe the QGP

➢ **RI:** Different conserved charges and higher orders to be studied

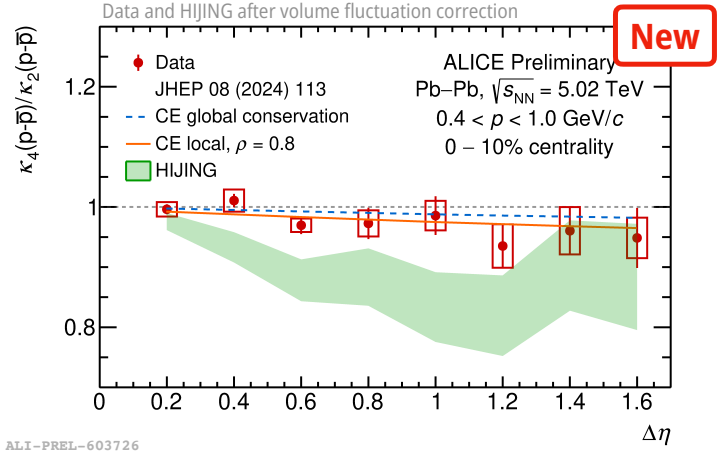
Acceptance & Baseline

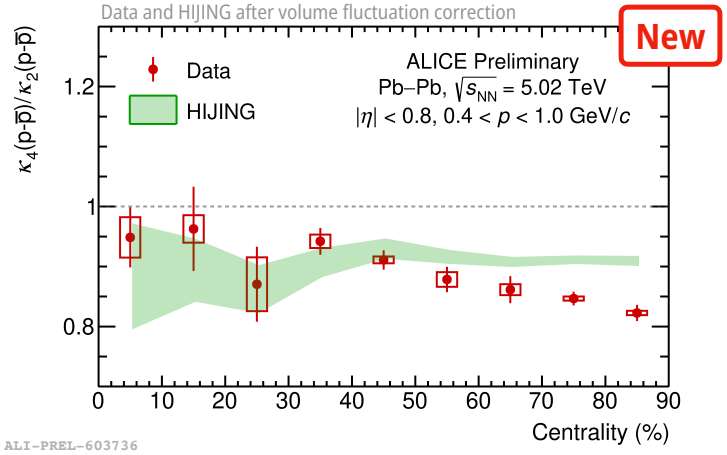
Canonical Ensemble (CE) with the Metropolis Algorithm



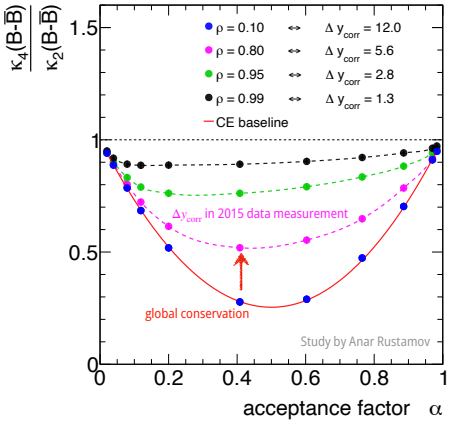
P. B.-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113

4th order cumulants of net-p in Pb–Pb

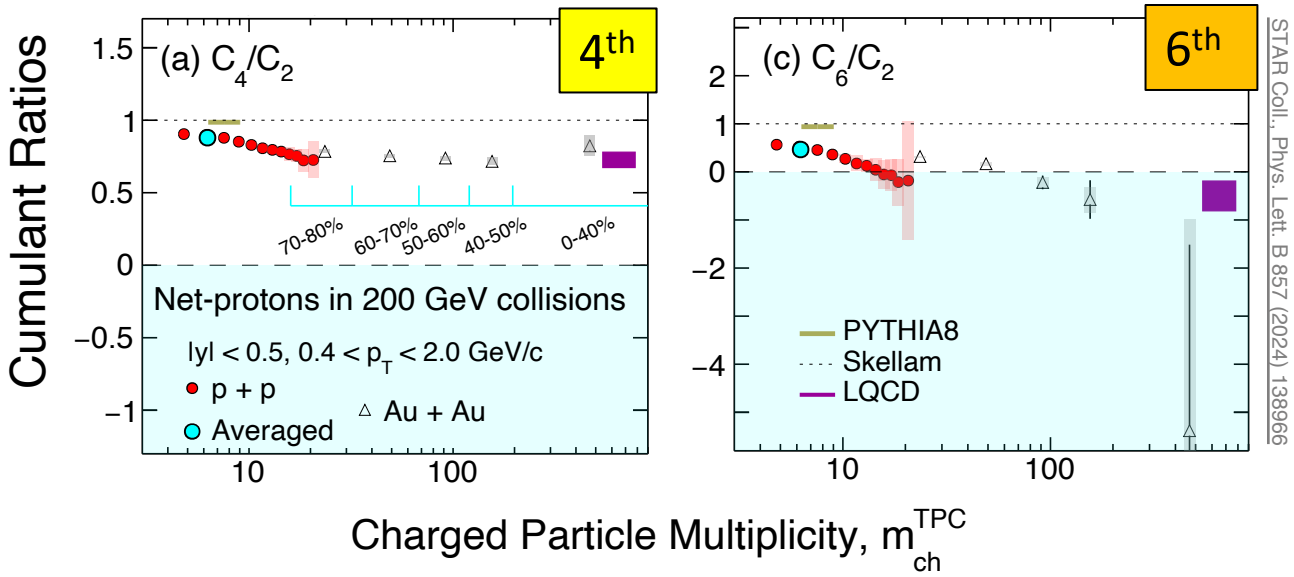




P. B.-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113



Higher order cumulants of net-p in small systems



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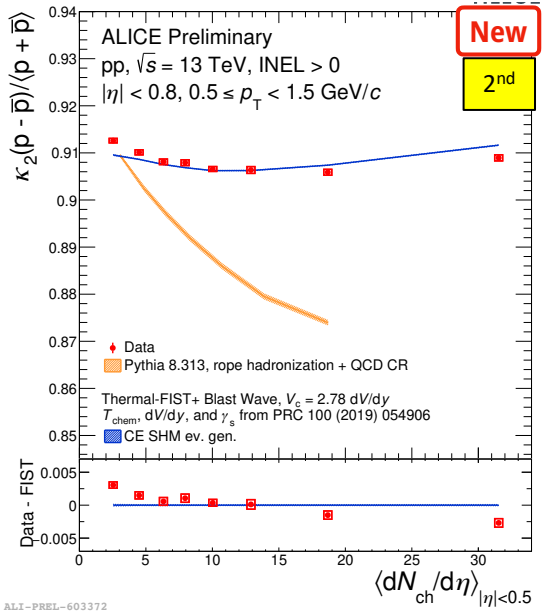
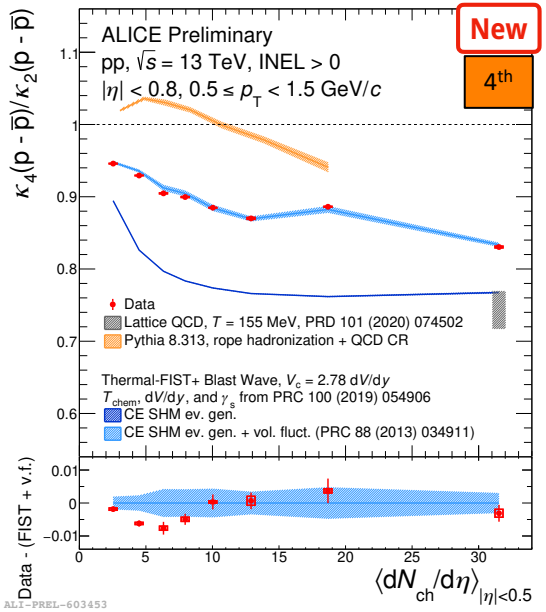
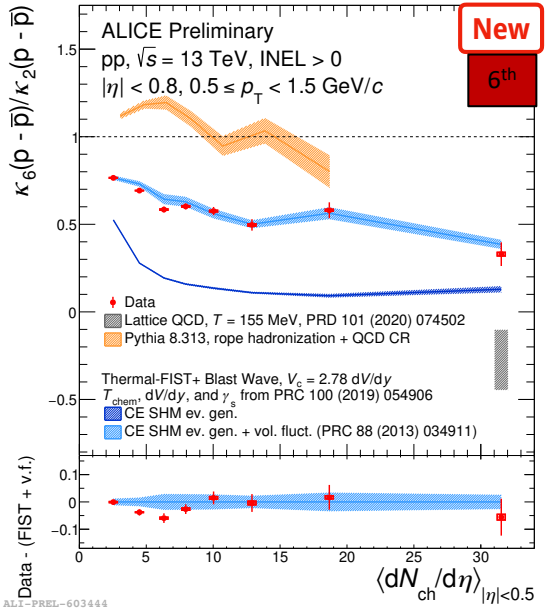


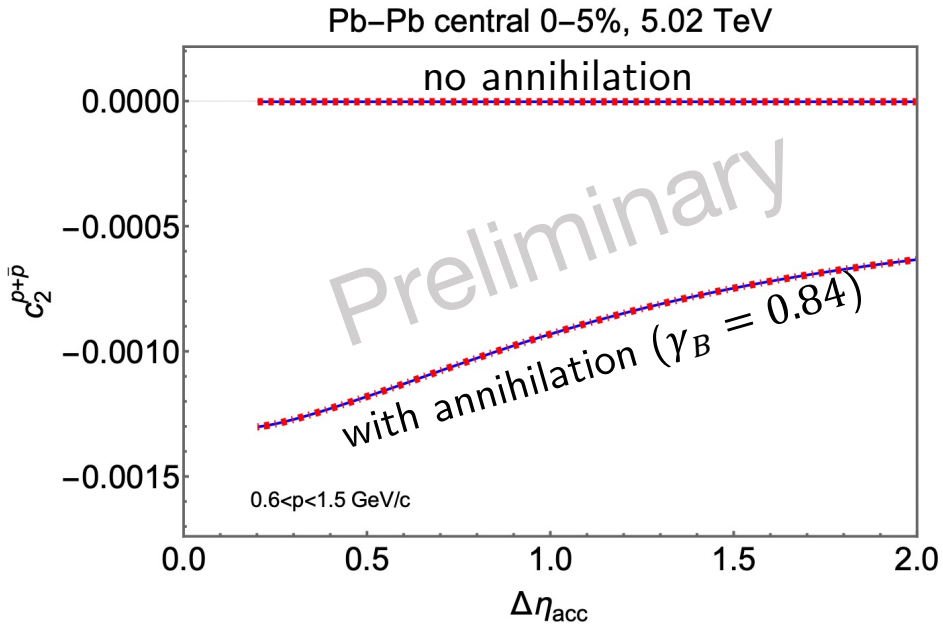




2nd order p: Annihilation

➢ **Question:** Can we test annihilation with cumulants?

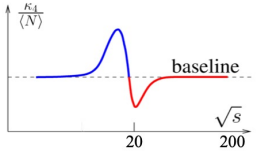
$$\frac{\hat{C}_2^{p+\bar{p}}}{\langle p+\bar{p}\rangle^2} = \hat{c}_2^{p+\bar{p}}$$

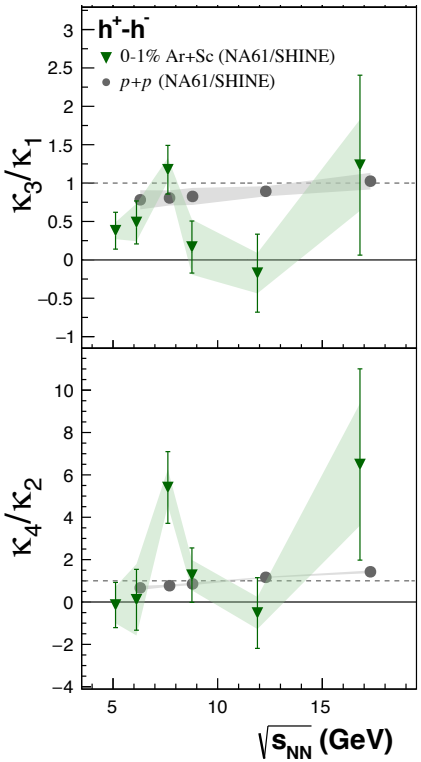


➢ **TA:** Clear signal and independent of baryon conservation → Worth to try

Net-Q: Nonmonotonic behavior

➢ **Question:** Do we see nonmonotonic behavior?



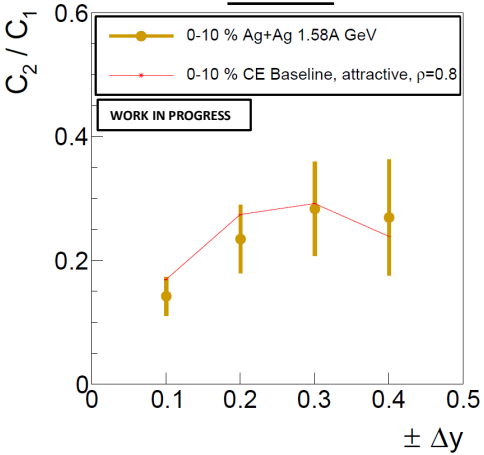


- **TAM:** Hint of non-monotonic behavior for κ_3/κ_1 and κ_4/κ_2
- **RFI:** VFC + higher precision needed

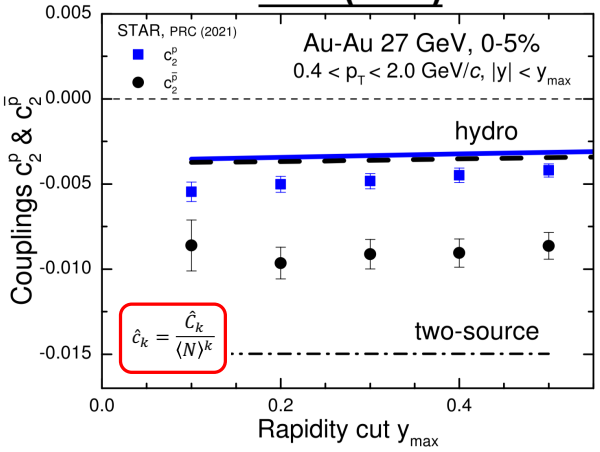
Understanding the stopped protons

➢ **Question:** How important is the acceptance dependence for the modelling the non-critical baseline?

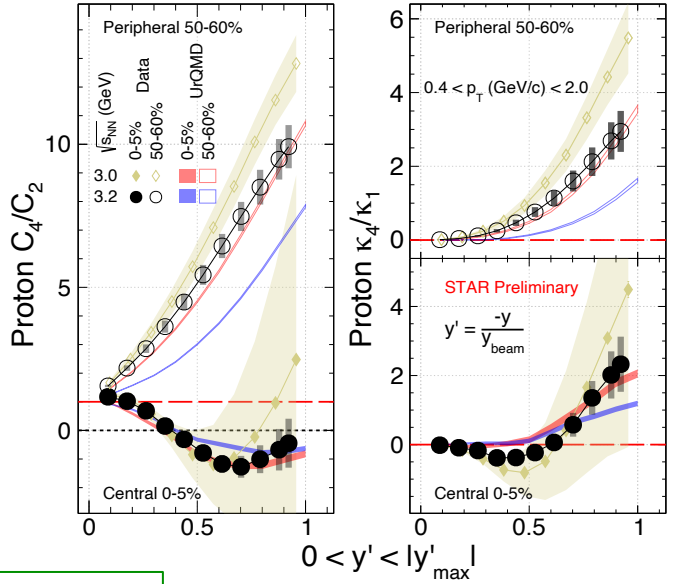
HADES



STAR (BESII)



STAR (Fixed target)

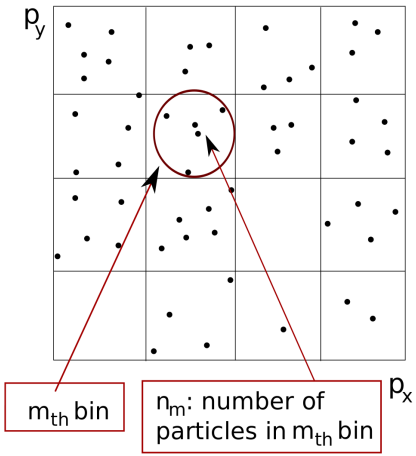


➢ **TA:** Differential measurement of the (factorial) cumulants is very essential

Intermittency as control observable

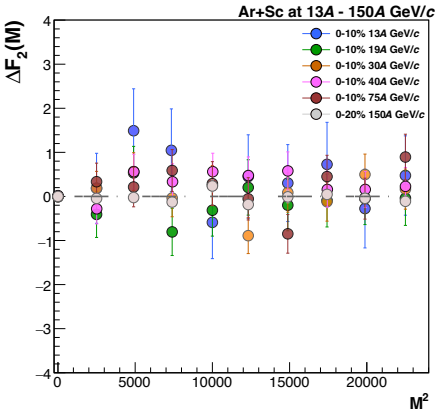
➢ **Question:** Do we confirm what we see in cumulants using intermittency?

$$F_r(M) = \frac{\left\langle \frac{1}{M^2}\sum_{m=1}^{M^2} n_m(n_m-1)\ldots(n_m-r+1) \right\rangle}{\left\langle \frac{1}{M^2}\sum_{m=1}^{M^2} n_m \right\rangle^r}$$



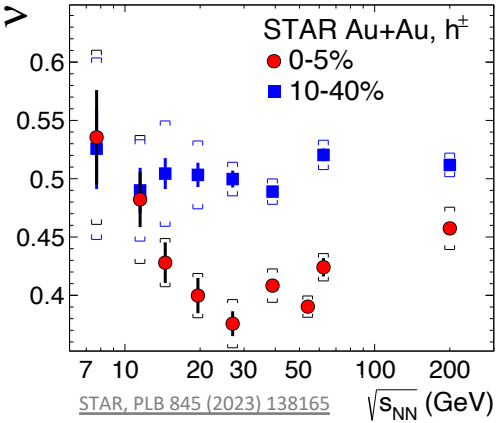
NA61

No sign of criticality



STAR

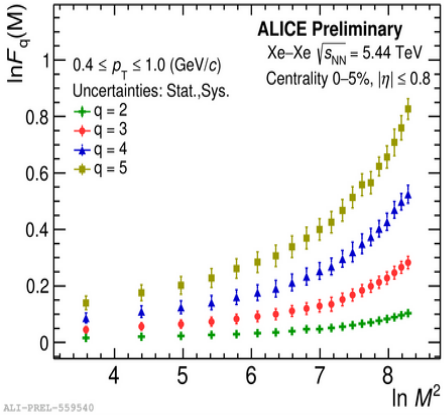
A minima at 27 GeV!



STAR, PLB 845 (2023) 138165

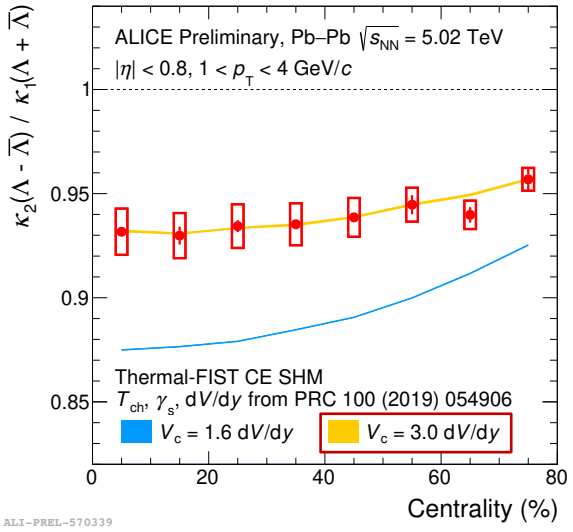
ALICE

Sign of power-law scaling

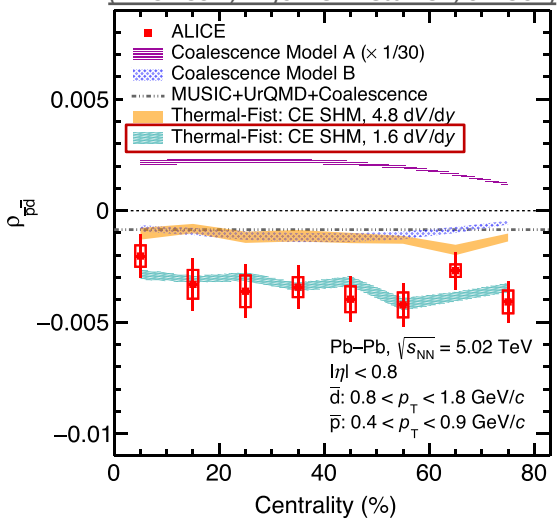


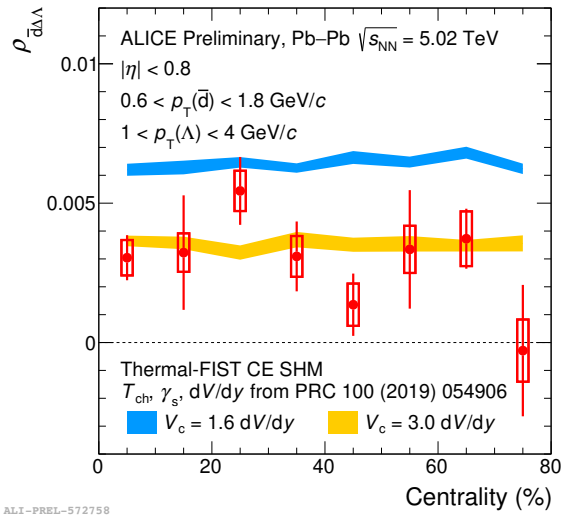
➢ **TAM:** No agreement among experiments

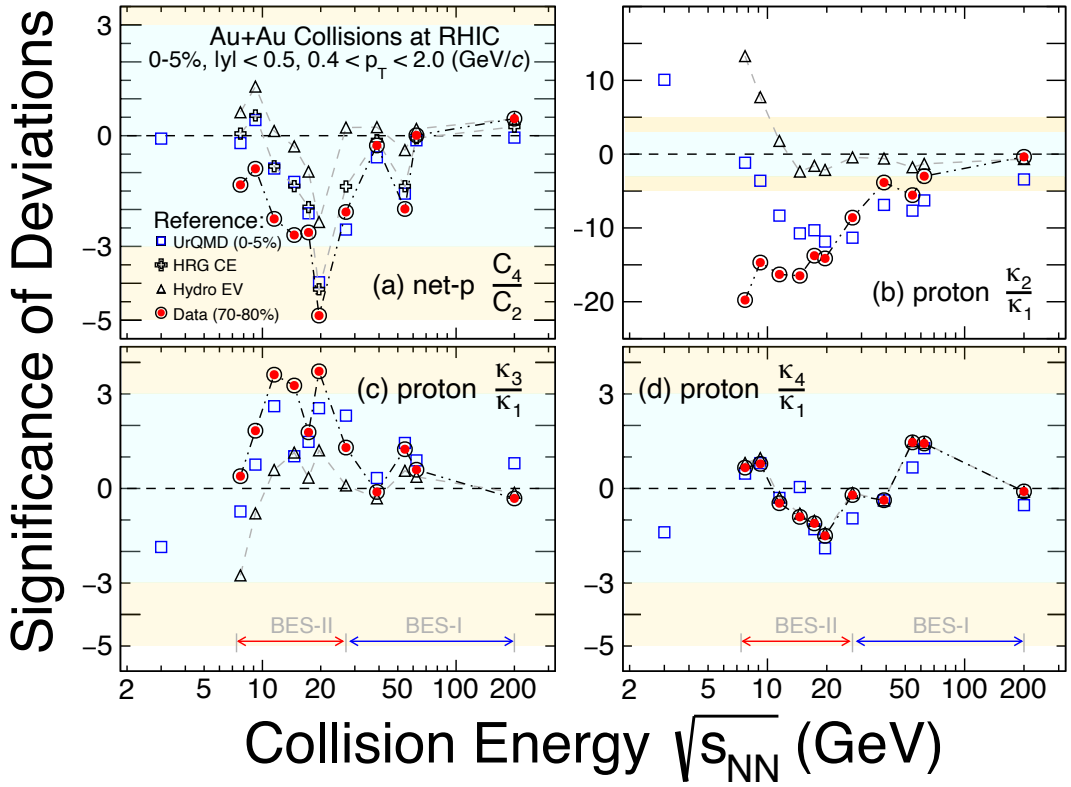
➢ RFI: What about trying different ideas; mesons in coordinate space, in narrow p_T window?
(R. C. Hwa, C. B. Yang, Phys. Rev. C **85**, 044914)



(ALICE Coll., Phys. Rev. Lett. 131, 041901)

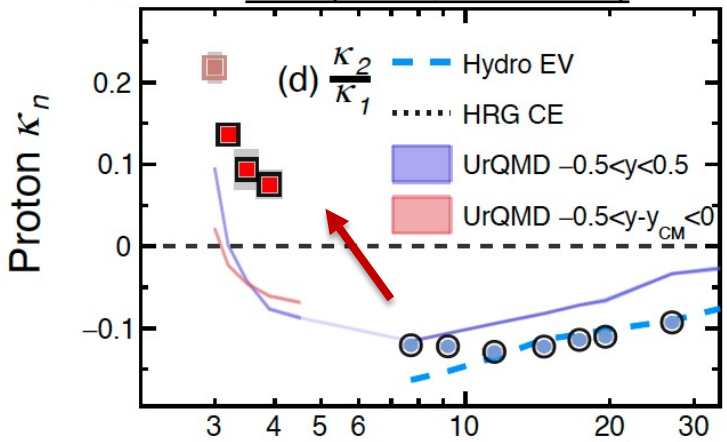





Au+Au Collisions at RHIC
0-5%, |y| < 0.5, 0.4 < p_T < 2.0 (GeV/c)
Reference:
UrQMD (0-5%)
HRG CE
Hydro EV
Data (70-80%)
(a) net-p C4/C2
(b) proton κ2/κ1
(c) proton κ3/κ1
(d) proton κ4/κ1
BES-II
BES-I
Significance of Deviations
Collision Energy √sNN (GeV)

Critical point or first order phase transition?

STAR (Au-Au, 3—7.7 GeV)



HADES (Ag-Ag 2.55 GeV)