Collectivity of rotational motion in ^{220}Rn and ^{226}Ra

A. Dobrowolski[1], K. Mazurek[2], K. Pomorski[3]

1. MCS Univ., PL-20031 Lublin, Poland 2. IFJ PAN, PL-31342 Krakow, Poland, 3. NCNR, PL-00801 Warsaw, Poland

Introduction

Calculations to reconstruct rotational level patterns in the ^{220}Rn and ^{226}Ra nuclei [1] have been performed using a collective quadrupole+octupole approach [2] with microscopic mass tensor and moments of inertia dependent on deformation and pairing degrees of freedom. The main objective is to quantitatively confirm the known experimental observations that the Rn nucleus passes from octupole vibrational to octupole deformed with increasing rotation frequency, while the Ra nucleus is relatively weakly affected by collective rotation, being octupole deformed from the beginning.

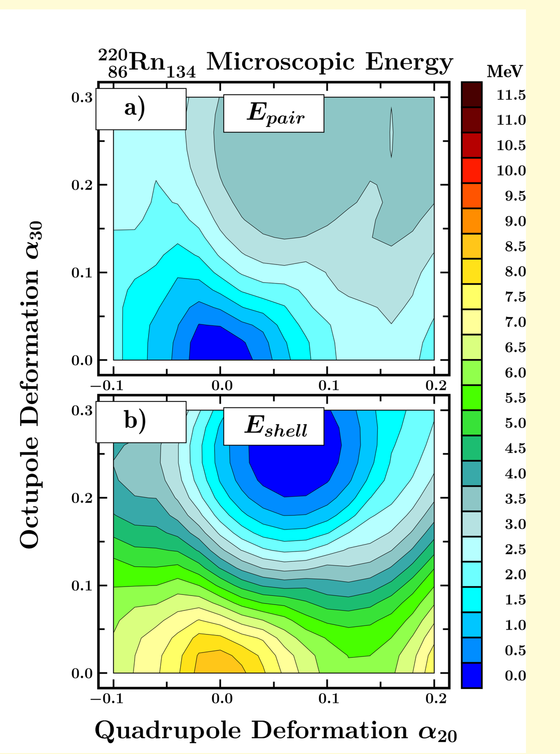
Mean Field Potential Energy

The potential energy and the inertia parameters are defined in the vibrational-rotational, nine-dimensional collective space of the multipole-deformation parameters and Euler angles. The symmetrization procedure applied to the eigenstates of the collective Hamiltonian ensures their uniqueness with respect to the laboratory coordinate system.

$$R(\vartheta,\varphi) = R_0 c(\alpha)\left[1+\sum_{\lambda=1}^{3}\sum_{\mu=-\lambda}^{\lambda}(\alpha^{\star}_{\lambda\mu})Y_{\lambda\mu}(\vartheta,\varphi),\right]$$

Total energy

$$E^{I}_{tot}(\alpha_2,\alpha_3,\Delta_n,\Delta_p;I) = V(\alpha_2,\alpha_3,\Delta_n,\Delta_p) + \frac{I(I+1)\hbar^2}{2J_{\perp}(\alpha_2,\alpha_3,\Delta_n,\Delta_p)}$$



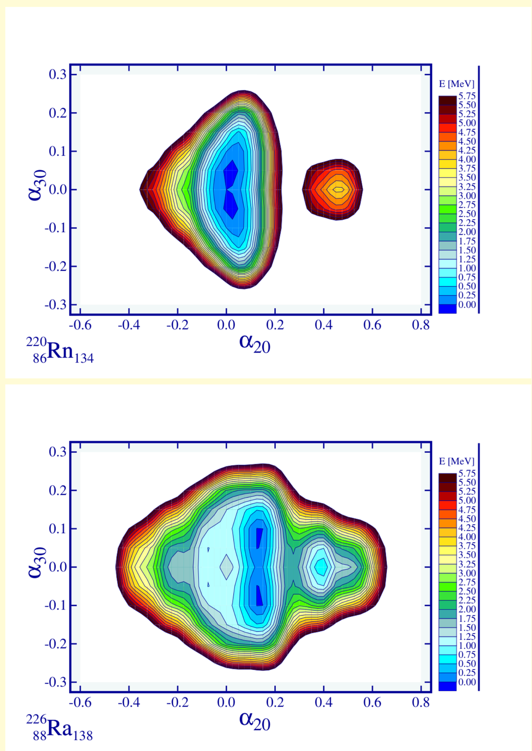


Fig. 1 (left): Microscopic (pairing and shell) energy of the ^{220}Rn in the quadrupole-octupole plane $(\alpha_{20},\alpha_{30})$.
Fig. 2 (right): Maps generated for the quadrupole-octupole $(\alpha_{20},\alpha_{30})$ deformations.

Conclusions

- The presented work is an investigation of the character of the octupole instabilities in two nuclei, ^{220}Rn and ^{226}Ra.
- These nuclei are not classical rotors, the Coriolis and centrifugal effects modify the single-particle structure.
- Our model allow to construct the positive and negative-parity collective states based on $\alpha_{2\mu}$ and $\alpha_{3\nu}$ degrees of freedom.
- A complex transition from vibrational to a stable deformed configuration is achieved, due to: the high precision calculations made on a very dense deformation and pairing Δ grid; the well-tested ingredients of the macroscopic-microscopic model in various fields of nuclear physics.
- The collective model used to determine the rotational states with spin-dependent moments of inertia is capable of reproducing, on average, the energies as well as the electric transitions with order-of-magnitude accuracy.

Realistic Collective Hamiltonian

A realistic collective Hamiltonian contains the potential-energy term obtained through the macroscopic-microscopic Strutinsky-like method with particle-number-projected BCS approach and deformation-dependent mass tensor.

$$\mathcal{H}_{coll}(\alpha_2,\alpha_3,\Omega) = \frac{-\hbar^2}{2}\left\{\frac{1}{\sqrt{|B_2|}}\sum_{\nu\nu'=0}^{2}\frac{\partial}{\partial\alpha_{2\nu}}\sqrt{|B_2|}\left[B_2^{-1}\right]^{\nu\nu'}\frac{\partial}{\partial\alpha_{2\nu'}} + \frac{1}{\sqrt{|B_3|}}\sum_{\mu\mu'=0}^{3}\frac{\partial}{\partial\alpha_{3\mu}}\sqrt{|B_3|}\left[B_3^{-1}\right]^{\mu\mu'}\frac{\partial}{\partial\alpha_{3\mu'}}\right\} + \hat{H}_{rot}(\Omega) + \hat{V}(\alpha_2,\alpha_3), \quad (1)$$

where α_2 and α_3 describe the sub-spaces of the quadrupole and octupole variables with metrics $B_2(\alpha_2)$, $B_3(\alpha_3)$ given in this approach as the quadrupole and octupole microscopic mass tensors, respectively.

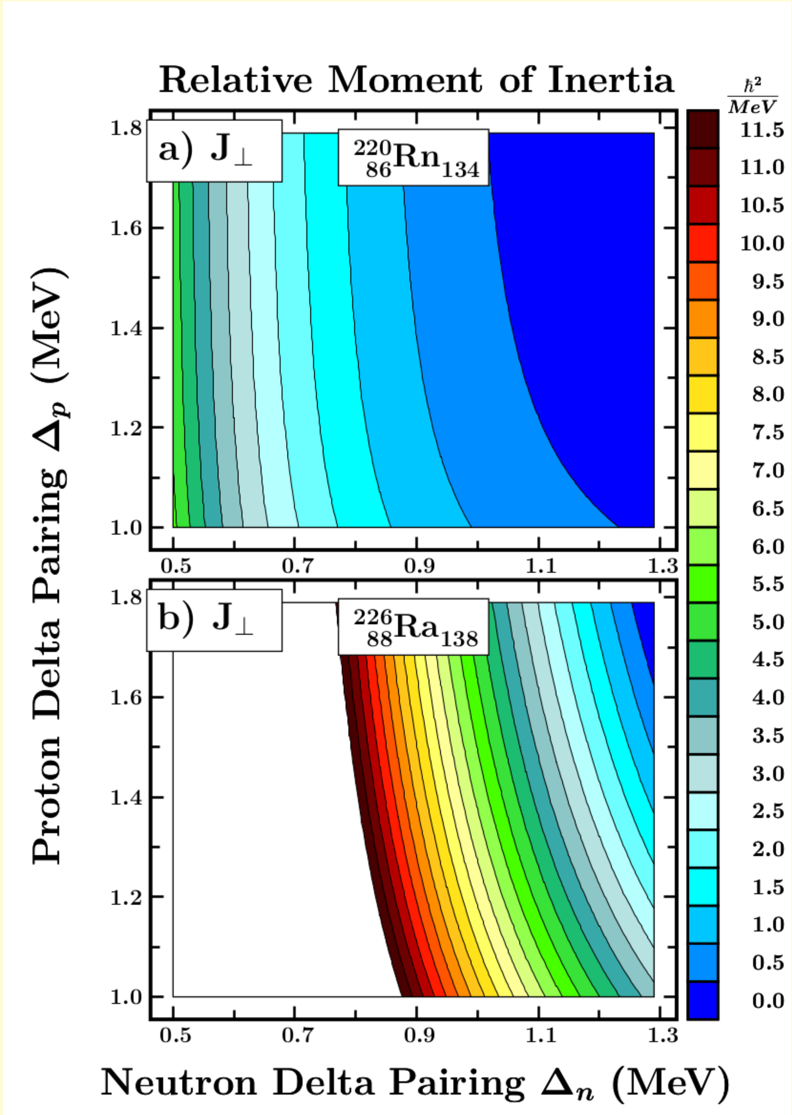


Fig. 3: Moment of inertia evolution for the minimum-energy deformation in ^{220}Rn and ^{226}Ra nuclei for various pairing Δ_n and Δ_p parameters.

Quantities $|B_2| = \det(B_2(\alpha_2))$ and $|B_3| = \det(B_3(\alpha_3))$ stand for square roots of the metric-tensor determinants. These microscopic mass tensors are determined via *cranking* method of Ref. [3]. Its covariant component, $B_{\lambda\nu,\lambda\nu'}$, for $\lambda=2$ or $\lambda=3$ and indices $\nu>0$ is given by the expression

$$B_{\lambda\nu,\lambda\nu'}(\{\alpha_{\lambda\mu}\}) = \sum_{kl}\frac{\langle\phi_k|\frac{\partial\hat{H}_{sp}}{\partial\alpha_{\lambda\nu}}|\phi_l\rangle\langle\phi_l|\frac{\partial\hat{H}_{sp}}{\partial\alpha_{\lambda\nu'}}|\phi_k\rangle}{(E_k+E_l)^3}\times\left(u_k v_l + v_k u_l\right)^2, \quad (2)$$

and the cranking moment of inertia

$$J_{\perp}(\alpha_2,\alpha_3,\Delta_n,\Delta_p) = 2\hbar^2\sum_{kl}\frac{|\langle k|\hat{j}_{\perp}|l\rangle|^2}{E_k+E_l}(u_k v_l - u_l v_k)^2,$$

where the double sum runs over the full set of the BCS quasi-particle (including time-reversed) states, obtained out of eigensolutions of the used mean-field Hamiltonian $\hat{H}_{sp}$ and chosen pairing model. Quantities v_n are the occupation probability amplitudes of the n^{th} quasi-particle state while u_n is given by the normalization relation $u_n^2 = 1 - v_n^2$. In the denominator of Eq. (3), E_k and E_l are the quasi-particle energies of k^{th} and l^{th} states.
The pairing vibration are taken into account by minimizing total energy over (Δ_n,Δ_p) pairing energy gaps.

Rotational properties of states

Even if the mass numbers in ^{220}Rn (a) and ^{226}Ra nuclei differ by six mass units (2 protons and 4 neutrons), the response of the pairing field on external rotation leads to significantly different ways of evolution of potential energy wells toward octupole deformation with increasing spin.

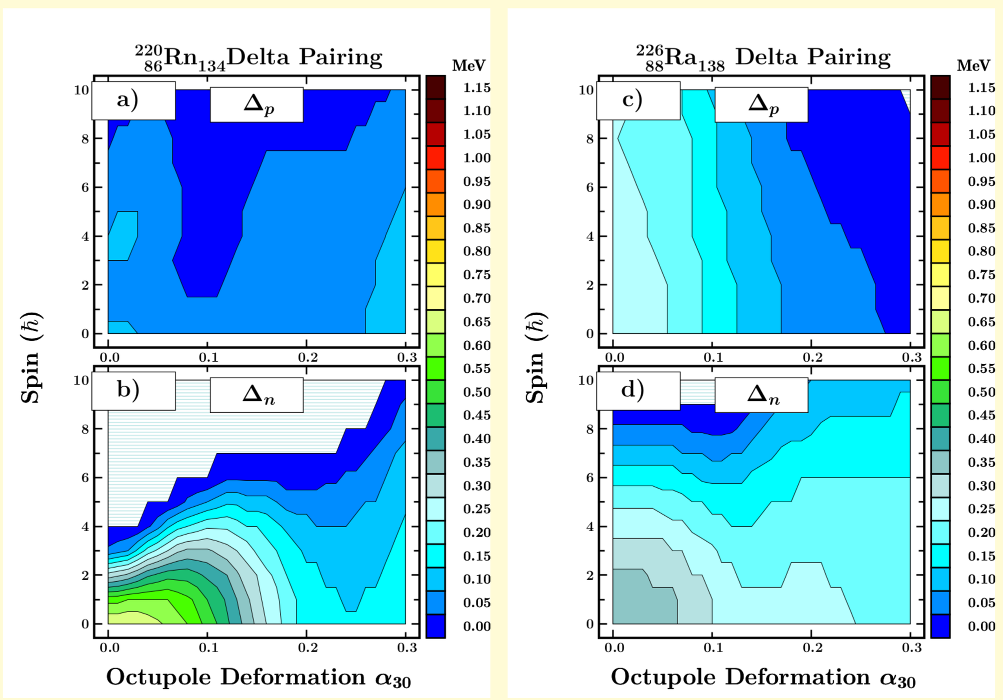


Fig. 5: Proton and neutron pairing Δ values in spin and deformation α_{30} plane compared for ^{220}Rn (a,b) and ^{226}Ra obtained by minimizing the total energy (Eq. 1) over all axial and non-axial quadrupole and octupole deformation parameters.

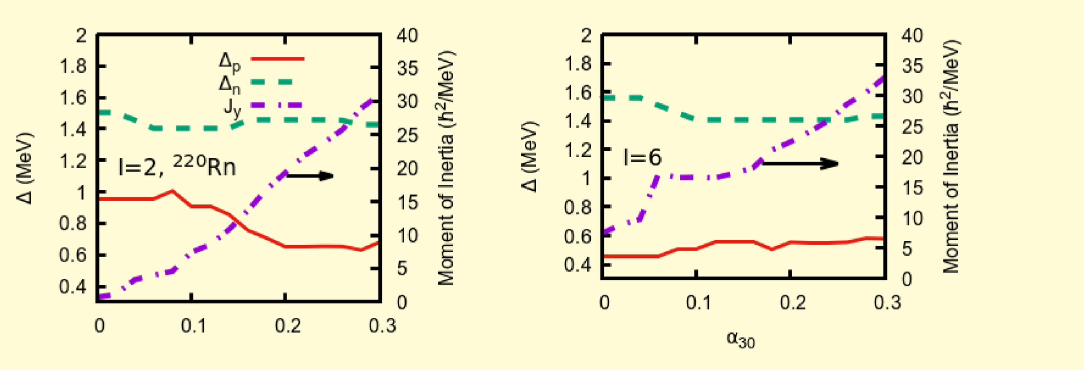


Fig. 6: Pairing Δ values for protons (red) and neutrons (green) obtained after energy minimization for $I = 2\,\hbar$ (a) and $6\,\hbar$ (b) for in ^{220}Rn. The moment of inertia evolution, $(J_{\perp})$, marks magenta line.

Negative-parity bands

The negative-parity states are created in the potential energy well based on the quadrupole-deformed ground-state configuration with quadrupole deformation $\alpha_{20} = 0.25$ and $\alpha_{22} = 0.0$. By consequence, the resulting octupole negative-parity states have a significant static quadrupole deformation producing large $B(E2)$ intra-band transition probabilities.

a) $J(\Delta_{n,p}$;I) exp. b) J_0 $J(\Delta_{n,p}$;I) exp.

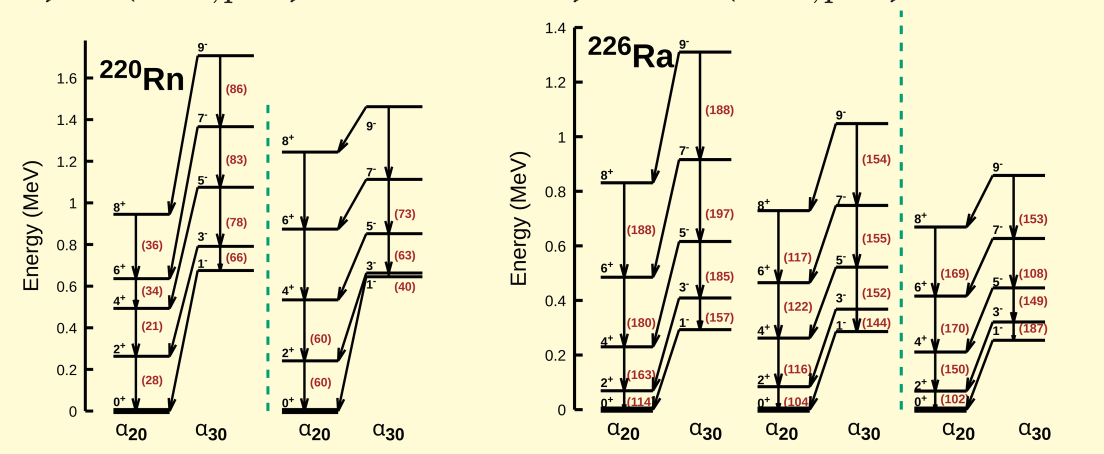


Fig. 4: Comparison of theoretical and experimental [4, 5] bands for ^{220}Rn (a) and ^{226}Ra (b) obtained with the spin-dependent moment of inertia $J(\Delta_{n,p};I)$ and, in case of ^{226}Ra, additionally with constant J_0 value. Transition probability B(E2)(efm) are marked in brackets.

References

[1] A. Dobrowolski, K. Mazurek, and K. Pomorski arXiv:2507.00758; Phys. Rev. C 112, (2025)
[2] A. Dobrowolski, K. Mazurek, and A. Góźdź Phys. Rev. C 97, 024321 (2018); **94**, 054322 (2016).
[3] D. R. Inglis, Phys. Rev. **96**, 1059 (1954); **103**, 1786 (1956); S. T. Balyaev, Nucl. Phys. **24**, 322 (1961).
[4] P.A. Butler, W. Nazarewicz, Rev. Mod. Phys. **68**, 349 (1996); J. Phys. G: Nucl. Part. Phys. **43**, 073002 (2016).
[5] J. F. C. Cocks et al., Phys. Rev. Lett. **78**, 2920 (1997); Nucl. Phys. A **645**, 61 (1999).