Evidence of light multineutron bound systems formation in the ^{159}Tb (p, x)^{157}Dy nuclear reactions

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Outline

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- Theoretical predictions
- Our experimental results to search for a bound dineutron
- Expected properties of a bound dineutron
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- Results of the search for a bound three-neutron
- Expected properties of a bound three-neutron
- Conclusions

Hypothesis on a bound dineutron existence

A second possible reaction is

$$_1H^3 + {}_1H^2 \rightarrow {}_2He^3 + 2\,{}_0n^1,$$

with either the creation of three particles with a continuous neutron energy range up to a maximum or the possible emission of a "di-neutron," $_0n^2$. The existence of the di-neutron has been discussed but no evidence for their existence has been found. If they do exist, then important knowledge concerning the binding energies can be obtained from this reaction.

Published in : M. Y. Colby and R. N. Little, Jr.
Possible Results of a New Reaction.
***Phys. Rev.* 70, 437 (1946)**

Theoretical predictions

SOVIET JOURNAL OF NUCLEAR PHYSICS VOLUME 16, NUMBER 2 FEBRUARY, 1973

TWO INTERACTING PARTICLES IN A POTENTIAL WELL

A. B. MIGDAL

L. D. Landau Theoretical Physics Institute of the USSR Academy of Sciences

Submitted February 16, 1972

Yad. Fiz. 16, 427–434 (August, 1972)

This paper contains the solution of the problem of finding the energy spectrum for two interacting particles in a potential well, when there is a resonance level with energy close to zero. It is shown that under certain circumstances there appears an additional bound state, which does not exist in perturbation theory. Possible applications to nuclear theory are discussed, in particular the possible existence of a dineutron near the surface of some nuclei.

1. FORMULATION OF THE PROBLEM

THE problem solved in the present paper is a special case of the three-body problem, when one of the three particles has a considerably larger mass than the other

tains only one parameter: the matrix element of H′ between the one-particle wave functions of the resonant state.

It is shown that in the case when the potential well is produced by a nucleus there appears a state which has

- **Migdal's paper: "Two interacting particles can form a bound state even though their own interaction is insufficient for this to occur" (~66 keV for the dineutron)**

Our experimental results to search for a bound dineutron

- **Steps of experimental search for a bound dineutron:**
  - **to irradiate some specific nuclei with neutrons or protons;**
  - **to keep the energy of impinging particles below the thresholds of the corresponding (*n*, 2*n*) and (*p*, 3*n*) nuclear reactions;**
  - **to identify nuclear reaction channels via the measurement of the induced activity of the residual nucleus in the output channel;**
  - **to make sure the results are statistically significant;**
  - **to follow a complete analogy with (*n*, *d*) and (*n*, *np*) or similar nuclear reactions kinematics as an example.**

Our experimental results to search for a bound dineutron

- **A bound dineutron was indirectly observed in the following nuclear reactions:**
  - ^{159}Tb(n,^{2}n)^{158}Tb for E_n = 6.85 MeV → σ = 75 ± 30 mb

    https://iopscience.iop.org/article/10.1209/0295-5075/114/42001/meta
  - ^{197}Au(n,^{2}n)^{196}Au for E_n = 6.19 MeV → σ = 0.18 ± 0.06 mb and for E_n = 6.275 MeV → σ = 0.037 ± 0.008 mb

    https://iopscience.iop.org/article/10.1209/0295-5075/131/52001/meta
  - ^{175}Lu(n,^{2}n)^{174g}Lu for E_n = 5.76 MeV → σ = 33.5 ± 6.0 mb

    https://www.sciencedirect.com/science/article/pii/S0370269324006580
  - ^{127}I(n,^{2}n)^{126}I for E_n = 7.2 MeV → σ = 0.25 ± 0.08 mb and for E_n = 8.78 MeV → σ = 0.24 ± 0.07 mb

    Poster (abstract ID161): poster session No.1 on September 23 at 19:40
- **Cross-sections are ranged from dozens μb to dozens mb**

Our experimental results to search for a bound dineutron

epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

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Possible observation of the dineutron in the ^{159}Tb (n, ^{2}n) ^{158g}Tb nuclear reaction

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PACS 21.10.-k – Properties of nuclei; nuclear energy levels
PACS 21.10.Dr – Binding energies and masses
PACS 27.10.+h – $A \leq 5$

Abstract – Experimental observation of the ^{159}Tb(n, ^{2}n) reaction product was performed with application of the activation technique. Tb specimen of natural composition was irradiated with (d, d) neutrons of 5.39 and 7 MeV energies. Instrumental spectra of Tb specimen were measured with HPGe spectrometer. An unexpected 944.2 keV γ-ray peak was observed. Other γ-ray lines due to ^{158g}Tb decay were identified as well. A bonded dineutron emission with the binding energy (B_{dn}) within limitations 1.3 MeV $< B_{dn} <$ 2.8 MeV is evidenced by the energy of incident neutrons and by the ^{158g}Tb presence in the output channel. The specific nuclear properties of ^{158}Tb as deformed nucleus were discussed to explain a bonded dineutron formation based on theoretical assumptions and calculations, using standard parameters for this mass region.

Introduction. – The purpose of this letter is to point out that the dineutron may exist as a bonded particle in the vicinity of the heavy nucleus in the output channel of nuclear reaction. The field of nuclear physics experienced a long history of searching for dineutron bound states. Numerous attempts had been made to look for the dineutron either as a structural component of light nuclei (the two-

the nuclear surface of this heavy nucleus. In this study our attention was focused on Terbium (Tb, $A = 158$–160, ^{158g}Tb: $T_{1/2} = 180$ y) as an element of the rare-earth group based on our previous research [5,6]. A reason to select Tb is its special nuclear properties, such as strong deformation in ground state. Suggestions for further nuclear physics research of the rare earths may include such

Our experimental results to search for a bound dineutron

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Statistically significant observation of and cross-sections for a new nuclear reaction channel on ^{197}Au with bound dineutron escape

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PACS 21.45.Bc – Two-nucleon system
PACS 25.90.+k – Other topics in nuclear reactions: specific reactions
PACS 27.10.+h – $A \leq 5$

Abstract – A new nuclear reaction channel on ^{197}Au with the neutron as a projectile and a bound dineutron (^2n) as an ejectile in the output channel is considered based on available experimental observations. The dineutron is assumed to be formed as a particle satellite, separated from the volume but not from the potential well of the ^{196}Au nucleus. The dineutron was identified by statistically significant radioactivity detection due to decay of ^{196g}Au nuclei. Cross-sections for the ^{197}Au $(n, ^2n)^{196g}$Au reaction are determined as $180 \pm 60\,\mu$b and $37 \pm 8\,\mu$b for [6.09–6.39] and [6.175–6.455] MeV energy ranges, correspondingly. Possible outcomes of dineutron detection near the surface of deformed nuclei are also raised and discussed.

Introduction. – The purpose of this letter is the discussion of the dineutron as a bound particle, or two-nucleon nucleus, consisting of the two neutrons only without any nucleus charge and formed in the outgoing channel of neutron-induced nuclear reaction on ^{197}Au near the ^{196}Au nucleus surface. Such a configuration is different from the classical description of nuclear reactions at low energies and reported in details in [1]. The dineutron of some nuclear reactions. First observation of this phenomenon was described in [3], but statistical significance of detection of ^{158g}Tb-induced activity as a product of the ^{159}Tb $(n, ^2n)$ nuclear reaction was not good enough due to the 180 years half-life of the ^{158g}Tb residual nucleus.

Thus, we decided to search for another nucleus with a reasonable half-life to make sure the statistical significance of its decay detection will meet the 5σ criteria. Another re-

Our experimental results to search for a bound dineutron

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Letter

Formation of bound dineutrons in the ^{175}Lu (n, ^{2}n) $^{174\ g}$Lu nuclear reaction and its cross-section

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Keywords:
Bound dineutron
Neutron-induced nuclear reaction
Lutetium-175
Reaction cross-section
Binding energy interval estimate

A B S T R A C T

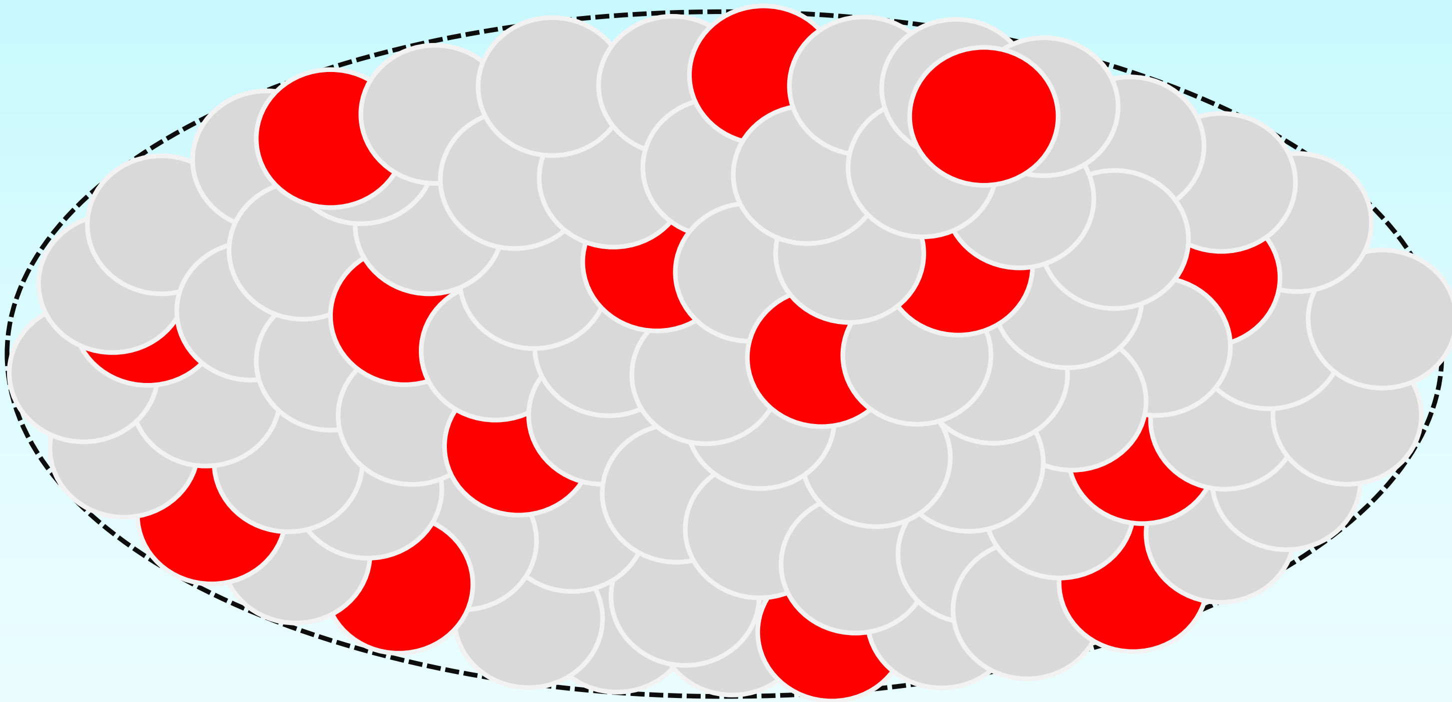
The dineutron as a bound chargeless nucleus of two identical nucleons has been attracting attention for many decades. In addition to the study of the formation of a bound dineutron in the outgoing channel of specific nuclear reactions, cross-section estimates have been gaining interest as research targets. We used a traditional neutron activation technique to irradiate Lu samples followed by measurements of the induced activity with HPGe spectrometer in order to detect gamma-peaks of the ^{175}Lu (n, 2n)$^{174\ g}$Lu reaction product when this reaction channel is not open for incident neutron energies. Based on two measurements of the instrumental gamma-ray spectra of Lu samples we reliably identified the presence of the $^{174\ g}$Lu isotope in the outgoing channel of the ^{175}Lu (n, ^{2}n)$^{174\ g}$Lu nuclear reaction. Also, the cross-section for this nuclear reaction was obtained and to be equal $33.5^{+7.0}_{-6.7}$ mb. For the first time, the dineutron as a nuclearreaction product was indirectly detected via statistically significant observation of the induced activity of $^{174\ g}$Lu with a corresponding cross-section estimate.

Expected properties of a bound dineutron

- The dineutron is a bound nucleus in the singlet state (0^+);
- The binding energy: 1.6 MeV < B_{dn} < 2.8 MeV ($\lesssim$ 2.5 MeV);
- The decay mode: $^2n \rightarrow d + e^- + \tilde{\nu}_e$
- The superallowed transition for the decay into the deuteron in the triplet state;
- The end-point energy of β^--spectrum: $E_{max\text{-}dn} \cong 0.56$ MeV;
- The half-life estimates are for:
  - the G-T transition with the deuteron in the triplet state: 1,215 s (twice as many for a single neutron);
  - the Fermi transition with the deuteron in the singlet state: 5,877 s;
- The radius estimate: $r_{dn} \cong 4.1$ fm (a "friable" structure!).

Expected properties of a bound dineutron

- **The nuclear reaction mechanism is assumed as follows:**
  - **a pre-equilibrium escape of the two paired-up neutrons to form the dineutron**



Results of a literature search for a bound three-neutron

- **Paper of S.M.Qaim, M.Ejaz, *J. Inorg. Nucl. Chem.*, 30 (1968), pp.2577-2581:**

  - **The neutron-activation technique, the nuclear reaction: $^{127}I(n, 3n)^{125}I$, the D-T neutron energy ~14.7 MeV;**
  - **The cross-section: σ = 40 ± 15 mb (E_{th} = 16.42 MeV);**
  - **The most likely our explanation: a new nuclear reaction type and channel is open:**

$$^{127}I(n, {}^{2}n+n)\ ^{125}I \qquad \text{OR} \qquad ^{127}I(n, {}^{3}n)\ ^{125}I$$

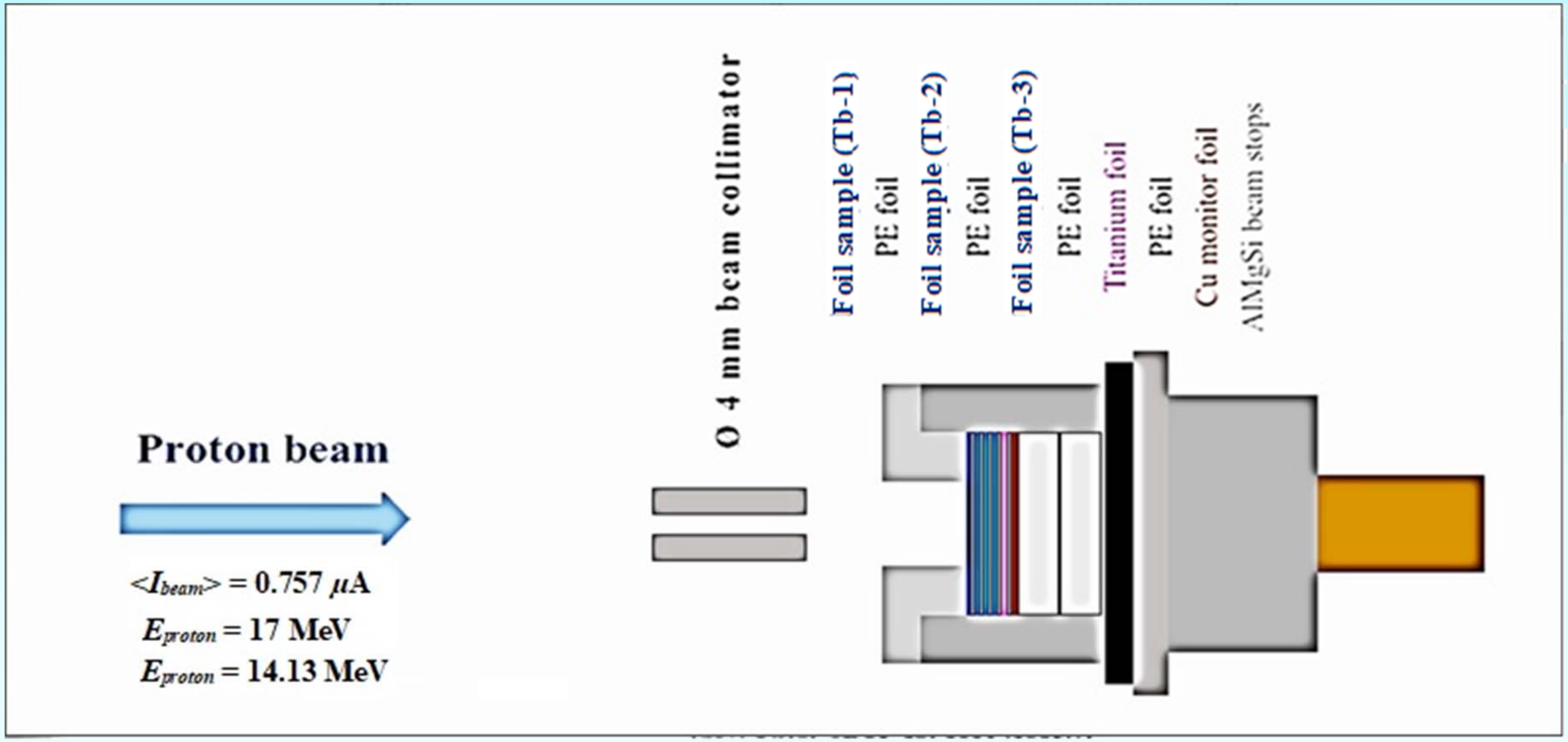
Results of a literature search for a bound three-neutron

- **Paper of F.Tárkányi, A.Hermanne, F.Ditrói, S.Takács, A.V.Ignatyuk, *Appl. Radiat. Isot.*, 127 (2017), pp.7-15:**

  - **The stack-foil technique, the nuclear reaction: ^{159}Tb (p, 3n) ^{157}Dy, the proton energy range in stack: 5.5 - 34 MeV for incident proton beam energy 34 MeV;**
  - **The cross-section: σ = 90 ± 10 μb for E_p=14.86 ± 0.85 MeV (<u>E_{th} = 17.14 MeV</u>);**
  - **The most likely our explanation: a new nuclear reaction type and channel is open:**

^{159}Tb (p, ^2n+n) ^{157}Dy OR ^{159}Tb (p, ^3n) ^{157}Dy

Experiments to search for a bound three-neutron



- Layout of the experiment to search for multineutron nuclei in a proton induced nuclear reactions on ^{159}Tb using the stack foil technique

Results of the search for a bound three-neutron

- **Nuclear reaction cross section:** $$\sigma_{reaction} = \frac{A_{EOB}}{\langle\varphi_{proton}\rangle * N_{atoms} * C_{sat}}$$

from the measured activity of an irradiated (activated) sample:

$$A_{EOB} = A_{sat} * \left(1 - e^{-ln2 * \frac{(t_{STOP;irr} - t_{START;irr})}{T_{1/2}}}\right) = A_{sat} * C_{sat}$$

where the saturation factor is $$C_{sat} = 1 - e^{-ln2 * \frac{(t_{STOP;irr} - t_{START;irr})}{T_{1/2}}}$$

- **The time integrated proton flux:** $$\langle\varphi_{proton}\rangle = \frac{A_{EOB;mon}}{\sigma_{mon\,reaction} * N_{atoms;mon} * C_{sat;mon}}$$

where saturation factor for the monitor nuclear reaction is

$$C_{sat;mon} = 1 - e^{-ln2 * \frac{(t_{STOP;irr} - t_{START;irr})}{T_{1/2;mon}}}$$

Results of the search for a bound three-neutron

where:

$<\varphi_{proton}>$ - time averaged proton flux (protons/cm^2/s);

N_{target} - number of the target atoms in the sample to be irradiated;

$t_{START;irr}$ - START date of the irradiation;

$t_{STOP;irr}$ - STOP date of the irradiation;

$T_{1/2}$ – the half-life of the product radioisotope;

A_{EOB} – the end of bombardment activity;

A_{sat} – the saturation activity;

$A_{EOB;mon}$ - the end of bombardment activity of the radioisotope produced via the proton induced monitor reaction;

$\sigma_{mon\ reaction}$ – the cross-section of the monitor nuclear reaction at the E_{mean} proton energy leading to the activation of the monitor foil in the stack;

E_{mean} – is calculated by the STACK-TS energy loss calculation code;

$N_{atoms;mon}$ – the number of the target atoms of the monitor nuclear reaction induced in the monitor foil by bombarding protons;

$T_{1/2;mon}$ – the half-life of the radioisotope produced via the proton induced monitor reaction.

Results of the search for a bound three-neutron

- The foils Nos. 1 and 3 showed no presence of ^{157}Dy with $T_{1/2}$ = 8.14 h
- The foil No.2 demonstrated the presence of 326.3 keV gamma peak with its area 2,154 ± 274 counts due to the decay of ^{157}Dy, counted during 56,950 s live time after proton irradiation with E_p = 16.23 ± 0.23 MeV
- The cross-section estimate is: 654 ± 150 nb
- Similar experiment with E_p = 13.87 ± 0.26 MeV (*below the upper limit of the binding energy of the dineutron: 3.01 MeV*) also resulted in 326.3 keV gamma-peak with its area 188 ± 95 counts due to the decay of ^{157}Dy in the instrumental spectrum counted during 3,838 s live time
- The cross-section estimate is: 581 ± 488 nb

Expected properties of a bound three-neutron

- **The three-neutron is a bound nucleus with its spin: $s = \frac{1}{2}^{+}$;**
- **The binding energy interval estimate : 3.27 MeV < B_{tn} < 9.77 MeV;**
- **The decay mode:** $^{3}n \rightarrow {}^{3}H + e^{-} + \tilde{\nu}_{e}$**;**
- **The allowed transition;**
- **The end-point energy of β^{-}-spectrum: $E_{max\text{-}tn}$ < 9.26 MeV;**
- **The half-life lower estimate is about: 0.0375 s;**
- **The radius estimate: 2.06 < r_{tn} < 3.55 fm (quite compact structure!)**

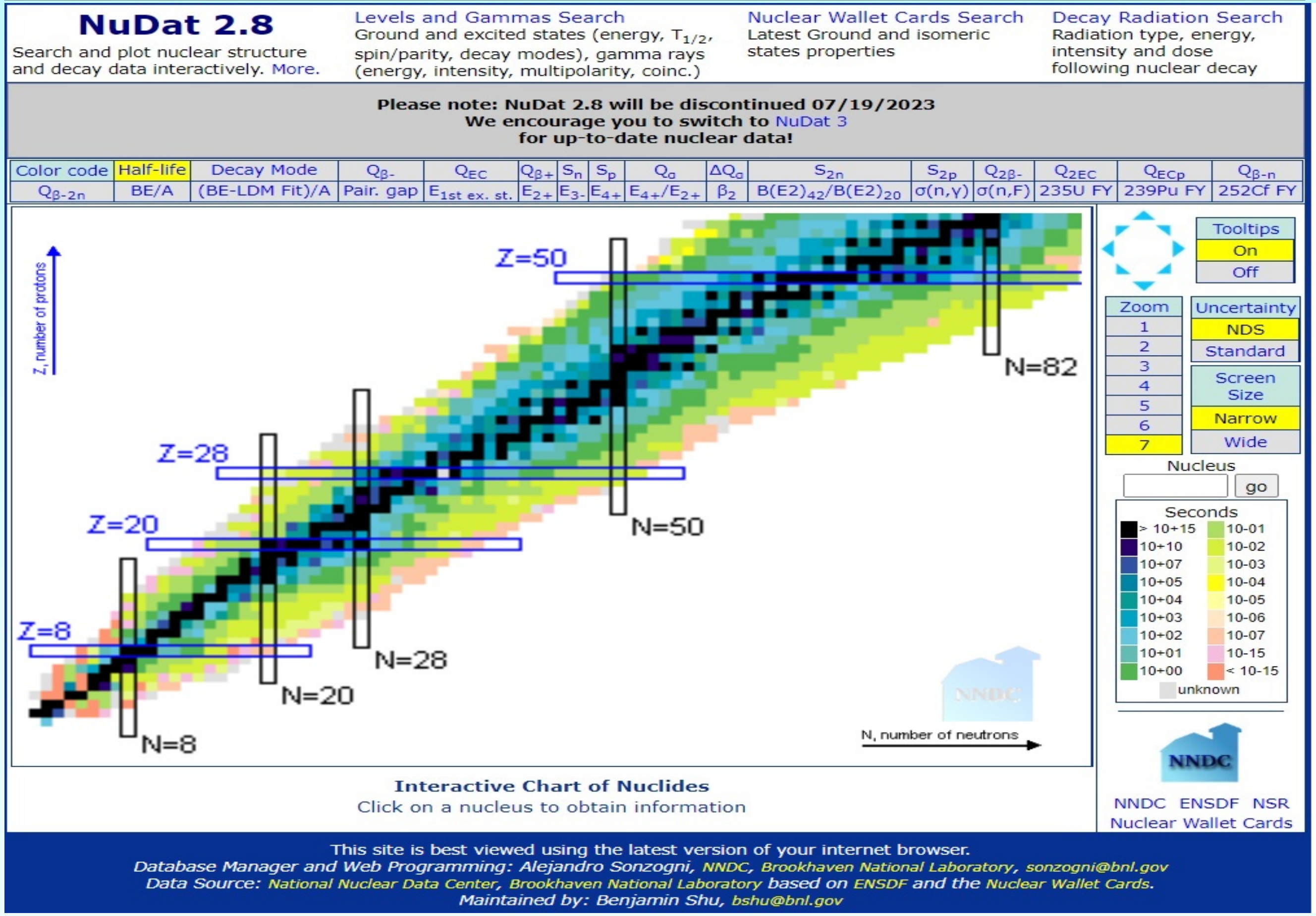
Conclusions

- Experimentally proved the possibility of dineutron existence in a bound state to be hosted by the recoil nucleus in the outgoing channel of nuclear reactions on ^{159}Tb with protons and neutrons in the input channel;
- Statistically and systematically significant observed the dineutron in the outgoing channel of the ^{197}Au(n, ^{2}n)^{196g}Au and ^{175}Lu (n, ^{2}n)^{174g}Lu nuclear reactions;
- The interval estimates are obtained for the binding energy, half-life and radius of the dineutron as well as cross-sections to generate the dineutron;
- Based on our experimental evidence, a possible interaction can take place between dineutron decay products and the heavy nucleus in the outgoing channel to fostering its "faster decay" and low energy nuclear fusion reactions under ambient conditions;

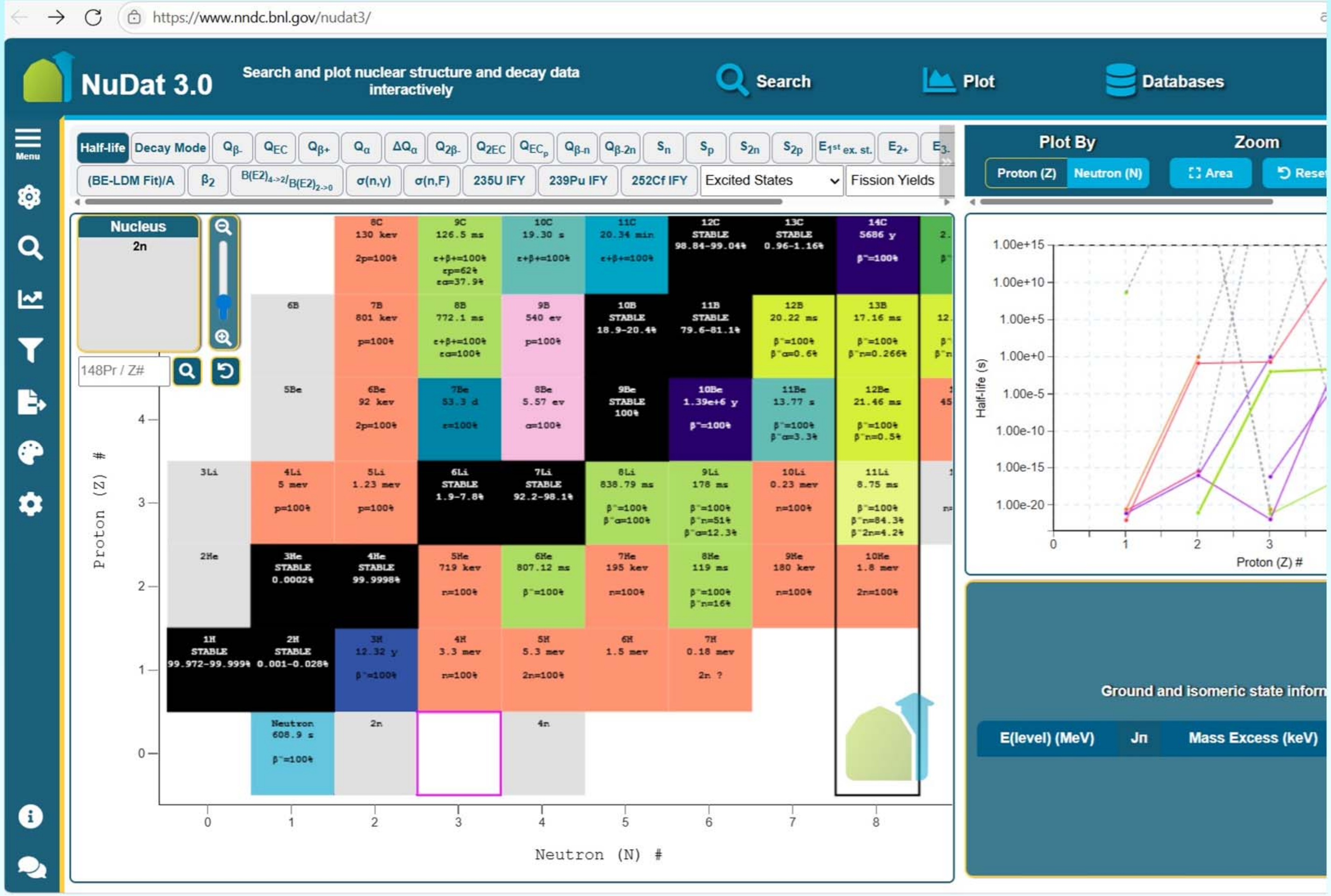
Conclusions

- **Results of this study support an idea about the existence of a bound dineutron in neutron and proton induced nuclear reactions;**
- **The existence of a bound three-neutron was implicitly envisaged in the original paper of A. Migdal;**
- **The formation of a bound three-neutron is evidenced in ^{159}Tb (p, ^{3}n)^{157}Dy nuclear reaction for the impinging protons of E_p = 13.87 ± 0.26 MeV and, very likely, for E_p = 16.23 ± 0.23 MeV;**
- **The interval estimates are obtained for the binding energy, half-life and radius of the three-neutron.**

Conclusions



Conclusions



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Many thanks for your attention and any questions!

Attachment - 1

part of λ for $\lambda^2 < \lambda_0^2/2$ is not taken into account.)

We define the value $\lambda_0 = \lambda_c$ where this supplementary branch of the spectrum terminates: $\lambda^2 = \lambda_c^2/2$. It follows from (22) that

$$\frac{2}{\pi}|H_0'|\ln\frac{2c_1a^2}{3\lambda_c^2} = 1 + 8|H_0'|, \qquad (23)$$

$$\lambda_c^2 \sim a^2 \exp\left\{-\frac{\pi}{2}\frac{1+8|H_0'|}{|H_0'|}\right\}.$$

Substituting into (22), we find

$$\lambda^2 - \lambda_0^2 = -\lambda_0^2 \Big/ \left[2 + \frac{1}{2\pi}\ln\frac{3\lambda_c^2}{2(\lambda^2+\lambda_0^2)}\right]. \qquad (22a)$$

For $\lambda_0^2 \ll \lambda_c^2$ (22a) implies for the additional branch that $\lambda^2 = \lambda_0^2$. Thus, the additional branch corresponding to a positive denominator (22a), starts at λ_0^2 passes into the region $\lambda_0^2 > \lambda^2 > \lambda^2/2$ and terminates at the point $\lambda_0^2 = \lambda_c^2$. As far as the regular branch is concerned, it is situated close to the line $\lambda^2 = \lambda_0^2$ for small H_0' and large λ_0^2, and for small values of λ_0^2 it tends to the finite limit $\lambda(0)$, which according to (22a) is determined by the condition

$$2 + \frac{1}{2\pi}\ln\frac{3\lambda_c^2}{2\lambda^2(0)}, \qquad \lambda_c^2 \simeq \lambda^2(0)e^{-4\pi}. \qquad (24)$$

The exact solution yields

$$\lambda_c^2 = \lambda^2(0)e^{-4\pi} = 4a^2\exp\left(\frac{\pi}{2H_0'} - \frac{3\pi}{2}\right). \qquad (25)$$

We now consider the case $\lambda_0 < 0$ (when the level of the one-particle problem is a virtual one). The spectrum is given by the expression (13)

$$1 + H_0'J_2 = 0,$$

corresponding to a zero of the denominator in (22a)

$$4\pi + \ln\frac{3\lambda_c^2}{2(\lambda^2+\lambda_0^2)} = 0.$$

From (25) we obtain

$$\ln\frac{2\lambda^2(0)}{2\lambda^2+\lambda_0^2} = 0; \qquad \lambda^2 = \lambda^2(0) - \frac{\lambda_0^2}{2}.$$

differs from the interaction of free particles. It is not hard to take this fact into account, making use of the experimentally determined parameters of the effective interaction[3].

We estimate the quantity H_0'. According to (5a) the function φ_0 has the form

$$u_0 = r\sqrt{4\pi}\varphi_0(r) = \pm\sin Kr,\ r < R,$$
$$u_0 = r\sqrt{4\pi}\varphi_0(r) = 1,\ r > R.$$

The boundary condition

$$K\,\mathrm{ctg}\,KR = 0$$

yields $\sin KR = \pm 1$; $\cos KR = 0$. Here $K = (2U_0)^{1/2}$, where U_0 is the depth of the well. Assuming the interaction to be a delta function we obtain

$$H_0' = \frac{v_1}{4\pi}\int_0^R\frac{\sin^2 Kr}{r^2}dr + \frac{v_2}{4\pi}\int_R^\infty\frac{dr}{r^2},$$

$$H'(\mathbf{r},\mathbf{r}') = v_0(r)\delta(\mathbf{r}-\mathbf{r}'),$$

v_1 is the value of $v_0(r)$ inside the nucleus, $v_2 = v_0(r)$ for $r > R$. The integration yields

$$H_0' = \frac{v_1}{4\pi}K\frac{\pi}{4} + \frac{v_2}{16}Kr_0 = \frac{c_1}{16}Kr_0 + \frac{c_2}{4\pi}\frac{r_0}{R},$$

where r_0 is the range of the forces, and c_1 and c_2 are numbers of order one.

For two neutrons with opposite spins we obtain, starting from the effective interaction within the nucleus[3] and the interaction in vacuum:

$$c_1 \approx 2.5 \pm 2, \quad c_2 \simeq -35 \pm 10$$

and consequently

$$H_0' \simeq -2.5A^{-1/3}.$$

The energy of the one-particle level $\lambda_c^2/2$ at which there appears the additional branch of the energy curve is ~ 0.4 MeV, as can be seen from Eq. (25). Thus, an additional energy level exists, and this level is to be interpreted as a dineutron and is situated in the range of one-particle energy levels between 0 and 0.4 MeV.

Attachment - 2

We now consider the case $\lambda_0 < 0$ (when the level of the one-particle problem is a virtual one). The spectrum is given by the expression (13)

$$1 + H'_0 J_1 = 0,$$

corresponding to a zero of the denominator in (22a)

$$4\pi + \ln \frac{3\lambda_c^2}{2(\lambda^2 + \lambda_0^2)} = 0.$$

From (25) we obtain

$$\ln \frac{2\lambda^2(0)}{2\lambda^2 + \lambda_0^2} = 0; \qquad \lambda^2 = \lambda^2(0) - \frac{\lambda_0^2}{2},$$

i.e., λ^2 decreases, as λ_0^2 increases, from $\lambda^2(0)$ to zero. The exact form of the decrease is somewhat distorted by our rough formulas. Thus, in the case of a virtual level there is only the usual solution. As can be seen from (25), an anomalous solution exists in the narrow range of well parameters, when

$$0 < \lambda_0 < \lambda_c.$$

After this qualitative discussion the exact solution obtained numerically appears natural (cf. the figure).

6. APPLICATION TO ATOMIC NUCLEI. DINEUTRONS AND MORE COMPLICATED PARTICLES NEAR THE NUCLEAR SURFACE

We estimate the magnitudes entering the theory having atomic nuclei in view. Above we have considered the case when the energy level of the one-particle problem which lies near the continuous spectrum has zero angular momentum. The theory can be easily reformulated for arbitrary angular momentum. In addition, one should remember that the interaction $H'(\mathbf{r}_1, \mathbf{r}_2)$ depends not only on the difference $\mathbf{r}_1 - \mathbf{r}_2$, but also on $\mathbf{r}_1$, since the effective interaction of particles within a nucleus[3] and the interaction in vacuum:

$$c_1 \approx 2.5 \pm 2, \quad c_1 \simeq -35 \pm 10$$

and consequently

$$H'_0 \simeq -2.5A^{-1/3}.$$

The energy of the one-particle level $\lambda_c^2/2$ at which there appears the additional branch of the energy curve is ~ 0.4 MeV, as can be seen from Eq. (25). Thus, an additional energy level exists, and this level is to be interpreted as a dineutron and is situated in the range of one-particle energy levels between 0 and 0.4 MeV.

This phenomenon can be observed in nuclear reactions, where it manifests itself as a resonant input channel. In particular, the spectra of several nuclei should exhibit states which are to be interpreted as dineutrons near the nuclear surface.

One might think that an analogous mechanism leads to bound states which are more complicated than the dineutron. Possible experimental consequences will be discussed in another paper.

The author expresses his gratitude to A. M. Dyugaev for interesting discussions.

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Translated by M. E. Mayer

Attachment - 3