Open quantum system approach for threshold effects

Marek Płoszajczak (GANIL)

European Nuclear Physics Conference 2025

Outline

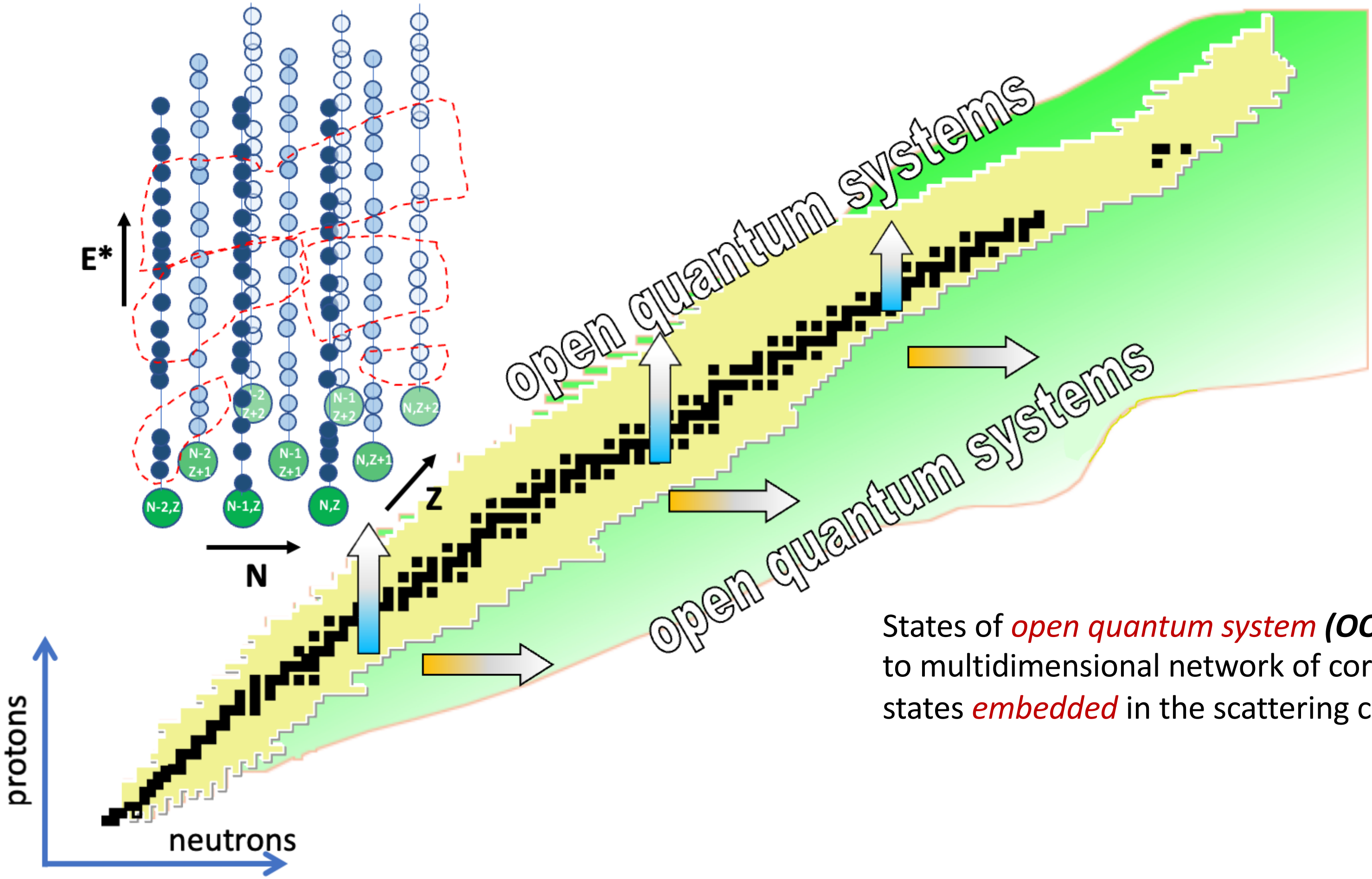
1. Why do we care about the continuum?

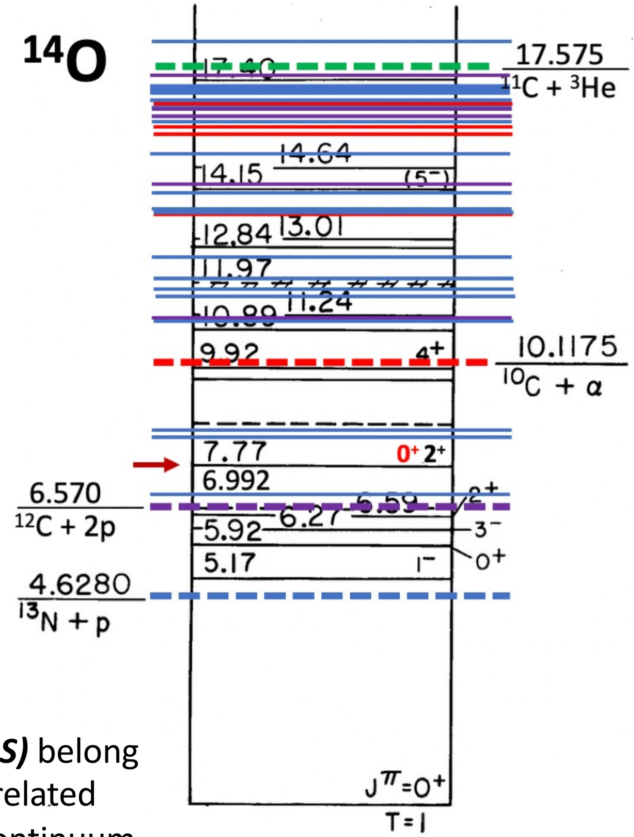
2. Shell model for open quantum systems
   - NN interaction in different regimes of binding
   - Configuration mixing in open quantum systems

3. Near-threshold states and origin of clustering
   - Near-threshold collectivization: γ-transitions
   - Role of near-threshold states in nuclear astrophysics
   - Mimicry mechanism in resonances
   - Rise and fall of α-clustering in ^{8}Be

4. Message to take

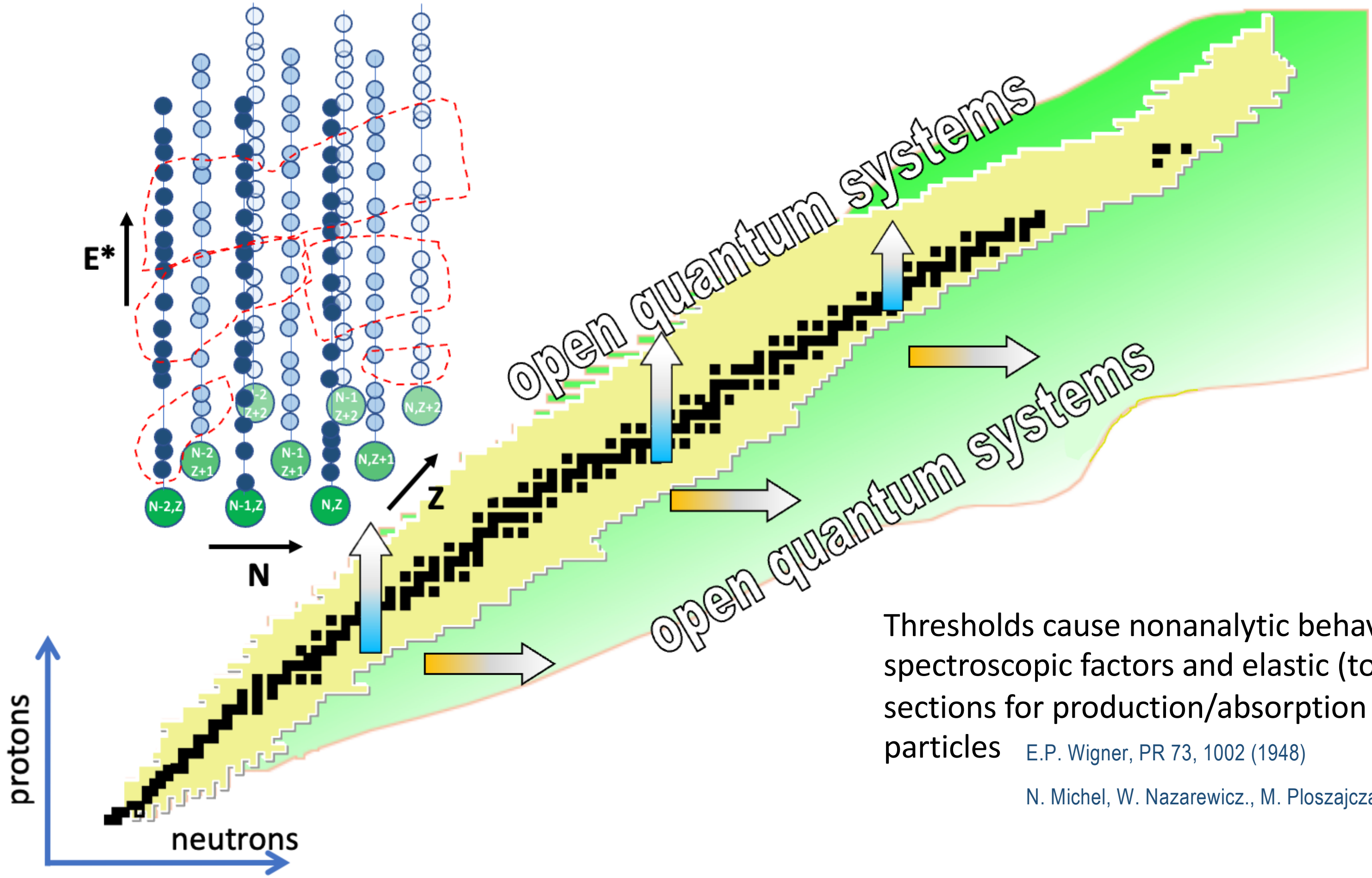
Why do we care about the continuum?

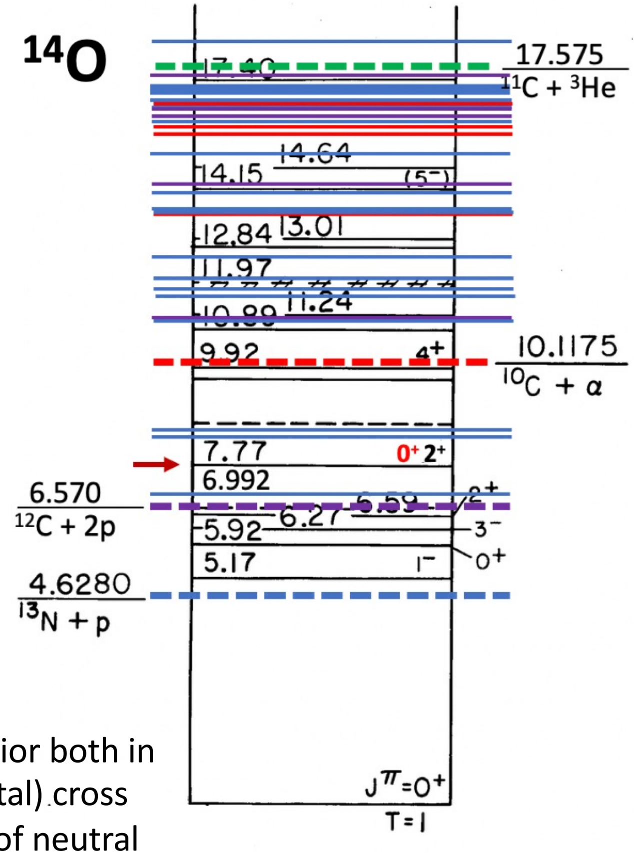




States of *open quantum system* ***(OQS)*** belong to multidimensional network of correlated states *embedded* in the scattering continuum

Why do we care about the continuum?



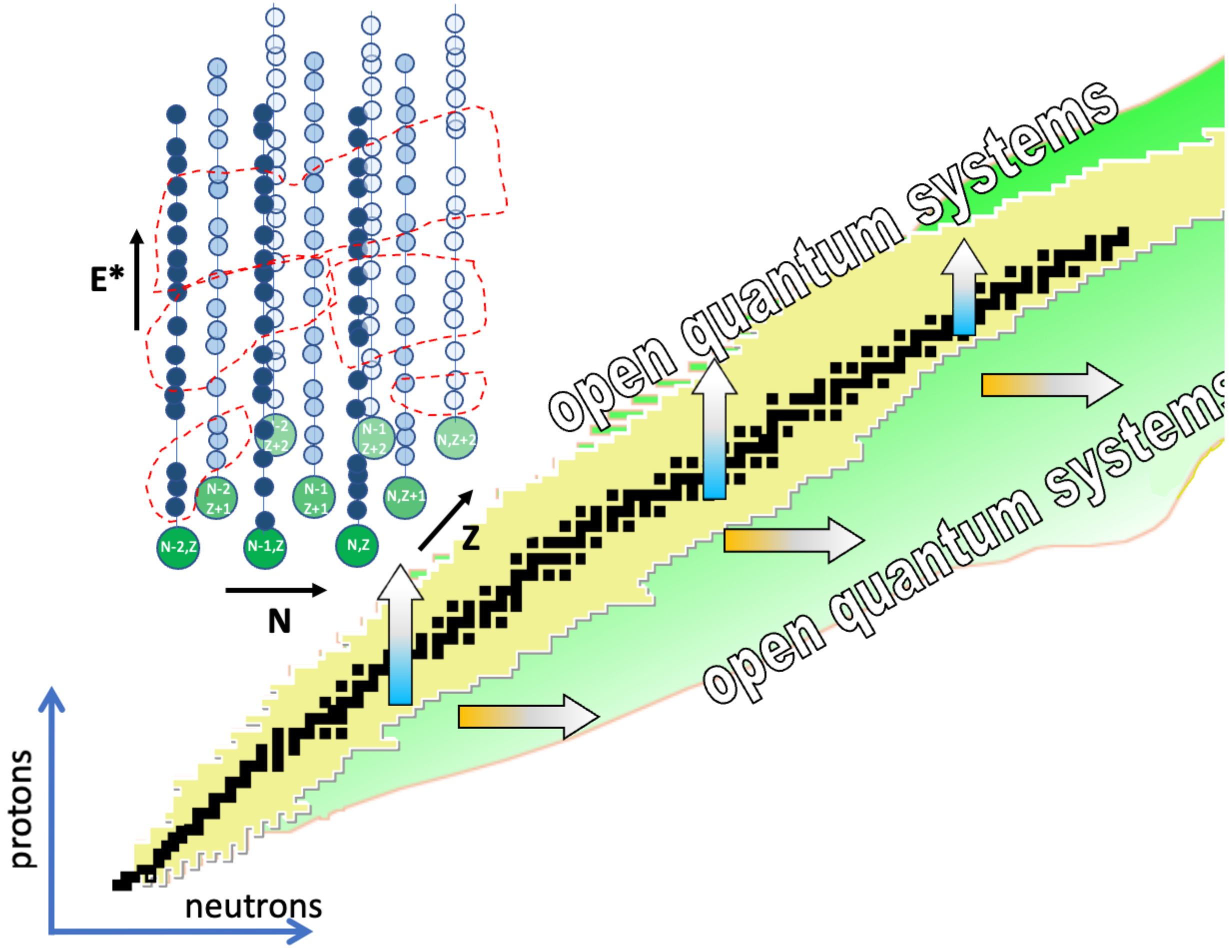


Thresholds cause nonanalytic behavior both in spectroscopic factors and elastic (total) cross sections for production/absorption of neutral particles

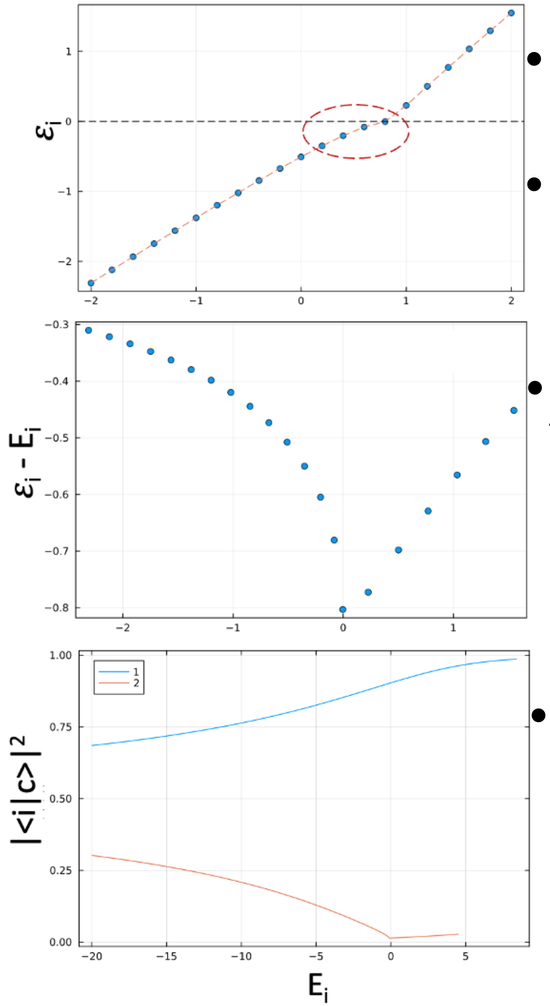
E.P. Wigner, PR 73, 1002 (1948)

N. Michel, W. Nazarewicz., M. Ploszajczak, PRC 75, 031301(R) (2007)

Why do we care about the continuum?

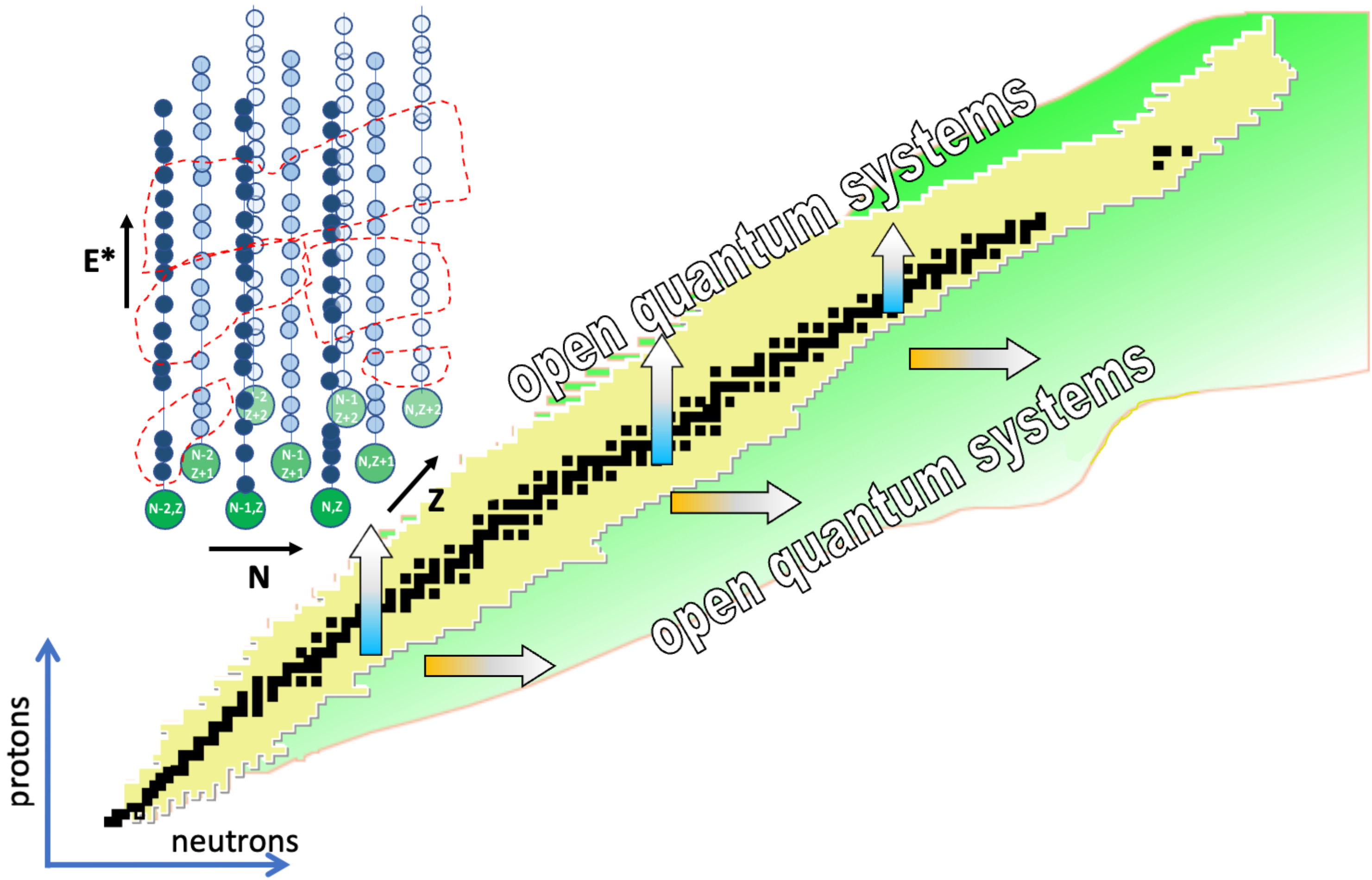


Picket fence model (ℓ=0) A. Volya, M. Ploszajczak (2025)



- Accumulation of states near threshold
- One state appears almost at the threshold
- Attractive energy correction for near-threshold states
- In the vicinity of a threshold, one state exhausts almost 100% of continuum coupling and shares properties of the decay threshold

Why do we care about the continuum?

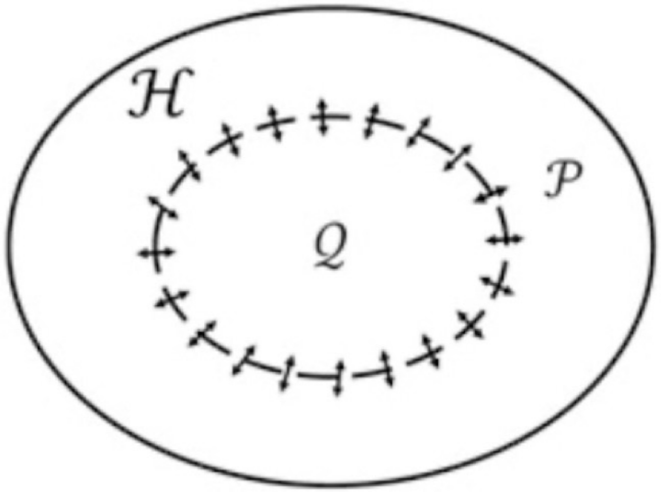


- Any comprehensive theory of threshold states should both ensure the ***unitarity and*** include ***coupling between discrete and scattering states***

Continuum shell model

Shell model for open quantum systems

Non-hermitian Quantum Mechanics in Hilbert space



C. Mahaux, H.A. Weidenmüller,
« *Shell Model Approach to Nuclear Reactions* »
(North-Holland Publishing Company, 1969)

J. Okołowicz, M. P., I. Rotter,
Physics Reports 374, 271 (2003)

Shell model embedded in the continuum (SMEC)

$$H^{(SM)} \rightarrow \mathcal{H}^{eff}(E) = H'(E) - (i/2)V(E)V^T(E)$$

[N×N] [N×N] [N×N] [N×k] [k×N]

$$= H^{(SM)} + u(E) - (i/2)w(E)$$

Hermitian Anti-hermitian

Complex-symmetric eigenvalue problem for *non-hermitian* Hamiltonian

Coupling to the environment (in P) cannot be reduced to refitting the Hamiltonian of the ***CQS***

Open QS solution in Q space

$$\mathcal{H}_{QQ}^{eff}|\Psi_\alpha\rangle = \mathcal{E}_\alpha(E)|\Psi_\alpha\rangle$$

$$\langle\Psi_{\tilde{\alpha}}|\mathcal{H}_{QQ}^{eff} = \mathcal{E}_\alpha^*(E)\langle\Psi_{\tilde{\alpha}}|$$

⟵ $\langle\Psi_{\tilde{\alpha}}|\Psi_\beta\rangle = \delta_{\alpha\beta}$

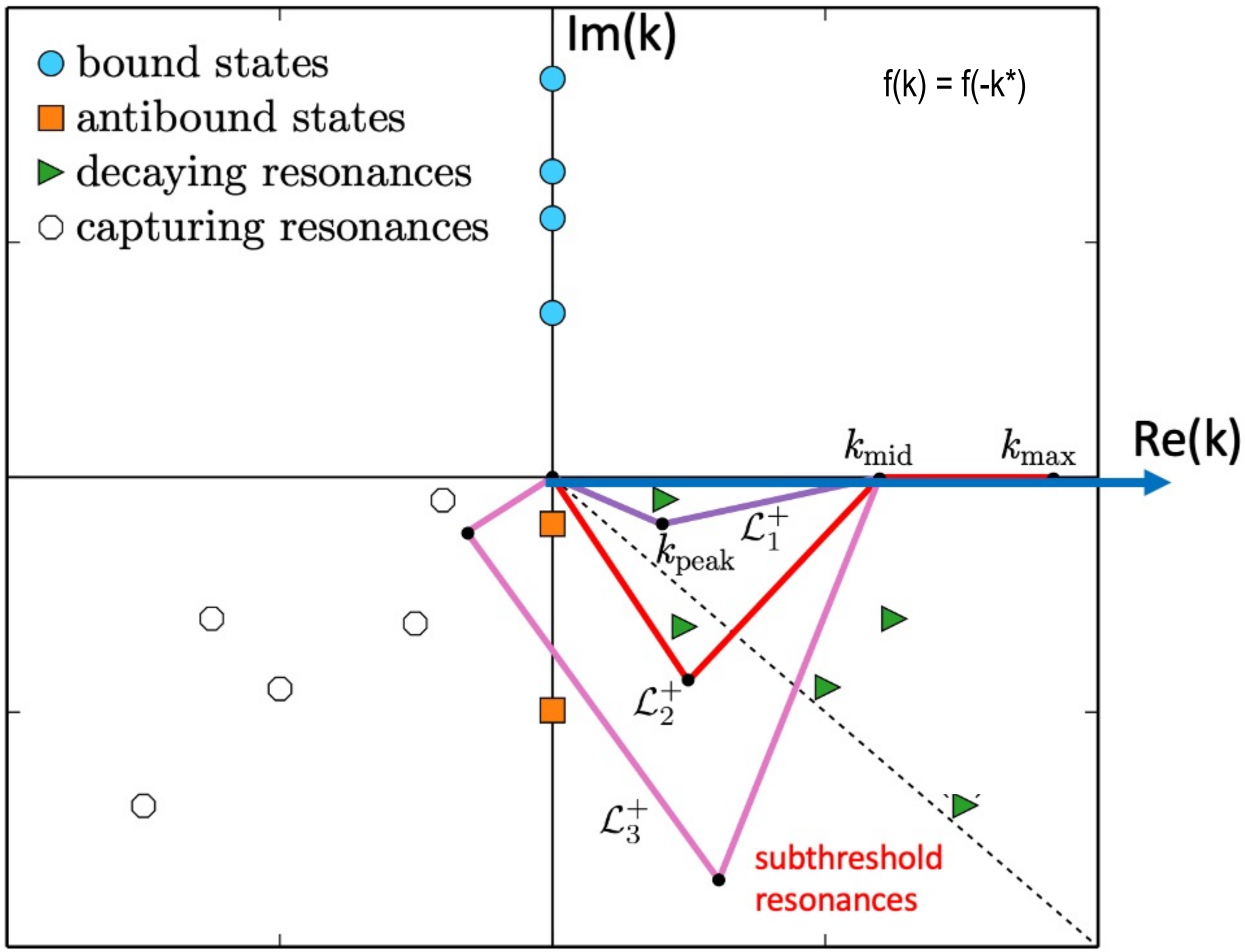
For bound states: $\mathcal{E}_\alpha(E)$ is real.
Physical resonances correspond to the poles of the scattering matrix

$$\Psi_\alpha = \sum_i b_{\alpha i}\Phi_i^{(SM)} \Rightarrow \Psi_E^c \sim \sum_\alpha c_\alpha \Psi_\alpha$$

- Entrance and exit reaction channels defined
  ⟶ Shell model and reaction theory reconciled

Shell model for open quantum systems

Gamow shell model (GSM)



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|dk = 1 \;;\; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

Shell model for open quantum systems

Gamow shell model (GSM)

Slater determinant representation

$$|SD_i\rangle = |u_{i_1} \dots u_{i_A}\rangle \Rightarrow \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

N. Michel et al, PRL 89, 042502 (2002)
N. Michel, et al, J. Phys. G37, 064042 (2010)

- *Complex-symmetric* eigenvalue problem for *hermitian* Hamiltonian
- Center-of-mass handled by recoil term in the Hamiltonian:

$$H \to H + \frac{1}{M_{\text{core}}} \sum_{(i<j)\in \text{val}} \mathbf{p}_i \cdot \mathbf{p}_j$$

- Unitary formulation of the nuclear Shell Model 😀
- No identification of reaction channels 😩

N. Michel, M.Płoszajczak,
«*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* »
Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021

Coupled-channel representation

$$|\Psi_M^J\rangle = \sum_c \int_0^{+\infty} |(c,r)_M^J\rangle \frac{u_c^{JM}(r)}{r} r^2 \, dr$$

$$|(c,r)\rangle = \hat{\mathcal{A}}[|\Psi_T^{J_T}; N_T, Z_T\rangle \otimes |r \; L_{CM} \; J_{\text{int}} \; J_P; n, z\rangle]_M^J$$

$$H|\Psi_M^J\rangle = E|\Psi_M^J\rangle \Rightarrow \sum_c \int_0^\infty r^2 \left(H_{cc'}(r,r') - E N_{cc'}(r,r')\right) \frac{u_c(r)}{r} = 0$$

$$H_{cc'}(r,r') = \langle (c,r)|\hat{H}|(c',r')\rangle$$

$$N_{cc'}(r,r') = \langle (c,r)|(c',r')\rangle$$

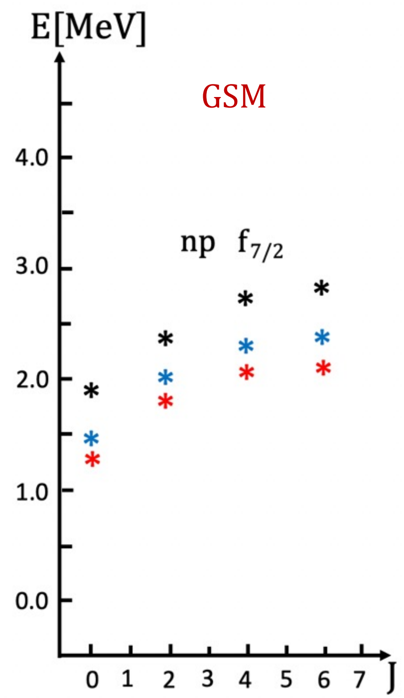
- Entrance and exit reaction channels defined
  ⇒ Unification of nuclear structure and reactions 😀

Y. Jaganathen et al, PRC 88, 044318 (2014)
K. Fossez et al., PRC 91, 034609 (2015)
A. Mercenne et al., PRC 99, 044606 (2019)

Shell model for open quantum systems

NN interaction in different regimes of binding

* (B(ja), B(jb)) = (-10, -10) MeV
* (B(ja), B(jb)) = (-1, -10) MeV
* (B(ja), B(jb)) = (+1, -10) MeV



Strong reduction of np interaction
in weakly bound/unbound nuclei:
~50% reduction in *p*-shell

Dependence of V_{nn}/V_{pp} on $S_n - S_p$ asymmetry

ℓ_j	J^{π}	S_p[MeV]	S_n[MeV]	V_{nn}/V_{pp}
$P_{1/2}$	2^{+}	10	-1	0.39
		1	-1	0.58
$d_{5/2}$	2^{+}	10	-1	0.83
		1	-1	0.835
	4^{+}	10	-1	0.75
		1	-1	0.84

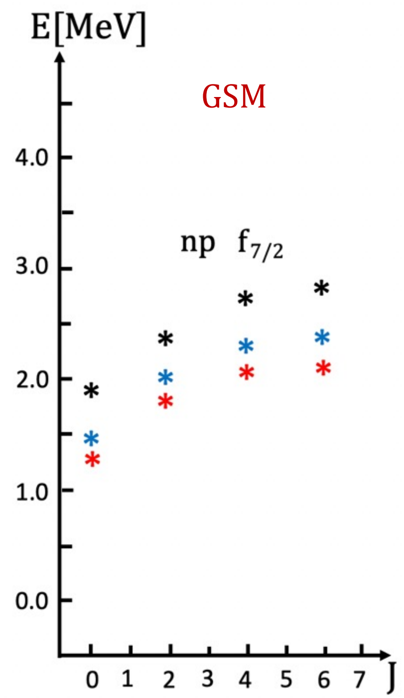
- Strong asymmetry of V_{nn} and V_{pp} for large $|S_n - S_p|$ and low ℓ_j

Shell model for open quantum systems

NN interaction in different regimes of binding

≫

* (B(ja), B(jb)) = (-10, -10) MeV
* (B(ja), B(jb)) = (-1, -10) MeV
* (B(ja), B(jb)) = (+1, -10) MeV



Strong reduction of np interaction
in weakly bound/unbound nuclei:
~50% reduction in *p*-shell

Dependence of V_{nn}/V_{pp} on $S_n - S_p$ asymmetry

ℓ_j	J^π	S_p[MeV]	S_n[MeV]	V_{nn}/V_{pp}
$P_{1/2}$	2^+	10	-1	0.39
		1	-1	0.58
$d_{5/2}$	2^+	10	-1	0.83
		1	-1	0.835
	4^+	10	-1	0.75
		1	-1	0.84

- Strong asymmetry of V_{nn} and V_{pp} for large $|S_n - S_p|$ and low ℓ_j

Dependence of spectroscopic factors on $S_n - S_p$ asymmetry

Spectroscopic factors for the knockout of a $p_{3/2}$ nucleon from the 3/2- g.s. of ^{9}C and ^{9}Li to the g.s. of ^{8}B, ^{8}He, ^{8}B, and ^{8}Li

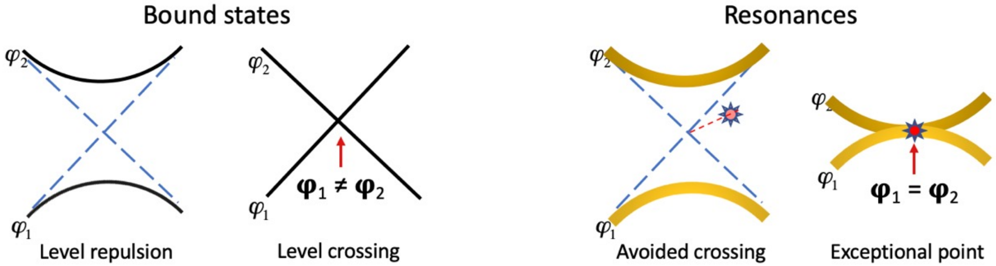
		$^9C \rightarrow {}^8C$	$^9Li \rightarrow {}^8He$	$^9C \rightarrow {}^8B$	$^9Li \rightarrow {}^8Li$
Model	N_{cont}	14.22	13.94	1.30	4.06
HO-SM	0	0.86	0.85	0.95	0.96
GSM-*ps*	3	0.67	0.67	0.98	0.98
GSM-*psd*	3	0.60	0.67	0.89	0.88
GSM-*psd*	4	0.48	0.65	0.89	0.88
GSM-psd_{res}	4	0.48	0.64	0.84	0.85

- If $S_n \gg S_p$, then neutron spectroscopic factor is reduced with respect to proton spectroscopic factor, and vice versa if $S_p \gg S_n$

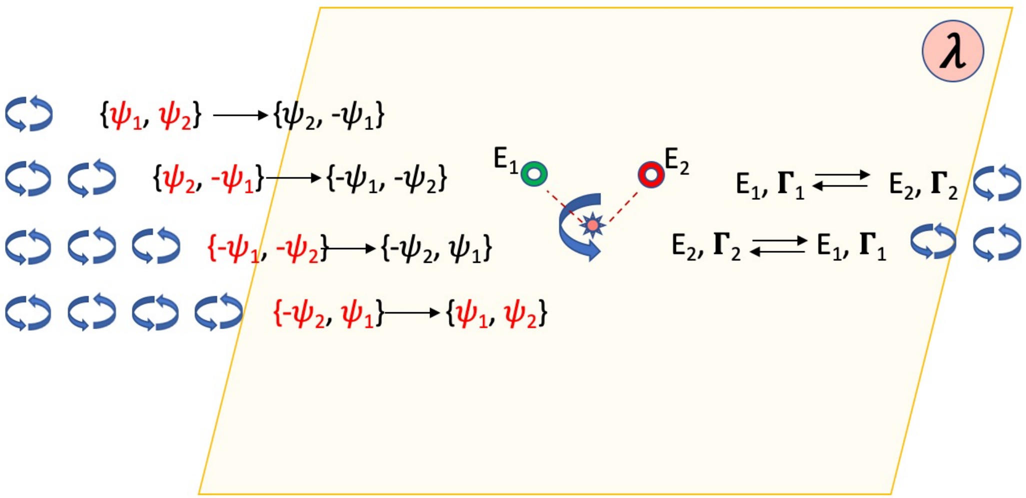
N. Michel, M.Płoszajczak,
«*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* »
Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021)
J. Wylie, J. Okolowicz et al, Phys. Rev. C 104, L061301 (2021)

Shell model for open quantum systems

Configuration mixing in open quantum systems



Topological features of the exceptional points



- Bose-Einstein condensation of gases with attractive 1/r – interaction
- Microwave cavity experiments
- Atoms coupled to radiation field
- Atom – cavity quantum composite
- Optical lattices
- Atomic nuclei

M.R. Zirnbauer et al., Nucl. Phys. A411 (1983) 161
C. Dembowski et al., PRL 86 (2001) 787; PRL 90 (2003) 034101
J. Okołowicz, M. Ploszajczak PRC 80 (2009) 034619

More in the talk of D. Cardona Ochoa on Monday

Near-threshold states and origin of clustering

α-clustering “…α-cluster states can be found in the proximity of α-particle decay threshold…”

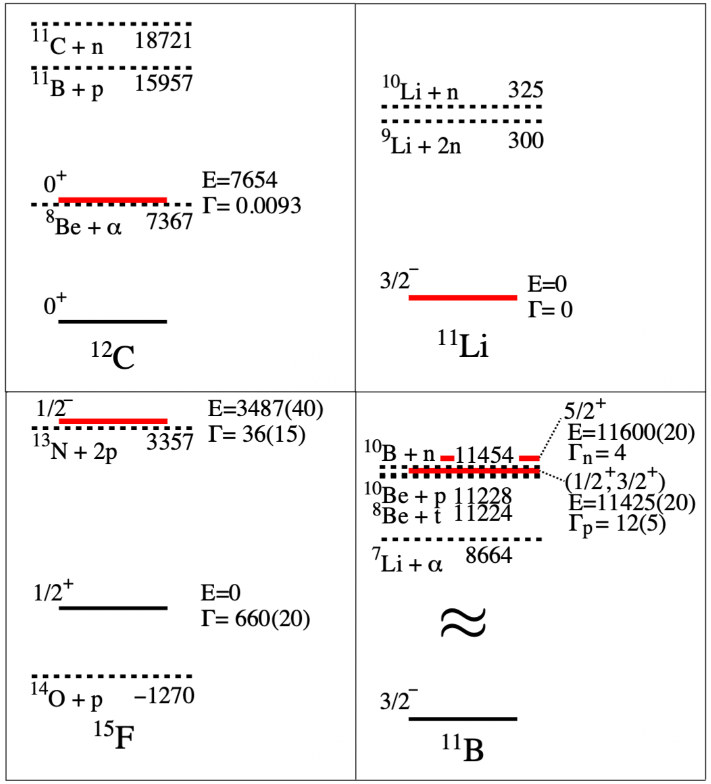
K. Ikeda, N. Takigawa, H. Horiuchi, Prog. Theor. Phys. Suppl. 464 (1968)

Near-threshold states and origin of clustering

α-clustering “…α-cluster states can be found in the proximity of α-particle decay threshold…”

K. Ikeda, N. Takigawa, H. Horiuchi, Prog. Theor. Phys. Suppl. 464 (1968)

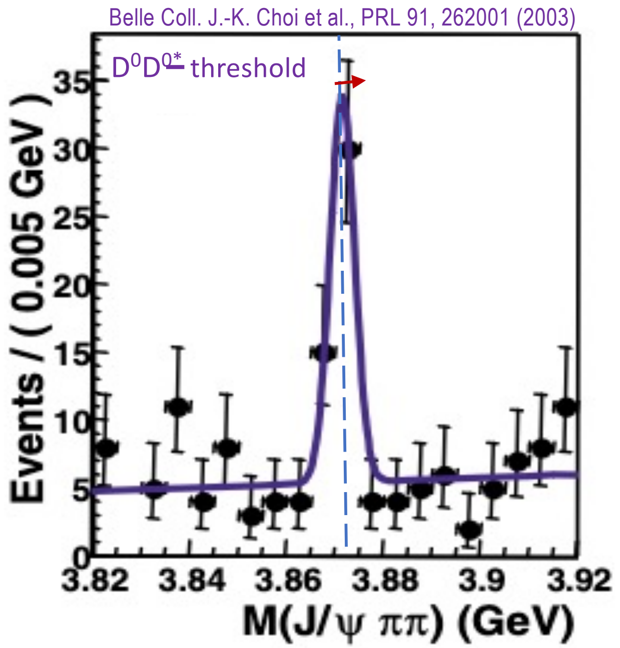
But this is only the tip of the iceberg!



- Other cases: ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, ^{11}O, ^{11}C, ^{17}O, ^{20}Ne, ^{26}O,…
- Various clusterings: 2p, 2n, ^{3}He, ^{3}H, …

Threshold effects in multiquark systems

Belle Coll. J.-K. Choi et al., PRL 91, 262001 (2003)



$$B^\pm \to K^\pm \pi^+ \pi^- J/\psi$$

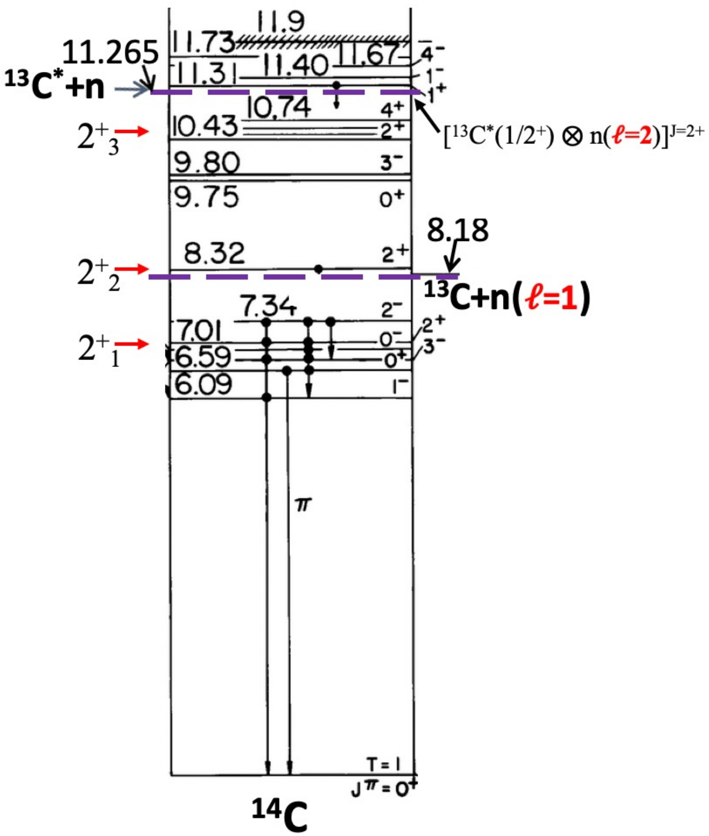
M_X = 3872 +/- 0.6 (stat) +/- 0.5 (syst) MeV

$(M_{D^0} + M_{D^{*0}})$ = 3871.1 +/- 1 MeV

- **Natural explanation provided by the OQS perspective:**
  - ➡ Correlated (clustered) states near the thresholds of the reaction channel are a consequence of the *collective rearrangement* of the wave functions caused by mutual coupling through the continuum

Near-threshold states and origin of clustering

Near-threshold collectivization: γ-transitions in ^{14}C

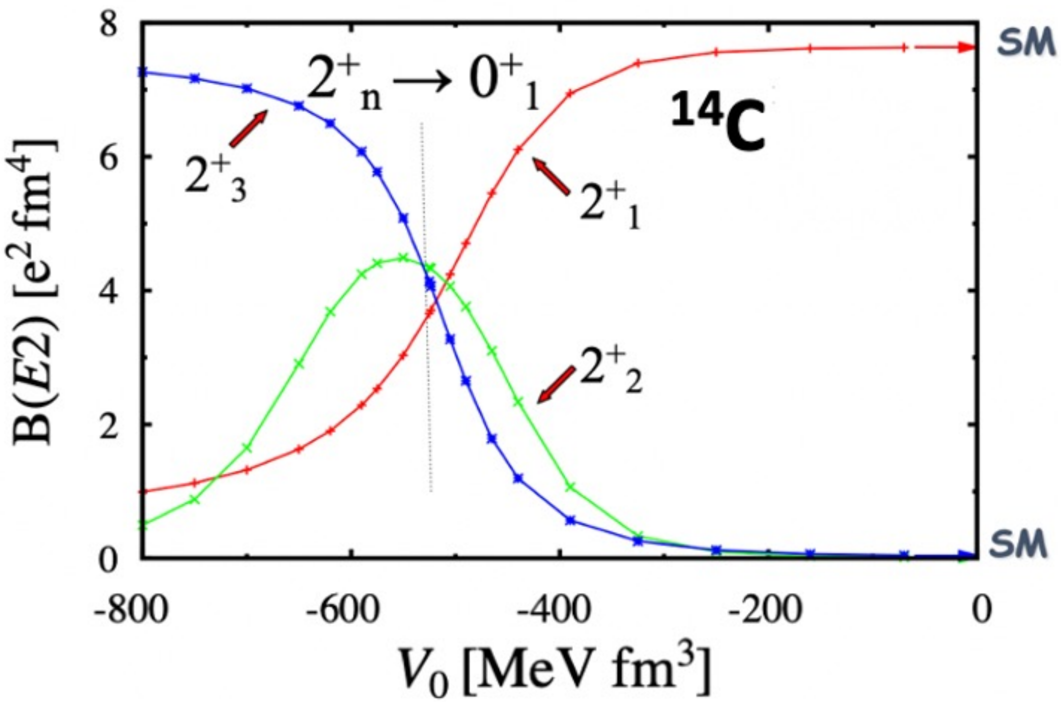


$[^{13}C(1/2^-) \otimes n(\ell=1)]^{J=2+}$

$[^{13}C(K^\pi) \otimes n(\ell_j)]^J$

$K^\pi = 1/2^+_1, 3/2^-_1, 5/2^+_1, 5/2^+_2, 3/2^+_1, 7/2^+_1, 5/2^-_1, 3/2$

M. Płoszajczak and J. Okołowicz, J. Phys.Conf. Ser. 1643, 012156 (2020)

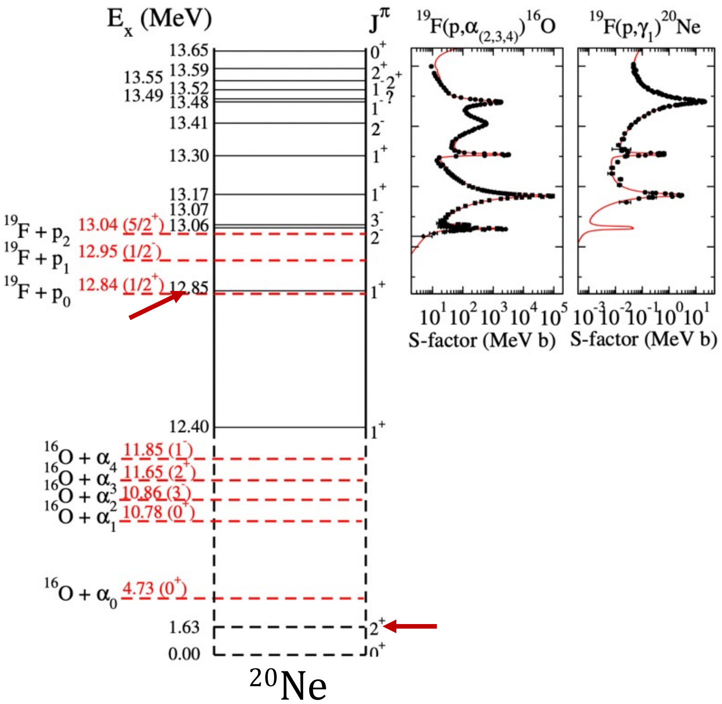


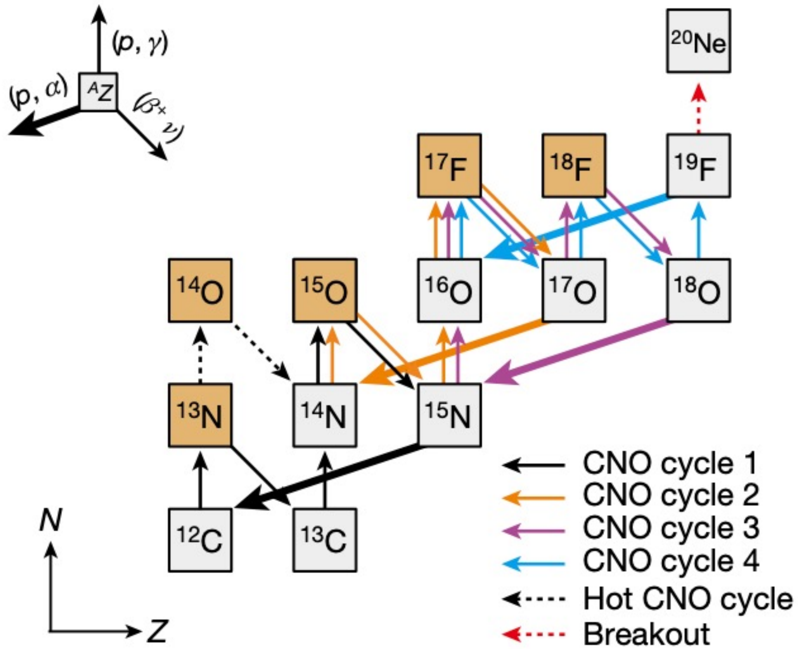
Strong collectivization of B(E2)[$0^+_1 \rightarrow 2^+_2$]
due to the nearby exceptional point

Near-threshold states and origin of clustering

Role of near-threshold states in nuclear astrophysics

R.J. DeBoer et al, Nature 610, 656 (2022)





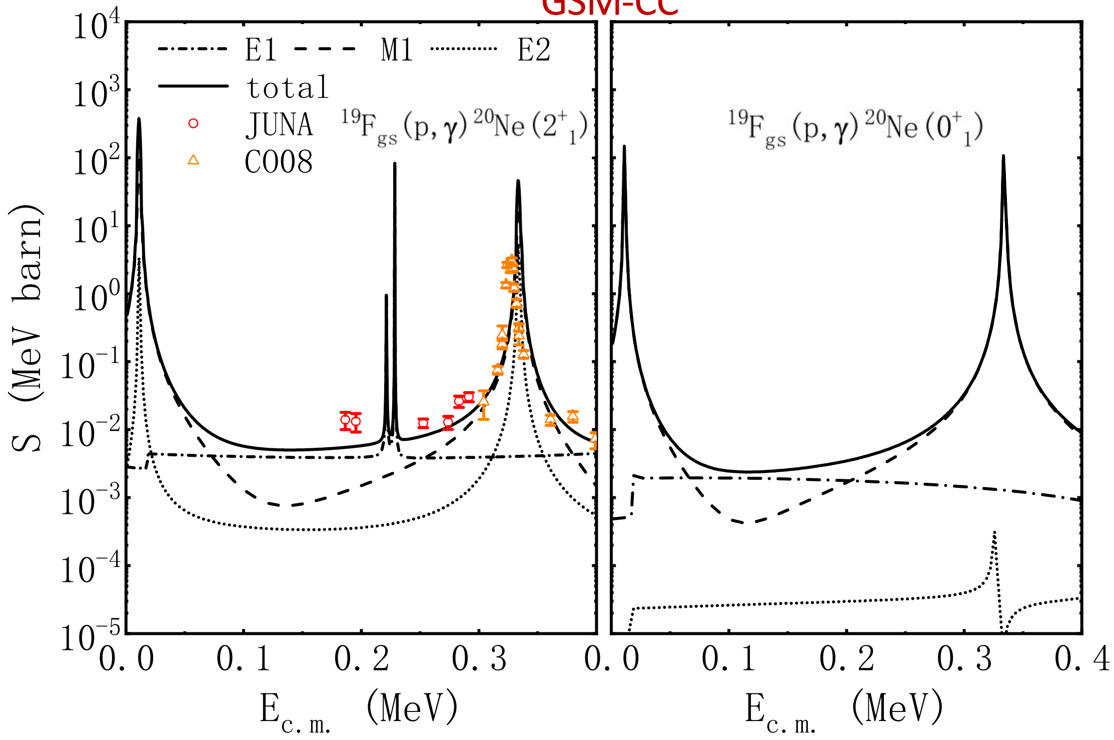
Liyong Zhang et al., Nature 610, 656 (2022)

What is the effect of 1⁺ resonance at ~10 keV above the proton emission threshold on the S-factor?

Near-threshold states and origin of clustering

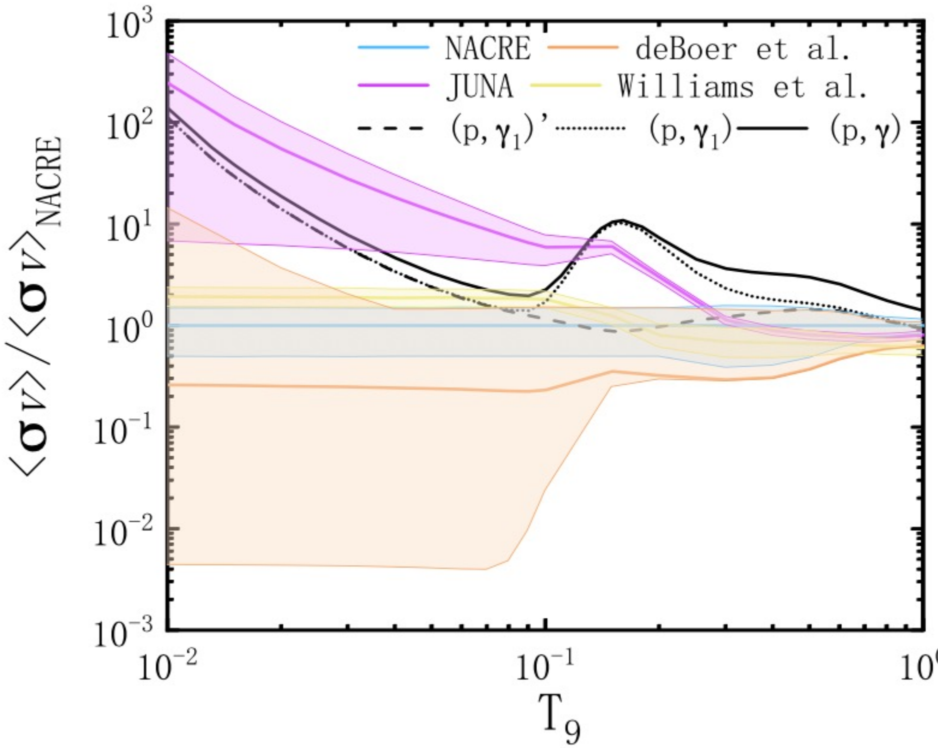
Role of near-threshold states in nuclear astrophysics

GSM-CC



Exp: Liyong Zhang et al., Nature 610, 656 (2022)

Exp: Liyong Zhang et al., Nature 610, 656 (2022)



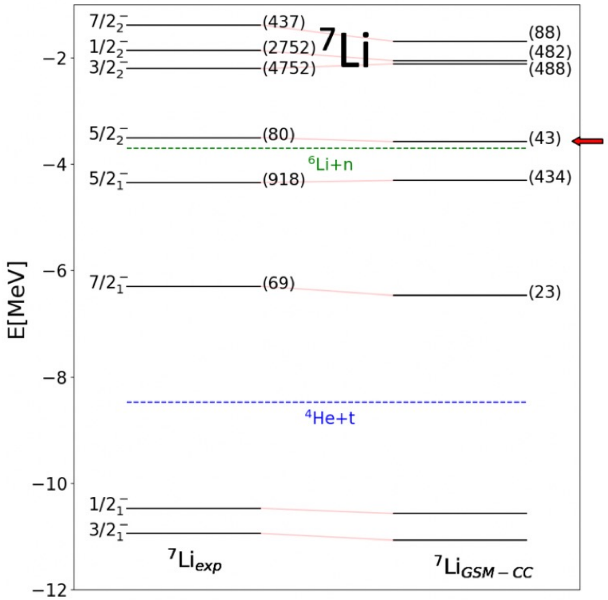
- The decay to the 2+ first excited state in ^{20}Ne dominates

X.B. Wang, et al, Phys. Rev. C 110, L061601 (2024)

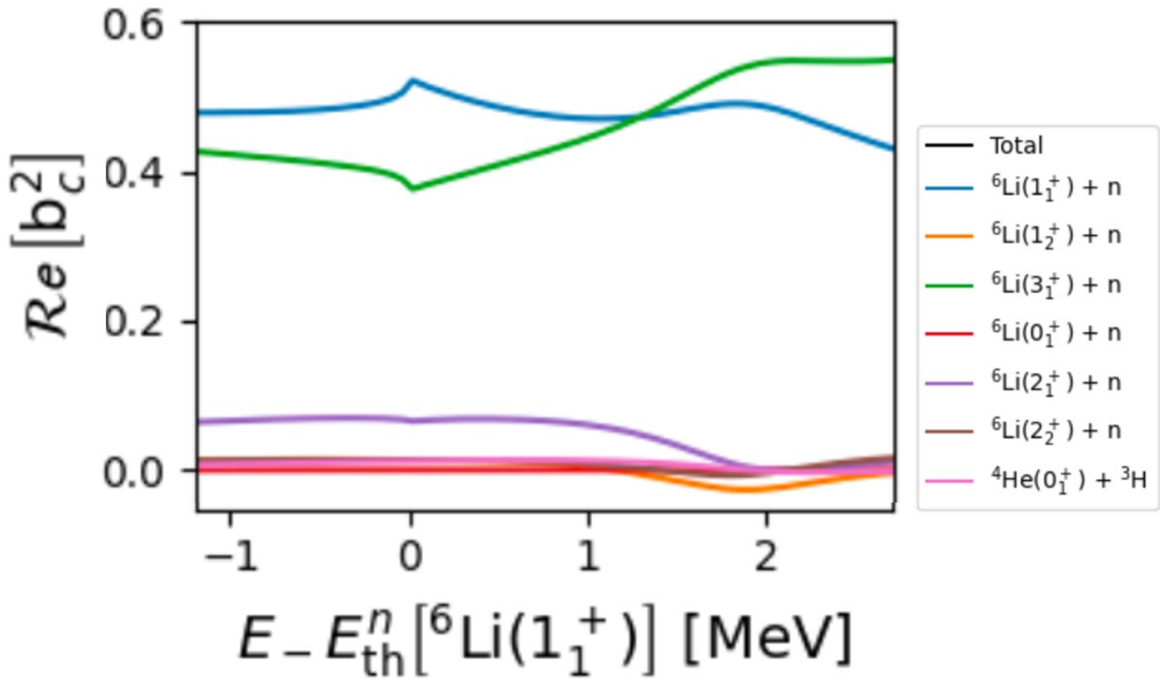
- GSM-CC reaction rates are significantly larger than in NACRE database

Near-threshold states and origin of clustering

Mimicry mechanism in resonances



$[|^4\text{He}\rangle \otimes |^3\text{H}\rangle], [|^6\text{Li}\rangle \otimes |n\rangle]$



- Hamiltonian: 1-body potential, 2-body FHT interaction

H. Furutani et al, Prog. Theor. Phys. 62, 981 (1979)

^{3}H wave functions calculated using N^{3}LO$_{(2\text{-body})}$ interaction

- Channels: ^{6}Li(K^π): $K^\pi = 1_1^+, 1_2^+, 3_1^+, 0_1^+, 2_1^+, 2_2^+$

  n: $\ell_j = s_{1/2}, p_{1/2}, p_{3/2}, d_{3/2}, d_{5/2}, f_{5/2}, f_{7/2}$

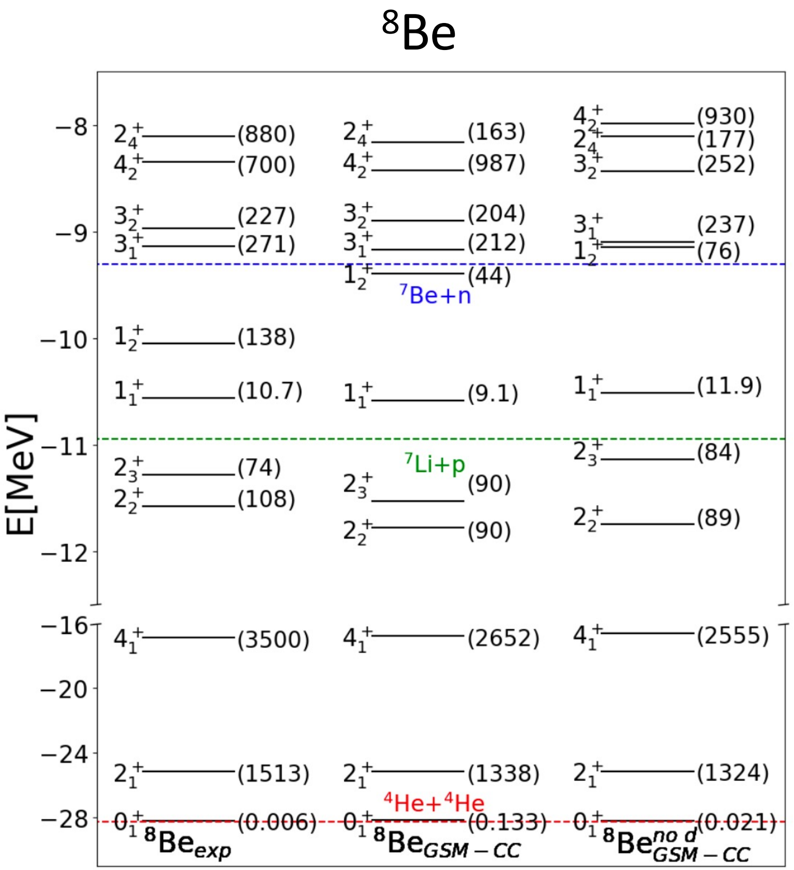
  ^{3}H(L): $L \equiv {}^{2J_{int}+1}[L_{CM}]_{J^P} = {}^2S_{1/2}, {}^2P_{1/2}, {}^2P_{3/2}, {}^2D_{3/2}, {}^2D_{5/2}, {}^2F_{5/2}, {}^2F_{7/2}$

- Structure of the resonance w.f. changes with changing energy as a result of the alignment (*mimicry*) with the nearby reaction channel

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Near-threshold states and origin of clustering

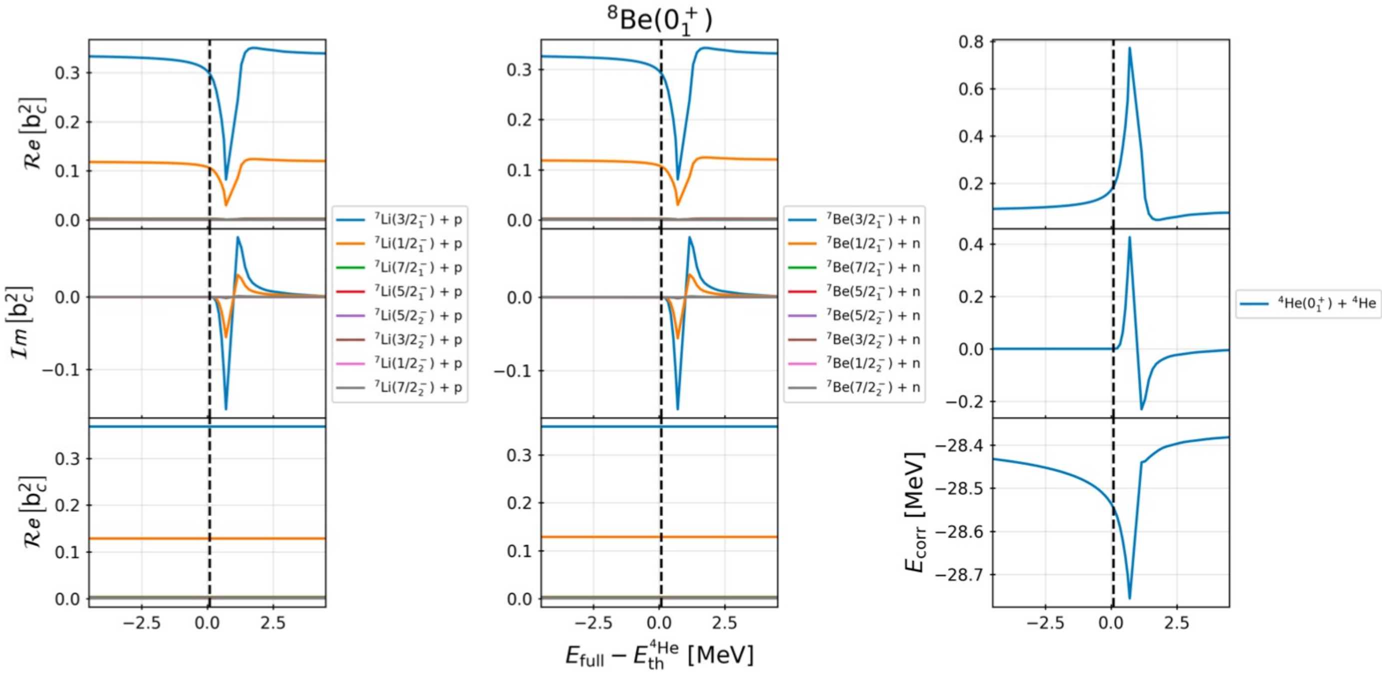
Rise and fall of α-clustering in ^{8}Be



Continuum coupling correlation energy (GSM-CC)

$$E^{(\mathrm{corr})}_{J^\pi,M} = \langle \tilde{\Psi}^J_M | H | \Psi^J_M \rangle - \langle \tilde{\Phi}^{J;(\alpha)}_M | H | \Phi^{J;(\alpha)}_M \rangle \equiv \mathcal{E}_{J^\pi,M} - \mathcal{E}^{(\alpha)}_{J^\pi,M}$$

$$|\Phi^{J;(\alpha)}_M\rangle = \sum_{\mathrm{c};\mathrm{c}\neq\alpha} \int_0^{+\infty} |(\mathrm{c},r)^J_M\rangle \frac{\bar{u}^{JM}_{\mathrm{c}}(r)}{r} r^2 \, dr$$



Mass partitions:

$[|^4\mathrm{He}\rangle \otimes |^4\mathrm{He}\rangle]$, $[|^7\mathrm{Li}\rangle \otimes |\mathrm{p}\rangle]$, $[|^7\mathrm{Be}\rangle \otimes |\mathrm{n}\rangle]$, $[|^6\mathrm{Li}\rangle \otimes |\mathrm{d}\rangle]$

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Message to take

- Near-threshold eigenstates of open quantum systems have unique properties which distinguish them from eigenstates of well-bound closed quantum systems

  The richness of nuclear interaction and the existence of nucleons in two distinct states (proton/neutron) make studies on the near-threshold phenomena in atomic nucleus unique

- Near-threshold phenomena are *terra incognita* of nuclear physics:
  - Collectivization of wave functions due to the coupling to decay channel(s)
  - Formation of clusters/correlations which carry an imprint of nearby decay channel(s)
  - Modification of NN interaction/spectroscopic factors
  - … …

  Essential role of *unitarity*!

- Deeper understanding of near-threshold phenomena in the shell model for open quantum systems will help to define new territory of nuclear spectroscopy studies:
  - γ-selection rules for in- and out-band transitions in resonance bands
  - Coupling of collective and single-particle motion in the continuum
  - Violation of mirror symmetry/isospin symmetry
  - New kinds of near-threshold clustering: ^2H, ^3H, ^3He, ^3n, ^4n, …
  - Effects of coalescing resonances in nuclear spectroscopy and reactions
  - … …

Special thanks:	*Nicolas*	Michel	IMP/CAS Lanzhou/Beijin, China
	Witek	Nazarewicz	MSU/FRIB East Lansing, USA
	David	Cardona Ochoa	GANIL, France
	Alan	Dassie	GANIL, France
	Jose Pablo	Linares	LSU Baton Rouge, USA
	Guoxiang	Dong	Huzhou University, China
	Xiaobao	Wang	Huzhou University, China
	Alexander	Volya	FSU, Tallahassee, USA

Thank You