

Effects of neutron emission during fission on fragment mass distribution calculated with dynamical model

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Introduction

Background

- Fission-fragment mass distributions (FFMDs) from low-energy fission of actinides from uranium to einsteinium show a **double humped shape**, with the heavy fragment mass around $A_H = 140$.
- These FFMDs have been interpreted to result from **effects of nuclear shell structure**.
- The measured FFMD from a the highly excited nucleus, is influenced by **the effects of multi-chance fission**, i.e. fission after the evaporation of neutrons [1,2].

Multi-chance fission (MCF)

The process by which an excited nucleus emits neutrons and undergoes fission. Competition between **neutron emission** and **fission**.

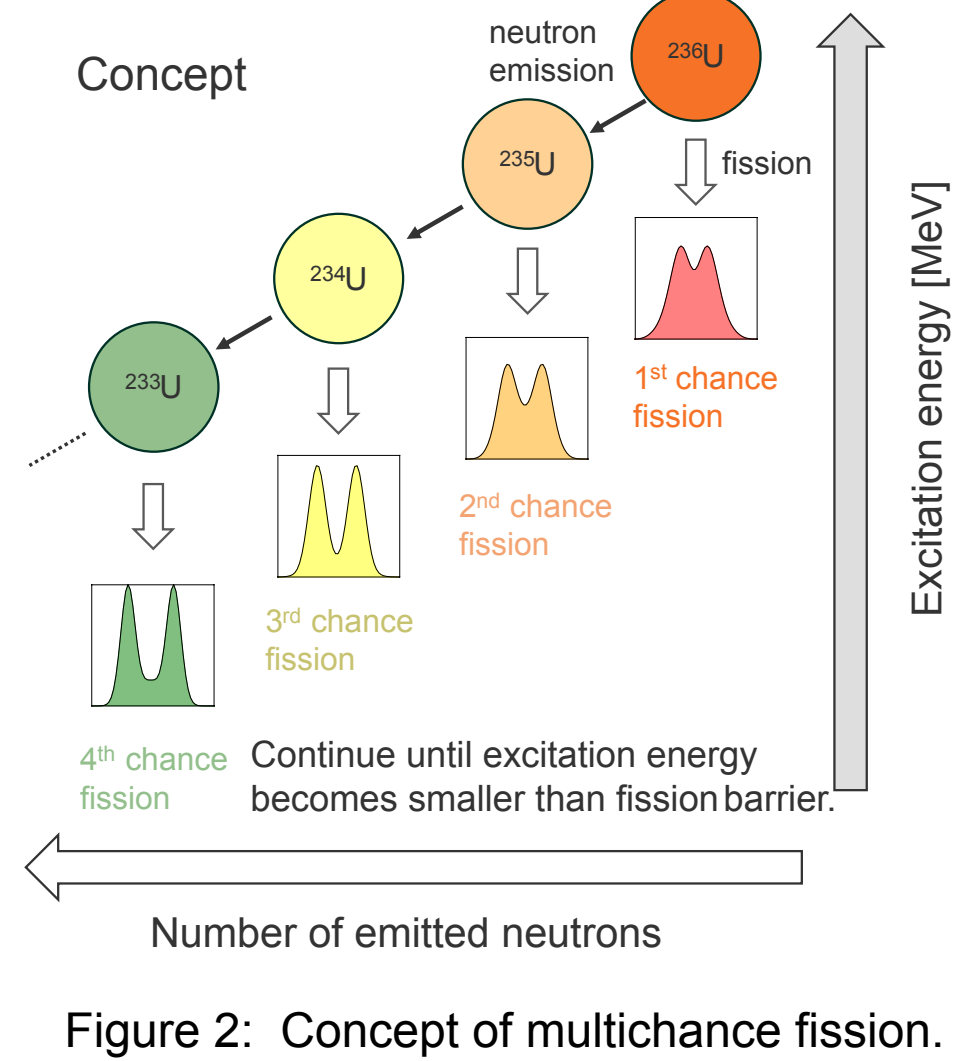


Figure 2: Concept of multichance fission.

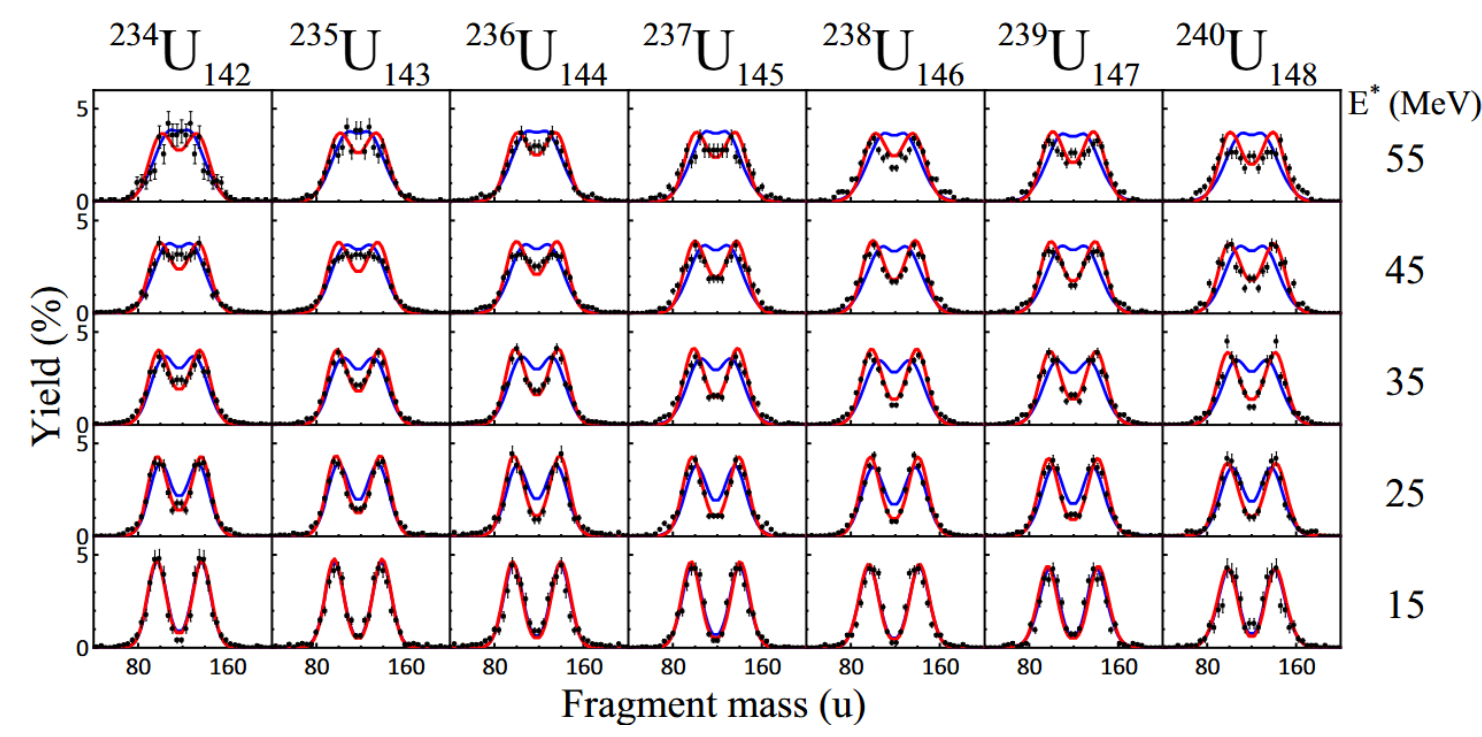


Figure 1: FFMDs of ²³⁴⁻²⁴⁰U compared with experimental data [1-3].

Previous study:

The neutron emission from compound nucleus is calculated by the GEF code [4], then the Langevin calculation starts to obtain FFMDs [1,3].

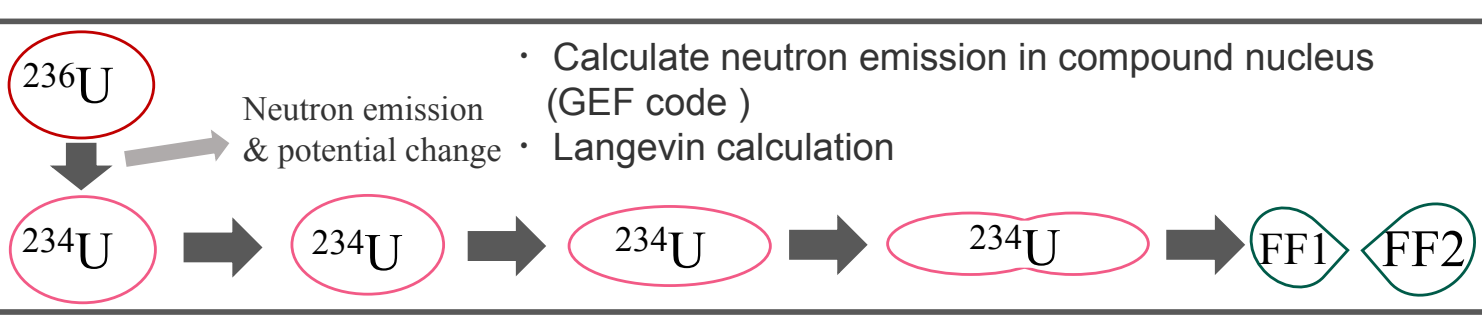


Figure 3: Previous study of MCF calculation [1,3].

Present study:

We calculate the neutron emission **dynamically** during fission process using by **Langevin equation** and **statistical model**. Neutrons can be evaporated at any stage of fission process. Neutron emission competes with nuclear shape changes.

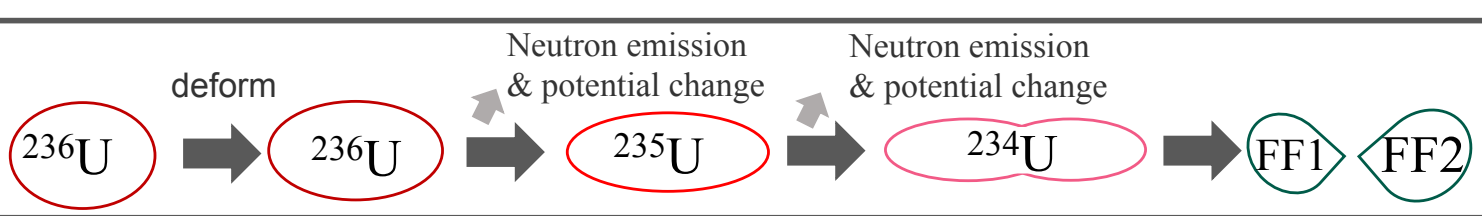


Figure 4: Present study of MCF calculation.

Models

Two-center Shell Model [5]

$$z = \frac{z_0}{BR}, \delta = \frac{3(a-b)}{2a+b}, \alpha = \frac{A_1-A_2}{A_1+A_2}, B = \frac{3+\delta}{3-2\delta}, R = r_0 A^{1/3}, r_0 = 1.2 \text{ [fm]}$$

z : two center distance, δ : deformation, α : mass asymmetry

Two-center parametrization: $q = \{z, \delta, \alpha\}$

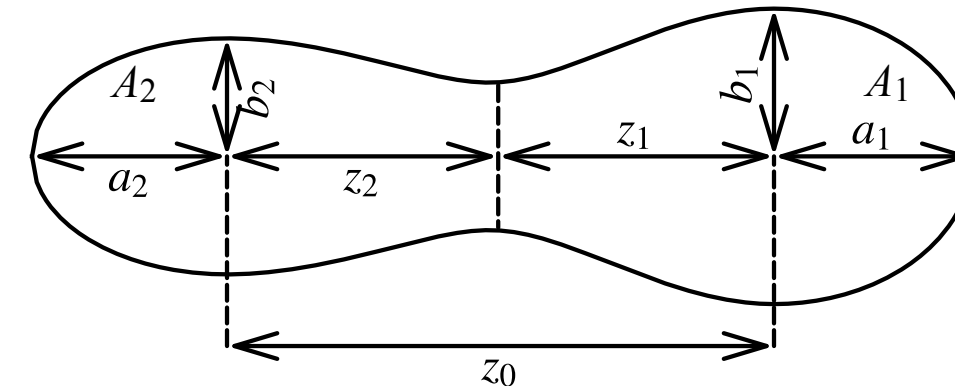


Figure 5: Two center Shell Model

Langevin Equation [7]

$$\begin{aligned} \frac{dq_i}{dt} &= (m^{-1})_{ij} p_j \\ \frac{dp_i}{dt} &= -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{d}{dt} (m^{-1})_{jk} p_j p_k \\ &\quad - \underbrace{\gamma_{ij} (m^{-1})_{jk} p_k}_{\text{dissipation}} + \underbrace{g_{ij} R_j(t)}_{\text{fluctuation}} \end{aligned}$$

$$\begin{aligned} \langle R_i(t) \rangle &= 0 \\ \langle R_i(t_1) R_j(t_2) \rangle &= 2\delta_{ij} \delta(t_1 - t_2) \quad (\text{white noise}) \\ \gamma_{ij} T &= \sum_k g_{ik} g_{jk} \quad (\text{Einstein relation}) \end{aligned}$$

Neutron emission

Langevin equation and statistical model are coupled to describe neutron emission during fission process [10].

Lifetime of neutron emission τ_n and time step of Langevin calculation Δt are competing. Neutron decay width Γ_n and level density ρ [11]

$$\begin{aligned} \frac{\Delta t}{\tau_n} &\geq \zeta, \quad \tau_n = \frac{\hbar}{\Gamma_n} \\ T_i &: \text{transmission probability, } U^*: \text{effective excitation energy,} \\ \sigma &: \text{spin dispersion parameter } (\sigma^2 = IT/\hbar^2), I: \text{moment of inertia,} \\ g_n &: 2s_n + 1 \quad (s_n: \text{spin of neutron } (=1/2)), \\ E_n &: \text{kinetic energy of evaporated neutron,} \\ \zeta &: \text{uniform random number } (0 \leq \zeta \leq 1) \end{aligned}$$

$$\begin{aligned} \Gamma_n &= \frac{1}{2\pi\rho(U^*, J)} g_n \\ &\quad \times \int_0^{U^*-B_n} \sum_\ell (2\ell+1) T_\ell(E_n) \rho(U^*-B_n-E_n, \ell) dE_n, \\ \rho(U^*, J) &= \frac{\pi}{12a_n^{1/4} U^{5/4}} \exp(2\sqrt{a_n U^*}) \\ &\quad \times \frac{2J+1}{2\sqrt{2\pi}\sigma^3} \exp\left\{-\frac{(J+1/2)^2}{2\sigma^2}\right\}. \end{aligned}$$

Result and discussion

Fission fragment mass distributions (FFMDs)

- FFMDs for 21 nuclides (²³⁴⁻²⁴⁰U, ²³⁶⁻²⁴⁰Np, and ²³⁸⁻²⁴⁴Pu) are shown with initial excitation energy range from $E^* = 15$ to 55 MeV (10 MeV bin).
- Calculations that do not take into account neutron emission agree at low excitation energies, but at high energies, **the distribution becomes a single peak**.
- When **neutron emission is taken into account**, the **distribution shows two peaks** even in the high-energy region, especially for neutron-rich nuclei.

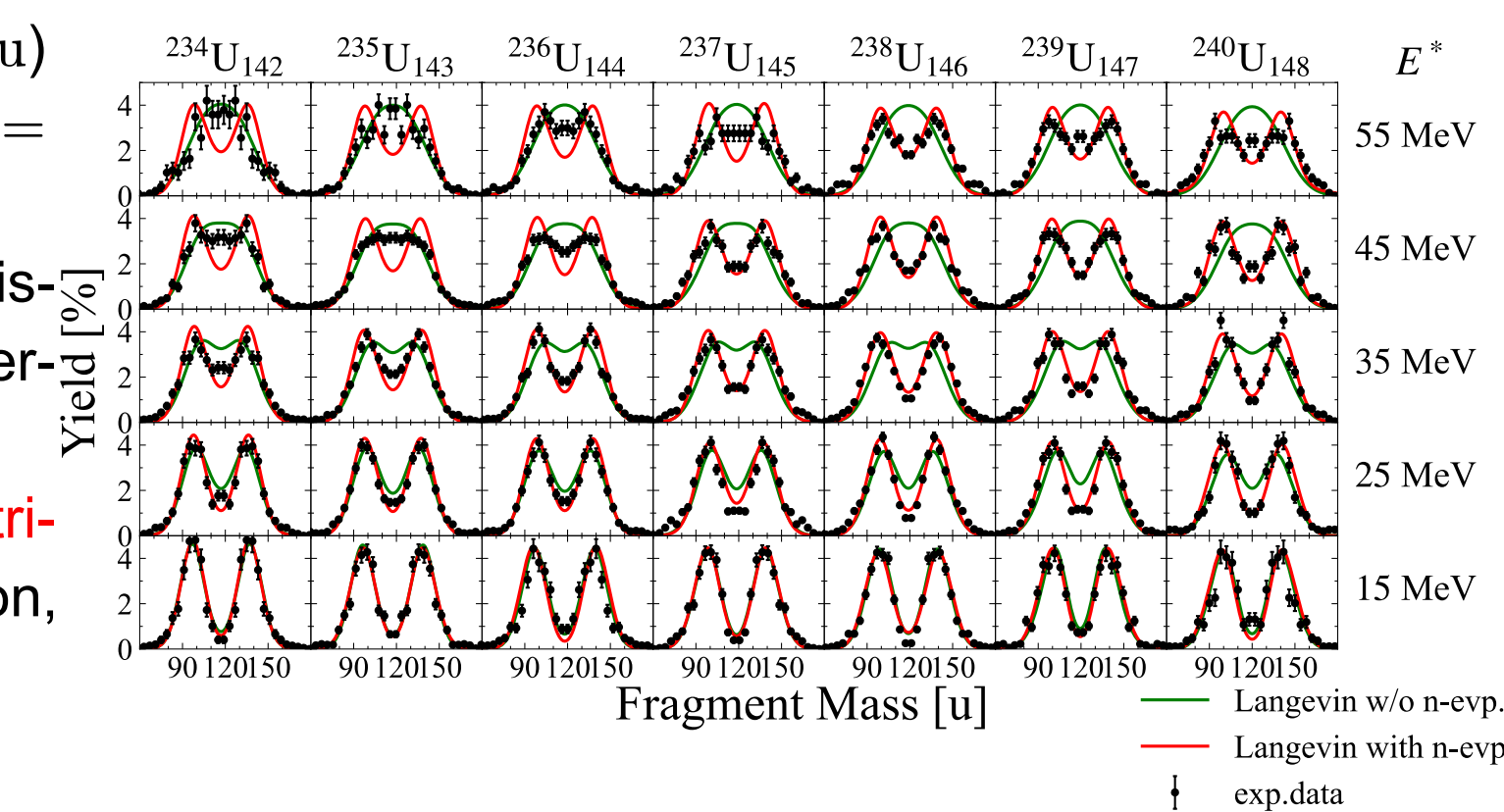


Figure 6: Fission fragment mass distributions for ²³⁴⁻²⁴⁰U, ²³⁶⁻²⁴⁰Np, and ²³⁸⁻²⁴⁴Pu with and without neutron emission compared with experimental data [1,2].

Precision neutron multiplicity

- The number of neutrons emitted **increases with the excitation energy and with the mass number** of the initial compound nucleus.
- At energies above 35 MeV, our Langevin calculations give higher multiplicities than in GEF calculations for all systems.
- Total number of neutron emission is almost same with GEF calculation up to $E^* = 35$ MeV for uranium and 25 MeV for neptunium and plutonium nuclides.

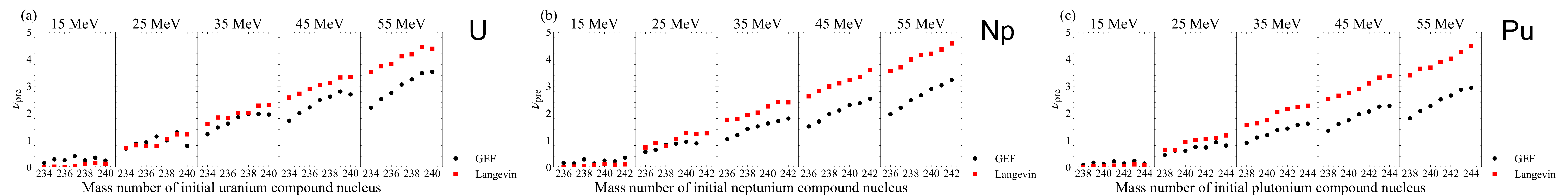


Figure 7: Comparison of the average number of neutrons emitted in fission of (a) ²³⁴⁻²⁴⁰U, (b) ²³⁶⁻²⁴⁰Np, and (c) ²³⁸⁻²⁴⁴Pu calculated by GEF [4] (black) and Langevin (red).

Probability of neutron emission

- The most of neutrons are emitted near the ground-state shape.
- The second minimum at $z = 0.7$ is the another neutron emission point.
- Neutrons are emitted at **any shape of nucleus from the saddle point to near the scission point of $z > 1$** .

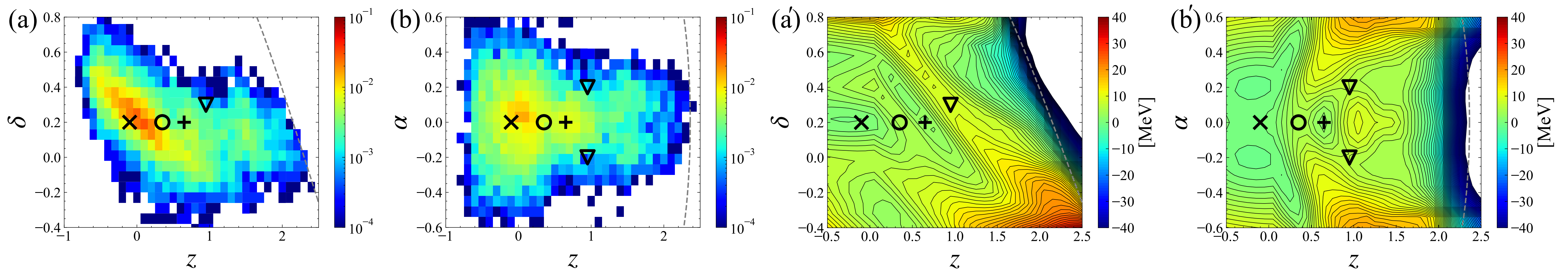


Figure 8: The left two figures show the probability of neutron emission in the deformation space of (a) (z, δ) and (b) (z, α) for ²³⁸U. The right two figures show the potential energy surface of ²³⁸U in the deformation space of (a') (z, δ) and (b') (z, α) . The gray line indicates the scission line.

Conclusion

- Fission fragment mass distributions are calculated in the Langevin equations for 21 U, Np, and Pu nuclei, and their excitation energy dependence is obtained from an initial excitation energy $E^* = 15$ to 55 MeV.
- Neutron emission is handled through **the evolution of nuclear shape from the ground state to the scission point**, in contrast to the usual statistical model.
- The present calculation treating neutron emission during fission demonstrates that measured FFMDs can adequately predict the FFMDs for high-energy fissions, thus can be **alternative to the traditional approach of treating multichance fission**.
- Concerning the origin point of neutrons, they are mostly emitted from **the ground-state shape**. However, neutrons are emitted in **all the shape of shape evolution down to the scission point**, by showing relatively large neutrons around the second minimum.

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