SU*(3) limit of the Interacting Boson Model and its relation to the realistic shell model

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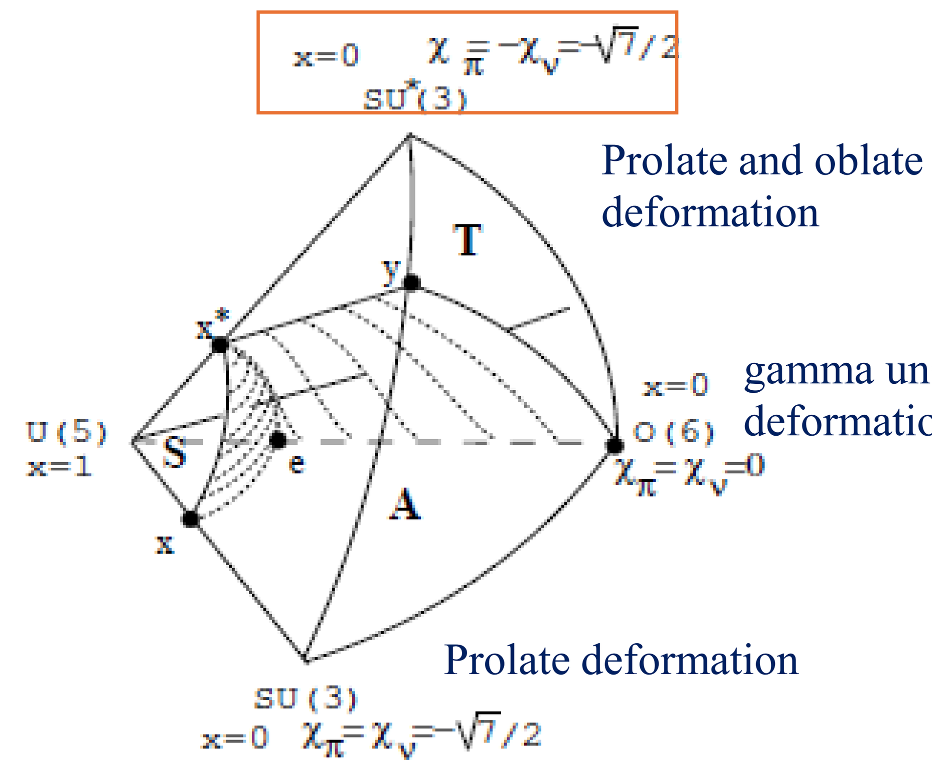
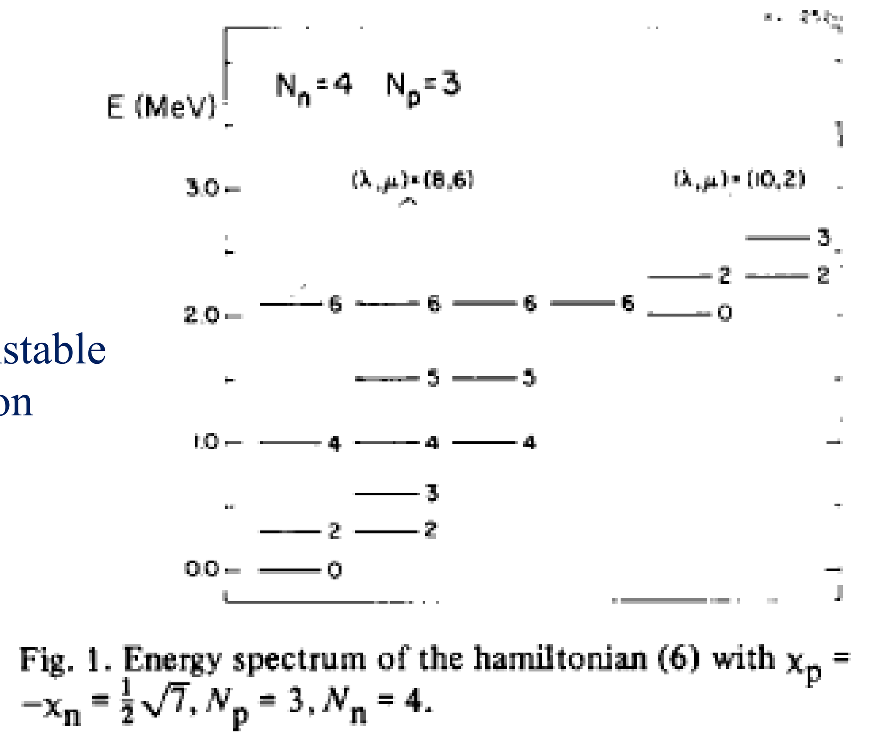
Previous research

Proton-neutron Interacting Boson Model (IBM2) : one of successful nuclear collective models [1]
U(5), SU(3), O(6) and SU(3)* symmetry limits in the extended symmetry triangle in IBM2
SU(3)* was discussed in [2] (1982) : triaxial nature
In 2004, renewed interests have been paid to symmetry triangle in IBM2: New symmetry limits are proposed. [4,5]

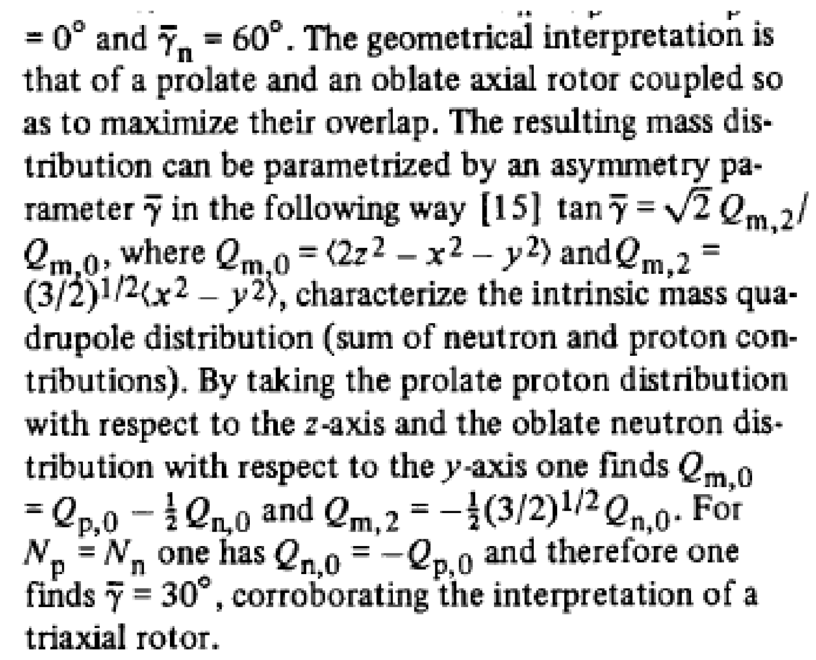


Phase diagram of IBM-2 Taken from reference [5]

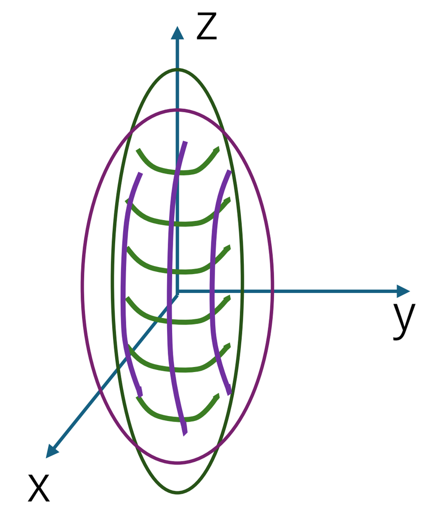


Example of energy level for SU(3)* Taken from reference [2]

This spectrum is unlikely in realistic nuclei but shows triaxial nature [2] below.


$= 0°$ and $\bar{\gamma}_n = 60°$. The geometrical interpretation is that of a prolate and an oblate axial rotor coupled so as to maximize their overlap. The resulting mass distribution can be parametrized by an asymmetry parameter $\bar{\gamma}$ in the following way [15] $\tan\bar{\gamma} = \sqrt{2}\,Q_{m,2}/Q_{m,0}$, where $Q_{m,0} = \langle 2z^2 - x^2 - y^2\rangle$ and $Q_{m,2} = (3/2)^{1/2}\langle x^2 - y^2\rangle$, characterize the intrinsic mass quadrupole distribution (sum of neutron and proton contributions). By taking the prolate proton distribution with respect to the z-axis and the oblate neutron distribution with respect to the y-axis one finds $Q_{m,0} = Q_{p,0} - \frac{1}{2}Q_{n,0}$ and $Q_{m,2} = -\frac{1}{2}(3/2)^{1/2}Q_{n,0}$. For $N_p = N_n$ one has $Q_{n,0} = -Q_{p,0}$ and therefore one finds $\bar{\gamma} = 30°$, corroborating the interpretation of a triaxial rotor.

Taken from reference [2]



However, IBM has no geometrical interpretation.

Present research

Revisit of SU(3)* limit in IBM-2 (present poster) and Nuclear shell model (near future presentation [6])
Method : Diagonalization, Mean-field, PES, Variation-Before-Projection, Variation-After-Projection
IBM-2 Hamiltonian

$$H_{IBM2} = -\kappa(Q_\pi(\chi_\pi) + Q_\nu(\chi_\nu))\cdot(Q_\pi(\chi_\pi) + Q_\nu(\chi_\nu)) \quad \text{where} \quad Q_\rho(\chi_\rho) = s_\rho^\dagger\widetilde{d_\rho} + d_\rho^\dagger\widetilde{s_\rho} + \chi_\rho(d_\rho^\dagger\widetilde{d_\rho})^{(2)} \quad \text{for} \quad \rho = \pi \text{ or } \nu$$

Mean-field coherent state wave function

$$|\psi\rangle = \frac{1}{\sqrt{N_\pi! N_\nu!}}\left[\frac{1}{\sqrt{1+\beta_\pi^2}}\left(s_\pi^\dagger + \beta_\pi cos\gamma_\pi d_{\pi 0}^\dagger + \frac{1}{\sqrt{2}}\beta_\pi\, sin\gamma_\pi\left(d_{\pi 2}^\dagger + d_{\pi -2}^\dagger\right)\right)\right]^{N_\pi}\left[\frac{1}{\sqrt{1+\beta_\nu^2}}\left(s_\nu^\dagger + \beta_\nu cos\gamma_\nu d_{\nu 0}^\dagger + \frac{1}{\sqrt{2}}\beta_\nu\, sin\gamma_\nu\left(d_{\nu 2}^\dagger + d_{\nu -2}^\dagger\right)\right)\right]^{N_\nu}|0\rangle$$

β_π , γ_π , β_ν, γ_ν are variational parameters in the wavefunction but are not β,γ of proton-neutron system. We define $Q = Q_\pi(\chi_\pi) + Q_\nu(\chi_\nu)$.

$$Q_0 = \frac{N_\pi}{1+\beta_\pi^2}\left(2\beta_\pi cos\gamma_\pi - \sqrt{\frac{2}{7}}\beta_\pi^2\chi_\pi\cos(2\gamma_\pi)\right) + \frac{N_\nu}{1+\beta_\nu^2}\left(2\beta_\nu cos\gamma_\nu - \sqrt{\frac{2}{7}}\beta_\nu^2\chi_\nu cos(2\gamma_\nu)\right),\ Q_{\pm 1} = 0$$

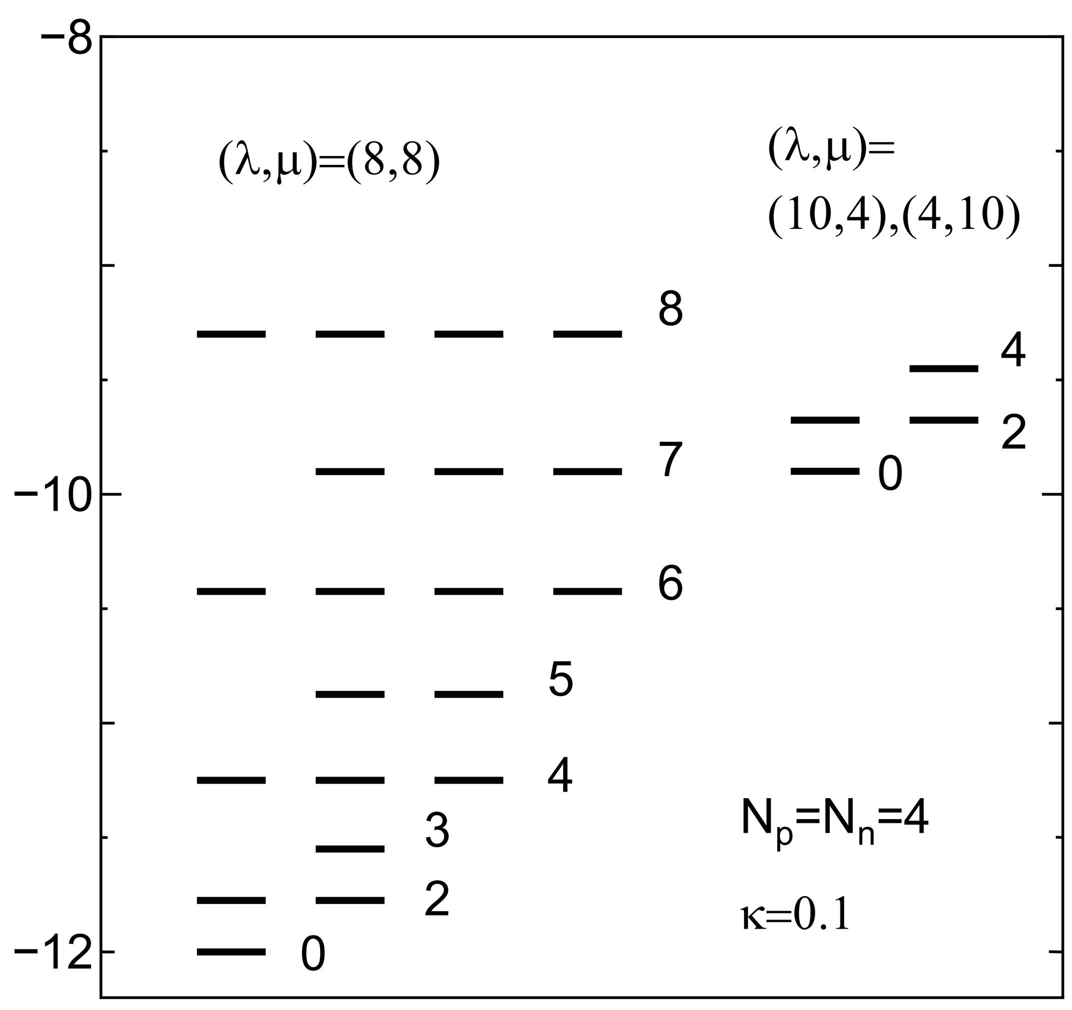
$$Q_{\pm 2} = \frac{N_\pi}{1+\beta_\pi^2}\left(\sqrt{2}\beta_\pi sin\gamma_\pi + \sqrt{\frac{1}{7}}\beta_\pi^2\chi_\pi\sin(2\gamma_\pi)\right) + \frac{N_\nu}{1+\beta_\nu^2}\left(\sqrt{2}\beta_\nu sin\gamma_\nu + \sqrt{\frac{1}{7}}\beta_\nu^2\chi_\nu sin(2\gamma_\nu)\right)$$

⇨ We can define β,γ of proton-neutron system [7] like shell model [8].
Prolate $\gamma_\pi = 0$ and oblate $\gamma_\nu = \frac{\pi}{3}$ yields triaxial $\gamma = \frac{\pi}{6.}$

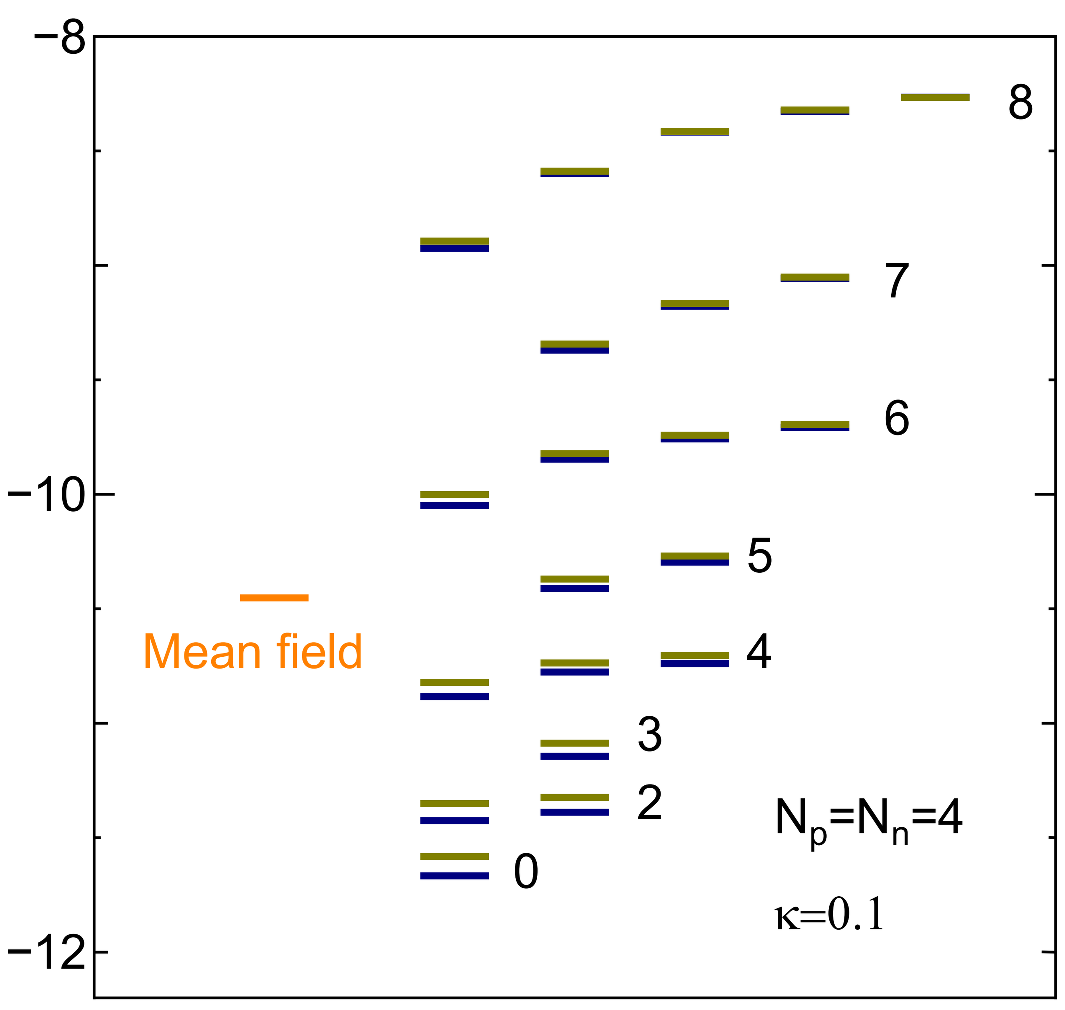
We introduce Variation-before-projection and Variation-after-projection beyond mean-field approximation [8]

$$E_{proj}^J = \frac{\langle\psi_J|H_{IBM2}|\psi_J\rangle}{\langle\psi_J|\psi_J\rangle} \quad \text{where} \quad |\psi_J\rangle = \sum g_{K'}P_{KK'}^J|\psi\rangle \quad \text{and} \quad P_{KK'}^J = \frac{2J+1}{8\pi^2}\int D_{KK'}^{J*}(\Omega)R(\Omega)d\Omega$$

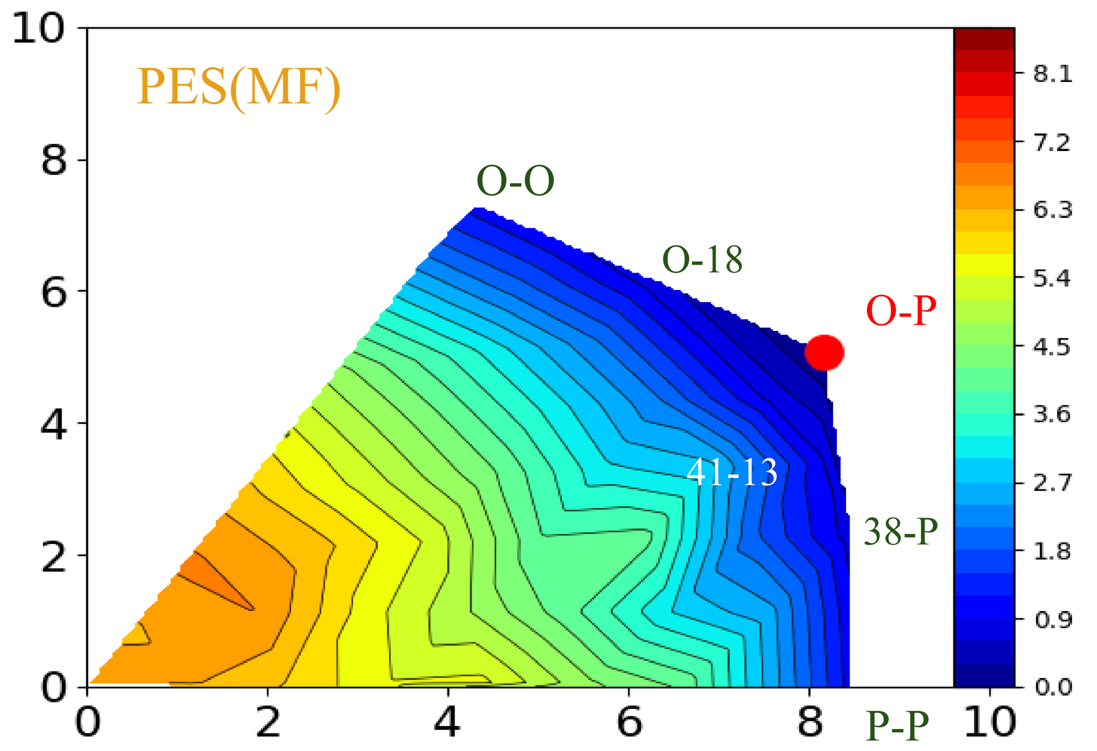
For example, SU(3)* states are projected out from prolate and oblate intrinsic state as below.

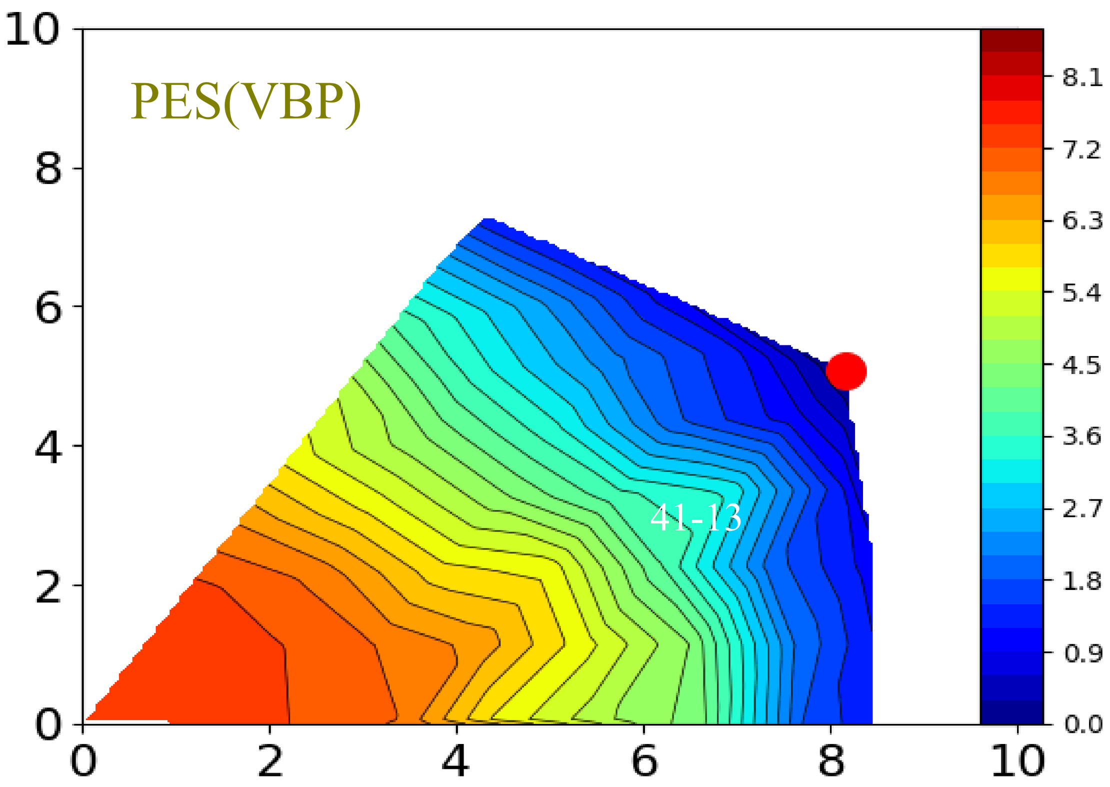


Exact Diag. / Group theory



Mean field VBP VAP





Stable triaxial deformation
~~gamma unstable deformation~~

Summary

Characteristic feature of SU(3)* limit in IBM2 including energy levels is clarified by Mean field, PES, VBP and VAP.
In IBM2, triaxial deformation with proton prolate and neutron oblate is confirmed to be realized in the SU(3)* limit.

Perspective

What are the implications when protons and neutrons favor different nuclear shapes?
Proton prolate and neutron oblate: (1) Triaxial deformation ? (2) Shape coexistence and shape driving effect ?
In nuclear shell model, we may expect exotic state.

References

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[7] T. Mizusaki, unpublished work.
[8] For example, T. Mizusaki et. al., Phys. Rev. C59, R1846 (1999).
[9] For example, P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).