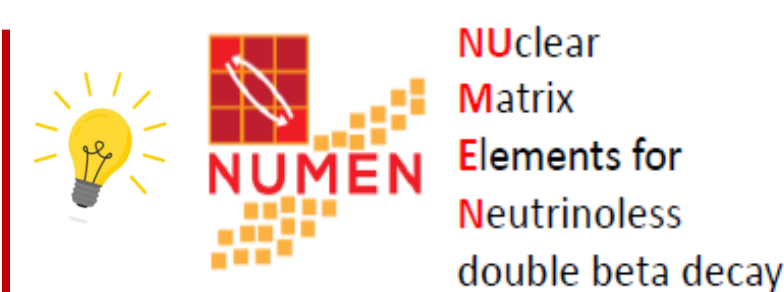
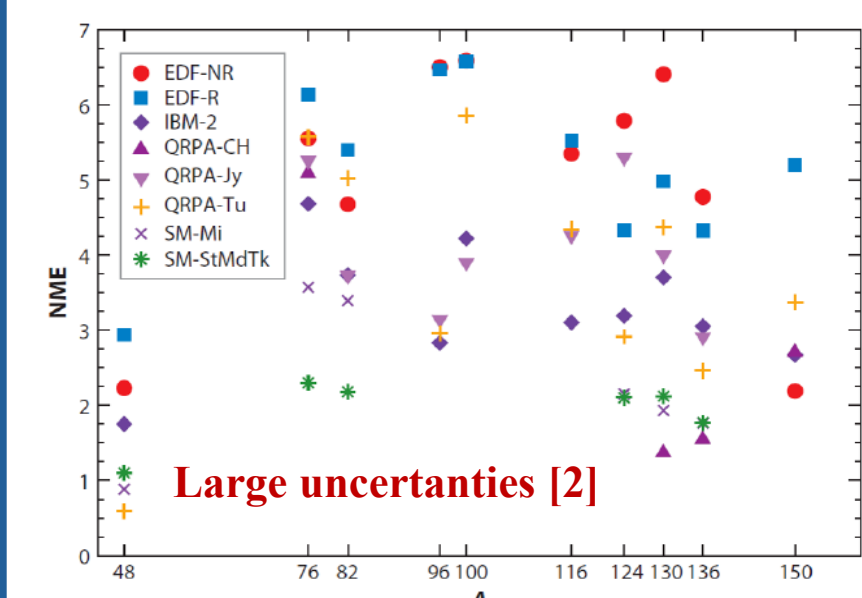


1. NUMEN PROJECT & DCE REACTION

decay time phase space factor neutrino effective mass

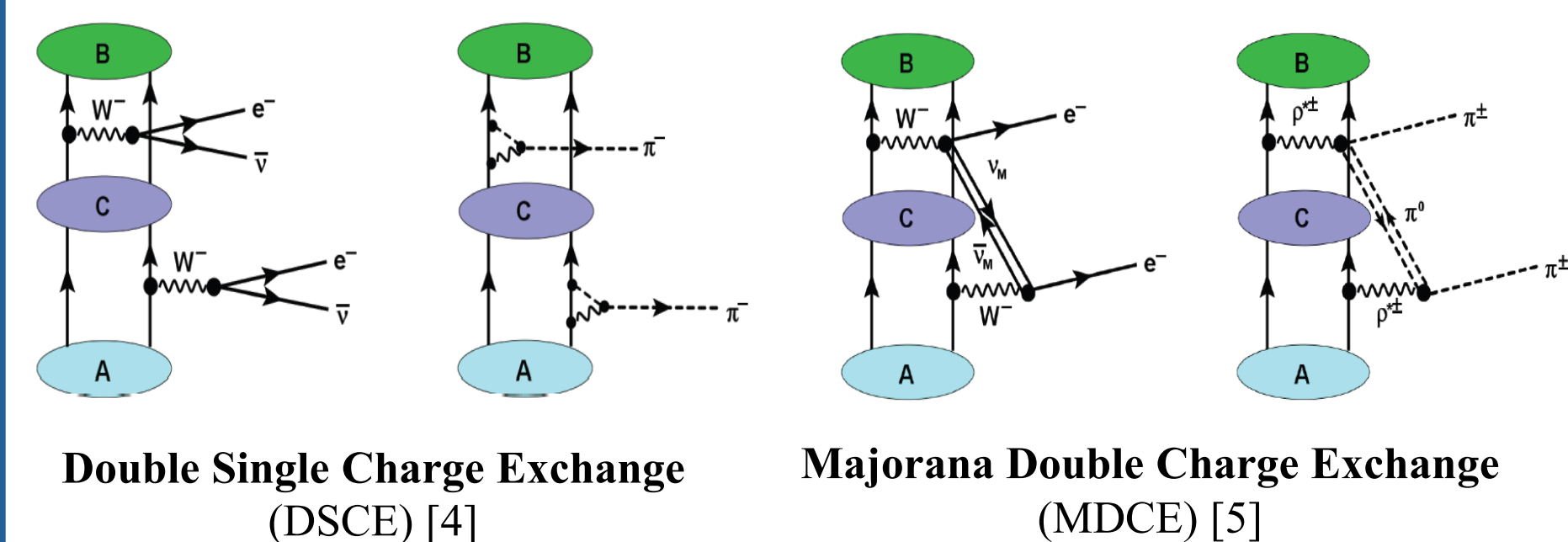
$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{0\nu} |M_{0\nu}|^2 \langle m_{\beta\beta} \rangle^2 \quad [1]$$

the $0\nu\beta\beta$ NME



[3]

$$M_{0\nu} = \langle \phi_f | \hat{O}^{0\nu\beta\beta} | \phi_i \rangle \longleftrightarrow M_{DCE} = \langle \phi_f | \hat{O}^{DCE} | \phi_i \rangle$$



There is also the **Multi-nucleon Transfer Double Charge Exchange (TDCE)** [6]

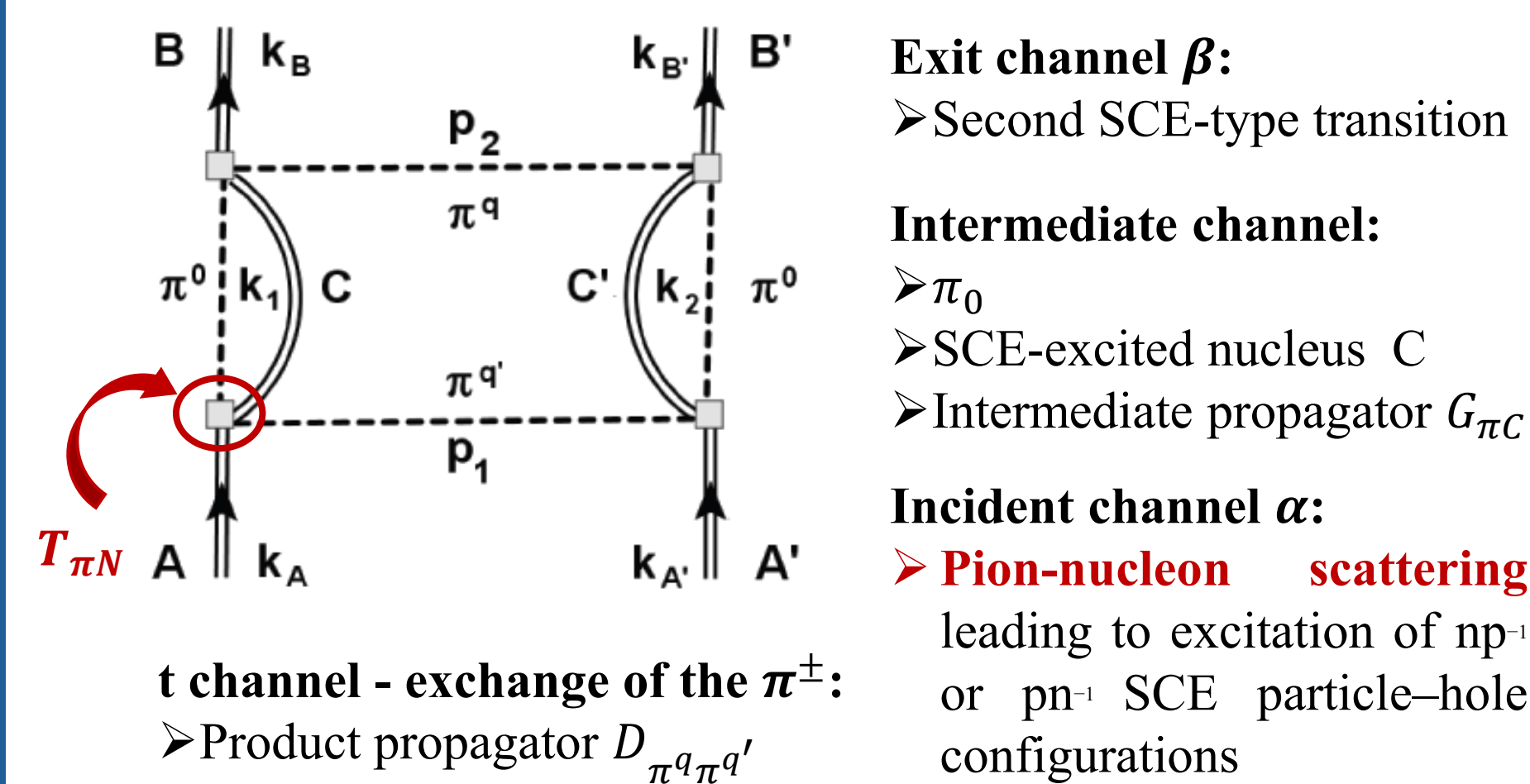
2. MDCE MECHANISM

DISTORTED WAVE REACTION AMPLITUDE

$$d\sigma_{\alpha\beta}^{(1)} \propto \sum_{M_A \in \alpha, M_B, M_C \in \beta} |M_{\alpha\beta}^{(1)}(\vec{k}_\alpha, \vec{k}_\beta)|^2 d\Omega_{\alpha\beta},$$

TRANSITION FORM FACTOR

$$M_{\alpha\beta}^{(1)}(\vec{k}_\alpha, \vec{k}_\beta) = \langle \chi_\beta^{(-)} | \langle B | T_{\pi N} G_{\pi C} T_{\pi N} | A \rangle D_{\pi q} D_{\pi q'} \langle B' | T_{\pi N} G_{\pi C'} T_{\pi N} | A' \rangle | \chi_\alpha^{(+)} \rangle$$



➤ Pion potentials

Strong counterpart of the neutrino potential:

$$U_\pi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} T_{\pi N} e^{i\vec{k}\cdot\vec{x}} g_{\pi C}^{(+)} T_{\pi N}$$

$$T_{\pi N}(\mathbf{p}, \mathbf{p}') = \left[\mathbf{T}_0(s_{\pi N}) + \frac{1}{m_\pi^2} (\mathbf{T}_1(s_{\pi N}) \mathbf{p} \cdot \mathbf{p}' + i \mathbf{T}_2(s_{\pi N}) \boldsymbol{\sigma} \cdot (\mathbf{p} \times \mathbf{p}')) \right] \mathbf{T}_\pi \cdot \boldsymbol{\tau}_N$$

➤ Transition Matrix Elements (TMEs)

$$W_{AB} = \langle B | T_{\pi N} G_{\pi C} T_{\pi N} | A \rangle \rightarrow W_{AB} \sim \sum_{ij} \mathcal{M}_{AB}^{(ij)} \quad \text{nine partial TMEs}$$

REFERENCES

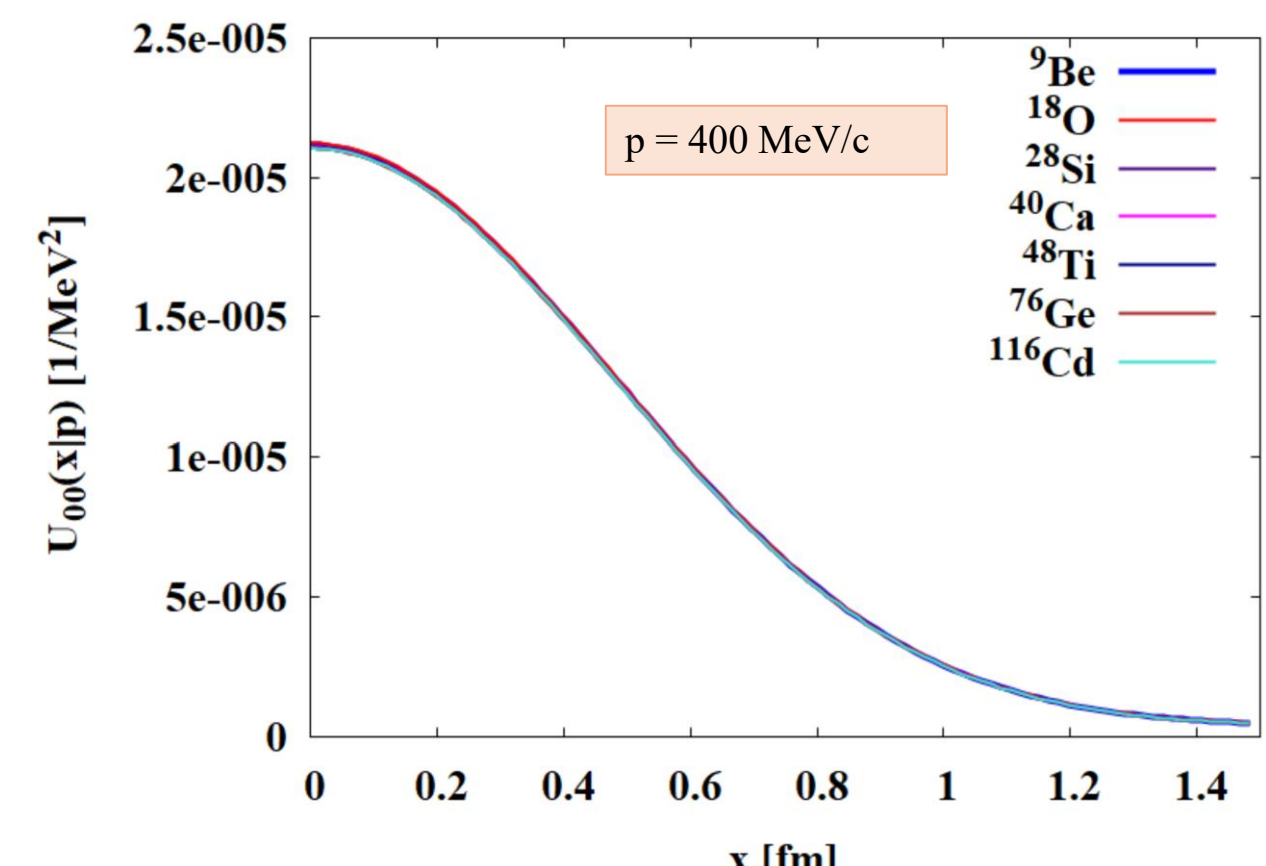
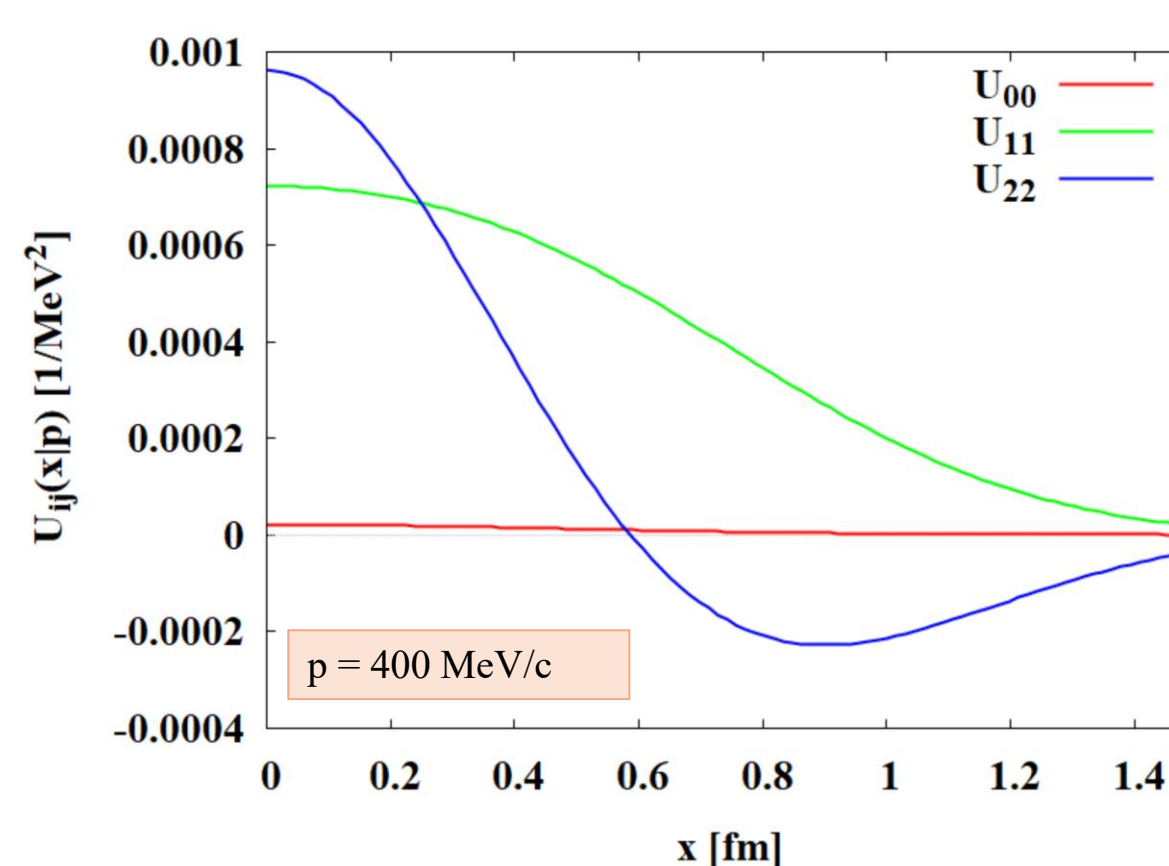
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- [3] F. Cappuzzello et al., EPJ A **54**, 72 (2018).
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3. NUMERICAL RESULTS

Note: U_π & TMEs were derived in **closure approximation** and using the **collinear approximation**.

➤ **P-wave** potential components dominate over the **S-wave** ones.

➤ **Short range – conjecture**



$$I_{ij}(n) = \int d^3r r^n U_{ij}(r)$$

$$\langle r \rangle_{ij} = \frac{\int d^3r r U_{ij}(r)}{\int d^3r U_{ij}(r)}$$

$$\langle r^2 \rangle_{ij} = \frac{\int d^3r r^2 U_{ij}(r)}{\int d^3r U_{ij}(r)}$$

$$v_{ij} = \langle r^2 \rangle_{ij} - \langle r \rangle_{ij}^2$$

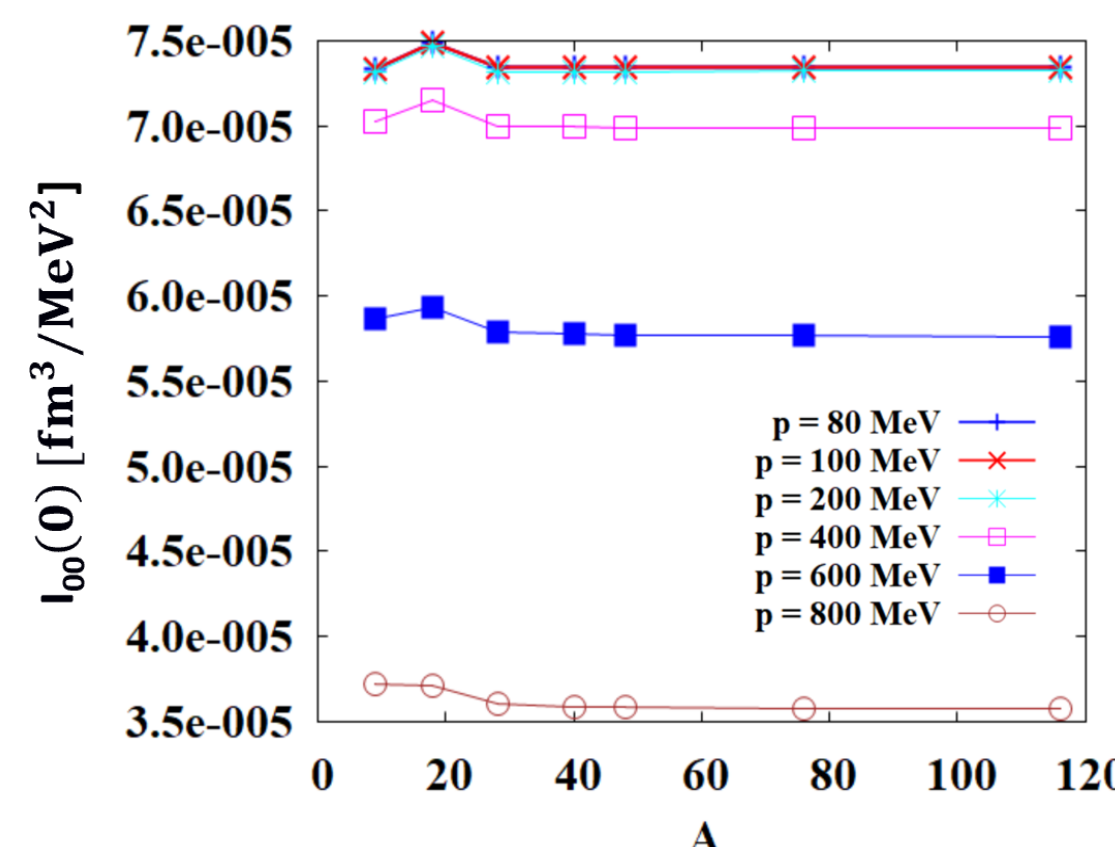
$n=0 \rightarrow$ strength of the potential
 $n=1, 2 \rightarrow$ potential spatial distribution

Normalized linear radial monopole moments → potential radius;

Quadratic radial monopole moments → extension of the potential with respect to the origin

Potential distribution variance → potential spatial extension

➤ Moments of pion potentials are near-independent with respect to the nuclear system



➤ The MDCE pion potential covers a radius of about **1 fm** & only a small dispersion around the mean value is present.

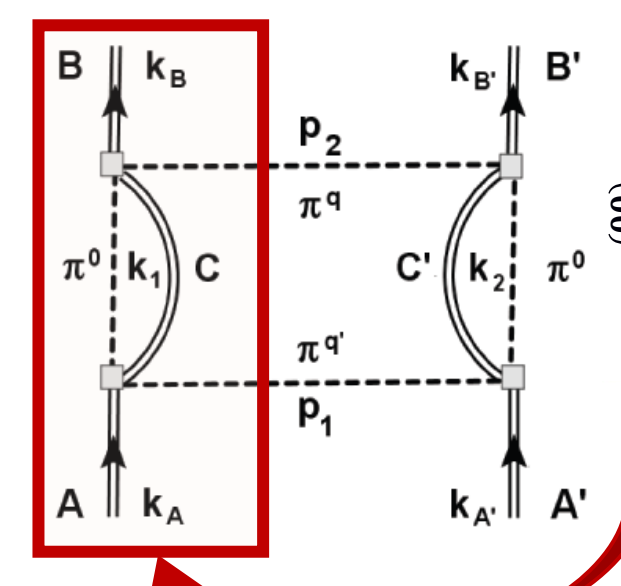
Components	$\langle \bar{r} \rangle_{ij}$ (fm)	v_{ij} (fm ²)
00	1.89	0.10
11	1.04	0.03
22	1.49	0.06
01	1.41	0.05
02	1.20	0.04
12	1.06	0.03

$\langle \bar{r} \rangle_{ij}$ is the mean value of the normalized moments with respect to A for each component, having averaged with respect to the momentum p.

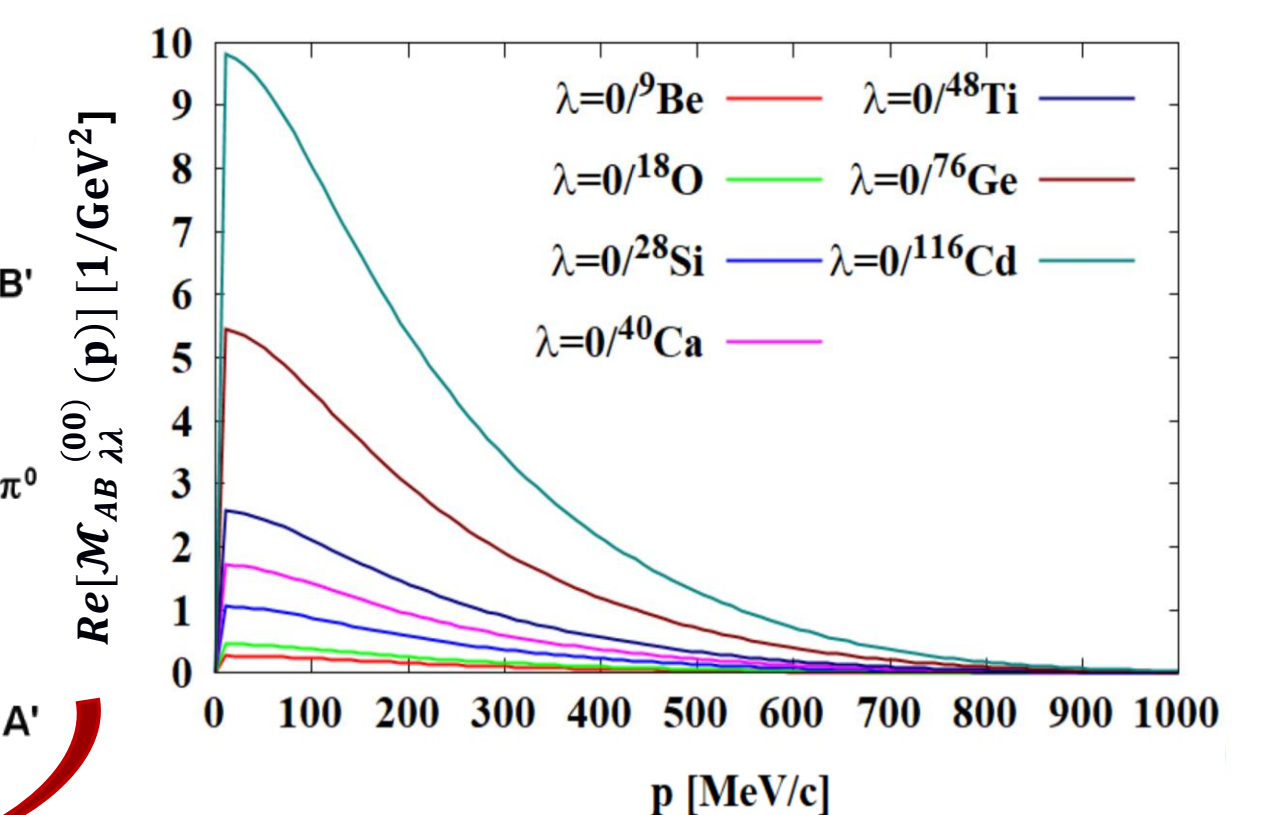
➤ The **P-wave** TME increase strongly with momentum, exceeding the strength of the **S-wave** TME by large factors.

➤ TMEs grow with the mass involved in the DCE transition

- ${}^9\text{Be} \rightarrow {}^9\text{He}$
- ${}^{18}\text{O} \rightarrow {}^{18}\text{Ne}$
- ${}^{28}\text{Si} \rightarrow {}^{28}\text{Mg}$
- ${}^{40}\text{Ca} \rightarrow {}^{40}\text{Ar}$
- ${}^{48}\text{Ca} \rightarrow {}^{48}\text{Ti}$
- ${}^{76}\text{Ge} \rightarrow {}^{76}\text{Zn}$
- ${}^{116}\text{Cd} \rightarrow {}^{116}\text{Pd}$



➤ The imaginary part of TMEs is of moderate strength



4. CONCLUSIONS & FUTURE PERSPECTIVES

➤ The pion potentials are of pronounced **short-range character** with radii of **~ 1 fm**;

➤ MDCE transitions proceed on **short-range NN correlations** as expected for $0\nu\beta\beta$ decay;

➤ MDCE is a universal mechanism becoming **independent of the nuclear system** for medium and heavy nuclei, at the level of a few per cent;

➤ Differential cross section to compare with the experimental data: **in progress!**

