



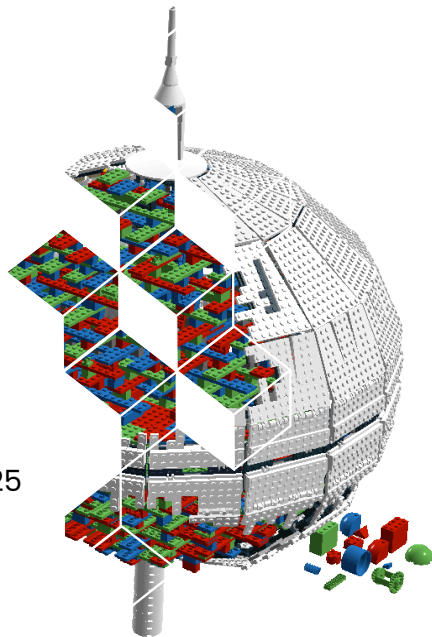
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Proton internal pressure from DVCS at the future EIC

Hervé Moutarde

European Nuclear Physics Conference 2025

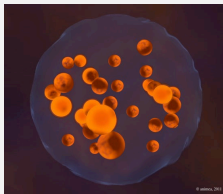
22 September 2025



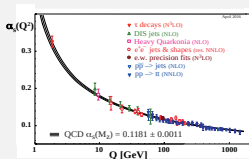
Hadron structure



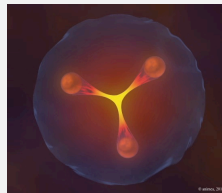
Perturbative QCD



Asymptotic freedom



Nonperturbative QCD



Interface between perturbative and nonperturbative regimes

- Define **universal** objects describing 3D hadron structure:
Generalized Parton Distributions (GPD).
- Relate GPDs to measurements using **factorization**:
Compton scattering (DVCS, TCS) or **meson production** (DVMP).
- Get **experimental knowledge** of hadron structure.

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1. Energy-momentum tensor

Energy-momentum tensor

- EMT defined from the invariance under space and time translations.
- **Quark** and **gluon** contributions

$$T_q^{\mu\nu} = \bar{q} \gamma^\mu \frac{i}{2} \overleftrightarrow{D} q$$
$$T_g^{\mu\nu} = -F^{\mu\lambda} F^\nu_{\lambda} + \frac{1}{4} \eta^{\mu\nu} F^2$$

with $\overleftrightarrow{D}$ the symmetric covariant derivative and $F^{\mu\nu}$ the field strength tensor.

- $T^{\mu\nu} = \sum_a T_a^{\mu\nu}$ ($a = q, g$).

Parameterization: massive spin-1/2 target

- **5 GFFs** in local, gauge-invariant, asymmetric EMT:

$$\begin{aligned} \langle p', s' | T_a^{\mu\nu}(0) | p, s \rangle = & \bar{u}(p', s') \left\{ \frac{P^\mu P^\nu}{M} A_a(t) + M \eta^{\mu\nu} \bar{C}_a(t) \right. \\ & + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t) \\ & + \frac{P^{\{\mu} i \sigma^{\nu\} \Delta}}{4M} [A_a(t) + B_a(t)] \\ & \left. + \frac{P^{[\mu} i \sigma^{\nu] \Delta}}{4M} D_a(t) \right\} u(p, s) \end{aligned}$$

with $P = (p' + p)/2$, $\Delta = p' - p$, $t = \Delta^2$ and polarizations s, s' . Shorthand notations: $a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$, $a^{[\mu} b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$, and $i \sigma^{\mu\Delta} = i \sigma^{\mu\lambda} \Delta_\lambda$

✍ Lorcé *et al.* (2019)

3D profile of GFFs

Localization in the Wigner sense

- Define distribution of a physical quantity inside a system, by first **localizing the system in both position and momentum** space.
- Breit frame where $P^\mu = (P^0, \vec{0})$ and $\Delta^\mu = (0, \vec{\Delta})$

$$\langle T_a^{\mu\nu} \rangle_{\text{BF}(\vec{r})} = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\vec{\Delta}\vec{r}} \left[\frac{\langle p', s | T_a^{\mu\nu}(0) | p, s \rangle}{2P^0} \right]_{\vec{P}=\vec{0}}$$

- Specific role of 3D Fourier transform of GFFs.

✍ Lorcé et al. (2019)

Definition of pressure

- **Matrix element** in the Breit frame ($a = q, g$):

$$\left\langle \frac{\Delta}{2} \left| T_a^{\mu\nu}(0) \right| - \frac{\Delta}{2} \right\rangle = M \left\{ \eta^{\mu 0} \eta^{\nu 0} \left[A_a(t) + \frac{t}{4M^2} B_a(t) \right] + \eta^{\mu\nu} \left[\bar{C}_a(t) - \frac{t}{M^2} C_a(t) \right] + \frac{\Delta^\mu \Delta^\nu}{M^2} C_a(t) \right\}$$

- Anisotropic fluid in **relativistic hydrodynamics**:

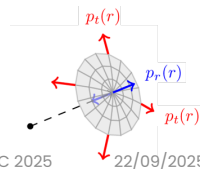
$$\Theta^{\mu\nu}(\vec{r}) = [\varepsilon(r) + p_t(r)] u^\mu u^\nu - p_t(r) \eta^{\mu\nu} + [p_r(r) - p_t(r)] \chi^\mu \chi^\nu$$

where u^μ and $\chi^\mu = x^\mu/r$ are unit timelike and spacelike orthogonal 4-vectors.

- Define **isotropic pressure** and **pressure anisotropy**:

$$p(r) = \frac{p_r(r) + 2 p_t(r)}{3}$$
$$s(r) = p_r(r) - p_t(r)$$

✍ Lorcé et al. (2019)



Pressure from gravitational form factors

- Write dictionary between quantum and fluid pictures:

$$\begin{aligned}\frac{\varepsilon_a(r)}{M} &= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2} [B_a(t) - 4C_a(t)] \right\} \\ \frac{p_{r,a}(r)}{M} &= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) - \frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d}{dt} \left(t^{3/2} C_a(t) \right) \right\} \\ \frac{p_{t,a}(r)}{M} &= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d}{dt} \left[t \frac{d}{dt} \left(t^{3/2} C_a(t) \right) \right] \right\} \\ \frac{p_a(r)}{M} &= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{2}{3} \frac{t}{M^2} C_a(t) \right\} \\ \frac{s_a(r)}{M} &= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} \left(t^{5/2} C_a(t) \right) \right\}\end{aligned}$$

✍ Lorcé et al. (2019)

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- Write dictionary between quantum and fluid pictures:

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✍ Lorcé et al. (2019)

Connection with experimental data

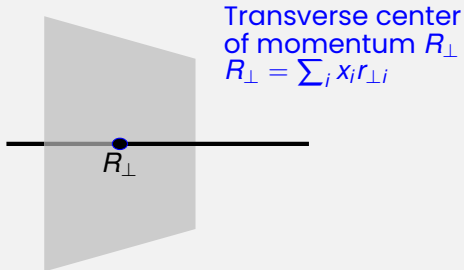
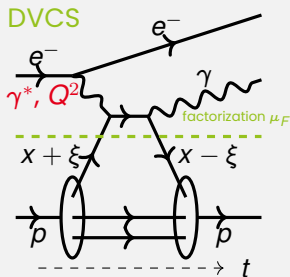
■ Link between GPDs and GFFs

$$\int dx x H^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - 4\xi^2 C^q(t)$$

✎ Ji (1997), ✎ Goeke (2001)

Deeply Virtual Compton Scattering (DVCS)



Connection with experimental data

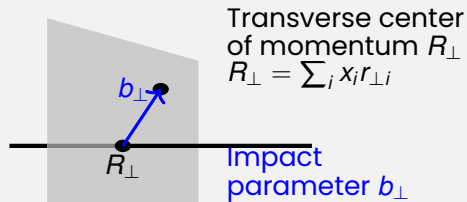
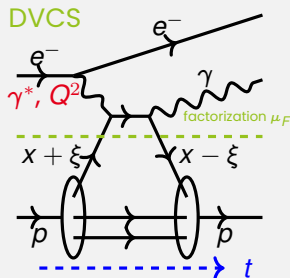
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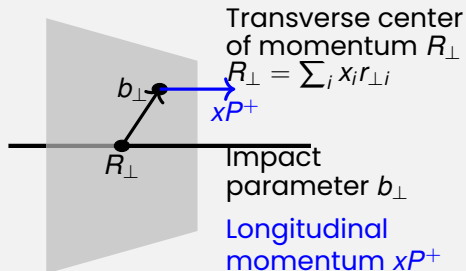
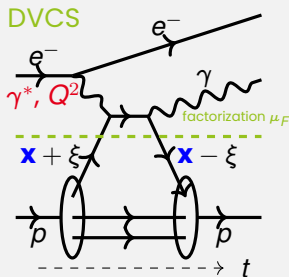
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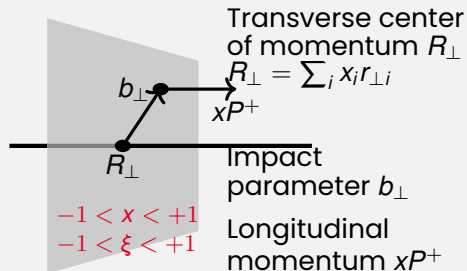
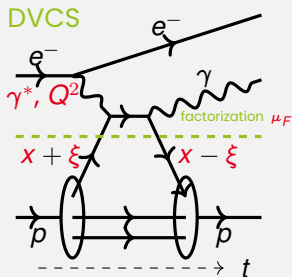
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✎ Ji (1997), ✎ Goeke (2001)

Deeply Virtual Compton Scattering (DVCS)



Compton Form Factors

Bjorken regime : large Q^2 and fixed $x_B \simeq 2\xi/(1 + \xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS.
- GPDs depend on a (arbitrary) factorization scale μ_F .
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^1 dx \, T\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F .

- Kernels T derived at NLO and (partially) NNLO.

✍ Belitsky and Müller (1998)

✍ Braun *et al.* (2022)

- CFF $\mathcal{F}$ is a **complex function**.



2. Phenomenology

What is the proton internal pressure?

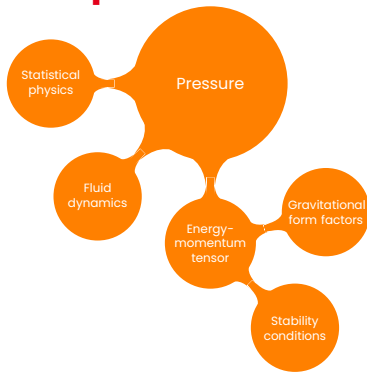
Pressure

Generalized
parton dis-
tributions

Measurements

DVCS

What is the proton internal pressure?

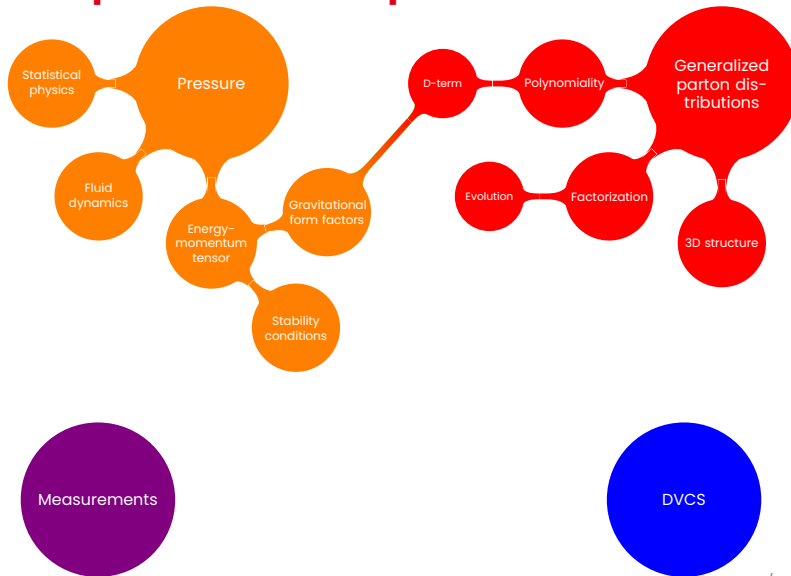


Generalized parton distributions

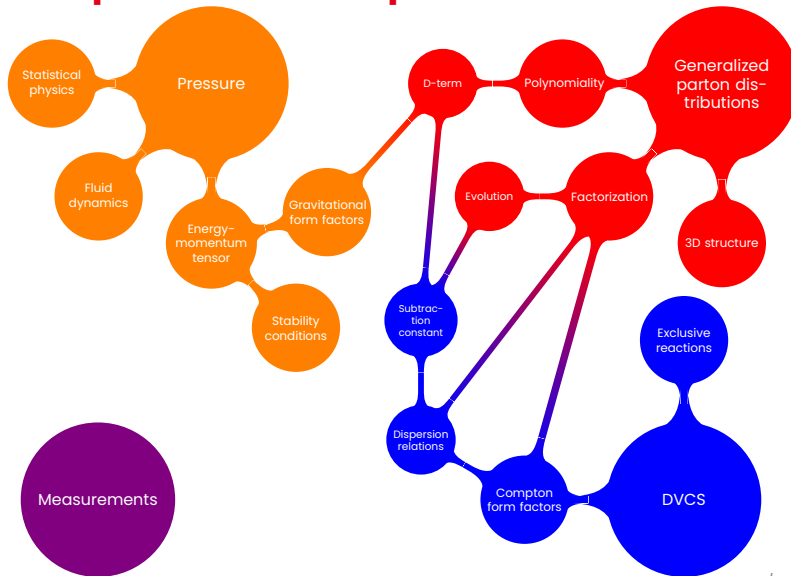
Measurements

DVCS

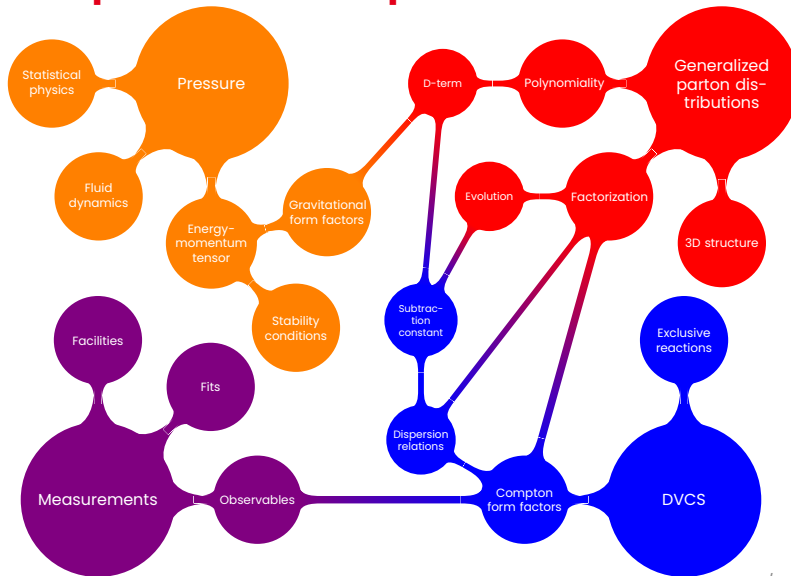
What is the proton internal pressure?



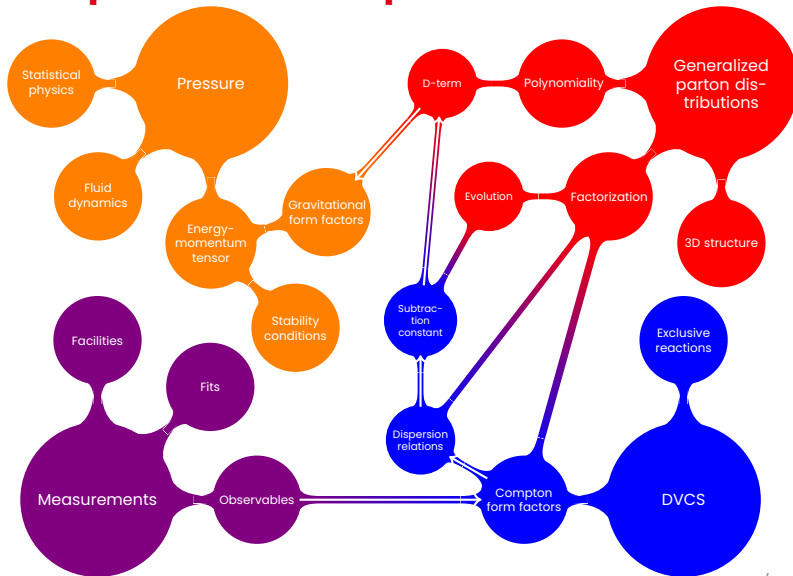
What is the proton internal pressure?



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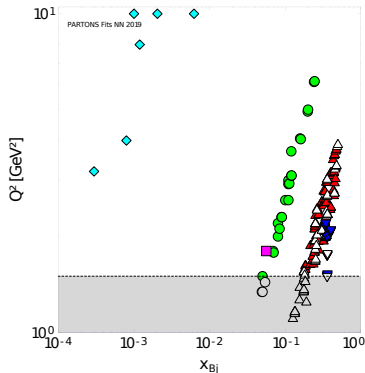
What is the proton internal pressure?



2600+ measurements, 30 observables, published 2001–2017

No.	Collab.	Year	Ref.	Observable	Kinematic dependence	No. of points used / all	
1	HERMES	2001	[40]	A_{LU}^+	ϕ	10 / 10	
2		2006	[41]	$A_C^{\cos i\phi}$	t	4 / 4	
3		2008	[42]	$A_C^{\cos i\phi}$	x_{BJ}	18 / 24	
				$A_{UT,DVCS}^{\sin(\phi-\phi_S)\cos i\phi}$	$i = 0$		
				$A_{UT,I}^{\sin(\phi-\phi_S)\cos i\phi}$	$i = 0, 1$		
				$A_{UT,I}^{\cos(\phi-\phi_S)\sin i\phi}$	$i = 1$		
4		2009	[43]	$A_{LU,I}^{\sin i\phi}$	$i = 1, 2$	x_{BJ}	35 / 42
$A_{LU,DVCS}^{\sin i\phi}$				$i = 1$			
$A_C^{\cos i\phi}$				$i = 0, 1, 2, 3$			
5		2010	[44]	$A_{UL}^{+, \sin i\phi}$	$i = 1, 2, 3$	x_{BJ}	18 / 24
$A_{LL}^{+, \cos i\phi}$				$i = 0, 1, 2$			
6		2011	[45]	$A_{LT,DVCS}^{\cos(\phi-\phi_S)\cos i\phi}$	$i = 0, 1$	x_{BJ}	24 / 32
$A_{LT,DVCS}^{\sin(\phi-\phi_S)\sin i\phi}$				$i = 1$			
$A_{LT,I}^{\cos(\phi-\phi_S)\cos i\phi}$				$i = 0, 1, 2$			
$A_{LT,I}^{\sin(\phi-\phi_S)\sin i\phi}$				$i = 1, 2$			
7		2012	[46]	$A_{LU,I}^{\sin i\phi}$	$i = 1, 2$	x_{BJ}	35 / 42
$A_{LU,DVCS}^{\sin i\phi}$				$i = 1$			
$A_C^{\cos i\phi}$				$i = 0, 1, 2, 3$			
8	CLAS	2001	[47]	$A_{LU}^{-, \sin i\phi}$	$i = 1, 2$	—	0 / 2
9		2006	[48]	$A_{UL}^{-, \sin i\phi}$	$i = 1, 2$	—	2 / 2
10	Hall A	2008	[49]	A_{LU}^-	ϕ		283 / 737
11		2009	[50]	A_{LU}^-	ϕ		22 / 33
12		2015	[51]	$A_{LU}^-, A_{UL}^-, A_{LL}^-$	ϕ		311 / 497
13		2015	[52]	$d^4\sigma_{UU}^-$	ϕ		1333 / 1933
14		2015	[34]	$\Delta d^4\sigma_{LU}^-$	ϕ		228 / 228
15		2017	[35]	$\Delta d^4\sigma_{LU}^+$	ϕ		276 / 358
16		COMPASS	2018	[36]	$d^3\sigma_{UU}^+$	t	
17	ZEUS	2009	[37]	$d^3\sigma_{UU}^+$	t		4 / 4
18	H1	2005	[38]	$d^3\sigma_{UU}^+$	t		7 / 8
19		2009	[39]	$d^3\sigma_{UU}^\pm$	t		12 / 12
SUM:						2624 / 3996	

2600+ measurements, 30 observables, published 2001-2017



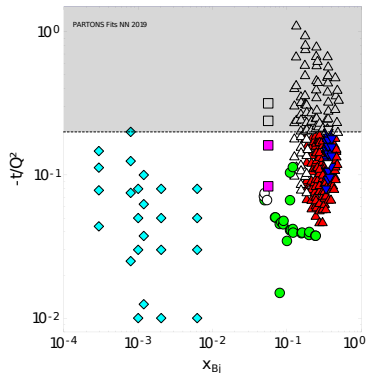
▼ Hall A

● HERMES

■ COMPASS

▲ CLAS

◆ H1 and ZEUS

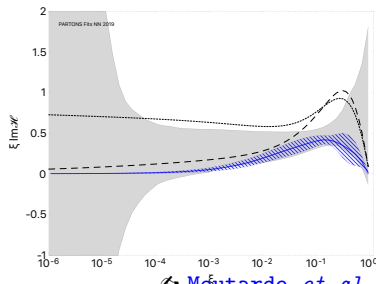
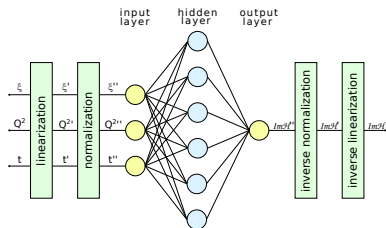


 Moutarde *et al.* (2019)

Need many more precise data at small x_B , large Q^2 and also as close as possible to the valence region.

Modeling of $\mathcal{H}$, $\tilde{\mathcal{H}}$, $\mathcal{E}$ and $\tilde{\mathcal{E}}$

- Real and imaginary parts of CFFs parameterized by **neural networks**.
- Propagation of uncertainties through **replica method** and evaluation of 68 % **confidence levels**.



 Moutarde *et al.* (2019)

Pressure forces from DVCS measurements

First-principle connection

1. Expand D-term on Gegenbauer polynomials

$$D_{\text{term}}^q(z, t, \mu_F^2) = (1 - z^2) \sum_{\text{odd } n} d_n^q(t, \mu_F^2) C_n^{3/2}(z)$$

2. Write dispersion relation for CFF (true at all pQCD orders)

$$C_H(t, Q^2) = \text{Re}\mathcal{H}(\xi) - \frac{1}{\pi} \int_0^1 d\xi' \text{Im}\mathcal{H}(\xi') \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$

3. Compute subtraction constant

$$C_H^{q,g}(t, Q^2) = \frac{2}{\pi} \int_1^{+\infty} d\omega \text{Im} T^{q,g}(\omega) \int_{-1}^1 dz \frac{D^{q,g}(z)}{\omega - z}$$

Diehl & Ivanov (2007)
Dutrieux et al. (2024)

📖 See V. Martinez-Fernandez's talk this afternoon!

4. Retrieve GFF

$$d_1^q(t, \mu_F^2) = 5C_q(t, \mu_F^2)$$

Pressure forces from DVCS measurements

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3. Compute subtraction constant at LO

$$C_H(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu_F^2 \equiv Q^2)$$

🔗 Diehl & Ivanov (2007)

🔗 Dutrieux *et al.* (2024)

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GFF extracting chain from experimental data

Experimental data

GFF C
Internal pressure

GFF extracting chain from experimental data

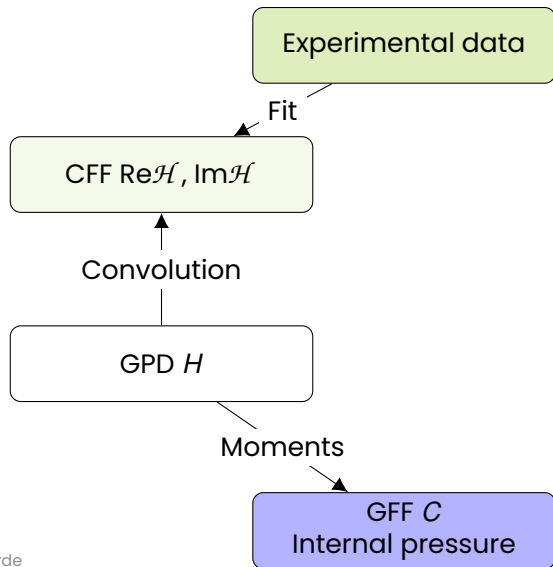
Experimental data

GPD H

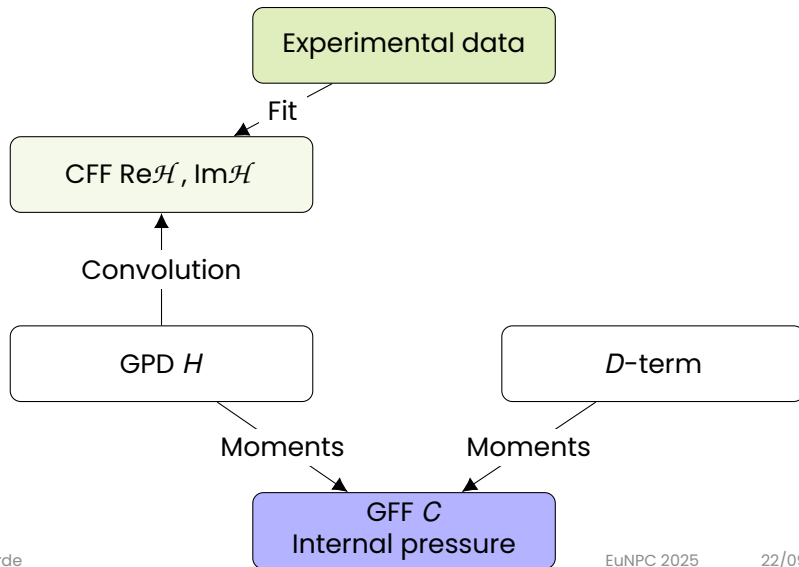
Moments

GFF C
Internal pressure

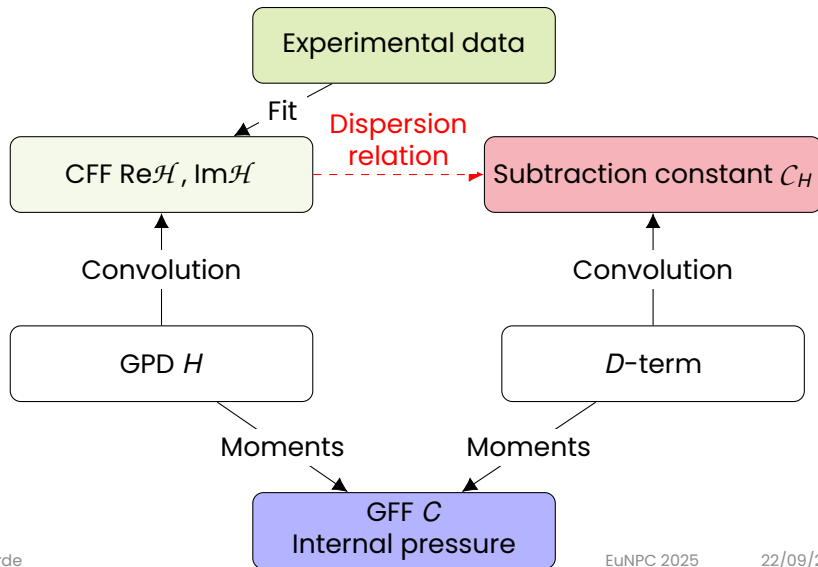
GFF extracting chain from experimental data



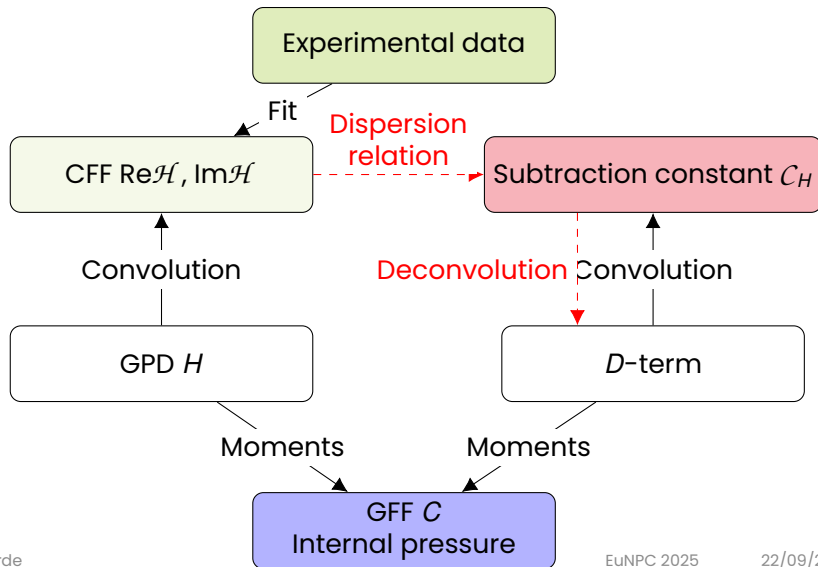
GFF extracting chain from experimental data



GFF extracting chain from experimental data



GFF extracting chain from experimental data



An aside on the deconvolution problem

- Assume CFF $\mathcal{H}$ is perfectly known. Solve inverse problem?

$$\mathcal{H}^q(\xi, Q^2) = \int_{-1}^1 \frac{dx}{\xi} T^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) H^q(x, \xi, \mu^2)$$

- Question raised about 20 years ago and has remained essentially open. Evolution proposed as a crucial element.

✍ Freund (2000)

- There exist **non-zero** GPDs with **vanishing forward limit** and **vanishing CFF** up to order α_s^2 .
- The DVCS deconvolution problem is **ill-posed**.

✍ Bertone *et al.* (2021)

- **Same conclusion holds** for several other hard exclusive processes. **Key role** of DDVCS: direct access to GPDs.
- **Large Q^2 lever arm** and **highly precise data** are much needed: unique feature of EIC.

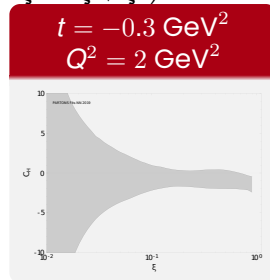
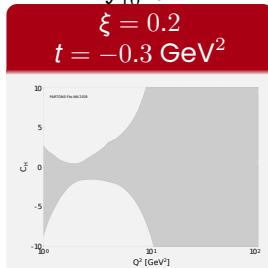
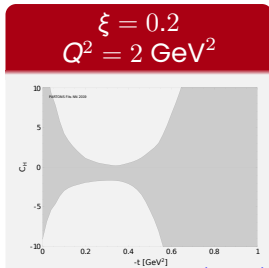
Subtraction constant from measurements.

- Range of kinematic variables in neural networks

$$\begin{array}{rclcl} 10^{-6} & < & \xi & < & 1 \\ 0 & < & -t & < & 1 \text{ GeV}^2 \\ 1 & < & Q^2 & < & 100 \text{ GeV}^2 \end{array}$$

- Implement DVCS dispersion relation

$$C_H(t, Q^2) = \text{Re}\mathcal{H}(\xi) - \frac{1}{\pi} \int_{10^{-6}}^1 d\xi' \text{Im}\mathcal{H}(\xi) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$



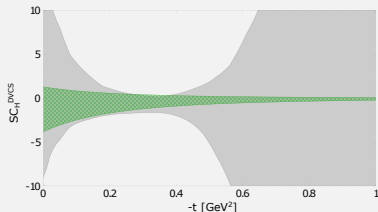
 Moutarde *et al.* (2019)

Pressure forces from DVCS measurements

Working assumptions

1. Subtraction constant assumed equal to d_1 .
2. Equal values for light quark contributions d_1^{uds} .
3. Radiative generation of gluon d_1^g and charm d_1^c contributions.
4. Tripole Ansatz for the t -dependence of d_1 .

Tripole Ansatz



Parameter	Value
$d_1^{uds}(\mu_F^2)$	-0.45 ± 0.92
$d_1^c(\mu_F^2)$	-0.0020 ± 0.0041
$d_1^g(\mu_F^2)$	-0.6 ± 1.3

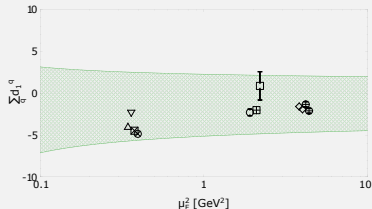
 [Dutrieux et al. \(2021\)](#)

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d_1 from DVCS data



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Summary of existing determinations

No.	Marker in Fig. 3	$\sum_q d_1^q(\mu_P^2)$	μ_P^2 in GeV^2	# of flavours	Type	Ref.
1	○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data	[13]
2	□	0.88 ± 1.69	2.2	2	from experimental data	[14]
3	◇	-1.59	4	2	t -channel saturated model	[55]
		-1.92	4	2	t -channel saturated model	[55]
4	△	-4	0.36	3	χ QSM	[30]
5	▽	-2.35	0.36	2	χ QSM	[10]
6	⊠	-4.48	0.36	2	Skyrme model	[56]
7	⊞	-2.02	2	3	LFWF model	[57]
8	⊗	-4.85	0.36	2	χ QSM	[58]
9	⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)	[59]
		-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)	[59]

✉ Dutrieux *et al.* (2021)

Relaxing some modelling assumptions

- **No justification to truncate the subtraction constant expansion to its first term** and assume that it is the d_1 coefficient related to the energy-momentum tensor.

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Relaxing some modelling assumptions

- **No justification to truncate the subtraction constant expansion to its first term** and assume that it is the d_1 coefficient related to the energy-momentum tensor.
- Leading contributions of d_1 and d_3 , higher order terms neglected.
- 3 active quark flavors (uds), and radiative generation of c contribution.
- 4 parameters

$$d_1^{uds}(\mu_0) \quad d_3^{uds}(\mu_0) \quad d_1^g(\mu_0) \quad d_3^g(\mu_0)$$

From leading order to next-to-leading order

Investigate 3 fitting scenarios

4
parameters

$$d_1^{uds}(0.1\text{GeV}^2)$$

$$d_1^g(0.1\text{GeV}^2)$$

$$d_3^{uds}(0.1\text{GeV}^2)$$

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$$d_3^{uds}(0.1\text{GeV}^2)$$

$$d_3^g(0.1\text{GeV}^2)$$

Free

Fixed

1. Nominal fit:

- 1 free parameter for light quarks.
- Gluon and charm radiatively generated.

	LO	NLO
$d_1^{uds}(2\text{GeV})$	-0.6 ± 1.1	-0.7 ± 1.3
$d_1^g(2\text{GeV})$	-0.8 ± 1.6	-0.9 ± 1.8
$d_1^c(2\text{GeV})$	-0.003 ± 0.005	-0.003 ± 0.006

 Dutrieux *et al.* (2024)

From leading order to next-to-leading order

Investigate 3 fitting scenarios

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$$d_1^{uds}(0.1\text{GeV}^2)$$

$$d_1^g(0.1\text{GeV}^2)$$

$$d_3^{uds}(0.1\text{GeV}^2)$$

$$d_3^g(0.1\text{GeV}^2)$$

Free

Fixed

2. Alternative fit:

- 2 free parameters: light quarks and gluons.
- Charm radiatively generated.

	LO	NLO
$d_1^{uds}(2\text{GeV})$	-0.6 ± 1.1	-1.1 ± 7.7
$d_1^g(2\text{GeV})$	-11 ± 132	-6 ± 78
$d_1^c(2\text{GeV})$	-0.04 ± 0.47	-0.02 ± 0.27

 Dutrieux *et al.* (2024)

From leading order to next-to-leading order

Investigate 3 fitting scenarios

4
parameters

$$d_1^{uds}(0.1\text{GeV}^2)$$

$$d_1^g(0.1\text{GeV}^2)$$

$$d_3^{uds}(0.1\text{GeV}^2)$$

$$d_3^g(0.1\text{GeV}^2)$$

Free

Fixed

3. Alternative fit:

- 2 free parameters: d_1 and d_3 for light quarks.
- Gluon and charm radiatively generated.

	LO	NLO
$d_1^{uds}(2\text{GeV})$	-2.1 ± 26.6	-1.7 ± 21
$d_1^g(2\text{GeV})$	-2.9 ± 37	-2 ± 30
$d_3^{uds}(2\text{GeV})$	1.5 ± 26.5	0.7 ± 15
$d_3^g(2\text{GeV})$	0.2 ± 4.1	0.1 ± 2.3

 Dutrieux *et al.* (2024)

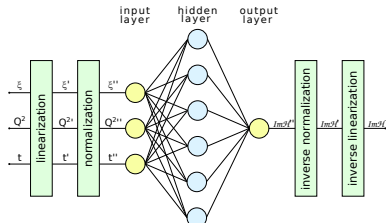


3. Areas for improvement

Increase the physics input in the global fit.

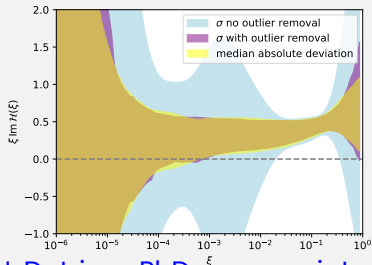
- So far the CFF fit gathering most of the world DVCS measurements relies on an **independent modeling** of the CFF real and imaginary parts by **neural networks**.
- Convenient because of the **dimensionality** of the problem but yields **large statistical uncertainties**.
✍ Moutarde et al. (2019)
- Conversely a fit to the same data with a **physically motivated** parameterization still required **ad hoc assumptions**.
✍ Moutarde et al. (2018)
- Many **first-principle constraints** expressed at the GPD level are not implemented at the CFF level.

Increase the physics input in the global fit.

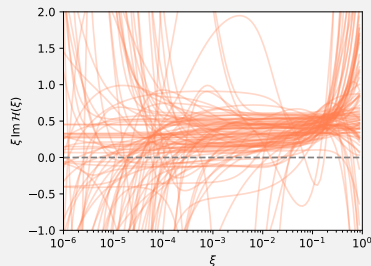


- Polynomiality?
- Positivity?
- Reduction to PDF or EFF?
- Evolution?

Outliers



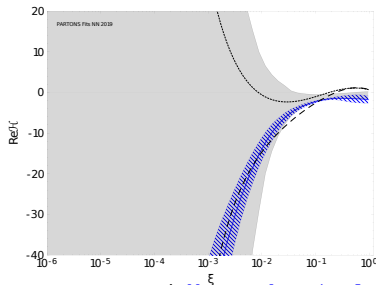
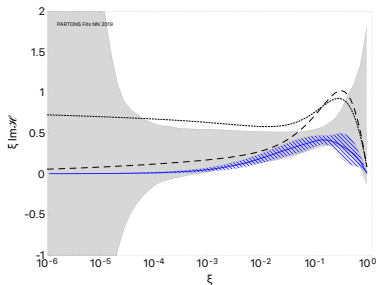
Replicas



From H. Dutrieux PhD manuscript

Increase the physics input in the global fit.

- Next step requires a (challenging) **GPD global fit** to world data.
- On the long run, need more experimental data to
 - Increase the Q^2 -**lever arm**.
 - Provide a better handle on the real part of $\mathcal{H}$.
 - Improve the **accuracy** of existing measurements.
 - Probe the kinematic regions insufficiently constrained.



 Moutarde *et al.* (2019)

Relax modeling assumptions on $d_1(t)$.

- Use **multipole Ansatz**

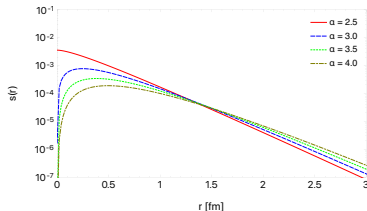
$$d_1(t, \mu_F) = \frac{d_1(\mu_F)}{\left(1 - \frac{t}{\Lambda^2}\right)^\alpha}$$

- Remind $d_1^q(t, \mu_F^2) = 5C_q(t, \mu_F^2)$.
- Plug in pressure anisotropy

$$\frac{s(r)}{M} \propto \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} \left(t^{5/2} d_1(t) \right) \right\}$$

- Normalization $d_1(\mu_F)$ set by fit.
- Position of node in r depends on Λ .

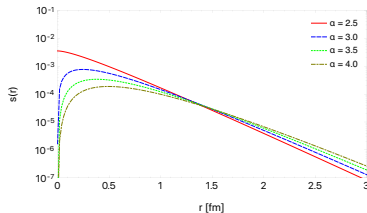
📄 [Dutrieux et al. \(2021\)](#)



Relax modeling assumptions on $d_1(t)$.

- Normalization set by fit.
- Position of node in r depends on Λ .

📄 Dutrieux *et al.* (2021)



- **Asymptotic** information on $|t|$ -dependence from perturbative QCD. *But how large is "asymptotic"?*
- **Factorization** constraint: $Q^2 \gg |t|$. *Most of the experimental data used as fit input has low $|t|$.*
- Need for more experimental data points.

Increase Q^2 -lever arm.

Evolution equations bring a slow $\log Q^2$ dependence

- Remind computation of subtraction constant at LO

$$C_H(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu_F^2 \equiv Q^2)$$

Diehl & Ivanov (2007)

- Plug LO evolution of D-term to obtain the following **pattern**

$$C_H(t, Q^2) \propto \sum_{\text{odd } n} d_n(t, \mu_F) \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n}$$

with γ_n computed in perturbative QCD.

- Since $\alpha_s(Q^2) \propto 1/\log Q^2$, an exact knowledge of $C_H(t, Q^2)$ on an Q^2 -interval allows to exactly retrieve d_n .

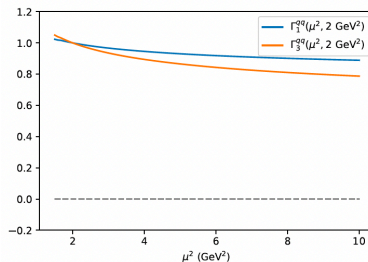
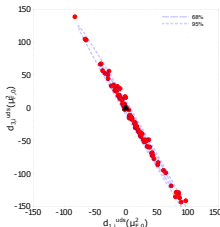
Increase Q^2 -lever arm.

Anomalous dimensions γ_n are small and take comparable values

- Introduce evolution operator Γ so that

$$d_n(\mu_1) = \Gamma_n(\mu_1, \mu_2) d_n(\mu_2)$$

- Probed Q^2 -range in CFF fit: $[1.5, 4] \text{ GeV}^2$.
- Γ_1 and Γ_3 are **numerically very close**.



- Evolution **"too slow"** to separate d_1 and d_3 for $Q^2 \in [1.5, 4] \text{ GeV}^2$.
- Experimental data mostly constrain $d_1 + d_3 + \dots$

 Dutrieux *et al.* (2021)

Anomalous dimensions γ_n and Q^2 -lever arm

Inverse problem and regularization

- Remind **pattern** of the problem

$$C_H(t, Q^2) \propto \sum_{\text{odd } n} d_n(t, \mu_F) \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n}$$

- If Q^2 -range is too small, a solution with $d_1(t, \mu_F) + d_3(t, \mu_F) + d_5(t, \mu_F) + \dots = 0$ can remain **hidden within experimental uncertainties** over the whole range $Q^2 \in [Q_{\min}^2, Q_{\max}^2]$.
- In practice: act as if the problem of retrieving $d_1, d_3, \dots$ from measurements has infinitely many solutions.
- Add extra **regularization** to select one solution **robust with respect to statistical uncertainties**.
- Today **cannot reliably estimate** the uncertainty associated to the neglect of $d_3, \dots$

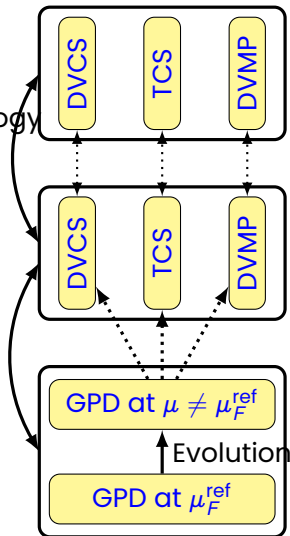
Computing chain design

Full processes

Experimental
data and
phenomenology

Small
distance
Computation
of
amplitudes

Large
distance
First
principles
and funda-
mental
parameters



PARTonic
Tomography
Of
Nucleon
Software

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

 [Berthou et al. \(2015\)](#)

Generic exclusive event generators EpiC.

Modular structure compatible with the architecture of PARTONS

- Includes treatment of radiative corrections.
- Can be extended to simulate other exclusive processes.
- **Already used** in the EIC community and run at BNL.
- **Publicly released** simultaneously with PARTONSv3.



EpiC: Monte Carlo generator for exclusive processes

This page:

[about](#)
[installation](#)
[usage](#)
[license](#)
[acknowledgements](#)
[contact](#)

View GitHub repository:

[link](#)

View Docker repository:

[link](#)

View Doxygen documentation:

[link](#)

View reference article:

[The EpiC framework](#)

2023-06-13 13:17:44 UTC

About

What is EpiC?

EpiC is a modern and versatile Monte Carlo generator used in studying exclusive processes. These processes are sensitive to generalised parton distributions (GPDs), which describe the 3D partonic structure of hadrons in the language of quantum chromodynamics (QCD). EpiC has been developed to support the current and future experimental programmes, like [electron-ion collider \(EIC\)](#) to be constructed in Brookhaven National Laboratory.

EpiC is based on [PARTONS framework](#), which provides the basic elements of its architecture. PARTONS is also used to evaluate Born cross-sections, which together with radiative corrections implemented in EpiC, are used to generate unweighted Monte Carlo events.

Exclusive processes currently available in EpiC are:

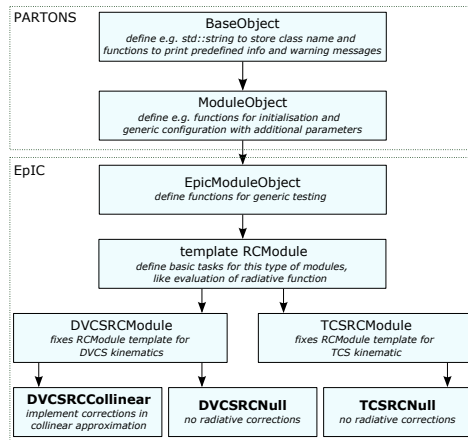
- deeply virtual Compton scattering (DVCS)
- time-like Compton scattering (TCS)
- deeply virtual meson production (DVMP), only n^0 case
- double deeply virtual Compton scattering (DDVCS)
- diphoton photoproduction (work in progress)

 [Aschenauer et al. \(2022\)](#)

Generic exclusive event generators EpIC.

Modular structure compatible with the architecture of PARTONS

- Includes treatment of radiative corrections.
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Impact study with ePIC.

- Latest **design** base on EIC Yellow Report and **simulation software** of ePIC detector.

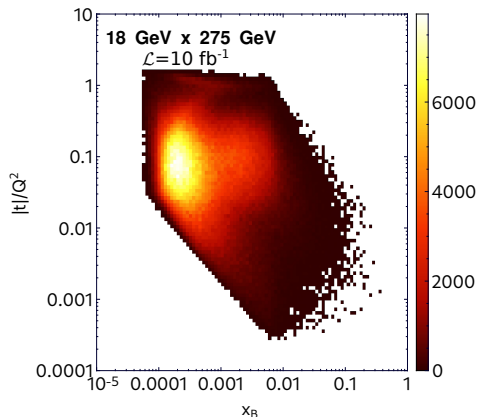
✍ [Abdul Khalek et al. \(2021\)](#)

- Assess impact of **radiative corrections** and **background contribution** from exclusive π^0 production.

- Essential **update** of previous (decade old) similar study.

✍ [Aschenauer et al. et al. \(2013\)](#)

- Machine learning based extraction of CFFs from pseudo data.



Distribution of reconstructed MC events.

✍ [Aschenauer et al. \(2025\)](#)

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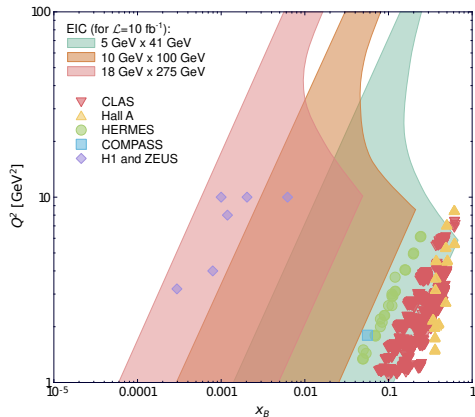
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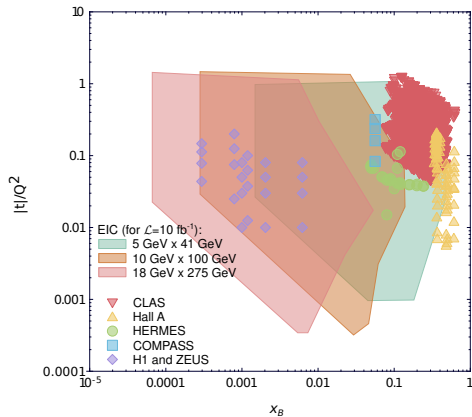
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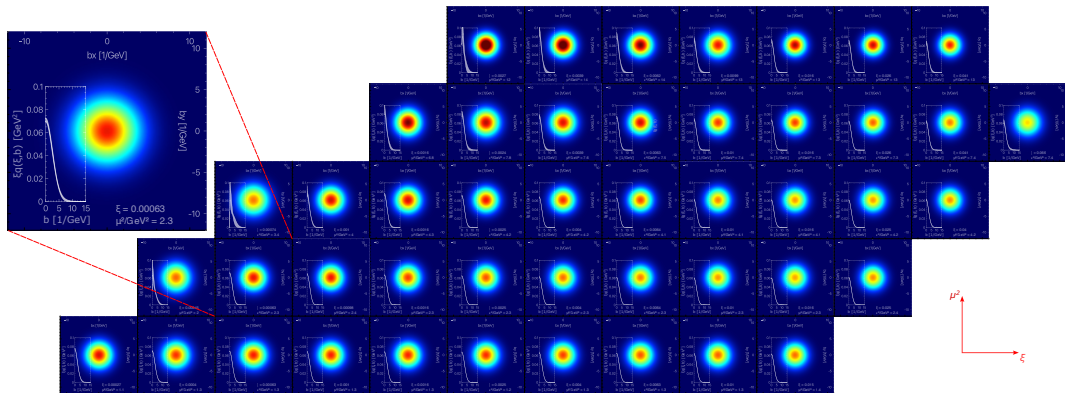
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✍ Aschenauer *et al.* (2025)

Impact study with EpIC.



Aschenauer *et al.* (2025)



4. Conclusion

Conclusion and prospects

The phenomenological quest towards proton internal pressure

- Concept **well-defined** and suitable for phenomenology.
- Strong **first-principle connection** between concept and experimental data.
- Need for **multi-channel** analysis **beyond LO** on a **wide kinematic coverage**.
EIC and ElcC much needed!
- The GPD deconvolution problem is **ill-posed**. **Huge sensitivity** to numerical noise or experimental uncertainties.



gg75478317 GoGraph.com

- Development of the **software ecosystem** PARTONS for 3D hadron structure studies.
- Need for **coordinated effort** involving fits, computing chains, continuum and lattice QCD to make the best from experiments.



A. Appendix

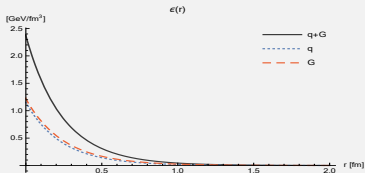
Abbreviations used in this presentation.

ANN	artificial neural network
CFF	Compton form factor
DDVCS	double deeply virtual Compton scattering
DVCS	deeply virtual Compton scattering
DVMP	deeply virtual meson production
DR	dispersion relation
EIC	electron-ion collider
ElcC	electron-ion collider in China
EFF	elastic form factor
GFF	gravitational form factor
GPD	generalized parton distribution
LO	leading order
NLO	next-to-leading order
PDF	parton distribution function
TCS	timelike Compton scattering

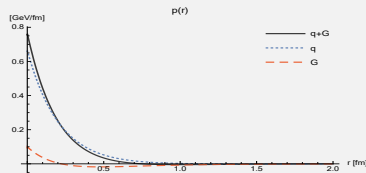
Equation of state

- Simple multiple models: dipole for GFFs A and $\bar{C}$, tripole for GFFs B and C .

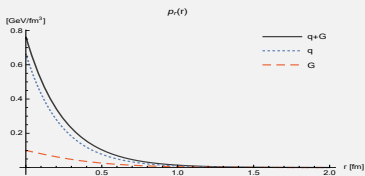
quark



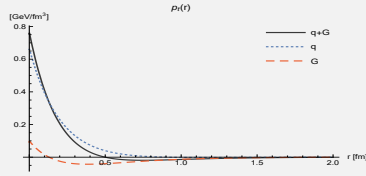
glue



quark + glue



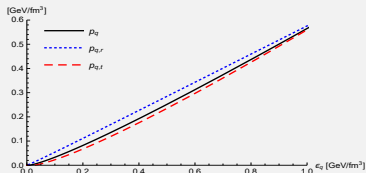
Neutron stars



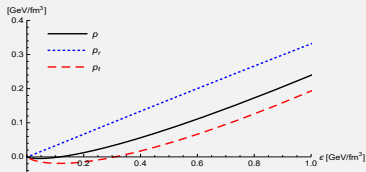
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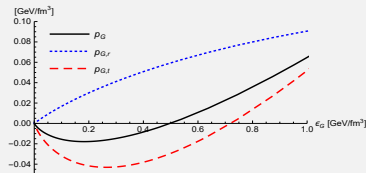
quark



quark + glue



glue



- Parametric plots of EOS

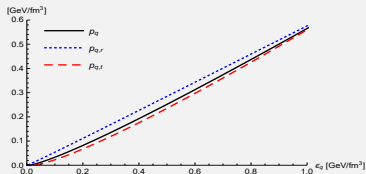
- $(\epsilon(r), p_r(r))$
- $(\epsilon(r), p_t(r))$
- $(\epsilon(r), p(r))$

- Quark and gluon contributions

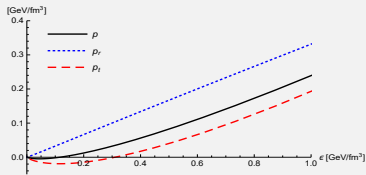
Equation of state

- From the nucleon to compact stars?

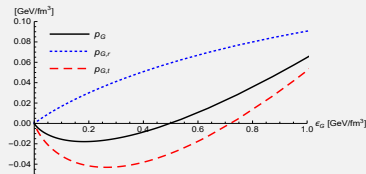
quark



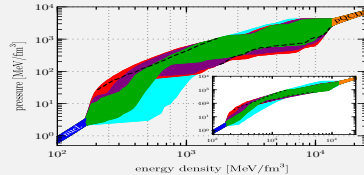
quark + glue



glue

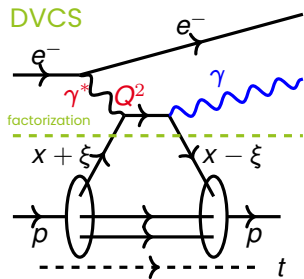


Neutron stars

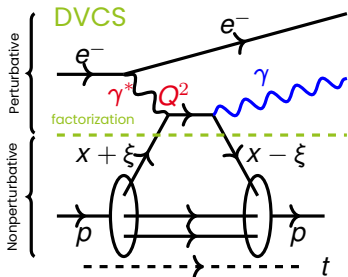


Annala et al. (2018)

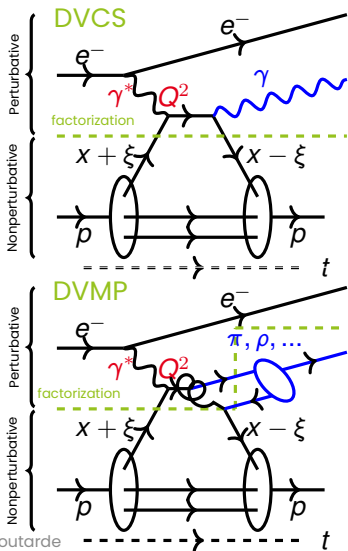
Exclusive processes of current interest



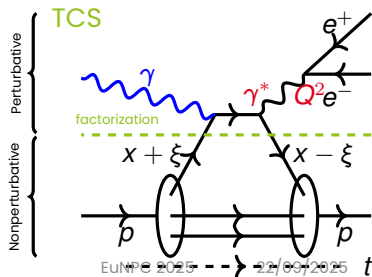
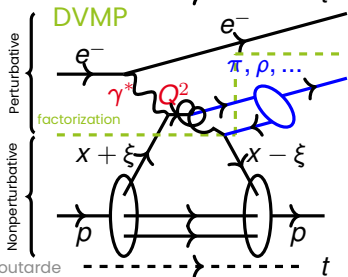
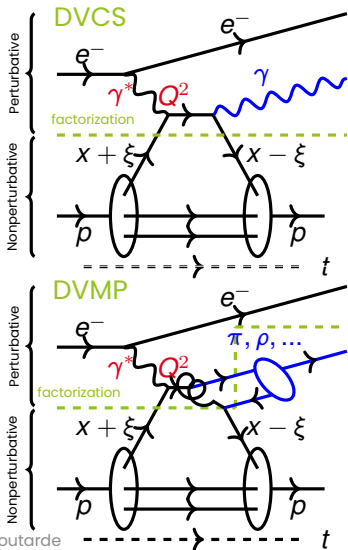
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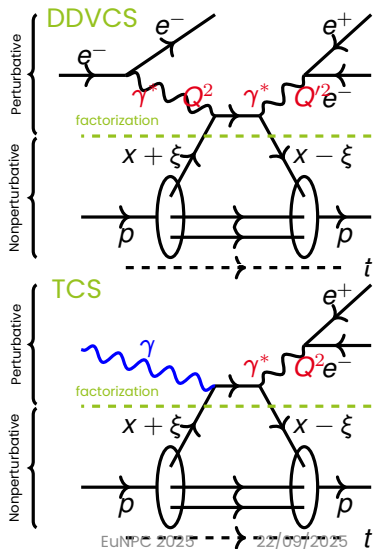
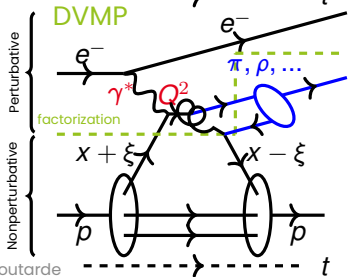
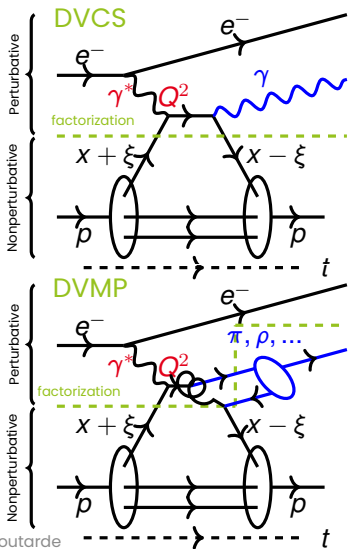
Exclusive processes of current interest



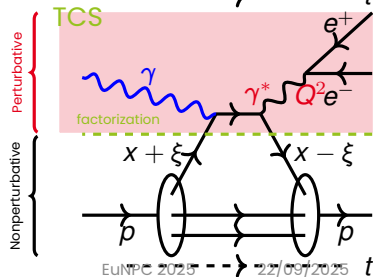
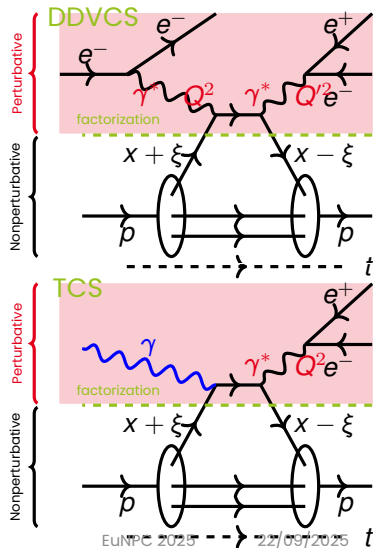
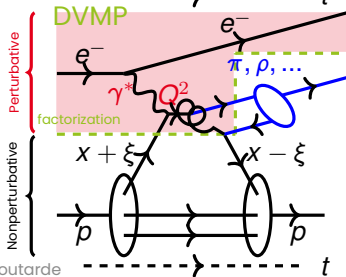
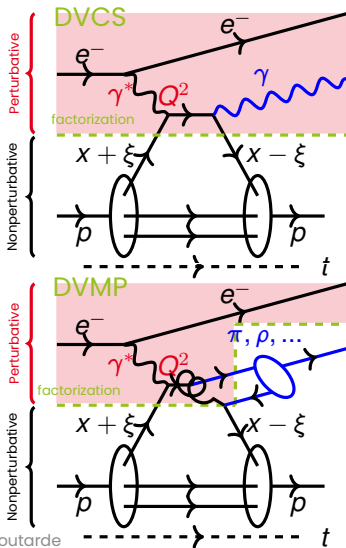
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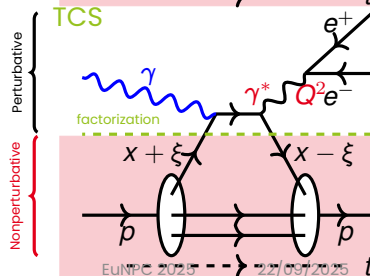
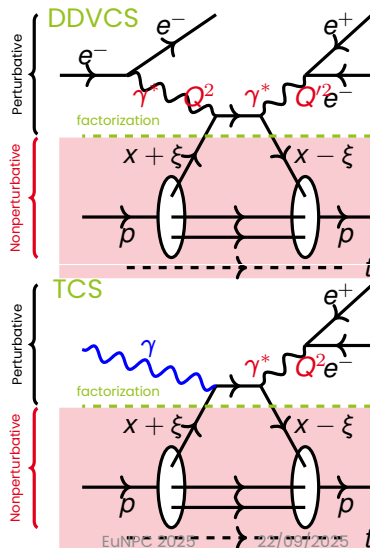
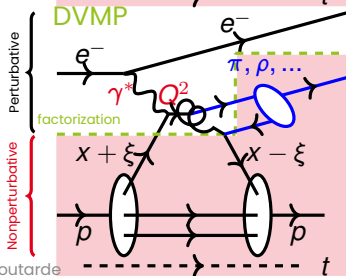
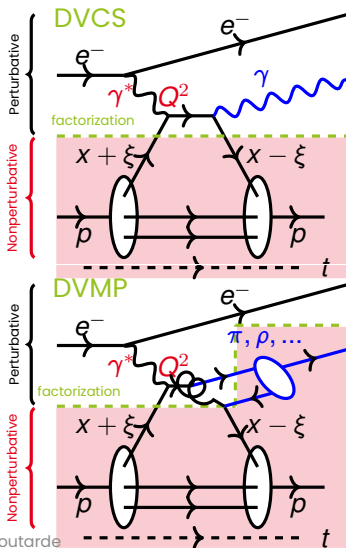
Exclusive processes of current interest



Exclusive processes of current interest

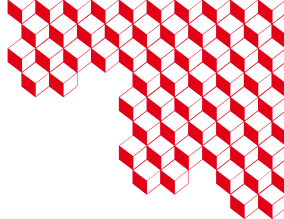


Exclusive processes of current interest





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Thank you !

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