Effective chiral lagrangian with thermal field fluctuations and broken scale invariance

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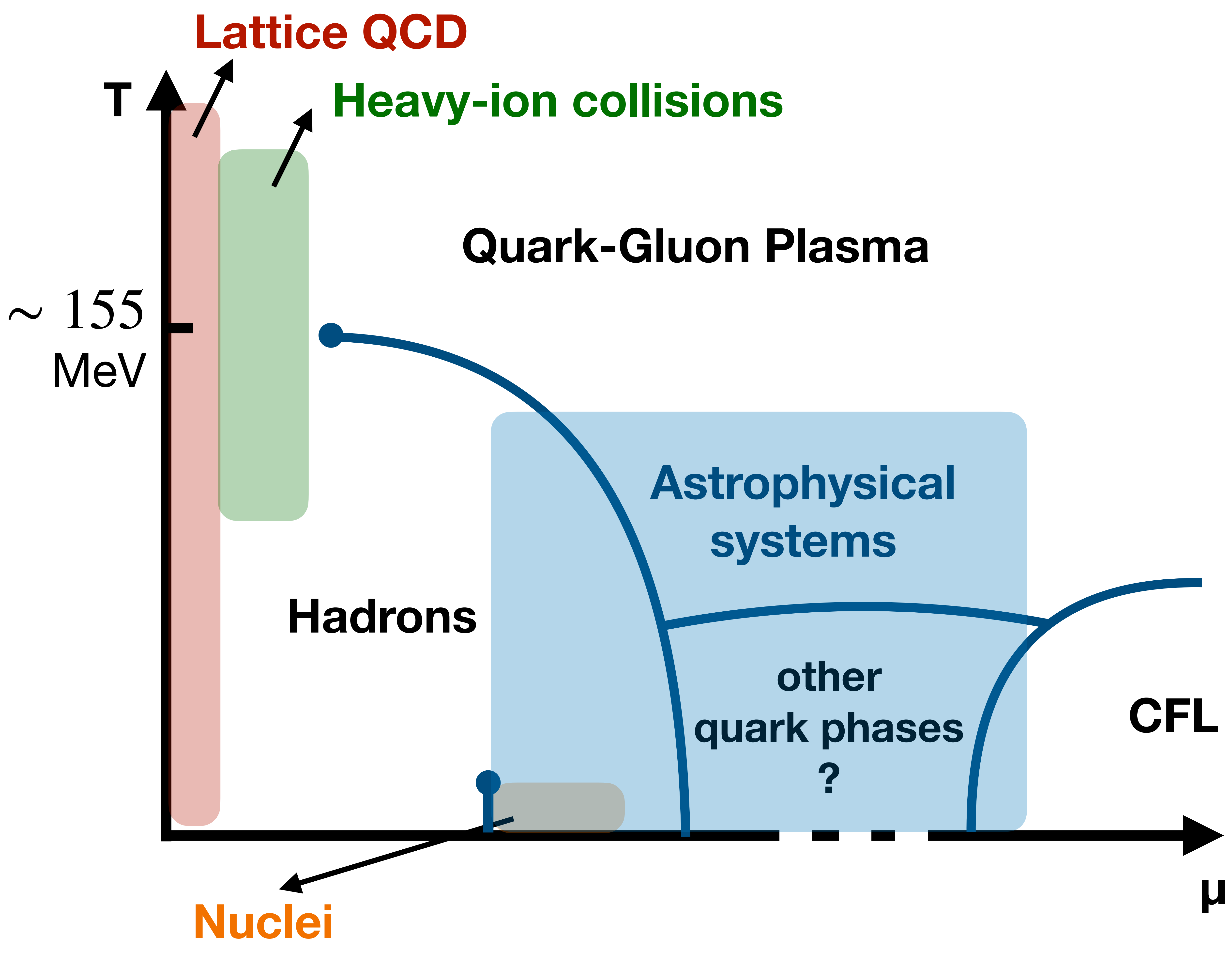
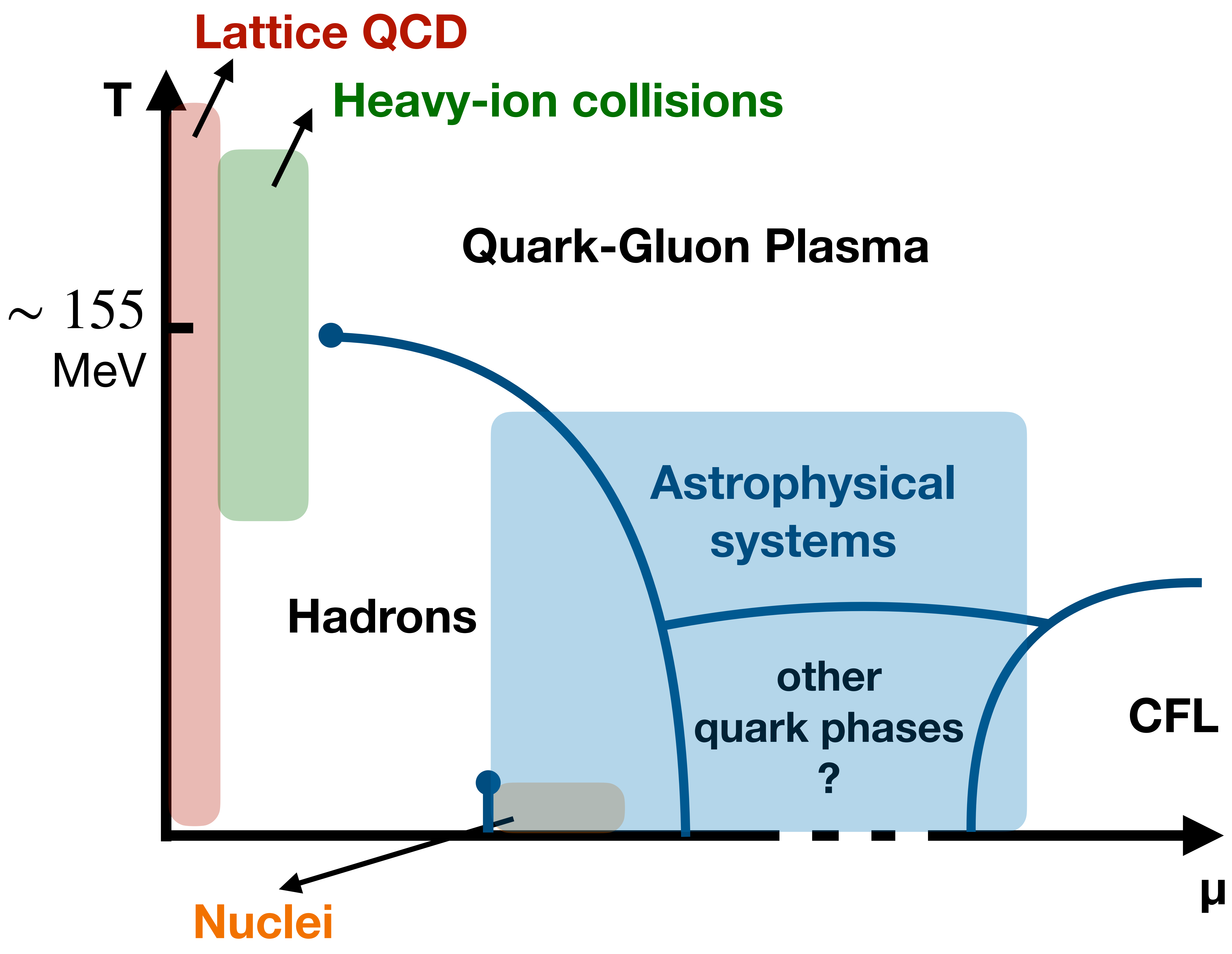
1 Introduction

QCD phase diagram



QCD phase diagram

- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions



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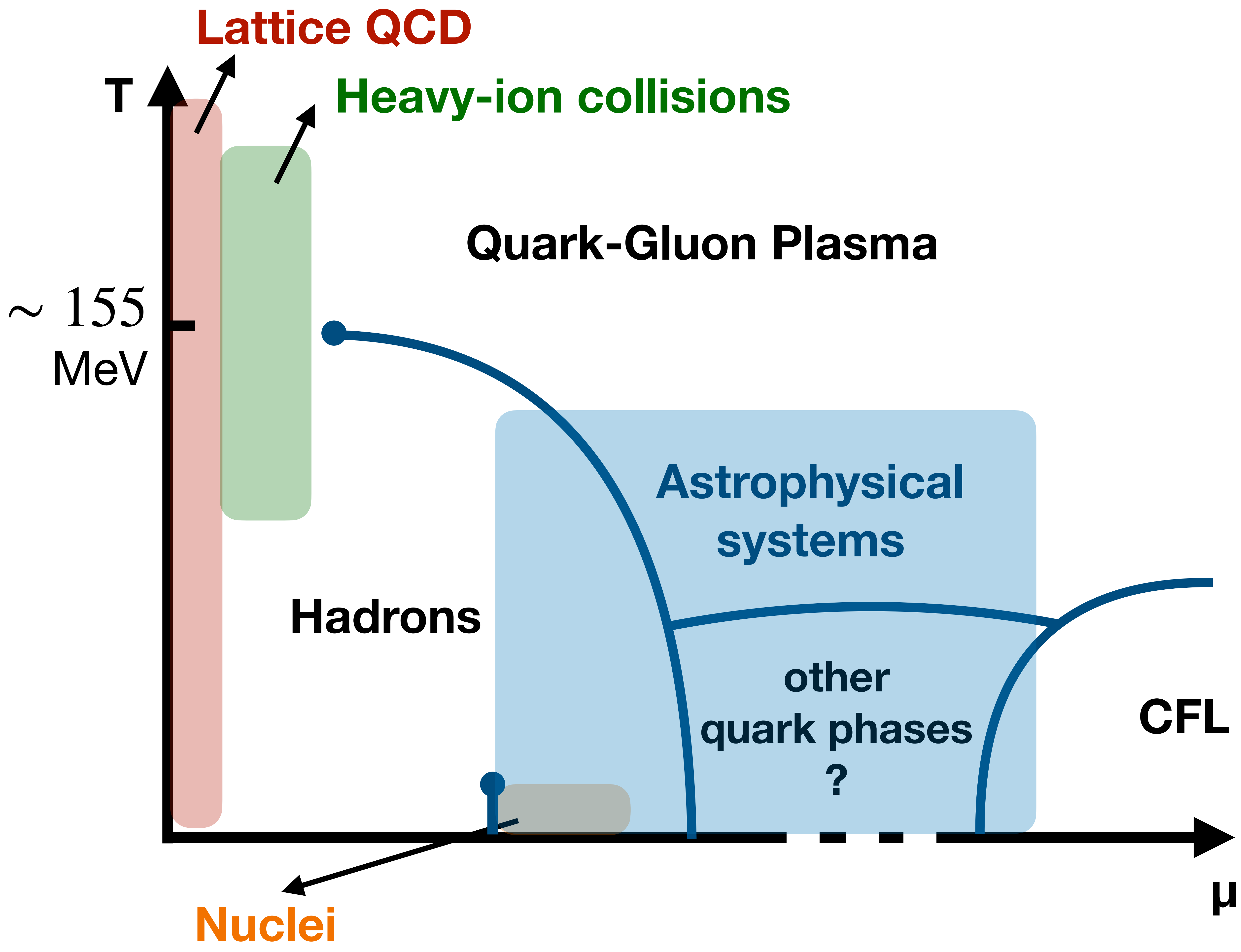
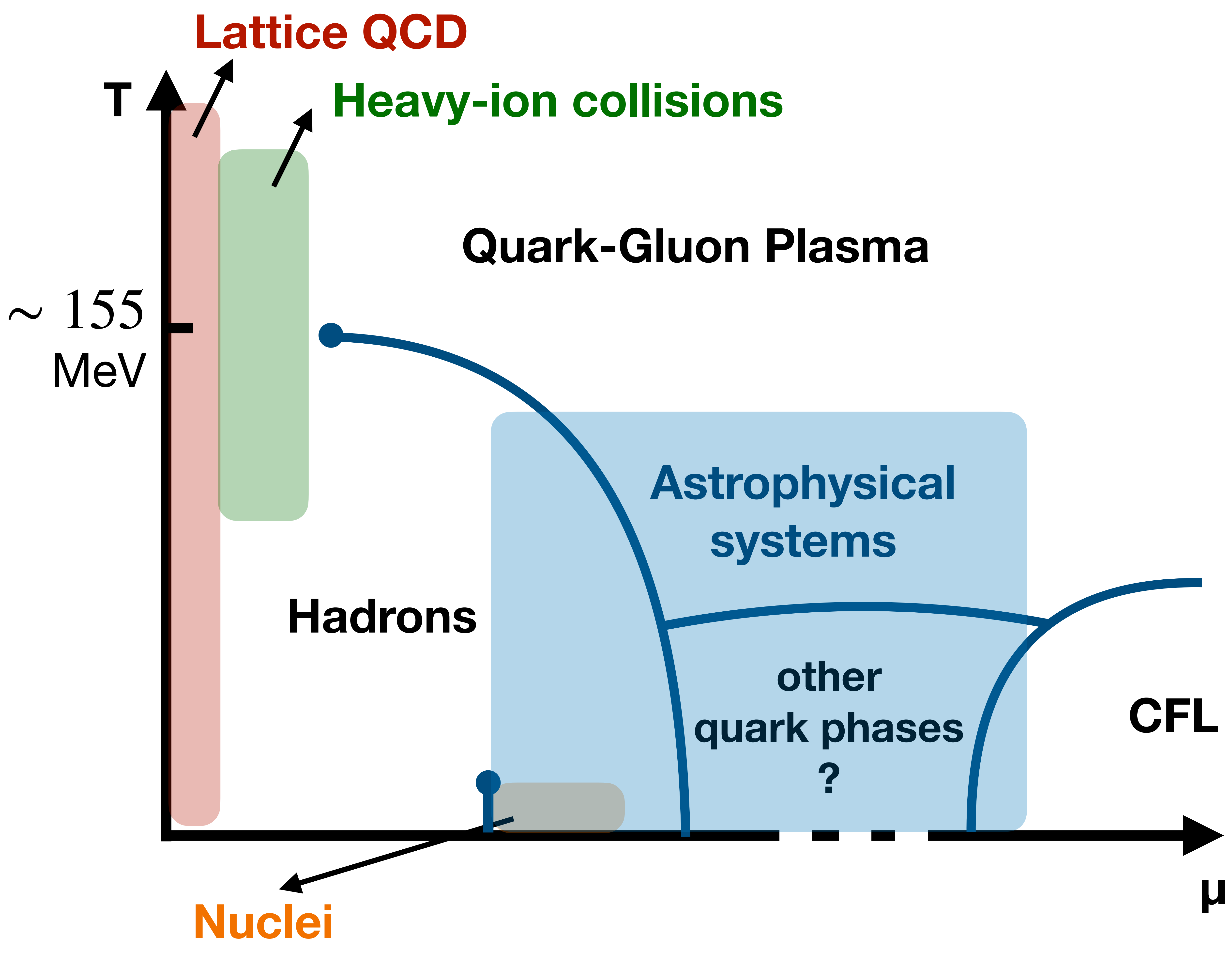


QCD phase diagram

- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions
- Regimes of low temperatures and high densities: neutron stars and compact stars
- Only a few EOS are studied in both regimes simultaneously and with several model-dependent parameters: a complete description is still challenging.



Research Overview

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   - Investigation of the phase diagram in the meson–baryon sector

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2 Pure Gauge $\mathrm{SU}(3)_\mathrm{c}$ sector

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Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \mathscr{V}, \qquad \mathscr{V} = \frac{B}{4}\left(\phi_0^4 - \phi^4 + 4\phi^4 \ln \frac{\phi}{\phi_0}\right) = \frac{B_0}{4}\left(1 - \chi^4 + 4\chi^4 \ln \chi\right).$$

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The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle\mathscr{V}(\chi)\rangle - P_{\mathrm{dil}}(\chi, T) - P_{\mathrm{q-free}}(\chi, T) - P_{\mathrm{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2}m_\chi^{*2}\langle\Delta_\chi^2\rangle\,.$$

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Infrared cut-off $\quad K(\chi) = \dfrac{A}{1-\chi}$

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It is possible to divide the glueball into two distinct components: the mean field $\bar{\chi}$ and the fluctuating part Δ_χ, with $\langle \Delta_\chi \rangle = 0$.

$$\chi = \bar{\chi} + \Delta_\chi$$

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$$\chi = \bar{\chi} + \Delta_\chi$$

$$\langle \Delta_\chi^2 \rangle = \frac{1}{2\pi^2 \phi_0^2} \int_0^\infty \frac{k^2}{\epsilon_\chi} \frac{1}{e^{\beta \epsilon_\chi} - 1} dk$$

$$P_\chi(z) = \sqrt{\frac{1}{2\pi \langle \Delta_\chi^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\chi^2 \rangle} \right)$$

$$\langle f(\chi) \rangle = \int_{-\infty}^{\infty} P_\chi(z) f(\bar{\chi} + z)\, dz$$

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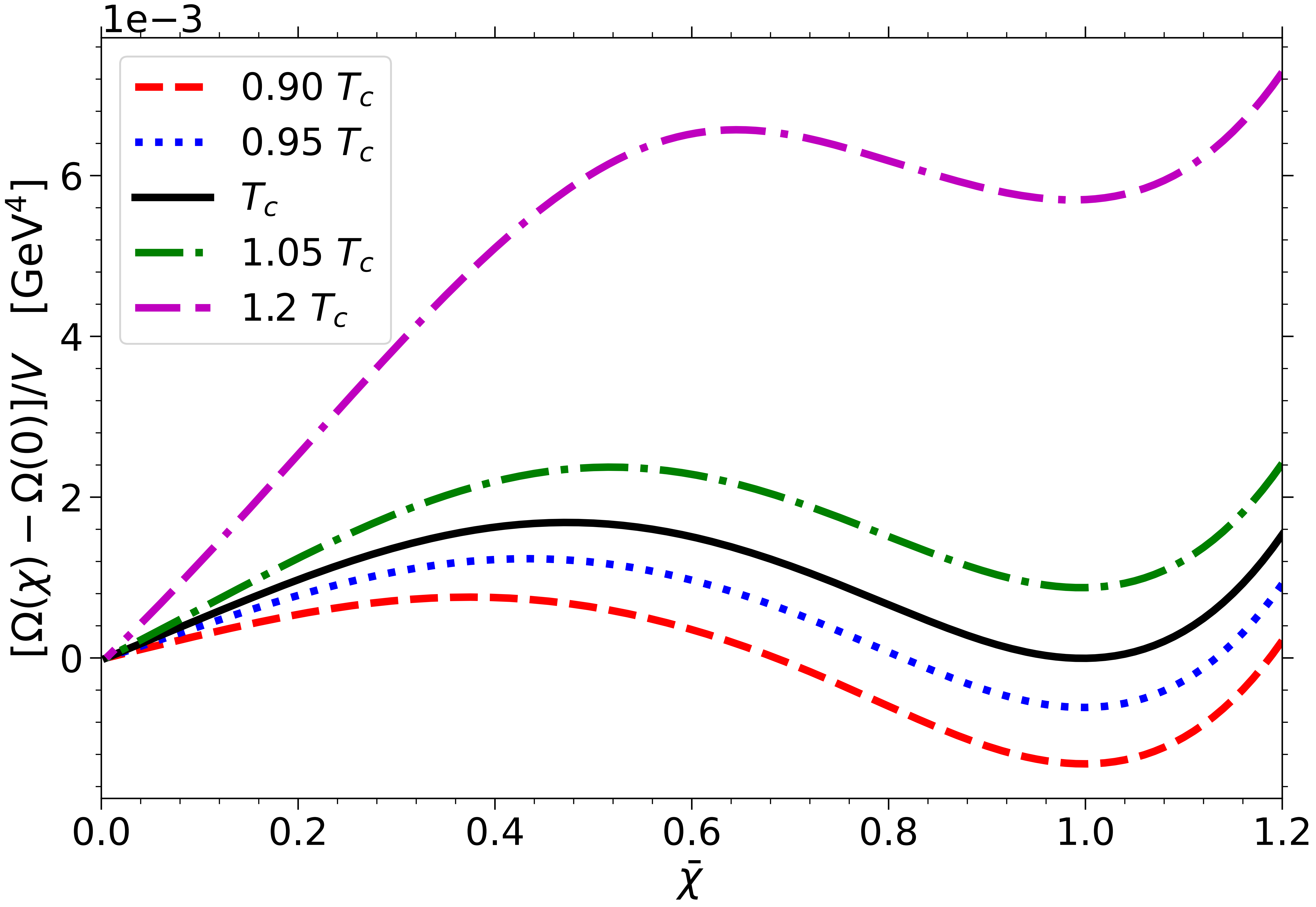

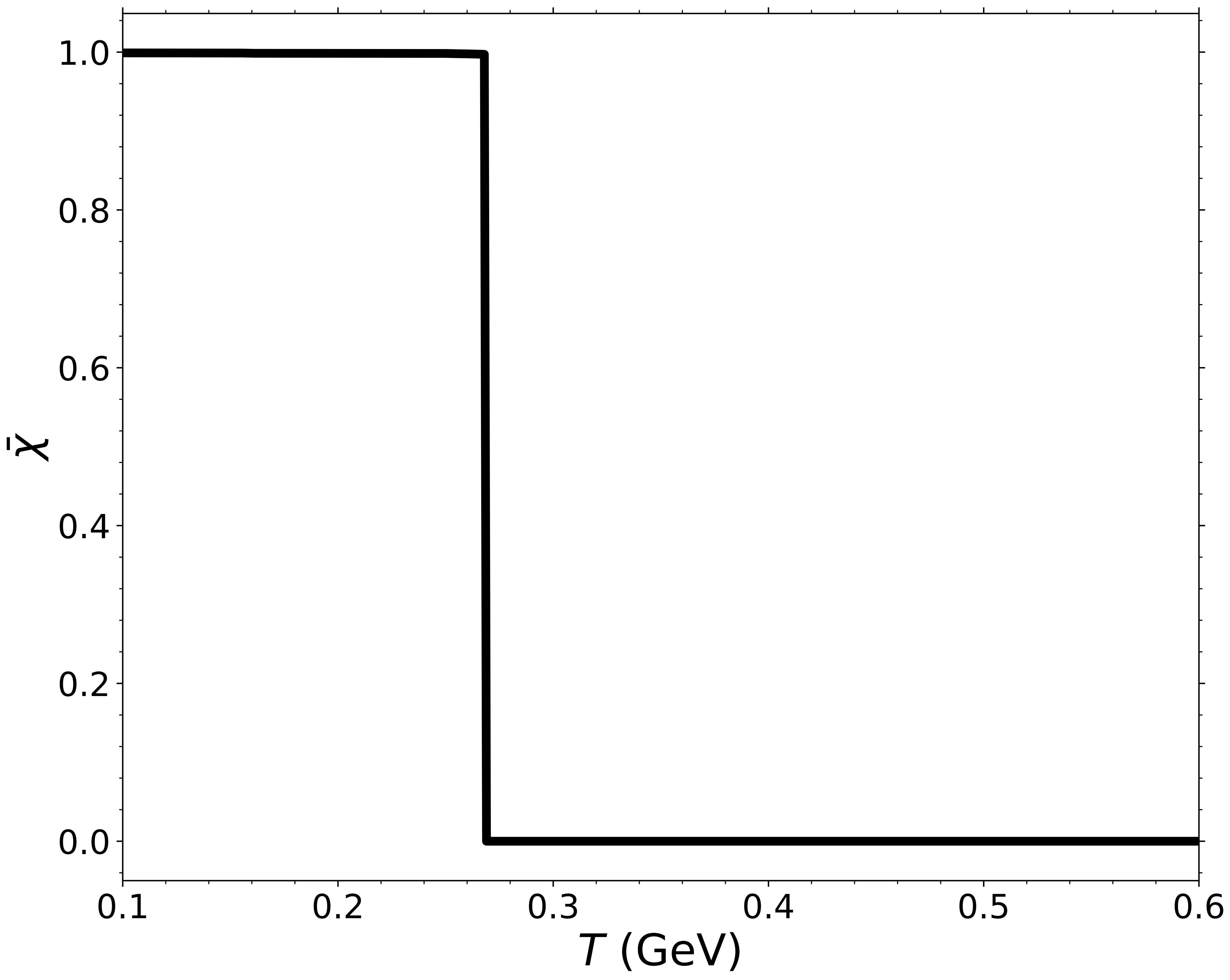

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Borsányi et al., JHEP 07 (2012) 056

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To improve accuracy with latticeQCD data:

- Inclusion **excited glueballs**:

- E. Trotti, S. Jafarzade, and F. Giacosa, *Eur.Phys.J.C* 83 (2023) 5, 390
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$$m^2\left(n = 1, J^{PC}\right) = a(n + J) + b_{J^{PC}}$$

J^{PC}	m [GeV]	$b_{J^{PC}}$ [GeV^2]
0^{++}	$1.647(25)$	-2.78 ± 0.21
2^{++}	$2.367(30)$	-10.87 ± 0.57
0^{-+}	$2.572(38)$	1.12 ± 0.27
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$$P_{\mathrm{dil}_J} = -(2J+1)T\int \frac{dk^3}{(2\pi)^3} \ln\left(1 - e^{-\beta\sqrt{k^2 + m_J^{*2}}}\right)$$

$$\frac{\tilde{\Omega}(\chi, T)}{V} = \frac{\Omega(\chi, T)}{V} - \sum_J^N P_{\mathrm{dil}_J}$$

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String tension term

$$\sigma(T) = \left(1 - \frac{1}{1+e^{-(T-\alpha)/\delta}}\right)$$

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To improve accuracy with latticeQCD data:

- Inclusion excited glueballs;
- Modification **infrared cut-off**:

$$I = \frac{s}{T^3} - \frac{4P}{T^4} \Rightarrow s/T^3 \gg 4P/T^4$$

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Modified infrared cut-off

$$\tilde{K}(\chi, T) = \frac{A}{1-\chi^2} = \frac{A}{1-\bar{\chi}^2 - \langle \Delta^2 \rangle \cdot F(T)}$$

Form factor

$$F(T) = \frac{\Theta(T-T_c)}{1 + \left(\frac{T-T_c}{\gamma}\right)^2}$$

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Meson and baryon sector at finite chemical potential

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$$\tilde{\mathscr{L}} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{4}\boldsymbol{B}_{\mu\nu}\cdot\boldsymbol{B}^{\mu\nu} + \frac{1}{2}G_{\omega\phi}\phi^2\omega_\mu\omega^\mu + \frac{1}{2}G_{b\phi}\phi^2\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu$$

$$+\left[(G_4)^2\omega_\mu\omega^\mu\right]^2 - \tilde{\mathscr{V}} + \bar{N}\left[\gamma^\mu\left(i\partial_\mu - g_\omega\omega_\mu - \frac{1}{2}g_\rho\boldsymbol{b_\mu}\cdot\boldsymbol{\tau}\right) - g\sqrt{\sigma^2+\pi^2}\right]N$$

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$$\tilde{\mathscr{V}} = \mathscr{V}(\chi) - \frac{1}{2}B_0\delta\chi^4\ln\frac{\sigma^2+\pi^2}{\sigma_0^2} + \frac{1}{2}B_0\delta\chi^2\left[\frac{\sigma^2+\pi^2}{\sigma_0} - \frac{\chi^2}{2}\right] - \frac{1}{4}\epsilon_1'\chi^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) - \chi^2\right] - \frac{3}{4}\epsilon_1'.$$

Meson and baryon sector at finite chemical potential

The **thermodynamical potential** is

$$\frac{\tilde{\Omega}}{V} = \langle \tilde{\mathscr{V}} \rangle - \frac{1}{2} m_\omega^2 \chi^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \chi^2 b_0^2 - \frac{G_4}{4} \omega_0^4 - \frac{1}{2} m_\sigma^{*2} \langle \Delta_\sigma^2 \rangle - \frac{1}{2} m_\pi^{*2} \langle \boldsymbol{\pi}^2 \rangle - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle$$

$$+ \frac{T}{2\pi^2} \int dk\, k^2 \left[\ln(1 - e^{-\beta\epsilon_\sigma}) + 3 \ln(1 - e^{-\beta\epsilon_\omega}) \right] + \frac{T}{2\pi^2} \int dk\, k^2 \left[\sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\pi^* - \mu_\pi^{*a})}) + 3 \sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\rho^* - \mu_\rho^{*a})}) \right]$$

$$- \frac{T}{\pi^2} \sum_{i=p,n} \int dk_i\, k_i^2 \left[\ln\left(1 + e^{-\beta(E_i^* - \mu_i^*)}\right) + \ln\left(1 + e^{-\beta(E_i^* + \mu_i^*)}\right) \right]$$

$$\tilde{\mathscr{V}} = \mathscr{V}(\chi) - \frac{1}{2} B_0 \delta \chi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2} + \frac{1}{2} B_0 \delta \chi^2 \left[\frac{\sigma^2 + \pi^2}{\sigma_0} - \frac{\chi^2}{2} \right] - \frac{1}{4} \epsilon_1' \chi^2 \left[\frac{4\sigma}{\sigma_0} - 2 \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) - \chi^2 \right] - \frac{3}{4} \epsilon_1' .$$

Meson and baryon sector at finite chemical potential

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ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\Big(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\Big)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\Big(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\Big)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle\Big[\rho_S^p + \rho_S^n\Big] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \Big(B_0\,\delta + \epsilon_1'\Big)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3\ln\chi - 2B_0\,\delta\chi^3\ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$
$$-2\,\epsilon_1'\,\bar{\chi}\,\frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\,\bar{\chi}\left(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle\right) - m_\rho^2\,\bar{\chi}\left(b_0^2 + \langle\Delta b_\mu\Delta b^\mu\rangle\right) = 0$$

Meson and baryon sector at finite chemical potential

$$\langle A(\mathbf{\Phi}_i + \Delta\Phi_i)\rangle = \int \prod_i dz_i\, P(z_i, \langle \Delta_i^2 \rangle)\, A(\mathbf{\Phi}_i + z_i)$$

$$\mathbf{\Phi} = (\phi, \sigma, \pi_0, \pi_+, \pi_-)$$

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σ field equation

$$\left\langle \frac{g \sigma_0^2 \sigma}{\sqrt{\sigma^2 + \pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n \right] - \left\langle \frac{B_0\, \sigma^2}{\sigma_0^4}\, \delta \chi^4\, \frac{\sigma}{\sigma^2 + \pi^2} + \left(B_0\, \delta + \epsilon_1' \right) \chi^2 \sigma - \epsilon_1'\, \frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\, \chi^3 \ln \chi - 2B_0\, \delta \chi^3 \ln\left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) + B_0\, \delta \chi\, \frac{\sigma^2 + \pi^2}{\sigma_0^2} - B_0\, \delta \chi^3 + \epsilon_1' \chi\, \frac{\sigma^2 + \pi^2}{\sigma_0} + \epsilon_1' \chi^3 \right\rangle$$

$$-2\, \epsilon_1'\, \bar{\chi}\, \frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\, \bar{\chi} \left(\omega_0^2 + \langle \Delta\omega_\mu \Delta\omega^\mu \rangle \right) - m_\rho^2\, \bar{\chi} \left(b_0^2 + \langle \Delta b_\mu \Delta b^\mu \rangle \right) = 0$$

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$$m_\rho^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)b_0 - g_\rho\,\rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \left(B_0\,\delta + \epsilon_1'\right)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\,\chi^3\ln\chi - 2B_0\,\delta\chi^3\ln\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right) + B_0\,\delta\chi\,\frac{\sigma^2+\pi^2}{\sigma_0^2} - B_0\,\delta\chi^3 + \epsilon_1'\,\chi\,\frac{\sigma^2+\pi^2}{\sigma_0} + \epsilon_1'\,\chi^3 \right\rangle$$

$$-2\,\epsilon_1'\,\bar{\chi}\,\frac{\bar{\sigma}}{\sigma_0} - m_\omega^2\,\bar{\chi}\left(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle\right) - m_\rho^2\,\bar{\chi}\left(b_0^2 + \langle\Delta b_\mu\Delta b^\mu\rangle\right) = 0$$

$$\langle A(\mathbf{\Phi}_i + \Delta\Phi_i)\rangle = \int \prod_i dz_i\,P(z_i,\langle\Delta_i^2\rangle)\,A(\mathbf{\Phi}_i + z_i)$$

$$\mathbf{\Phi} = (\phi,\sigma,\pi_0,\pi_+,\pi_-)$$

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Meson and baryon sector at finite chemical potential

ω field equation

$$4G_4\,\omega_0^3 + m_\omega^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)\omega_0 - g_\omega\,\rho_B = 0$$

ρ field equation

$$m_\rho^2\left(\bar{\chi}^2 + \langle\Delta_\chi^2\rangle\right)b_0 - g_\rho\,\rho_3 = 0$$

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$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2+\pi^2}} \right\rangle \left[\rho_S^p + \rho_S^n\right] - \left\langle \frac{B_0\,\sigma^2}{\sigma_0^4}\,\delta\chi^4\,\frac{\sigma}{\sigma^2+\pi^2} + \left(B_0\,\delta + \epsilon_1'\right)\chi^2\sigma - \epsilon_1'\,\frac{\chi^2}{\sigma_0} \right\rangle = 0$$

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$$m_\pi = m_\pi(\chi,\sigma,\pi,\omega,\rho,\langle\Delta_i^2\rangle)$$
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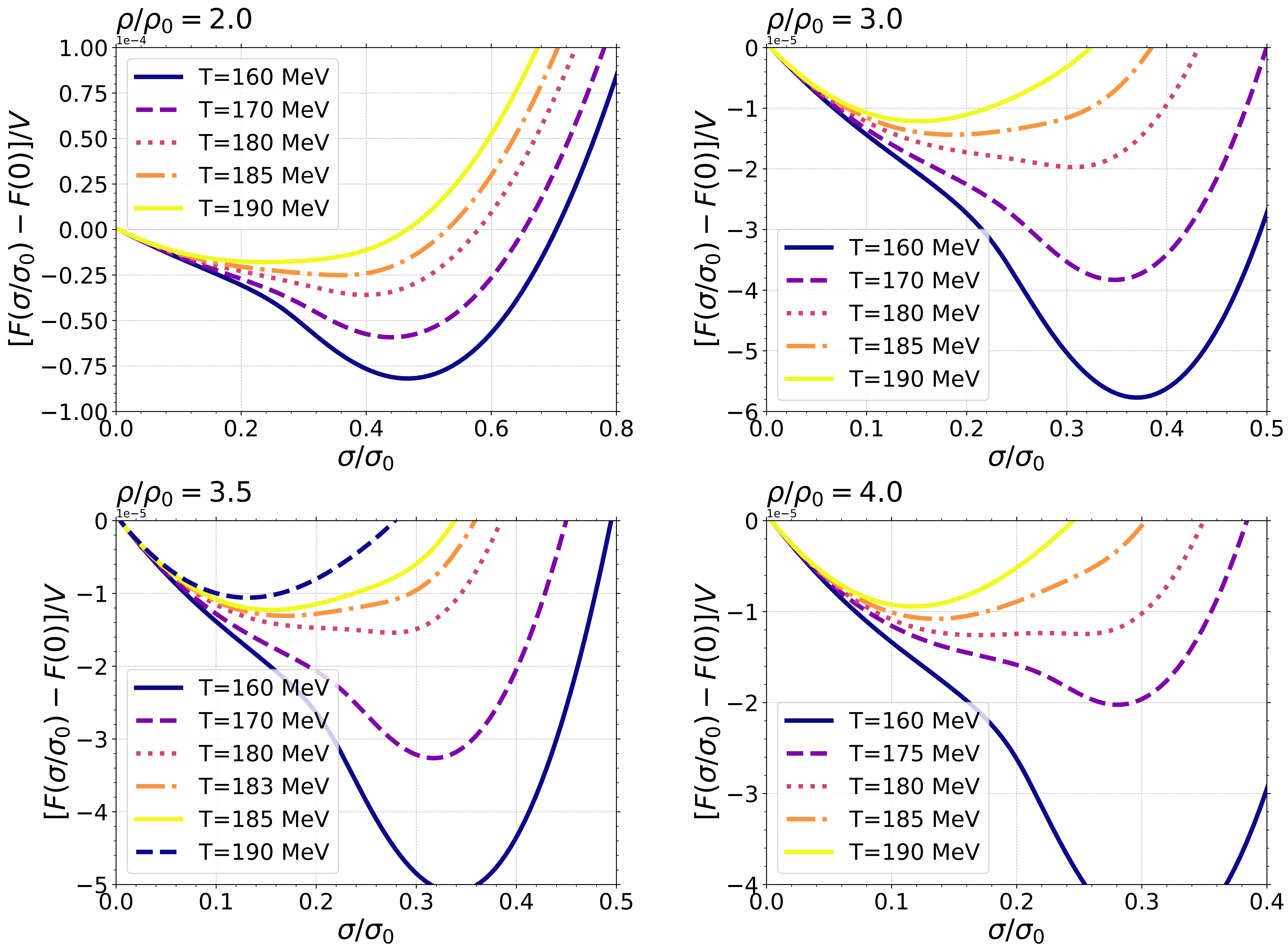
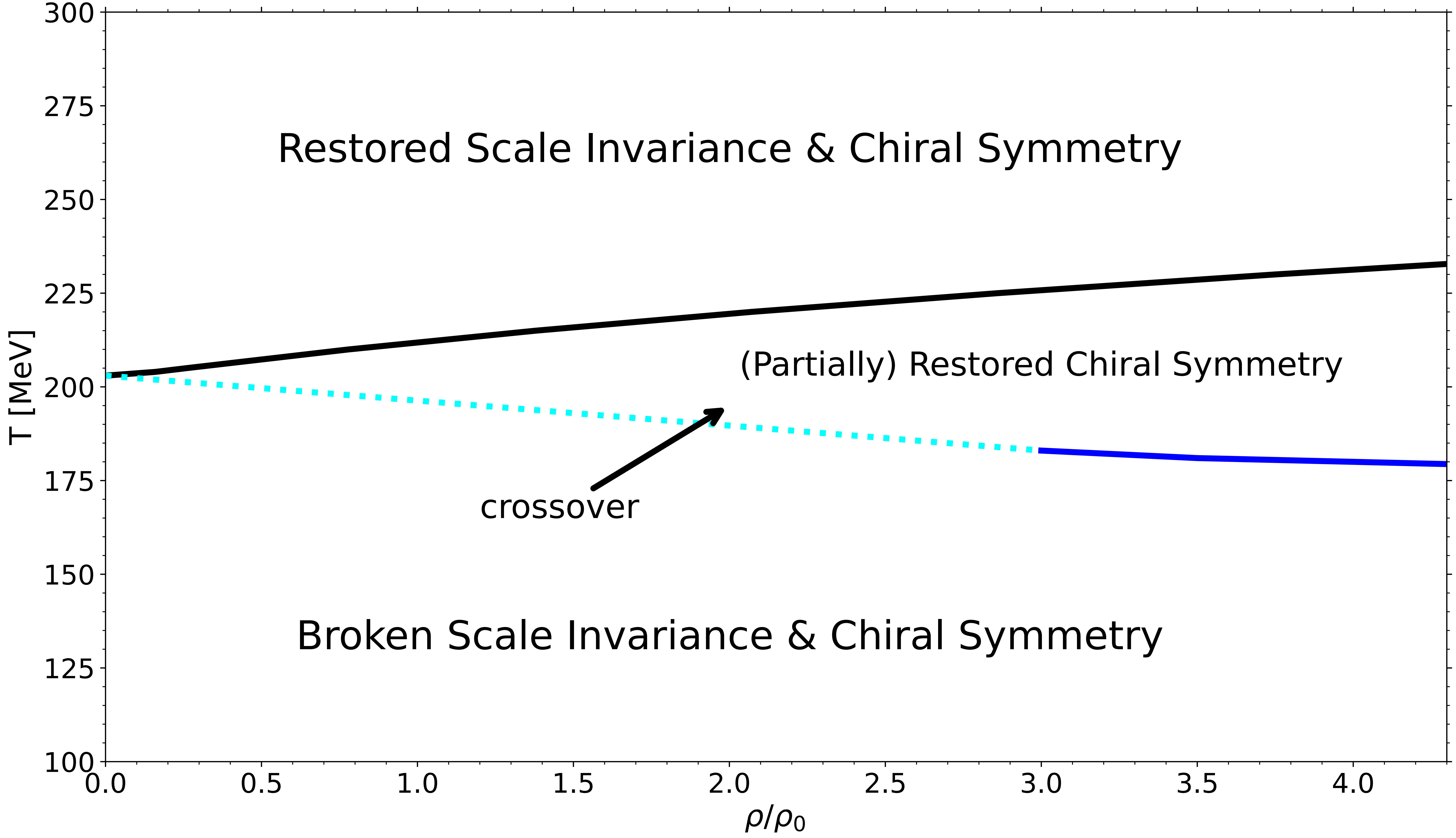
Meson and baryon sector at finite chemical potential

Meson and baryon sector at finite chemical potential



Meson and baryon sector: phase diagram



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3 Meson and baryon sector

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- A new numerical technique has been used to calculate **thermodynamic fluctuations** for the mesonic and baryonic sectors at finite chemical potential

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3 Meson and baryon sector

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- Investigations into high-energy astrophysical phenomena, such as the phenomenology of **neutron star** structure and **cosmological trajectories (little inflation scenario)** in the QCD epoch of the early universe

Thank you

Backup slides

Numerical details

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4 field equations:

$\chi, \sigma, \omega, \rho$

6 mass equations:

$$m_\chi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

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$$\begin{cases} \cdots \\ \cdots \\ \langle \Delta_i^2 \rangle = \dfrac{n_i}{2\pi^2} \displaystyle\int_0^\infty dk \, \dfrac{k^2}{e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle)} \dfrac{1}{\exp\left[\beta\left(e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle) - \mu_i^*\right)\right] - 1} \\ \cdots \\ \cdots \end{cases}$$

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2 conservation equation:

Total baryonic charge

Total electric charge

$$\begin{cases} \vdots \\ \langle \Delta_i^2 \rangle = \dfrac{n_i}{2\pi^2} \displaystyle\int_0^\infty dk \, \dfrac{k^2}{e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle)} \dfrac{1}{\exp\left[\beta\left(e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle) - \mu_i^*\right)\right] - 1} \\ \vdots \end{cases}$$

$$e_i^* = \sqrt{k^2 + m_i^{*2}}$$

Meson and baryon sector: thermal fluctuations

$$\langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty \frac{k^2}{\epsilon_i^*} \frac{1}{e^{\beta(\epsilon_i^* - \mu_i^*)} - 1} dk$$

$$n_i = \begin{cases} 1 & \text{for } \ \phi,\, \sigma, \pi_0,\, \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu,\, b_{0\mu},\, b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

$$P_\sigma(z) = \sqrt{\frac{1}{2\pi\langle \Delta_\sigma^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\sigma^2 \rangle} \right),$$

$$P_{\boldsymbol{\pi}}(y) = \sqrt{\frac{2}{\pi}} \left(\frac{3}{\langle \Delta_{\boldsymbol{\pi}}^2 \rangle} \right)^{(3/2)} \exp\left(-\frac{3y^2}{2\langle \Delta_{\boldsymbol{\pi}}^2 \rangle} \right),$$

$$\langle O(\bar{\sigma} + \Delta_\sigma, \boldsymbol{\pi}^2) \rangle = \int_{-\infty}^{\infty} dz P_\sigma(z) \int_0^\infty dy y^2 P_{\boldsymbol{\pi}}(y) O(\bar{\sigma} + z, y^2) \ .$$

The thermodynamical potential in the mean field approximation

$$\frac{\Omega(\chi,T)}{V} = \langle \mathscr{V}(\chi) \rangle - P_{\rm dil}(\chi,T) - P_{\rm q-free}(\chi,T) - P_{\rm int}(\chi,T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \left\langle \Delta_\chi^2 \right\rangle$$

Where

$$P_{\rm dil}(\chi,T) = \frac{1}{6\pi}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_\chi^{*2}}} \frac{1}{e^{\beta\sqrt{k^2+m_\chi^{*2}}}-1}$$

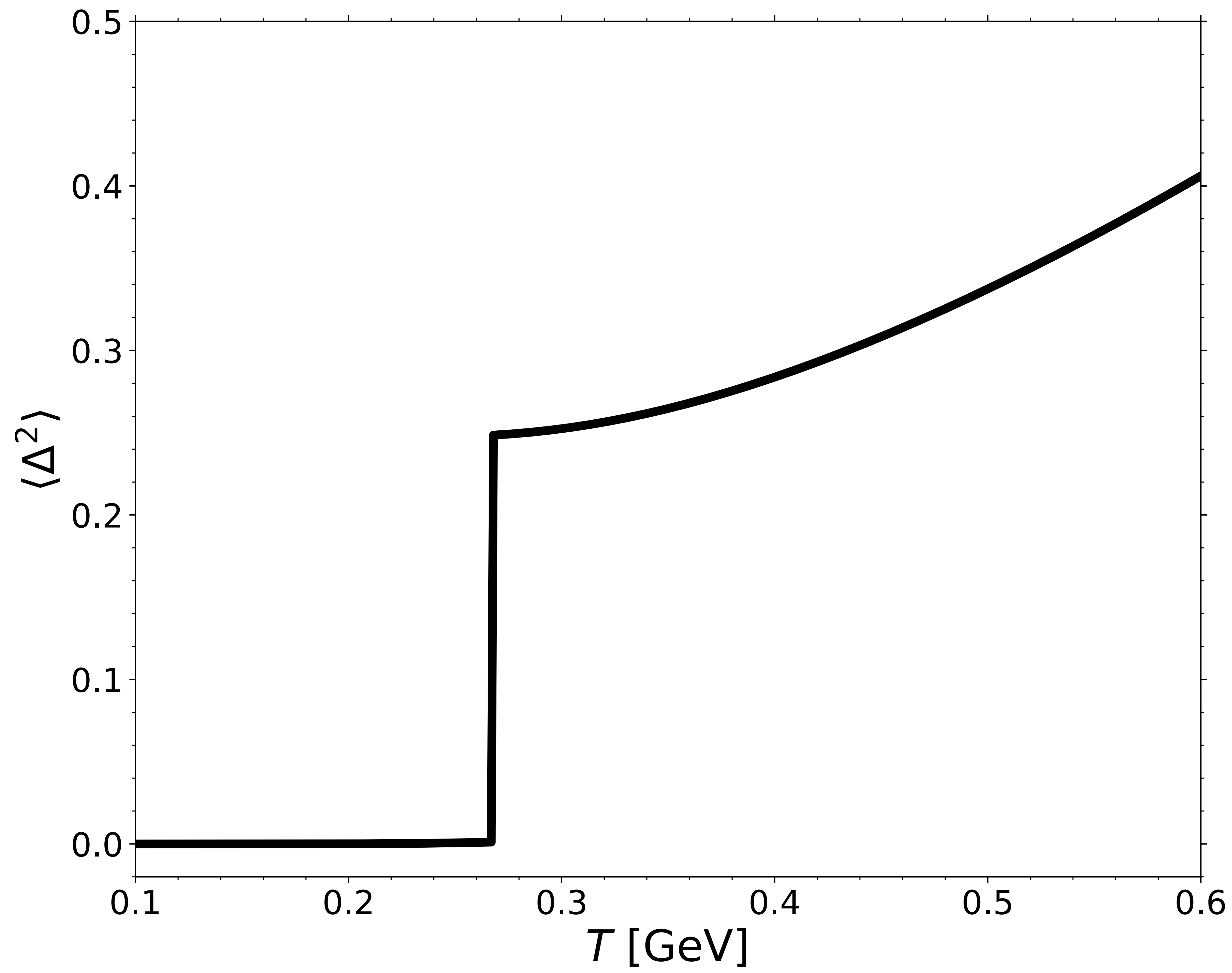
$$P_{\rm q-free}(\chi,T) = -2(N_c^2-1)T\int \frac{d^3k}{(2\pi)^3} \ln\left(1-e^{-k/T}\right)\Theta\left(k-K(\chi)\right)$$

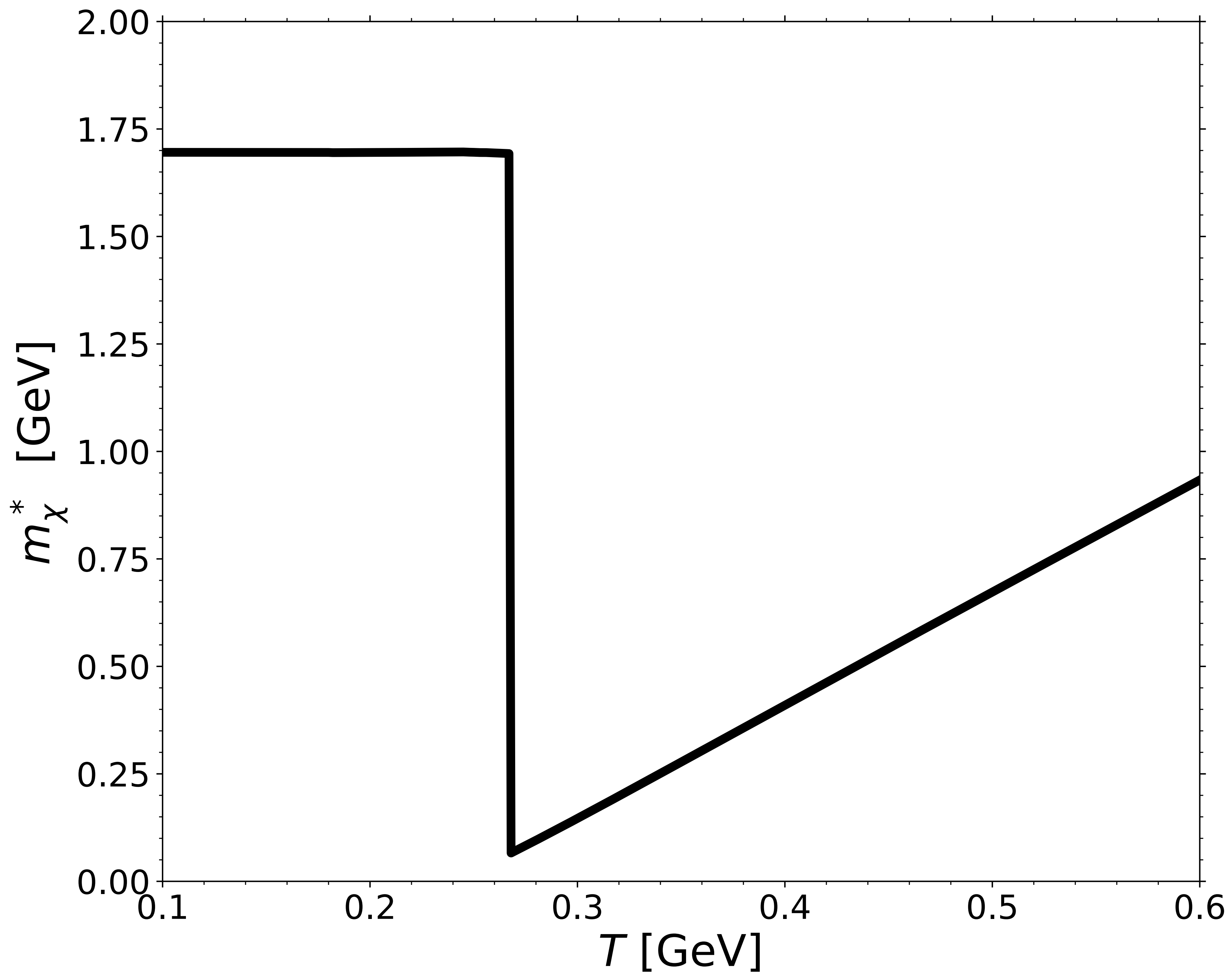
$$K(\chi) = \frac{A}{(1-\chi)}$$

$$g^2(T) = \frac{48\pi^2}{11N_c \ln\left(\frac{T^2+S^2}{\Lambda^2}\right)}$$

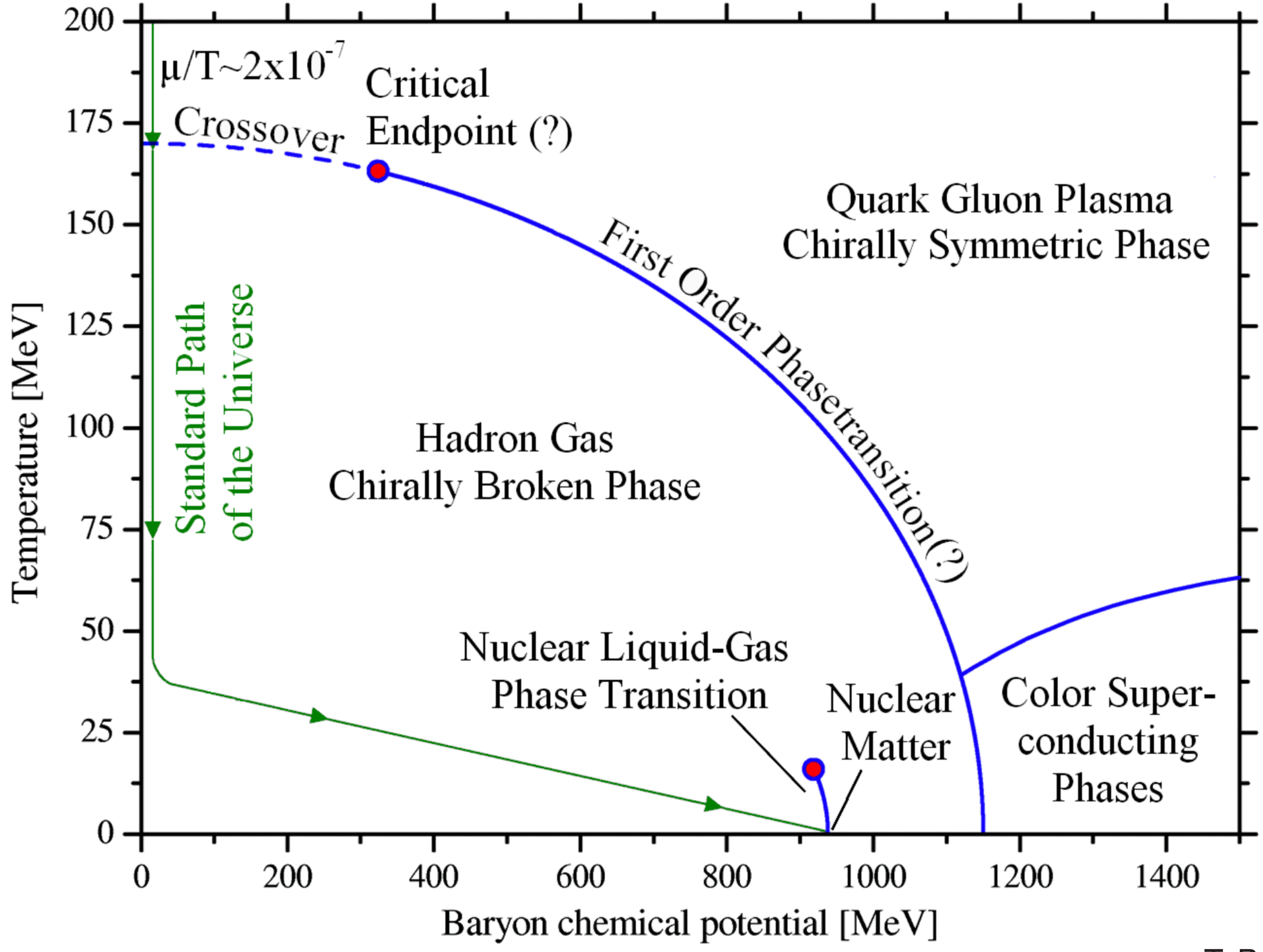
$$P_{\rm int}(\chi,T) = g^2 N_c(N_c^2-1)\Bigg\{(-3)\left[\int \frac{d^3k}{(2\pi)^3}\frac{1}{k}N_B\left(\frac{k}{T}\right)\Theta\left(k-K(\chi)\right)\right]^2 + \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{1}{k_1k_2}N_B\left(\frac{k_1}{T}\right)N_B\left(\frac{k_2}{T}\right)$$
$$\times\Theta\left(k_1-K(\chi)\right)\Theta\left(k_2-K(\chi)\right)\times\left[\frac{9}{4}\Theta\left(|\boldsymbol{k}_1+\boldsymbol{k}_2|-K(\chi)\right)-\frac{1}{4}\Theta\left(|\boldsymbol{k}_1-\boldsymbol{k}_2|-K(\chi)\right)\right]\Bigg\}$$

Pure gauge $\mathrm{SU(3)_c}$ sector



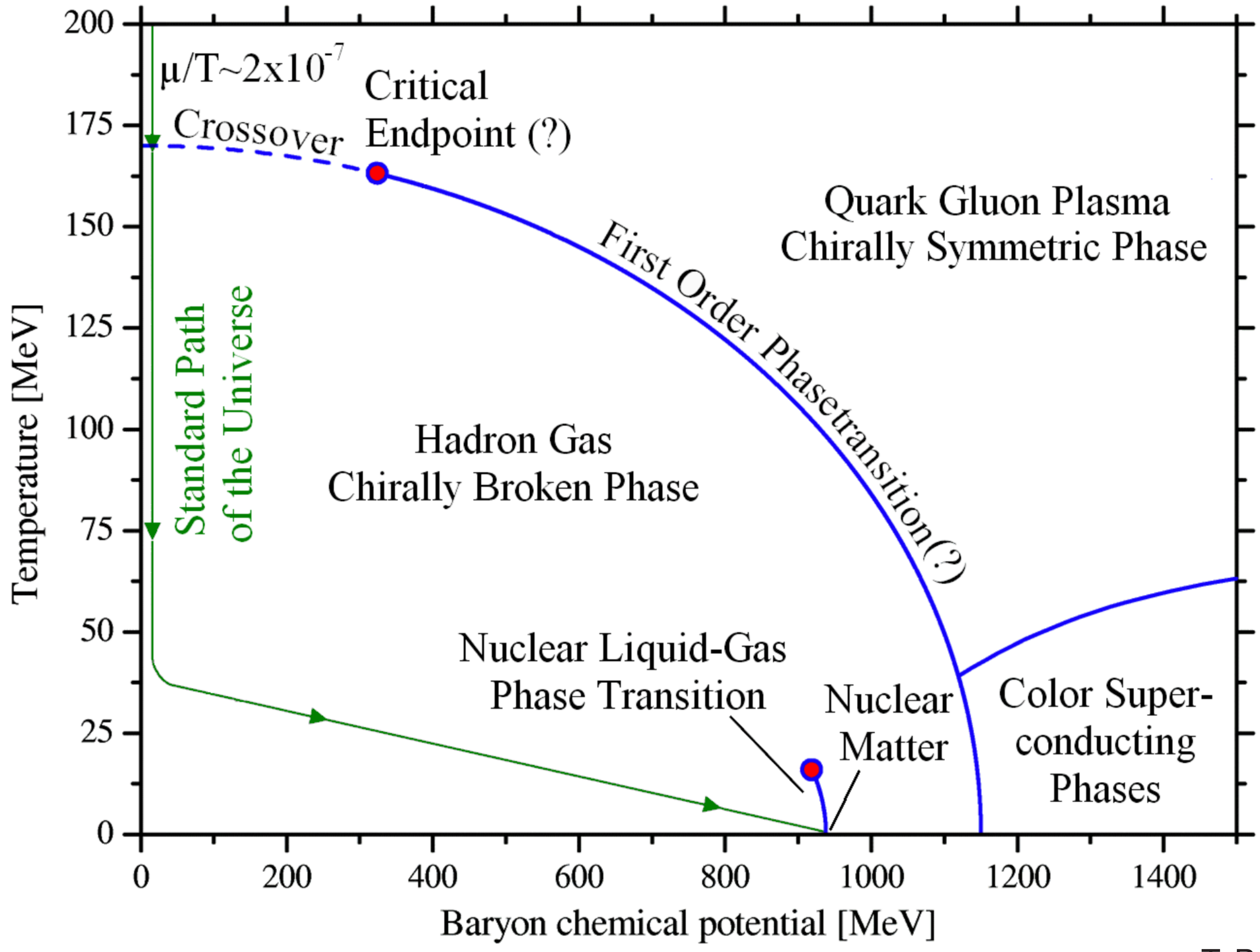


Little inflation scenario



T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Little inflation scenario



- Negligible baryon density: $\mu_B \ll T$

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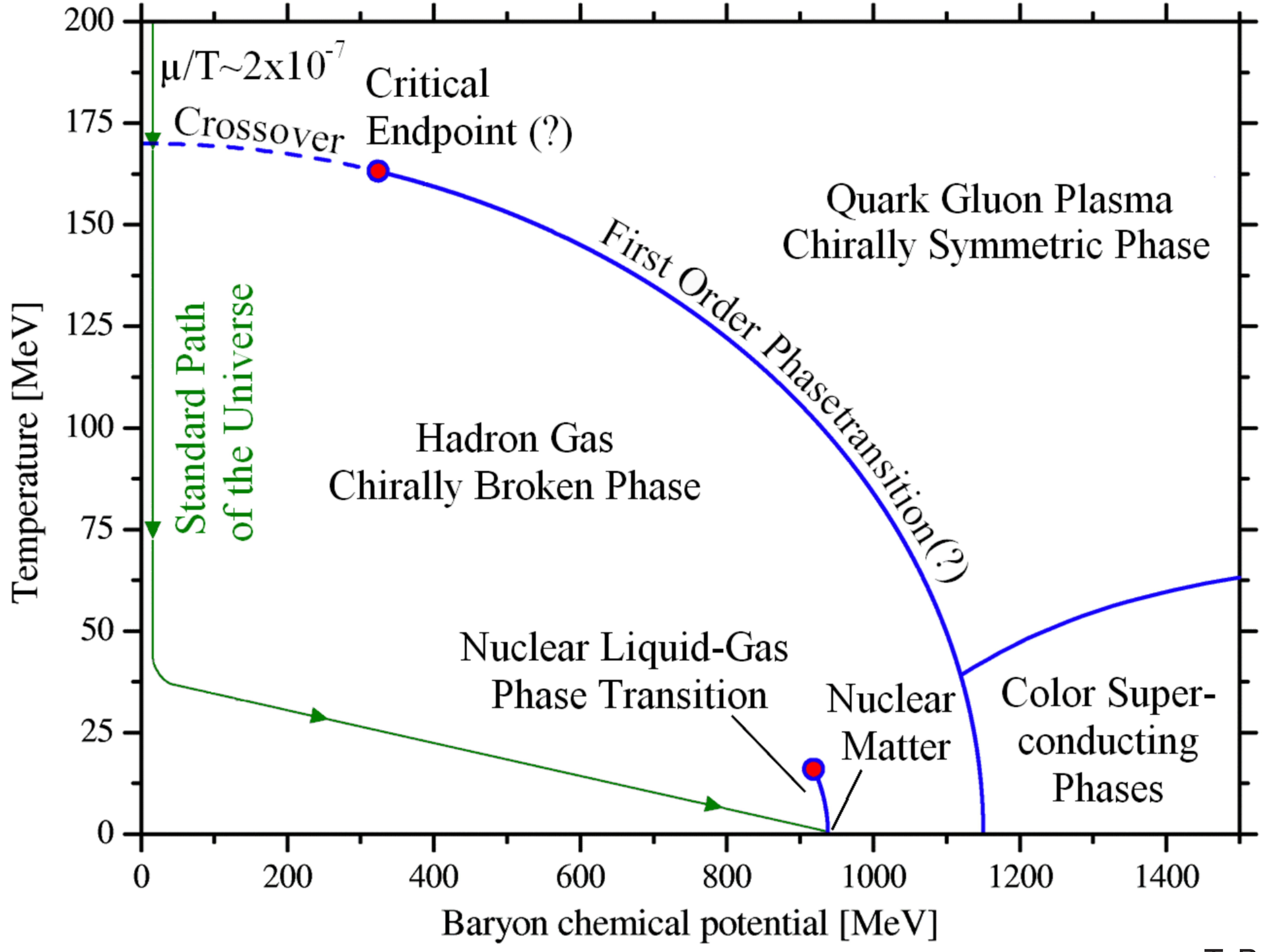

- Negligible baryon density: $\mu_B \ll T$
- QCD phase transition: **crossover**

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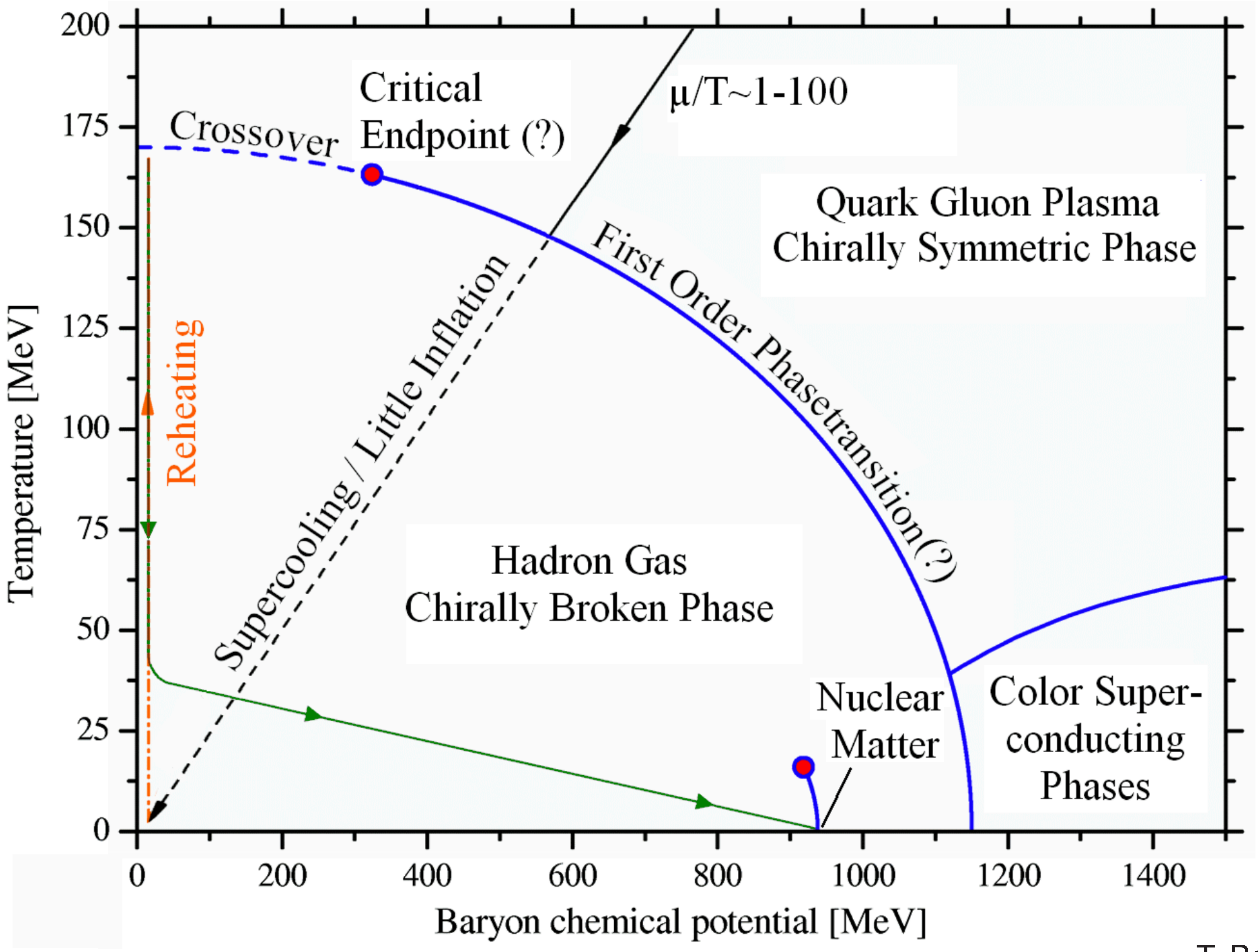

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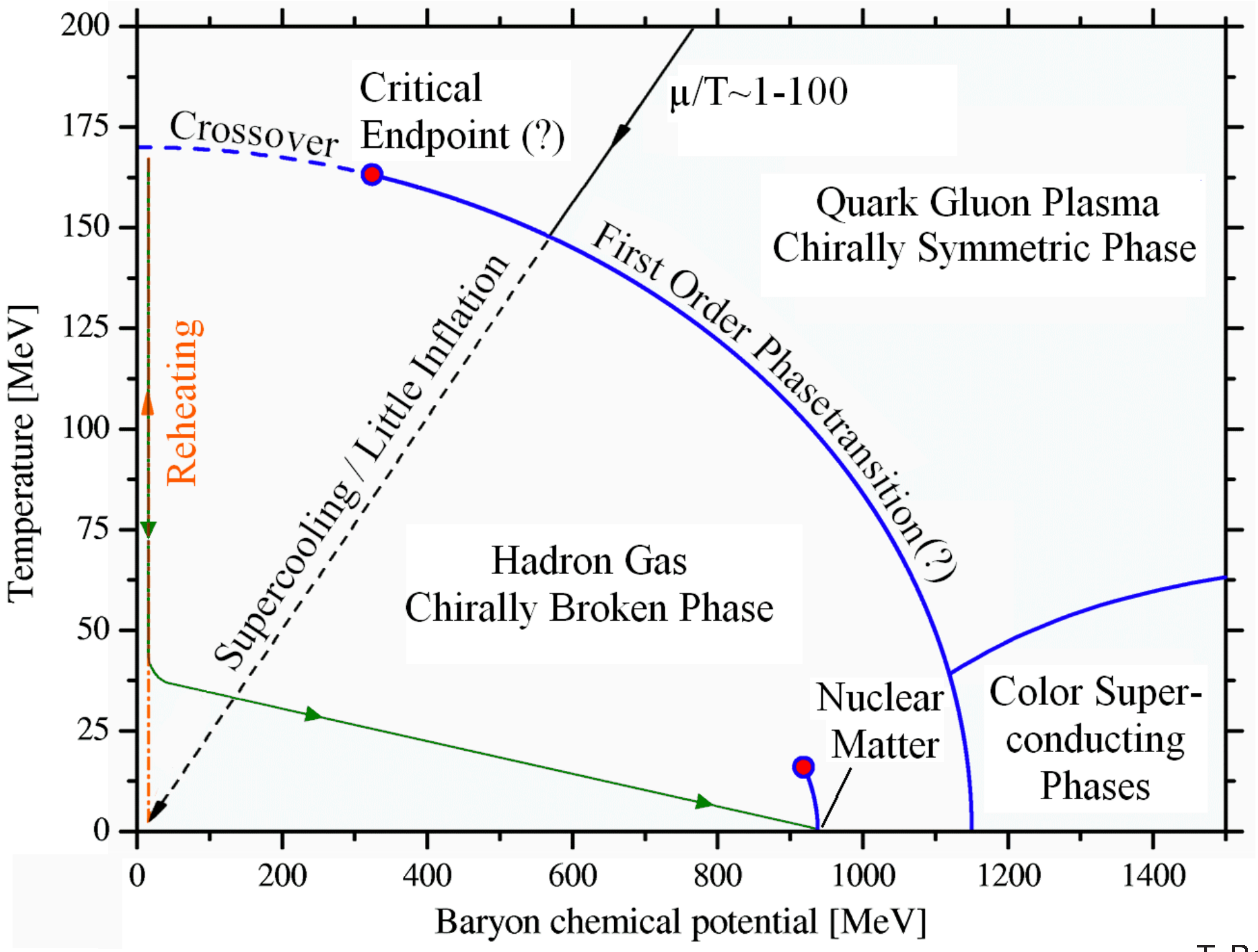

- Non vanishing μ_B: $\mu_B/T \sim O(1)$

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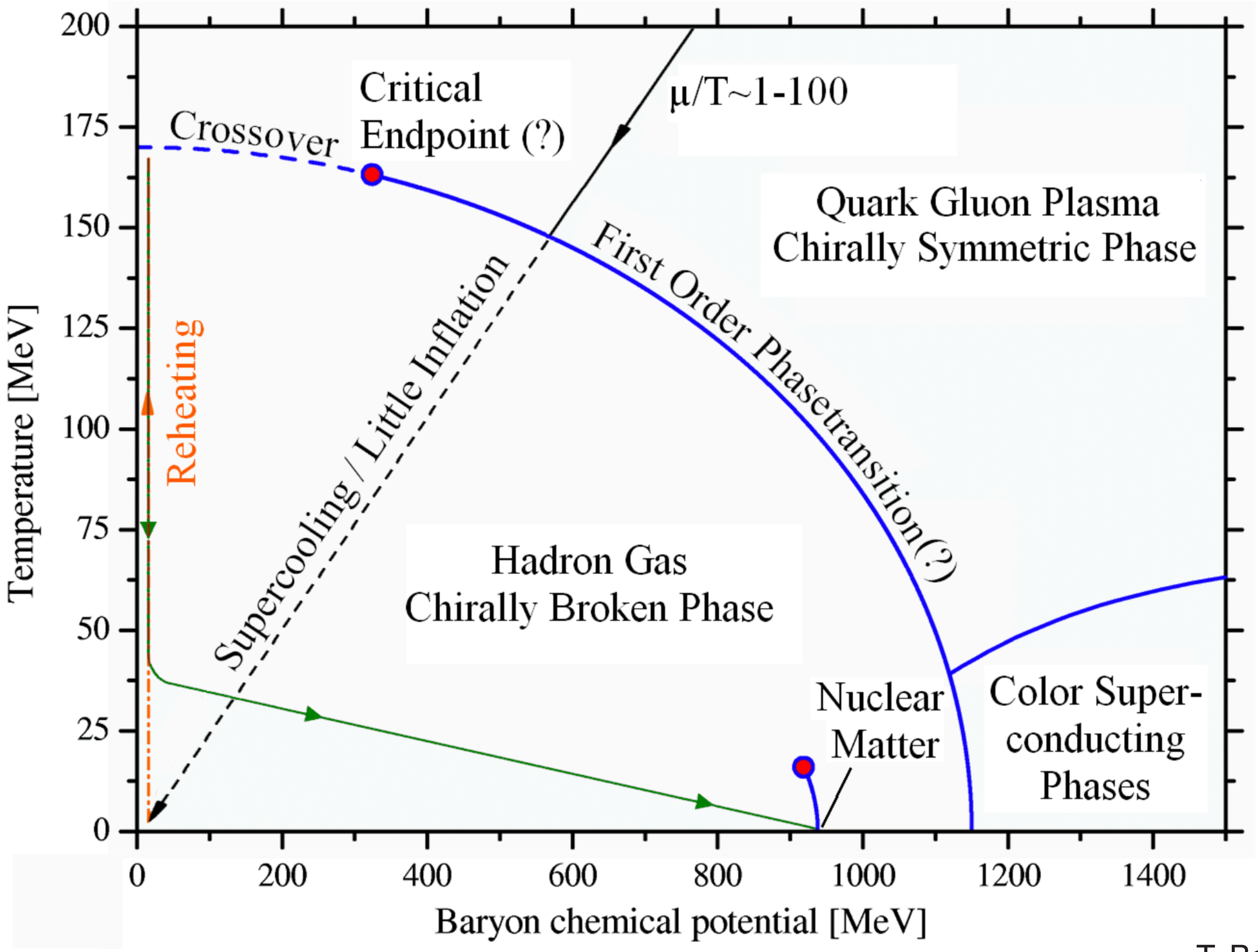

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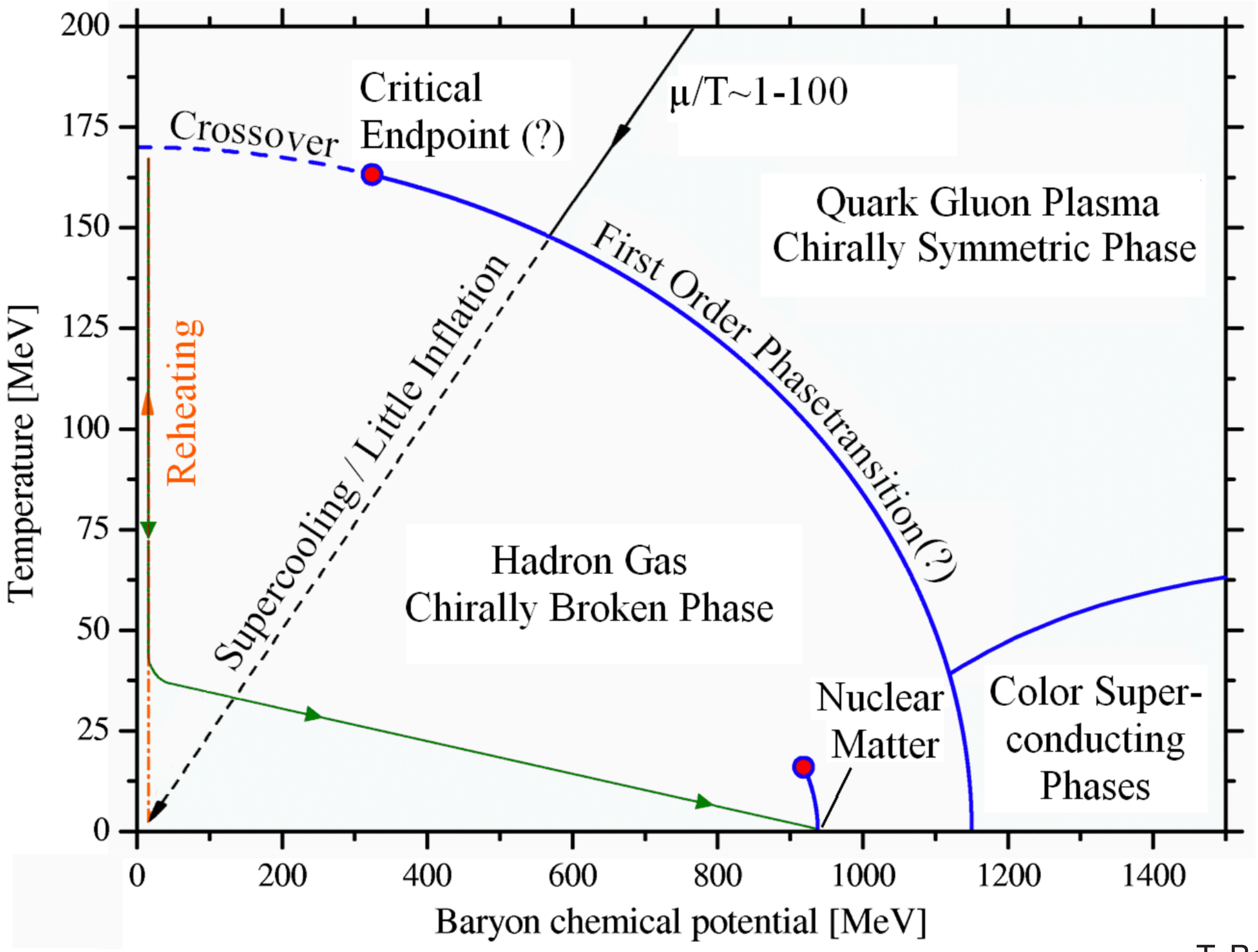

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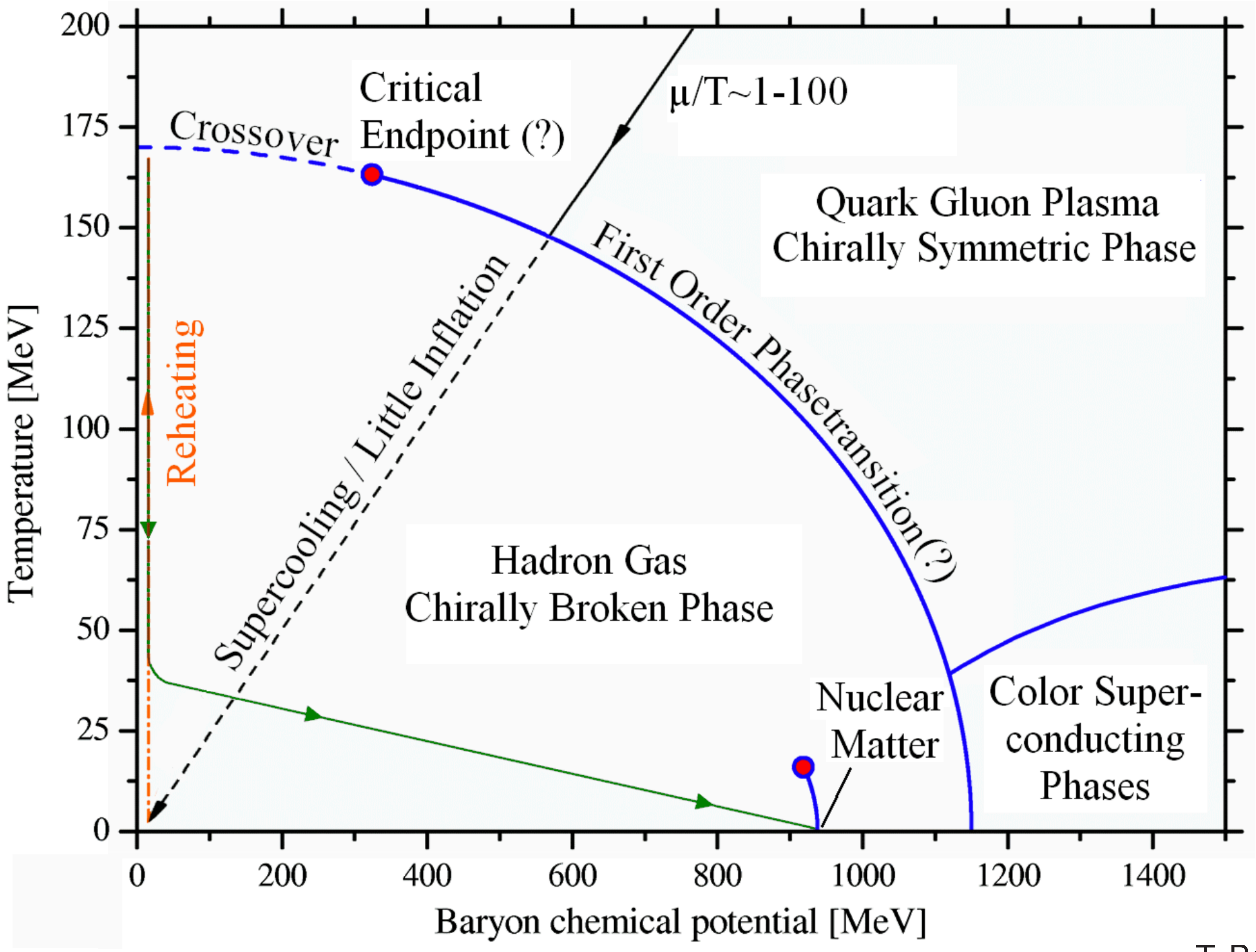

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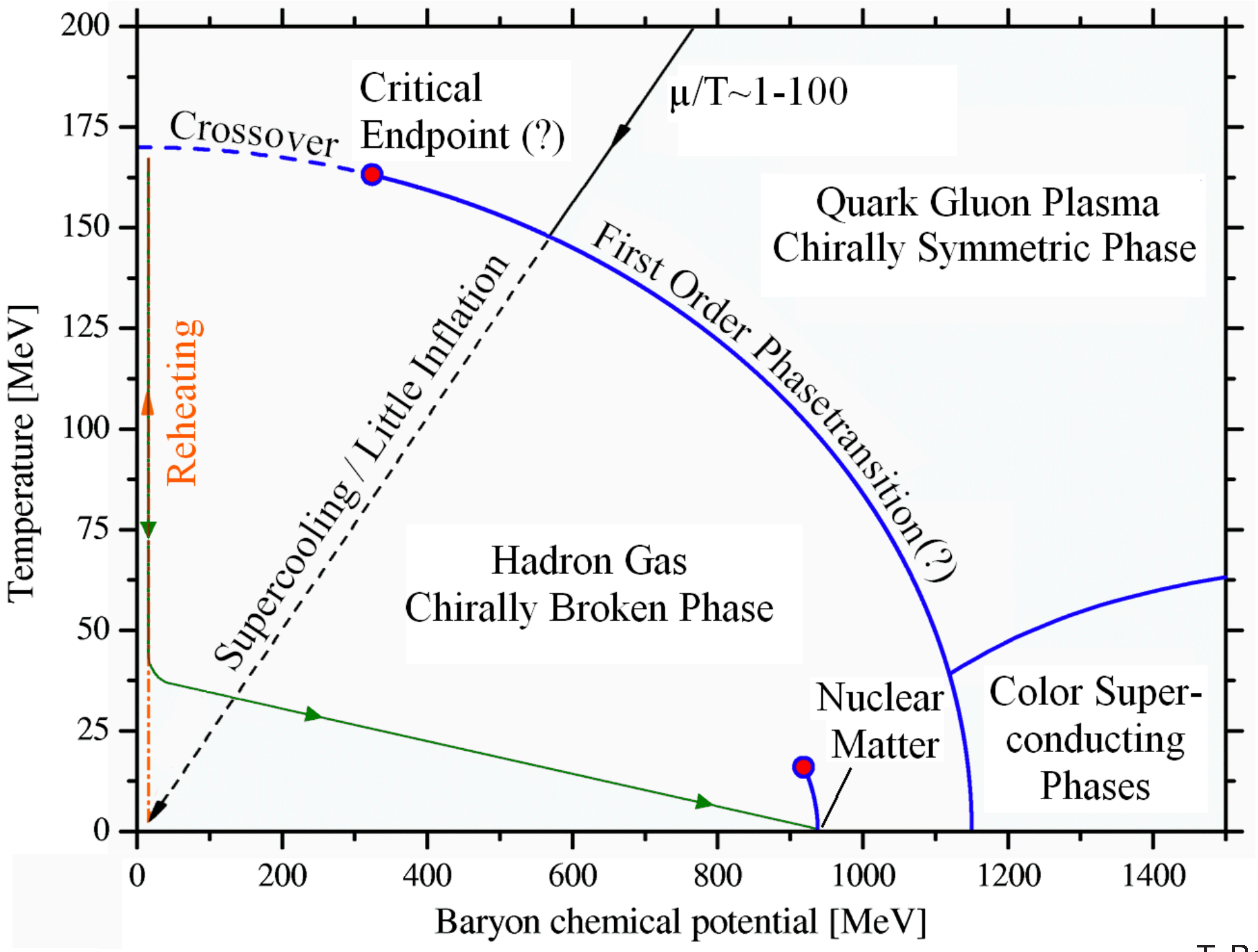

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T. Boeckel and J. Schaffner-Bielich, PRD 85, 103506 (2012)

Little inflation scenario

Equation of State for: cosmic trajectories

When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory

Mandy M. Wygas et al., PRL 121, 201302 (2018)

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Mandy M. Wygas et al., PRL 121, 201302 (2018)

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5 conservation laws

→5 equations

→5 chemical potentials

$\mu_{L_\alpha}, \mu_B, \mu_Q$

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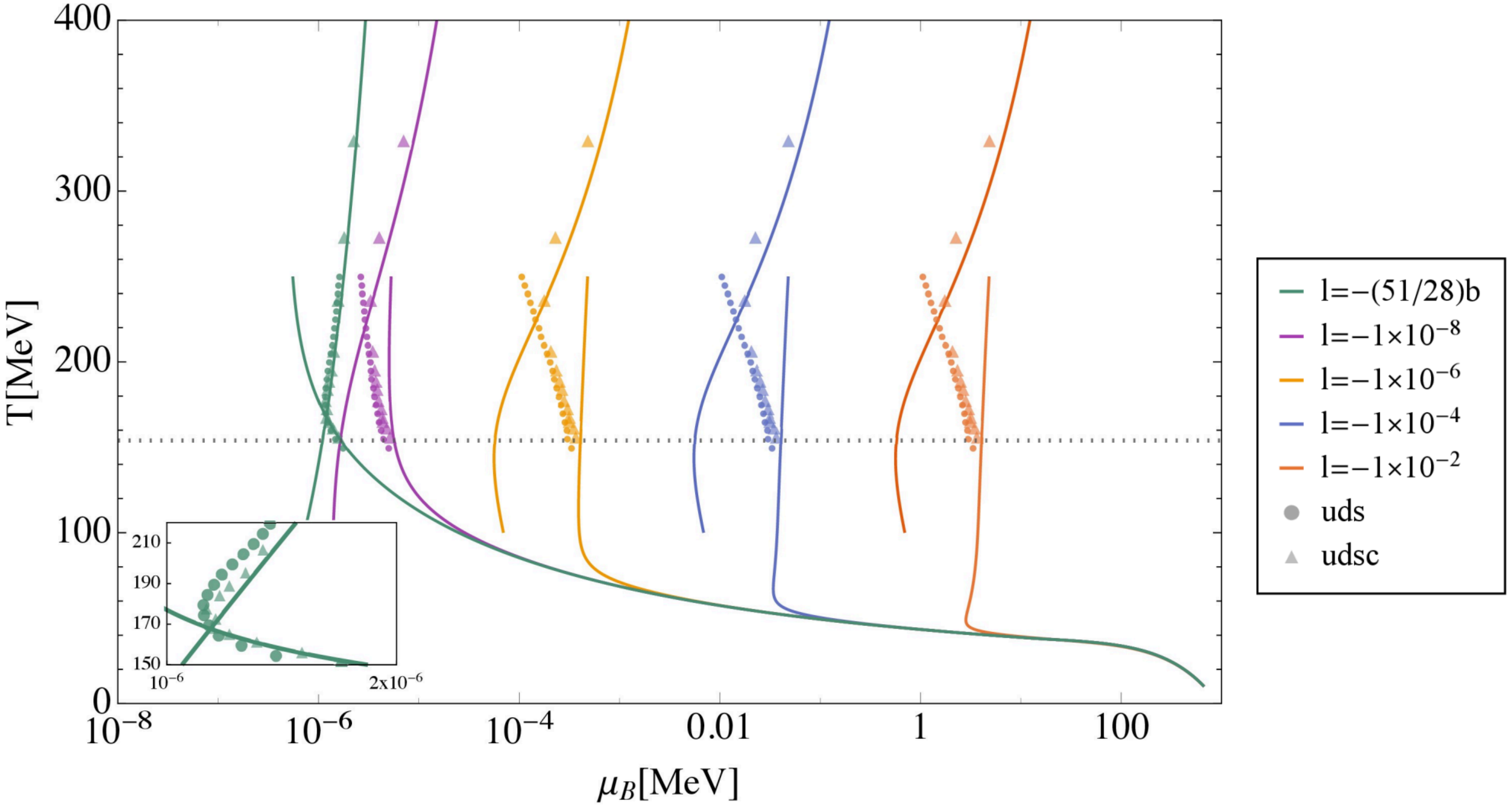

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