Perturbative calculations of few-nucleon observables in chiral EFT

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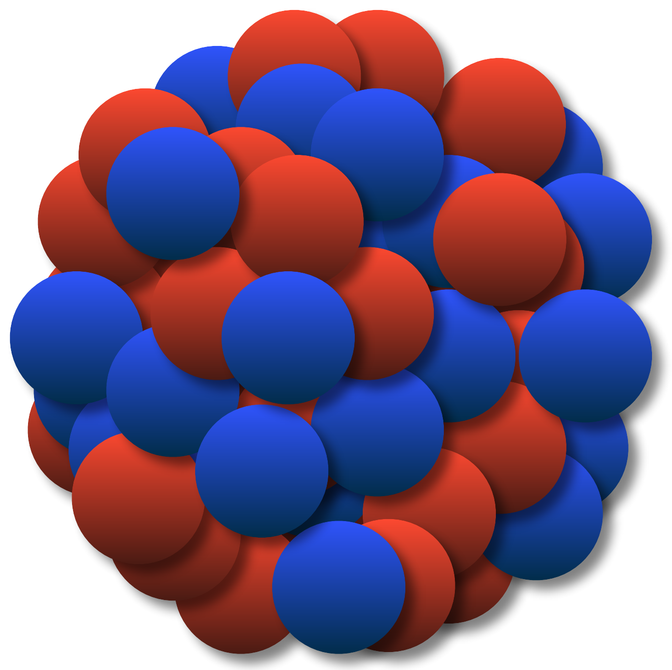
Ab initio nuclear theory

$\sim 10^{-15}$ m



$$H |\psi\rangle = E |\psi\rangle$$

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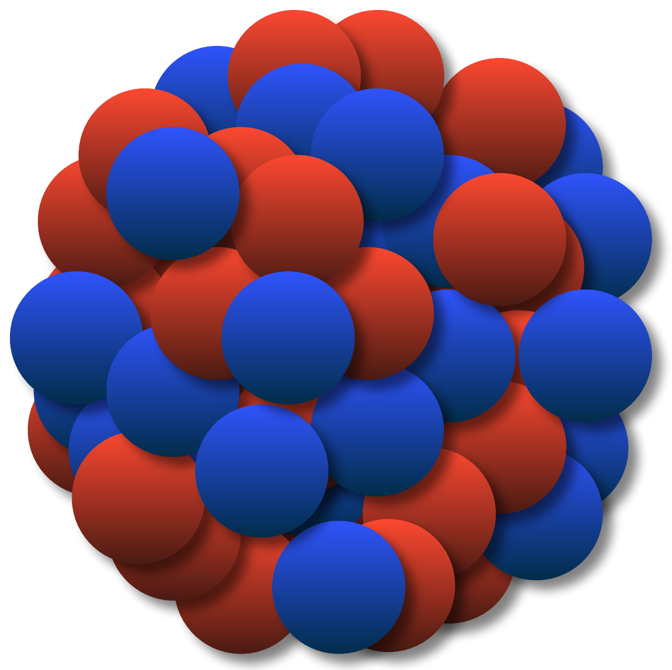

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Key questions

Ab initio nuclear theory

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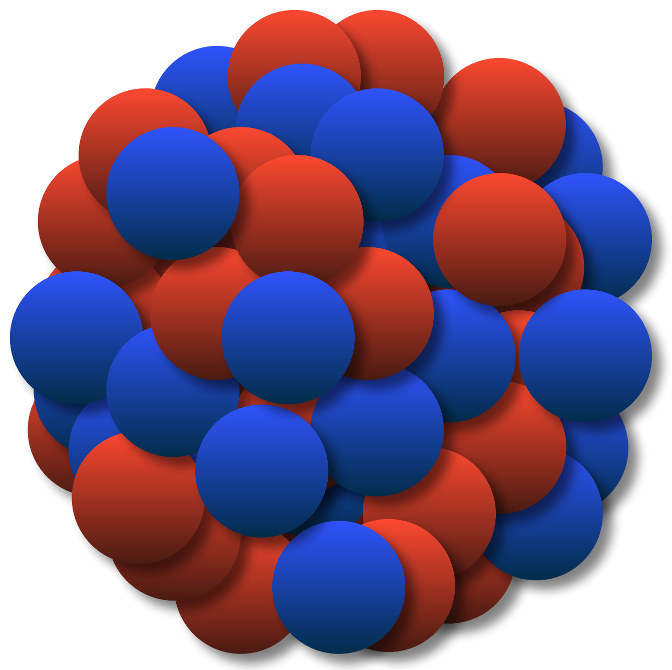


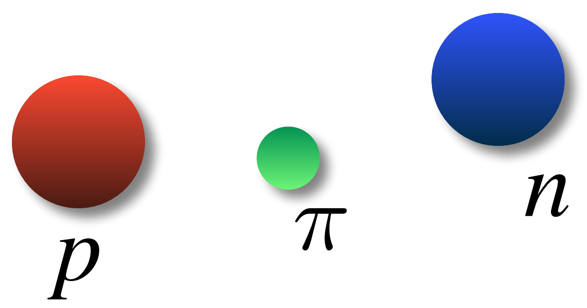
$$H |\psi\rangle = E |\psi\rangle$$

Key questions

- How to construct H to keep the connection to QCD?
- How to obtain precise predictions for nuclear observables with quantified theoretical error?

The nuclear force from EFT

- Weinberg, 90's: S. Weinberg, (1979), (1990), (1991)
  - Use protons, neutrons and pions as degrees of freedom.



  - Formulate the most general dynamics consistent with low-energy symmetries of QCD.
  - Perturbative expansion in $\frac{m_\pi \, , p}{\Lambda_b} \equiv \frac{Q}{\Lambda_b}$

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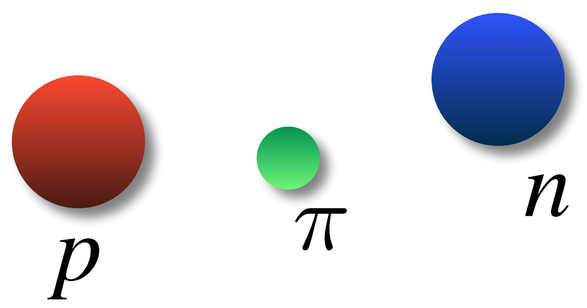

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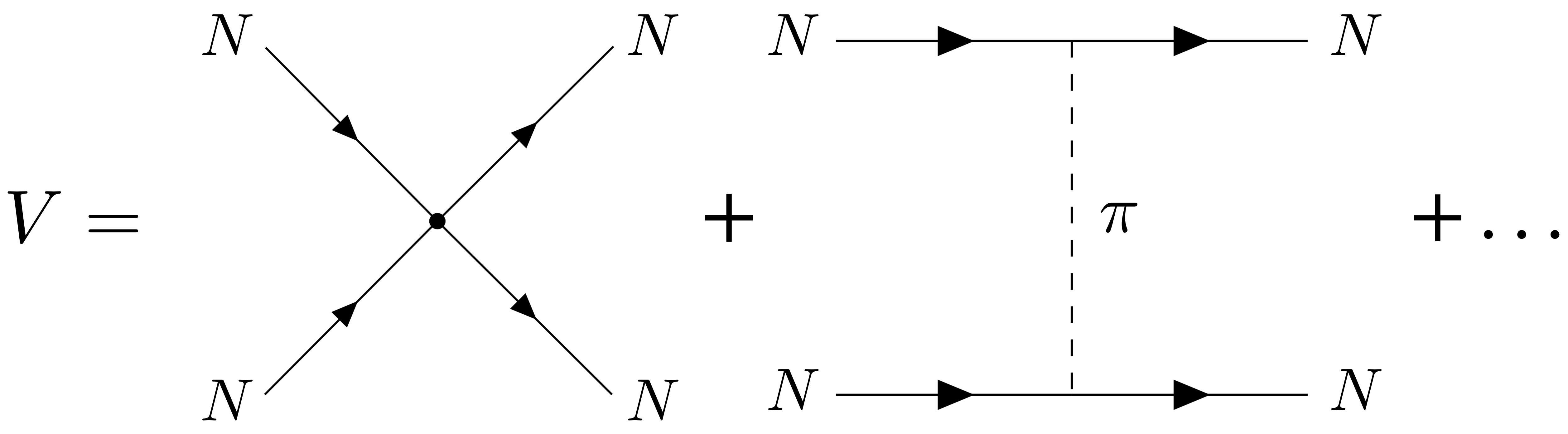

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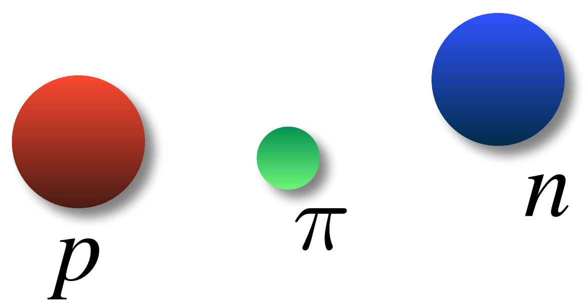

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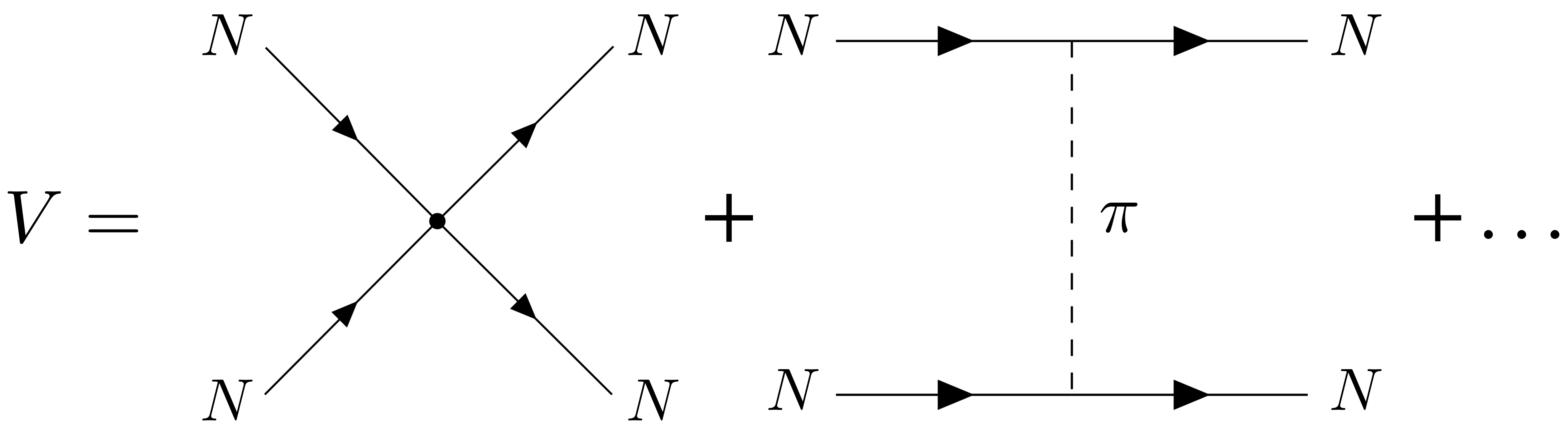
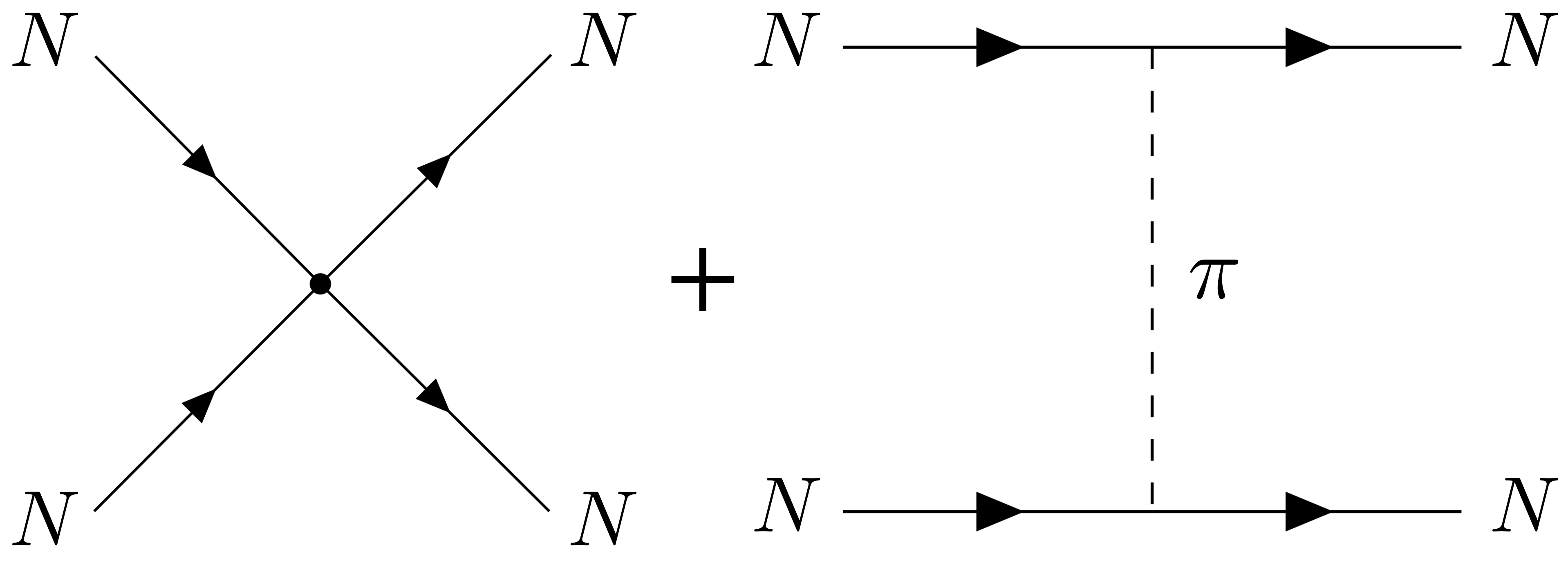


χEFT

The nuclear force from EFT

$V =$ 

$+ \dots$

- ✓ EFT description rooted in QCD.
- ✓ Systematic expansion with **quantifiable theoretical error.**

The nuclear force from EFT

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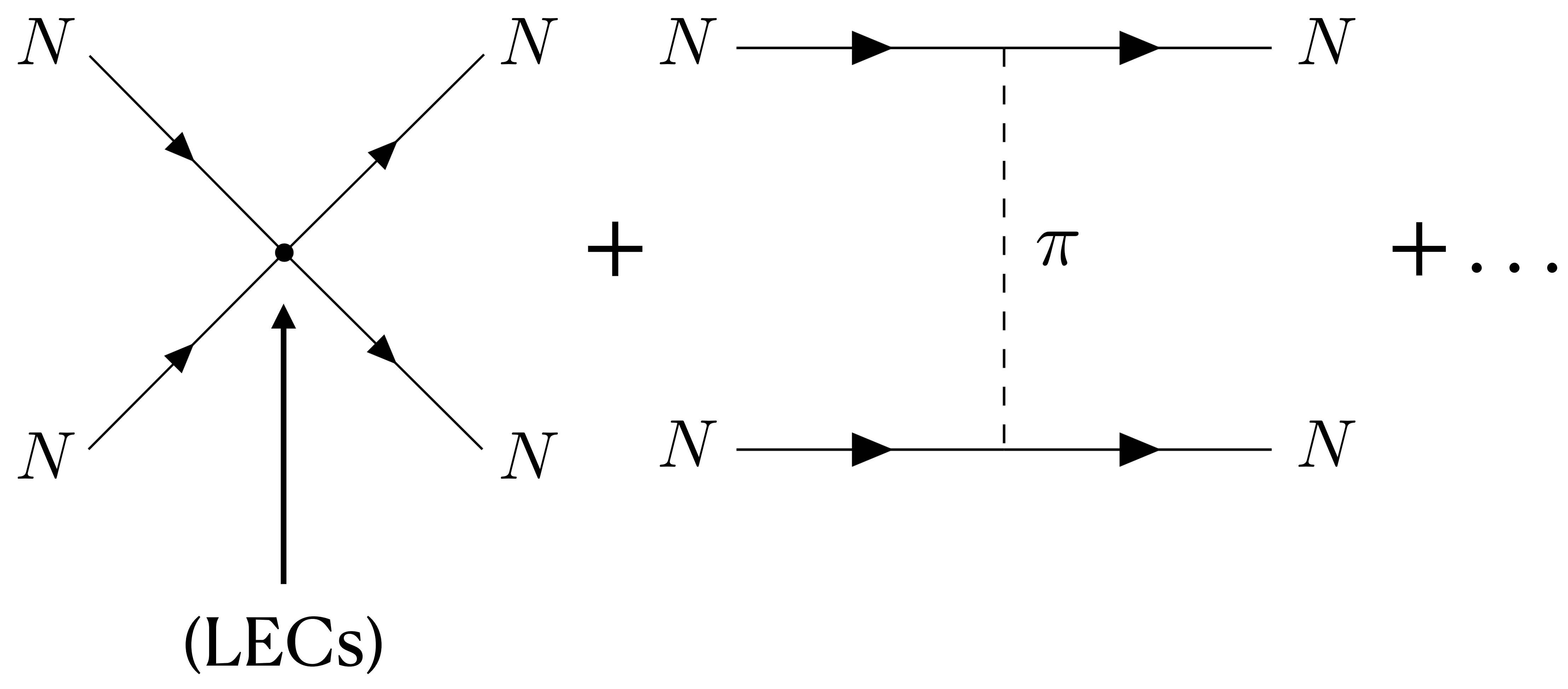


✓ EFT description rooted in QCD.

✓ Systematic expansion with **quantifiable theoretical error.**

🤔 Unknown values of low-energy constants (LECs).

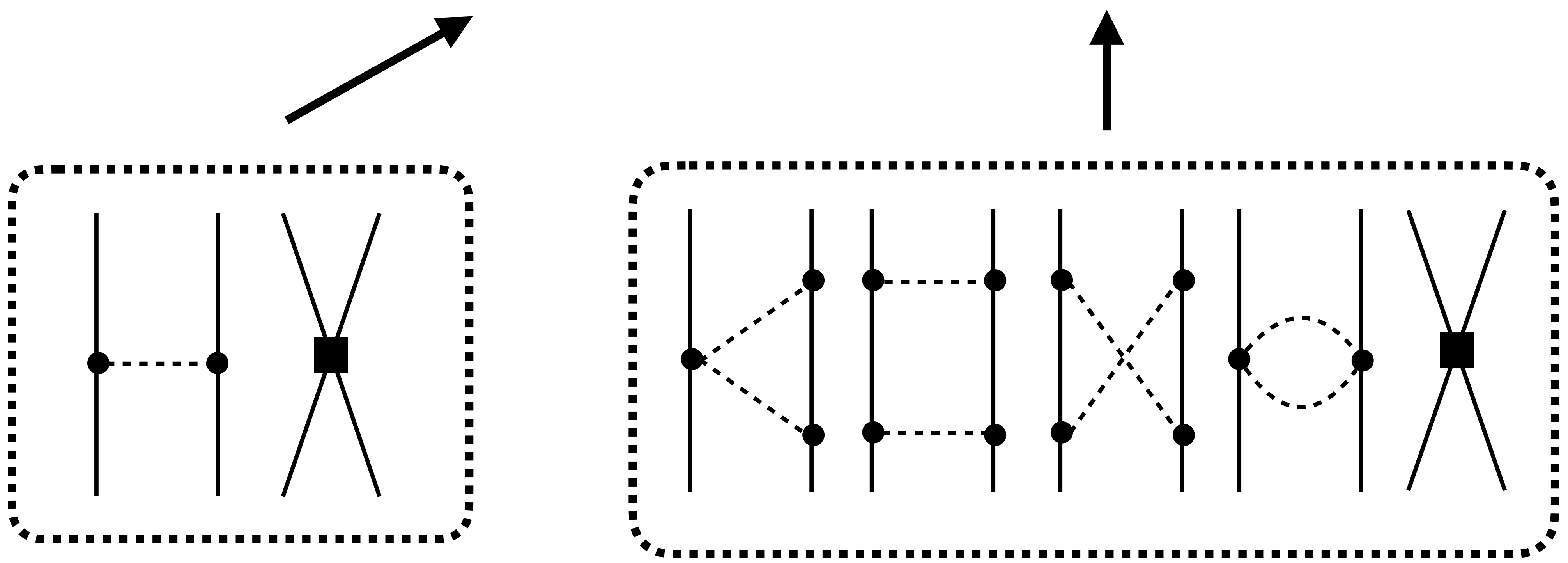
🤔 Importance of interactions: Power Counting (PC).

Weinberg Power Counting

- Construct nucleon-nucleon **potentials**:

R. Machleidt and D. R. Entem, Phys. Rep. **503** (2011)

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + \dots$$



$\nu = 0$ $\quad$ $\nu = 2$

- Organize diagrams: $(Q/\Lambda_b)^{\nu}$

  $Q \sim m_\pi,\ \Lambda_b \sim 1\ \mathrm{GeV}$

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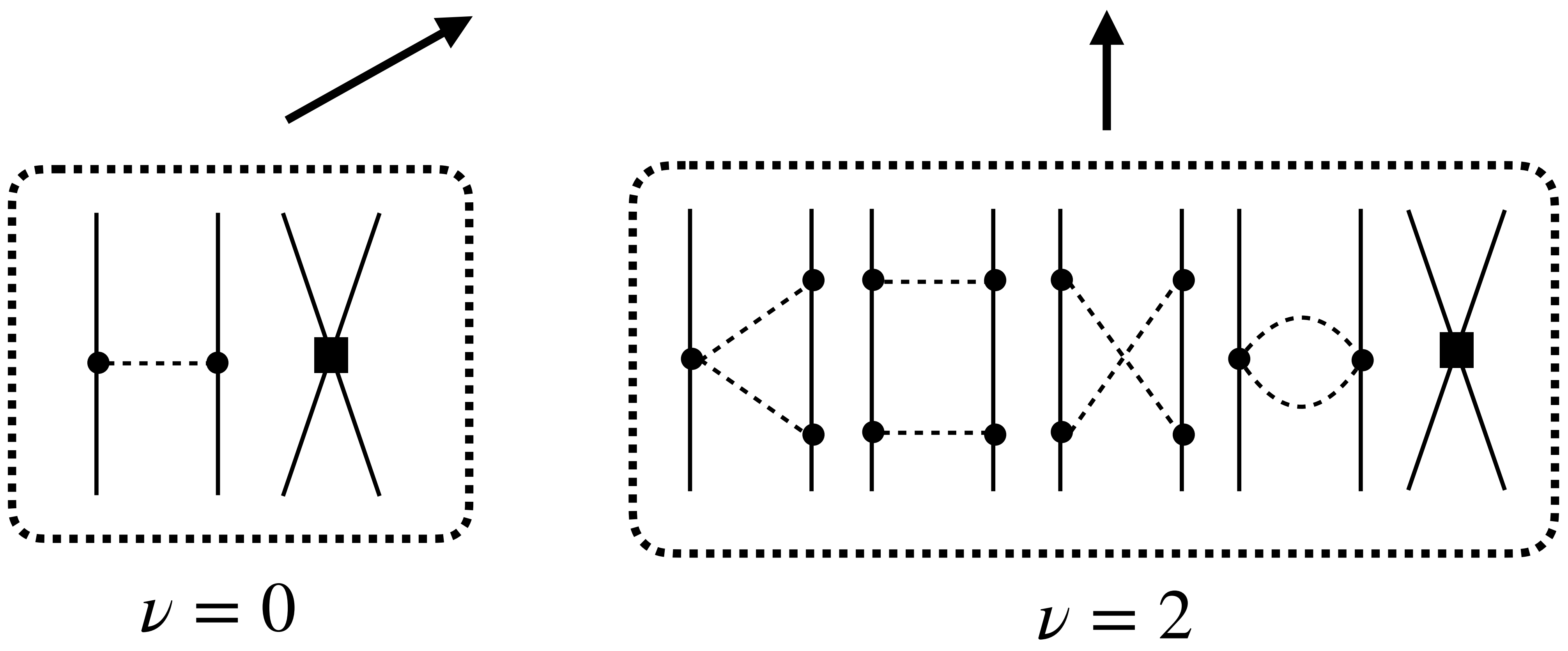

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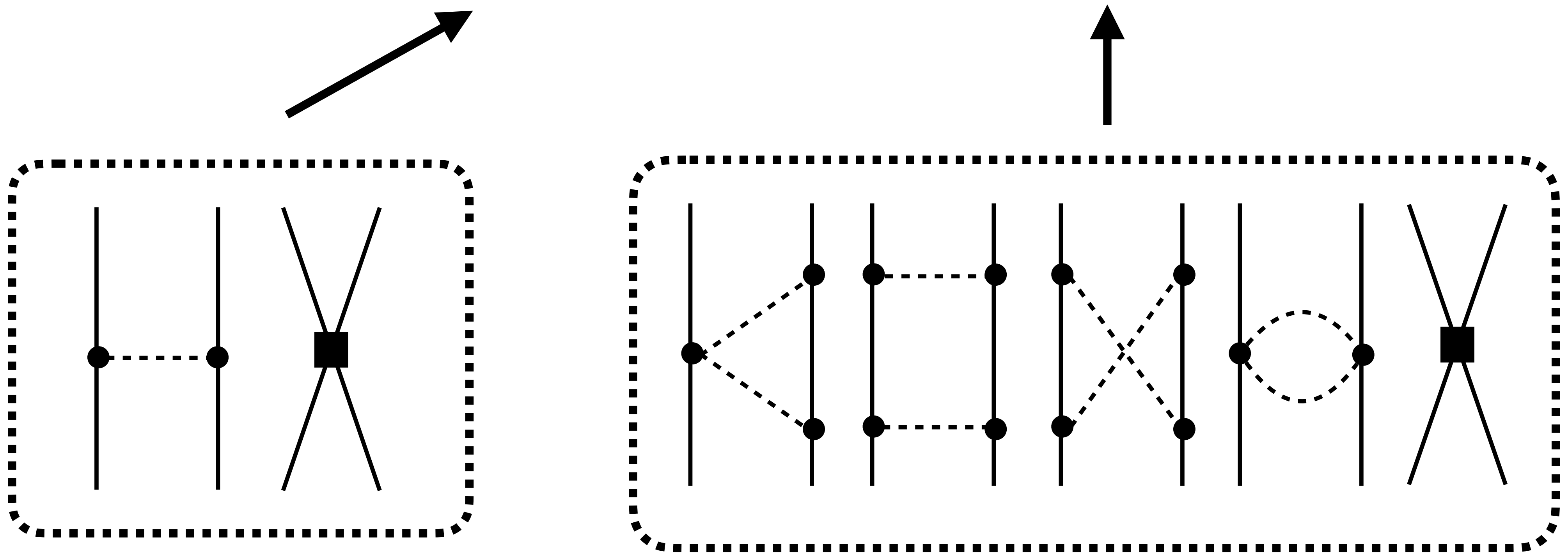

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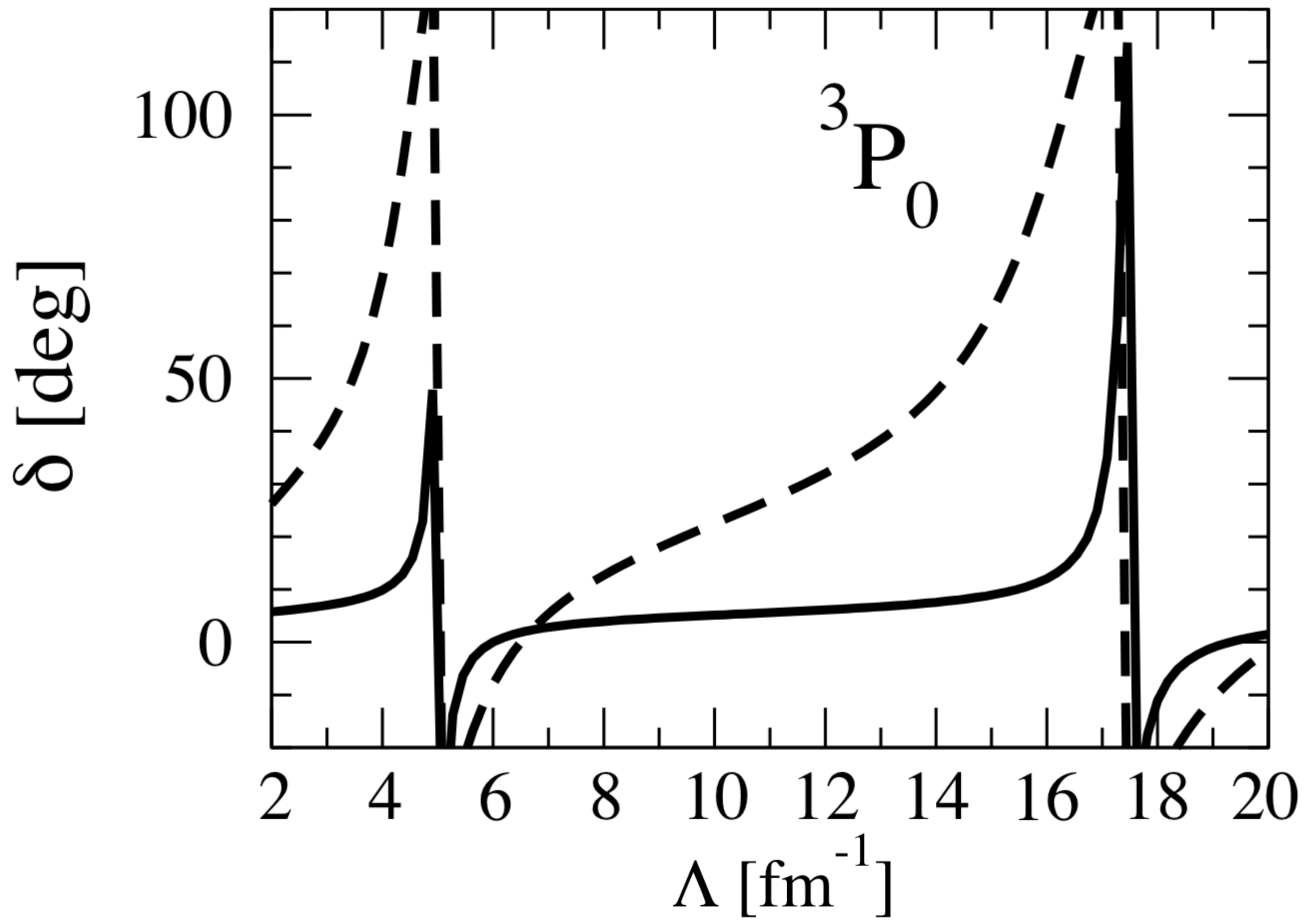
- χEFT with WPC: Successful descriptions of two- and three-nucleon forces and interaction currents.
- Predictions of observables **depend on Λ** (= not RG invariant)



A. Nogga *et al.,* Phys. Rev. C **72**, (2005)

Modified Weinberg PC
(Partly perturbative pions)

- What happens if we impose Λ**-independence** and promote counterterms to lower orders to achieve this?

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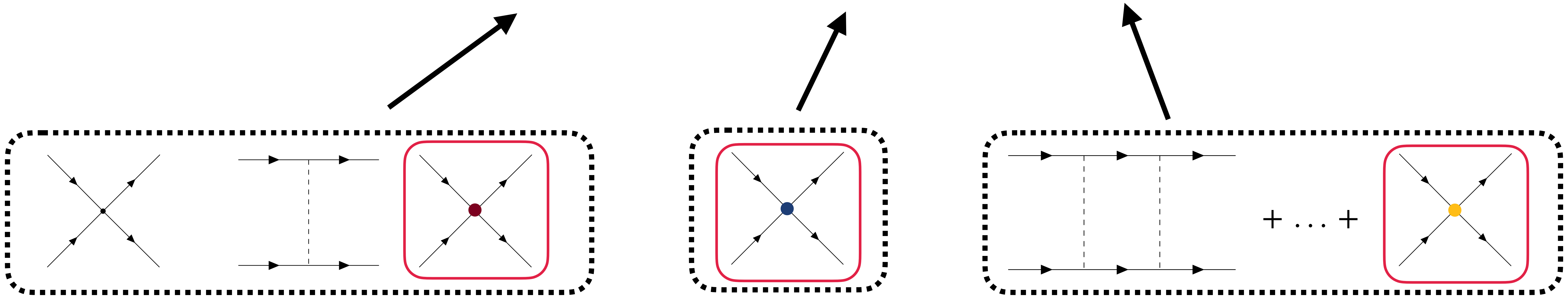

B. Long and U. van Kolck, Ann. Phys. **323**, (2008)

B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)

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Modified Weinberg PC
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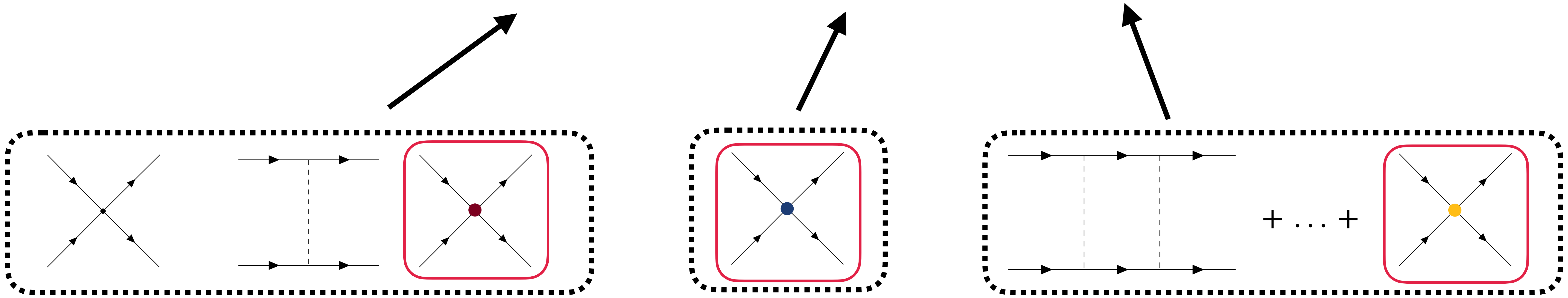
- What happens if we impose **Λ-independence** and promote counterterms to lower orders to achieve this?

Treated perturbatively

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\mathrm{NN}}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \ldots$$



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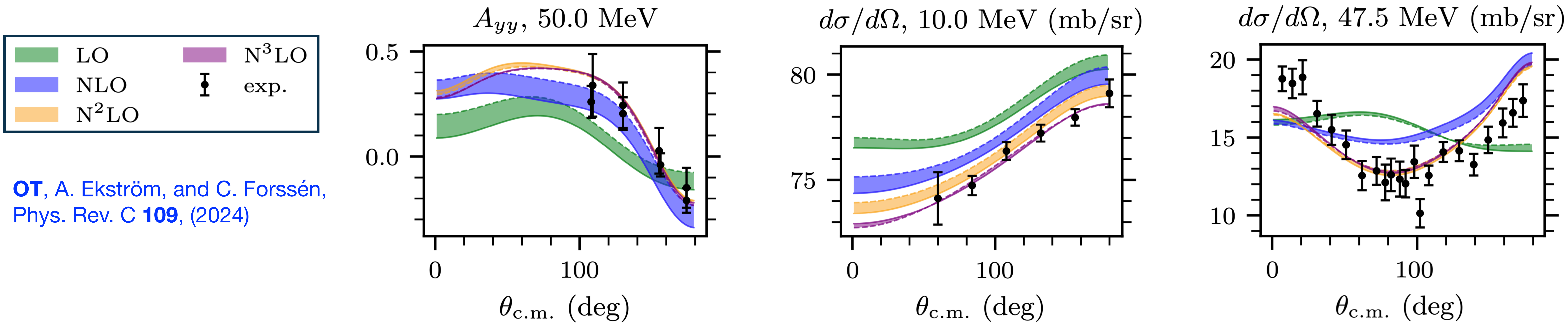
Neutron-proton scattering



OT, A. Ekström, and C. Forssén,
Phys. Rev. C **109**, (2024)

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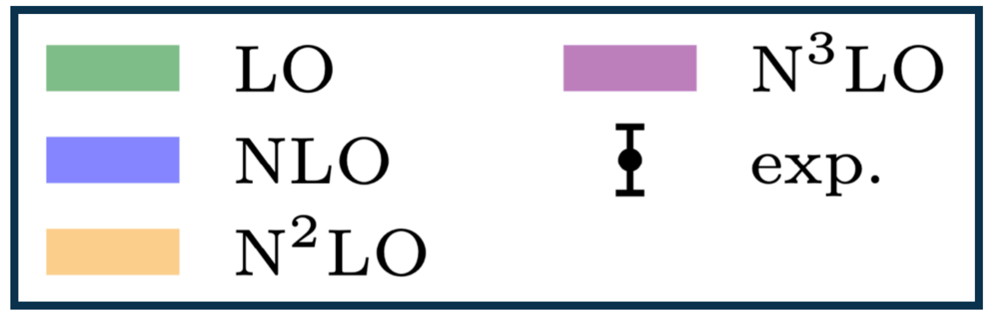

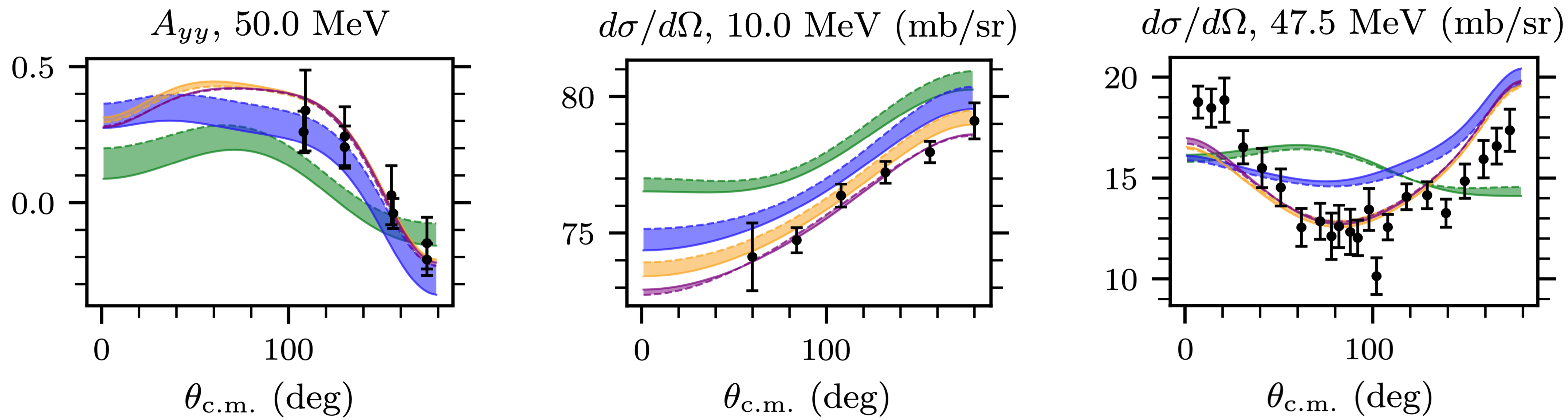

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- Also studied low-energy theorems (LETs).

Predicted effective range parameters

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	−23.735(16)	2.68(3)	−0.48(2)	3.9(1)	−19.6(5)
$\Lambda = 500$ MeV, (CIB)					
LO	*	1.68(0)	−1.55(0)	6.63(0)	−31.64(8)
NLO	*	*	−0.45(0)	3.42(0)	−18.95(8)
N^2LO	*	2.70(0)	−0.55(0)	3.77(0)	−18.8(2)
N^3LO	*	2.68(0)	−0.50(0)	4.02(0)	−19.8(2)

OT, Few-Body Syst. 65, 69 (2024)

Question:

Can χEFT with **partly perturbative pions** describe nuclear observables in A>2 systems?

- Some studies of LO and NLO for $A > 2$ observables, we are developing the machinery to go up to N^3LO.

Y.-H. Song, R. Lazauskas, and U. van Kolck, Phys. Rev. C **96**, (2017)

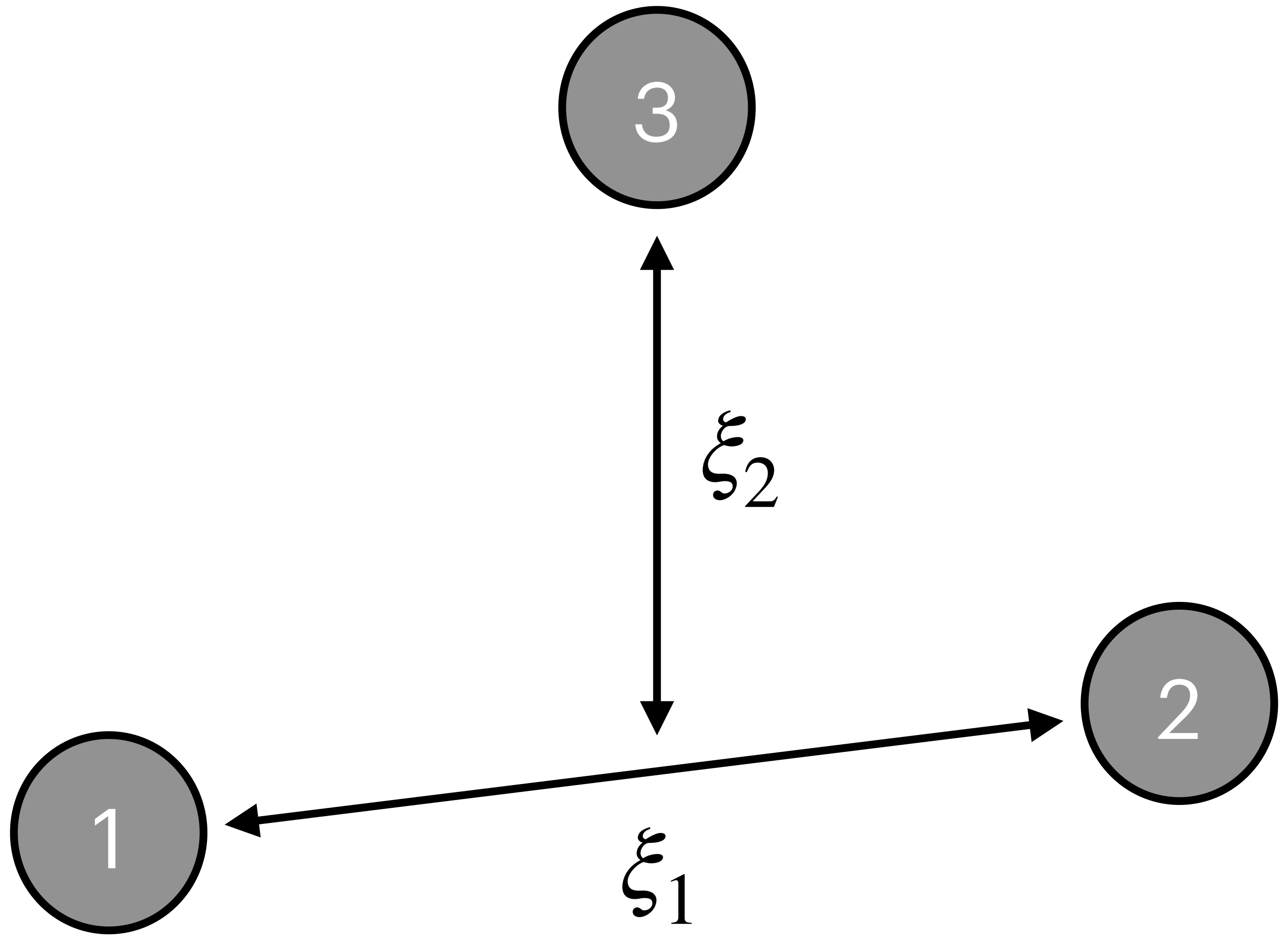
C. J. Yang *et al.*, Phys. Rev. C **103**, (2021)

Bound-state computations in perturbation theory

$$H|\psi\rangle = E|\psi\rangle$$

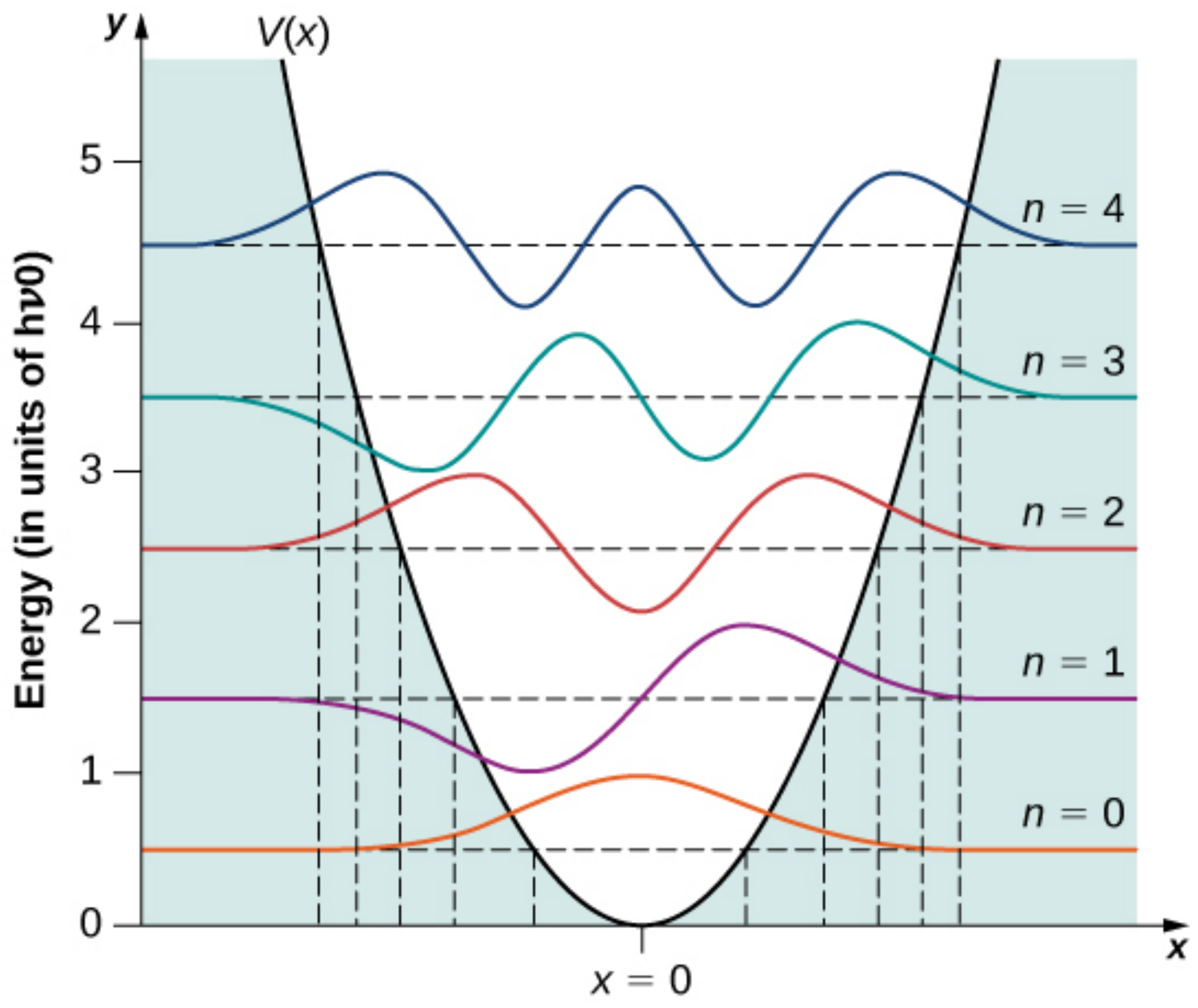
- Requirements:
  - Method that is easy to implement in existing codes.
  - Small computational overhead.
- Idea:
  - Start with ^{3}H: can implement both the Rayleigh-Schrödinger and Hellmann–Feynman method.

No-core shell model for ^{3}H



$$H = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m_N} + \sum_{i<j=3}^{A} V_{ij}$$

- Basis characterized by $(\hbar\Omega, N_{\max})$
  - $N_{\max}$ — number of basis states
  - $\hbar\Omega$ — shape of basis states



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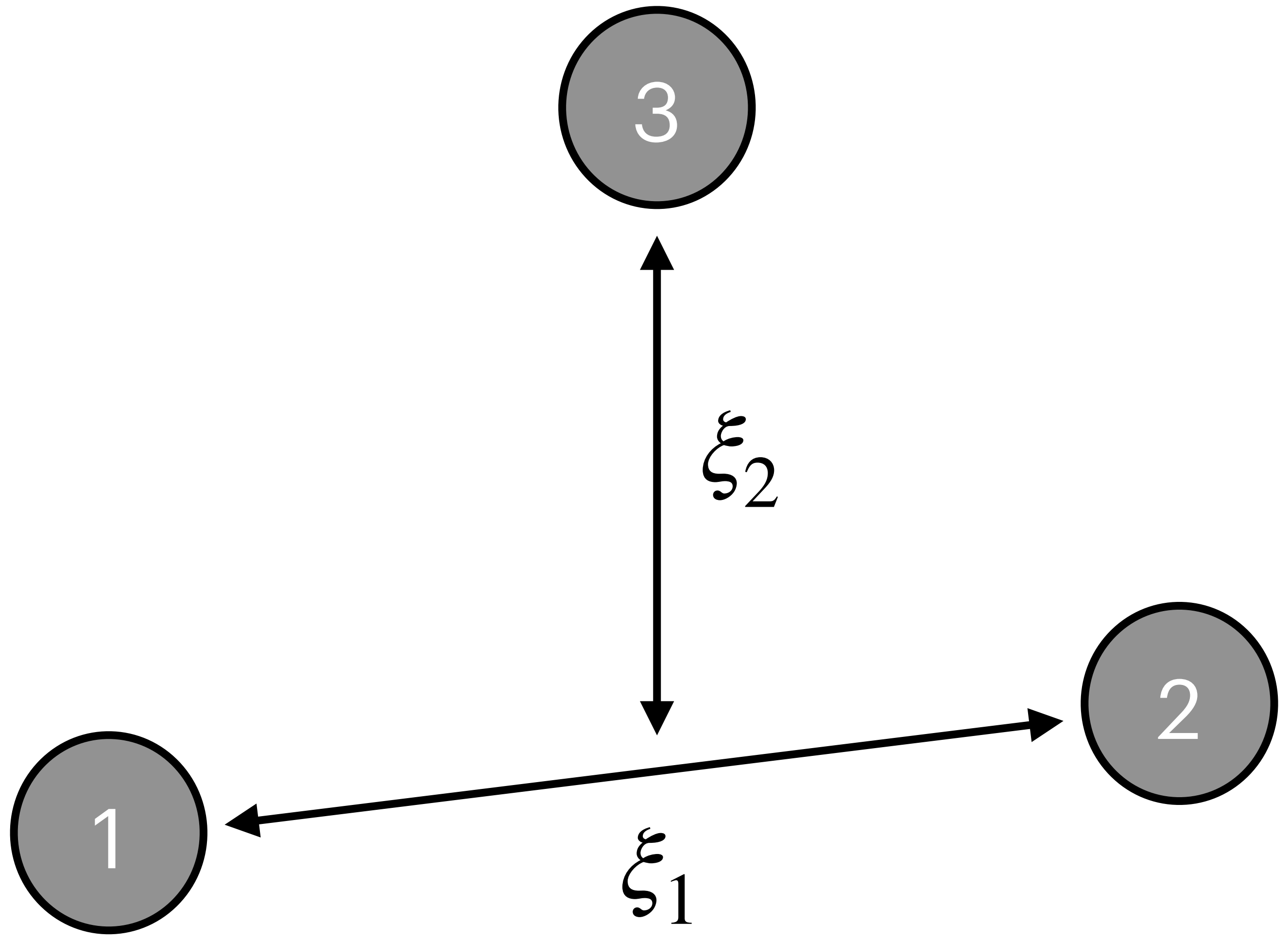

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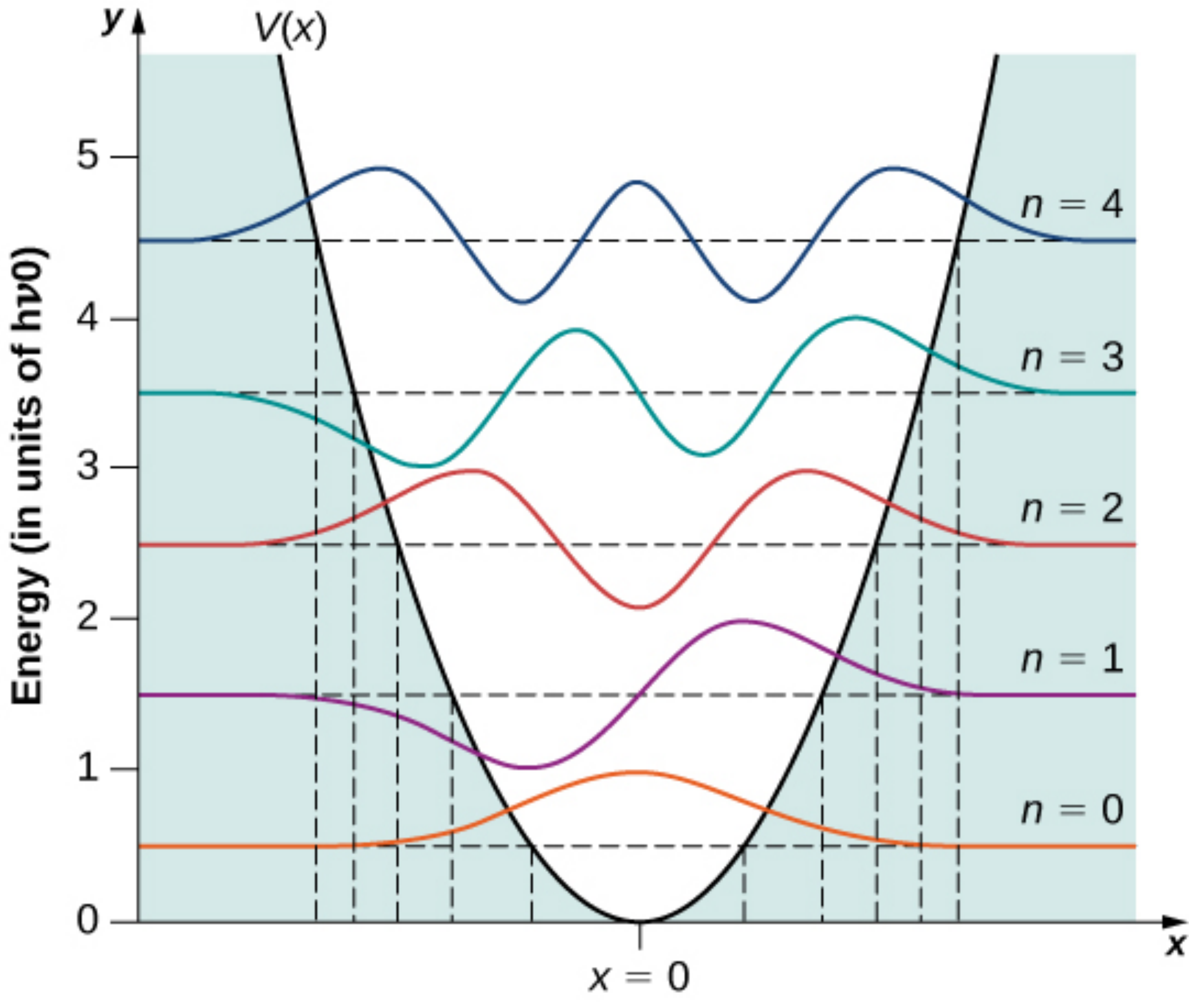
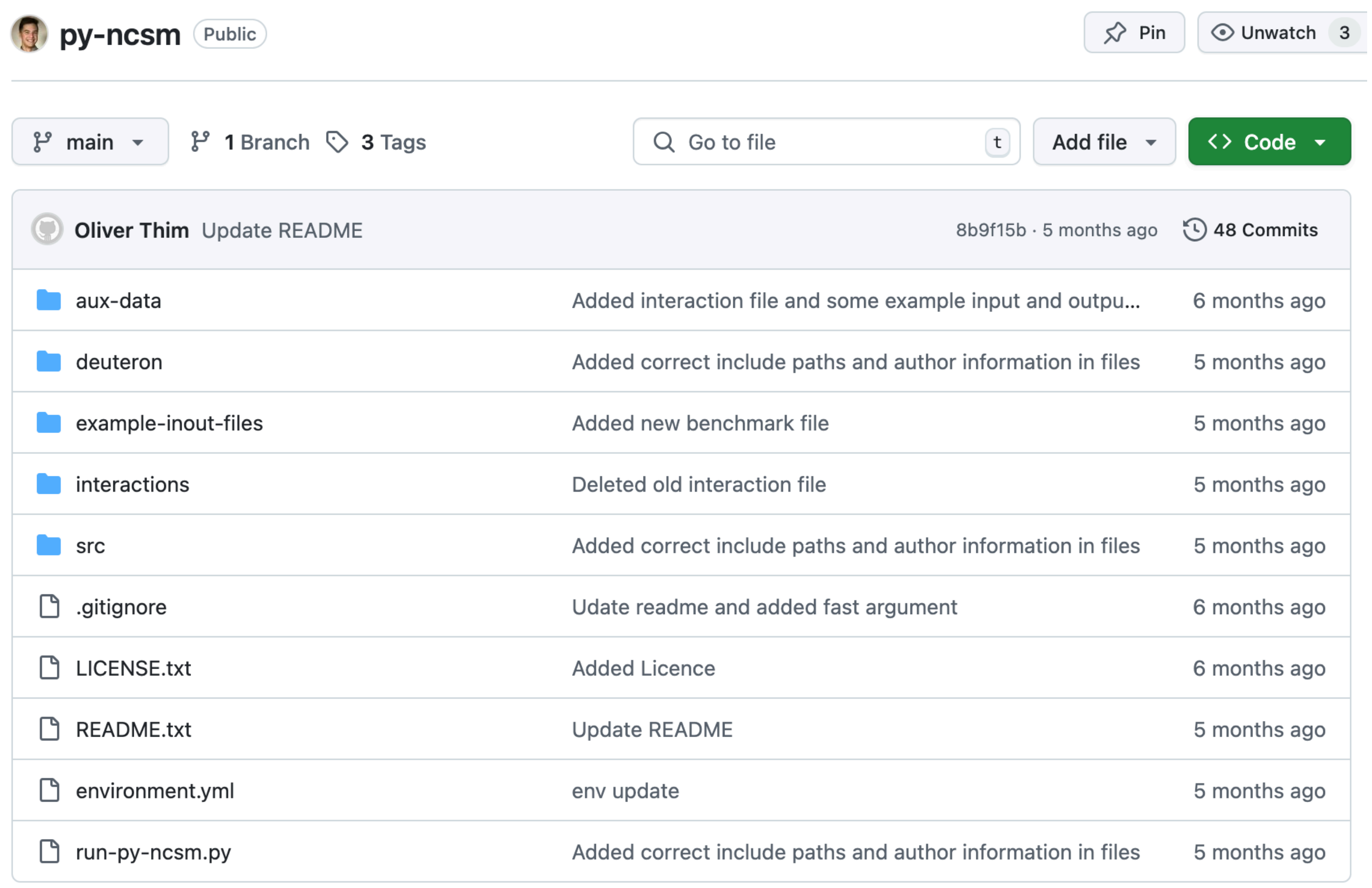


Code available:

https://github.com/othim/py-ncsm



Now: perturbation theory!

Treated perturbatively

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- Rayleigh-Schrödinger (RS):

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N²LO: $$E^{(2)} = \langle \Psi_0^{(0)} | V_{\text{NN}}^{(2)} | \Psi_0^{(0)} \rangle + \sum_{n \neq 0} \frac{| \langle \Psi_0^{(0)} | V_{\text{NN}}^{(1)} | \Psi_n^{(0)} \rangle |^2}{E_0^{(0)} - E_n^{(0)}}$$

N³LO: …

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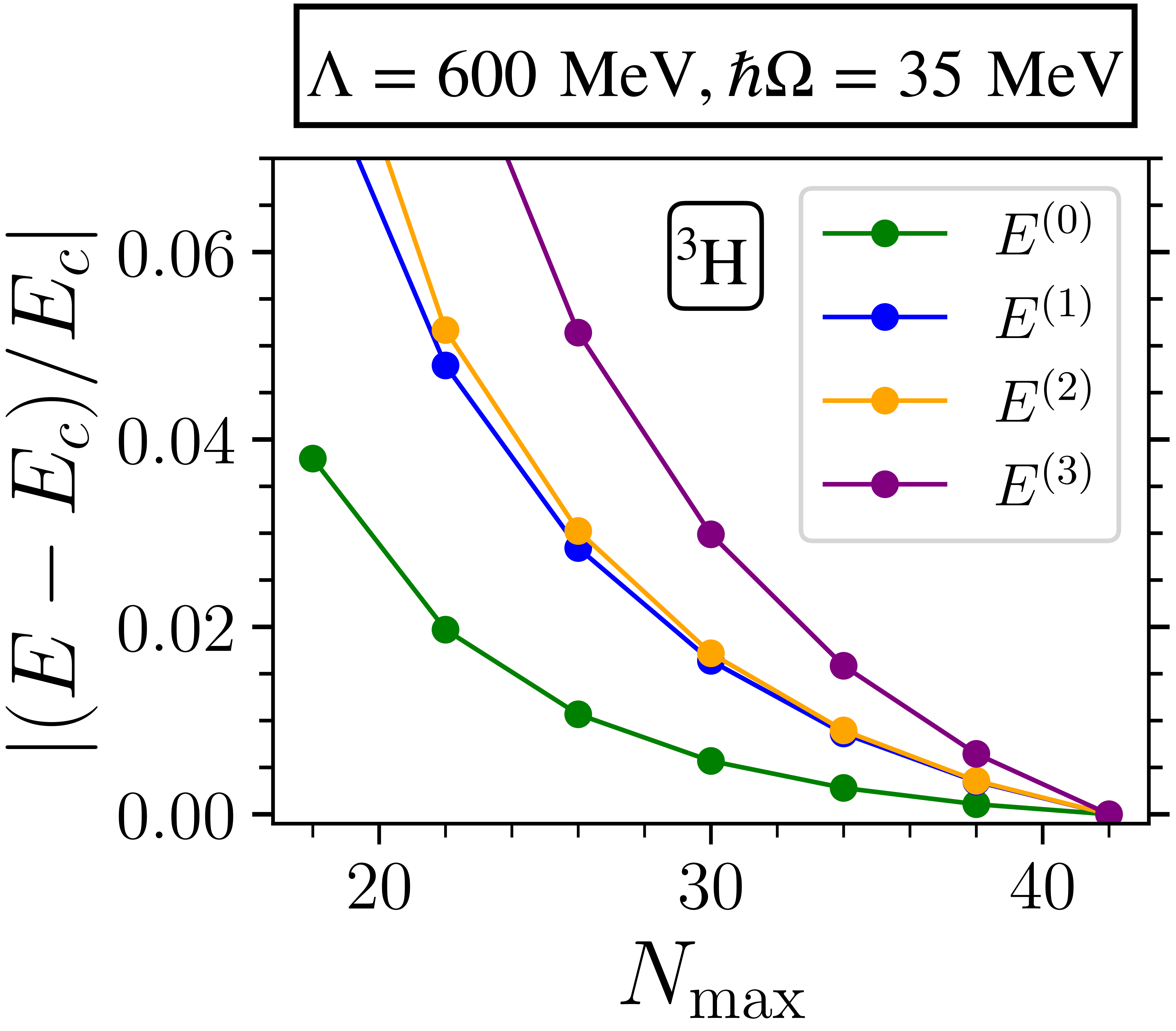
N^2LO: $$E^{(2)} = \langle\Psi_0^{(0)}|V_{\mathrm{NN}}^{(2)}|\Psi_0^{(0)}\rangle + \sum_{n\neq 0}\frac{|\langle\Psi_0^{(0)}|V_{\mathrm{NN}}^{(1)}|\Psi_n^{(0)}\rangle|^2}{E_0^{(0)} - E_n^{(0)}}$$

N^3LO: ...

− Need full LO spectrum beyond NLO!

+ Can implement for ^{3}H in the NCSM.

Convergence with basis size, $N_{\max}$



Perturbative corrections

$$E = E^{(0)} + E^{(1)} + E^{(2)} + \ldots$$

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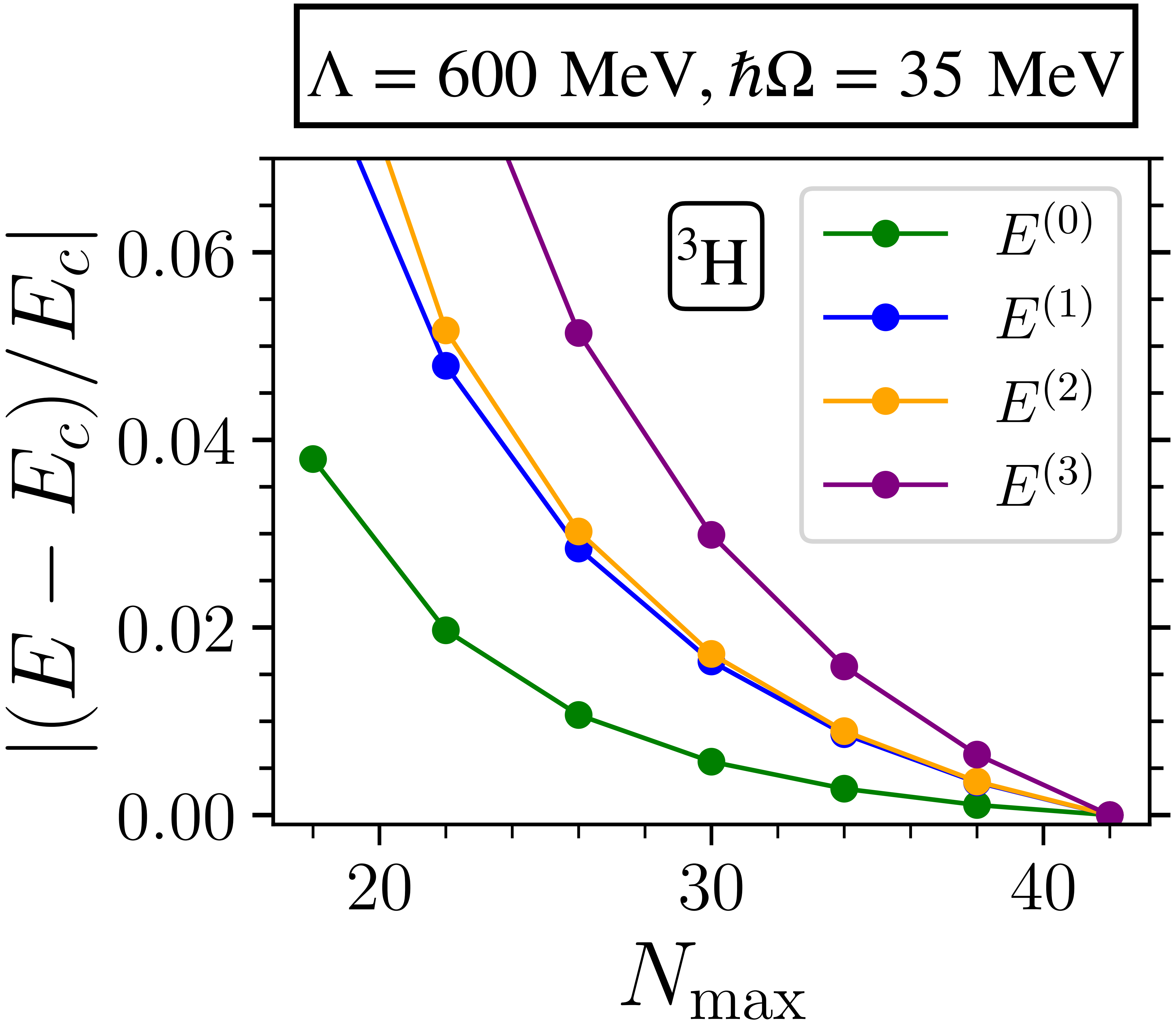

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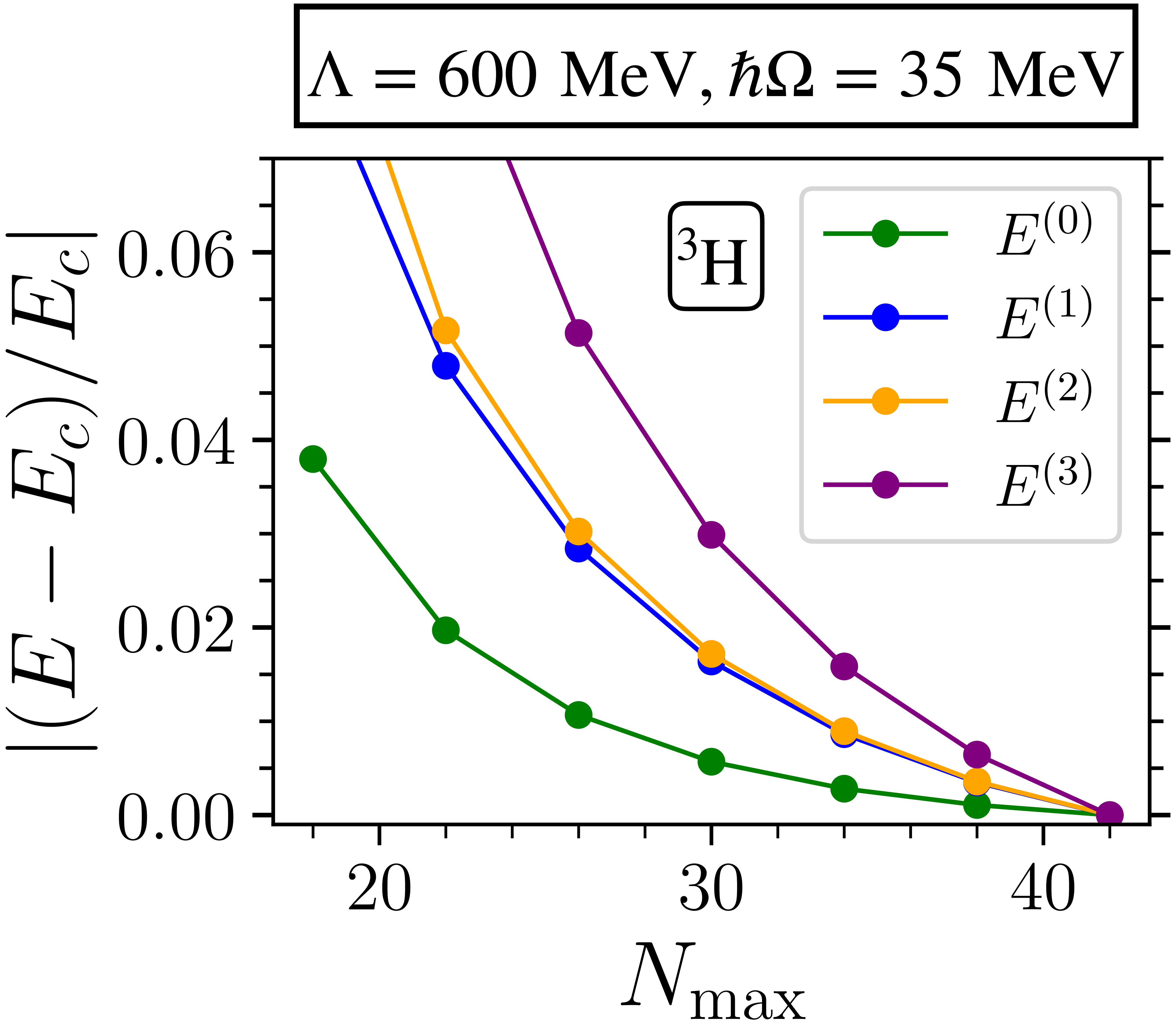

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- **Percent-level** precision beyond $N_{\max} = 38$.

Hellmann-Feynman method

$$V(\vec{x}) = V_{\mathrm{NN}}^{(0)} + x_1 \cdot V_{\mathrm{NN}}^{(1)} + x_2 \cdot V_{\mathrm{NN}}^{(2)} + x_3 \cdot V_{\mathrm{NN}}^{(3)}$$

Can solve exactly for $\vec{x} \in \mathbb{R}^3$

LO:

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$$\frac{dE^{(0)}(\vec{x})}{dx_1} \overset{\text{H.F.}}{=} \langle\Psi^{(0)}(\vec{x})|\frac{dH(\vec{x})}{dx_1}|\Psi^{(0)}(\vec{x})\rangle$$

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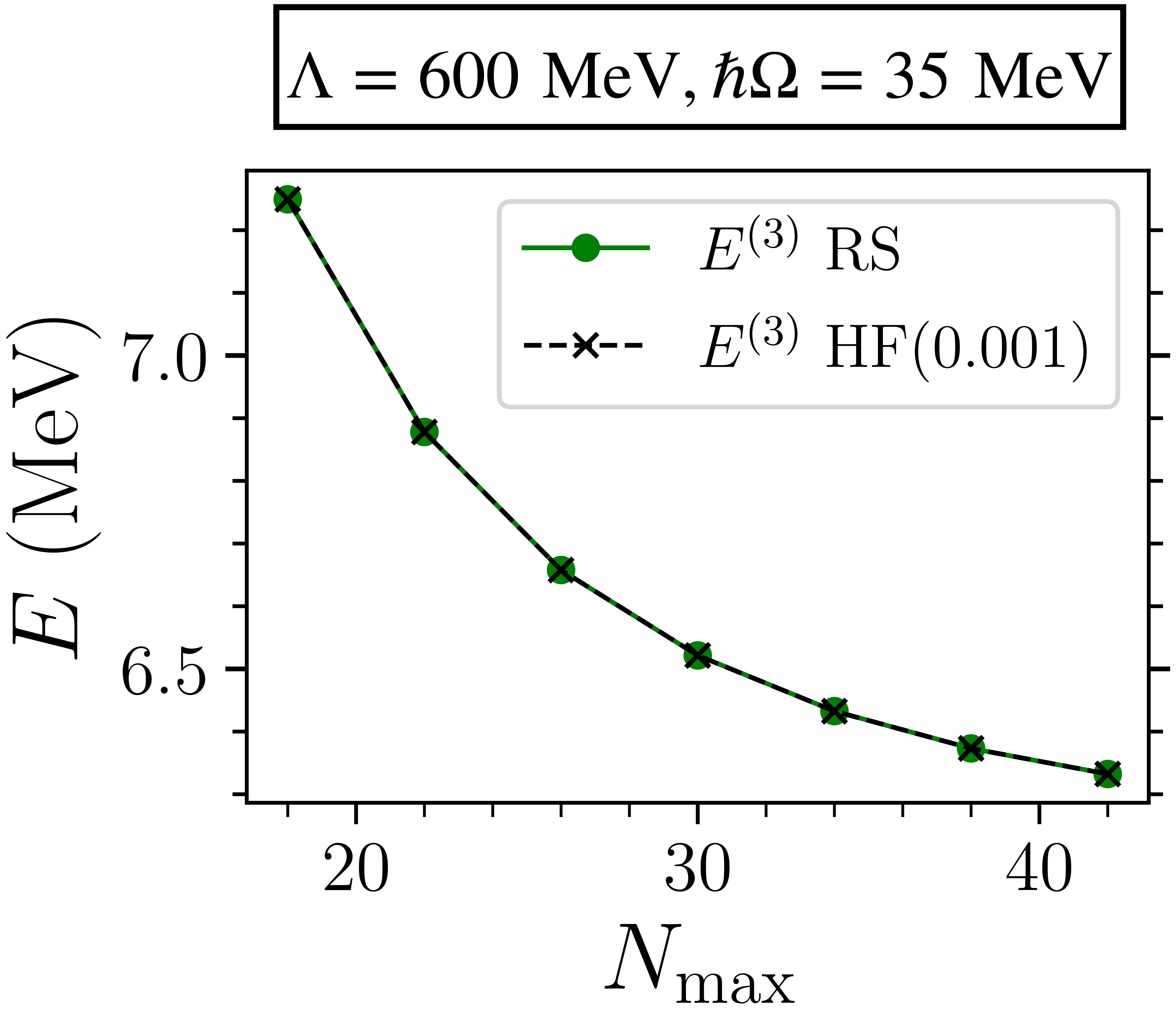
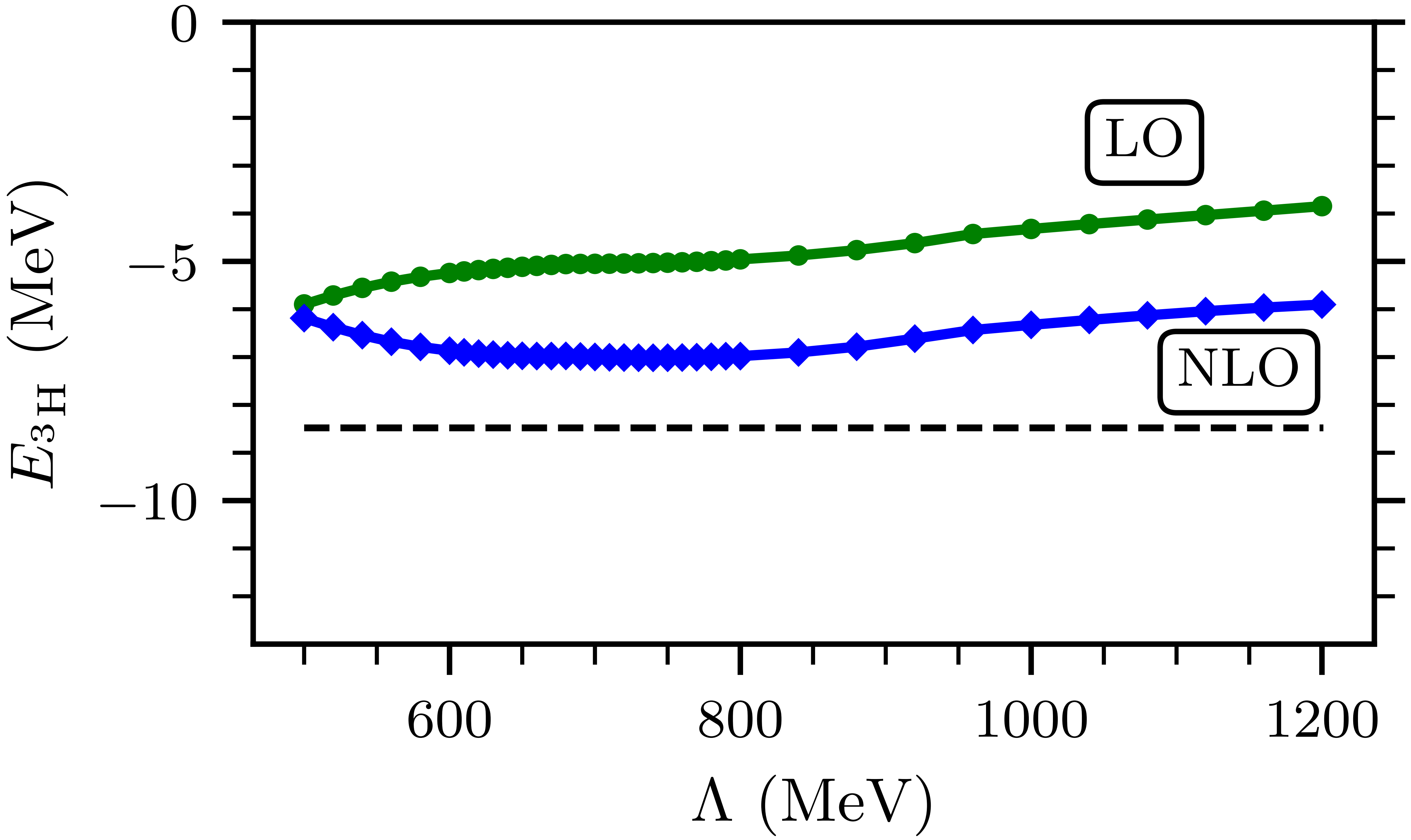
How reliably can the third-order derivatives be computed?

Comparing perturbative calculations: ^{3}H



Cutoff dependence: ^{3}H



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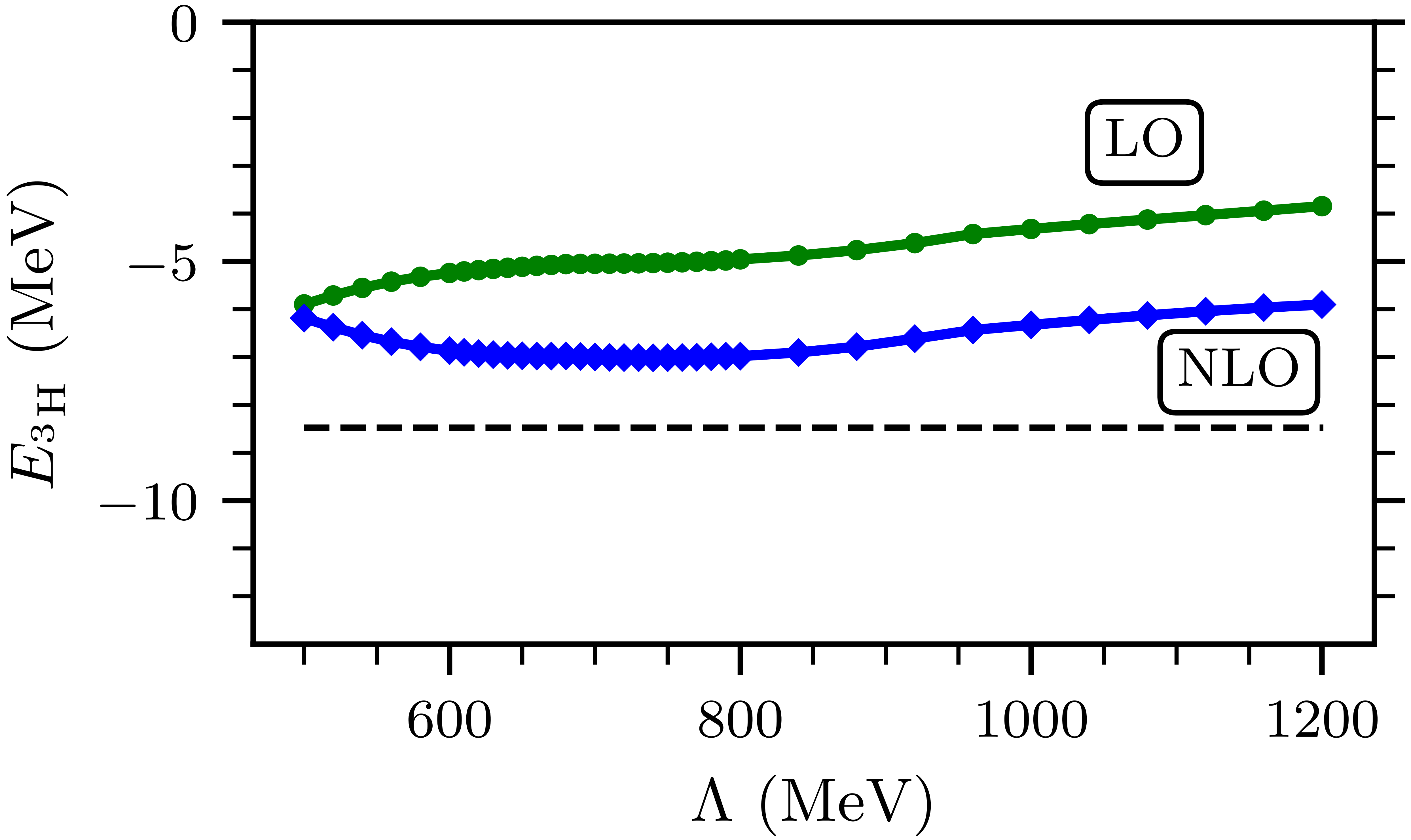

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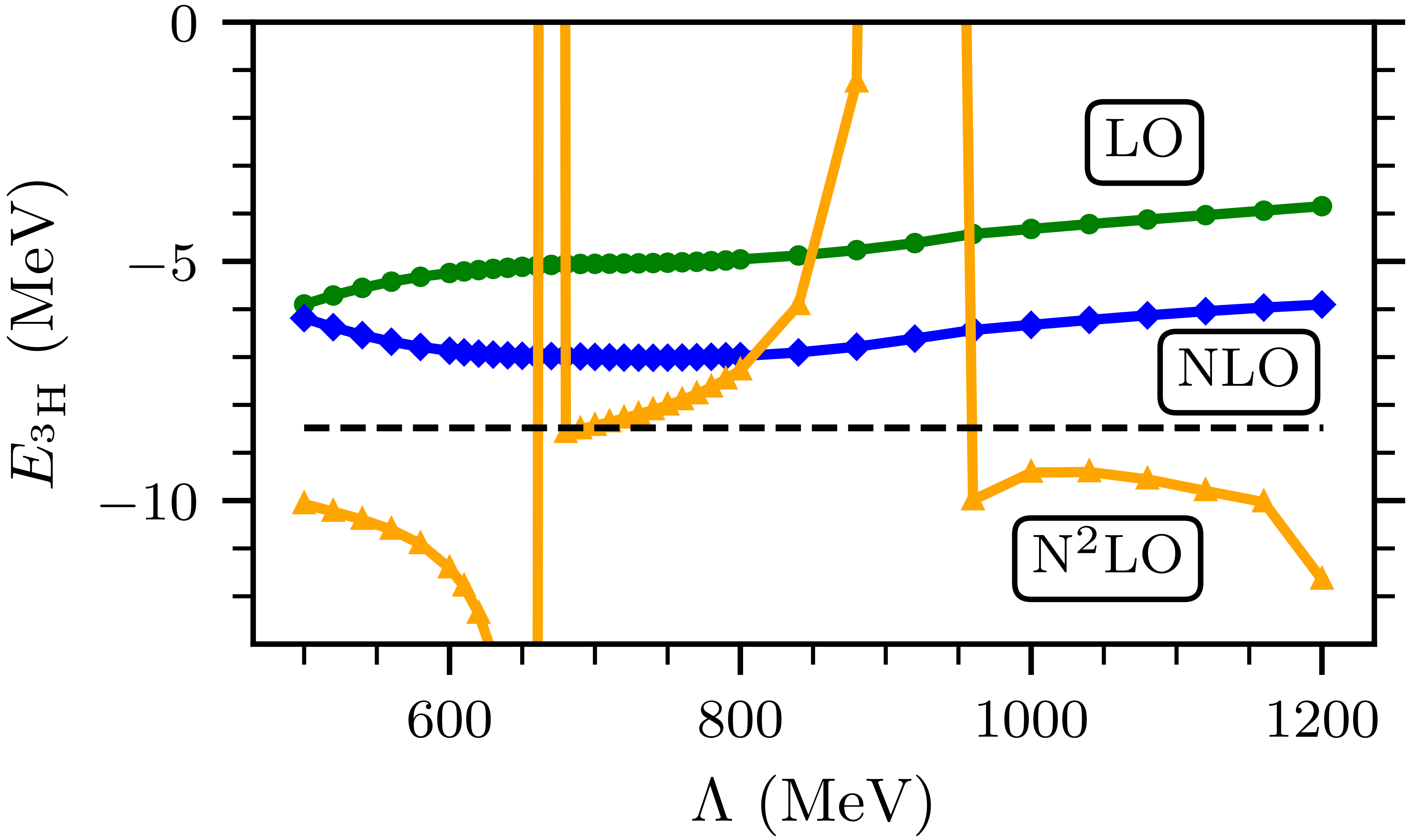

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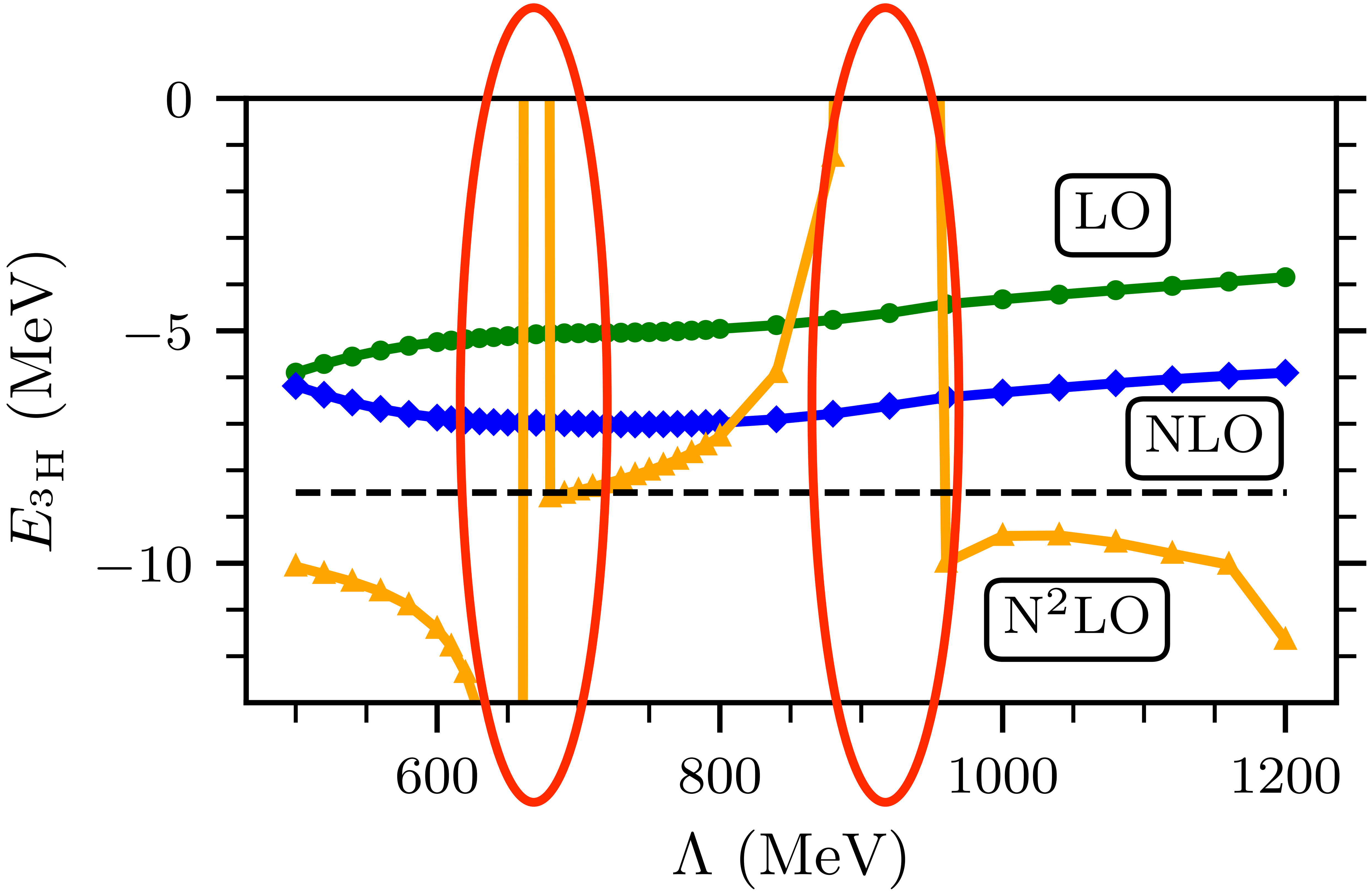

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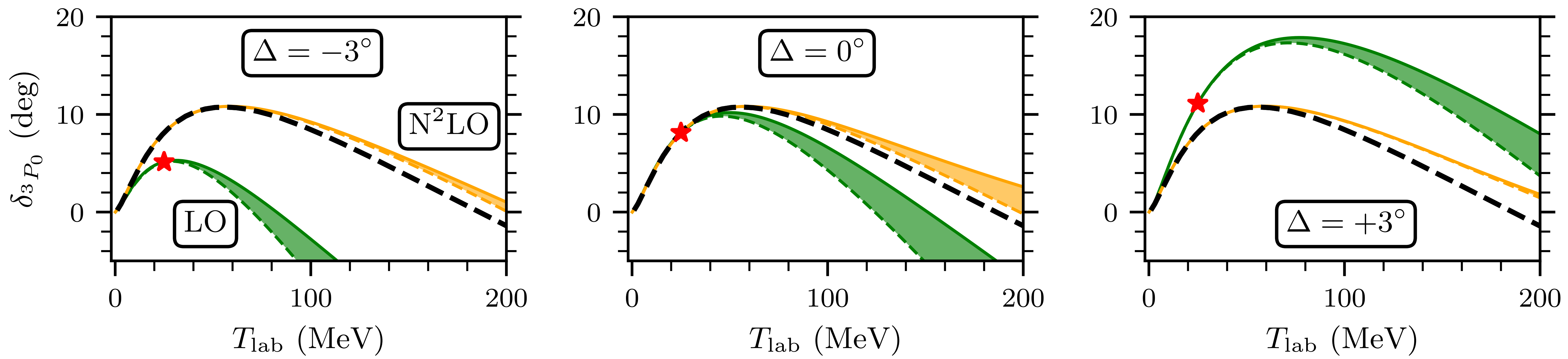
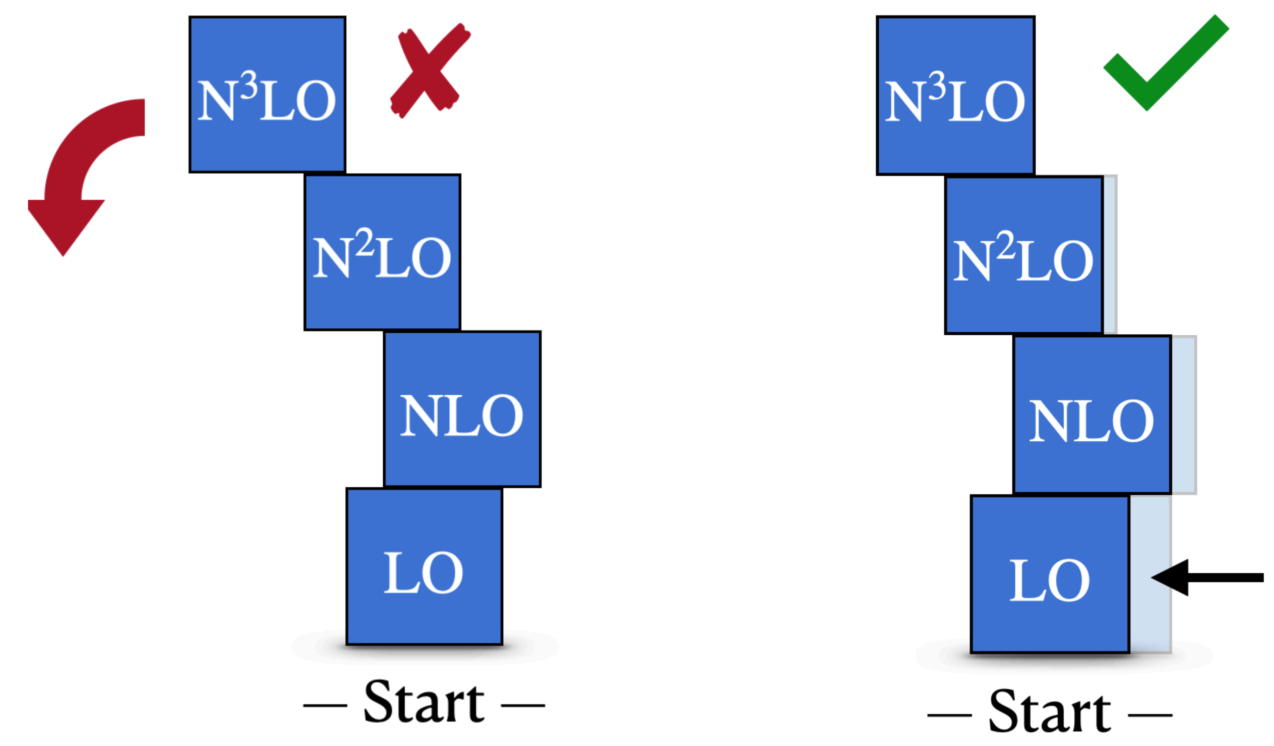
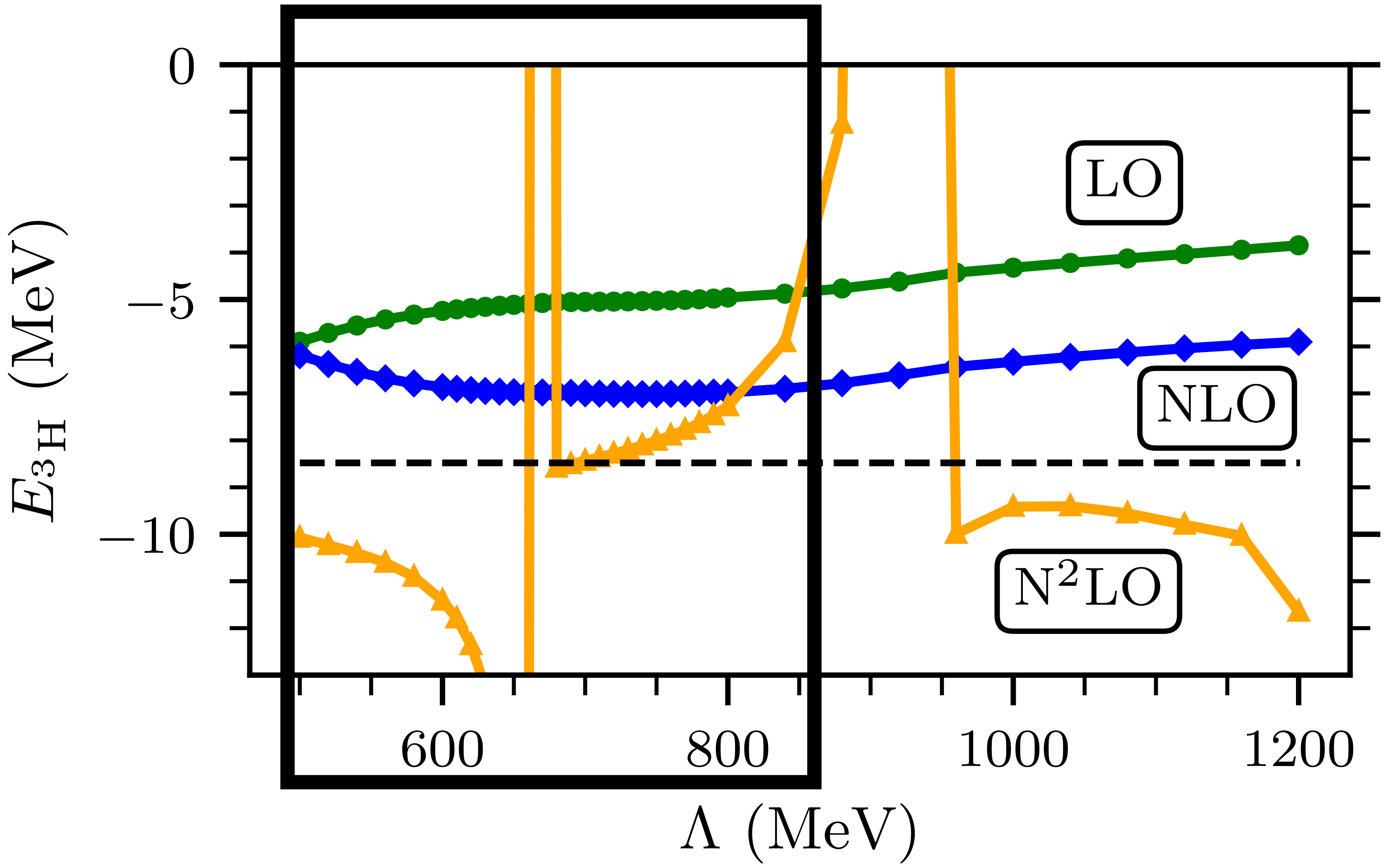
- Problematic regions in the vicinity of *exceptional cutoffs*.

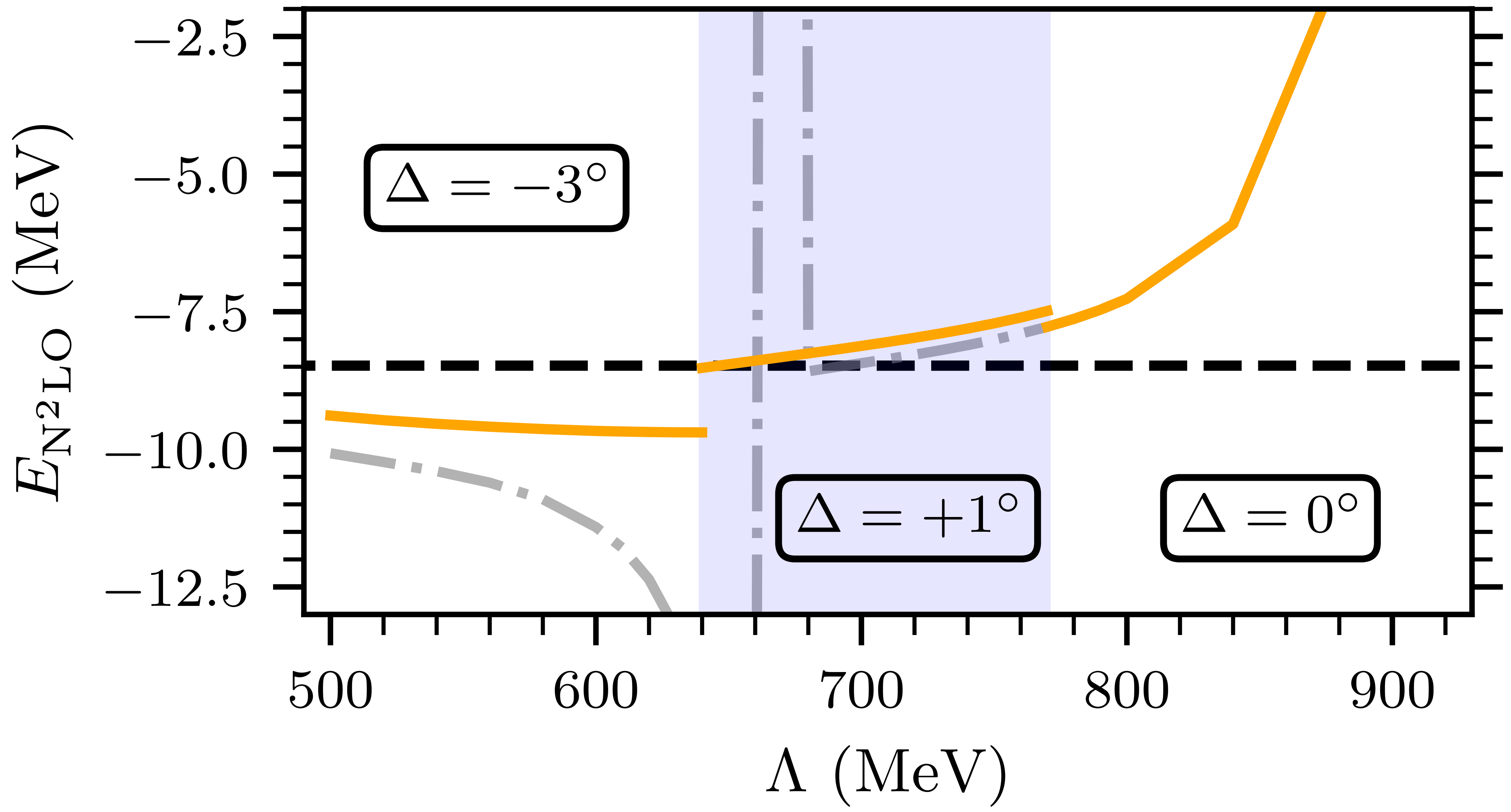
A. M. Gasparyan and E. Epelbaum, Phys. Rev. C **107**, (2023)

Strategies to avoid the exceptional cutoffs









R. Peng *et al.*, Phys. Rev. C **110**, (2024)

Summary and Outlook

- Chiral symmetry constrains the nucleon-nucleon interaction: χEFT.
- Explored a **modified** Weinberg power counting up to N^3LO for ^3H.
- Converged **perturbative** computations for triton with $\Lambda \leq 1200$ MeV — we find problematic regions at exceptional cutoffs.

Next:

- Study the **exceptional cutoffs**!
- **Quantify uncertainties** and study the **few-nucleon** sector beyond ^3H.

Summary and Outlook

- Chiral symmetry constrains the nucleon-nucleon interaction: χEFT.
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Next:

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Thank you!

Extra slides

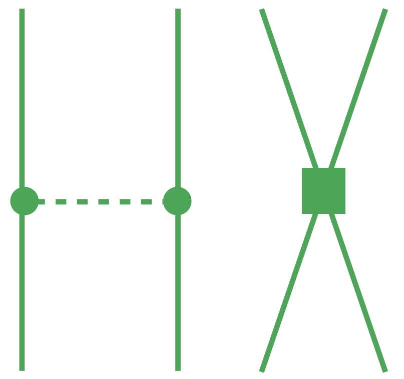
Modified Weinberg PC

B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)

Non-perturbative one-pion-exchange: $^1S_0, ^3S_1 - ^3D_1, ^3P_{0,1}, ^3P_2 - ^3F_2, ^1P_1$

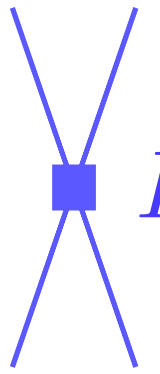
All other partial waves:

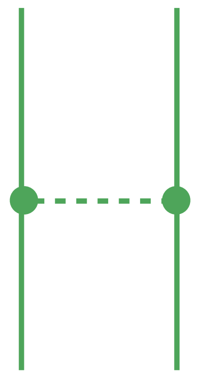
perturbative contritutions

LO g_A  $C_{^1S_0}, C_{^3S_1}, C_{^3P_0}, C_{^3P_2}$ — 4 | 0

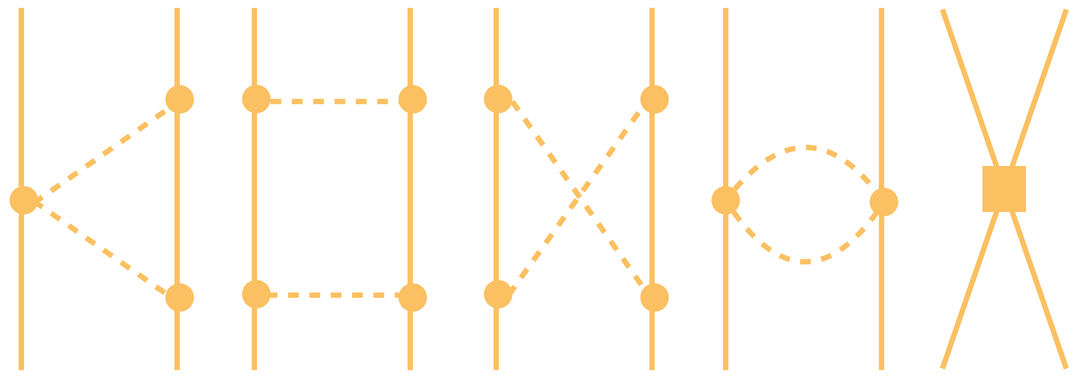
All other partial waves: —

Σ 4

NLO  $D_{^1S_0}$ — 1 | 1

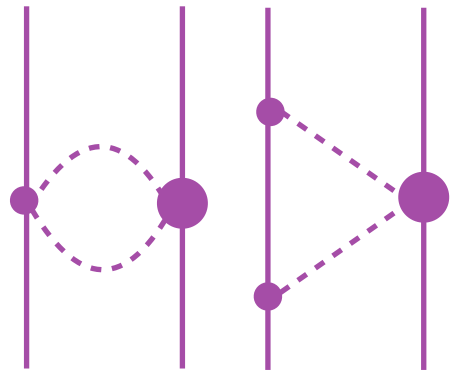
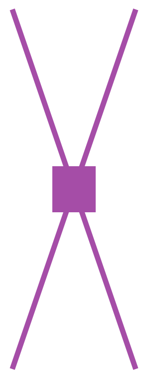
All other partial waves: g_A 

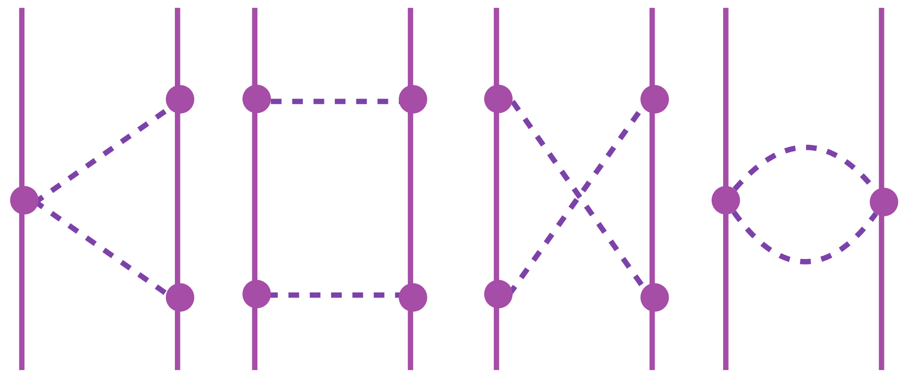
Σ 6

N2LO  $E_{^1S_0}, D_{^3S_1}, D_{^3S_1-^3D_1}, D_{^1P_1}, D_{^3P_1}, E_{^3P_0}, E_{^3P_2}, E_{^3P_2-^3F_2}$ — 8 | 5

All other partial waves: —

Σ 19

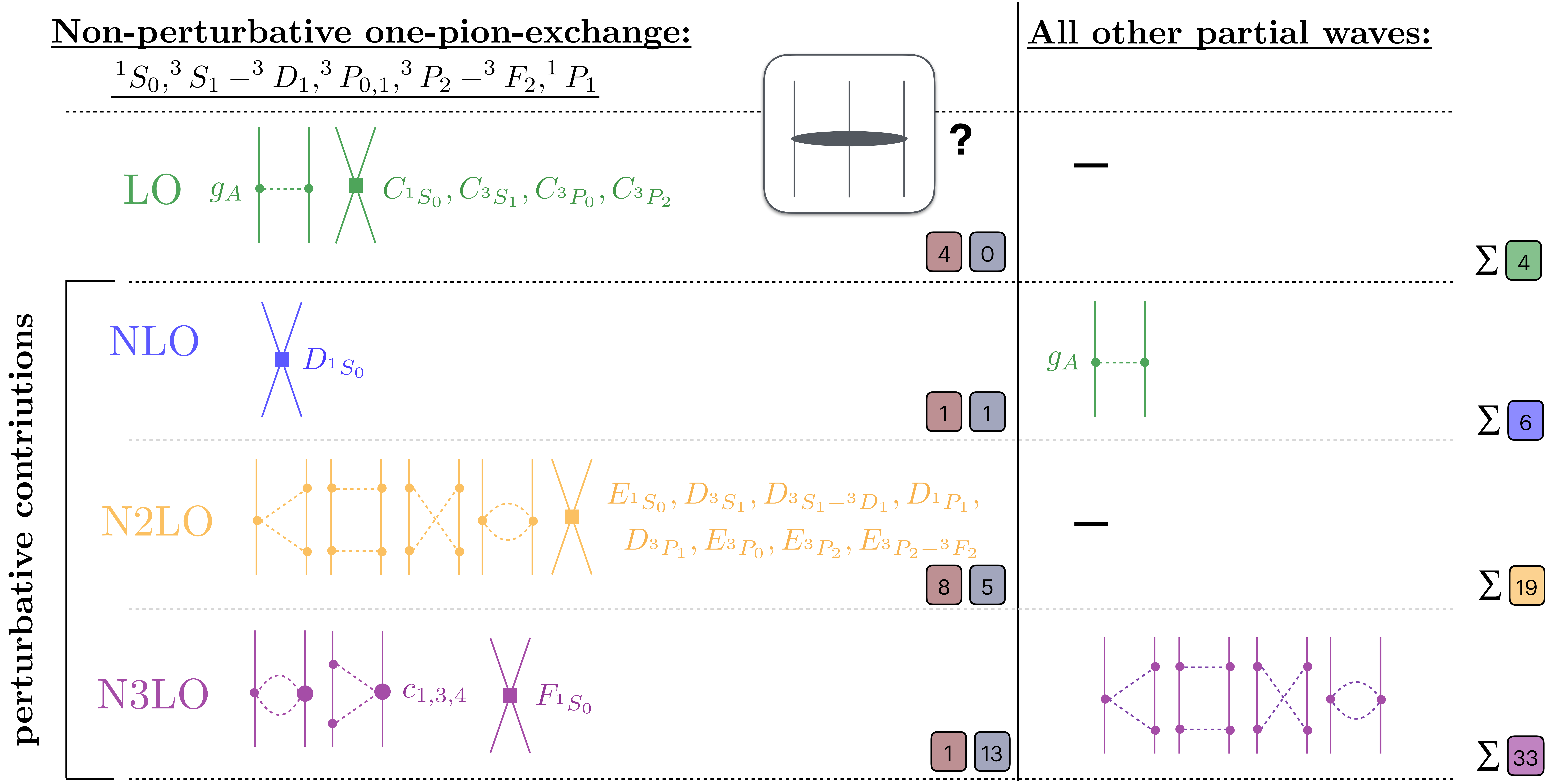
N3LO  $c_{1,3,4}$  $F_{^1S_0}$ — 1 | 13

All other partial waves: 

Σ 33

Modified Weinberg PC

B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)



Determining the values of LECs

- Compute NN scattering amplitudes perturbatively:

$$T = T^{(0)} + T^{(1)} + T^{(2)} + T^{(3)} + \dots$$

$$T^{(0)} > T^{(1)} > T^{(2)} > T^{(3)}$$

- Match to empirical phase shifts to fix LECs.

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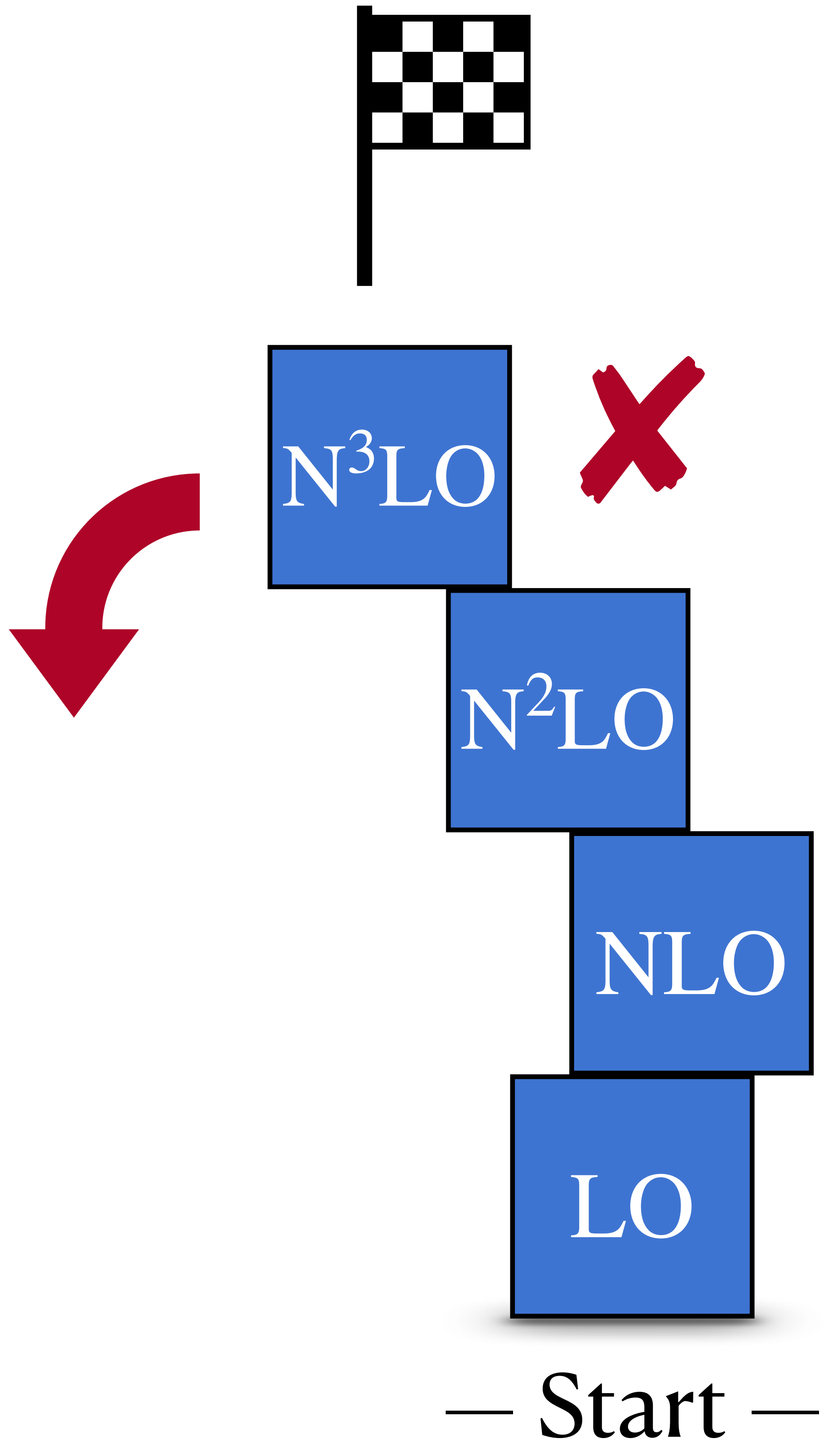

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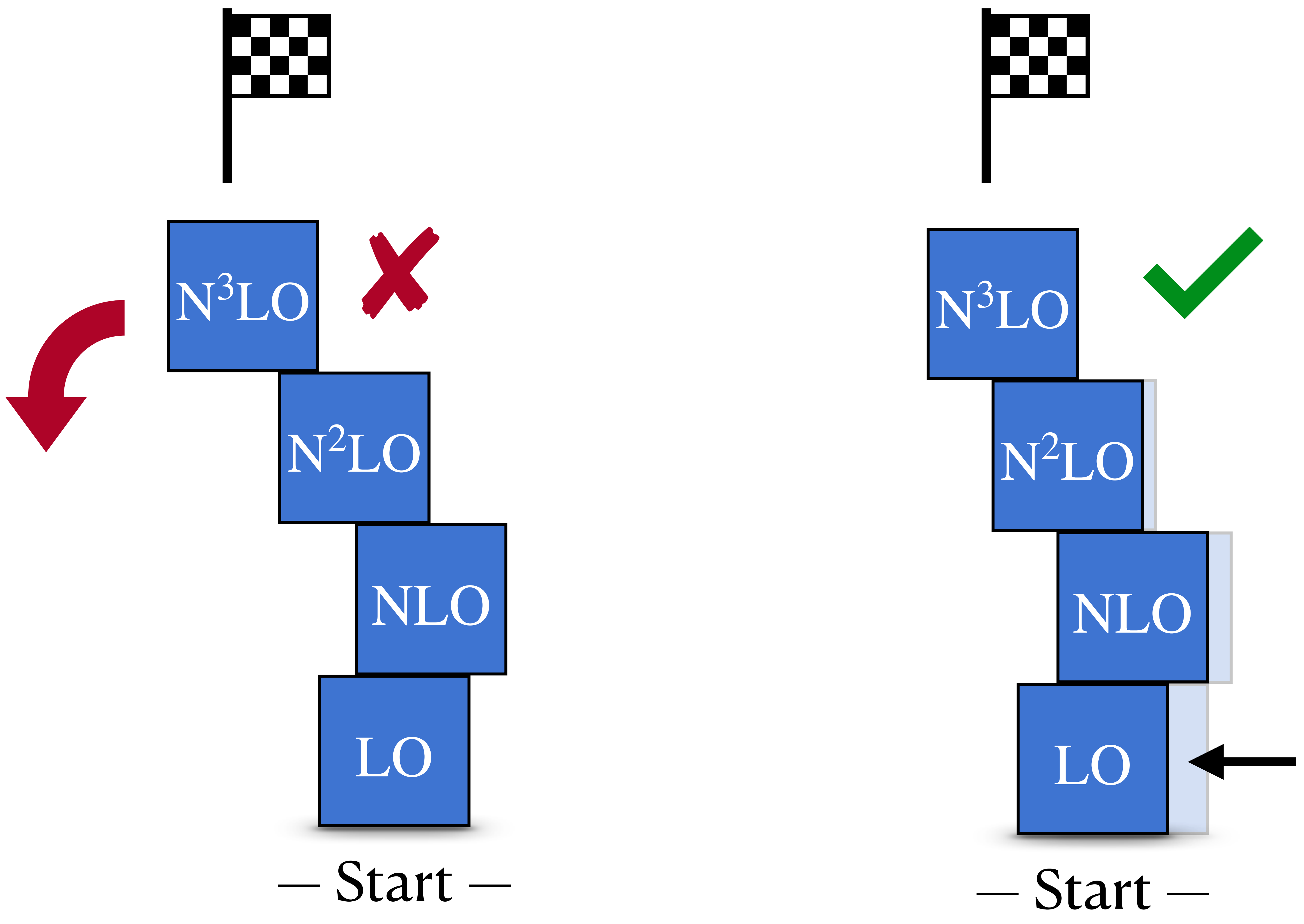

Determining the values of LECs

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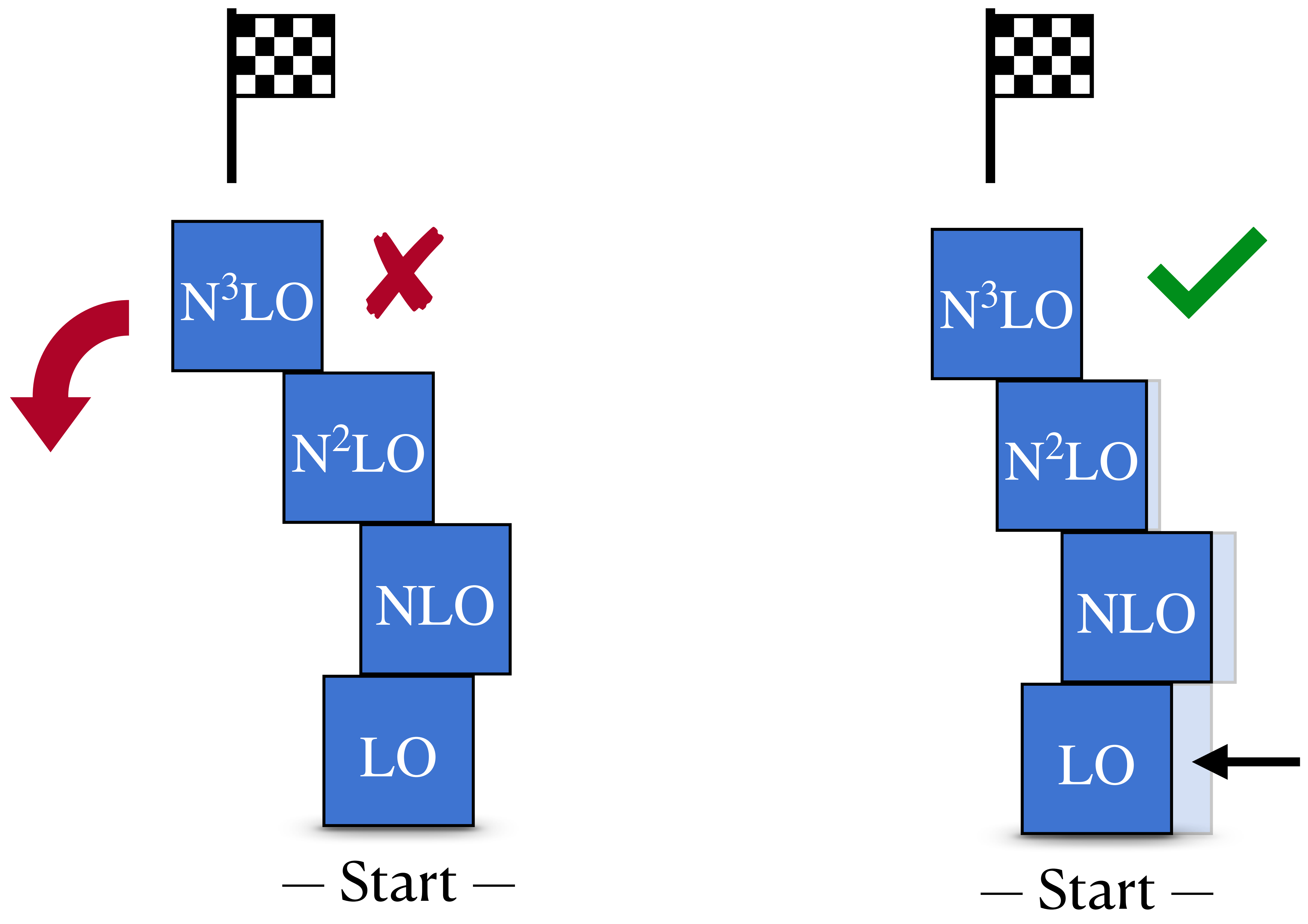
- Match to empirical phase shifts to fix LECs.

- Conclusion: The foundation (LO) is very important!



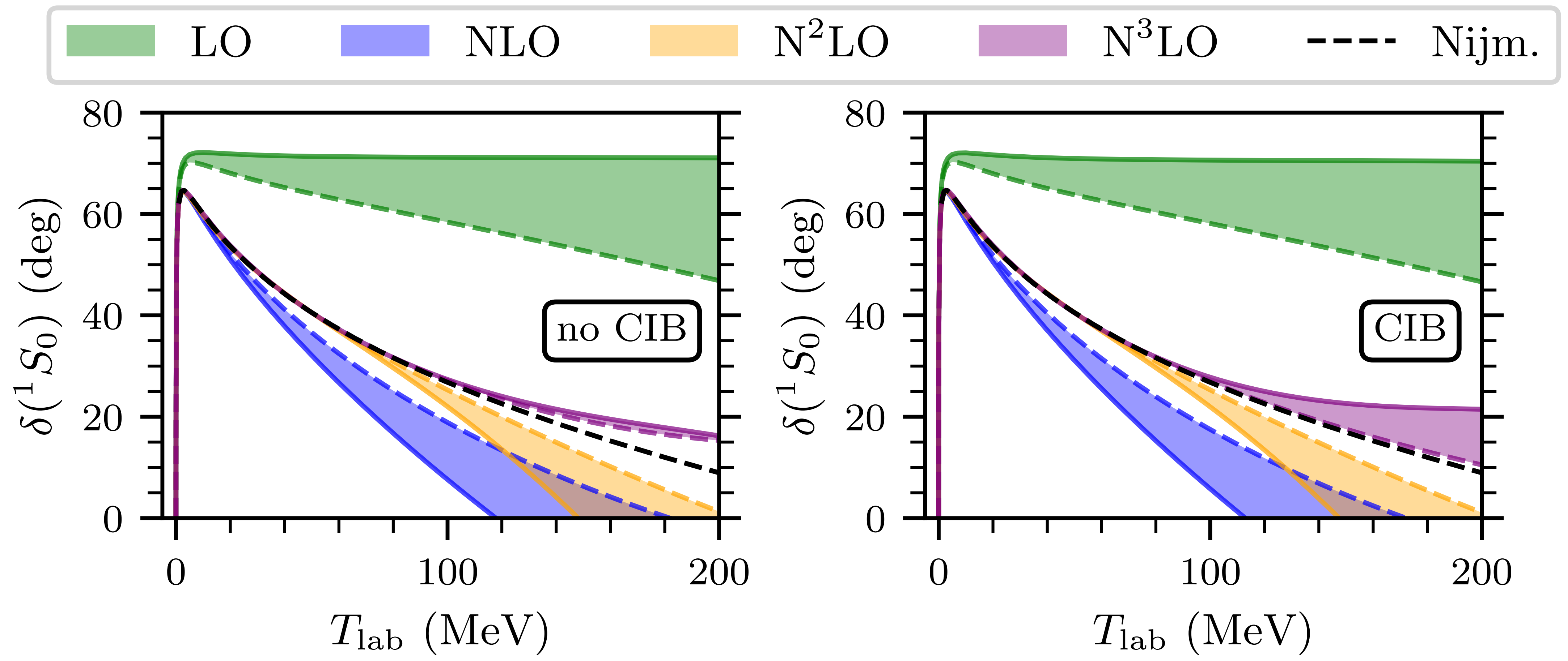
Perturbation theory for amplitudes

$$
\begin{aligned}
T^{(1)} &= \Omega_-^\dagger V^{(1)} \Omega_+, \\
T^{(2)} &= \Omega_-^\dagger \left(V^{(2)} + V^{(1)} G_1^+ V^{(1)} \right) \Omega_+, \\
T^{(3)} &= \Omega_-^\dagger \Big(V^{(3)} + V^{(2)} G_1^+ V^{(1)} + V^{(1)} G_1^+ V^{(2)} + \\
&\quad + V^{(1)} G_1^+ V^{(1)} G_1^+ V^{(1)} \Big) \Omega_+
\end{aligned}
$$

$$
\begin{aligned}
\Omega_+ &= \mathbb{1} + G_0^+ T^{(0)} \\
\Omega_-^\dagger &= \mathbb{1} + T^{(0)} G_0^+,
\end{aligned}
$$

Low-energy theorems: $^1S_0$

Phase shifts in $^1S_0$



Predicted effective range parameters (LETs)

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	−23.735(16)	2.68(3)	−0.48(2)	3.9(1)	−19.6(5)
$\Lambda = 500$ MeV, (no CIB)					
LO	*	1.71(0)	−1.77(0)	8.54(0)	−47.0(3)
NLO	*	*	−0.64(0)	4.79(0)	−29.9(2)
N^{2}LO	*	2.72(0)	−0.71(0)	5.05(0)	−29.3(2)
N^{3}LO	*	2.69(0)	−0.66(0)	5.42(0)	−31.0(2)
$\Lambda = 500$ MeV, (CIB)					
LO	*	1.68(0)	−1.55(0)	6.63(0)	−31.64(8)
NLO	*	*	−0.45(0)	3.42(0)	−18.95(8)
N^{2}LO	*	2.70(0)	−0.55(0)	3.77(0)	−18.8(2)
N^{3}LO	*	2.68(0)	−0.50(0)	4.02(0)	−19.8(2)

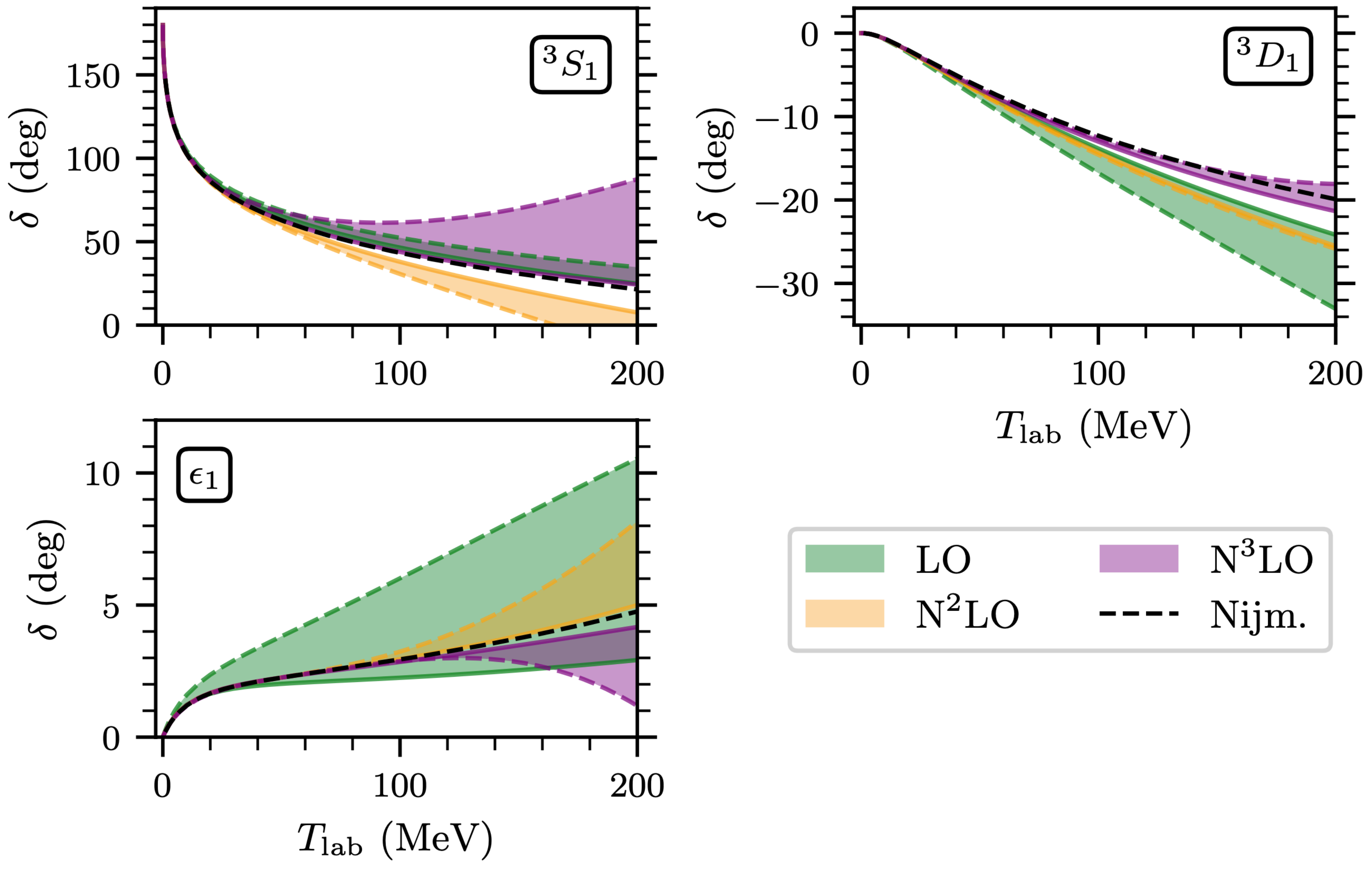
OT, Few-Body Syst. 65, 69 (2024)

- Leading CIB (pion mass splitting) in the one-pion exchange is **significant** in $^1S_0$.

✓ Both phase shift and LETs are accurate.

Low-energy theorems: $^3S_1$

Phase shifts in $^3S_1$— $^3D_1$



Predicted effective range parameters (LETs)

$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	5.42	1.75	0.045	0.67	−3.94
$\Lambda = 500$ MeV					
LO	*	1.58(0)	−0.10(0)	0.89(0)	−5.5(2)
N^{2}LO	*	*	0.14(0)	0.80(0)	−4.2(2)
N^{3}LO	*	*	−0.06(0)	0.46(0)	−3.7(2)
$\Lambda = 2500$ MeV					
LO	*	1.66(0)	−0.01(0)	0.79(0)	−4.7(2)
N^{2}LO	*	*	0.09(0)	0.74(0)	−4.2(7)
N^{3}LO	*	*	0.04(0)	0.67(2)	−4.0(9)

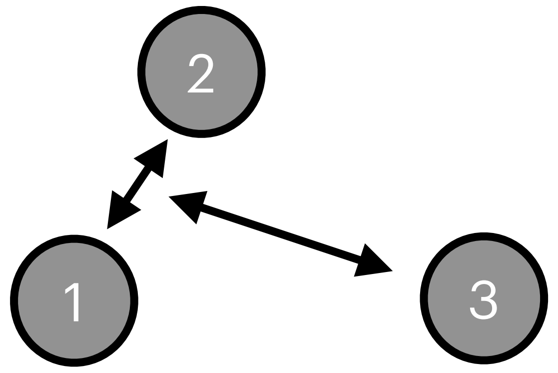
- CIB in one-pion exchange is **not** significant in $^3S_1$.
- Cutoff independence for $\Lambda \gtrsim 750$ MeV.

✓ Both phase shift and LETs are accurate, and improved for high cutoffs.

OT, Few-Body Syst. 65, 69 (2024)

No-core shell model (NCSM) for ^{3}H

- Three-particle antisymmetrizer $\hat{\mathcal{A}} = \frac{1}{3!}(1 - P_{12} - P_{23} - P_{13} + P_{12}P_{23} + P_{13}P_{23})$



- Diagonalize ${}_{12}\langle n\mathcal{N}\alpha|\hat{\mathcal{A}}|n'\mathcal{N}'\alpha'\rangle_{12}$ in basis: $|n\mathcal{N}\alpha\rangle_{12} \equiv |n\ell sjt, \mathcal{N}\mathcal{L}\mathcal{J}; JT\rangle$

- Keep only states with eigenvalue one!

$N_{\max}$	N	M
0	2	1
10	322	108
20	1892	632
30	5712	1906
40	12782	4263

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}$

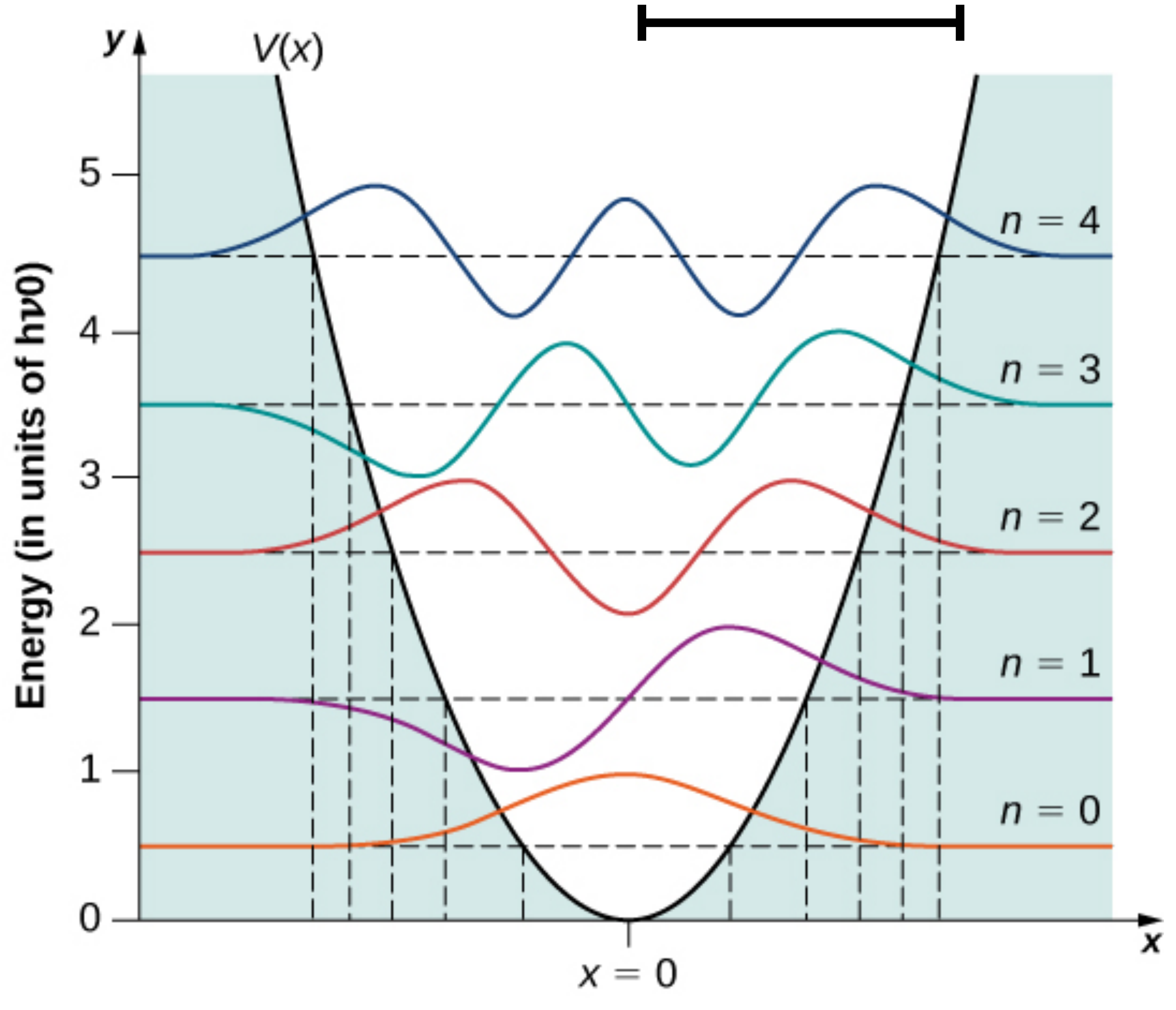
Limitations of the HO basis

$$R_{n\ell}(r;b) = \sqrt{\frac{2(n!)b^3}{\Gamma(n+l+\frac{3}{2})}} \left(\frac{r}{b}\right)^{\ell} e^{-\frac{1}{2}\left(\frac{r}{b}\right)^2} L_n^{\ell+1/2}\left(\frac{r^2}{b^2}\right)$$

- Basis characterized by $(\hbar\Omega, N_{\text{max}})$
  - N_{max} — number of basis states
  - $\hbar\Omega$ — shape of basis states

- IR scale $L_{\text{eff}} \approx \sqrt{2N}b$
- UV scale $\Lambda_{\text{eff}} \approx \sqrt{2N}\frac{\hbar}{b}$

$L_{\text{eff}} \approx \sqrt{2N}b$



$b = \sqrt{\frac{\hbar}{m_N\Omega}}$

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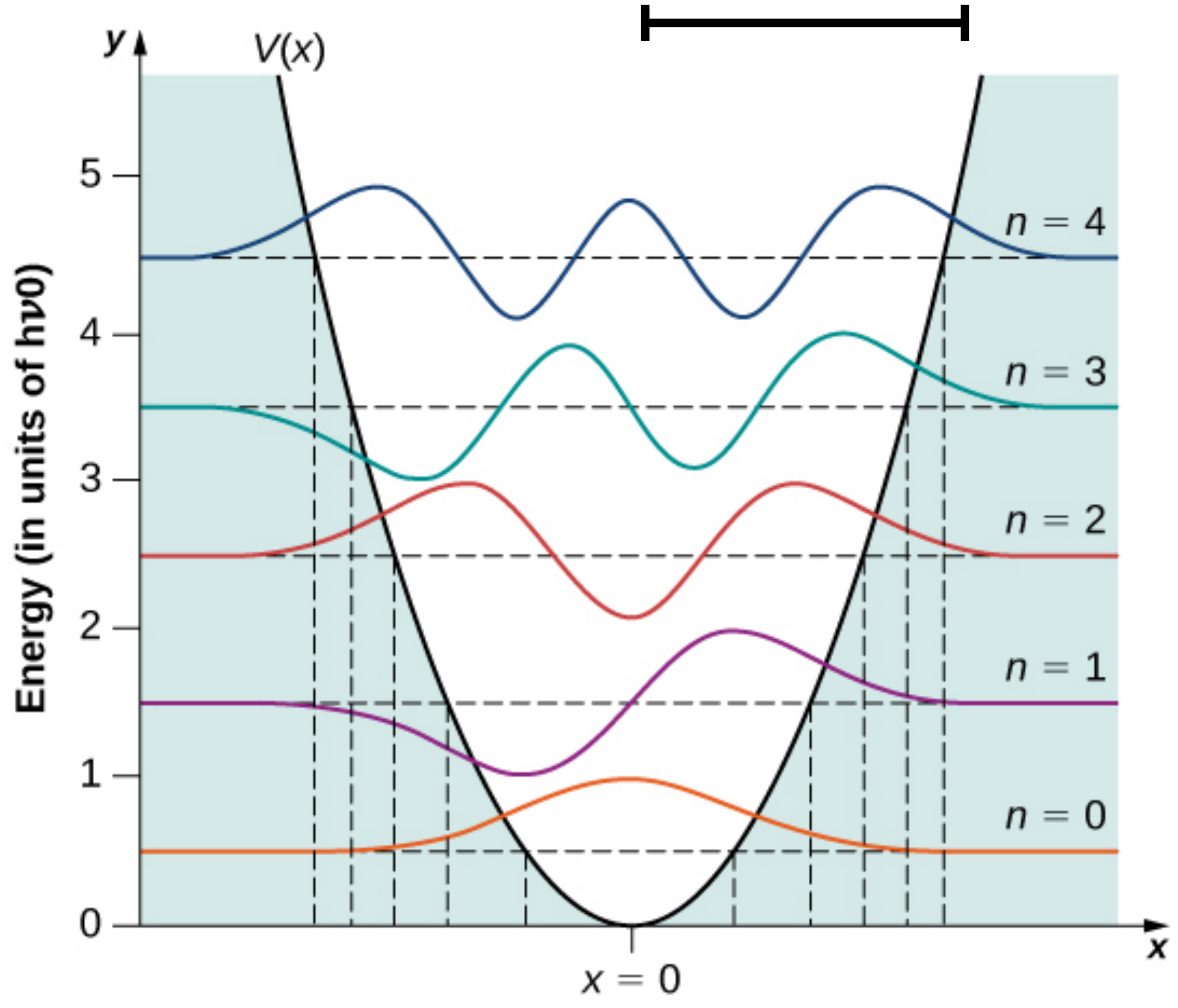
- IR scale $L_{\rm eff} \approx \sqrt{2N}b$
- UV scale $\Lambda_{\rm eff} \approx \sqrt{2N}\frac{\hbar}{b}$

Only way to converge in both is to increase $N_{\rm max}$!

$L_{\rm eff} \approx \sqrt{2N}b$



$b = \sqrt{\frac{\hbar}{m_N\Omega}}$

Convergence: ^{3}H