The two and three-nucleon correlation functions

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Introduction

- When a high-energy pp or p–nucleus collision occurs, particles are produced and emitted at relative distances of the order of the nuclear force
- The effect of the mutual interaction between hadrons is reflected as a correlation signal in the momentum distributions of the detected particles which can be studied using correlation functions
- The correlation function incorporate information on the emission process as well as on the final state interaction of the emitted pairs
- By measuring correlated particle pairs or triplets at low relative energies and comparing the yields to theoretical predictions, it is possible to study the hadron dynamics.

The two-particle correlation function

- The two-particle correlation function is defined as the ratio of the yield of a particle pair to the product of the single-particle yields.

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{\mathcal{P}\left(\vec{p}_1, \vec{p}_2\right)}{\mathcal{P}\left(\vec{p}_1\right)\mathcal{P}\left(\vec{p}_2\right)}$$

- $\mathcal{P}(\vec{p}_1, \vec{p}_2)$ is the probability of finding a pair with momenta $\vec{p}_1$ and $\vec{p}_2$
- $\mathcal{P}(\vec{p}_i)$ is the probability of finding each particle with momentum $\vec{p}_i$.
- In absence of correlations, the two-particle probability factorizes, $\mathcal{P}(\vec{p}_1, \vec{p}_2) = \mathcal{P}(\vec{p}_1)\mathcal{P}(\vec{p}_2)$, and the correlation function is equal to unity.

The two-particle correlation function

- The correlation between the pair is related to the particle emission and the subsequent interaction of the pair

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{1}{\Gamma} \sum_{m_1, m_2} \int d^3 r_1 \, d^3 r_2 S_1\left(r_1\right) S_1\left(r_2\right) \times \left|\Psi_{m_1, m_2}(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2)\right|^2$$

- $S_1(r)$ describes the spatial shape of the source for single-particle emissions. It can be approximated as a Gaussian probability distribution with a width R_M

$$S_1(r) = \frac{1}{(2\pi R_M^2)^{\frac{3}{2}}} e^{-r^2/2R_M^2}$$

- The integration on the CM coordinates leads to the Koonin-Pratt relation for two-particle correlation function

$$C(k) = \frac{1}{\Gamma} \int d^3 r \, S(r) |\psi_k\left(\vec{r}\right)|^2$$

The pp correlation function

- $S(r)$ is the two-particle emission source, given by

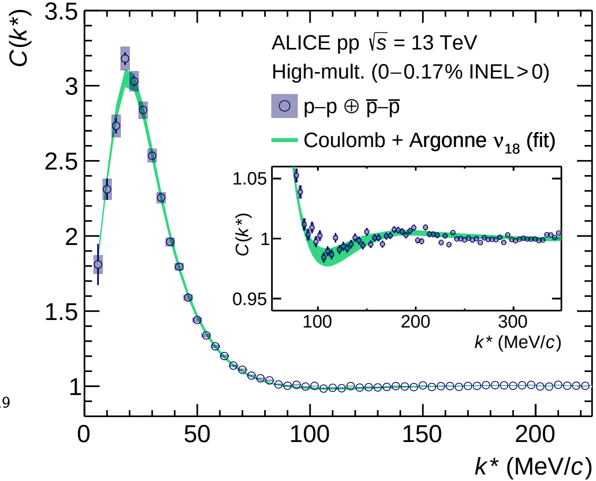
$$S(r) = \left(\frac{1}{4\pi R_{\mathrm{M}}^2}\right)^{3/2} e^{-\frac{r^2}{4R_{\mathrm{M}}^2}}$$

- $\psi_k(\vec{r})$ is the two-particle scattering wave function at $E = \hbar^2 k^2/2\mu$
- The scattering wave function is expanded in partial waves

$$\psi_k = 4\pi \sum_{JJ_z} \sum_{\ell m S S_z} i^{\ell}(kr)^{-1} u_\ell(kr) \mathcal{Y}_{[\ell S]}(\hat{r})(\ell m S S_z | J J_z) Y^*_{\ell m}(\hat{k})$$

- In the case of two protons $u_\ell(kr) \to F_\ell(\eta, kr) + T_{\ell\ell}\mathcal{O}_\ell(kr)$

The pp Correlation Function



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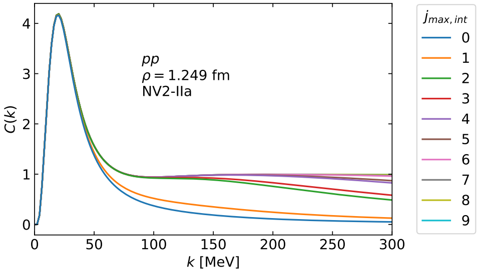
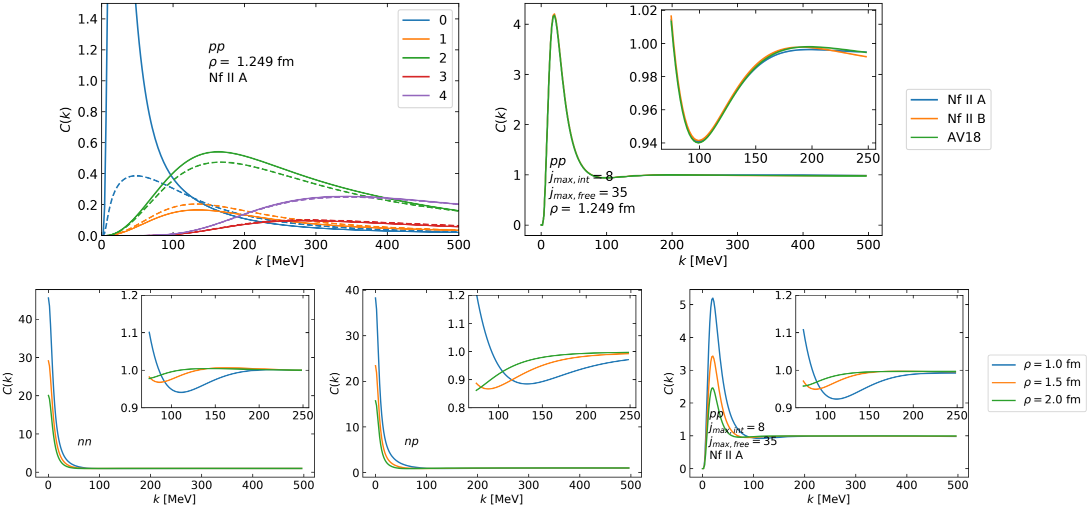
Phys. Lett. B 805, (2020) 135419

The pp correlation function

$$C(k) = \frac{1}{N}\left[\sum_{j\leq j_m}\sum_{l,l',s,t}\int dr\, r^2 S(r)\left|\Psi^{(l')}_{k;l,s,j,t}(r)\right|^2 + \sum_{j_m<j\leq j_M}\sum_{l,s,t}\int dr\, r^2 Sr\left|\Psi_{\mathrm{free};k;l,s,j,t}(r)\right|^2\right]$$



The *pp* correlation function

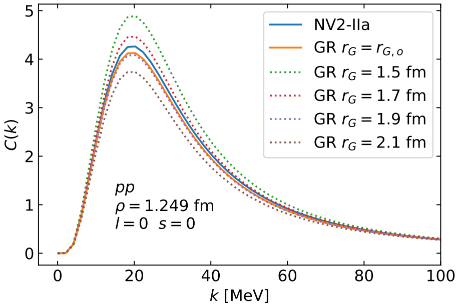


Matthias Göbel and A.K., Phys. Lett. B 869, 139835 (2025)

The pp correlation function from a Gaussian representation

$$V_{pp}(^1S_0) = V_0 e^{-(r/r_G)^2} + \frac{e^2}{r}$$

with V_0 fixed to reproduce the pp scattering length. When $r_G = r_{G,o}$ the pp effective range is described too.



The pd Correlation Function

- We now consider the pd correlation function:

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int d^3r_1 d^3r_2 d^3r_3 \; S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2,m_1}|^2$$

- the probability of deuteron formation

$$A_d = \frac{1}{3} \sum_{m_2} \int d^3r_1 d^3r_2 \; S_1(r_1) S_1(r_2) |\phi_{m_2}|^2$$

- the single particle source function

$$S_1(r) = \frac{1}{(2\pi R_\mathrm{M}^2)^{\frac{3}{2}}} e^{-r^2/2R_\mathrm{M}^2}$$

The pd Correlation Function

- the pd correlation function results

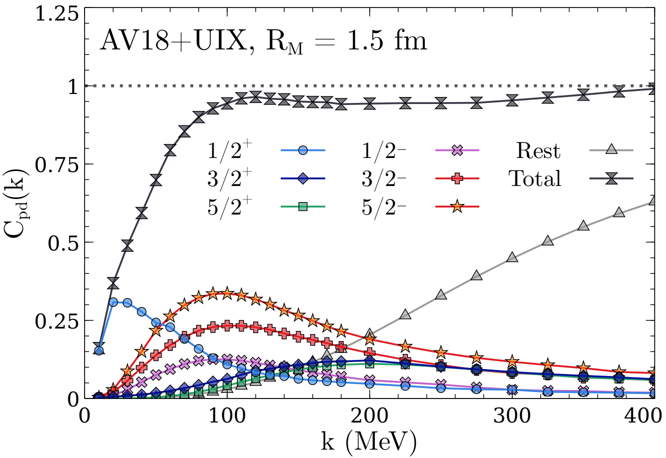
$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3} |\Psi_{m_2,m_1}|^2$$

$$\Psi_{m_2,m_1} = \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 \mid S J_z)(L 0 S J_z \mid J J_z) \Psi_{LSJJ_z}$$

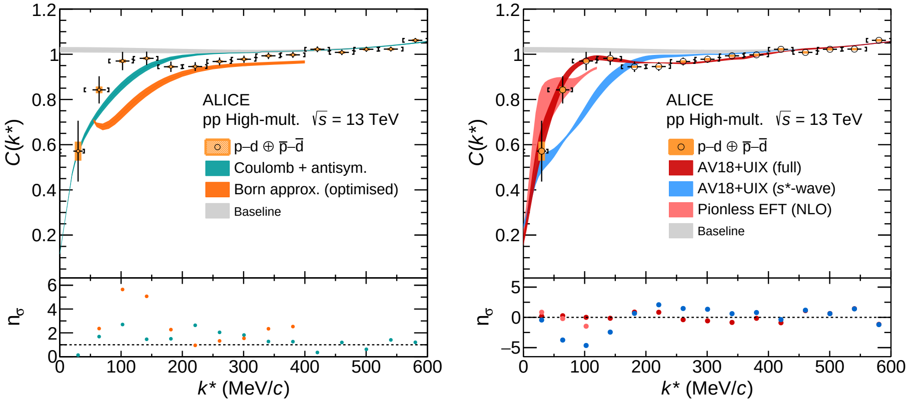
- the Jacobi coordinates: $\mathbf{x}_\ell = \mathbf{r}_j - \mathbf{r}_i$, $\quad \mathbf{y}_\ell = \mathbf{r}_\ell - \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$
- the hyperspherical coordinates $\rho = \sqrt{x_1^2 + (4/3) y_1^2}$, $\Omega \equiv [\alpha_1, \hat{x}_1, \hat{y}_1]$
- The scattering wave function is expanded in partial waves using the HH basis

$$\Psi_{LSJJ_z} = \rho^{-5/2} \sum_{[K]} u_{[K]}(\rho) \mathcal{Y}_{[K]}^{LSJJ_z}(\Omega)$$

The pd Correlation Function: partial-wave contributions



The pd Correlation Function: comparison to experiment



M. Viviani, S. König, A. Kievsky, L.E. Marcucci, B. Singh, O. Vázquez Doce, Phys. Rev. C 108, 064002 (2023)
ALICE collaboration, Physical Review X 14, 031051 (2024)

The ppp correlation function

- Now we consider the ppp correlation function:

$$C_{ppp}(Q) = \int \rho^5 d\rho d\Omega \; S_{\rho_0}(\rho) |\Psi_{ppp}|^2$$

with Q the hyper-momentum, S_{ρ_0} the source function defined as

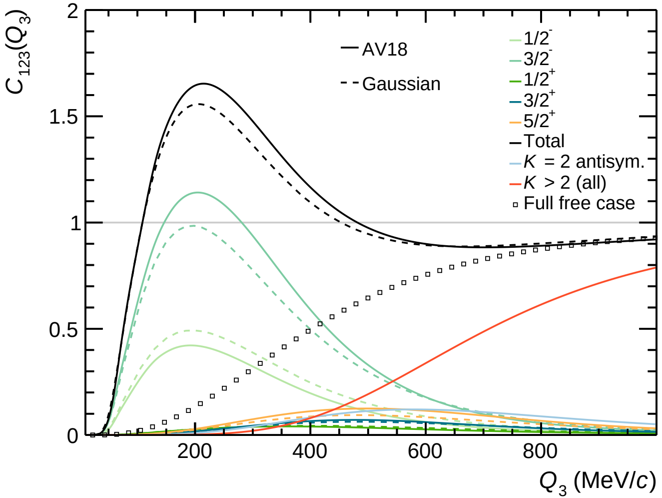
$$S_{\rho_0}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

Ψ_{ppp} is the ppp scattering wave function

$$\Psi_{ppp} = \sum_{[K]} u_{[K]}(\rho) \mathcal{B}_{[K]}(\Omega) = \Psi^0 + \sum_{J,[K]}^{\overline{J},\overline{K}} \Psi^J_{[K]}$$

To be noticed that Ψ^0 is not well known. In $\Psi^J_{[K]}$ the interaction has been considered up to $\overline{J}$ and $\overline{K}$

The ppp correlation function



Some remarks

- To compare the experimental and the theoretical correlation functions some corrections have been considered
- For the *pp* case the corrected correlation function is defiend as

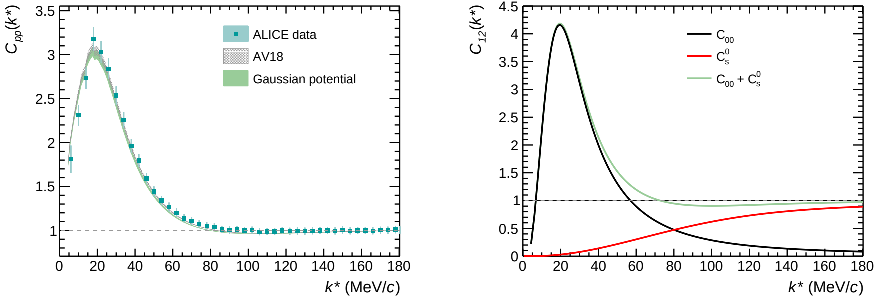
$$C(k) = \lambda_{pp} C_{pp}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(k)$$

- primary protons $\lambda_{pp} = 0.67$, secondary protons produced mainly in the decay of the Λ, $\lambda_{pp_\Lambda} = 0.203$, misidentification contributions $\lambda_x = 0.127$
- For the *ppp* case the corrected correlation function is defiend as

$$C(Q_3) = \lambda_{ppp} C_{ppp}(Q_3) + \lambda_{ppp_\Lambda} C_{ppp_\Lambda}(Q_3) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(Q_3)$$

- primary protons $\lambda_{ppp} = 0.618$, secondary protons produced mainly in the decay of the Λ, $\lambda_{ppp_\Lambda} = 0.196$, misidentification contributions $\lambda_x = 0.186$

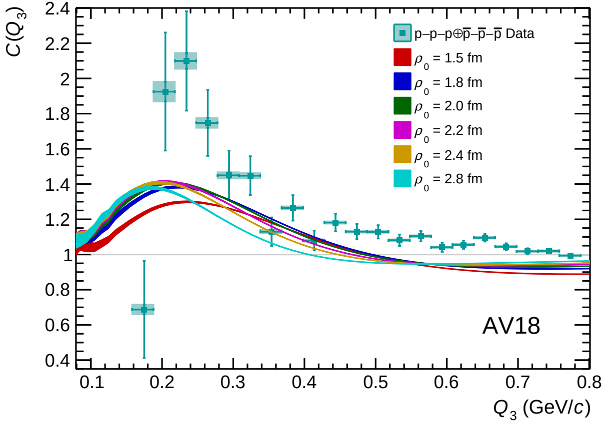
The pp correlation function



$$C_{12}(k) = C_s^0 + C_{00} = \int d\mathbf{r} S_{12}(r) \left[|\Psi_s^0|_\Omega^2 - \frac{1}{2}\left(\frac{F_0(\eta, kr)}{kr}\right)^2 + \frac{1}{2}\left(\frac{u_0(kr)}{kr}\right)^2 \right]$$

$$C_{pp}(k) = \lambda_{pp} C_{12}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\rm X} C_{\rm X}(k)$$

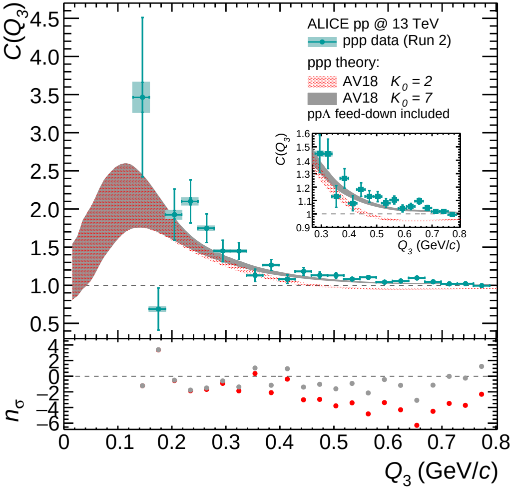
The *ppp* correlation function



$$C(Q_3) = \lambda_{ppp} C_{ppp}(Q_3) + \lambda_{pp p_\Lambda} C_{pp p_\Lambda}(Q_3) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(Q_3)$$

A. Kievsky, E. Garrido, M. Viviani, L. E. Marcucci, L. Šerkšnytė, and R. Del Grande Phys. Rev. C 109, 034006 (2024)

The *ppp* correlation function



E. Garrido, A. Kievsky, R. Del Grande, L. Šerkšnytė, M. Viviani, L. E. Marcucci, Phys. Lett. B 868, 139731 (2025)

The $p\Lambda$ and $pp\Lambda$ correlation functions

- The $p\Lambda$ correlation function is defined as

$$C(k) = \int d^3r\, S(r) |\psi_{p\Lambda}(\vec{r})|^2$$

- $\psi_{p\Lambda}$ is the scattering $p\Lambda$ wave function. It is governed by the $p\Lambda$ interaction which is not very well kown
- The few $p\Lambda$ scattering data can be described in the context of the EFT at different orders (see for example J. Haidenbauer et al. Eur. Phys. J. A 59 (2023) 63)
- At different cutoffs different sets of low-energy scattering parameters appear

	NLO13						NLO19				SMS N2LO		
C(MeV)	450	500	550	600	650	700	500	550	600	650	500	550	600
a_0 (fm)	-2.90	-2.91	-2.91	-2.91	-2.90	-2.90	-2.91	-2.90	-2.91	-2.90	-2.80	-2.79	-2.80
r_e^0 (fm)	2.64	2.86	2.84	2.78	2.65	2.56	3.10	2.93	2.78	2.65	2.82	2.89	2.68
a_1 (fm)	-1.70	-1.61	-1.52	-1.54	-1.51	-1.48	-1.52	-1.46	-1.41	-1.40	-1.56	-1.58	-1.56
r_e^1 (fm)	3.44	3.05	2.83	2.72	2.64	2.62	2.62	2.61	2.53	2.59	3.16	3.09	3.17

The $p\Lambda$ effective interaction

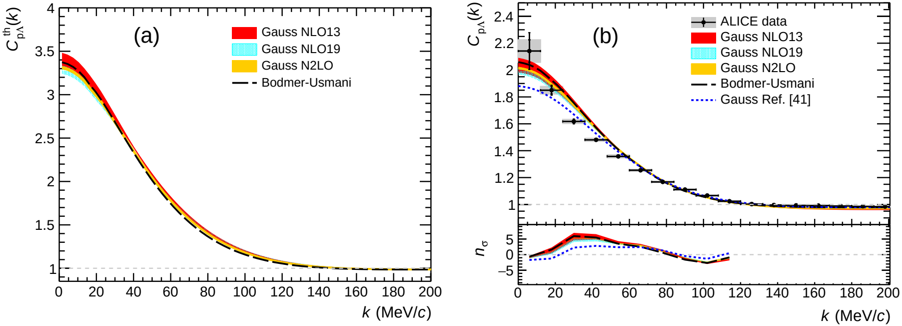
Using the Gaussian form, we define the effective $p\Lambda$ interaction as

$$V_{p\Lambda}(r) = \sum_{S=0,1} V_S e^{-(r/r_S)^2} \mathcal{P}_S$$

	NLO13				NLO19				SMS N2LO		
C(MeV)	500	550	600	650	500	550	600	650	500	550	600
V_0 (MeV)	-30.180	-30.574	-31.851	-34.831	-25.954	-28.817	-31.851	-34.831	-31.140	-29.753	-34.273
r_0 (fm)	1.467	1.459	1.434	1.380	1.563	1.495	1.434	1.380	1.439	1.466	1.382
V_1 (MeV)	-29.205	-33.839	-36.258	-38.455	-38.984	-39.470	-42.055	-40.373	-27.544	-28.609	-27.392
r_1 (fm)	1.338	1.247	1.216	1.183	1.178	1.163	1.126	1.143	1.361	1.344	1.364
$B(^3_\Lambda\mathrm{H})$ (MeV)	2.8729	2.87956	2.92508	2.98499	2.79212	2.83929	2.90455	3.25522	2.81932	2.79875	2.8785
W_3 (MeV)	11.83	11.733	12.32	12.873	10.545	11.056	11.795	12.294	10.65	10.375	11.4
ρ_3 (fm)	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

The $p\Lambda$ correlation function: $C_{p\Lambda}(k) = \int d^3r\ S(r)\ |\Psi_{p\Lambda}|^2$

$$C_{p\Lambda}(k) = \lambda_{p\Lambda} C^{\rm th}_{p\Lambda}(k) + \lambda_{\rm p\Lambda_{\Sigma^0}} C_{p\Lambda_{\Sigma^0}}(k) + \lambda_{p\Lambda_{\Xi}} C_{p\Lambda_{\Xi}}(k) + \lambda_{\rm flat}$$



$V^{BU}_{p\Lambda} = V_C(r)(1 - \epsilon + \epsilon P_x) + 0.25\, V_G\ T^2_\pi(r)\, \sigma_\Lambda \cdot \sigma_p$
$a_0 = -2.88$ fm, $r^0_e = 2.87$ fm
$a_1 = -1.66$ fm, $r^1_e = 3.67$ fm
$B(^3_\Lambda\mathrm{H}) = 2.73$ MeV

$V_{p\Lambda}(r) = \sum_{S=0,1} V_S\, e^{-(r/r_S)^2} \mathcal{P}_S$
$a_0 = -2.10$ fm, $r^0_e = 3.21$ fm
$a_1 = -1.54$ fm, $r^1_e = 3.16$ fm
$B(^3_\Lambda\mathrm{H}) = 2.40$ MeV

The $pp\Lambda$ system

Jacobi coordinates for two nucleons of mass m and the Λ of mass M in $\mathbf{r}_3$

$r-$space $\qquad\qquad\qquad\qquad$ $q-$space

$$\begin{cases} \mathbf{x} = \mathbf{r}_2 - \mathbf{r}_1 \\ \\ \mathbf{y} = \sqrt{\frac{4}{(1+2m/M)}}\,(\mathbf{r}_3 - \frac{\mathbf{r}_1+\mathbf{r}_2}{2}) \end{cases} \qquad \begin{cases} \mathbf{k} = \frac{1}{2}(\mathbf{p}_2 - \mathbf{p}_1) \\ \\ \mathbf{q} = \sqrt{\frac{m}{M}}\sqrt{\frac{m}{2m+M}}\,(\mathbf{p}_3 - \frac{M}{m}\frac{\mathbf{p}_1+\mathbf{p}_2}{2}) \end{cases}$$

The hyperradius $\rho = (x^2 + y^2)^{1/2}$ $\qquad$ the hypermomentum $Q = (k^2 + q^2)^{1/2}$

$[\Omega_\rho \equiv \hat{x}, \hat{y}, \alpha = \arctan(x/y)]$ $\qquad$ $[\Omega_Q \equiv \hat{k}, \hat{q}, \tilde{\alpha} = \arctan(k/q)]$

In terms of the particle distances $\frac{\rho^2}{2} = r_1^2 + r_2^2 + \frac{M}{m} r_3^2 - \frac{M+2m}{m} R^2$

The hypermomentum is related to the total energy $E = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2M} = \frac{Q^2}{m}$

The $pp\Lambda$ source function

The correlation function for three particles is given by

$$C_{123}(Q) = \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \; S_1(\mathbf{r}_1) S_2(\mathbf{r}_2) S_3(\mathbf{r}_3) |\Psi_s|^2$$

The source function $S_i(\mathbf{r}_i)$ is approximated by a Gaussian probability distribution. The widths of the proton and Λ distributions as R_m and R_M, respectively.

$$S_1(\mathbf{r}_1) S_1(\mathbf{r}_2) S_1(\mathbf{r}_3) = \frac{e^{-(\frac{\rho^2}{2} - (\frac{R_m^2}{R_M^2} - \frac{M}{m}) r_3^2 + \frac{M+2m}{m} R^2)/2R_m^2}}{(2\pi R_m^2)^3 (2\pi R_M^2)^{\frac{3}{2}}}$$

with the condition $R_m^2/R_M^2 = M/m$ after integrating our the center of mass

$$S_{123}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

with $\rho_0 = 2R_m$

The $pp\Lambda$ correlation function

$$C_{123}(Q) = \frac{1}{\pi^3 \rho_0^6} \int e^{-(\rho/\rho_0)^2} |\Psi_s|^2 \rho^5 d\rho d\Omega_\rho$$

With the three-body scattering wave function

$$\Psi_s = \frac{1}{\sqrt{N_S}} \frac{(2\pi)^3}{(Q\rho)^{5/2}} \sum_{JJ_z} \sum_{K\gamma} \Psi_{K\gamma}^{JJ_z} \sum_{M_L M_S} (LM_L S M_S | JJ_z) \mathcal{Y}_{KLM_L}^{\ell_x \ell_y}(\Omega_Q)^*$$

N_S is the number of spin states and $\gamma \equiv \{\ell_x, \ell_y, L, s_x, S\}$. The coordinate wave functions, $\Psi_{K\gamma}^{JJ_z}$, in the HH formalism take the general form

$$\Psi_{K\gamma}^{JJ_z} = \sum_{K'\gamma'} \Psi_{K\gamma}^{K'\gamma'}(Q,\rho) \Upsilon_{JJ_z}^{K'\gamma'}(\Omega_\rho)$$

$$\Upsilon_{JJ_z}^{K\gamma}(\Omega_\rho) = \sum_{M_L M_S} (LM_L S M_S | JJ_z) \mathcal{Y}_{KLM_L}^{\ell_x \ell_y}(\Omega_\rho) \chi_{SM_S}^{s_x}.$$

The $pp\Lambda$ correlation function

$$|\Psi_s|^2_\Omega = \frac{1}{\pi^6}\int d\Omega_\rho \int d\Omega_Q |\Psi_s|^2$$

For non-interacting particles $\Psi^{K'\gamma'}_{K\gamma}(Q,\rho) = i^K\sqrt{Q\rho}J_{K+2}(Q\rho)\delta_{KK'}\delta\gamma\gamma'$
and the norm results, with N_{ST} the number of states for a given K

$$|\Psi^0_s|^2_\Omega = \frac{2}{N_S}\frac{2^6}{(Q\rho)^4}\sum_K J^2_{K+2}(Q\rho)N_{ST}(K)$$

$$C_{pp\Lambda}(Q) = \frac{1}{4}\frac{2^6}{Q^4\rho_0^6}\int \rho\, d\rho\, e^{-\frac{\rho^2}{\rho_0^2}}\left(\sum_J (2J+1)\left|\frac{u^J_{n_0}}{\sqrt{Q\rho}}\right|^2 + \sum_{K>1} J^2_{K+2}(Q\rho)N_{ST}(K)\right)$$

where the sum over J includes the states $J^\pi = 1/2^+, 1/2^-, 3/2^-, 5/2^-$ with $u^J_{n_0}$ the corresponding wave function.

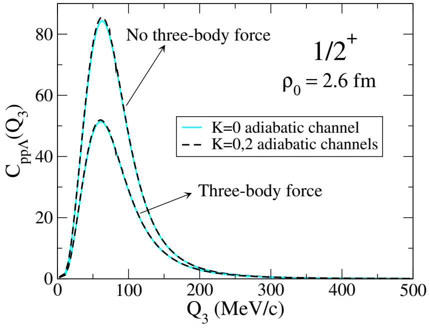
The $pp\Lambda$ three-body force

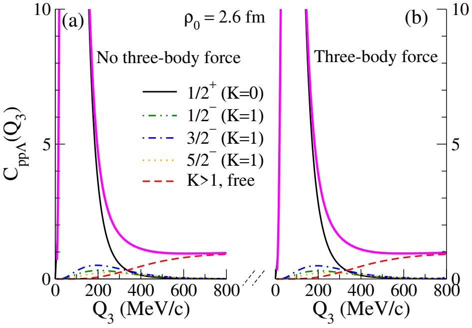
- The optimized LO $p\Lambda$ potential hase been constructed to describe the scattering length and effective range in the two spin channels
- Going to $NN\Lambda$ system this description has to completed including a three-body force
- This is related to what is called The three-body parameter as in pion-less EFT
- Accordingly, when describing the $pp\Lambda$ system we consider the following three-body force

$$W(r_{12}, r_{13}) = W_0 e^{-(r_{12}/\rho_0)^2-(r_{13}/\rho_0)^2}$$

- W_0, ρ_0 fixed to describe the hypertriton and if possible the $N = 4, 5$ hypernuclei

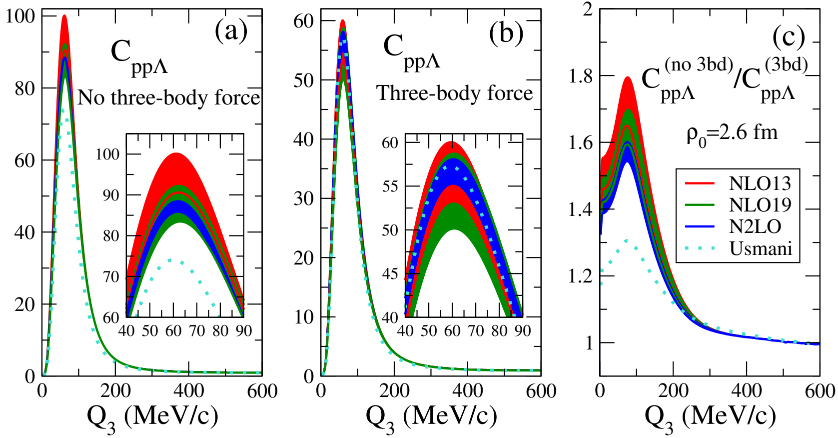
The $pp\Lambda$ correlation function



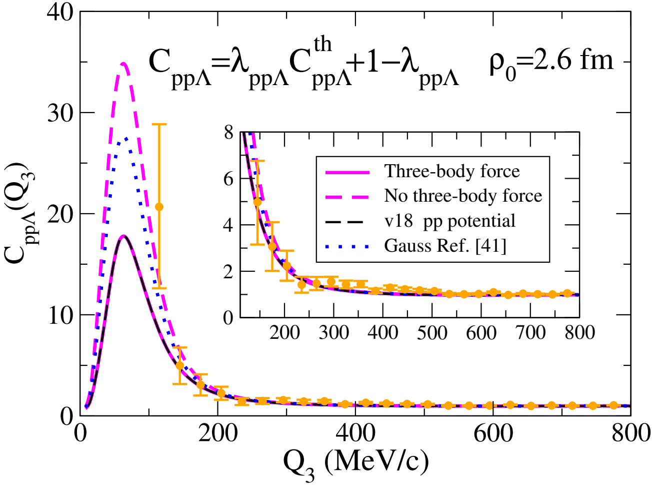


Contribution of the different partial waves

The $pp\Lambda$ correlation function



The $pp\Lambda$ correlation function



- A $NN\Lambda$ three-body force is included fixed to describe the $B(^3_\Lambda H)$

Summary

- Although its apparent simplicity, the three-body problem is of great complexity
- Measurements of the correlation function allow for new tests of the NN, NNN, $N\Lambda$, $NN\Lambda$,... interactions
- In the ppp case the Coulomb interaction couples the asymptotic dynamics increasing the difficulties of the numerical treatment
- The corrections of the computed pp and ppp correlation functions needs the kwonledge of the $p\Lambda$ and $pp\Lambda$ correlation functions
- The $N\Lambda$ and $NN\Lambda$ interactions are not very well known
- It would be important to link the correlation function data and the potential

- Studies on the $p\Lambda$ and $pp\Lambda$ correlation functions have been started
- The $pp\Lambda$ correlation functions could be sensitive to the $NN\Lambda$ three-body force, an important ingredient in the studies of compact systems