Influence of Exceptional Points on Nuclear Structure and Reactions

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- **Theoretical Framework**
  - **Gamow Shell Model**
  - **GSM-CC**
  - **Exceptional Points (EPs)**
- **Results**
  - **Spectrum and EP**
  - **Effects on Observables**
- **Conclusions**

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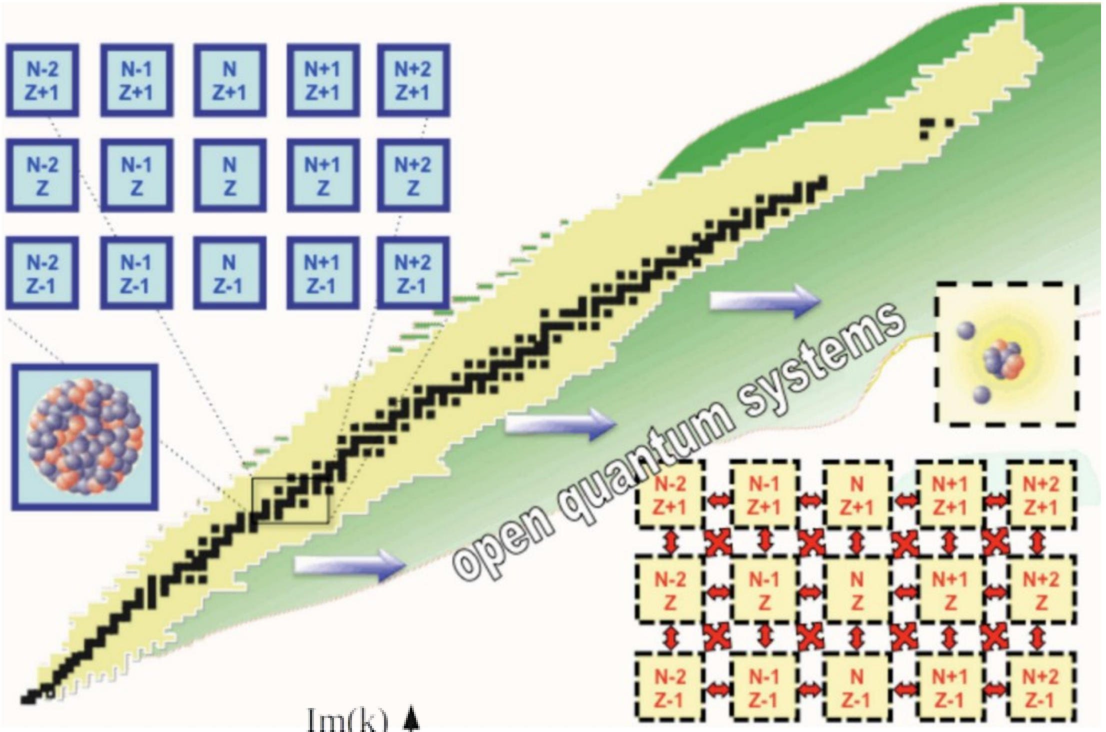
Gamow Shell Model



$$\sum_{n\in(b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|\,\mathrm{d}k = 1$$

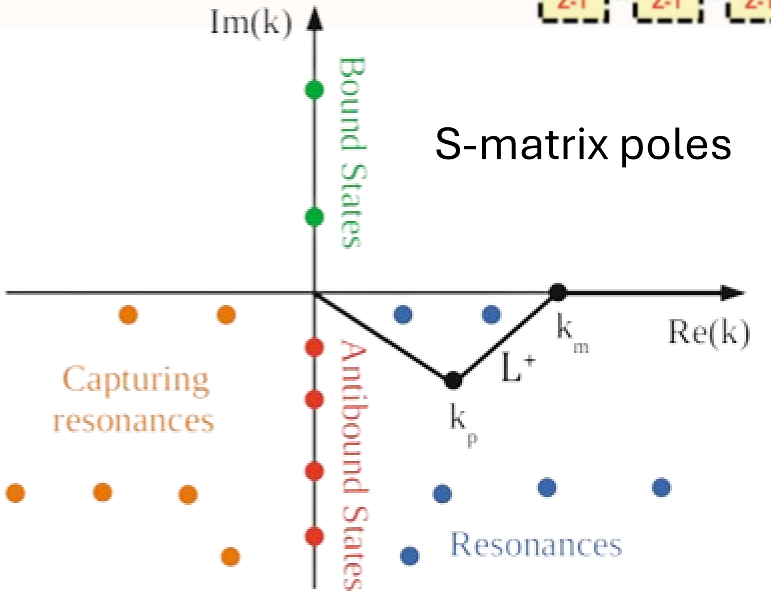
Berggren Completeness relation

$$\sum_{\mathrm{n}} |\mathrm{SD_n}\rangle\langle\widetilde{\mathrm{SD}}_\mathrm{n}| \approx \mathbb{1} \;\; ; \;\; |\mathrm{SD_n}\rangle = |u_1, u_2, \ldots u_N\rangle$$

Many-body completeness relation

$$|\Psi\rangle = \sum_{\mathrm{n}} \mathrm{b_n}|\mathrm{SD_n}\rangle$$

Generalization of the Shell Model that **preserves unitarity** at the opening of the thresholds



S-matrix poles

$$\hat{\mathrm{H}}|\Psi\rangle = E|\Psi\rangle$$

$$\langle\psi_\mathrm{k}|\psi_\mathrm{k}\rangle \Longrightarrow \langle\psi_\mathrm{k}^*|\psi_\mathrm{k}\rangle$$

Matrix Representation is Complex Symmetric

Non-Hermitian

Gamow Shell Model

$$\sum_{n\in(b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|\,\mathrm{d}k = 1$$

Berggren Completeness relation

$$\sum_{\mathrm{n}} |\mathrm{SD_n}\rangle\langle\widetilde{\mathrm{SD}}_\mathrm{n}| \approx \mathbb{1}\,;\ |\mathrm{SD_n}\rangle = |u_1, u_2, \dots u_N\rangle$$

Many-body completeness relation

$$\underbrace{|\Psi\rangle = \sum_{\mathrm{n}} \mathrm{b_n}|\mathrm{SD_n}\rangle \qquad \hat{\mathrm{H}}|\Psi\rangle = E|\Psi\rangle}_{\text{Reaction channels cannot be well defined}}$$

$\Downarrow$

Prime Tool for Structure studies

Coupled Channels Gamow Shell Model

In the coupled-channels representation the A-body wavefunction is decomposed into reaction channels:

$$|\Psi_M^J\rangle = \sum_c \int_0^\infty |(r,c)_M^J\rangle \frac{u_c^{JM}(r)}{r} r^2 dr$$

$\Downarrow$

$$|(c,r)_M^J\rangle = \hat{\mathcal{A}}[\underbrace{|\Psi_T^{J_T}\rangle}_{\text{Target}} \otimes \underbrace{|\Psi_P^{J_P}\rangle}_{\text{Projectile}}]_M^J$$

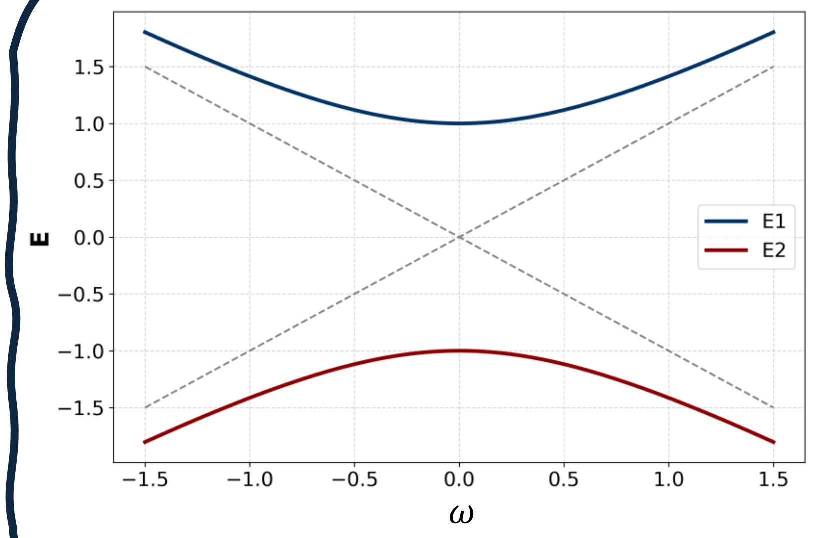
Then the coupled-channel equations can be obtained from $\hat{\mathrm{H}}\left|\psi_\mathrm{M}^\mathrm{J}\right\rangle = E\left|\psi_\mathrm{M}^\mathrm{J}\right\rangle$ as:

$$\sum_c \int_0^\infty r^2\left(H_{cc'}(r,r') - EN_{cc'}(r,r')\right)\frac{u_c(r)}{r} = 0$$

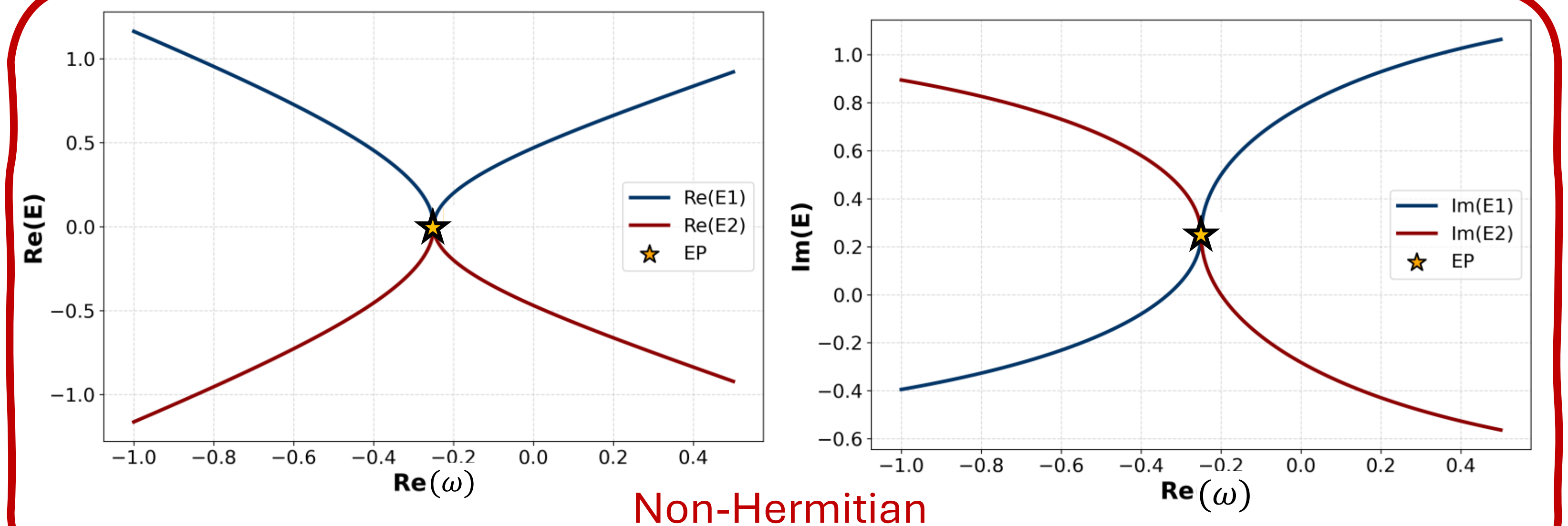
From which one can obtain resonant and scattering solution with **clear asymptotics**

Unifies Nuclear Structure and Reaction Studies

Exceptional Points



Hermitian



Non-Hermitian
Two state example

At an exceptional point two or more **complex eigenvalues** coalesce and the eigenfunctions become identical.

$$\left.\begin{array}{l}|\psi_1(\omega)\rangle \\ \\ |\psi_2(\omega)\rangle\end{array}\right] \rightarrow |\psi_{EP}(\omega_{\mathrm{EP}})\rangle$$

$$\left.\begin{array}{l}E_1(\omega) \\ \\ E_2(\omega)\end{array}\right] \rightarrow E_{EP}(\omega_{\mathrm{EP}})$$

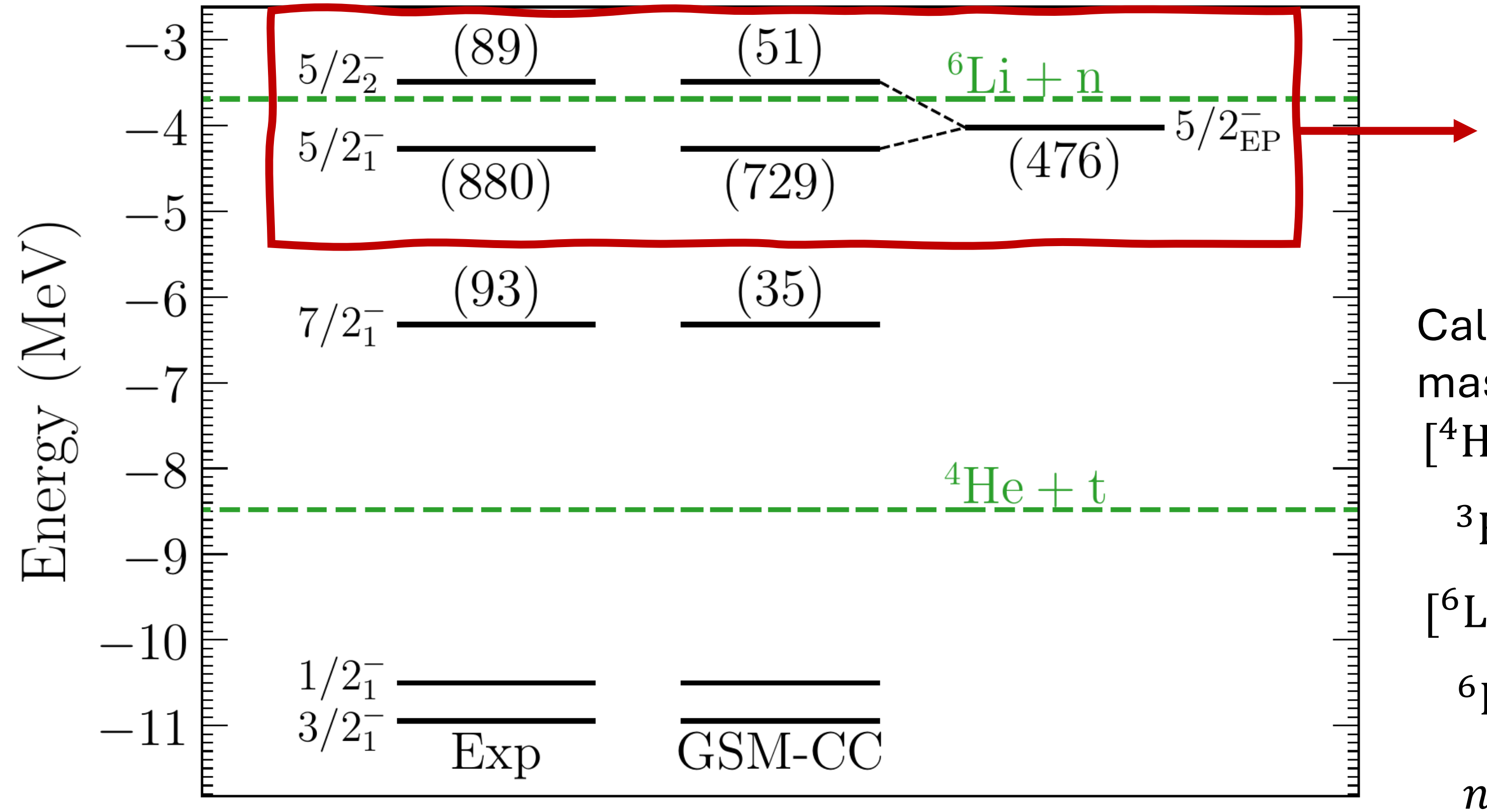
$$\langle \psi_{EP}^{*} | \psi_{EP} \rangle = 0$$

The state becomes self orthogonal
Under non-Hermitian norm

Exceptional points have been studied experimentally and theoretically in many areas of physics:

- Microwave cavities. *Phys. Rev.Lett. 86, 787 (2001).*
- Photonics *Phys. Rev. Lett. 108, 173901 (2012).*
- Molecular physics. *Phys. Rev. Lett. 103, 123003 (2009).*
- Quantum phase transitions *J. Phys. A: Math. Gen. 38 1843 (2005)*
- Atomic physics *J. Phys. B: At. Mol. Opt. Phys. 32 1669 (1999)*

^{7}Li GSM-CC



Discuss the effects of the presence of an exceptional point on quantities associated to the $5/2^-$ doublet

Calculations of 7Li were done using three different mass partitions

$[{}^4\mathrm{He}(0_1^+) \otimes {}^3\mathrm{H}(L_j)]^{J^\pi}$

^{3}H: $L_j = S_{1/2}, P_{1/2}, P_{3/2}, D_{3/2}, D_{5/2}, F_{5/2}, F_{7/2}$

$[{}^6\mathrm{Li}(K_i^\pi) \otimes n(l_j)]^{J^\pi}$

^{6}Li: $K_i^\pi = 1_1^+, 3_1^+, 0_1^+ 2_1^+, 2_2^+, 1_2^+$

n: $l_j = s_{1/2}, p_{1/2}, p_{3/2}, d_{3/2}, d_{5/2}, f_{5/2}, f_{7/2}$

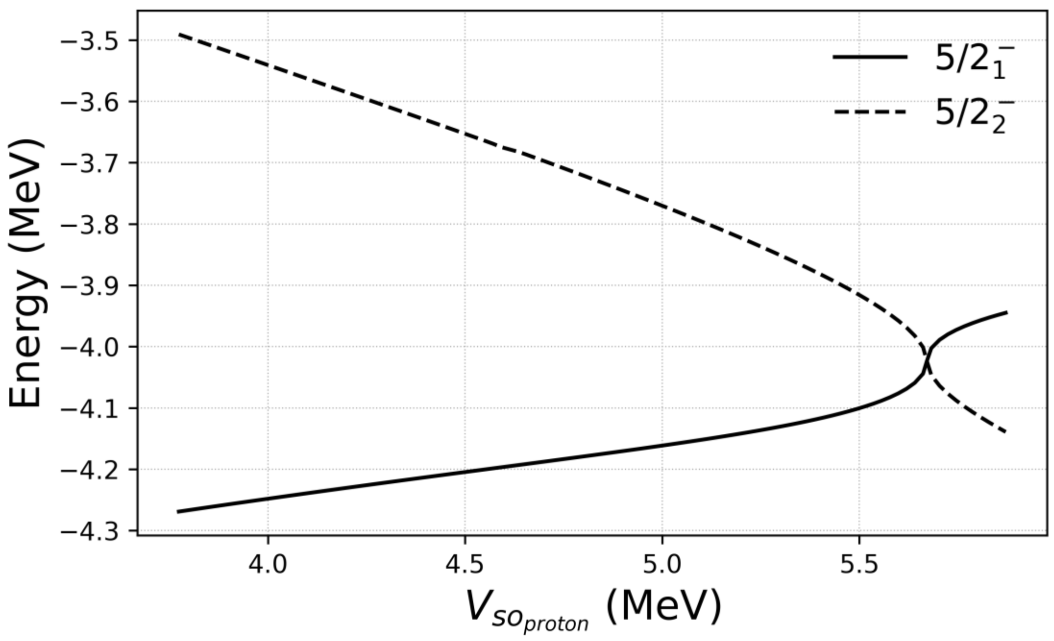
$[{}^5\mathrm{He}(K_i^\pi) \otimes {}^2\mathrm{H}(L_j)]^{J^\pi}$

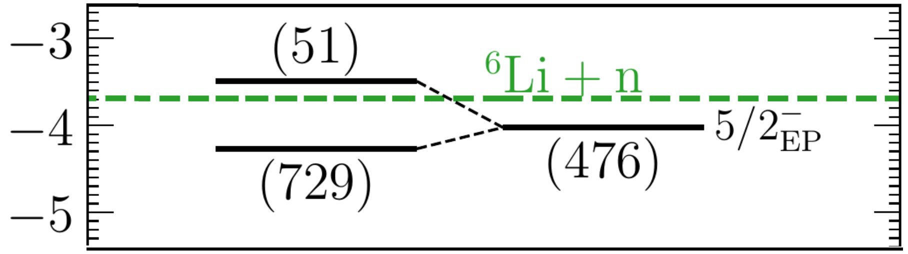
^{5}He: $L_j = 3S_1, 3P_0, 3P_1, 3P_3, 3D_1, 3D_2, 3D_3$

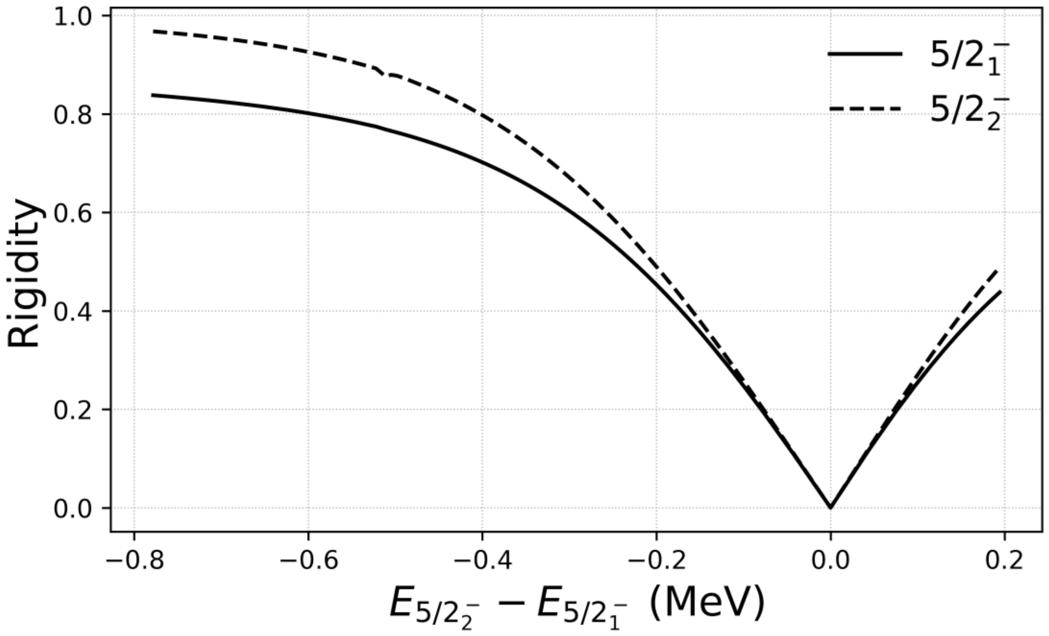
- Hamiltonian:
  - 1-body potential: Woods-Saxon + spin-orbit + Coulomb
  - 2-body FHT interaction: Central+spin-orbit+tensor+Coulomb

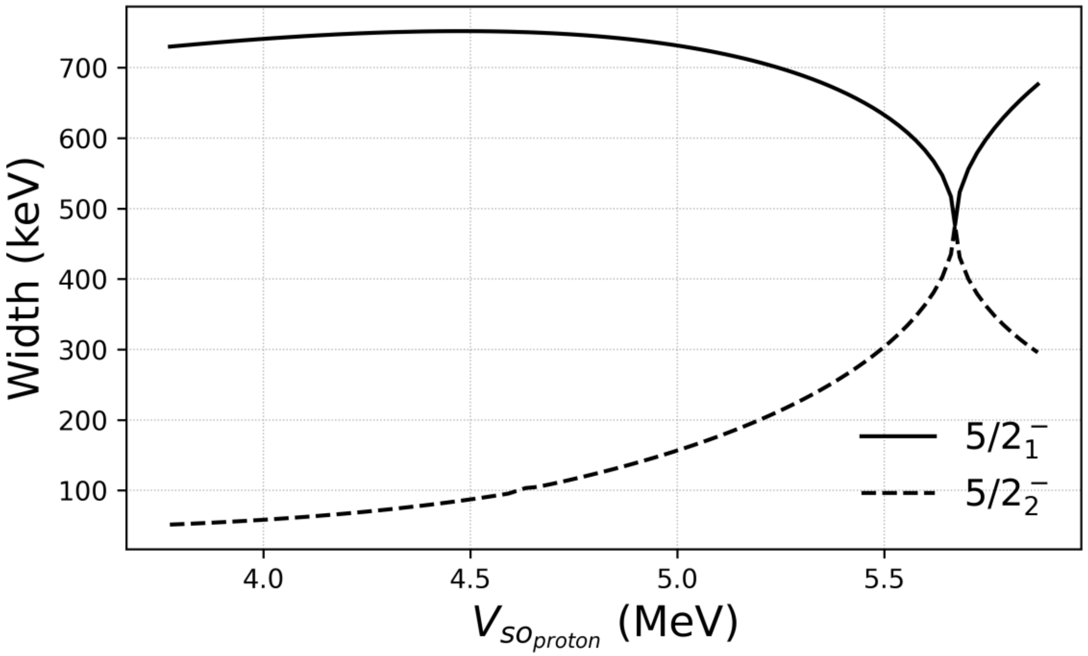
We used the <u>$\ell = 1$</u> part of the <u>**1-body spin-orbit terms**</u> of protons and neutrons as the variable parameters to find the Exceptional Point

$^{7}\mathrm{Li}$ GSM-CC





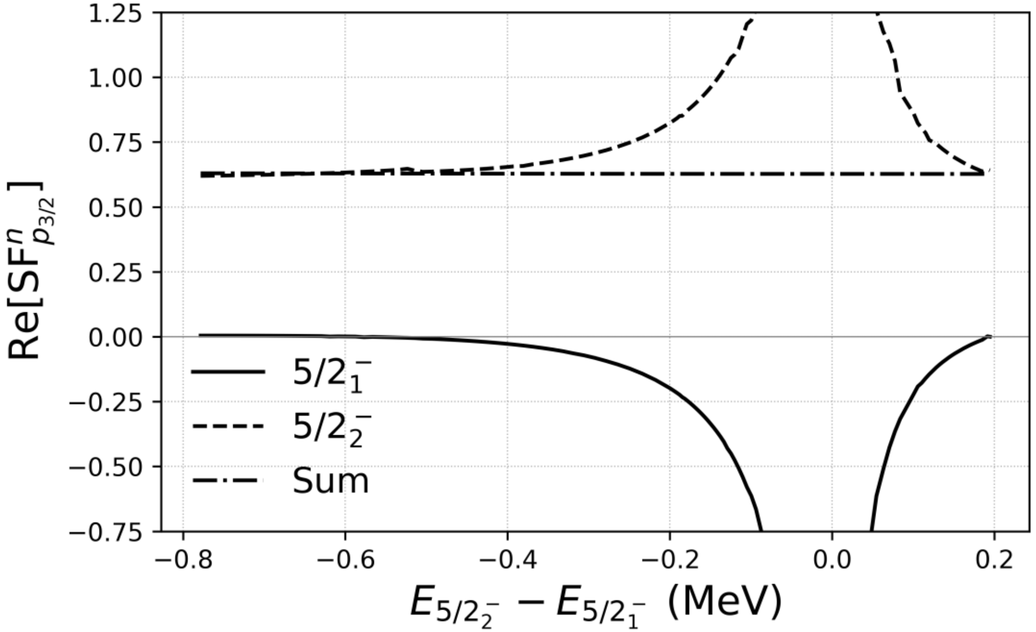


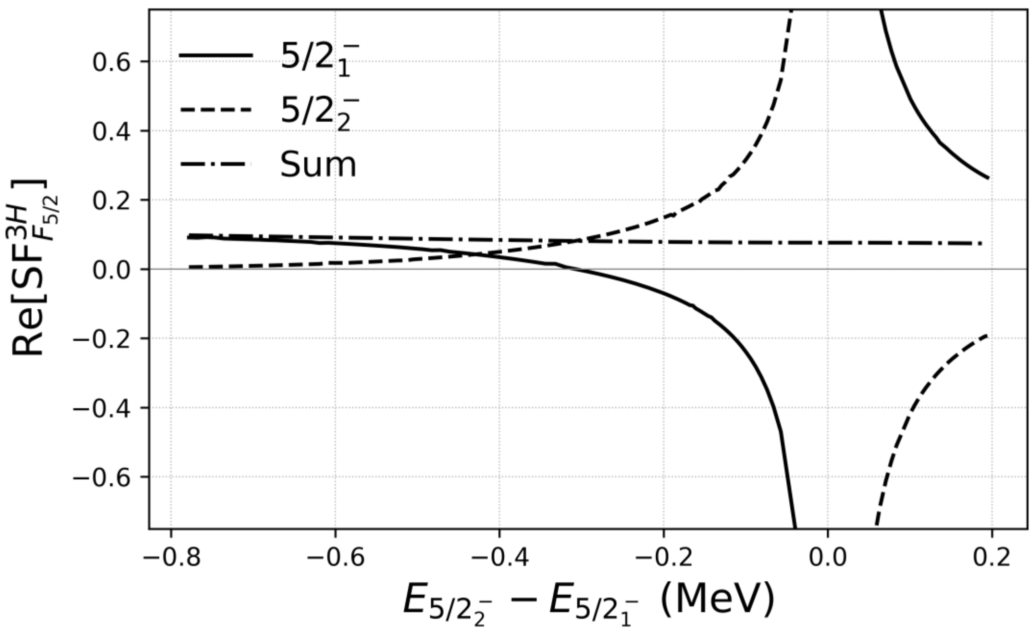


Indicator of strength of non-Hermitian effects

$$\mathrm{r_k} \equiv \frac{\langle \psi_\mathrm{k}^* | \psi_\mathrm{k} \rangle}{\langle \psi_\mathrm{k} | \psi_\mathrm{k} \rangle}$$

^{7}Li GSM-CC: Spectroscopic Factors





There are strong effects on expectation values of operators that do not commute with the Hamiltonian

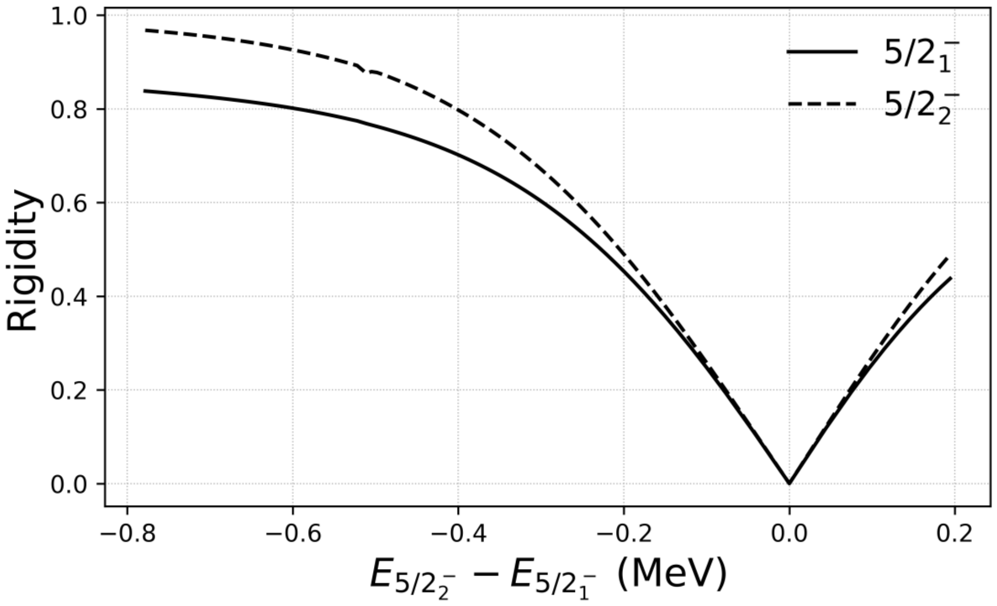
$$\langle \hat{O}_1 \rangle = \frac{\langle \psi_1^* | \hat{O} | \psi_1 \rangle}{\langle \psi_1^* | \psi_1 \rangle} \qquad \xrightarrow{\text{EP}} \qquad \frac{\langle \psi_{EP}^* | \hat{O} | \psi_{EP} \rangle}{\langle \psi_{EP}^* | \psi_{EP} \rangle} \to \infty$$

$$\langle \hat{O}_2 \rangle = \frac{\langle \psi_2^* | \hat{O} | \psi_2 \rangle}{\langle \psi_2^* | \psi_2 \rangle}$$

since, $\langle \psi_{EP}^* | \psi_{EP} \rangle = 0$

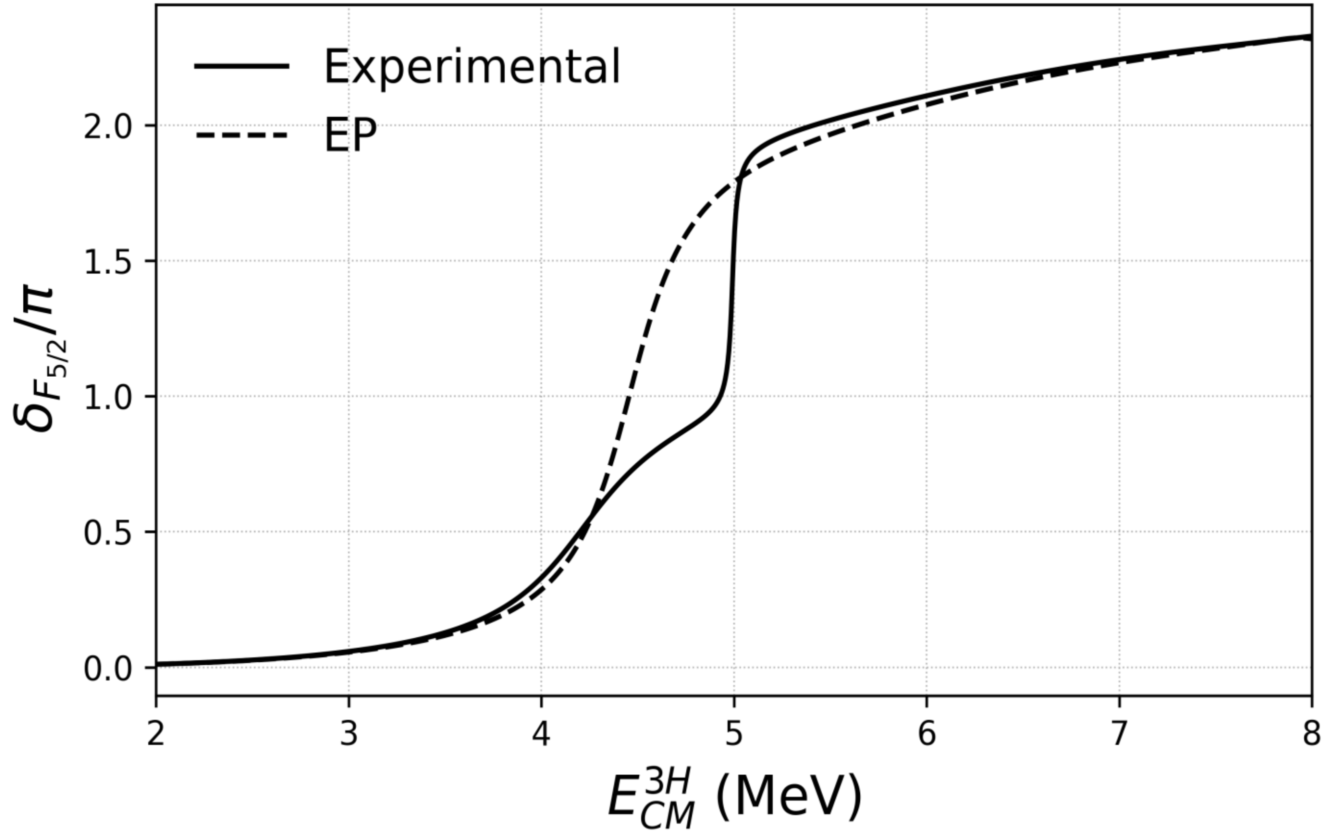
$\Downarrow$

High sensitivity to changes in parameters



$^{7}\mathrm{Li}$ GSM-CC: Phase Shift

$^{4}\mathrm{He}(^{3}\mathrm{H}\,,\,^{3}\mathrm{H})^{4}\mathrm{He}$



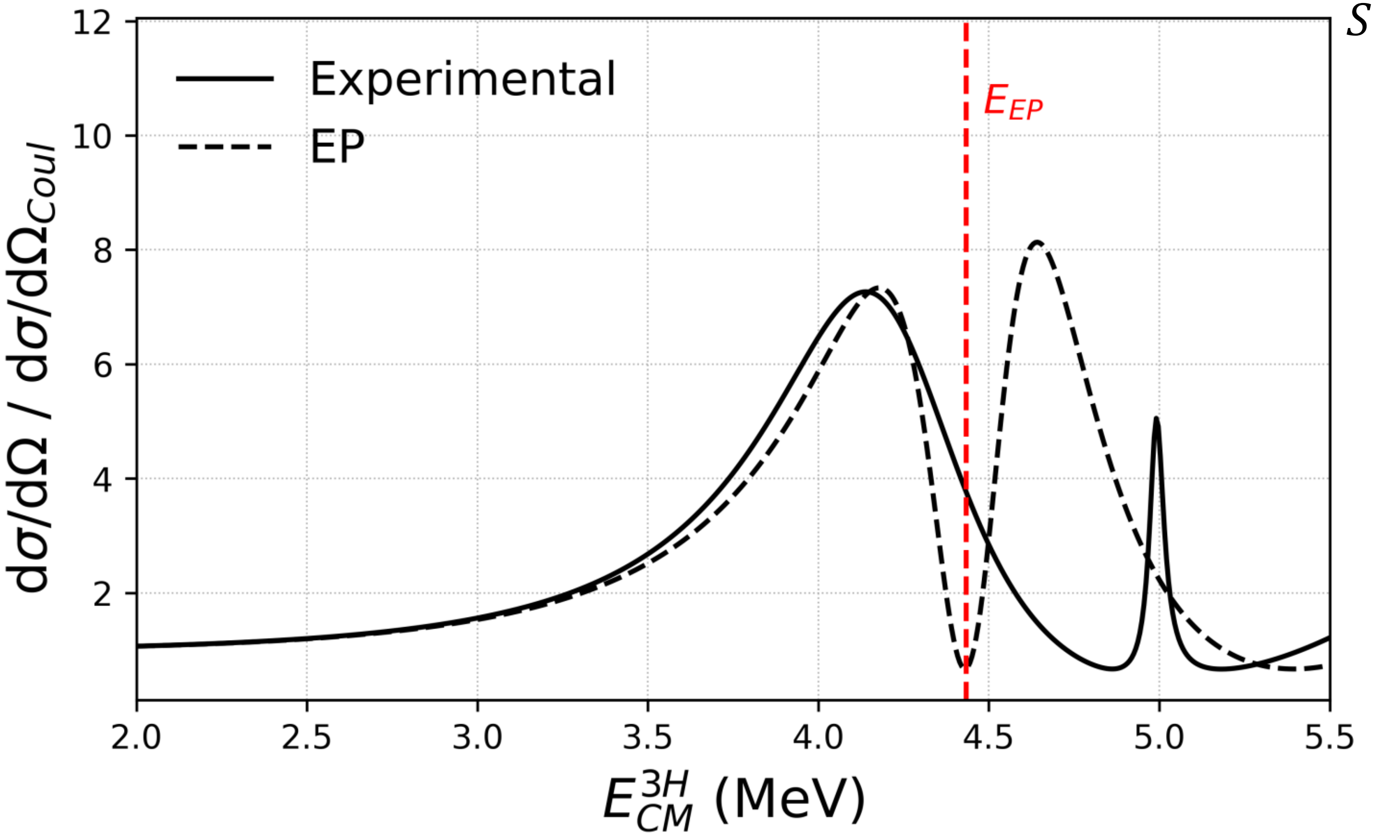
At the EP the S-matrix has a double pole

$$S(E) = \frac{(E - E_{EP} - i\Gamma_{EP}/2)^2}{(E - E_{EP} + i\Gamma_{EP}/2)^2} = e^{2i\delta_{EP}(E)}$$

$$\delta_{EP}(E) = \arctan\left(\frac{\Gamma_{EP}}{E_{EP} - E}\right) + \arctan\left(\frac{\Gamma_{EP}}{E_{EP} - E}\right)$$

Resulting in a single 2π jump at E_{EP}

^{7}Li GSM-CC: Elastic Cross Section



For a double pole one obtains

$$S(E) = \underbrace{\boxed{1 + 2i\,\frac{\Gamma_{EP}}{E - E_{EP} - \mathrm{i}/2\Gamma_{EP}}}}_{\text{Single resonance shape}} - \underbrace{\boxed{\frac{\Gamma_{EP}^2}{(E - E_{EP} - \mathrm{i}/2\Gamma_{EP})^2}}}_{\text{Interference term}}$$

Plugging into the elastic scattering cross section:

$$\sigma_{el} = \frac{\pi}{k^2}|S - 1|^2$$

$$\sigma_{el} = \frac{4\pi}{k^2}\underbrace{\frac{4(E - E_{EP})^2}{(E - E_{EP})^2 - \frac{\Gamma_{EP}^2}{4}}}_{\text{Interference factor}\ \xrightarrow{E \to E_{EP}} 0}\ \underbrace{\frac{\frac{\Gamma_{EP}^2}{4}}{(E - E_{EP})^2 - \frac{\Gamma_{EP}^2}{4}}}_{\text{Breit-Wigner shape}}$$

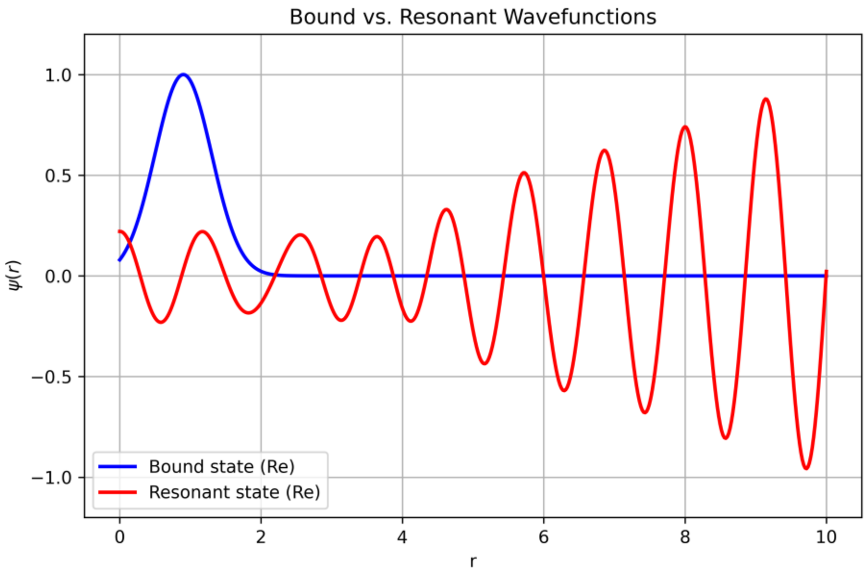
Conclusions

- Exceptional point in ^{7}Li $(5/2^{-}$ doublet) revealed with GSM-CC by varying spin–orbit terms
- At the EP: states merge in energy/width, phase rigidity vanishes, and non commuting operator diverge
- Reaction observables show clear signatures: split-peak elastic cross section and a single 2π phase-shift jump
- Although in nuclei, EPs cannot be accessed through direct parameter variation, proximity to them induces strong variation of observables. These indirect effects can be exploited to study them experimentally.

Thank you!

Supplementary Material

Resonances



$$\sum_{n\in(b.d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|\,\mathrm{d}k = 1$$

Berggren Ensemble

Resonant states are not square-integrable

$$\psi_{res}(r) \sim e^{+ikr} \qquad \mathrm{Im}(k) < 0$$

so $\int |\psi_{res}|^2 dr$ diverges, thus they don't belong to the Hilbert space.

Resonant states are regularized via complex scaling

$$\int_0^\infty \psi_{res}(r)dr = \int_0^R \psi_{res}(r)dr + \int_R^\infty \psi_{res}(R + xe^{i\theta})dx$$

The energies of resonant states are complex

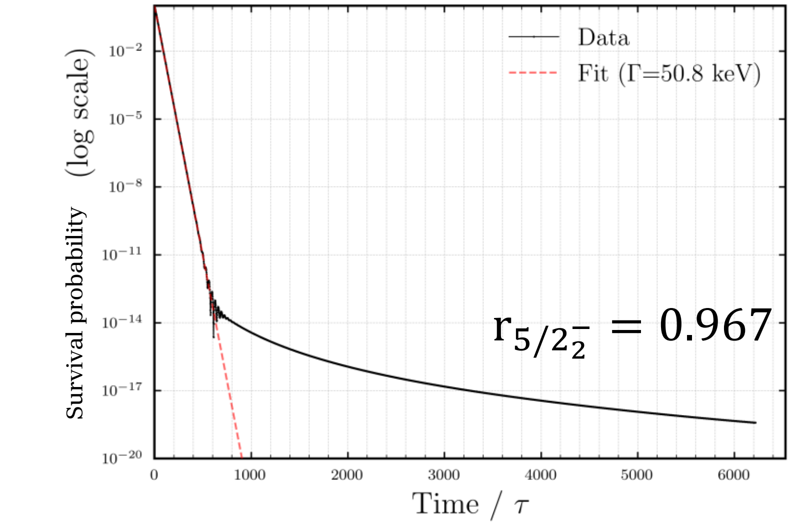
$$E_{res} = E - i/2\Gamma$$

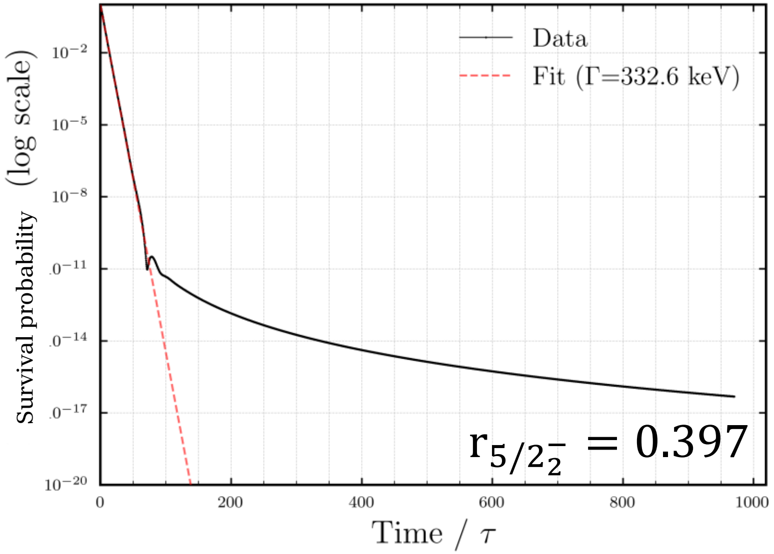
Then $\psi_{res}(r)$ and $\psi^*_{res}(r)$ are not related to the same complex eigenvalue $E_{res} \neq E^*_{\mathrm{res}}$.

The standard inner product is replaced by

$$\langle\psi^*_{res}|\psi_{res}\rangle = \int_{REG} \psi^2{}_{res}(r)dr$$

Rigged Hilbert space
Complex symmetric Hamiltonian matrix





The survival probability :

$$P(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2 = |\langle\Psi(0)|e^{-iHt}|\Psi(0)\rangle^2$$