Bayesian inference on nuclear data and neutron star observations for the nuclear equation of state

European Nuclear Physics conference, Caen, France

22nd-26th September

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22/9/2025

Collaborators

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Marco **Antonelli** (L.P.C. Caen)

Structure of the presentation

Bayesian inference on nuclear data and neutron star observations for the nuclear equation of state

- First Part: constraints on EoS from nuclear experiments[1]

- Bayesian inference
- Skyrme Interaction

- Second Part: constraints on EoS from Neutron Stars observations[2]

- Second Bayesian inference

[1] https://doi.org/10.1103/PhysRevC.111.014311 [2] https://doi.org/10.48550/arXiv.2505.16929

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Bayesian inference in a nutshell

Bayes' theorem

$$p(x, y) \rightarrow p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

Given a set of experimental data X and the parameters θ of our model M

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)}$$

Bayesian inference in a nutshell

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Prior distribution

Assumption on the model before considering experimental evidences

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)}$$

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Bayes' theorem

$$p(x, y) \rightarrow p(x\,|\,y) = \frac{p(y\,|\,x)p(x)}{p(y)}$$

Given a set of experimental data X and the parameters θ of our model M

$$\mathcal{L}(\theta) = \prod_i^N \exp\left(-\frac{(X_i - M_i(\theta))^2}{2\sigma_i^2}\right)$$

Likelihood $\mathcal{L}(\theta)$

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Evidence

Normalization factor for comparing different models; it does not depend on θ

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Assumption on the model before considering experimental evidences

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Posterior distribution

Probability distribution of model parameters; cannot be' computed analytically (MC sampling techniques)

Evidence

Normalization factor for comparing different models; it does not depend on θ

Parameters of the model and prior

Parameters (θ)

$n_{sat}, E_{sat}, K_{sat}, E_{sym}, L_{sym}$	Nuclear matter parameters
G_0, G_1	Surface term parameters
W_0, v_0	Spin-orbit parameter and pairing strength
$m_0^*/m, m_1^*/m$	Effective masses

0 = isoscalar; 1 = isovector

1-to-1 correspondence with usual Skyrme parameters[1]!

[1] L.-W. Chen et al. Phys. Rev. C 80, 014322 (2009)

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Prior distribution $p(\theta)$

	Units	Lower limit	Upper limit
n_{sat}	[fm^{-3}]	0.150	0.175
E_{sat}	[MeV]	-16.50	-15.50
K_{sat}	[MeV]	180.00	260.00
E_{sym}	[MeV]	24.00	40.00
L_{sym}	[MeV]	-20.00	120.00
G_0	[MeV fm^5]	90.00	170.00
G_1	[MeV fm^5]	-90.00	70.00
W_0	[MeV fm^5]	60.00	190.00
m_0^*/m	[-]	0.70	1.10
m_1^*/m	[-]	0.60	0.90
v_0	[MeV fm^3]	150	350

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Observables (X) and uncertainties (σ)

$$\mathscr{L}(\theta) = \prod_{i}^{N} \exp\left(-\frac{(X_i - M_i(\theta))^2}{2\sigma_i^2}\right)$$

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Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	$1636.4 \pm 2.0^*$	$5.50 \pm 0.05^*$	$2.02 \pm 0.50^*$
^{48}Ca	$416.0 \pm 2.0^*$	$3.48 \pm 0.05^*$	$1.72 \pm 0.50^*$
^{68}Ni	$590.4 \pm 2.0^*$	-	-
^{132}Sn	$1102.8 \pm 2.0^*$	4.71 ± 0.05	-
^{90}Zr	$783.9 \pm 2.0^*$	4.27 ± 0.05	-

Data from open shell nuclei			
	$B.E.$ [MeV]	R_{ch} [fm]	Δ_n [MeV]
^{50}Ca	$427.5 \pm 2.0^*$	$3.52 \pm 0.05^*$	-
^{46}Ca	$398.8 \pm 2.0^*$	-	-
^{44}Ca	$381.0 \pm 2.0^*$	-	-
^{42}Ca	$361.9 \pm 2.0^*$	-	-
^{120}Sn	$1020.5 \pm 2.0^*$	$4.65 \pm 0.05^*$	$1.3 \pm 0.2^*$
^{112}Sn	$953.5 \pm 2.0^*$	-	-
^{124}Sn	$1050.0 \pm 2.0^*$	-	-

$B.E.$: Binding Energy;
R_{ch} : Charge radius
ΔE_{SO} : Spin-orbit splitting
Δ_n: Neutron pairing gap

*Theoretical error

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Isoscalar resonances		
	$E^{\mathrm{IS}}_{\mathrm{GMR}}$ [MeV]	$E^{\mathrm{IS}}_{\mathrm{GQR}}$ [MeV]
^{208}Pb	$13.5 \pm 0.5^*$	$10.9 \pm 0.5^*$
^{90}Zr	$\mathbf{18.7 \pm 0.5}^*$	-

$B.E.$: Binding Energy;
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E^{IS}_{GMR} : IsoScalar Giant monopole resonance excitation energy (constrained)
E^{IS}_{GQR} : IsoScalar Giant quadrupole resonance excitation energy (centroid)

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Isovector properties			
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	$\mathbf{528 \pm 18}$
^{48}Ca	2.07 ± 0.22	-	$\mathbf{2550 \pm 113}$

$B.E.$: Binding Energy;
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E^{IS}_{GMR} : IsoScalar Giant monopole resonance excitation energy (constrained)
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α_D: Nuclear polarizability
$m(1)$: EWSR of IVGDR
A_{PV}: Parity violating asymmetry

*Theoretical error

[1] X. Roca-Maza, D H. Jakubassa-Amundsen Phys. Rev. Lett. 134, 192501 (2025)

Sampling of the posterior

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)} \longrightarrow$$

<u>Metropolis-Hastings algorithm:</u>
MCMC, explores parameter space focusing on zones with high $\mathscr{L}$

Sampling of the posterior

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)}$$ → Metropolis-Hastings algorithm: MCMC, explores parameter space focusing on zones with high $\mathscr{L}$

$$\mathscr{L}(\theta) = \prod_i^N \exp\left(-\frac{(X_i - M_i(\theta))^2}{2\sigma_i^2}\right)$$

"hfbcs-qrpa[1]" code to compute observables from parameters ($M(\theta)$)

Sampling of the posterior

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)}$$ ⟶

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Computing all the observables ⟶ ~ 2 hours!

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MH algorithm ⟶ 10^{6-7} model evaluations!

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Computing all the observables $\longrightarrow$ ~ 2 hours!

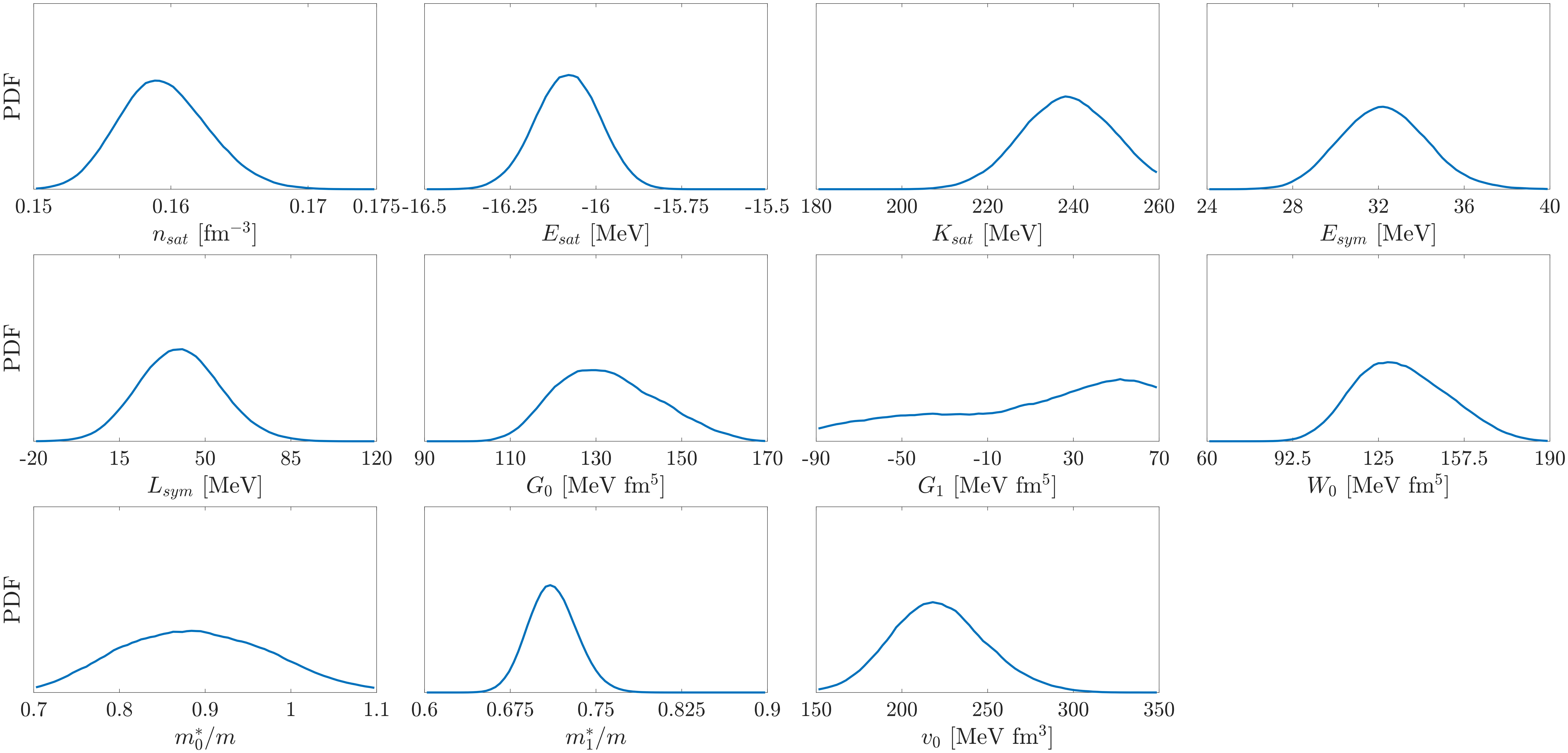
MH algorithm $\longrightarrow$ 10^{6-7} model evaluations!

2 h. x 10'000'000 points…

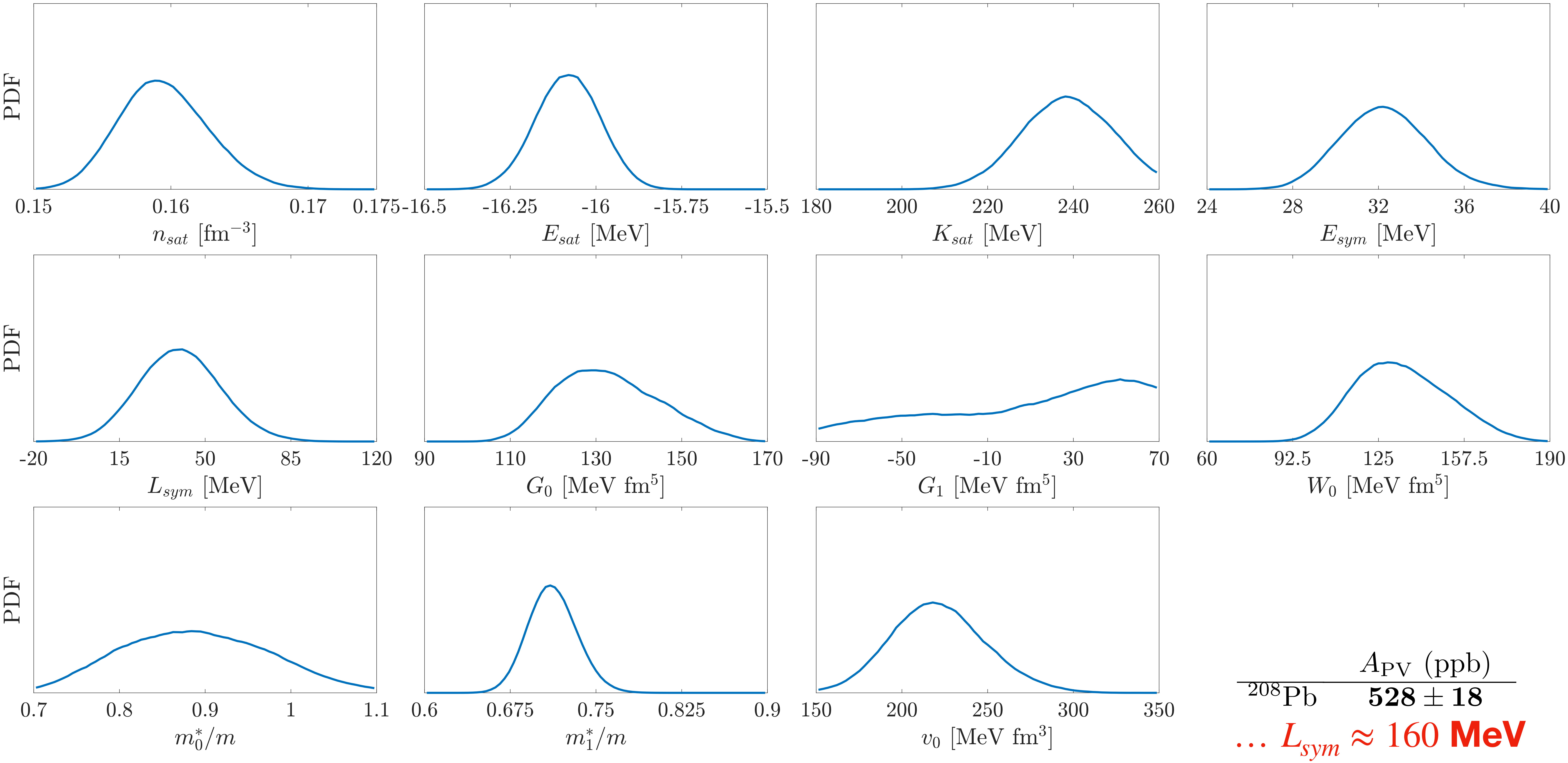
MADAI package[1]
(Emulator for Bayesian inference)

[1]https://madai.phy.duke.edu/

Progressive marginalized posteriors



Progressive marginalized posteriors



	$A_{\rm PV}$ (ppb)
^{208}Pb	$\mathbf{528 \pm 18}$

… $L_{sym} \approx 160$ **MeV**

Progressive marginalized posteriors

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^{208}Pb	$\mathbf{528 \pm 18}$

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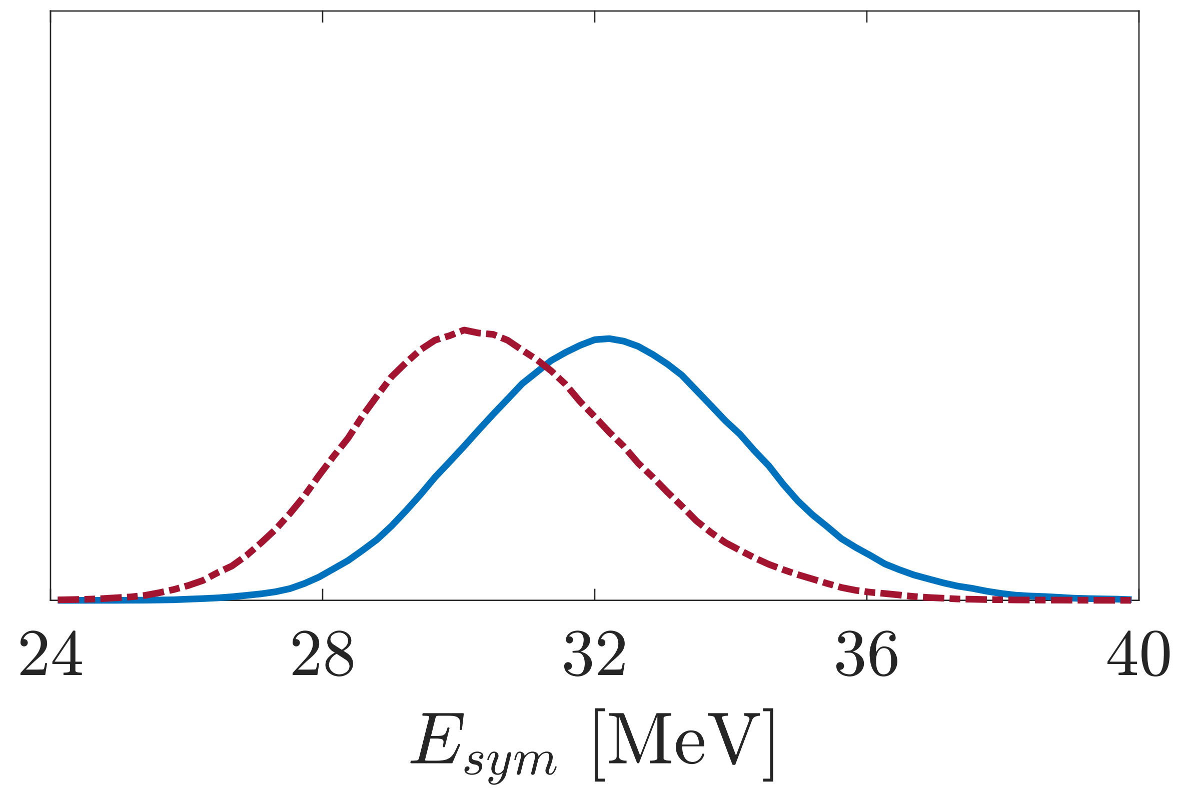
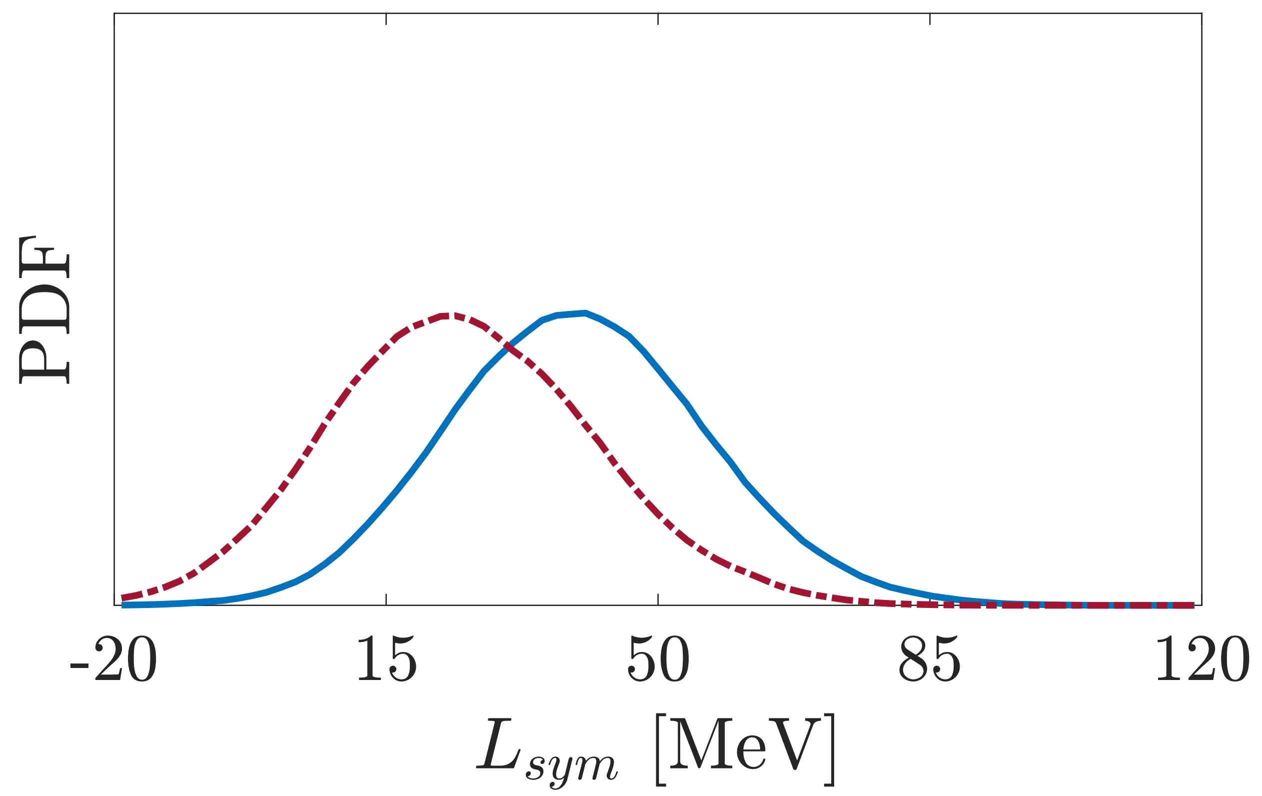
Progressive marginalized posteriors

		All	All but ^{208}Pb A_{PV}
n_{sat}	μ	0.160	0.160
	σ	0.003	0.003
E_{sat}	μ	-16.08	-16.05
	σ	0.09	0.09
K_{sat}	μ	238	236
	σ	10	10
E_{sym}	μ	32.3	30.5
	σ	2.0	2.0
L_{sym}	μ	39.8	24.9
	σ	16.6	16.7
G_0	μ	133	129
	σ	12	11
G_1	μ	8	2
	σ	44	44
W_0	μ	134	132
	σ	17	17
m_0^*/m	μ	0.89	0.91
	σ	0.08	0.08
m_1^*/m	μ	0.71	0.71
	σ	0.02	0.02
v_0	μ	222	223
	σ	28	27





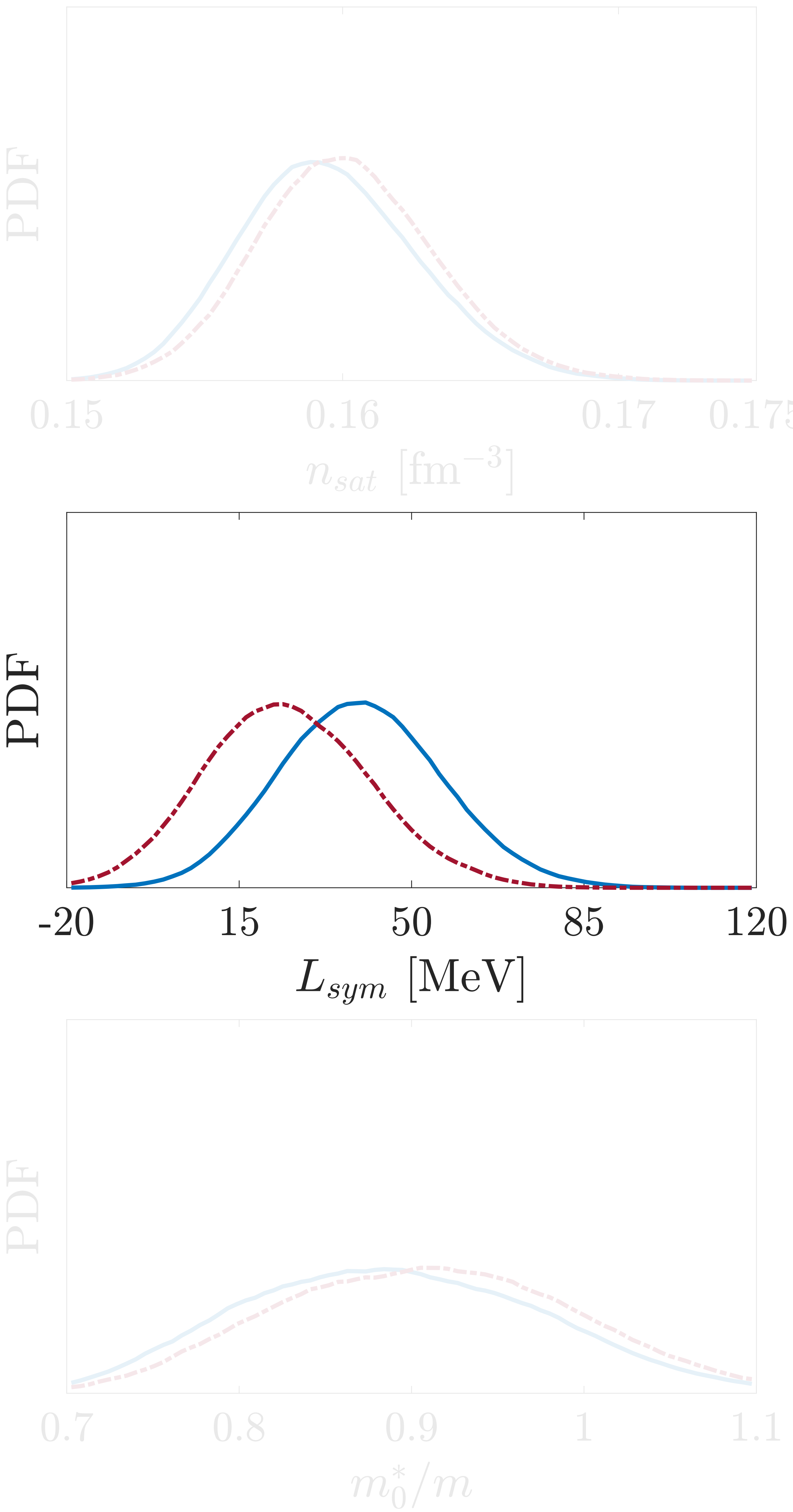
— All data

-·- All data except for ^{208}Pb A_{PV}

	A_{PV} (ppb)
^{208}Pb	**528 ± 18**

… $L_{sym} \approx 160$ **MeV**

Progressive marginalized posteriors



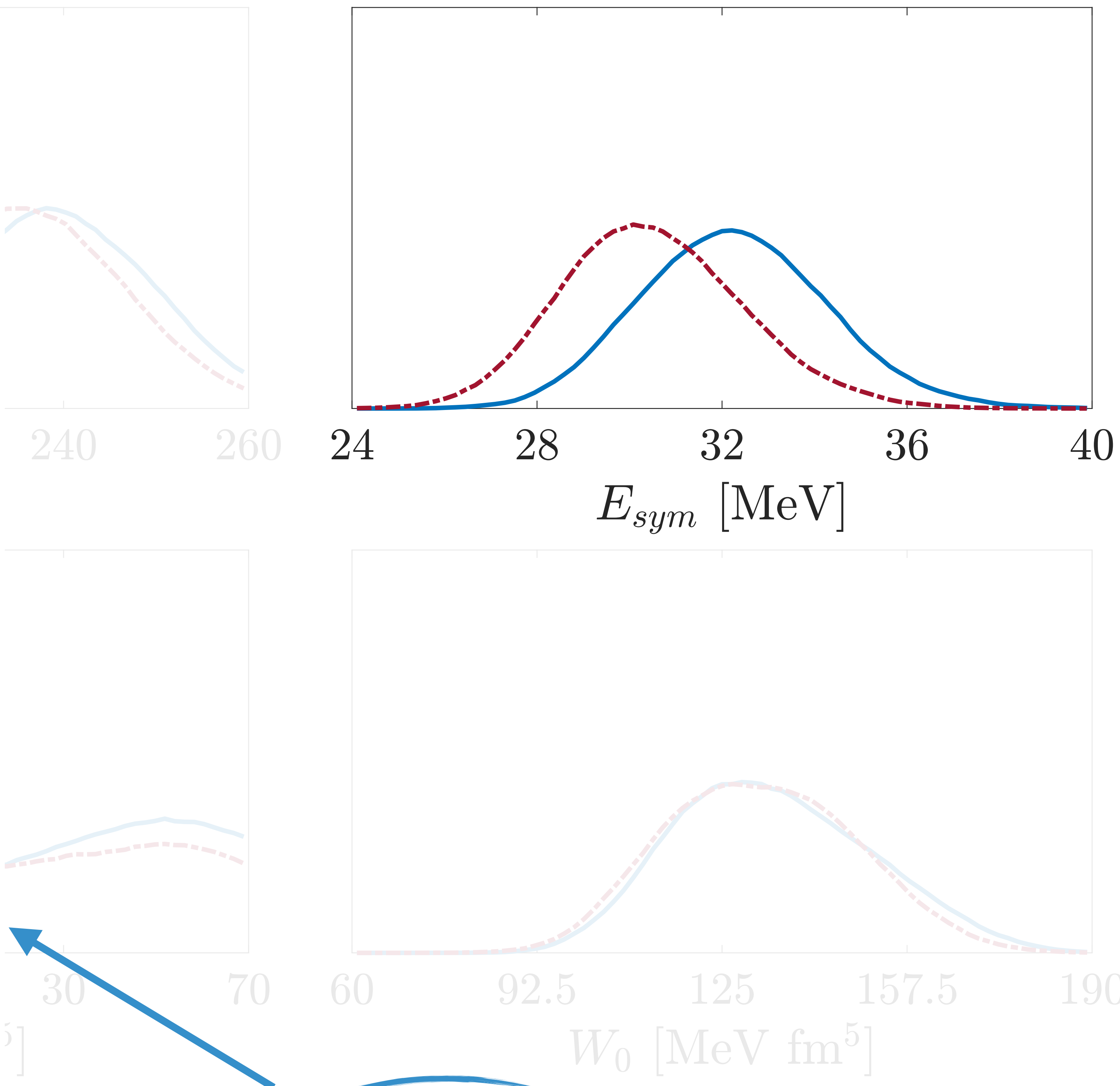
Fit Observables posterior (All data)

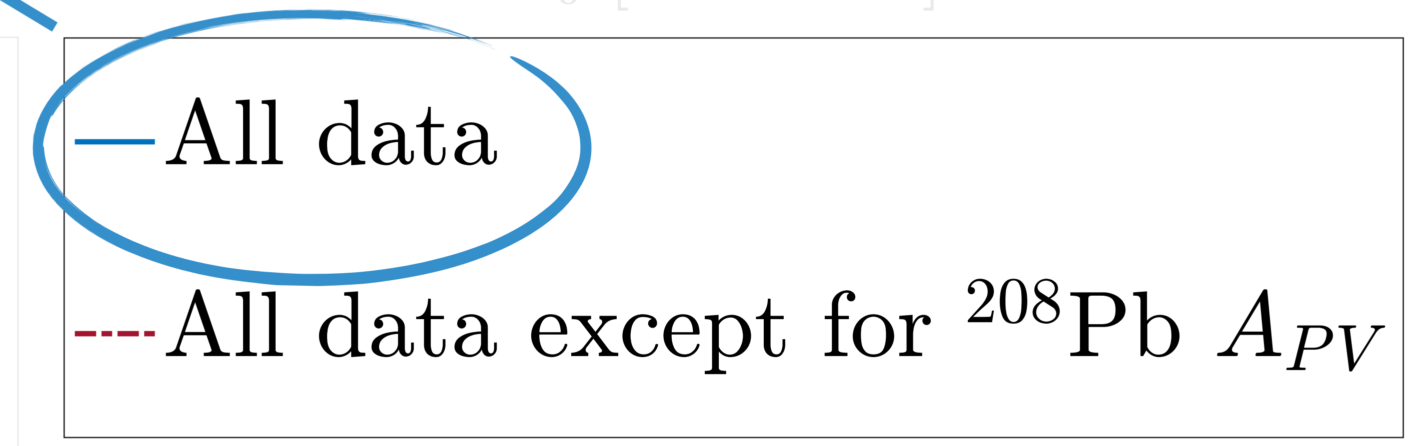
Ground-state properties			
	$B.E.$ [MeV]	$R_{\rm ch}$ [fm]	$\Delta E_{\rm SO}$ [MeV]
^{208}Pb	1637 ± 1.8	5.49 ± 0.03	2.41 ± 0.23
^{48}Ca	416 ± 1.0	3.50 ± 0.02	1.95 ± 0.21
^{68}Ni	589 ± 0.9	-	-
^{132}Sn	1101 ± 1.7	4.71 ± 0.02	-
^{90}Zr	784 ± 1.2	4.27 ± 0.02	-

Isoscalar resonances		
	$E^{\rm IS}_{\rm GMR}$ [MeV]	$E^{\rm IS}_{\rm GQR}$ [MeV]
^{208}Pb	14.0 ± 0.3	10.9 ± 0.4
^{90}Zr	18.4 ± 0.4	-

Isovector properties			
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} [p.p.b.]
^{208}Pb	19.9 ± 0.5	958 ± 22	579 ± 6
^{48}Ca	2.36 ± 0.08	-	2499 ± 57

Data from open shell nuclei			
	$B.E.$ [MeV]	$R_{\rm ch}$ [fm]	Δ_n [MeV]
^{50}Ca	428 ± 1.0	3.52 ± 0.02	-
^{46}Ca	400 ± 0.8	-	-
^{44}Ca	382 ± 0.9	-	-
^{42}Ca	362 ± 1.2	-	-
^{120}Sn	1020 ± 1.0	4.63 ± 0.02	1.26 ± 0.18
^{112}Sn	954 ± 1.6	-	-
^{124}Sn	1050 ± 0.9	-	-





	$A_{\rm PV}$ (ppb)
^{208}Pb	**528 ± 18**

… $L_{sym} \approx 160$ **MeV**

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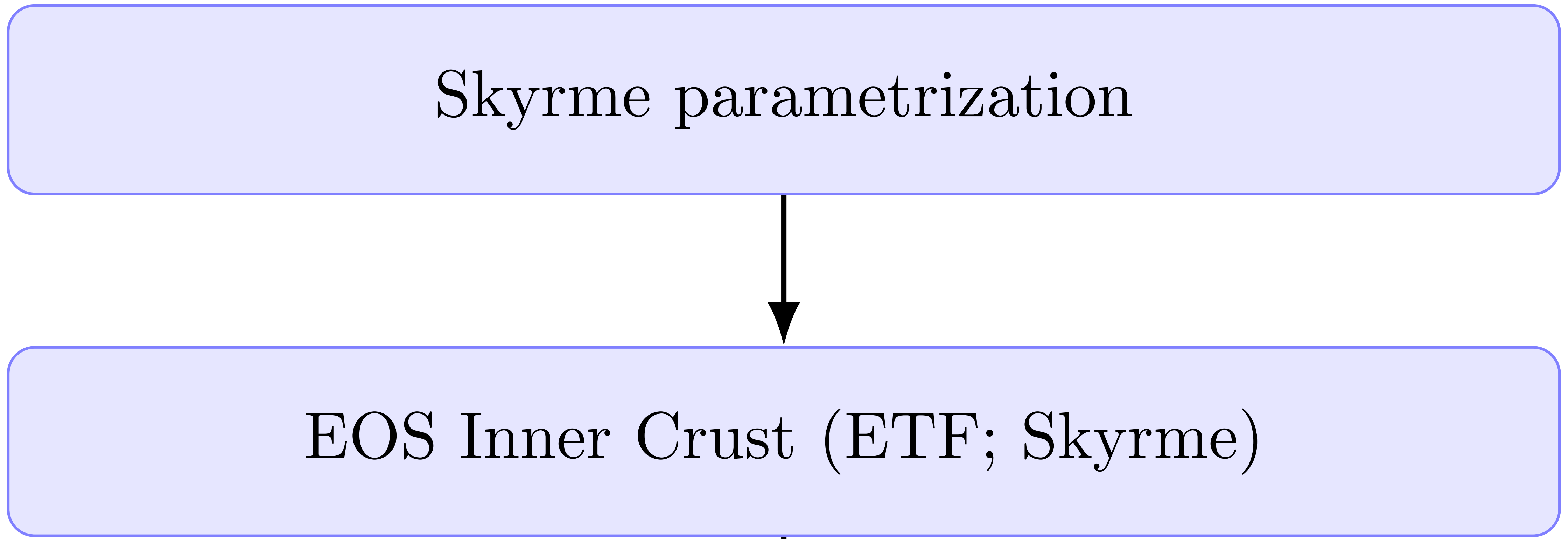
Computation of the NS EOS

Skyrme parametrization

Computation of the NS EOS



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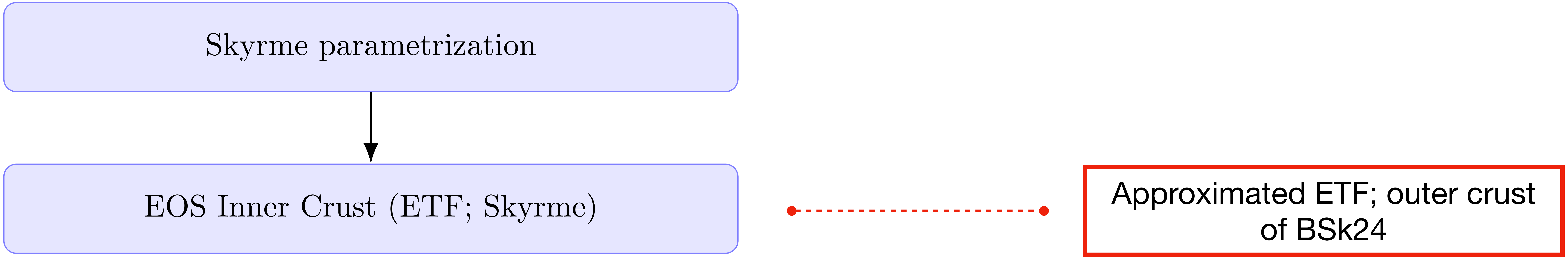

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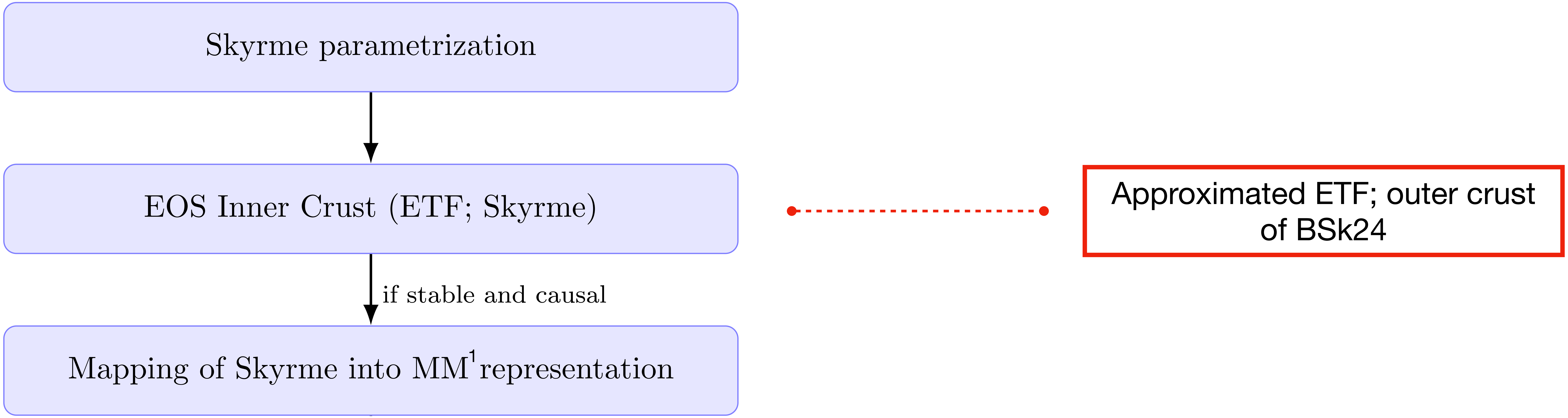
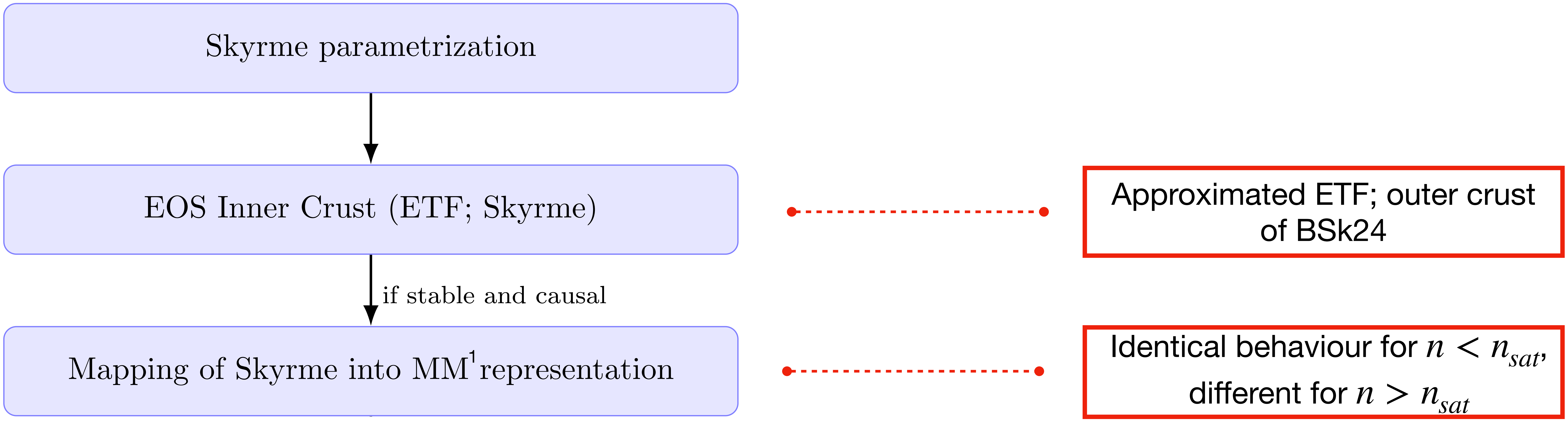


[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

Computation of the NS EOS



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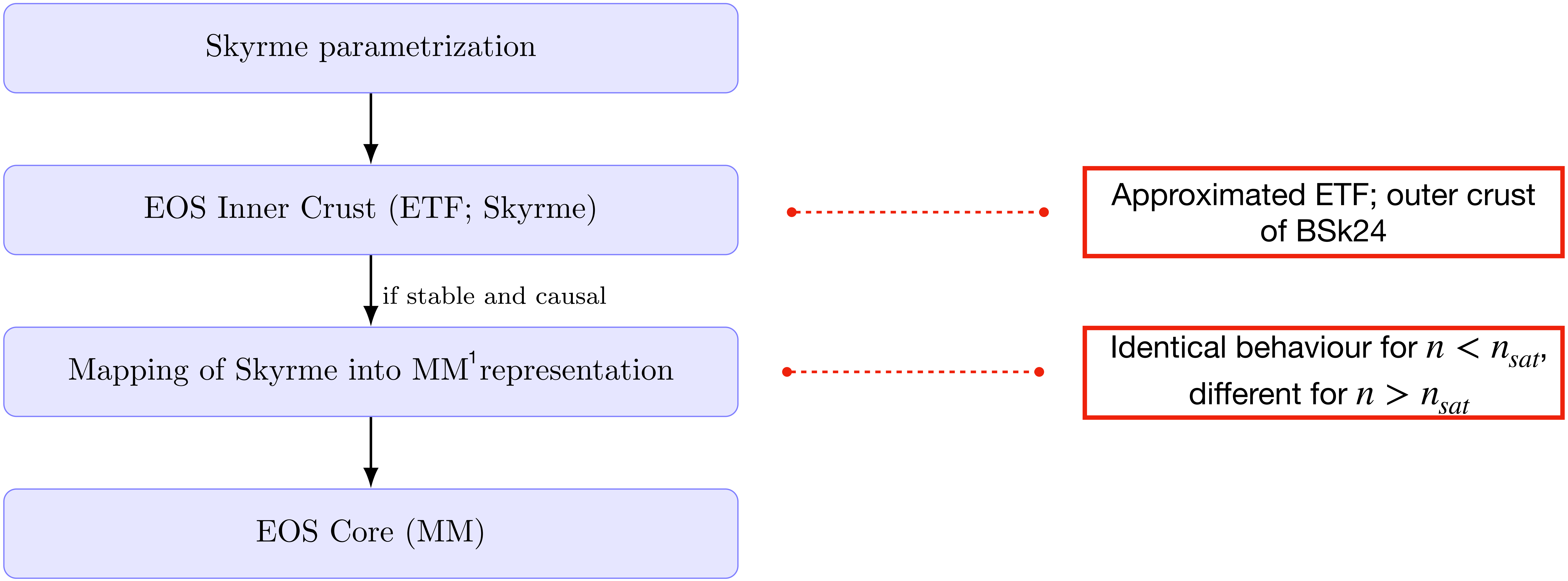

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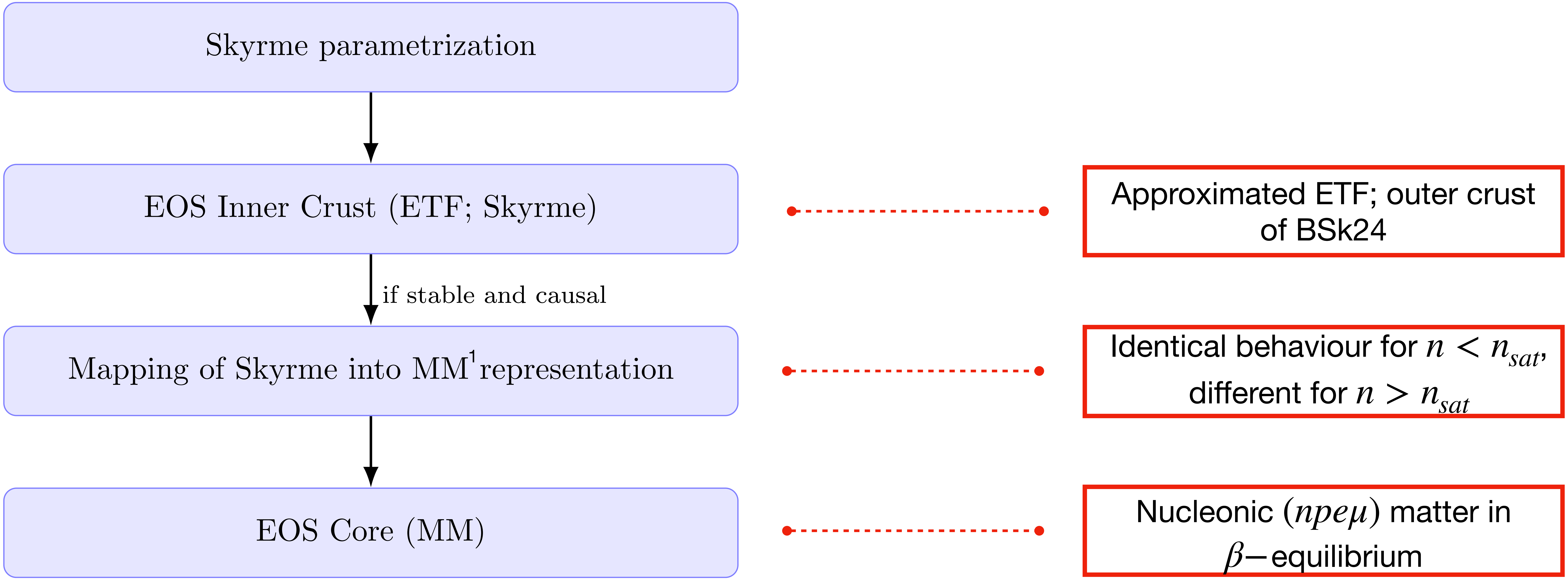

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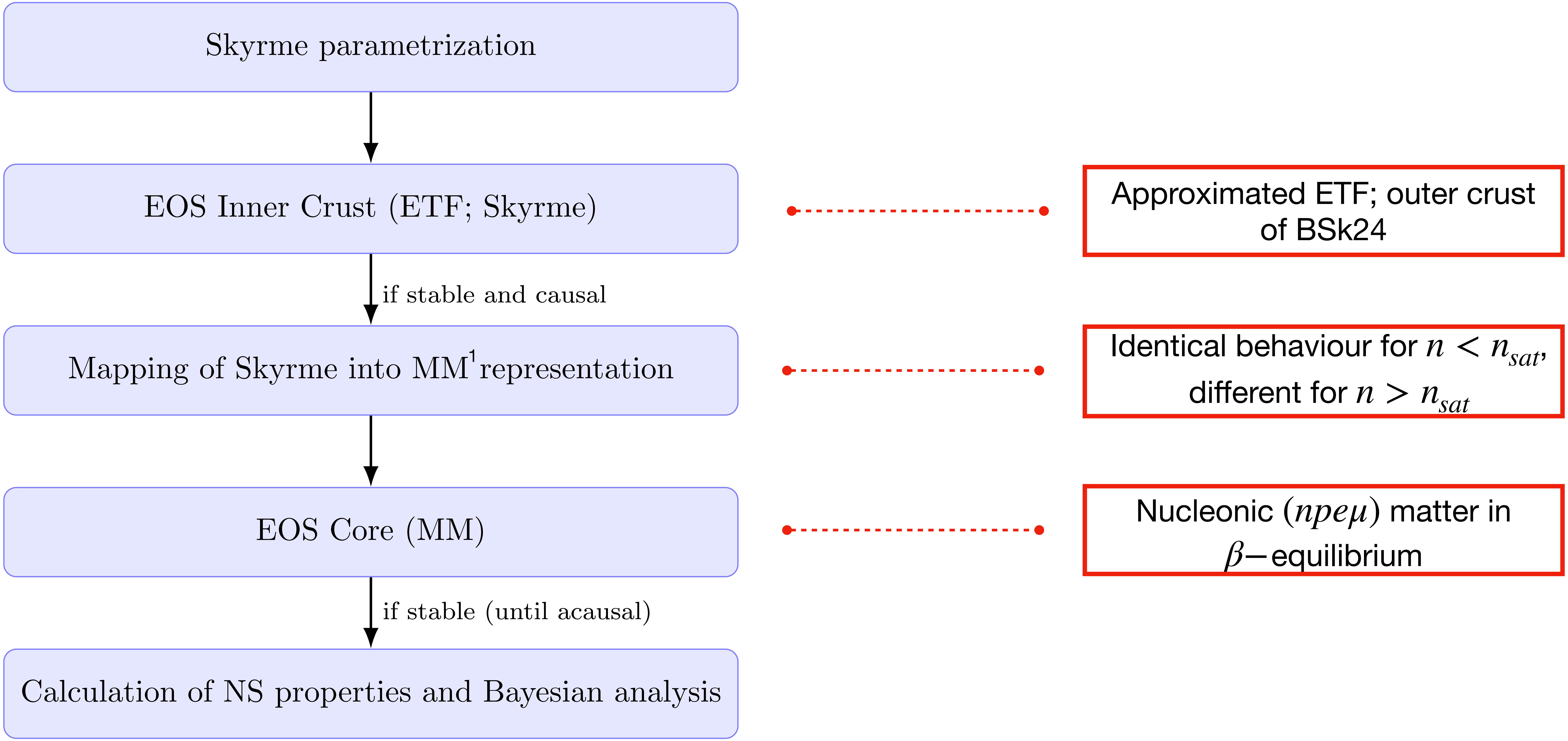

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Bayesian setup: prior and constraints

Prior distribution

E_{sat}	[MeV]	*
n_{sat}	[fm^{-3}]	*
K_{sat}	[MeV]	*
Q_{sat}	[MeV]	[-2000,2000]
Z_{sat}	[MeV]	[-3000,3000]
E_{sym}	[MeV]	*
L_{sym}	[MeV]	*
Q_{sym}	[MeV]	[-4000,4000]
Z_{sym}	[MeV]	[-5000,5000]
m^*_{IS}	[-]	*
m^*_{IV}	[-]	*
$w0$	[MeV fm^5]	*
G_0	[MeV fm^5]	*
G_1	[MeV fm^5]	*

Bayesian setup: prior and constraints

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$w0$	[MeV fm^5]	*
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Observational constraints

- Maximum mass of Neutron Star ($\mathscr{L}_{\mathrm{J0348}}$);
- Tidal deformability results ($\mathscr{L}_{\mathrm{LVC}}$);
- NICER mission mass-radius measurements ($\mathscr{L}_{\mathrm{NICER}}$);
- $\chi-$EFT computations of PNM at low density ($\mathscr{L}_{\chi}$).

Bayesian setup: prior and constraints

Prior distribution

E_{sat}	[MeV]	*
n_{sat}	[fm^{-3}]	*
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m^*_{IS}	[-]	*
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$w0$	[MeV fm^5]	*
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- $\chi-$EFT computations of PNM at low density ($\mathscr{L}_{\chi}$).

Prior distribution:

10^5 extractions from nuclear posterior

↓

NS EOS computation

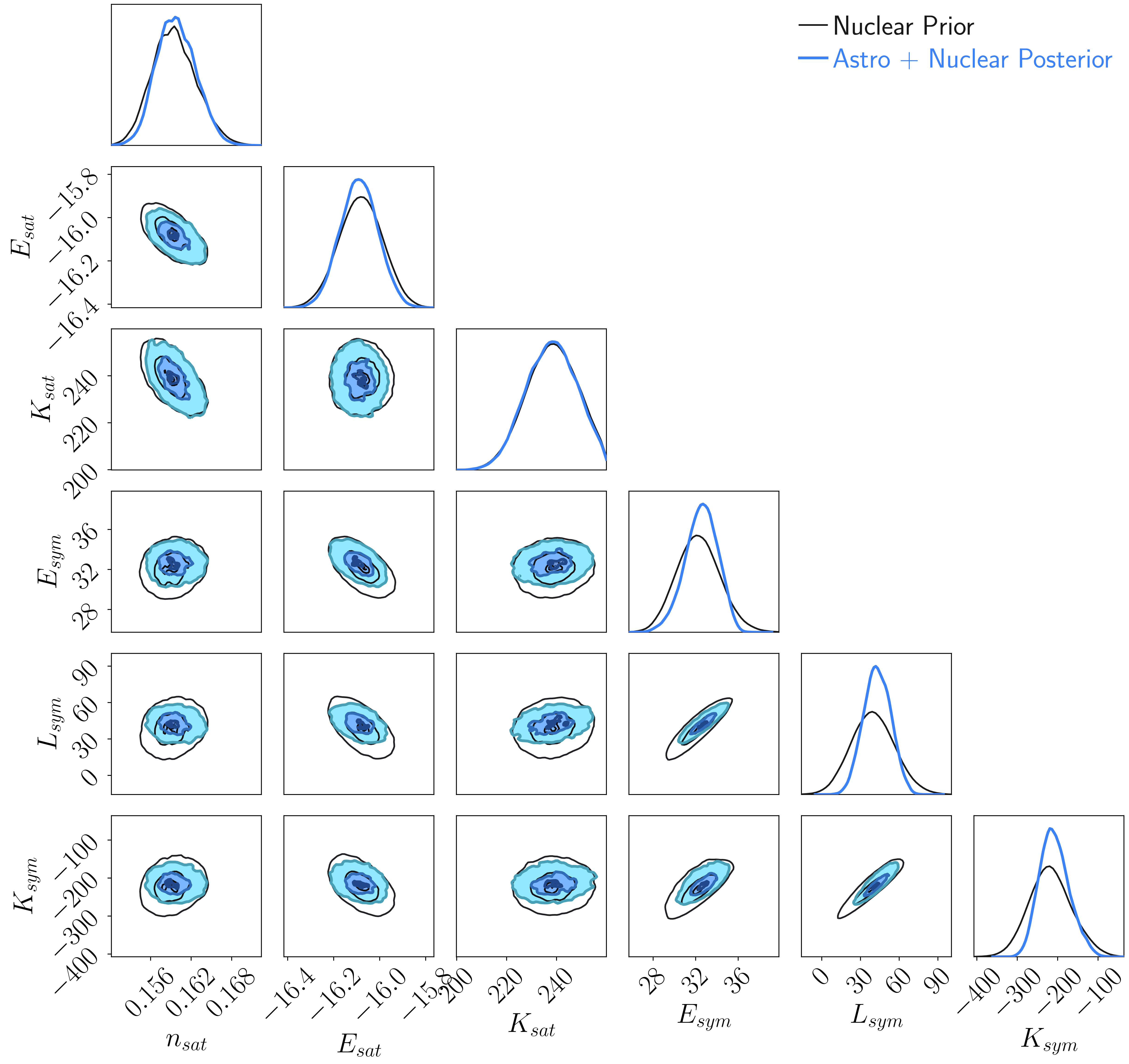
↓

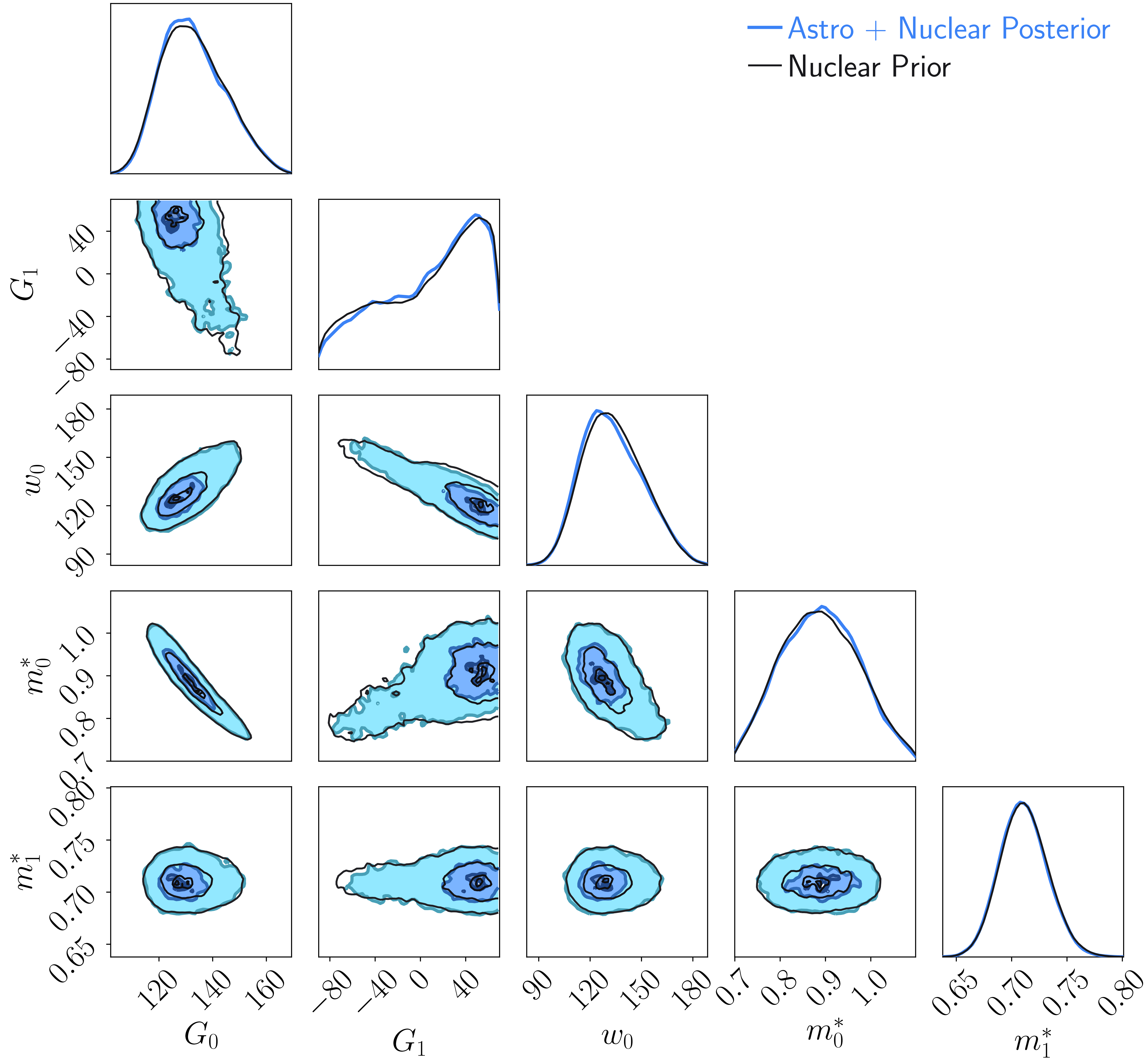
Posterior distribution:

Prior distribution weighted with

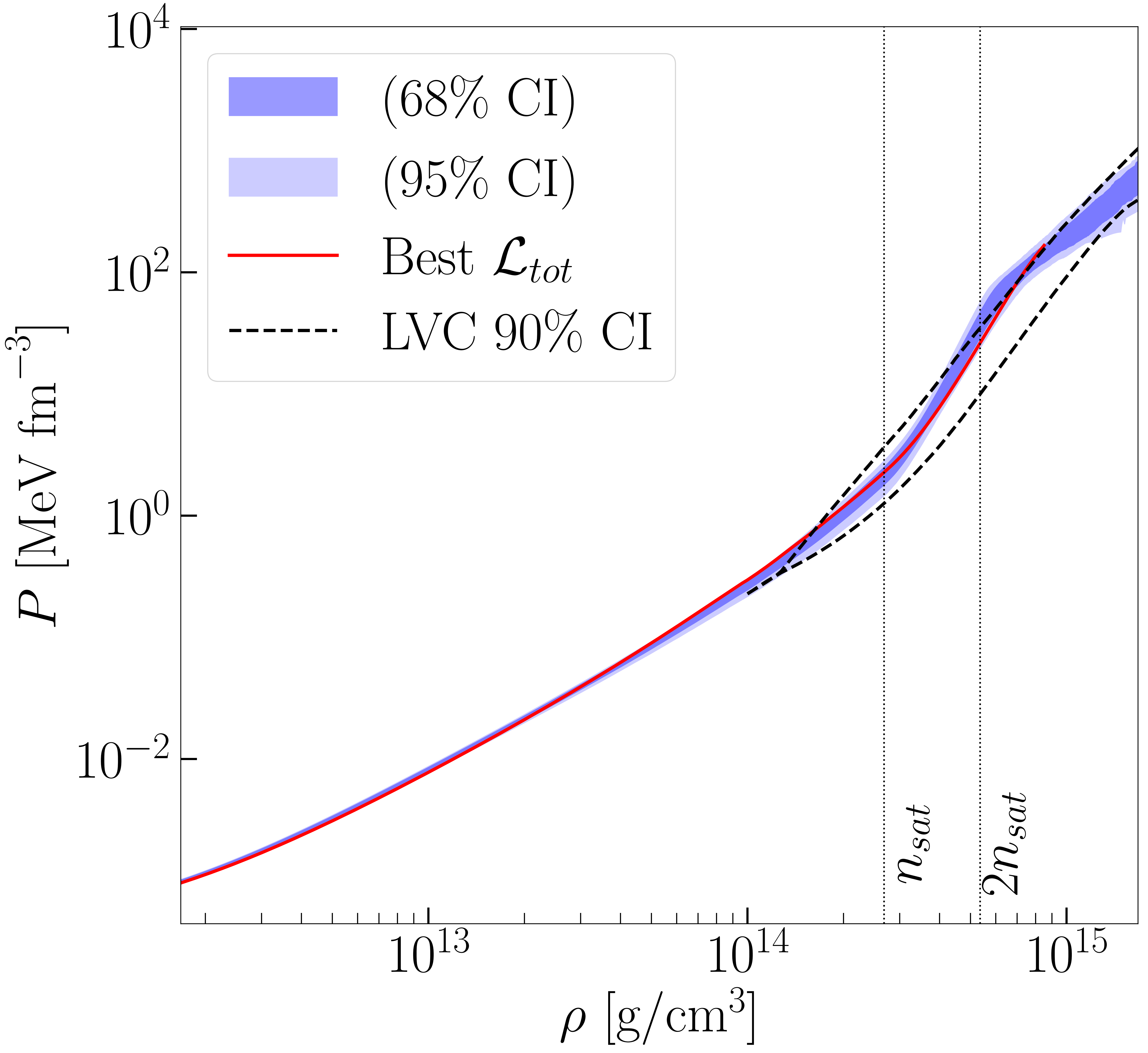
$$\mathscr{L}_{tot} = \prod_i \mathscr{L}_i$$

Corner plots: prior vs posterior





Equation of State



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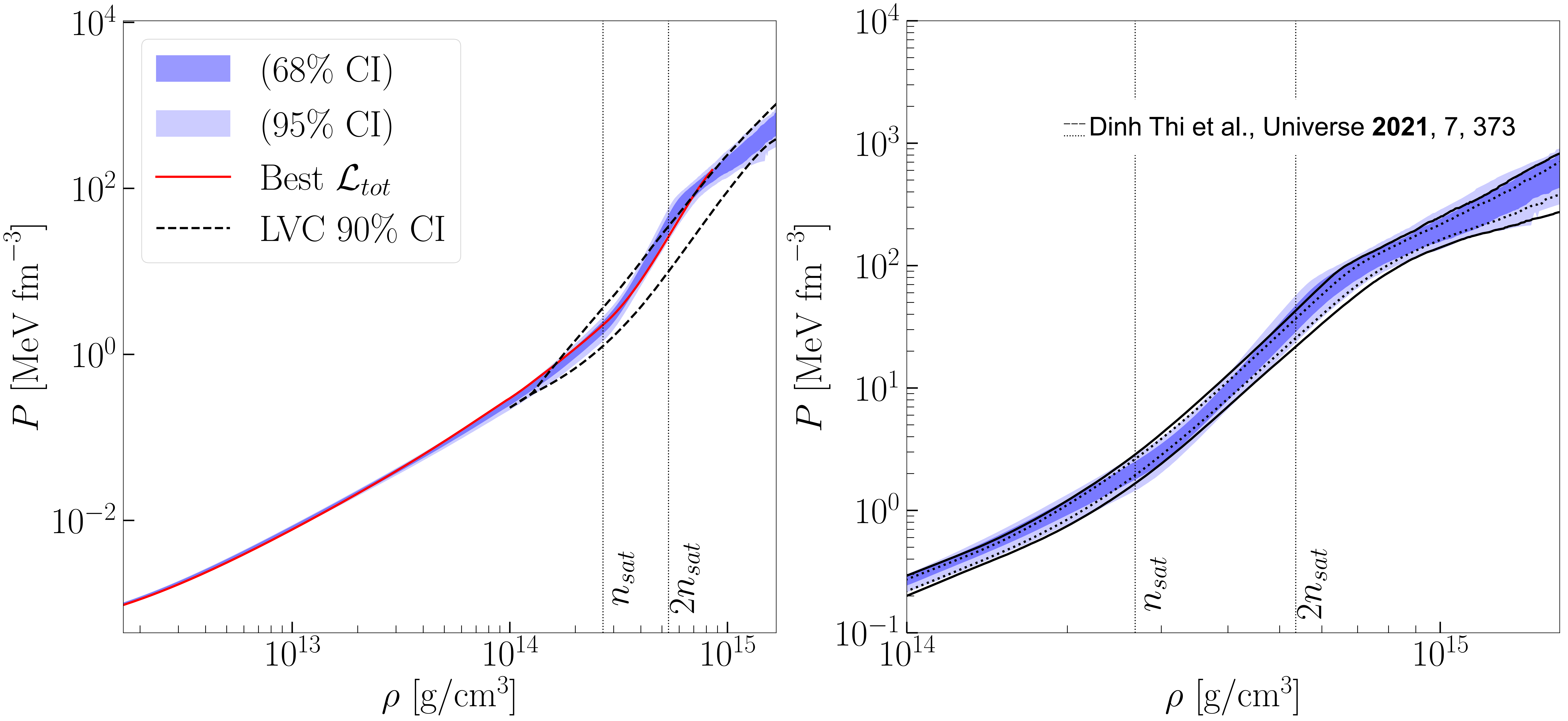
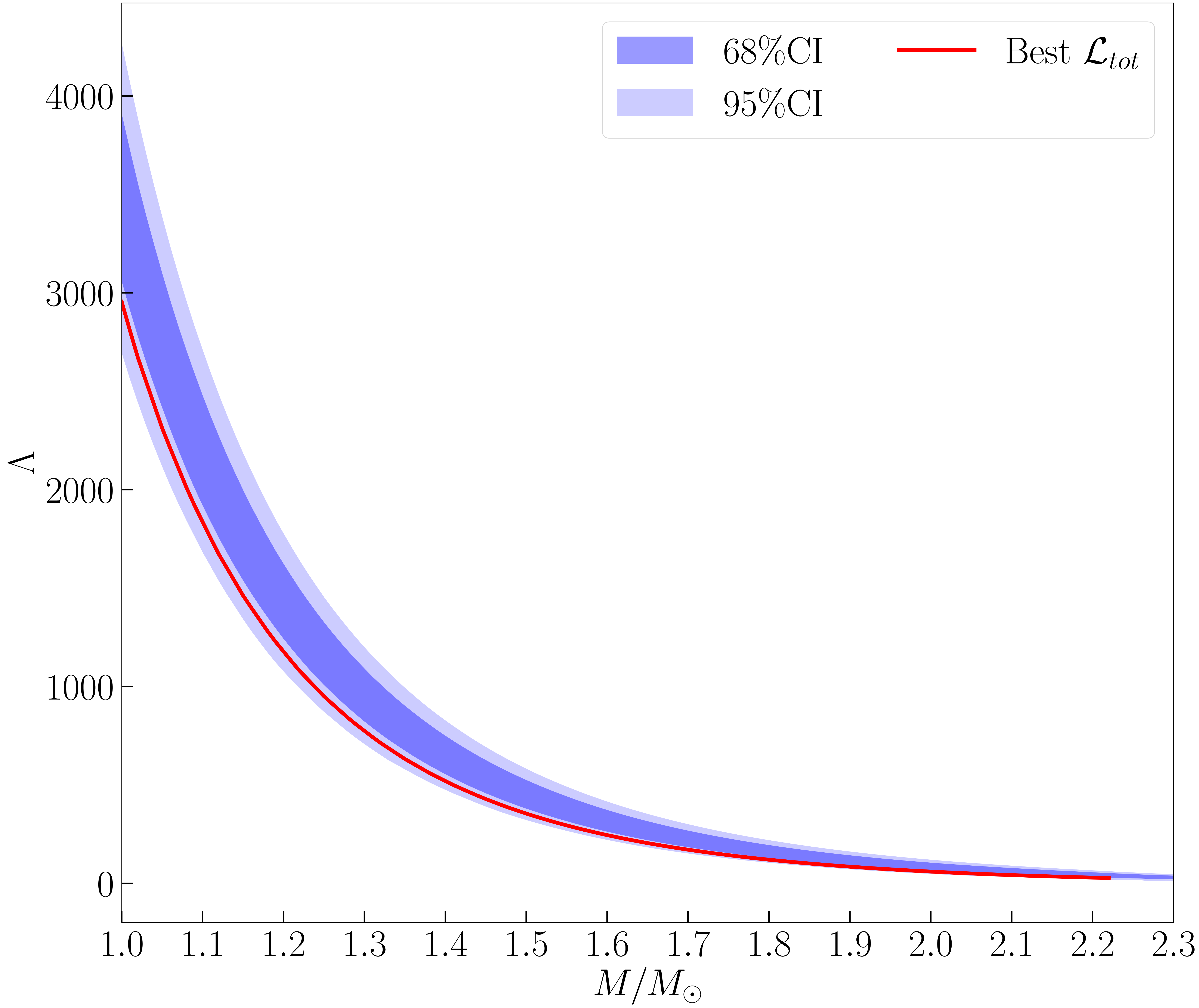
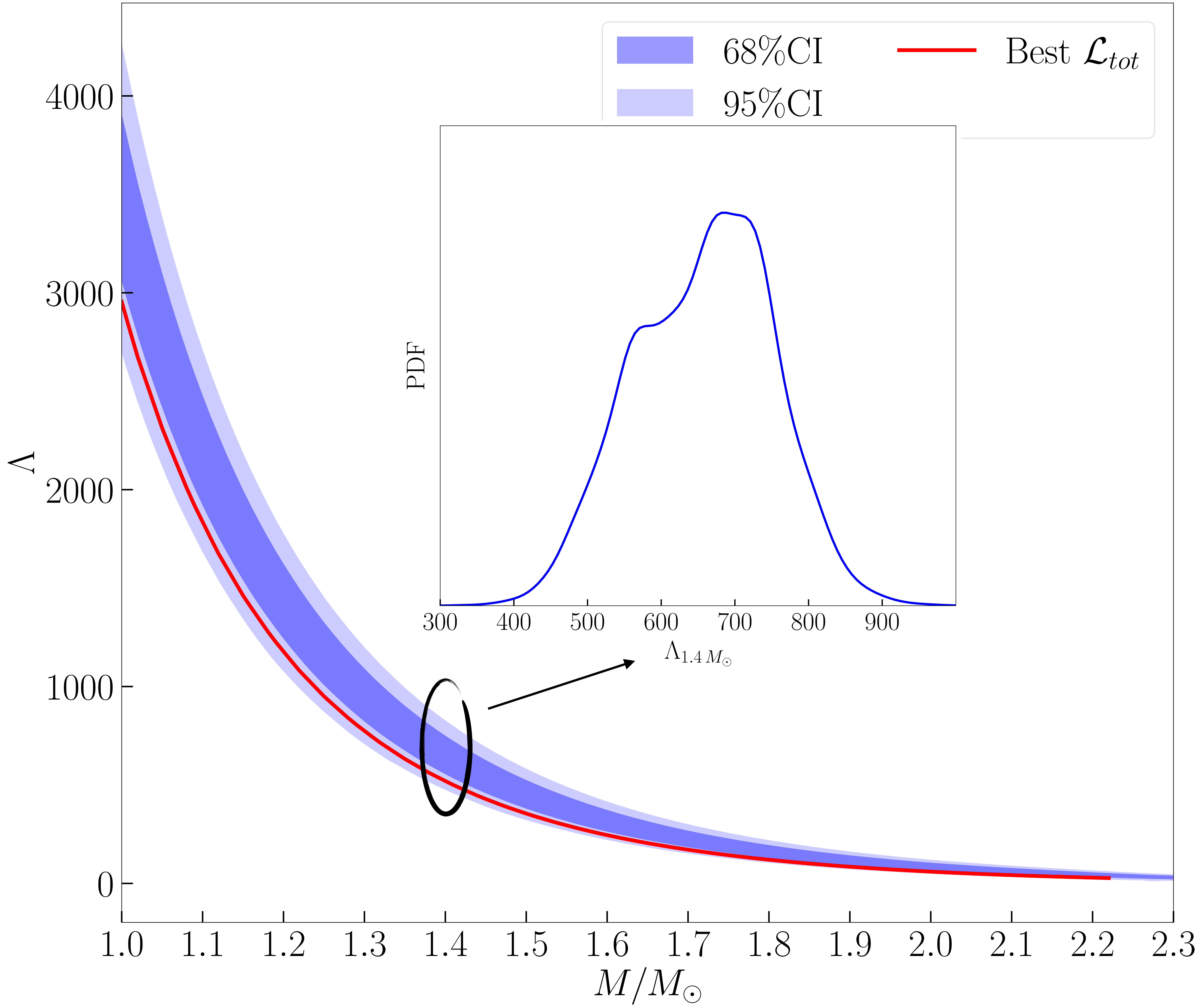


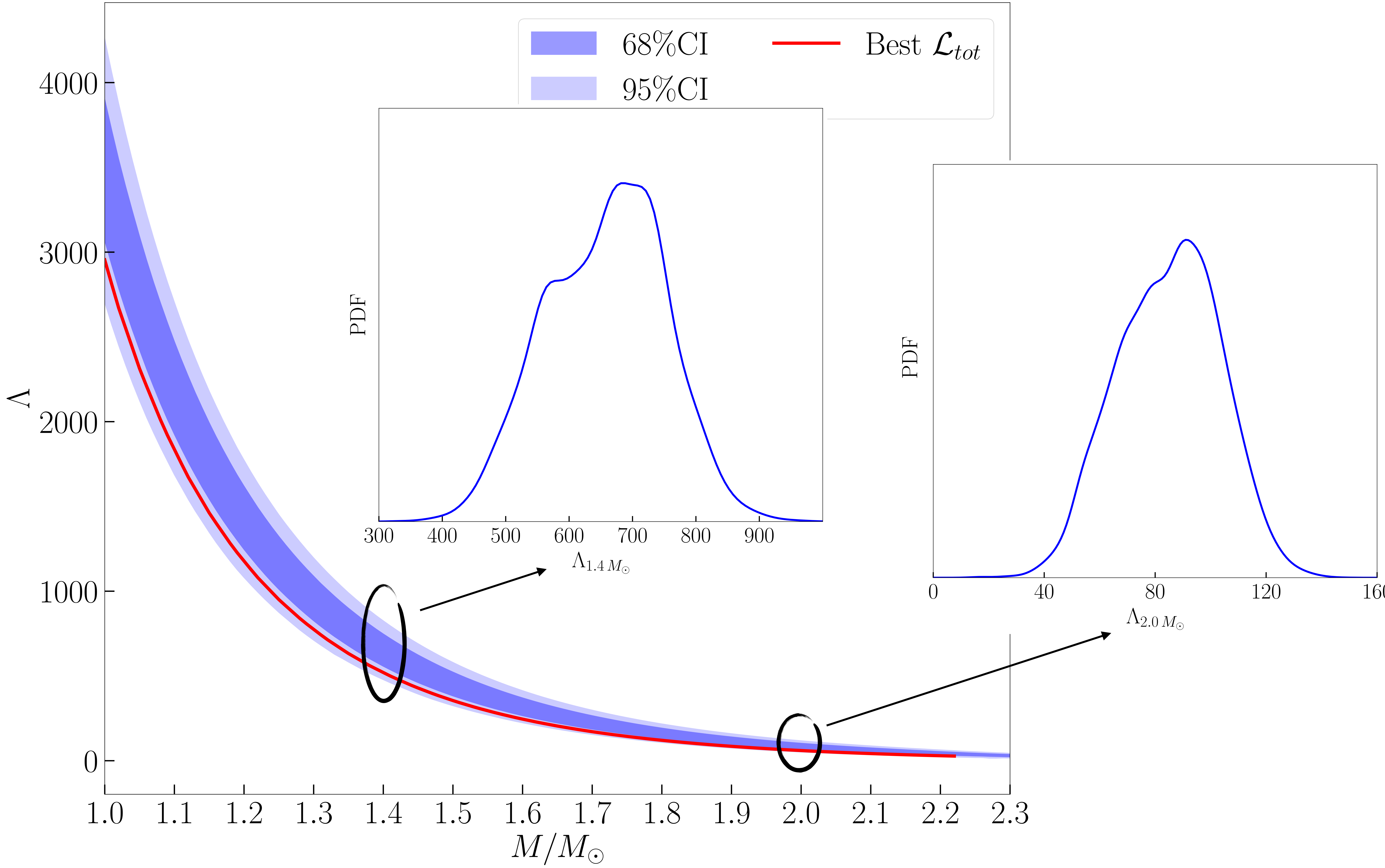
Tidal deformability



Tidal deformability



Tidal deformability



Conclusions

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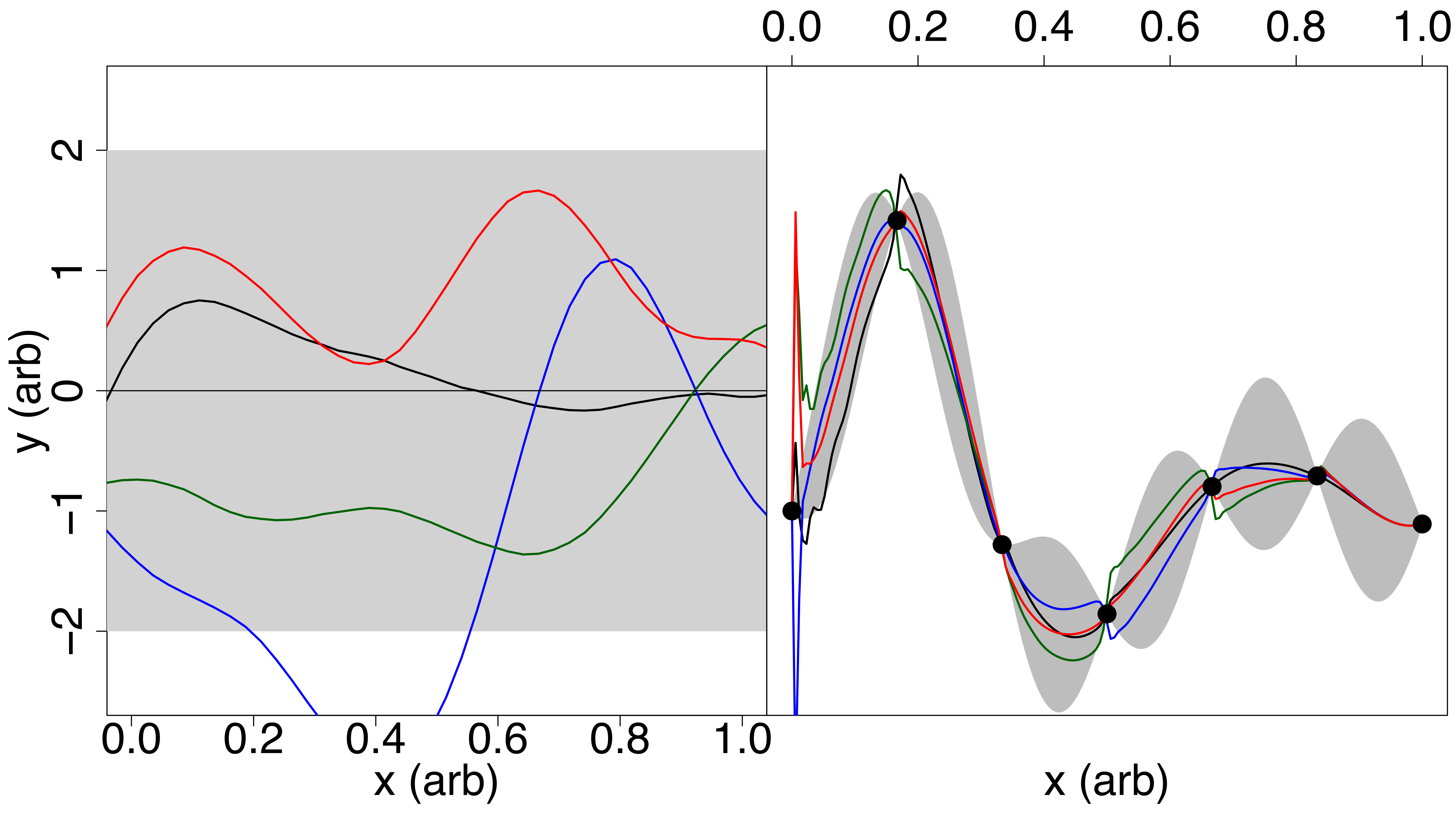
  - Effect on structure of the P between n_{sat} and $2n_{sat}$ due to nuclear informed prior

Thank you for your attention!

Gaussian process (GP) emulator

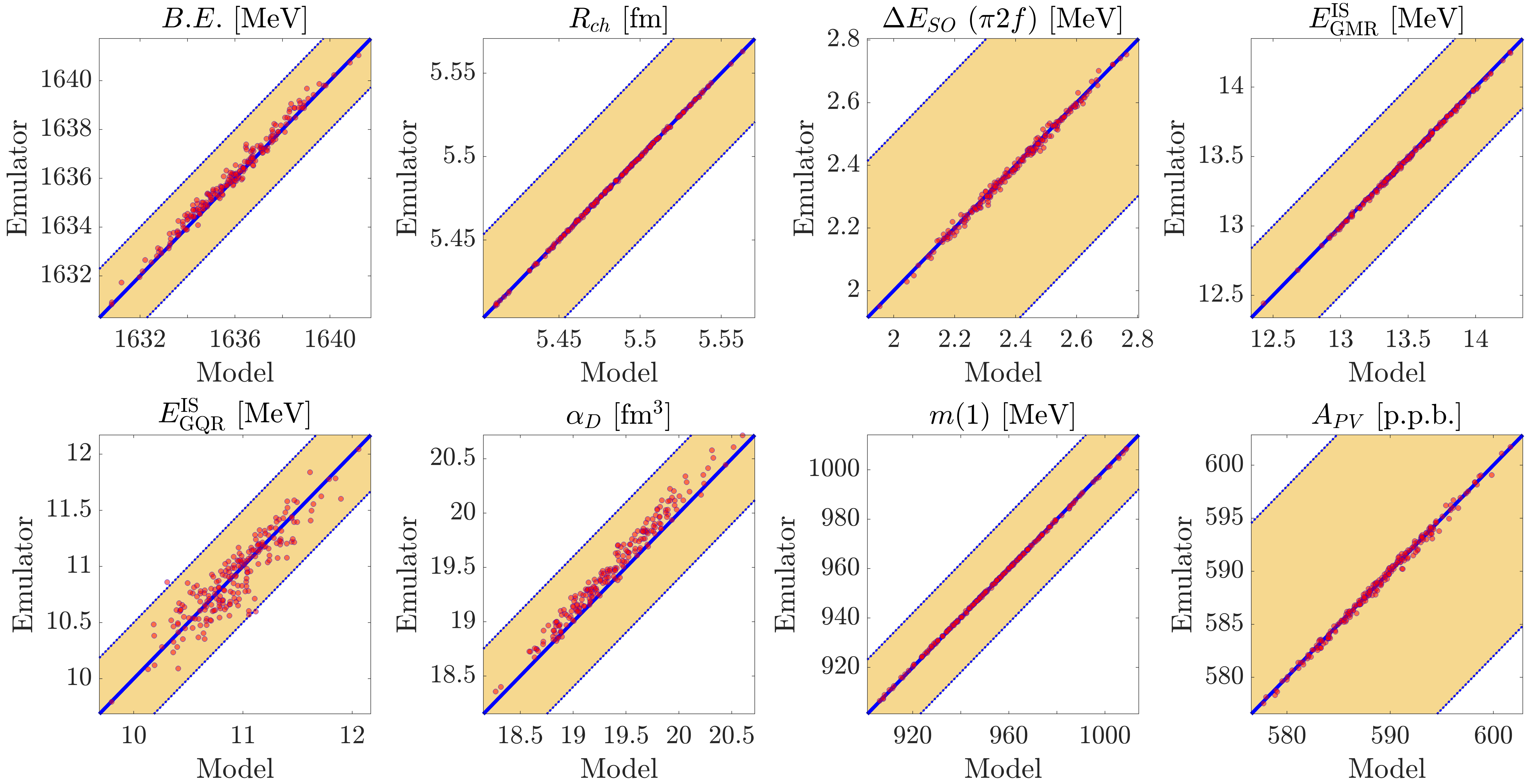


From MADAI user manual

The MADAI package:

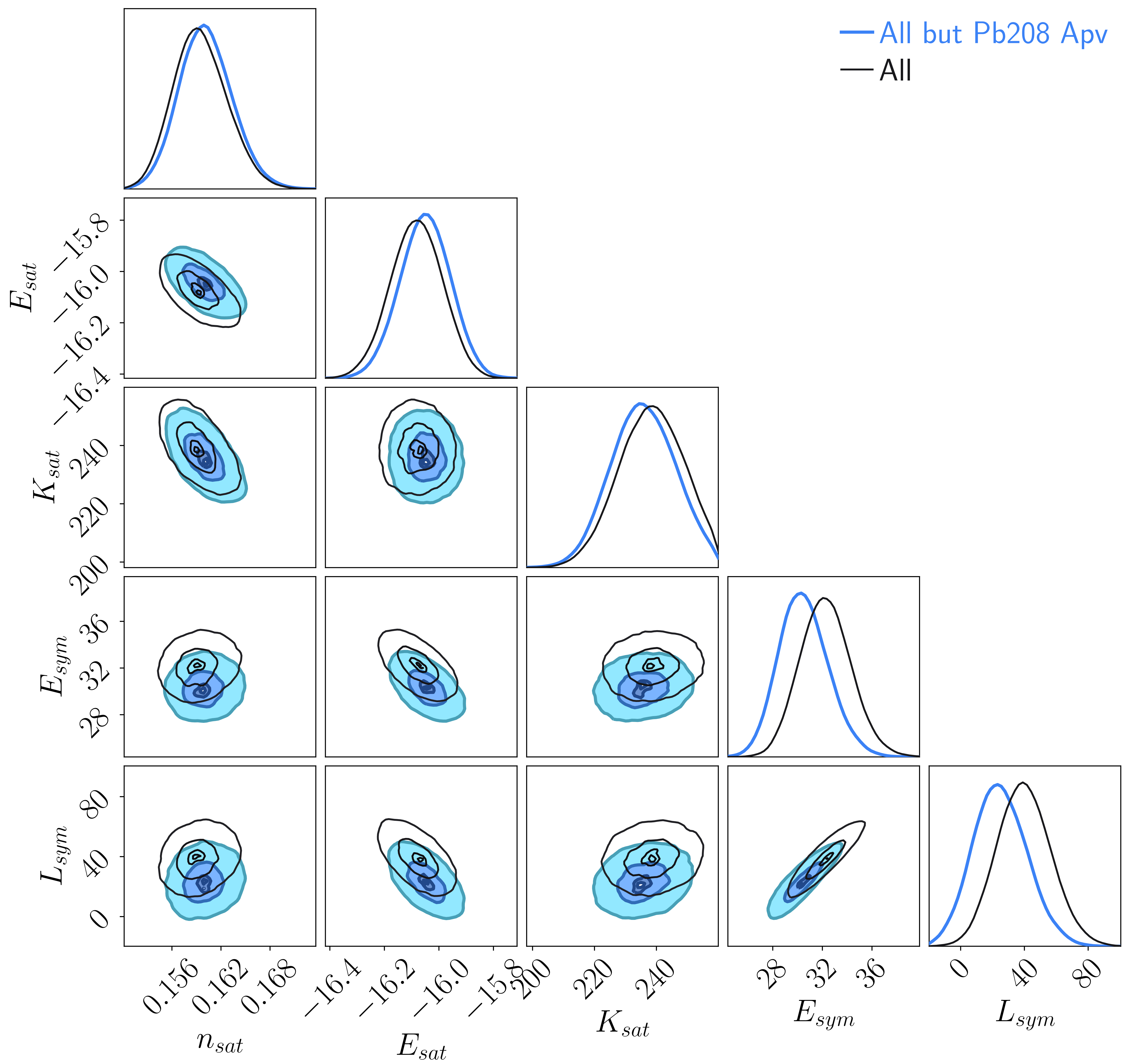
- was built for GP applied to bayesian inference
- given the parameters prior distributions, it automatically builds the grid
- it does a MCMC to estimate the posterior distribution
- it extracts parameters sample following the posteriors

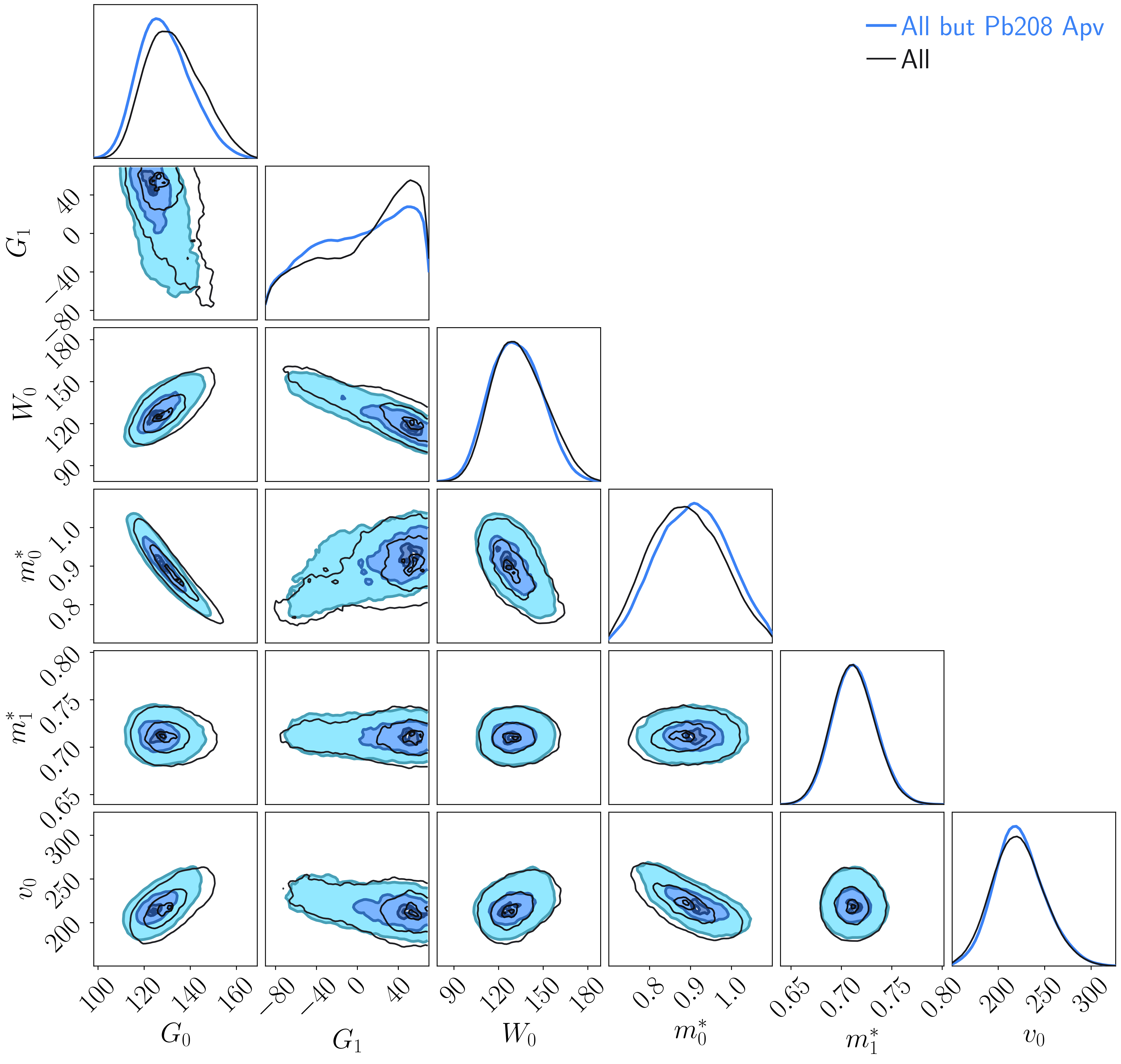
Validation



B2

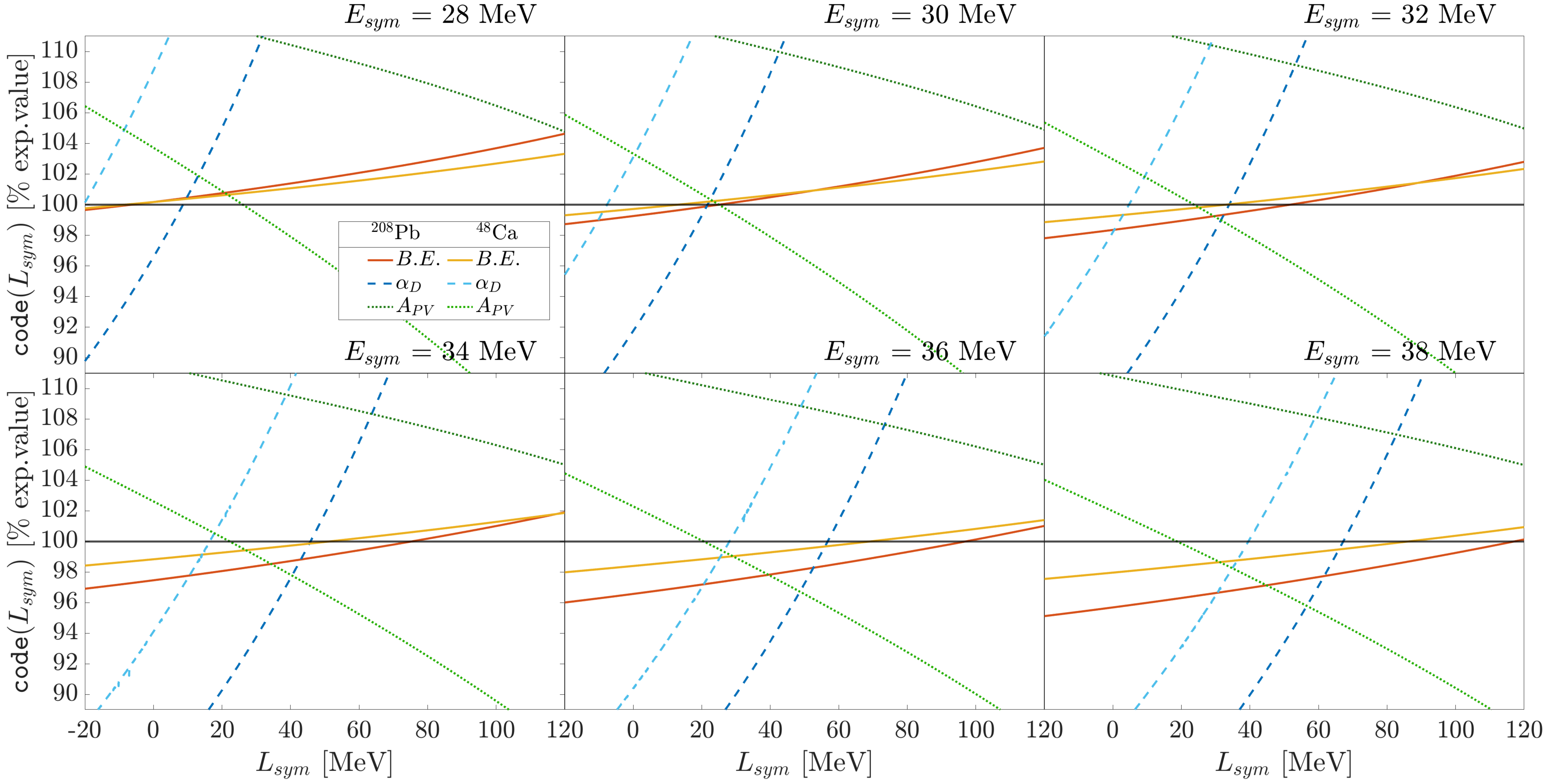
Corner plots





Why is L_{sym} so small?

- L_{sym} only free parameter
- E_{sym} fixed to (28,…,38) MeV
- Other parameters fixed at best log(Likelihood) values



B4

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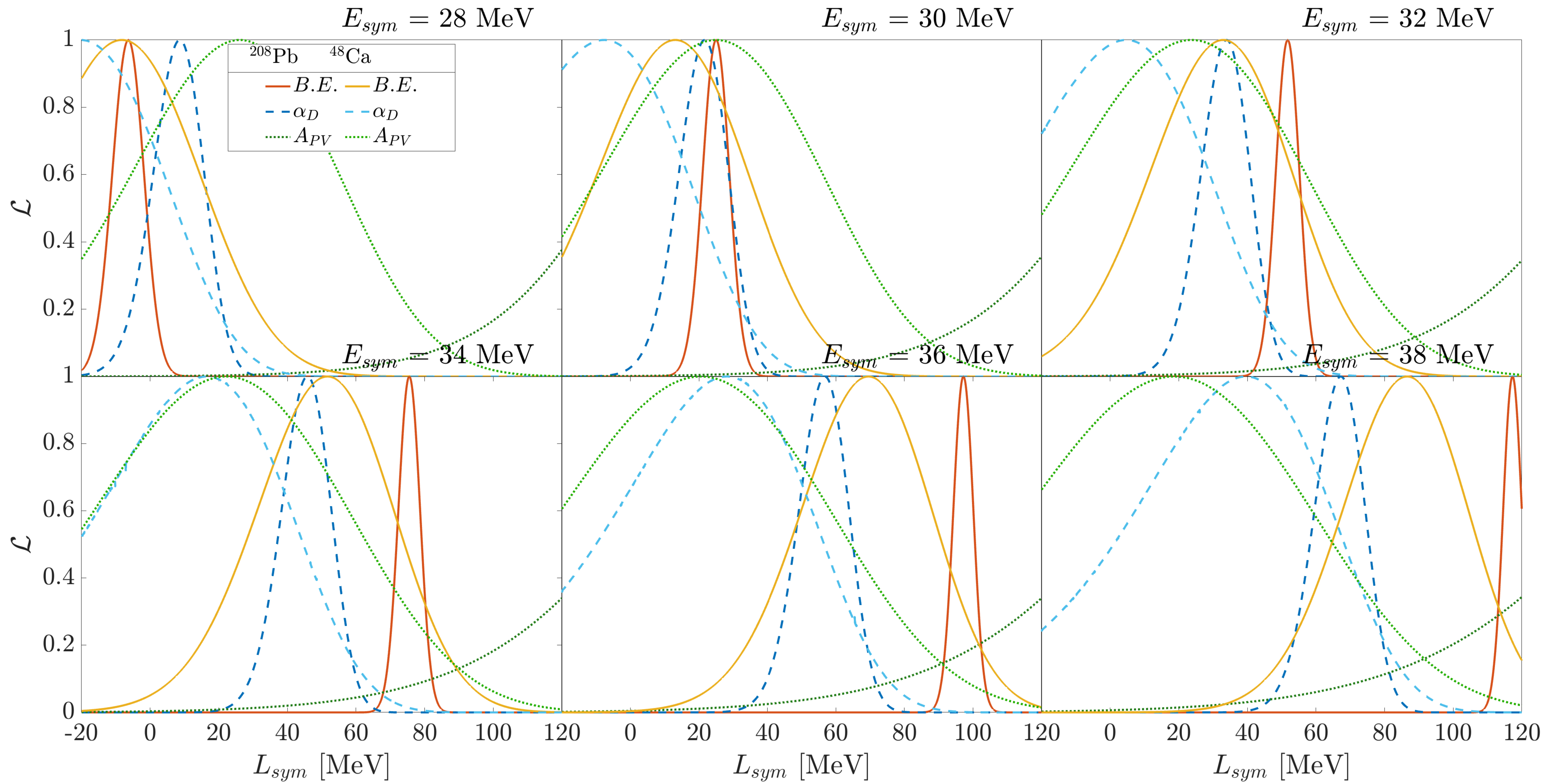
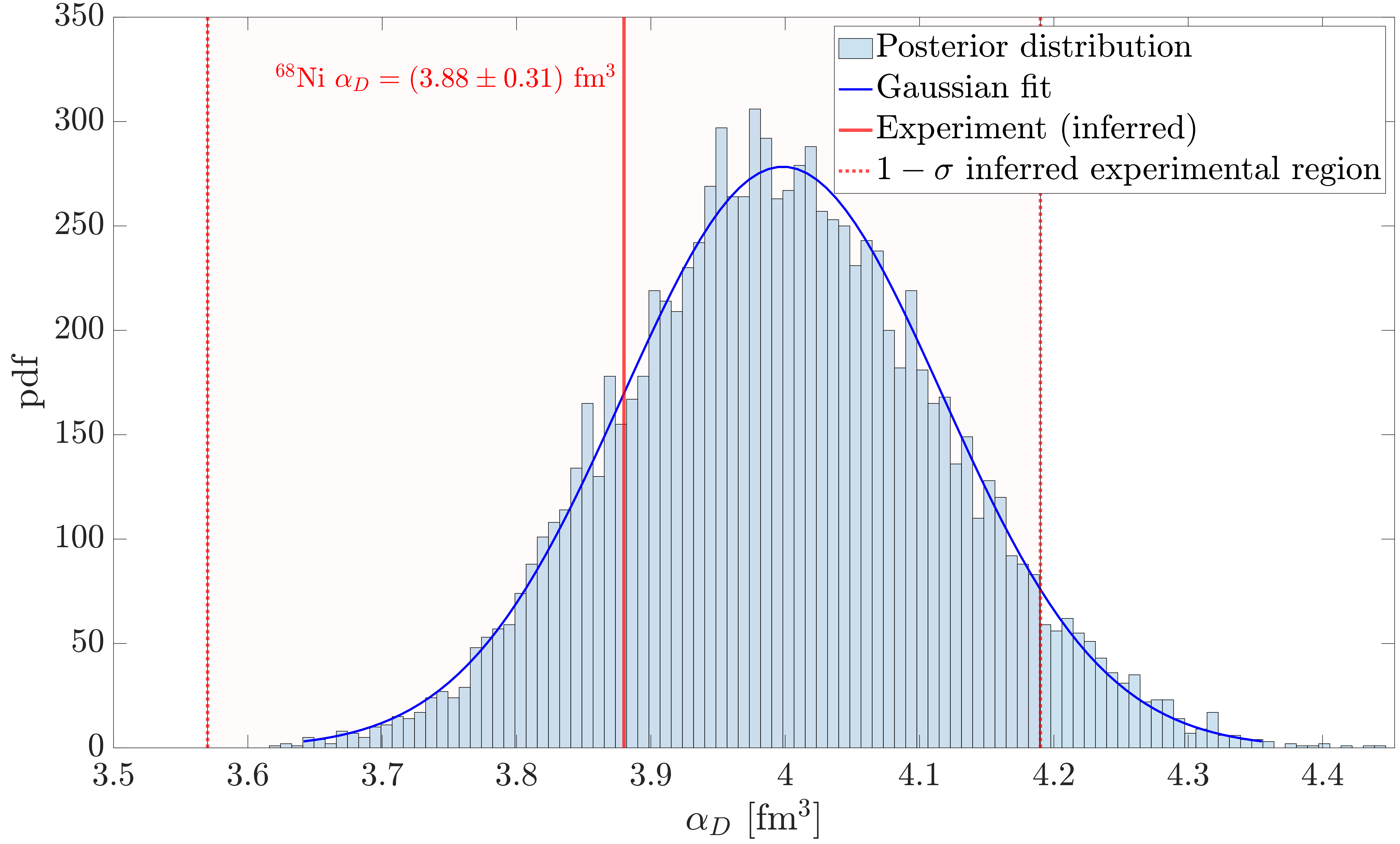


^{68}Ni α_D posterior distribution



NS EOS computation: Mapping of Skyrme into M.M.

Skyrme's parameters[1]

$$n_{sat}, E_{sat}, K_{sat}$$

$$E_{sym}, L_{sym}$$

$$G_0, G_1, W_0, m_0^*/m, m_1^*/m$$

M.M.'s parameters

$$n_{sat}, E_{sat}, K_{sat}, Q_{sat}, Z_{sat}$$

$$E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym}$$

$$m_0^*/m, m_1^*/m$$

[1] 1-to-1 correspondence with usual Skyrme's parameters (L.-W. Chen et al. Phys. Rev. C 80, 014322 (2009))

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$$K_{sym} = K_{sym}(n_{sat}, E_{sat}, K_{sat}, \ldots)$$

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M.M.'s parameters

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$E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym}, Q^*_{sym}, Z^*_{sym}$

$m_0^*/m, m_1^*/m$

Skyrme's formula $n < n_{sat}$

$$Q_{sat} = Q_{sat}(n_{sat}, E_{sat}, \ldots) \qquad Z_{sat} = Z_{sat}(n_{sat}, E_{sat}, \ldots)$$
$$Q_{sym} = Q_{sym}(n_{sat}, E_{sat}, \ldots) \qquad Z_{sym} = Z_{sym}(n_{sat}, E_{sat}, \ldots)$$

Randomly extracted $n > n_{sat}$

$Q^*_{sat,sym}, Z^*_{sat,sym}$

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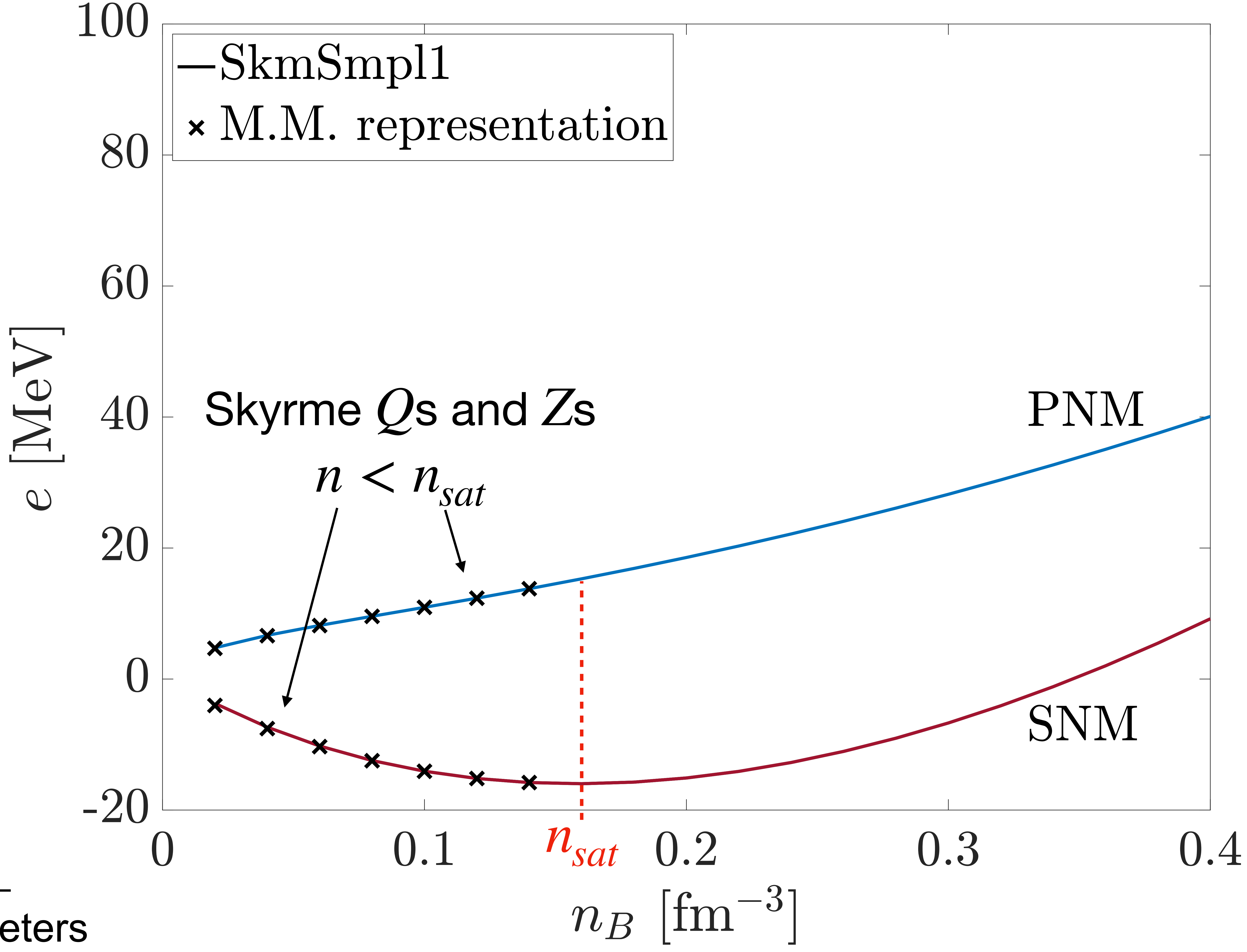

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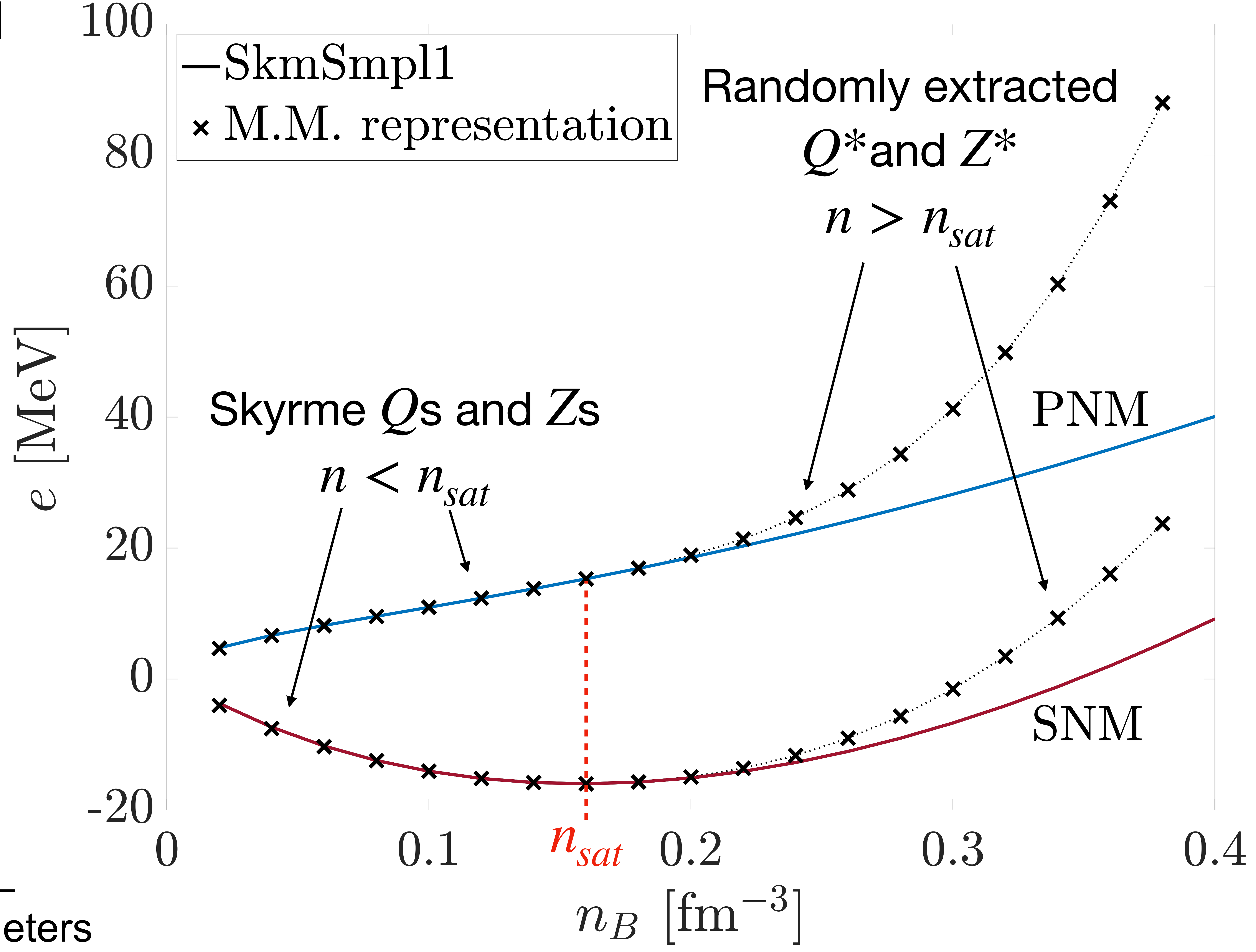

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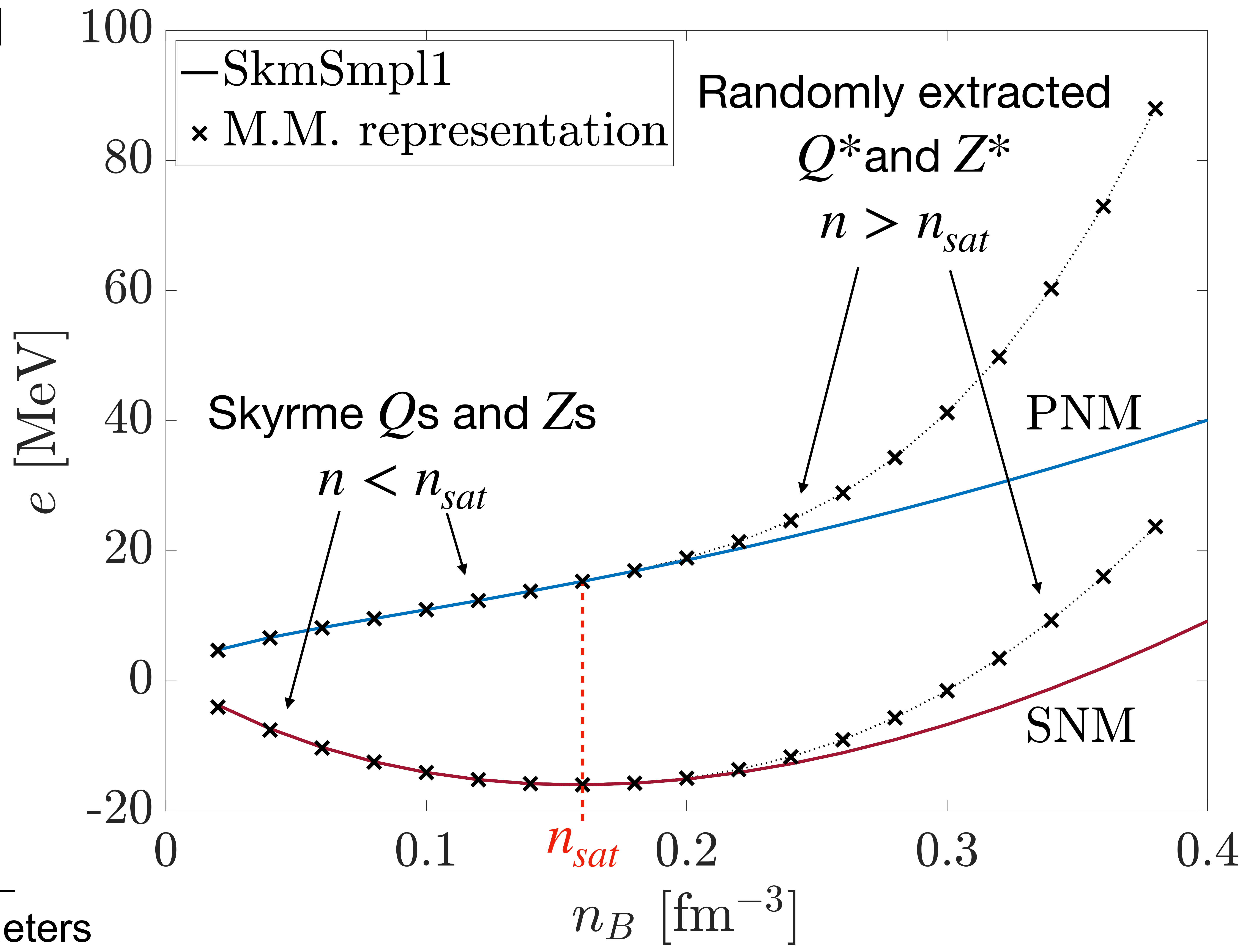

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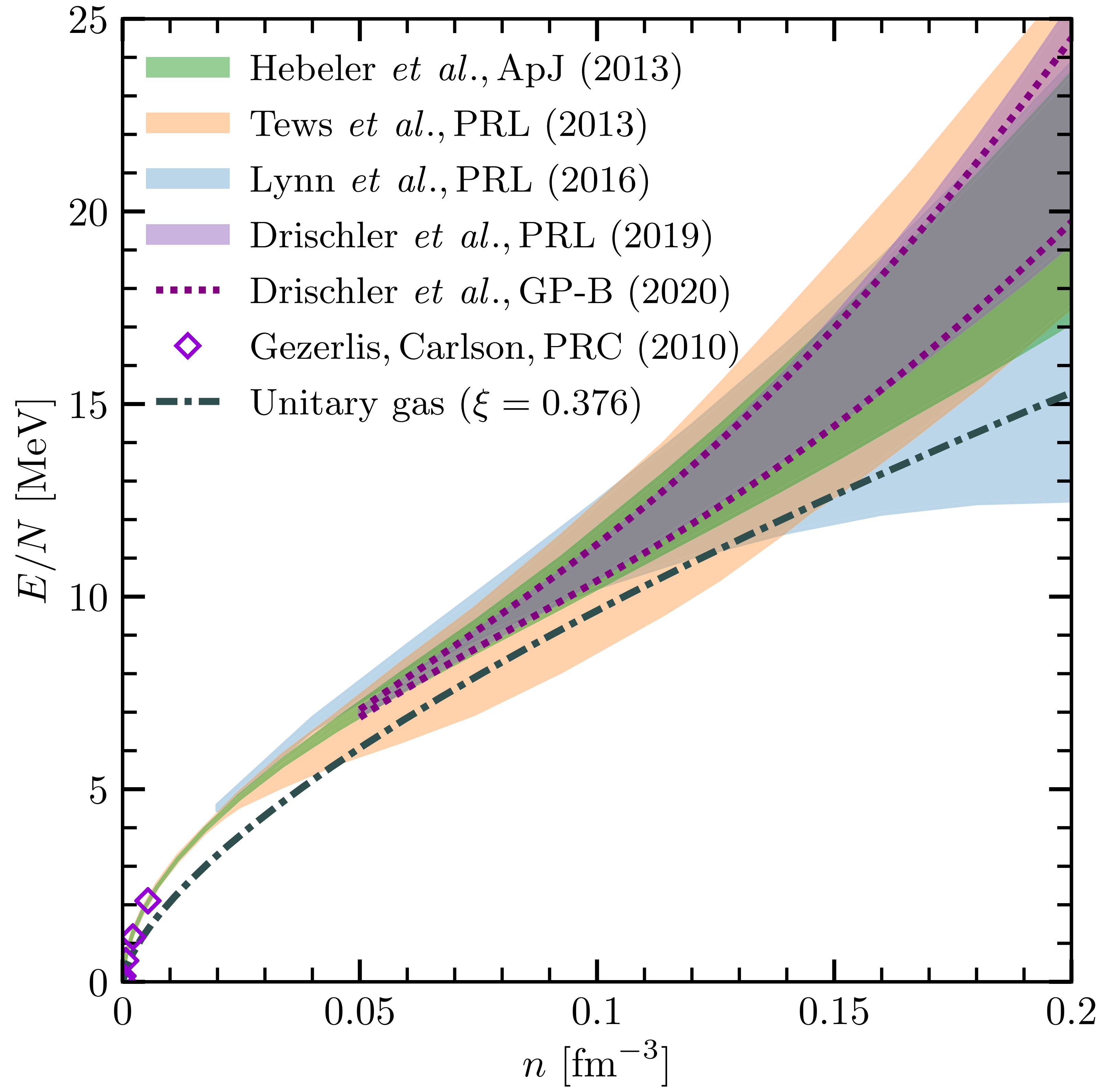
Likelihoods

$$\mathscr{L}_\chi : \quad p(e \mid n) = \begin{cases} \exp\left(-\frac{\left(e - e_-(n)\right)^2}{2\sigma_n^2}\right) & \textbf{if } e \in \left(-\infty, e_-(n)\right] \\ 1 & \textbf{if } e \in \left(e_-(n), e_+(n)\right] \\ \exp\left(-\frac{\left(e - e_+(n)\right)^2}{2\sigma_n^2}\right) & \textbf{if } e \in \left(e_+(n), \infty\right) \end{cases}$$

$$\sigma_n = \frac{e_+(n) - e_-(n)}{9\sqrt{2\pi}}$$

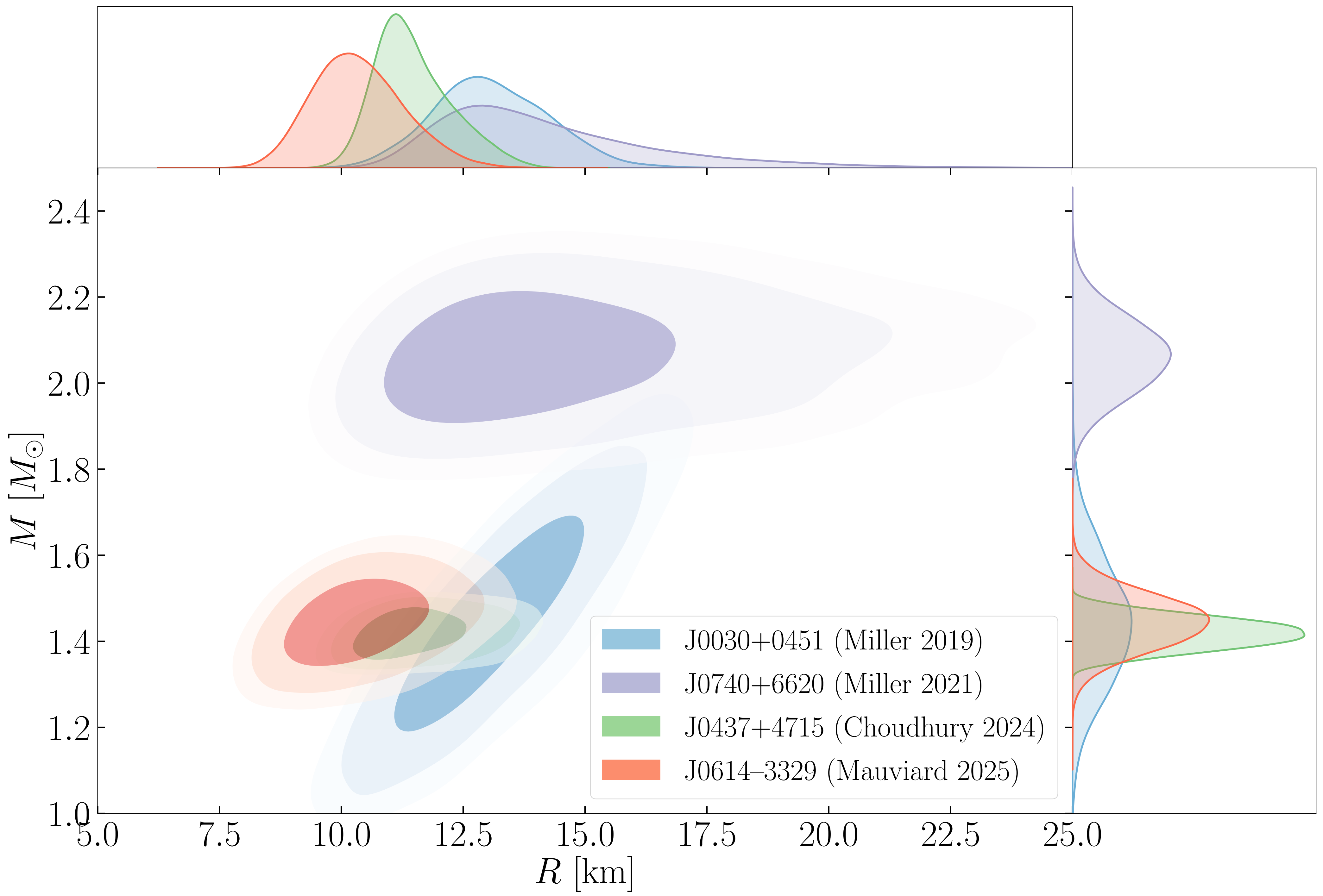
$$\mathscr{L}_{J0348} : \frac{1}{\sqrt{2\pi}\,\sigma} \int_0^{M_{max}/M_\odot} dx \, \exp\left(-\frac{(x - 2.01)^2}{2\sigma^2}\right)$$

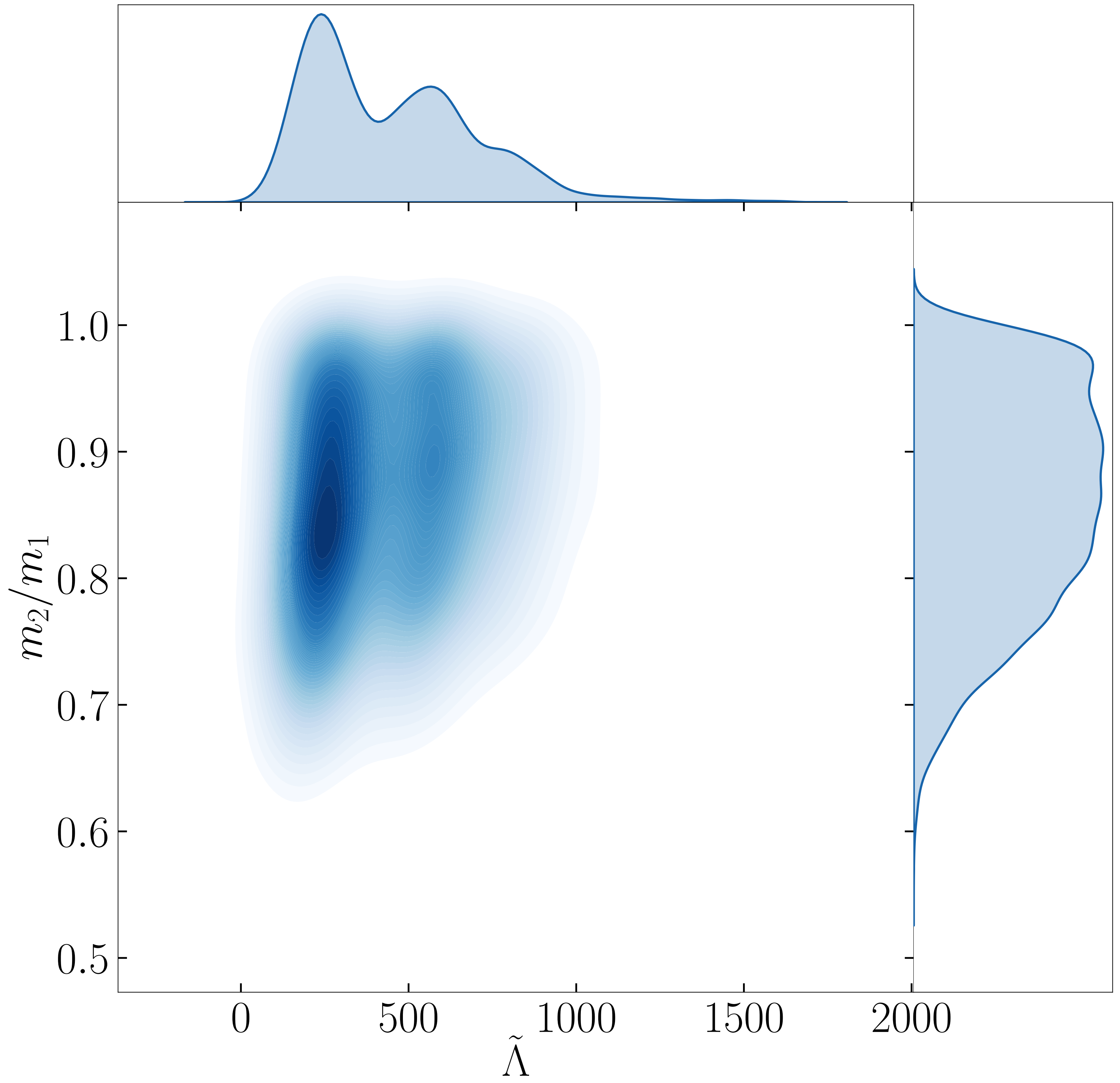


Phys. Rev. C **103**, 025803

Likelihoods

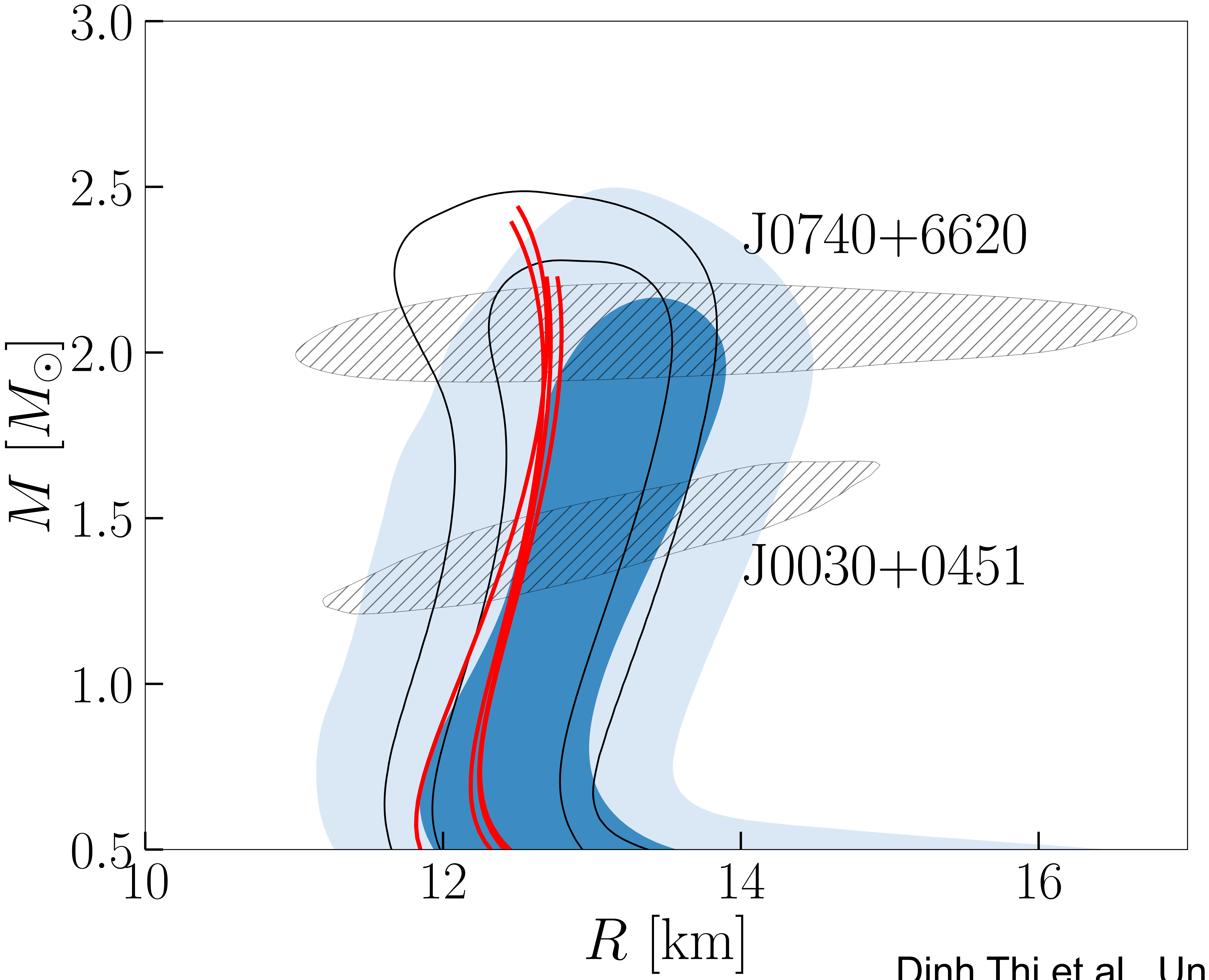
$$\mathscr{L}_{NICER}: \int dM\, P_{N19}(M, R(M)) \cdot \int dM\, P_{N21}(M, R(M))$$





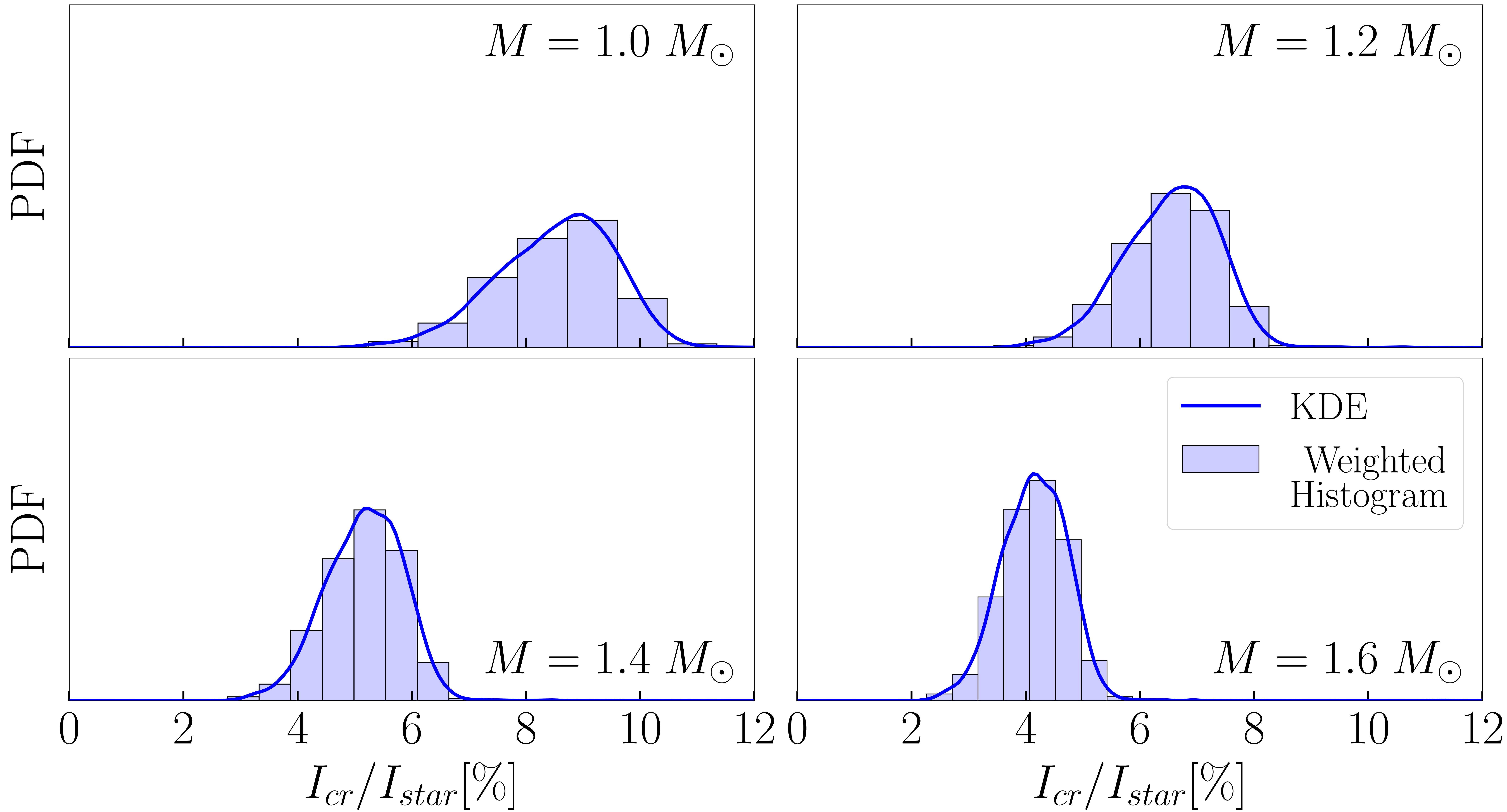
$$\mathscr{L}_{LVC}: \int dq\, P(\tilde{\Lambda}(q), q)$$

$M - R$ relation



Dinh Thi et al., Universe **2021**, 7, 373

Moment of inertia of the crust



Crust-Core transition properties