MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES

Cofinanciado por
la Unión Europea

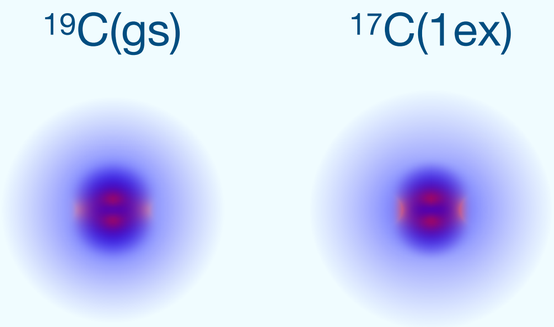
AGENCIA
ESTATAL DE
INVESTIGACIÓN

Transfer and breakup reactions of neutron-rich carbon isotopes described within few-body models

P. Punta, J. A. Lay and A. M. Moro
Departamento de Física Atómica Molecular y Nuclear, Universidad de Sevilla

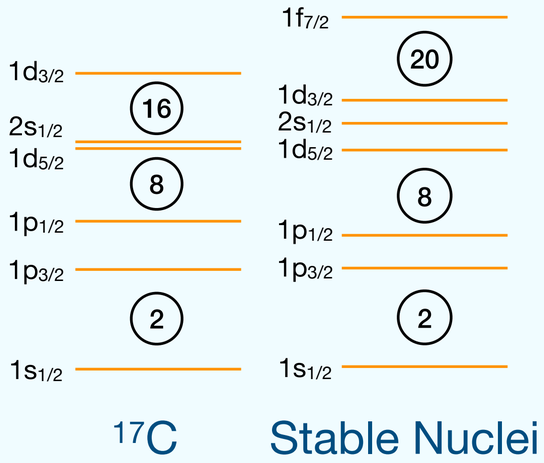
Exotic carbon isotopes ^{17}C and ^{19}C

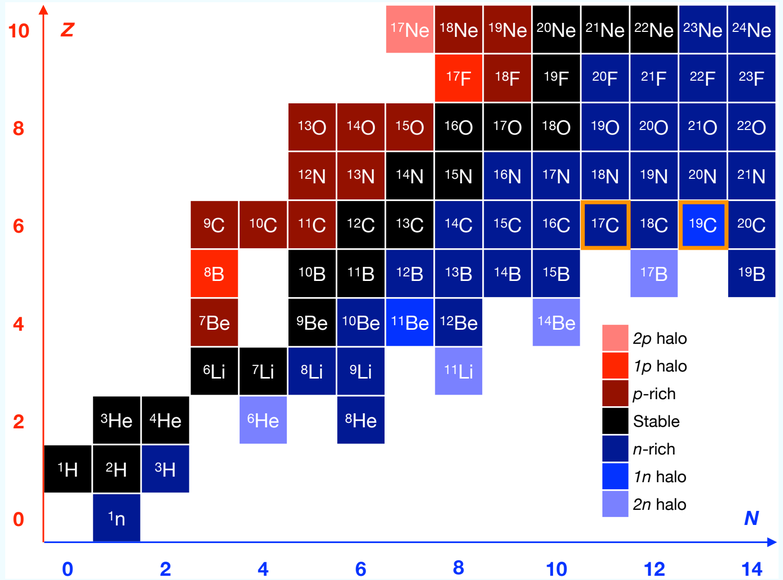
- Weakly bound exotic nuclei
  - Halo nature
  - N=16 shell closure



- Reactions involving them
  - Advances in radioactive beam facilities
  - Realistic but manageable models

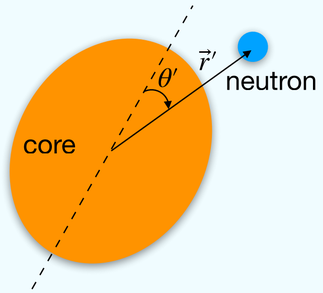
- Few-body models
  - Significant effect of deformation
  - Deformed core + neutron structure





Nilsson+AMD model (NAMD)

P. Punta *et al.* PRC**111** (2025) 064614



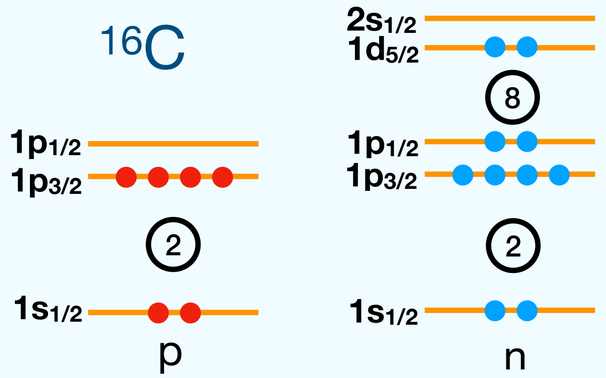
- Deformed two-body model
  - Combination of Nilsson and PAMD [P. Punta *et al.* PRC**108** (2023) 024613]
- Nilsson Hamiltonian $H_N = T + V_0(r) + V_{\ell s}(r)(\overrightarrow{\ell} \cdot \vec{s}) + V_2(r)Y_{20}(\theta')$
- Semi-microscopic coupling potentials with AMD densities [J. A. Lay *et al.* PRC**89** (2014) 014333]

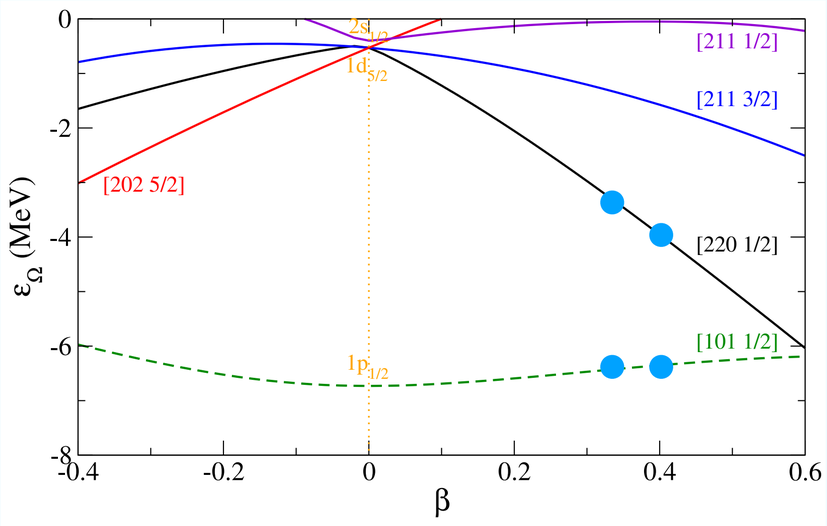
$$V_\lambda(r) = \int dr_1 r_1^2 \left\{ v_{IS}^\lambda(r_1, r)[\rho_{\lambda^+\to 0^+}^{n\lambda}(r_1) + \rho_{\lambda^+\to 0^+}^{p\lambda}(r_1)] + v_{IV}^\lambda(r_1, r)[\rho_{\lambda^+\to 0^+}^{n\lambda}(r_1) - \rho_{\lambda^+\to 0^+}^{p\lambda}(r_1)] \right\}$$

- Collective rotational term $H = H_N + \frac{\hbar^2}{2\mathcal{I}}\vec{I}^2$
- Diagonalization in the THO basis [S. Karataglidis *et al.* PRC**71** (2005) 064601] $\phi_{nl}^{THO}(r) = \sqrt{\frac{ds}{dr}}\phi_{nl}^{HO}[s(r)]$

Pauli exclusion principle

- Factorization of the system does not allow complete antisymmetrization
- States occupied by core nucleons should be blocked for the valence nucleons
- There are partially filled spherical single-particle levels
- Deformation gives rise to Nilsson levels with twofold degeneracy

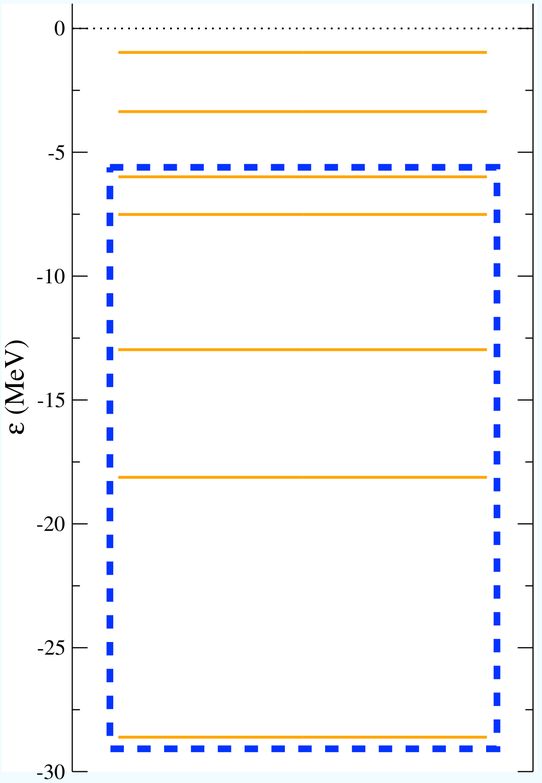




Pauli-blocking methods

$$H_N|\nu\rangle = \epsilon_\nu|\nu\rangle \qquad \langle\nu|H|\nu'\rangle = \epsilon_\nu\delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle\nu|\vec{I}^2|\nu'\rangle$$

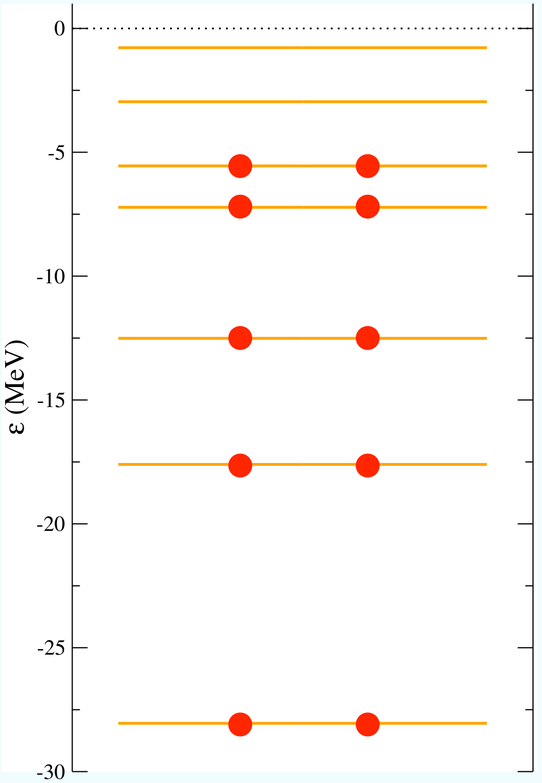
- No Blocking (NoB)
  - There is no blocking in the diagonalization of the complete Hamiltonian
  - The states with a dominant contribution from the lowest Nilsson states are discarded



Pauli-blocking methods

$$H_N|\nu\rangle = \epsilon_\nu|\nu\rangle \qquad \langle\nu|H|\nu'\rangle = \epsilon_\nu\delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle\nu|\vec{I}^2|\nu'\rangle$$

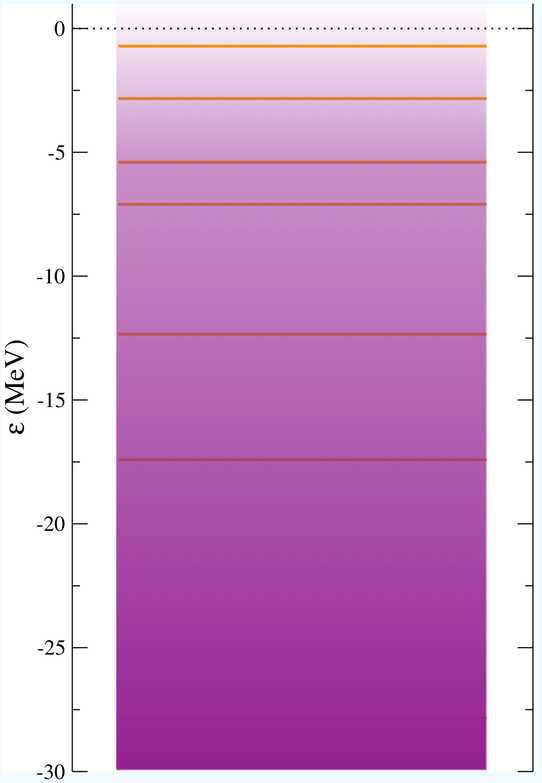
- No Blocking (NoB)
  - There is no blocking in the diagonalization of the complete Hamiltonian
  - The states with a dominant contribution from the lowest Nilsson states are discarded
- Total Blocking (TB)
  - The occupied Nilsson states are excluded in the diagonalization of the complete Hamiltonian



Pauli-blocking methods

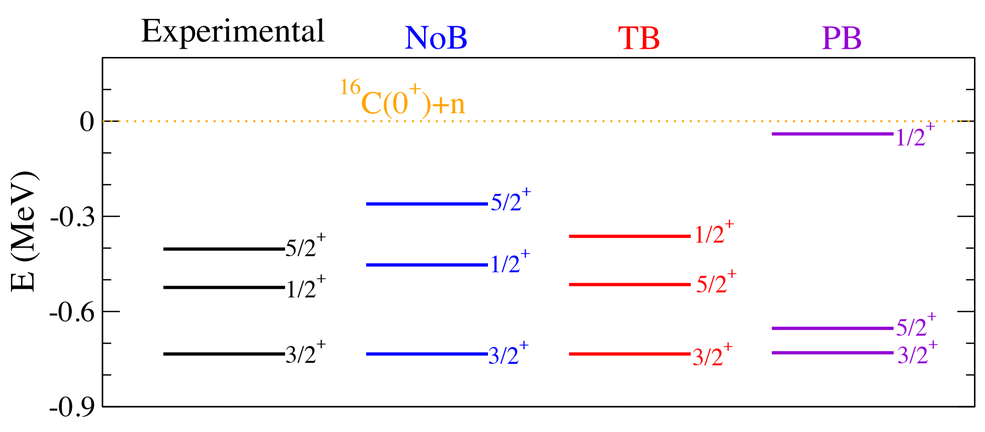
$$H_N|\nu\rangle = \epsilon_\nu|\nu\rangle \qquad \langle\nu|H|\nu'\rangle = \epsilon_\nu\delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle\nu|\vec{I}^2|\nu'\rangle$$

- No Blocking (NoB)
  - There is no blocking in the diagonalization of the complete Hamiltonian
  - The states with a dominant contribution from the lowest Nilsson states are discarded
- Total Blocking (TB)
  - The occupied Nilsson states are excluded in the diagonalization of the complete Hamiltonian
- Partial Blocking (PB)
  - Pairing correlations are taken into account
  - Nilsson states are partially blocked according to their occupation
  - Full implementation in reactions is in progress

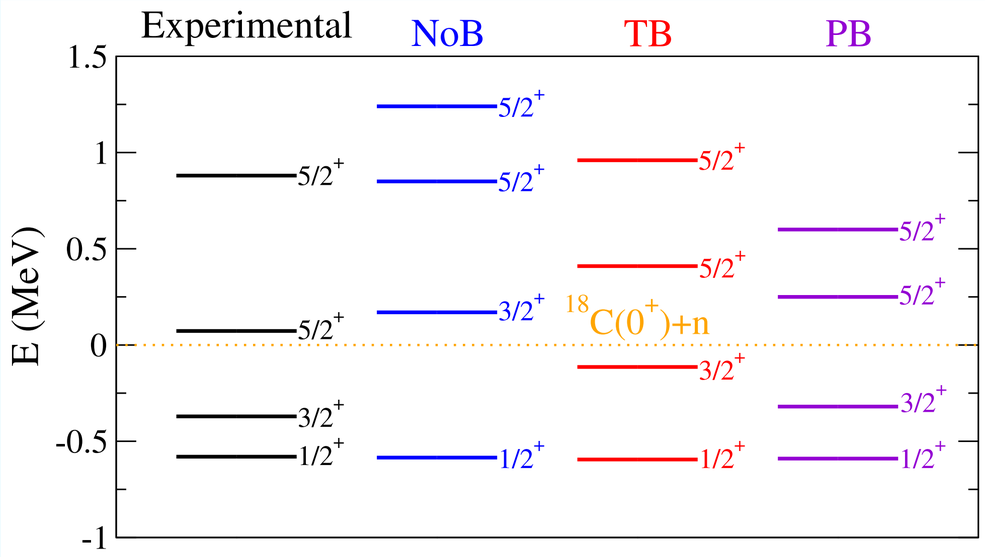


Application to ^{17}C and ^{19}C

^{17}C

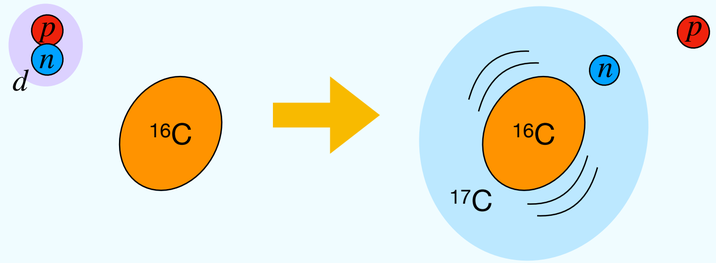


^{19}C



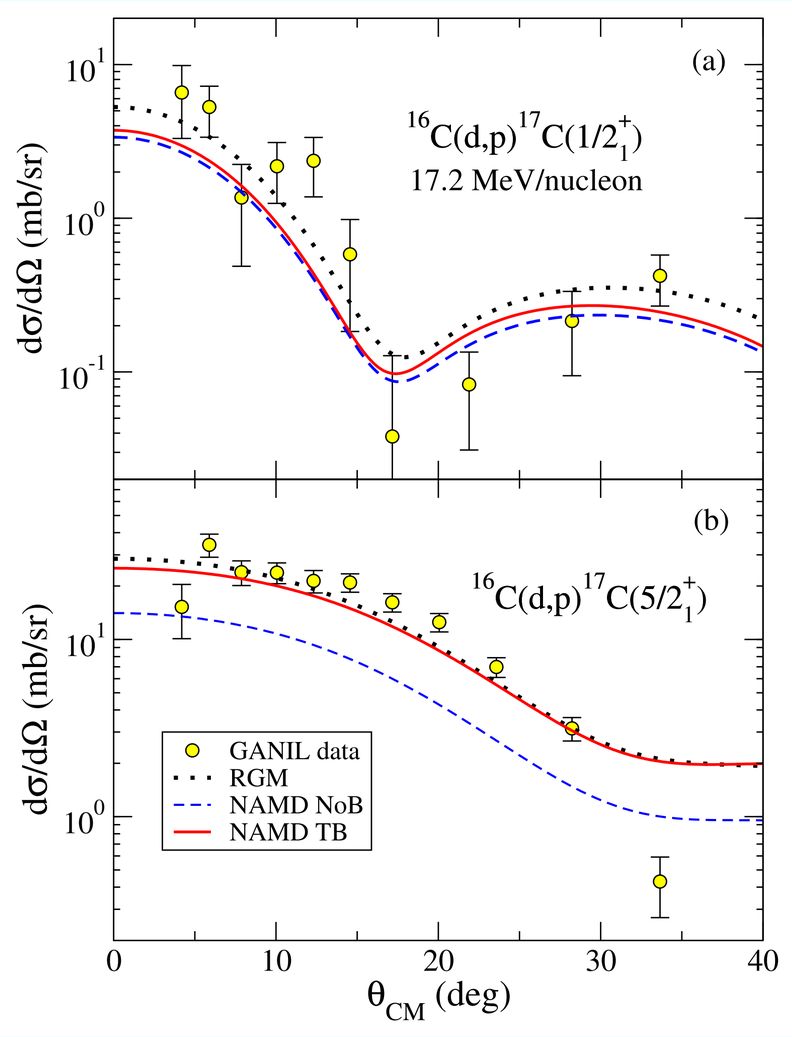
P. Punta *et al.* PRC**111** (2025) 064614

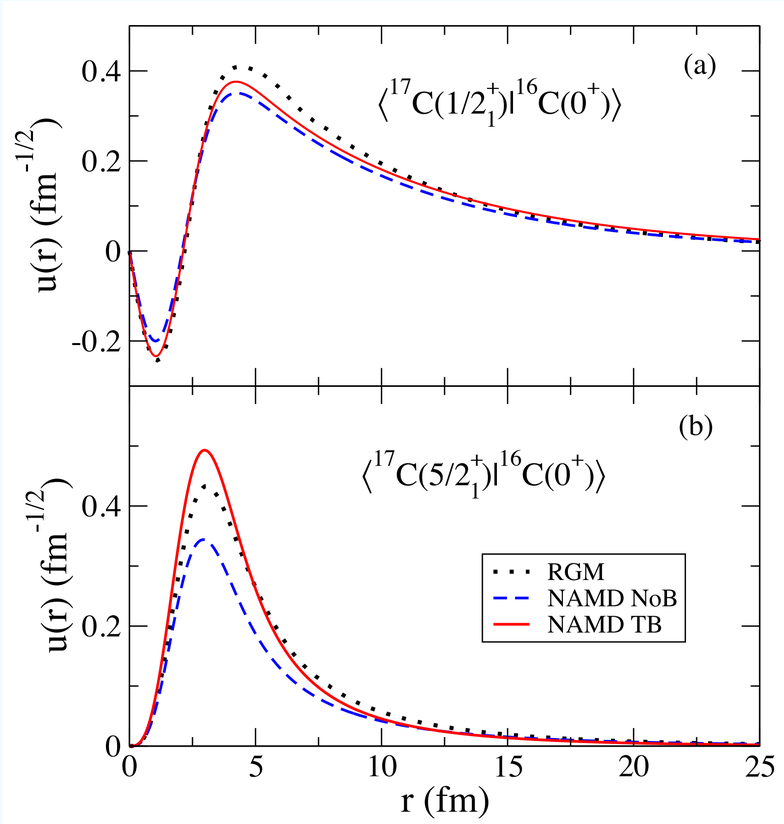
Application to neutron transfer reactions



- ^{16}C(d,p)^{17}C reaction is studied applying the adiabatic distorted wave Approximation (ADWA)
- Transfer to bound states: in the *post* form, the calculations only require the $\langle {}^{17}\mathrm{C} | {}^{16}\mathrm{C}(0^+) \rangle$ overlaps
- Transfer to the continuum: In the *prior* form, the calculations involves the vertex functions $\langle {}^{17}\mathrm{C} | V_{n-{}^{16}\mathrm{C}} | {}^{16}\mathrm{C}(0^+) \rangle$.
- Calculations are compared with experimental data from GANIL
    - Bound states: X. Pereira-López *et al.* PLB**811** (2020) 135939
    - Unbound states: J. Lois-Fuentes *et al.* PLB**867** (2025) 139600

Application to $^{16}C(d,p)^{17}C$ (bound states)

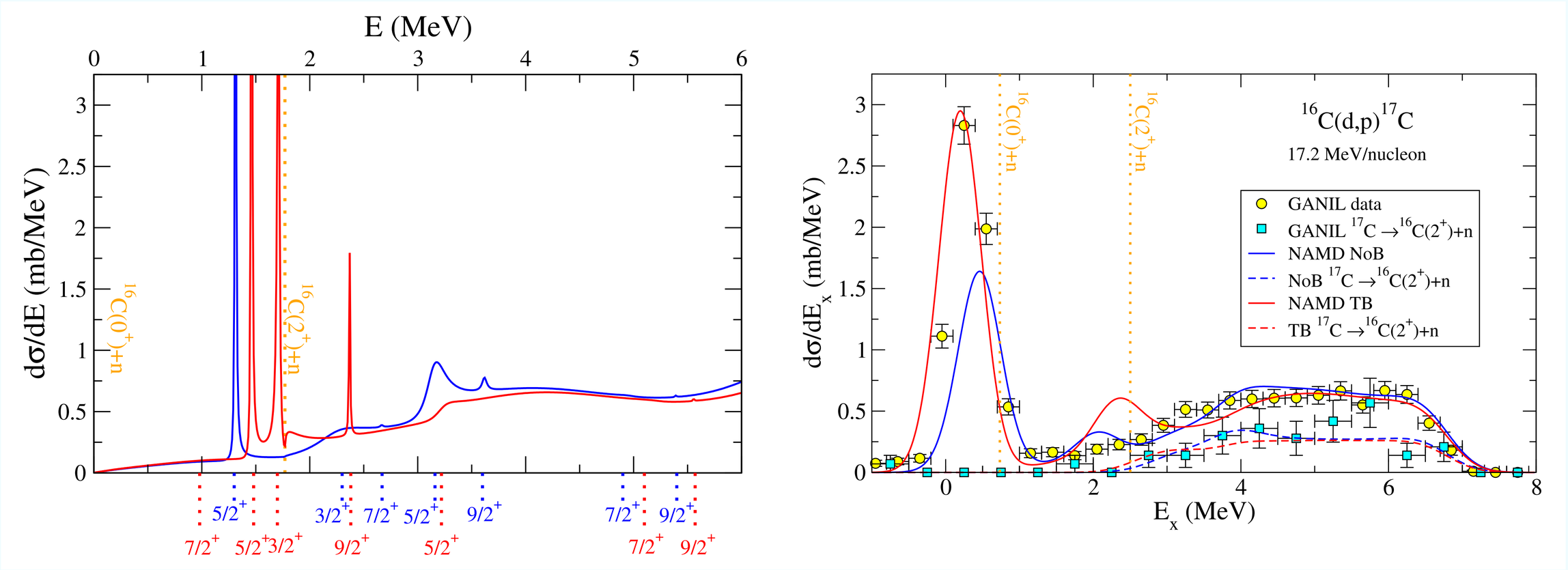




Resonating Group Method calculations (RGM):

L. H. Chien and P. Descouvemont PRC**108** (2023) 044605

$^{16}C(d,p)^{17}C$: transfer to the continuum

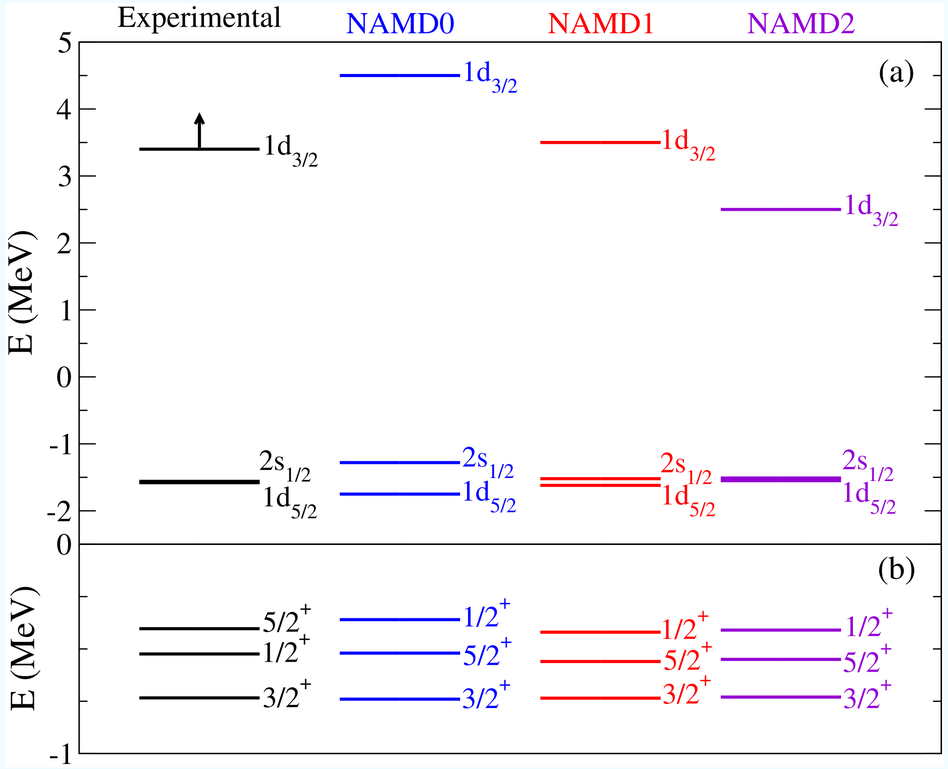


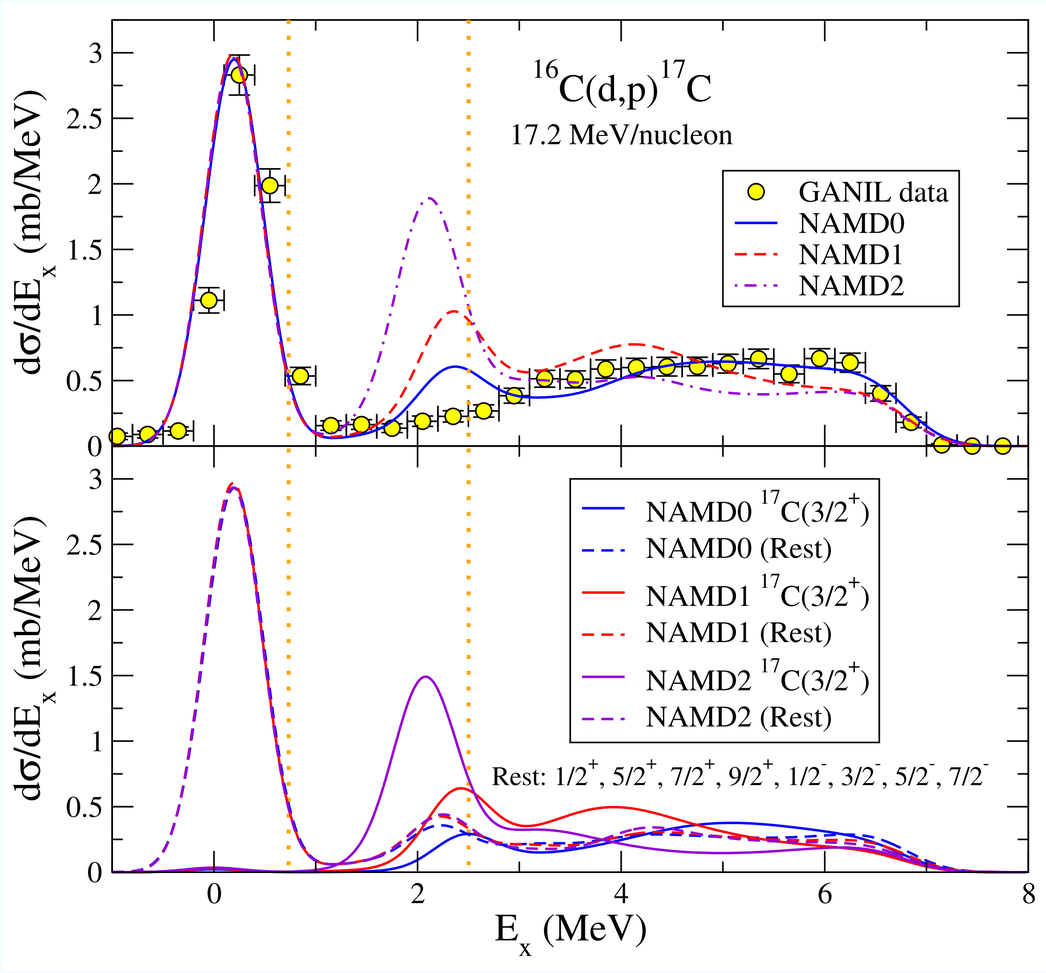
Energy with respect to $^{16}C(0^+)+n$ threshold

Excitation energy of ^{17}C

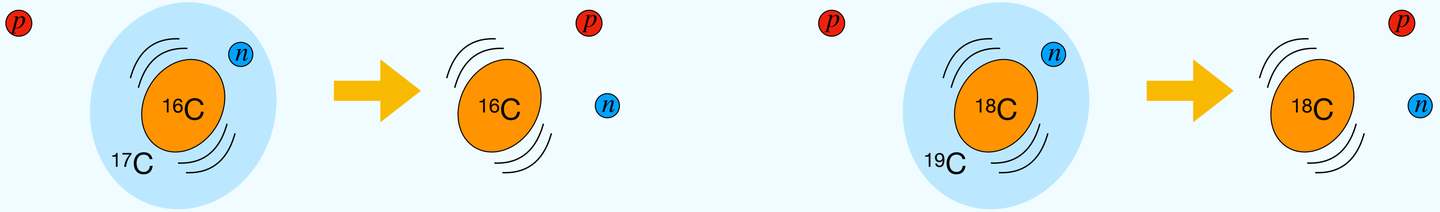
N=16 shell gap

Spin-orbit splitting variations on the NAMD model (TB)





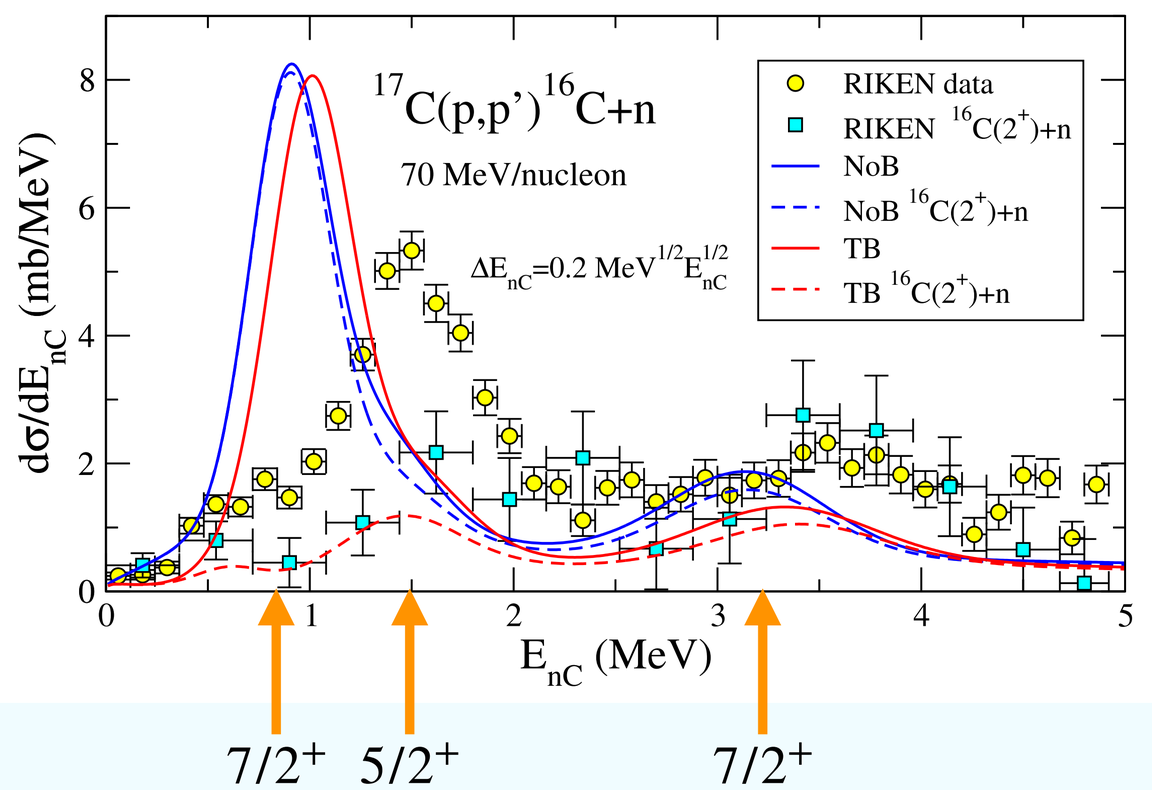
Application to breakup reactions



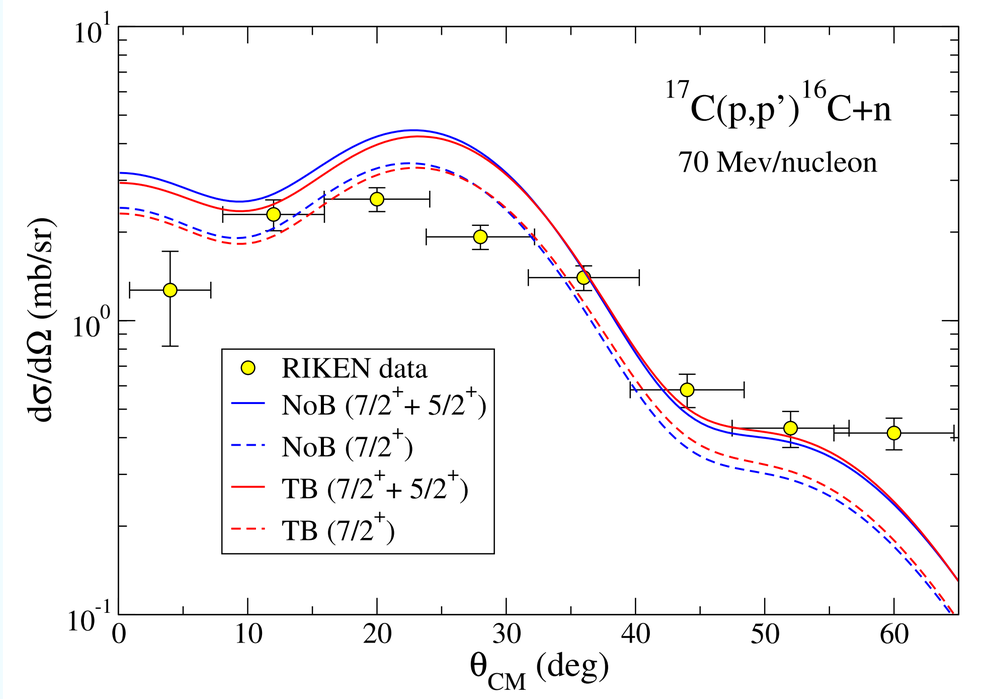
- Breakup reactions $^{17}C(p,p')^{16}C+n$ and $^{19}C(p,p')^{18}C+n$ studied using extended continuum-discretized coupled-channel method (XCDCC) [PRC**95** (2017) 044611]
- ^{17}C and ^{19}C described using the NAMD model
- p-^{16}C and p-^{18}C interactions calculated by convoluting the AMD densities with an effective N-N interaction
- Gaussian n-p potential [A. M. Moro and R. Crespo, PRC**85** (2012) 054613]
- Calculations are compared with experimental data from RIKEN [Y. Satou *et al.* PLB**660** (2008) 320]

Application to $^{17}C(p,p')^{16}C+n$

Energy distribution

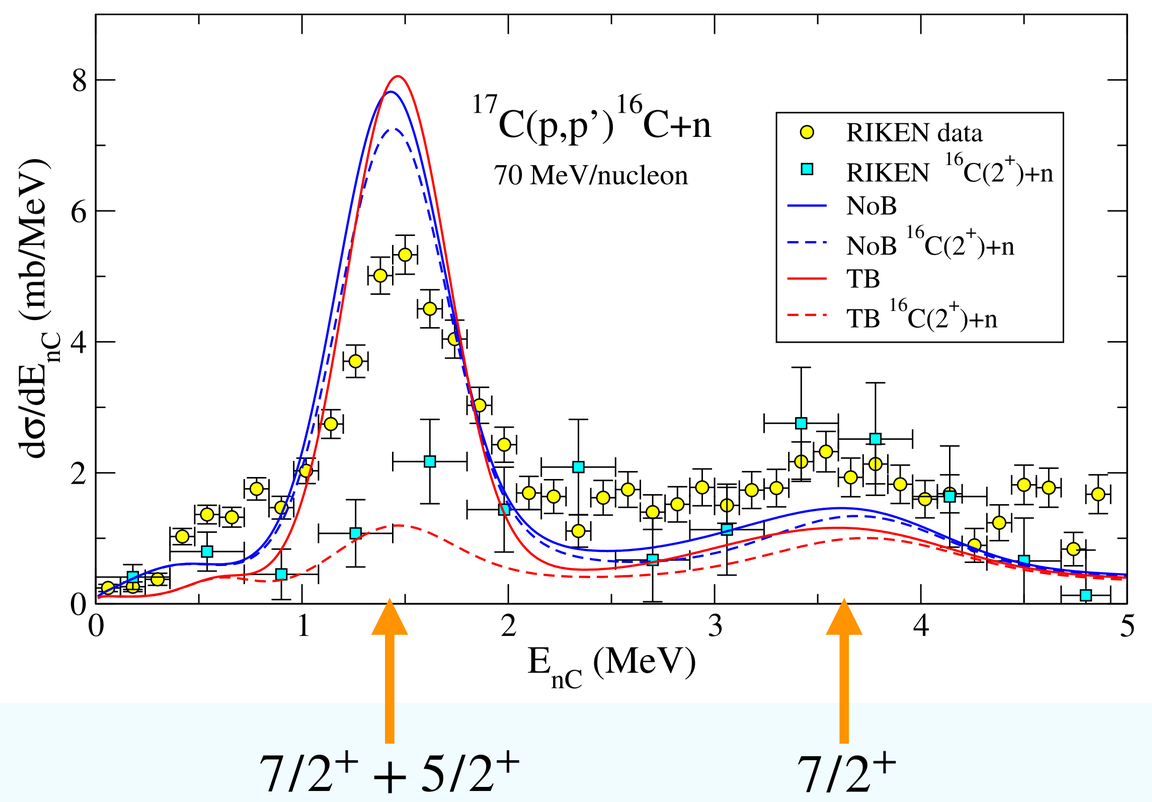


Resonant breakup

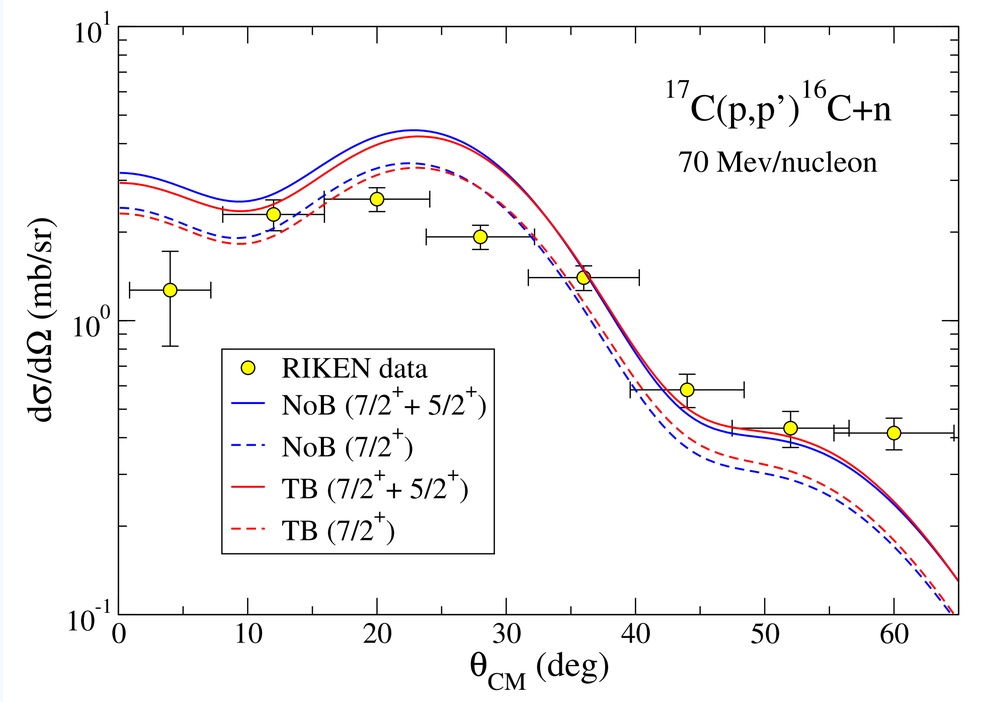


Application to $^{17}C(p,p')^{16}C+n$

Energy distribution (States $7/2^+$ shifted)



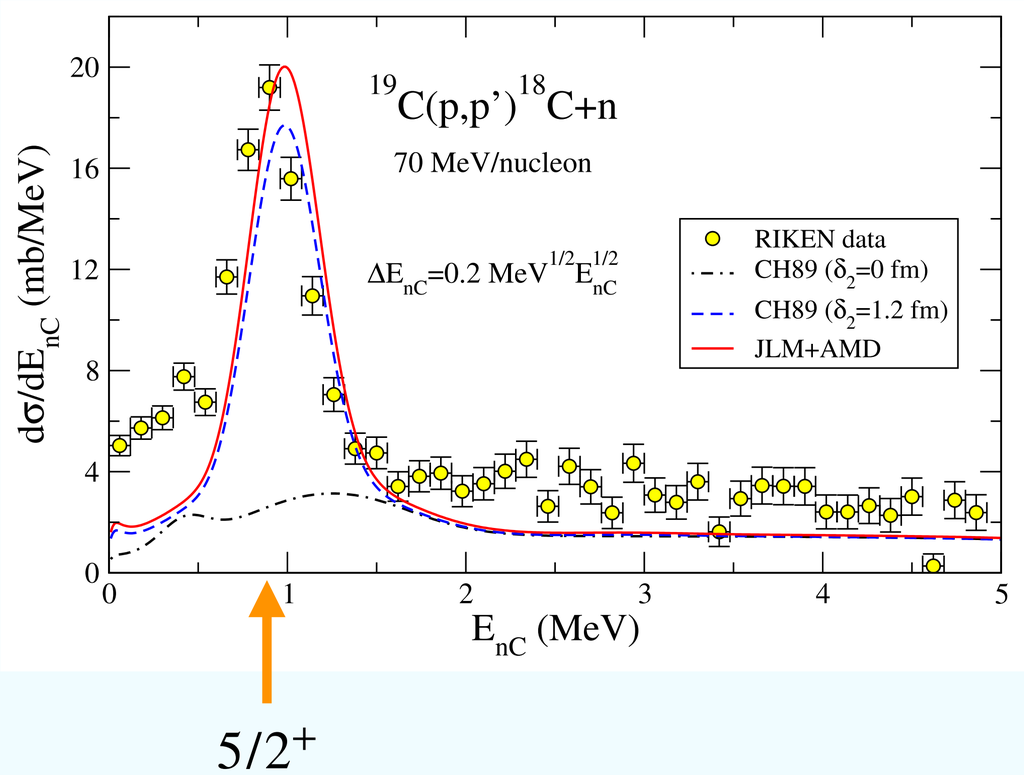
Resonant breakup



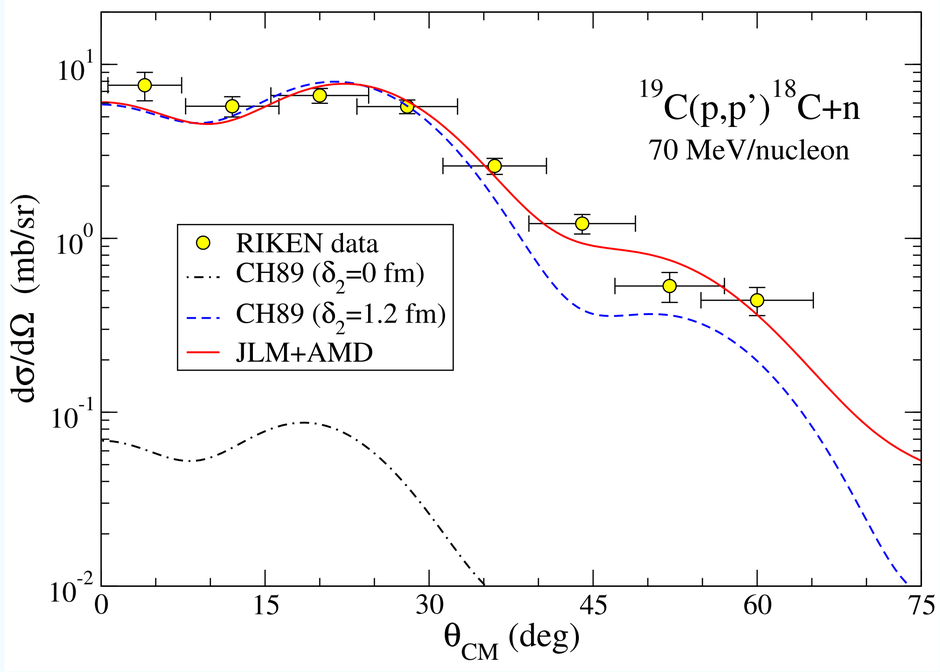
Application to $^{19}C(p,p')^{18}C+n$

XCDCC calculations with NAMD TB model using different p-^{18}C *interaction*

Energy distribution



Resonant Breakup



Conclusions

- The **NAMD** model shows **promising results** in the description of **^{17}C** and **^{19}C**
- **Significant effects** of **Pauli blocking** related to occupied Nilsson states have been found
- Transfer reaction **^{16}C(*d*,*p*)^{17}C** populating **bound** and **unbound** **states** has been studied
  - **Pauli blocking** methods are **needed** to reproduce **^{16}C(*d*,*p*)^{17}C($5/2^+_1$)** cross section
  - The results for the **transfer to the continuum** suggest a ***N*=16 shell gap** larger than 5 MeV
- **Successful results** for breakup reactions **^{17}C(*p*,*p'*)^{16}C+*n*** and **^{19}C(*p*,*p'*)^{18}C+*n***
  - The population of **7/2$^+$ resonances** plays a **key role** in the **^{17}C breakup**.
  - The excitation to the **5/2$^+$ resonance** of **^{19}C** is clearly **dominated** by the **core excitation** mechanism

Thanks for your attention

Proyecto PID2023-146401NB-I00 financiado por

MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES

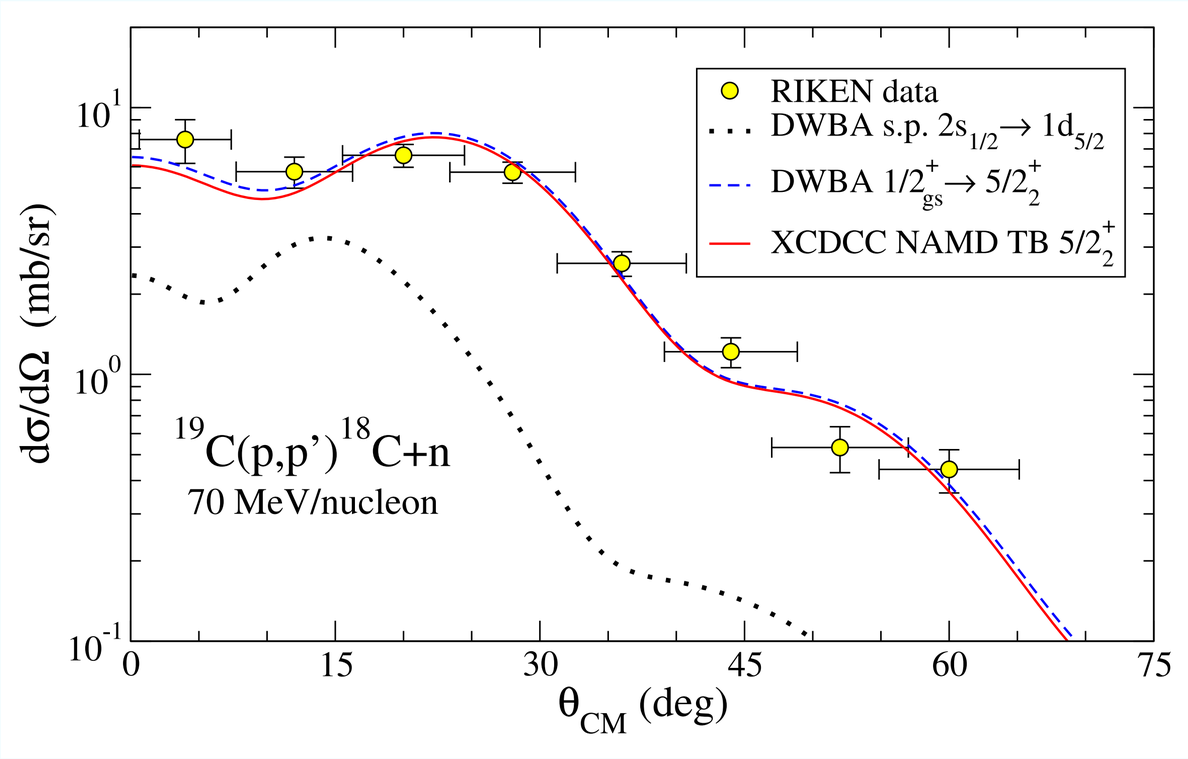
Cofinanciado por
la Unión Europea

AGENCIA
ESTATAL DE
INVESTIGACIÓN

Ph.D. grant FPU21/03931

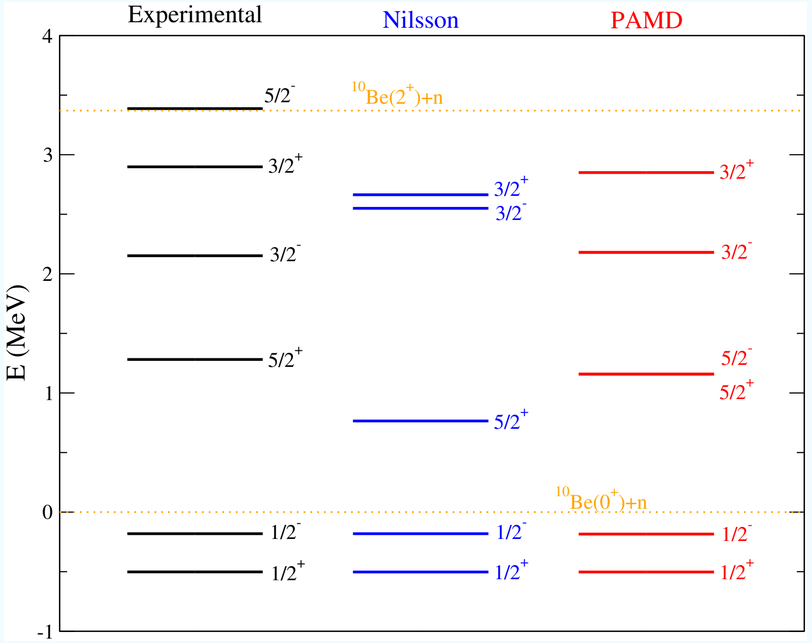
Backup

^{19}C(*p*,*p*')^{18}C+*n*: resonant breakup



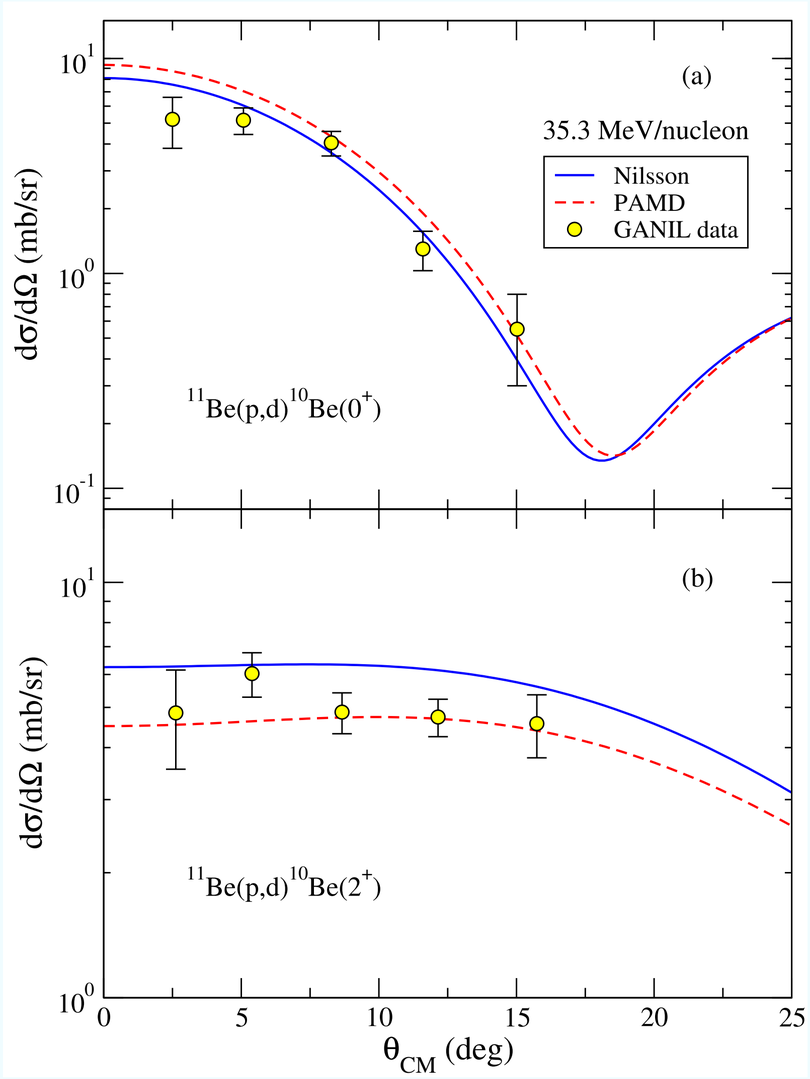
- Good agreement between experimental data and XCDCC calculations using the NAMD model with TB
- Similar results using the distorted wave Born approximation (DWBA)
- Single-particle calculations are not consistent with the data

Application to ^{11}Be



P. Punta, *et* al. PRC**108** (2023) 024613

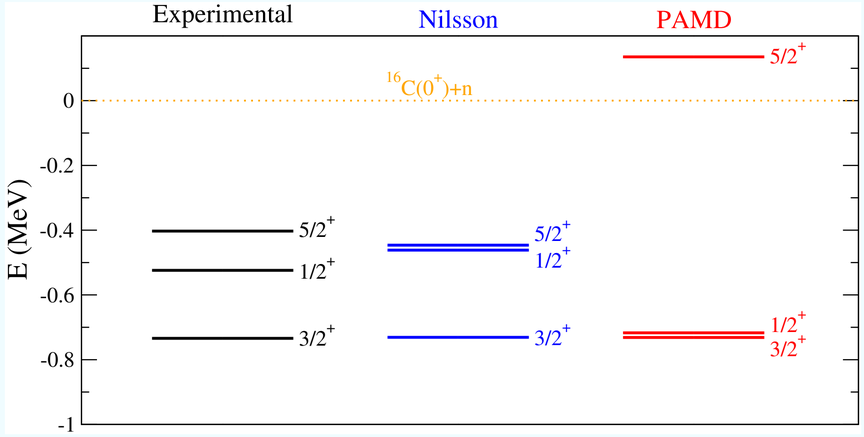
GANIL data [NPA**683** (2001) 48]



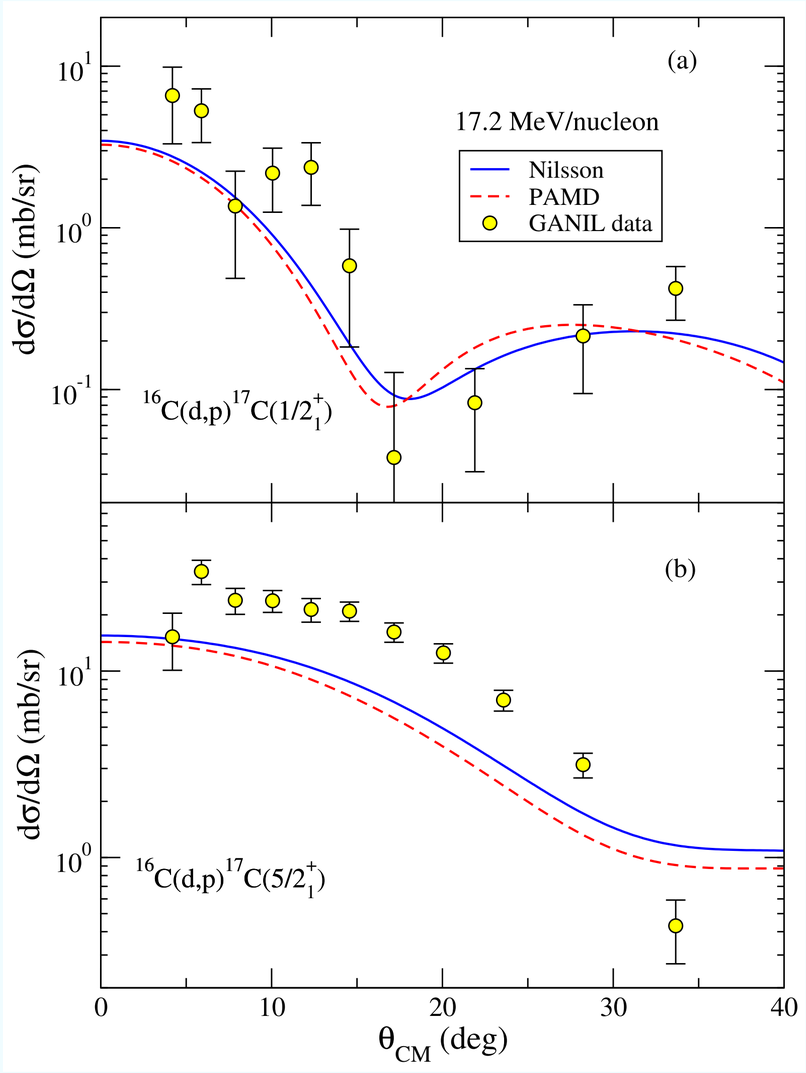
Application to ^{17}C

- Deformed two-body models
  - Nilsson
  - PAMD

P. Punta, *et* al. PRC**108** (2023) 024613

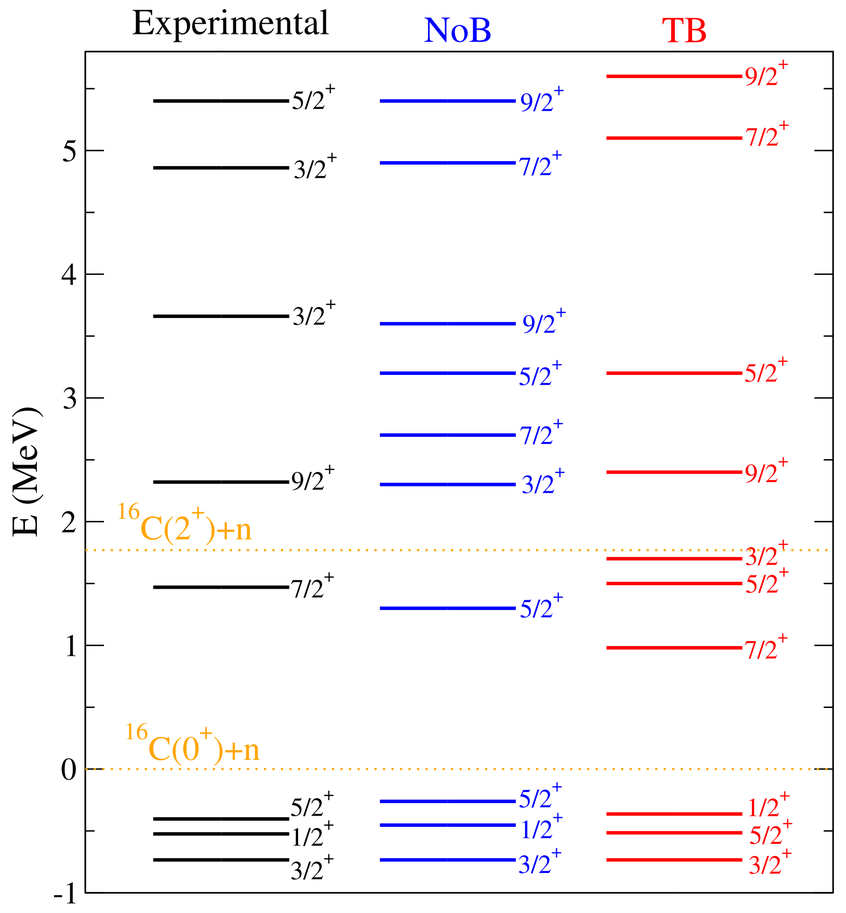


GANIL data [PLB**811** (2020) 135939]

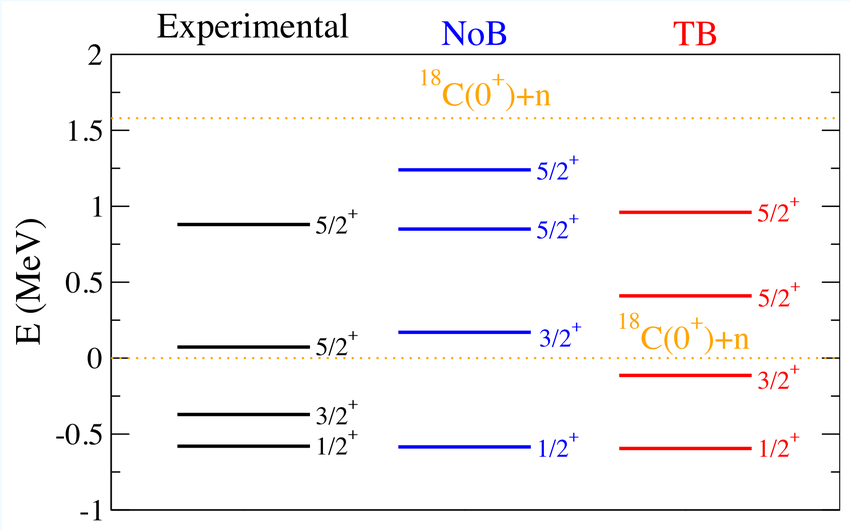


Application to ^{17}C and ^{19}C

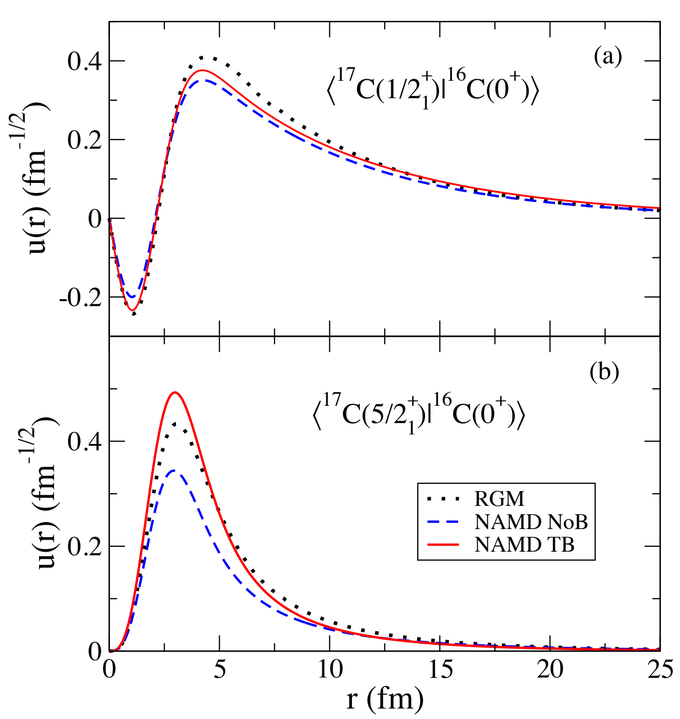
^{17}C



^{19}C

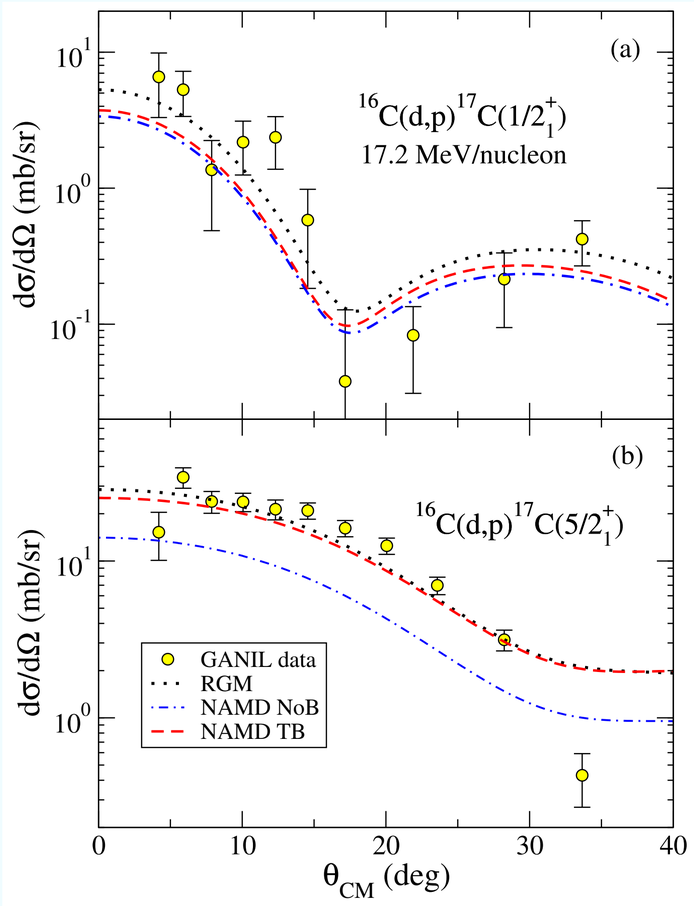


Application to $^{16}C(d,p)^{17}C$ (bound states)



Spectroscopic Factors
for $(\ell s)j \otimes 0^+$

	$3/2^+_1$	$1/2^+_1$	$5/2^+_1$
Exp	$0.03\ ^{+0.05}_{-0.03}$	0.64±0.18	0.62±0.13
RGM	0.01	0.94	0.56
NoB	0.00	0.68	0.33
TB	0.01	0.80	0.66



Resonating Group Method calculations (RGM): L. H. Chien and P. Descouvemont PRC**108** (2023) 044605

BCS implementation

- Nilsson eigenstates

$$H_N|\nu J\rangle = \epsilon_\nu |\nu J\rangle$$

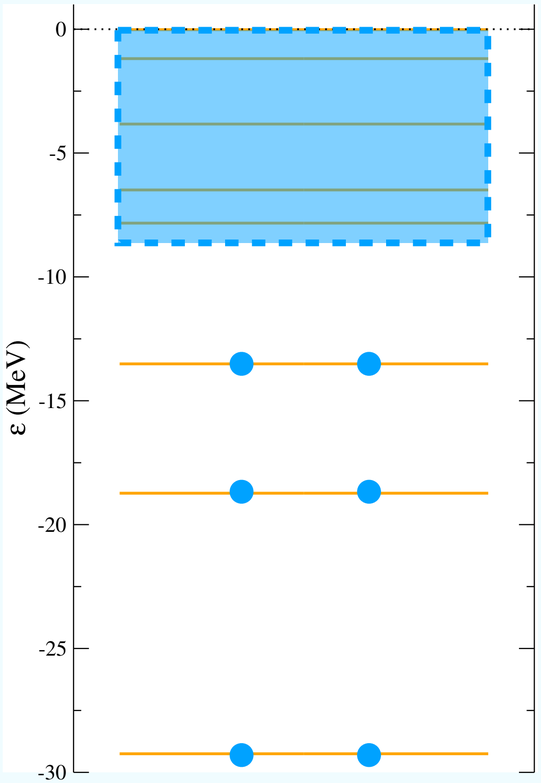
- Nilsson+rotational

$$\langle \nu J|H|\nu' J\rangle = \epsilon_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle \nu J|\vec{I}^2|\nu' J\rangle$$

- Transformation to quasi-particles

$$\langle BCS, \nu J|H'|BCS, \nu' J\rangle = E_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle \nu J|\vec{I}^2|\nu' J\rangle F_{\nu\nu'}$$

- $F_{\nu\nu'}$ account for BCS occupation probabilities

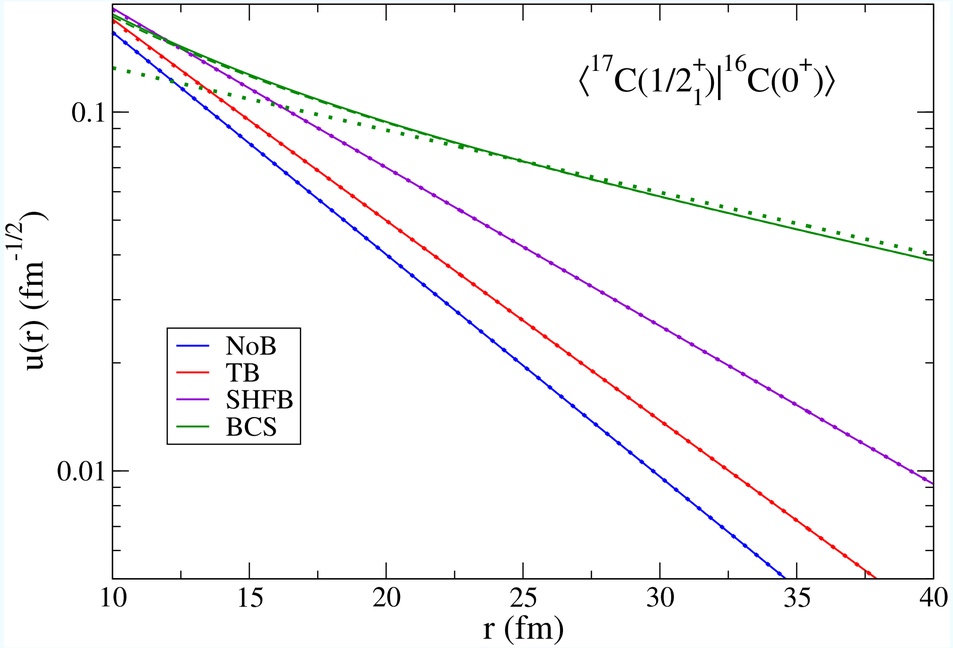


$$F_{\nu'\nu} = u_{\nu'}u_\nu - v_{\nu'}v_\nu + (1-2v_\nu)\delta_{\nu'\nu}$$

Asymptotic behavior

- The NAMD+SHFB formalism is a variation of the NAMD+BCS formalism from P. Punta, *et al.* PRC**111** (2025) 064614

- The radial pairing interaction maintains the expected asymptotic behavior



Simplified HFB formalism

- SHFB equations [I. Hamamoto, *et al.* PRC**73** (2006) 044317]

$$\begin{cases} (H_N - \lambda)\psi^u(\vec{r}') + \Delta(\vec{r}')\psi^v(\vec{r}') = E\psi^u(\vec{r}') \\ \Delta(\vec{r}')\psi^u(\vec{r}') + (\lambda - H_N)\psi^v(\vec{r}') = E\psi^v(\vec{r}') \end{cases}$$

- Pairing interaction derived from AMD densities

$$\Delta(\vec{r}') = \Delta_0(r) + \Delta_2(r)Y_{20}(\theta') \qquad \Delta_\lambda(r) \propto \rho^{n\lambda}_{\lambda^+\to 0^+}(r_1)$$

- Nilsson eigenstates as basis

$$H_N\psi_\nu(\vec{r}') = \epsilon_\nu\psi_\nu(\vec{r}') \qquad \begin{pmatrix} \psi^u_\mu \\ \psi^v_\mu \end{pmatrix} = \sum_\nu \begin{pmatrix} u^\mu_\nu \\ v^\mu_\nu \end{pmatrix} \psi_\nu$$

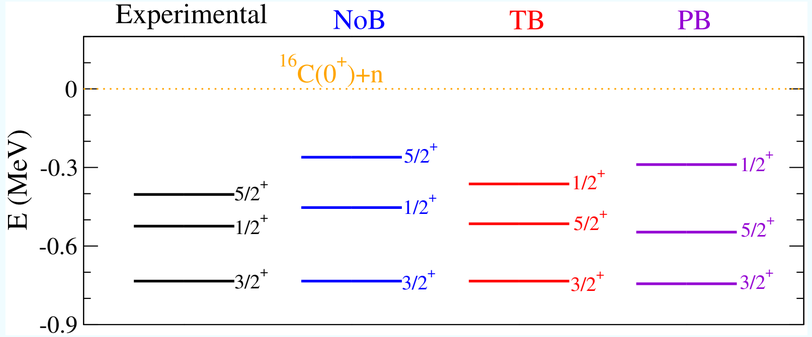
- One quasi-particles states

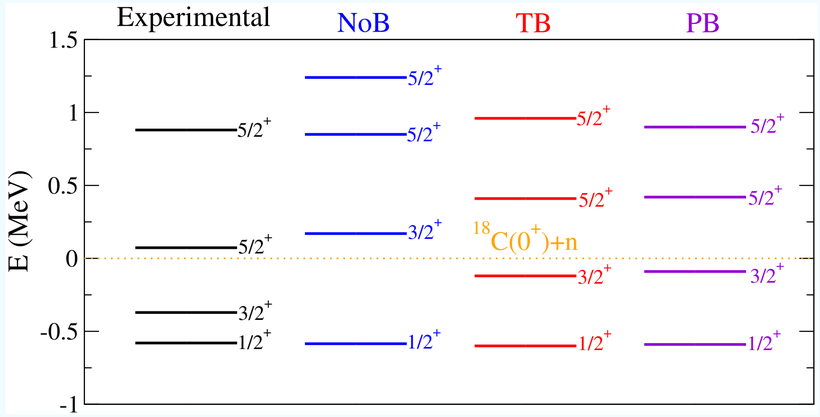
$$H'_N\,|\,\mathrm{SHFB},\mu\rangle = E_\mu\,|\,\mathrm{SHFB},\mu\rangle \qquad \sum_{\mu\nu} |\,v^\mu_\nu\,|^2 = \frac{N}{2}$$

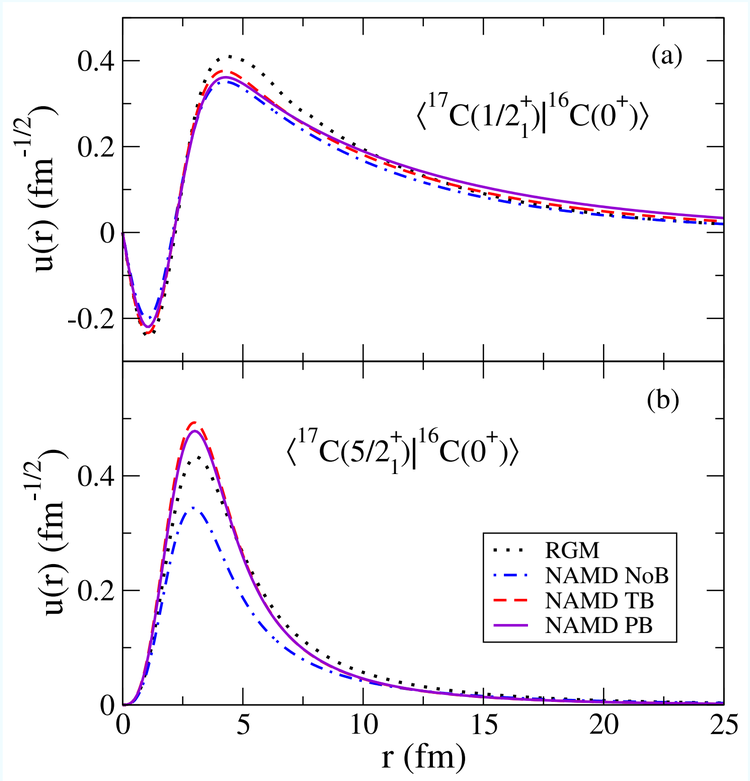
- Rotational term

$$\langle \mathrm{SHFB},\mu\,|\,H'\,|\,\mathrm{SHFB},\mu'\rangle = E_\mu\delta_{\mu\mu'} + \frac{\hbar^2}{2\mathcal{J}}\sum_{\nu\nu'}\langle\nu\,|\,\vec{I}^2\,|\,\nu'\rangle(u^\mu_\nu u^{\mu'}_{\nu'} - v^\mu_\nu v^{\mu'}_{\nu'})$$

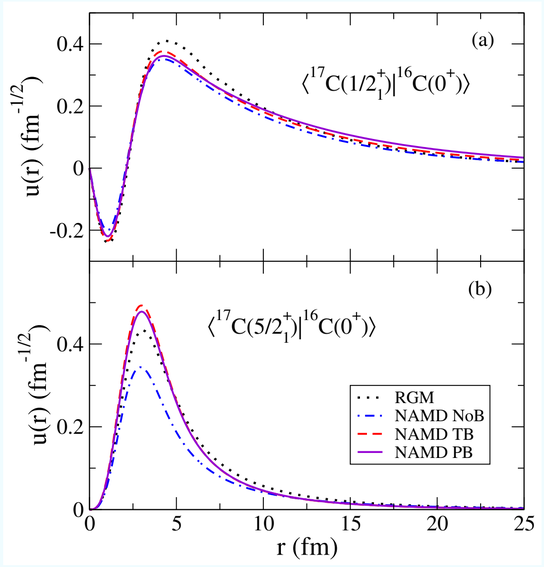
Partial blocking with SHFB formalism





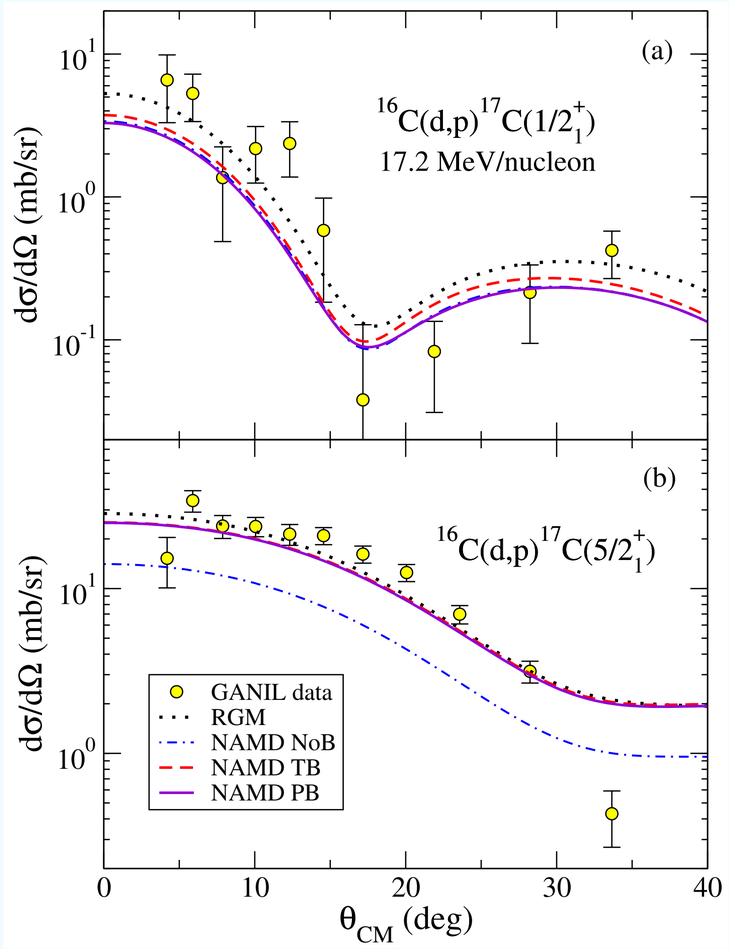


Application to $^{16}C(d,p)^{17}C$ (bound states)



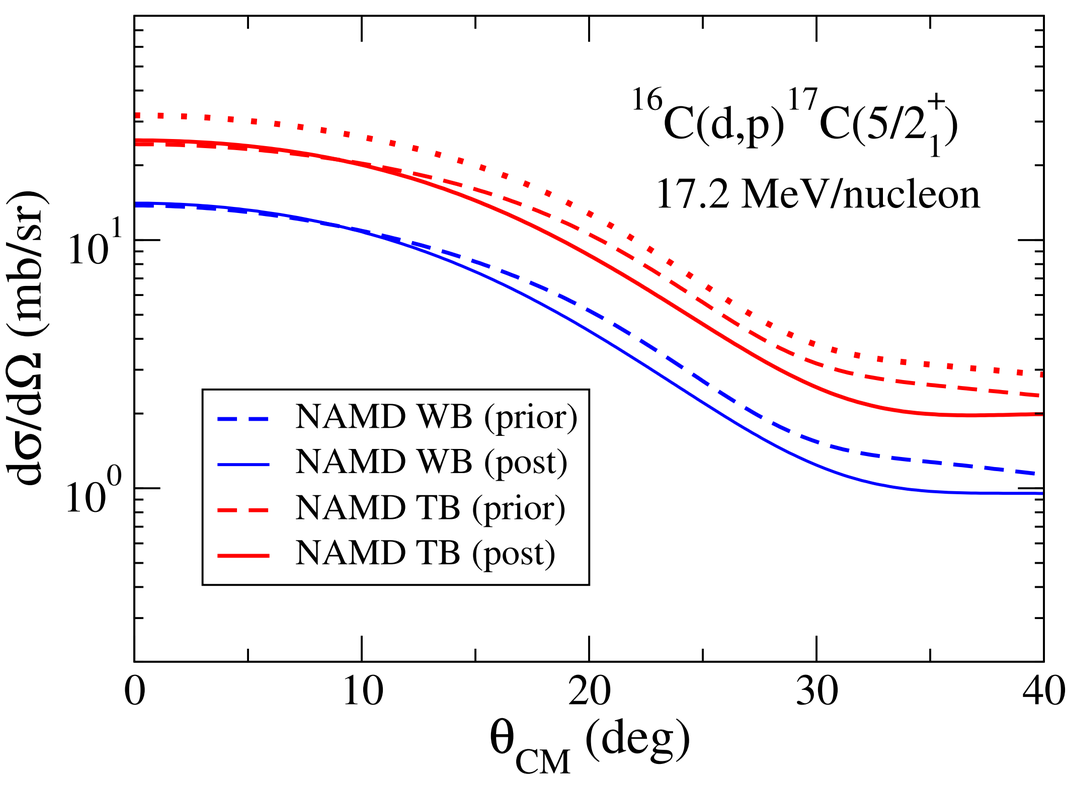
Spectroscopic Factors for $(\ell s)j \otimes 0^+$

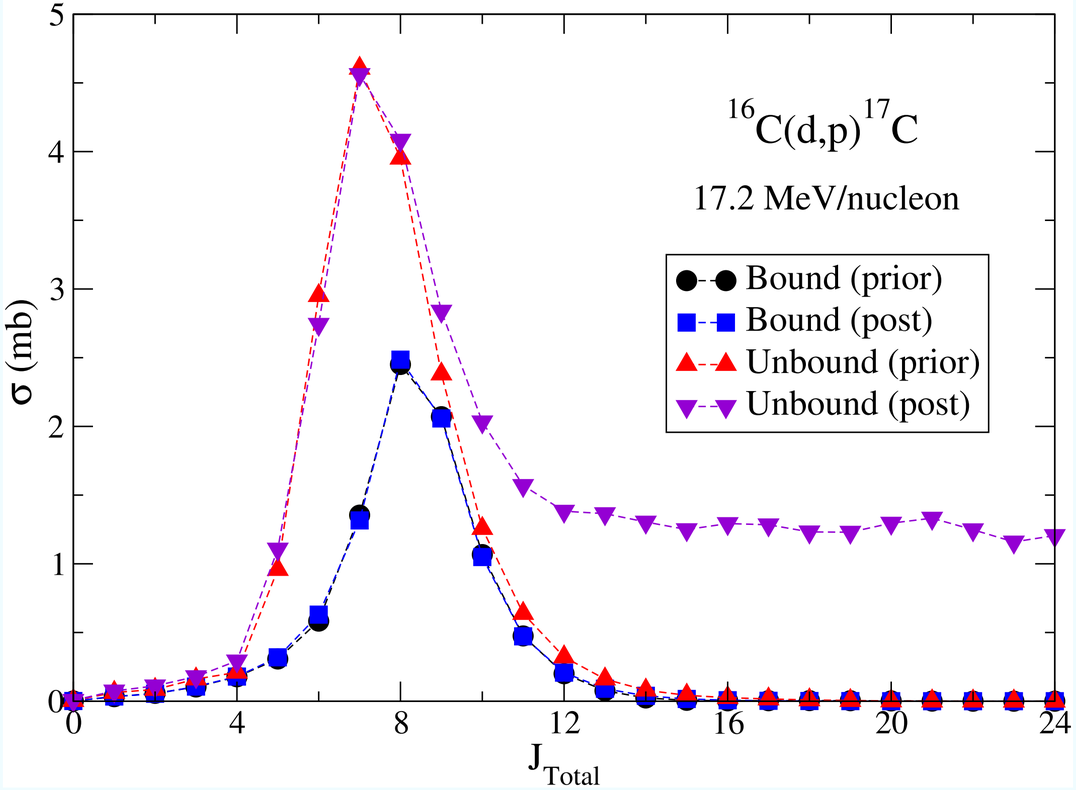
	$3/2^+_1$	$1/2^+_1$	$5/2^+_1$
Exp	$0.03\ ^{+0.05}_{-0.03}$	0.64±0.18	0.62±0.13
RGM	0.01	0.94	0.56
NoB	0.00	0.68	0.33
TB	0.01	0.80	0.66
PB	0.01	0.80	0.63



GANIL data: X. Pereira-López *et al.* PLB**811** (2020) 135939

Prior/Post


^{16}C(d,p)^{17}C(5/2$^+_1$)
17.2 MeV/nucleon
NAMD WB (prior)
NAMD WB (post)
NAMD TB (prior)
NAMD TB (post)
dσ/dΩ (mb/sr)
θ$_{CM}$ (deg)


^{16}C(d,p)^{17}C
17.2 MeV/nucleon
Bound (prior)
Bound (post)
Unbound (prior)
Unbound (post)
σ (mb)
J$_{Total}$

Transfer to the continuum

- The *prior* form of the ADWA approximation is used

  X $\langle {}^{17}\mathrm{C} | {}^{16}\mathrm{C} \rangle \rightarrow$ Too much spatial extension

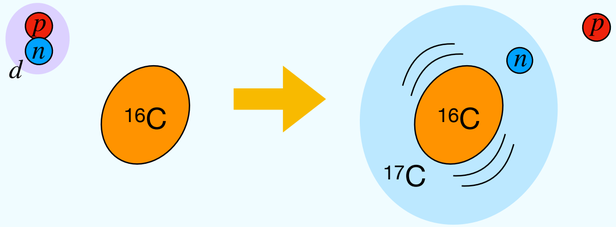
  ✓ $\langle {}^{17}\mathrm{C} | V_{n-{}^{16}C} | {}^{16}\mathrm{C} \rangle \rightarrow$ Little spatial extension

- The continuum is discretized with the pseudo-states method
  - Scattering amplitudes for a discrete number of pseudo-states
  - Convolution with the *exact n*+core scattering states

$$\mathscr{F}(k,\theta) \approx \sum_n \langle \Psi(k,r) | \Psi_n^{THO}(r) \rangle \mathscr{F}_n^{THO}(\theta)$$

Pseudo-states discretization method

$^{16}C(d,p)^{17}C$



$$\mathcal{F}(k,\theta) = \sqrt{\frac{\mu_i \mu_f k_f}{(2\pi\hbar^2)^2 k_i}} \langle \chi_f^{(-)} \varphi_p \Psi_{^{17}C} \,|\, U \,|\, \chi_i^{(+)} \psi_d \phi_{^{16}C} \rangle$$

- Scattering amplitudes for a discrete number of pseudo-states

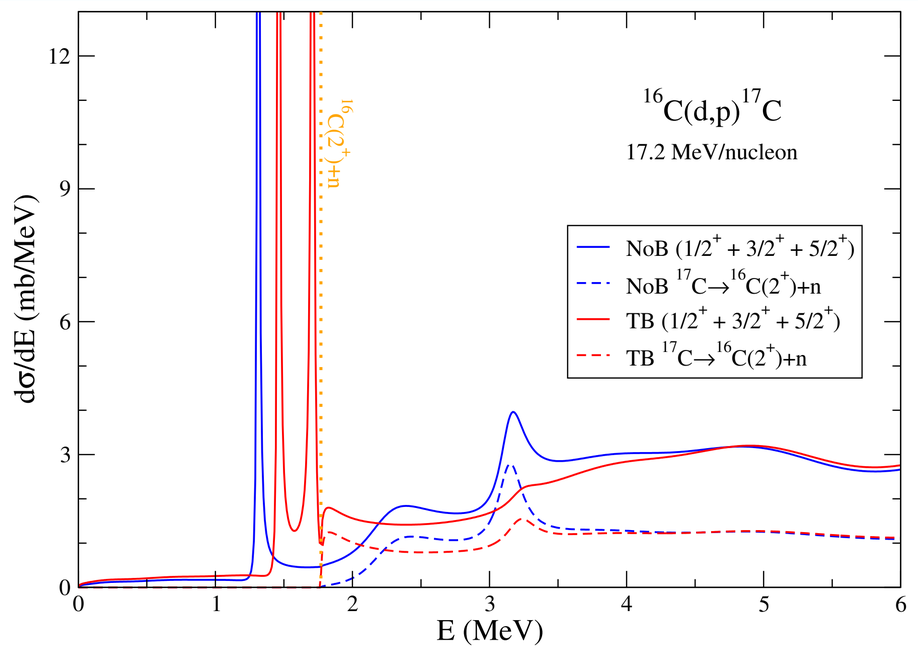
$$\mathcal{F}_n^{THO}(\theta) = \sqrt{\frac{\mu_i \mu_f k_n}{(2\pi\hbar^2)^2 k_i}} \langle \chi_f^{(-)} \varphi_p \Psi_n^{THO} \,|\, U \,|\, \chi_i^{(+)} \psi_d \phi_{^{16}C} \rangle$$

- Convolution with the *exact* *n*+core scattering states

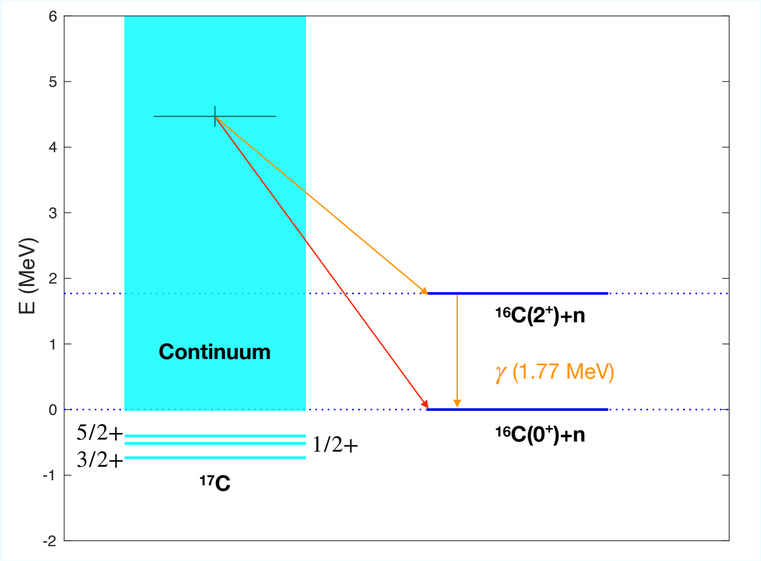
$$\mathcal{F}(k,\theta) \approx \sum_n \langle \Psi_{^{17}C}(k,r) \,|\, \Psi_n^{THO}(r) \rangle \mathcal{F}_n^{THO}(\theta)$$

Transfer to the continuum: Decay mode

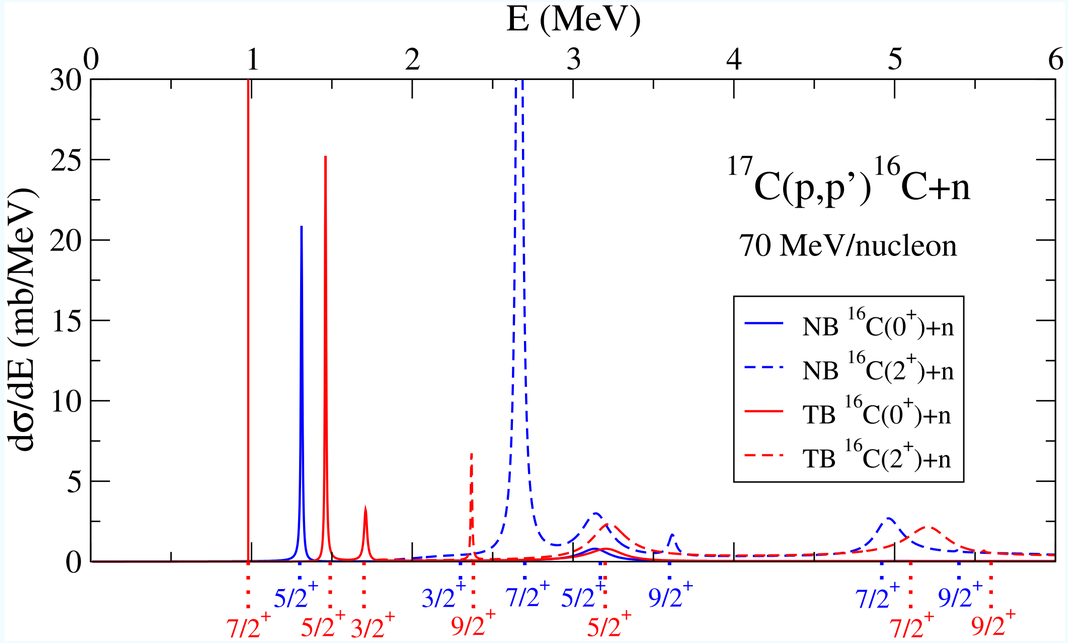
- The part of the cross section that involves a decay from an unbound state of ^{17}C to ^{16}C(2^{+})+*n* is calculated



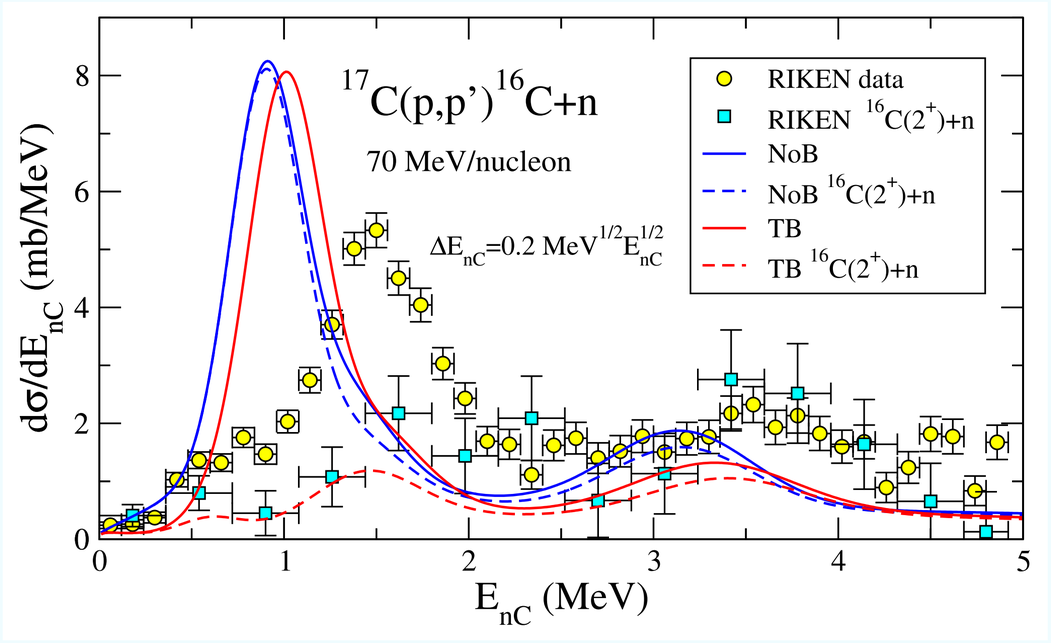
^{17}C → ^{16}C+*n*



Application to $^{17}C(p,p')^{16}C+n$

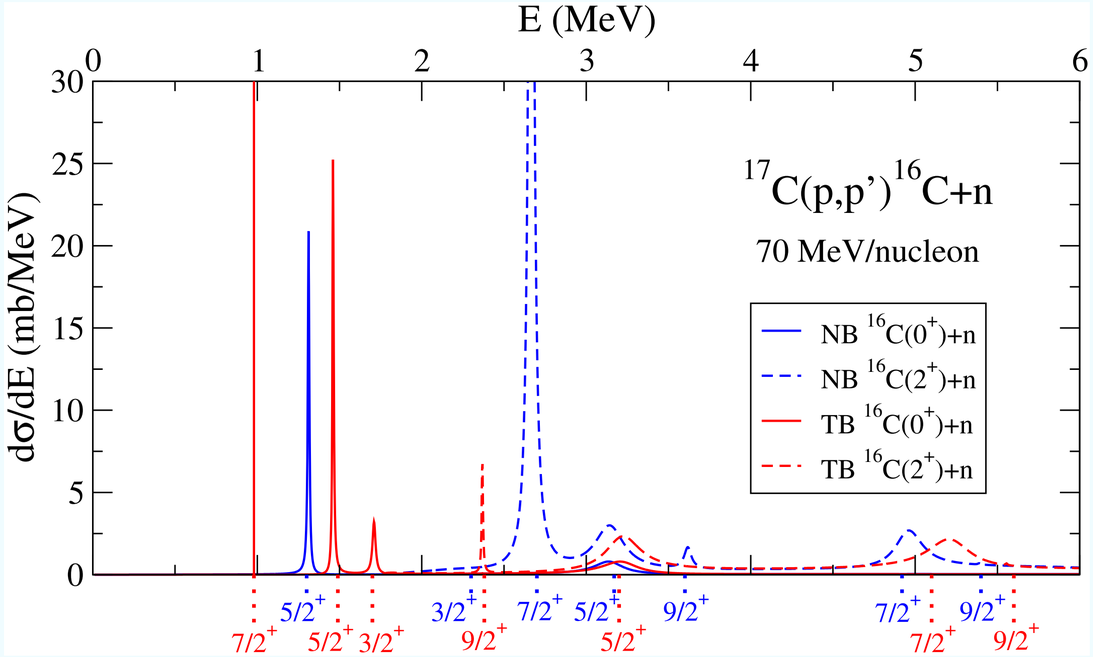


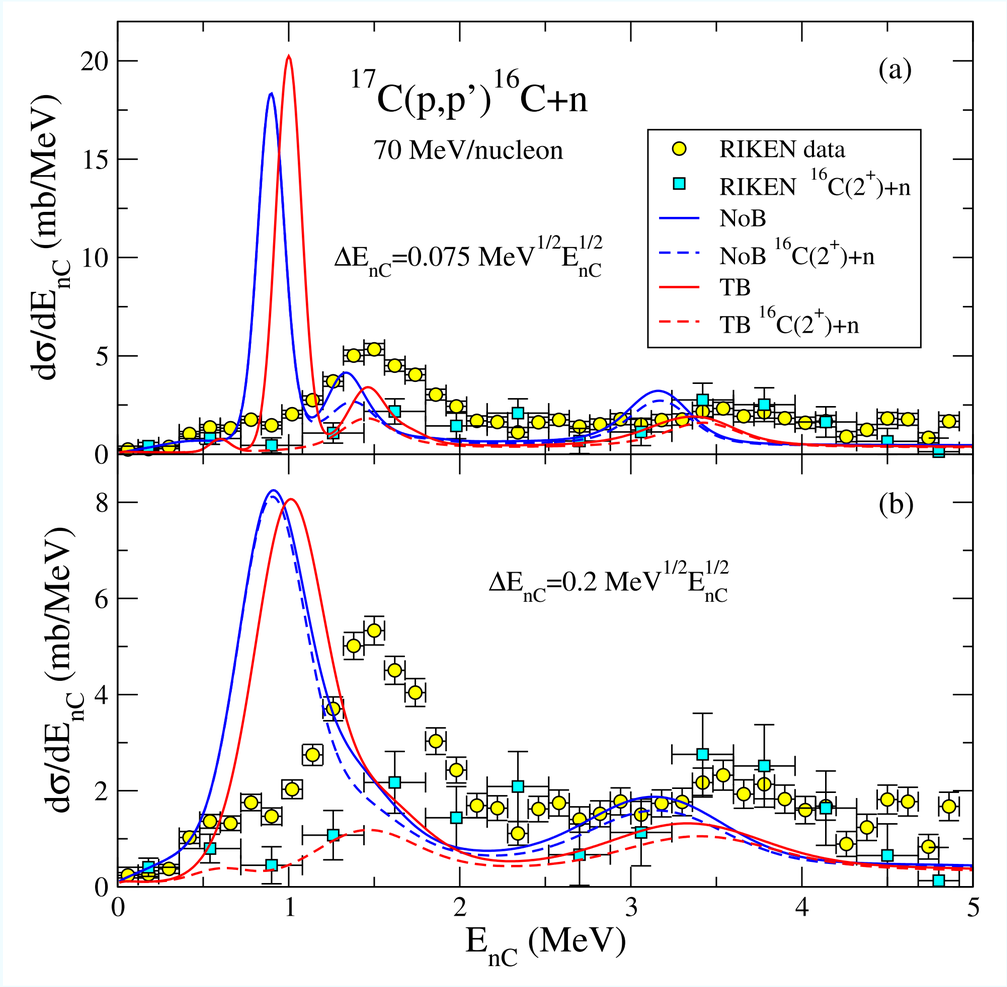
Energy with respect to $^{16}C(0^+)+n$ threshold



Relative energy of the ^{16}C-n system

Application to $^{17}C(p,p')^{16}C+n$





^{19}C(*p*,*p*')^{18}C+*n*: effect of ^{18}C+*p* *interaction*

Resonant Breakup