Caen, September 22-26, 2025

EuNPC2025

Two-neutron decays within the hyperspherical framework: Application to ^{13}Li, ^{16}Be and ^{21}B

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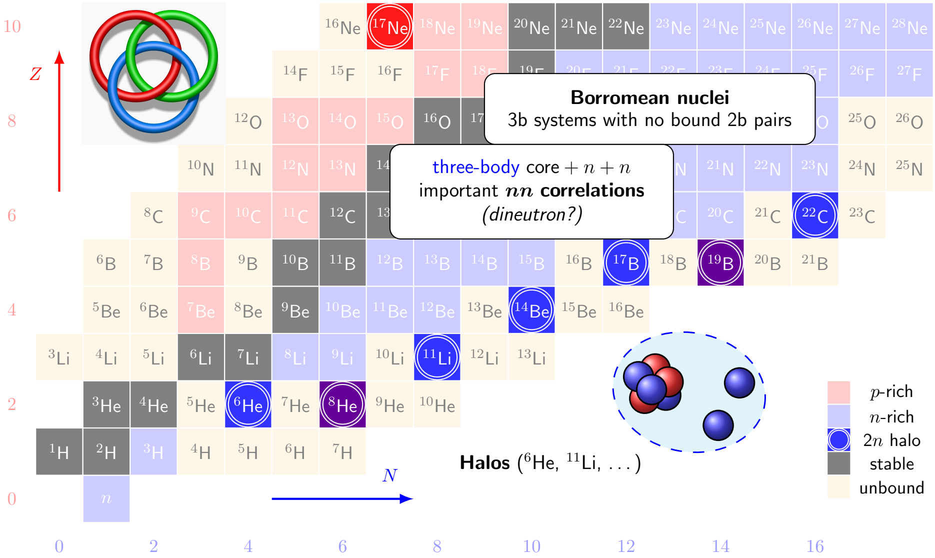
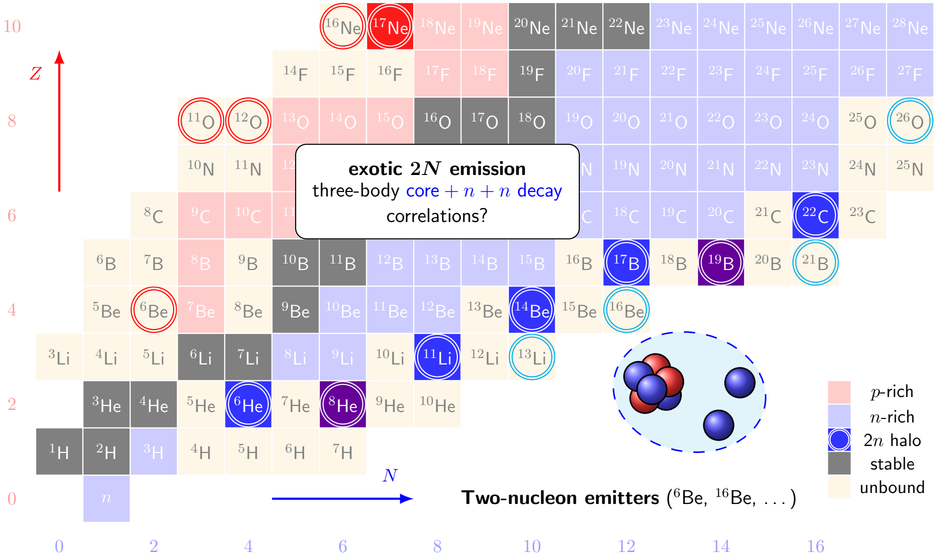
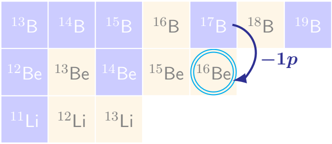

Borromean nuclei
3b systems with no bound 2b pairs
three-body core + n + n
important nn correlations
(dineutron?)
Halos (^{6}He, ^{11}Li, ...)
Z
N
p-rich
n-rich
$2n$ halo
stable
unbound



Two-nucleon emitters (^{6}Be, ^{16}Be, . . .)



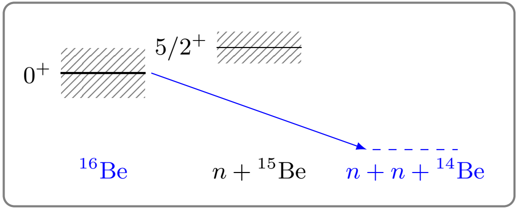
Example: ^{16}**Be**

$1n$ bound
$2n$ unbound

Possible $2n$ decay paths:

- Sequential
- Simultaneous ("dineutron"-like ??)

two-neutron correlations in the g.s. resonance?



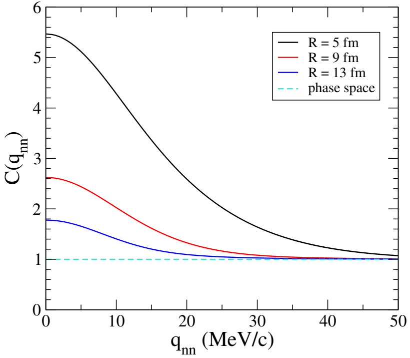
➢ Study the link between dineutron correlations in initial state and the dynamics of two-neutron decay

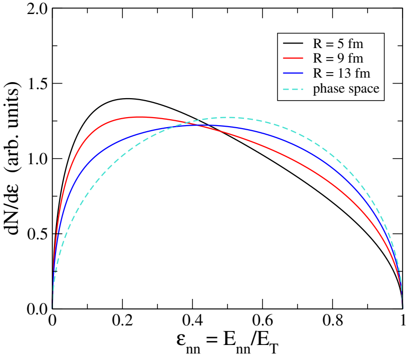
A simple picture of $2n$ decay - **Lednicky model**

➢ FSI model, s-wave dominated (large scattering length)

$$a_{nn} \approx -18.5 \text{ fm}$$
$$d_{nn} \approx 2.8 \text{ fm}$$

Sensitivity to the size of the (Gaussian) source R.

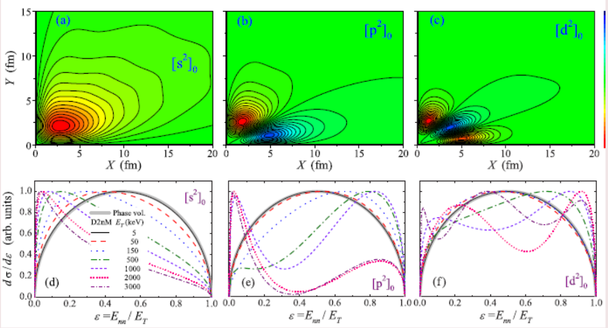




[$C(q_{nn})$ correlation function from: Lednicky, SJNP 35 (1982) 770]

Grigorenko *et al.* [PRC 97 (2018) 034605]

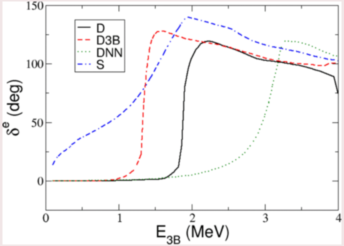
Dynamical dineutron model (Green's function)



A. Lovell *et al.* [PRC 95 (2017) 034605]

Hyperspherical R-matrix method
"true" continuum ^{16}Be (0^+ res.)

$E_{3B} = 1.35$ MeV, $\Gamma = 0.17$ MeV
(and clear $2n$ configuration)

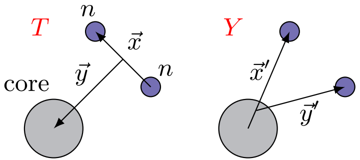


S. Wang and W. Nazarewicz [PRL 126 (2021) 034605]

Time-dependent approach ^{6}Be, ^{6}He'
⇒ dynamics of two-nucleon emission at long times and large distances

time evolution favors low relative angular momenta

Three-body description: Jacobi and hyperspherical coordinates



$$\boxed{\Psi^{j\mu}(\rho,\Omega) = \rho^{-5/2}\sum_{\beta}\chi^{j}_{\beta}(\rho)\mathcal{Y}^{j\mu}_{\beta}(\Omega)}$$

$$\rho = \sqrt{x^2+y^2}, \quad \tan\alpha = x/y$$

$\mathcal{Y}^{j\mu}_{\beta}(\Omega)$ expanded in hyperspherical harmonics; $\beta \equiv \{K, l_x, l_y, l, S\}_j$

$$\left[\frac{-\hbar^2}{2m}\left(\frac{d^2}{d\rho^2} - \frac{15/4 + K(K+4)}{\rho^2}\right) - \varepsilon\right]\chi^{j}_{\beta}(\rho) + \sum_{\beta'} V^{j\mu}_{\beta'\beta}(\rho)\chi^{j}_{\beta'}(\rho) = 0$$

➢ three-body barrier determined by hypermomentum K (even if $K=0$)

➢ $V^{j\mu}_{\beta'\beta}(\rho)$: binary potentials + three-body force

Pseudostate method $\Rightarrow$ expand $\chi^j_\beta(\rho)$ in a discrete basis (diagonalize H)

$$\chi^j_{\beta n}(\rho) = \sum_{i=0}^{N} C^j_{i\beta n} U_{i\beta}(\rho)$$

- $\varepsilon_n < 0$ **bound states**
- $\varepsilon_n > 0$ **discretized continuum**

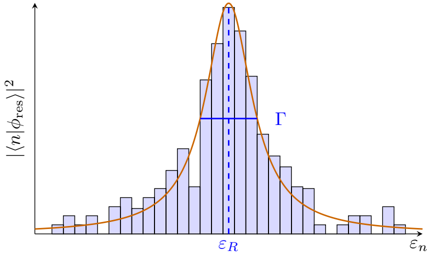
➢ use of THO basis [J.C., PRC 88 (2013) 014327]

$\Rightarrow$ Diagonalize a **resonance operator** in a pseudostate basis $\{|n\rangle\}$

$$\boxed{\widehat{M} = \widehat{H}^{-1/2}\widehat{V}\widehat{H}^{-1/2}, \qquad \widehat{M}|\phi_{\text{res}}\rangle = m|\phi_{\text{res}}\rangle; \qquad |\phi_{\text{res}}\rangle = \sum_n \mathcal{C}_n |n\rangle}$$

- Identify resonances as localized states with $m \ll 0$.
- The expansion allows to build energy distributions.

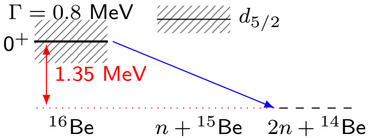
[J.C., J. Gómez-Camacho, PRC **99** (2019) 014604]

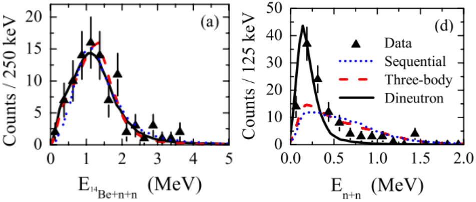


^{16}Be (^{14}Be $+ n + n$)

Spyrou et al. [PRL**108**(2012)102501]

^{9}Be target; 53 AMeV @MSU





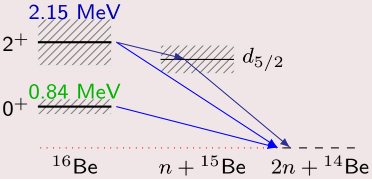
CLAIMS: Decay of a dineutron!

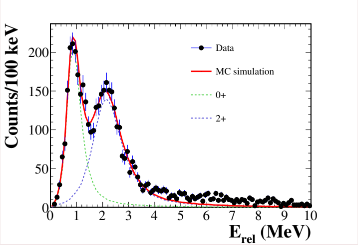
(Simple model, e.g. Lednicky (1982):
More dineutron $\Rightarrow$ stronger E_{nn} signal)

Populated via proton removal from ^{17}B

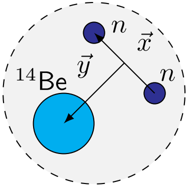
Monteagudo et al. PRL132(2024)082501

[NEW] p target; 280 AMeV @RIKEN





Three-body calculations for ^{16}Be

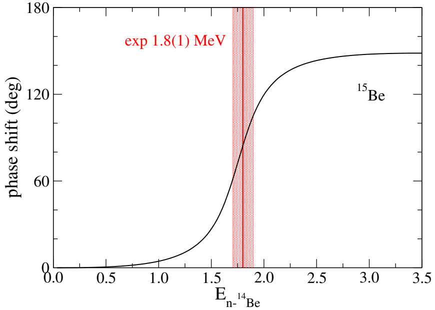


Ingredients for the calculations

- $V_{\text{c-}n}$: WS adjusted to ^{15}Be resonance ($d_{5/2}$)
- $V_{n\text{-}n}$: Gogny-Pires-Tourreil (GPT) tensor interaction
- V_{3b}: three-body force to correct g.s. energy

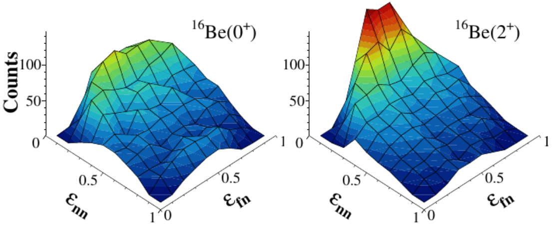
➢ Inert ^{14}Be core (0^+)
[no excited bound states ✓]

➢ Pauli states removed via supersymmetric transformation
[phase-equivalent potentials ✓]



[B. Monteagudo *et al.*, PRL132(2024)082501]

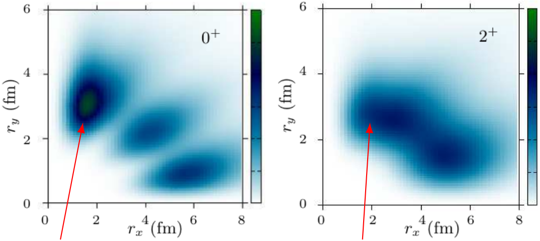
Dalitz plots:



3b model:

0^+ g.s.

$E_r = 0.85$ MeV
$\Gamma = 0.10$ MeV



dineutron correlation

weaker dineutron

signature of these structure properties in the decay?

2^+

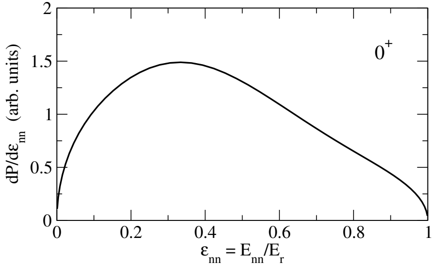
$E_r = 2.15$ MeV
$\Gamma = 0.42$ MeV

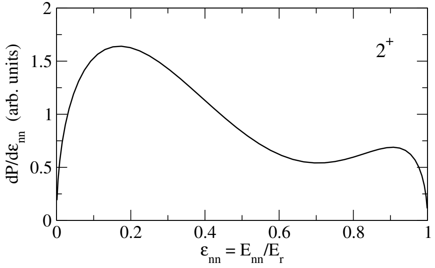
Solve inhomogeneous equation: $(H_0 - E_r)|\Psi_r\rangle = \lambda|\Phi_s\rangle$

$$|\Phi_s\rangle = \sum_{nm} |n\rangle \left(\frac{-\langle n|V|m\rangle}{\varepsilon_m - E_r} + \delta_{nm} \right) \langle m|\phi\rangle$$

- $|\Phi_s\rangle$: short-range, from $\widehat{M}$ eigenstate $|\phi\rangle$
- $|\Psi_r\rangle$: resonance at all distances $\rightarrow$ fragment correlations (e.g., E_{nn})

Asymptotically: $\langle \rho, \beta|\Psi_r\rangle \rightarrow \mathcal{A}_\beta (k\rho)^{1/2} H^+_{K+2}(k\rho)$ (J.C., J.G.C, in preparation)





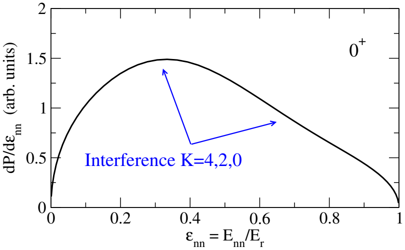
$$P(\alpha) \propto \sum_{l_x,l_y,l,S} \sum_{K,K'} \mathcal{A}_\beta (\mathcal{A}_{\beta'})^* (-i)^{K-K'} \varphi_K^{l_x,l_y}(\alpha)\varphi_{K'}^{l_x,l_y}(\alpha),$$

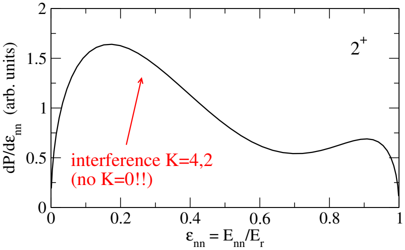
- $\varphi_K^{l_x,l_y}(\alpha) \propto$ Jacobi polynomials in $\cos(2\alpha)$ of order $n = \frac{K-l_x-l_y}{2}$; hyperangle $\alpha \longrightarrow$ relative E_{nn}

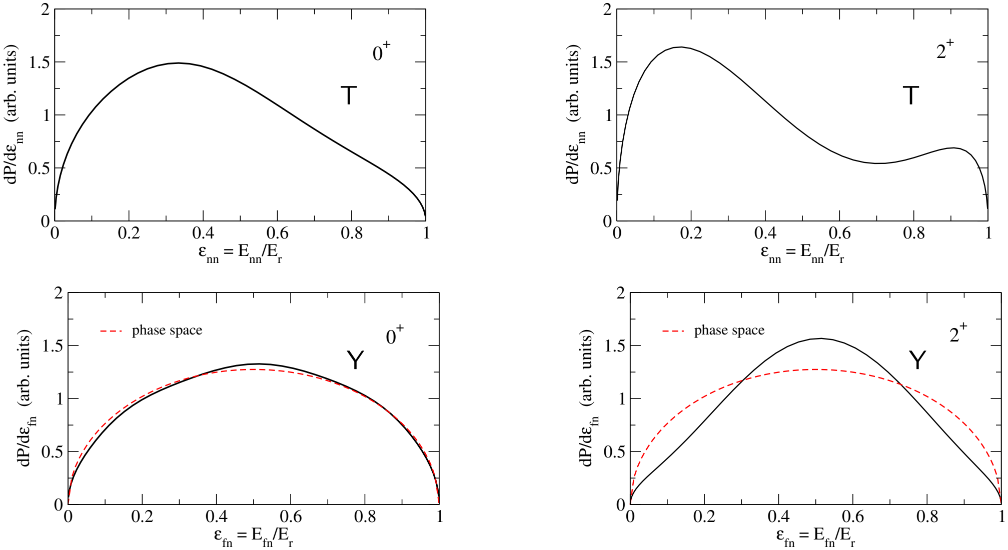
$\mathcal{A}_\beta$ for lowest possible K values are favored [consistent with Wang & Nazarewicz PRL126(2021)034605]

- For ^{16}Be, $K=4$ mostly. Asymptotically, $K=2$ and 0 compete
- $j^\pi = 0^+ \Rightarrow K_{min} = 0$; $j^\pi = 2^+ \Rightarrow K_{min} = 2$

$\Rightarrow P(\varepsilon_{nn})$ for the 2^+ state is more asymmetric (no $K=0$ terms)



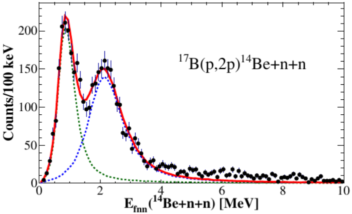


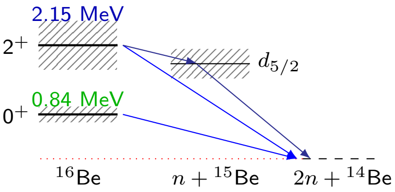


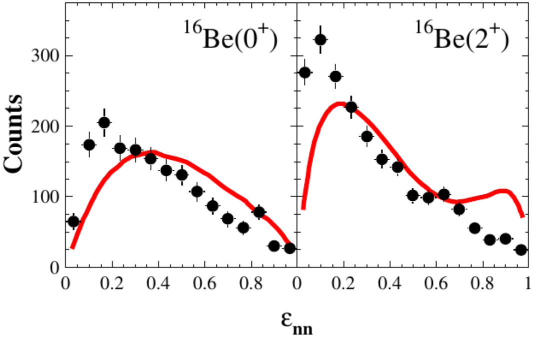
Consistent with direct decay! (no distinct peaks in ε_{fn} dist.)

Comparison with data

[B. Monteagudo *et al.*, PRL132(2024)082501]







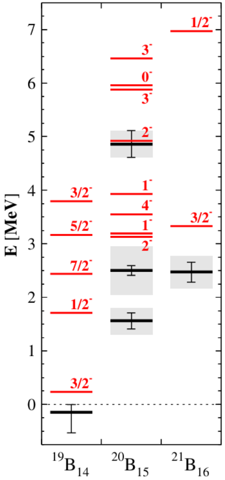
Overall trend reproduced

Possible improvement:

➢ deformed core??

2^+ more pronounced peak
(though less dineutron corr.)

^{21}B (^{19}B $+ n + n$)

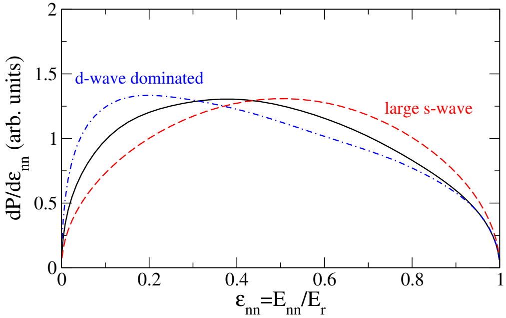


[Data: Leblond PRL121(2018)262502]

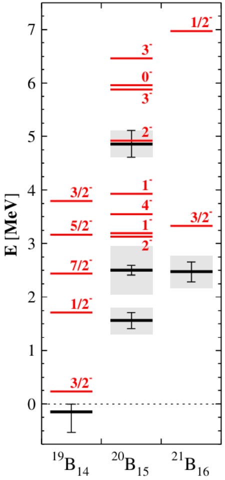
Consistent with g.s. res. at $E_{\text{res}} \approx 2.4$ MeV

Preliminary 3b calculations:

➢ $^{19}\text{B} + n$: $s_{1/2}$, $d_{5/2}$
Sensitivity to partial-wave content



^{21}B (^{19}B $+ n + n$)

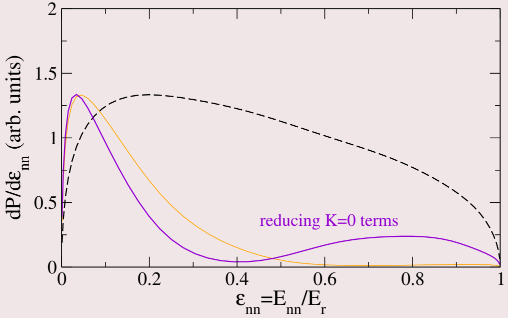


[Data: Leblond PRL121(2018)262502]

Consistent with g.s. res. at $E_{\text{res}} \approx 2.4$ MeV

Preliminary 3b calculations:

ε_{nn} data under analysis show minimum ~ 0.5
[Oliveira, Marqués, private communication]



stay tuned!

Summary and outlook

- The **continuum of three-body systems**, such as $2n$ emitters, can be described using pseudostates within the hyperspherical framework and the eigenstates of a resonance operator.
- ^{16}Be ($^{14}\text{Be} + n + n$): 0^+ g.s. resonance and excited 2^+

  $\Rightarrow$ distinct nn configurations
- Decay-energy distributions: Solve inhom. equation $\rightarrow$ asymptotics

  Results for ^{16}Be; data trend reproduced

  $\Rightarrow$ ε_{nn}, ε_{fn} suggest direct decay

  $\Rightarrow$ a more compact dineutron does not imply larger ε_{nn} signal

  [Results published in PRL132(2024)082501]
- Very preliminary ^{21}B ($^{19}\text{B} + n + n$): low-lying g.s. resonance

  $\Rightarrow$ Sensitivity to partial-wave content
- *To be done:*

  - Pauli / core excitation effects (deformed ^{14}Be, ^{19}B)

Collaborators:

J. Gómez-Camacho (Universidad de Sevilla)

B. Monteagudo (Hope Univ.), E. Oliveira, F. M. Marqués (LPC Caen), *et al.*

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Grant agreement 101023609

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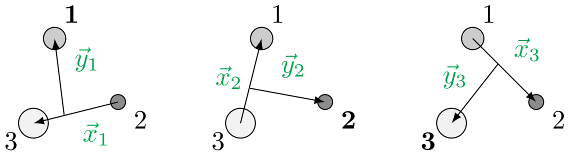
PID2023-146401NB-I00



Jacobi coordinates
$\{x, y, \widehat{x}, \widehat{y}\}$

Hyperspherical coordinates $\{\rho, \alpha, \widehat{x}, \widehat{y}\}$ $\rho = \sqrt{x^2 + y^2}$ $\alpha = \arctan \dfrac{x}{y}$

$$\Psi^{j\mu}(\rho, \Omega) = \rho^{-5/2} \sum_{\beta} \chi_{\beta}^{j}(\rho) \mathcal{Y}_{\beta}^{j\mu}(\Omega) \qquad \beta \equiv \{K, l_x, l_y, l, S_x, J; I\}_j$$

Hyperspherical Harmonics (HH) expansion — hypermomentum $\widehat{K}$

$$\mathcal{Y}_{\beta}^{j\mu}(\Omega) = \left[\left(\Upsilon_{Klm_l}^{l_x l_y}(\Omega) \otimes \kappa_{S_x}\right)_J \otimes \phi_I\right]_{j\mu}$$

$$\Upsilon_{Klm_l}^{l_x l_y}(\Omega) = \varphi_K^{l_x l_y}(\alpha) \left[Y_{l_x}(\widehat{x}) \otimes Y_{l_y}(\widehat{y})\right]_{lm_l}$$

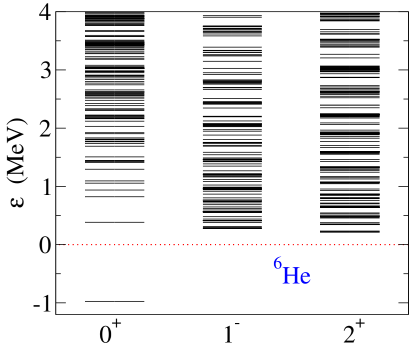
$$\varphi_K^{l_x l_y}(\alpha) = N_K^{l_x l_y} (\sin\alpha)^{l_x} (\cos\alpha)^{l_y} P_n^{l_x+\frac{1}{2}, l_y+\frac{1}{2}}(\cos 2\alpha)$$

Pseudostate method $\Rightarrow$ expand $\chi^j_\beta(\rho)$ in a discrete basis (diagonalize H)

$$\chi^j_{\beta n}(\rho) = \sum_{i=0}^{N} C^j_{i\beta n} U_{i\beta}(\rho)$$

- $\varepsilon_n < 0$ **bound states**
- $\varepsilon_n > 0$ **discretized continuum**

$U_{i\beta}(\rho)$: Analytical Transformed Harmonic Oscillator (THO) basis
$\Rightarrow$ enables high concentration of discretized states close to threshold

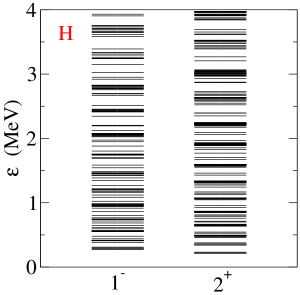


(e.g., J.C. *et al.* [PRC 88 (2013) 014327])

- suitable for reactions
- excitations into continuum

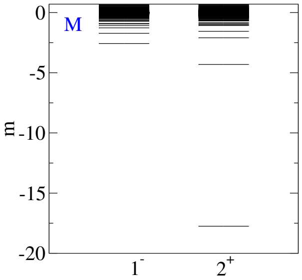
But no simple characterization of resonant states!

Identifying and characterizing few-body resonances: A new approach



Ex: ^{6}He $(\alpha + n + n)$
non-res. 1^-
2^+ resonance

$\widehat{H}|n\rangle = \varepsilon_n|n\rangle$
mix res. and non-res.



⇒ Diagonalize a **resonance operator** in a PS basis $\{|n\rangle\}$

$$\widehat{M} = \widehat{H}^{-1/2}\widehat{V}\widehat{H}^{-1/2}, \qquad \widehat{M}|\phi\rangle = m|\phi\rangle; \qquad |\phi\rangle = \sum_n \mathcal{C}_n|n\rangle$$

- Selection of resonant states, which are more localized
- The expansion in terms of $|n\rangle$ allows to build energy distributions.

[J.C., J. Gómez-Camacho, PRC **99** (2019) 014604]

Decay $\Rightarrow$ time evolution:

$$|\psi(t)\rangle = \sum_n \mathcal{C}_n e^{-i\varepsilon_n t}|n\rangle$$

Amplitudes:

$$a(t) = \langle\psi(0)|\psi(t)\rangle = \sum_n |\mathcal{C}_n|^2 e^{-i\varepsilon_n t}$$

For "ideal" BW:

$$a_r(t) = e^{-i\varepsilon_r t - \frac{\Gamma}{2}t}$$

Resonance quality parameter:

$$\boxed{\delta^2\left(\varepsilon_r,\Gamma\right) = \frac{\int_0^\infty e^{-xt}|a(t)-a_r(t)|^2 dt}{\int_0^\infty e^{-xt}|a(t)|^2 dt}}$$

($1/x$: relevant timescale for the resonance formation)

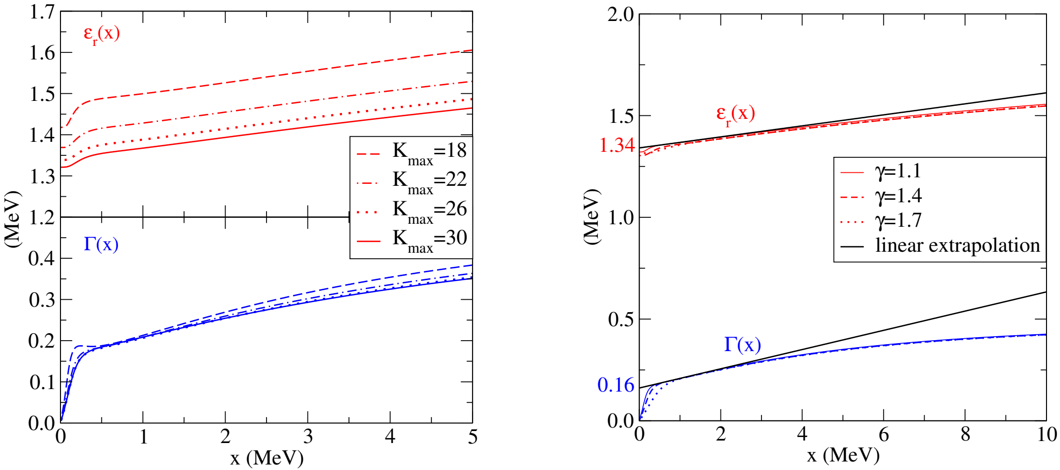
In order to find the resonance parameters ε_r and Γ which best describe the time evolution $a(t)$, we perform a minimization

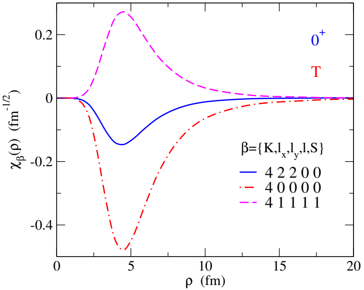
$$\frac{\partial}{\partial\varepsilon_r}\delta^2\left(\varepsilon_r,\Gamma\right) = 0, \quad \frac{\partial}{\partial\Gamma}\delta^2\left(\varepsilon_r,\Gamma\right) = 0$$

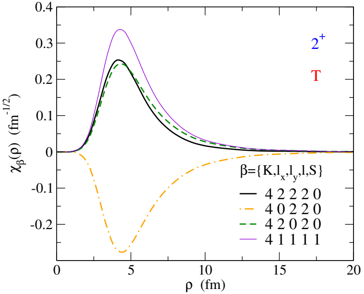
$\Rightarrow$ as a function of x, i.e., $\varepsilon_r(x), \Gamma(x)$

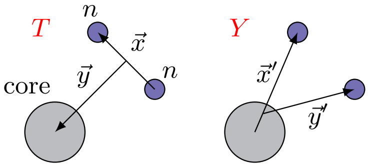
$x \to 0$ limit means long times

Resonance operator: ^{16}Be; Resonance parameters (0^+)









- both 0^+ and 2^+ governed by $K = 4$
- 94.8% and 96.5% $[d^2]$ waves (Jacobi Y)

- very different l_x content (Jacobi T)

$j^\pi \setminus l_x$	0	1	2
0^+	70.4%	20.7%	7.0%
2^+	24.0%	34.7%	33.9%

Free propagator: $G_0(E_r) = (H_0 - E_r)^{-1} \quad \Rightarrow \quad |\psi_r\rangle = \lambda G_0(E_r)|\Phi_s\rangle$

In the hyperspherical case, for outgoing boundary conditions:

$$\langle \beta, \rho | G_0(E_r) | \beta, \rho' \rangle = \frac{4mi}{\pi \hbar^2 k} F_\beta(k\rho_<) H_\beta^+(k\rho_>); \qquad \frac{\hbar^2 k^2}{2m} = E_r$$

No Coulomb: $F_\beta(x) = x^{1/2} J_{K+2}(x); \quad H_\beta^+(x) = x^{1/2} H_{K+2}^+(x)$

The hyperradial behaviour for each β:

$$\langle \beta, \rho | \psi_r \rangle = \lambda \int d\rho' \langle \beta, \rho | G_0(E_r) | \beta, \rho' \rangle \langle \beta, \rho' | \Phi_s \rangle$$

At $\rho >> \rho'$ (associated to the source term):

$$\langle \beta, \rho | \psi_r \rangle = \frac{4mi\lambda}{\pi \hbar^2 k} (k\rho)^{1/2} H_{K+2}^+(k\rho) \mathcal{A}_\beta$$

$$\mathcal{A}_\beta = \int d\rho (k\rho)^{1/2} J_{K+2}(k\rho) \langle \beta, \rho | \Phi_s \rangle$$