Effective mass of a nucleus immersed in superfluid neutrons in the inner crust of a neutron star

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23rd September 2025, EuNPC2025, Caen

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Agata Zdanowicz

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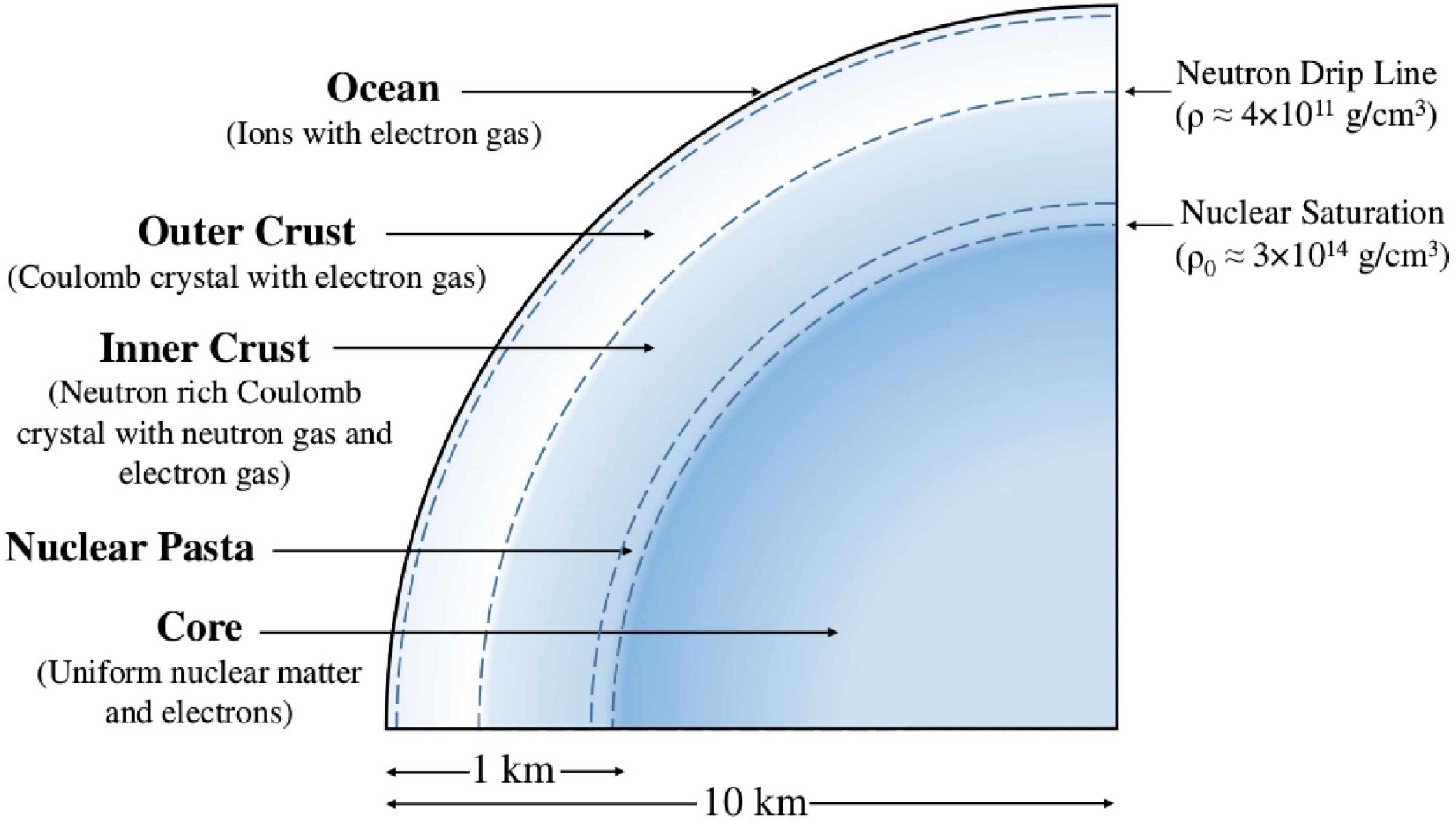
Nicolas Chamel

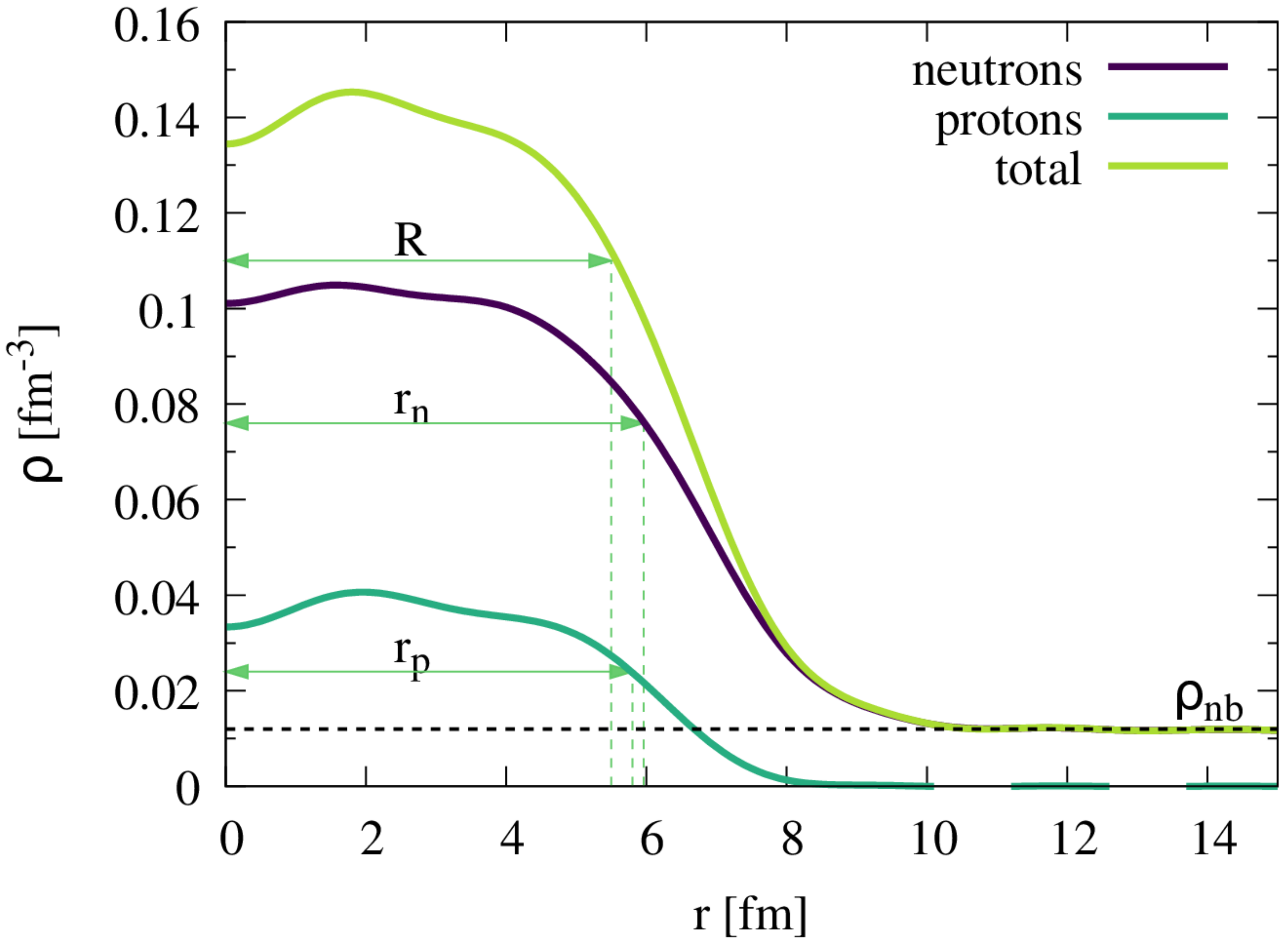
Physical Review X
14, 041054 (2024)

LUMI PLGRID NARODOWE CENTRUM NAUKI

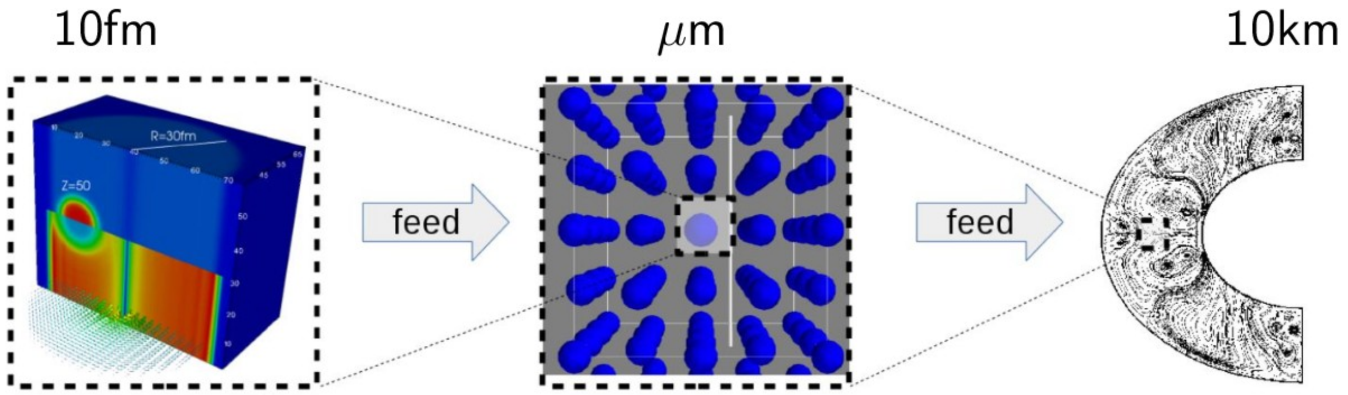
PLGrid LUMI PLL/2022/03/016433 ★ NCN 2021/40/C/ST2/00072

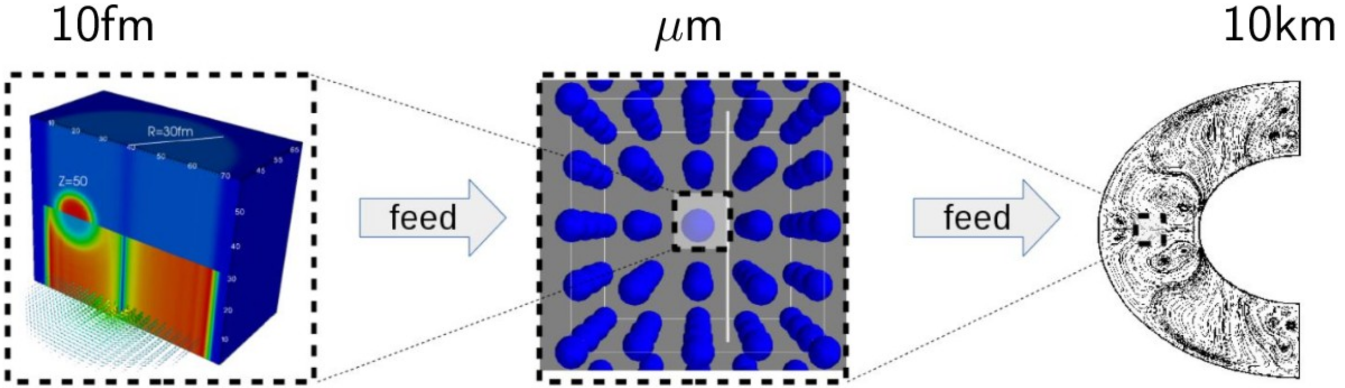


Caplan, M. E., and C. J. Horowitz, Reviews of Modern Physics 89, 041002 (2017)



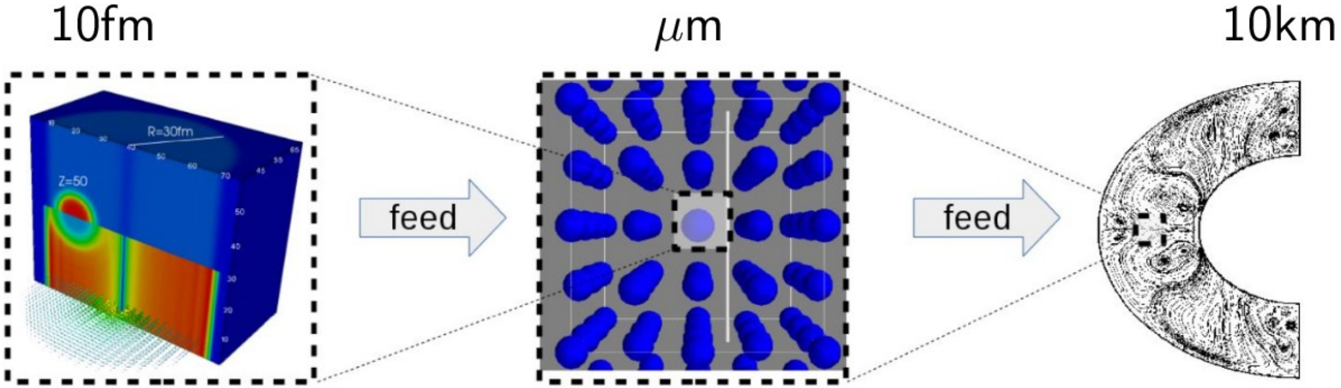
neutrons
protons
total
R
r_n
r_p
ρ_{nb}
ρ [fm^{-3}]
r [fm]
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
10
12
14


10fm
μm
10km
feed
feed
R=30fm
Z=50



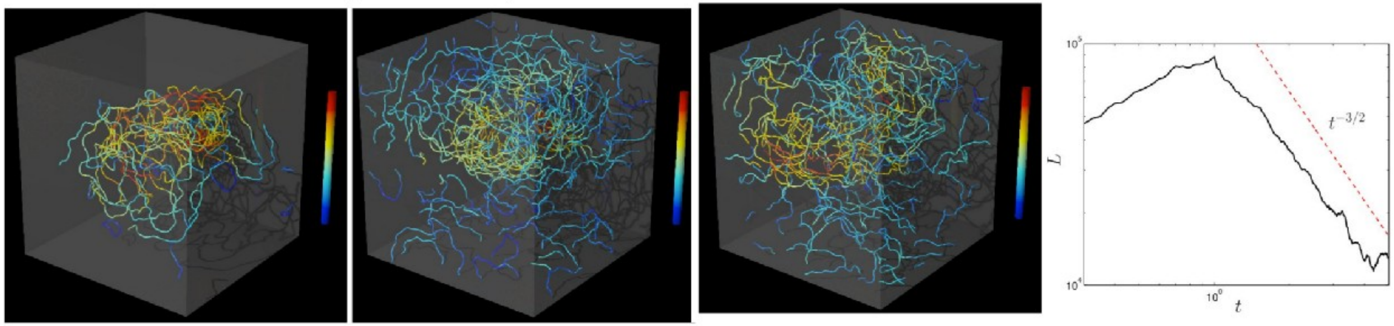
LUMI, Finland (#5 Top 500)

Piz Daint, Switzerland (#37 Top 500)



LUMI, Finland (#5 Top 500)

Piz Daint, Switzerland (#37 Top 500)



R. Hänninen, A. W. Baggaley, Proc. Nat. Acad. Sci. U.S.A. 111, 4667 (2014)

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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$$\rho(r) = \sum_k |v_k(r)|^2$$

$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$

$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$
$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla\frac{\delta\varepsilon}{\delta\tau}\nabla - \frac{i}{2}\left\{\frac{\delta\varepsilon}{\delta\boldsymbol{j}}, \nabla\right\}$$
$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

Superfluid **L**ocal **D**ensity **A**pproximation

A. Bulgac, Physical Review A **76**, 040502 (2007)

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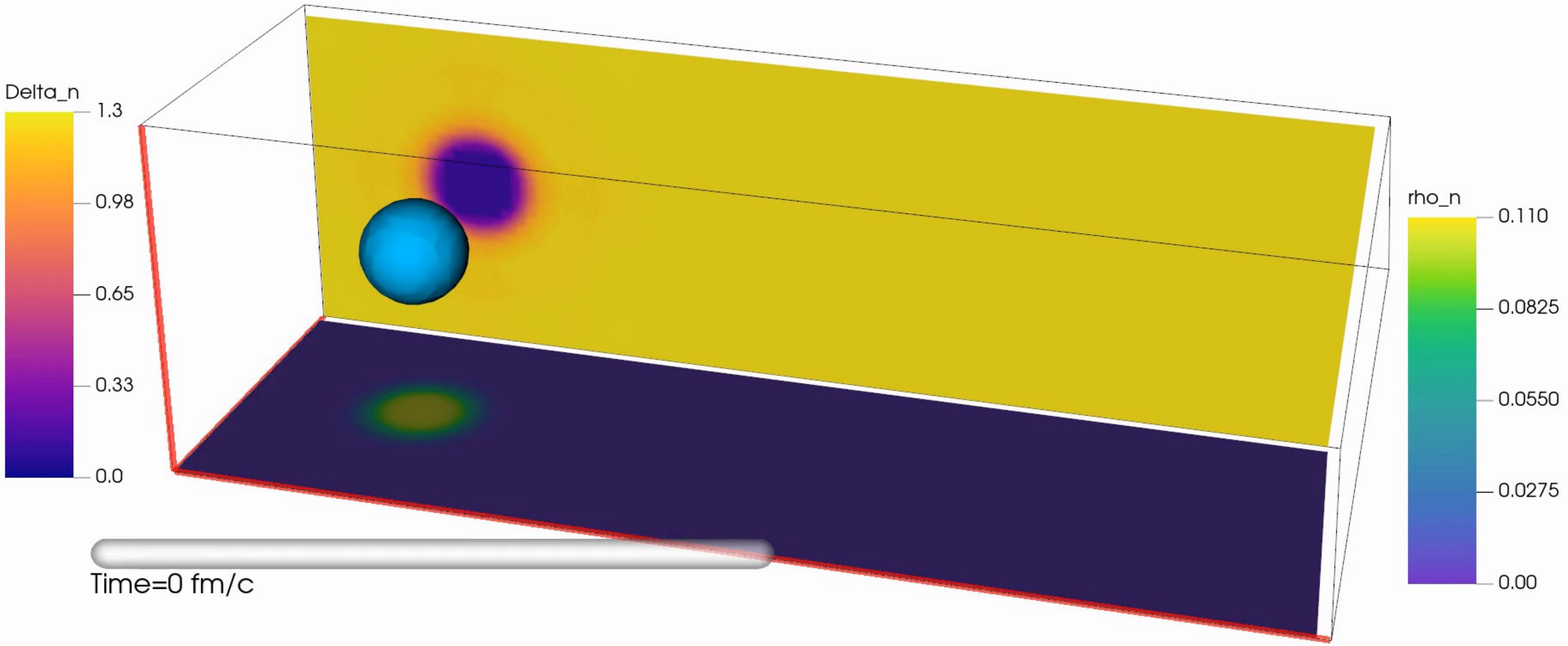
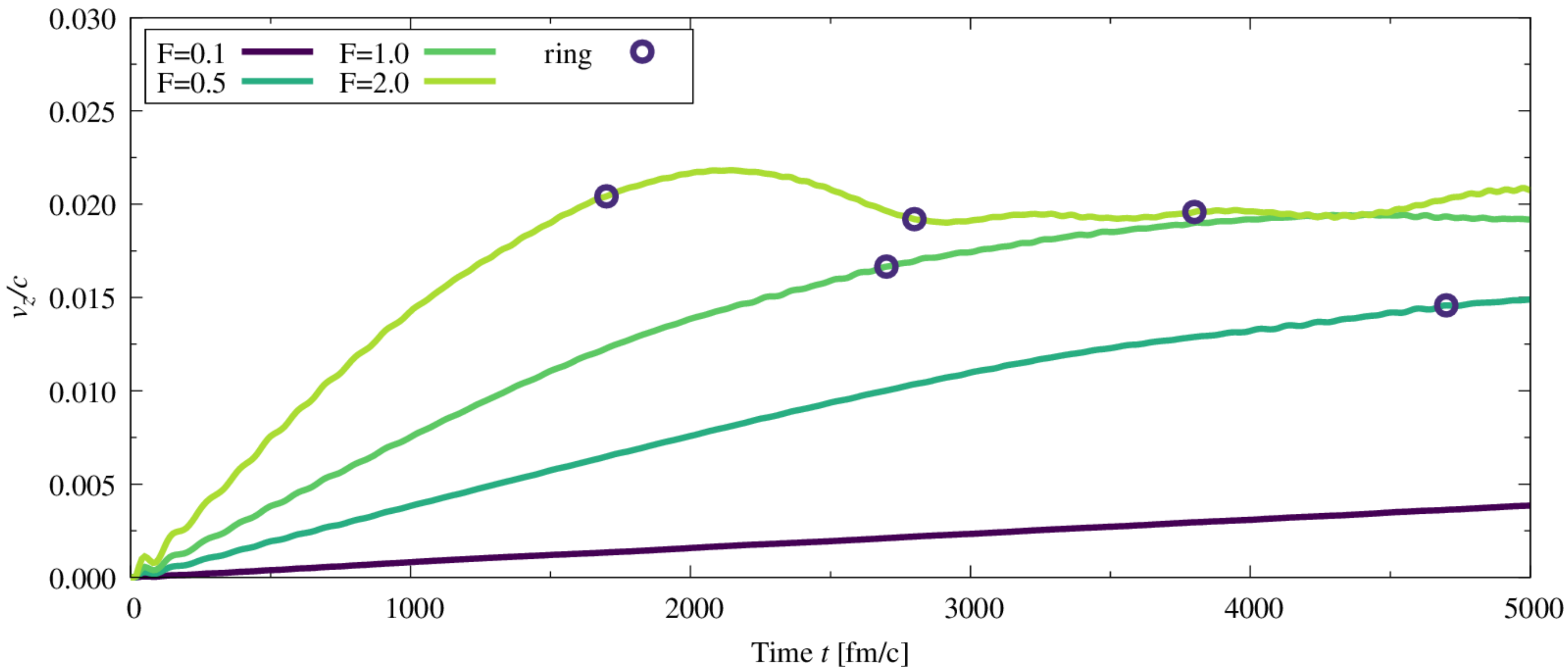
Superfluid **L**ocal **D**ensity **A**pproximation

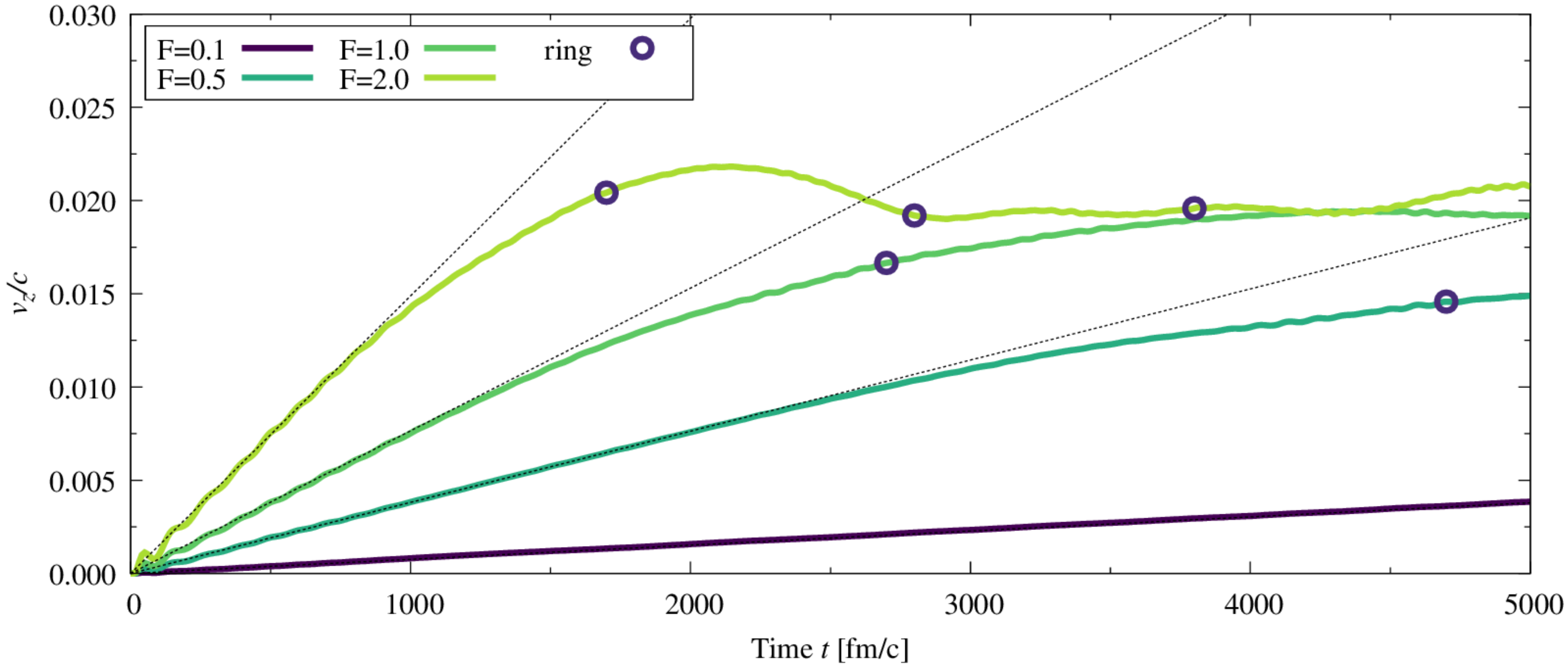
A. Bulgac, Physical Review A **76**, 040502 (2007)

Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

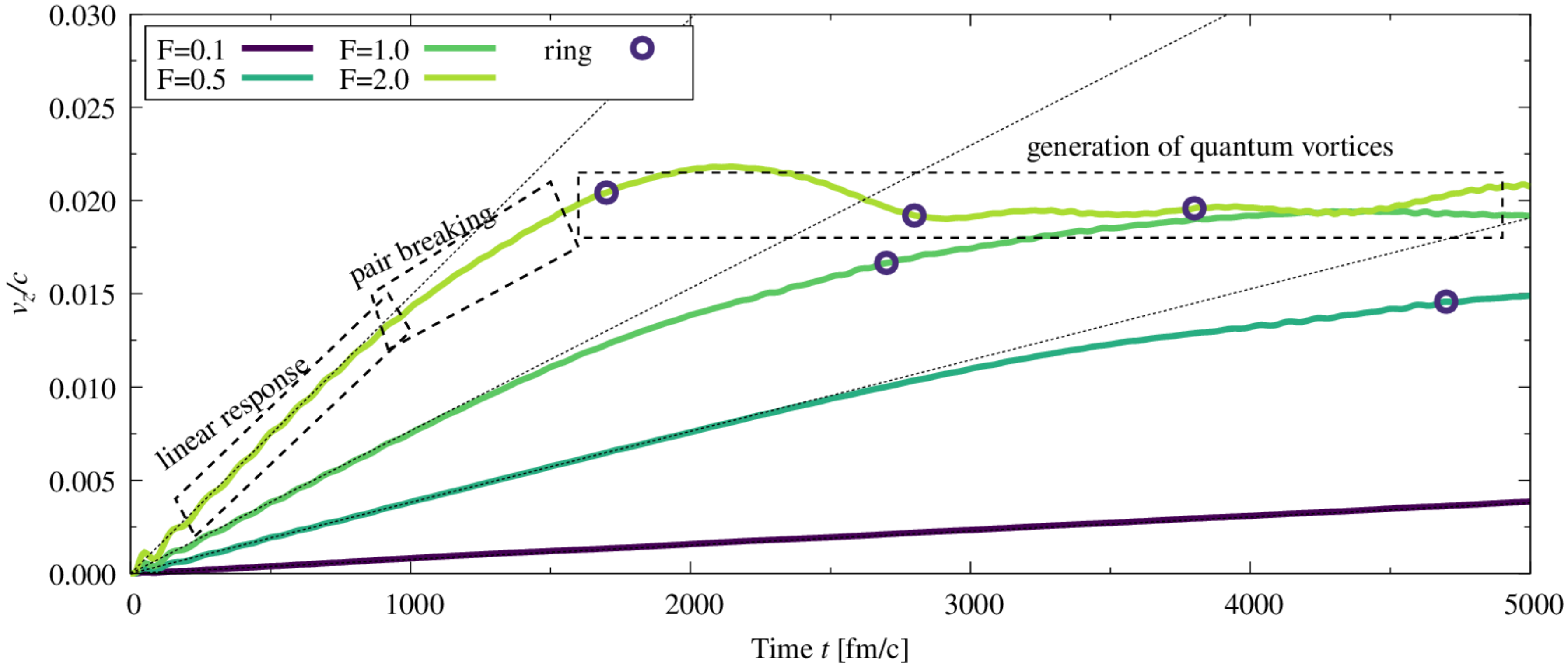

Delta_n
1.3
0.98
0.65
0.33
0.0
rho_n
0.110
0.0825
0.0550
0.0275
0.00
Time=0 fm/c


F=0.1
F=0.5
F=1.0
F=2.0
ring
0.030
0.025
0.020
0.015
0.010
0.005
0.000
v_z/c
0
1000
2000
3000
4000
5000
Time t [fm/c]



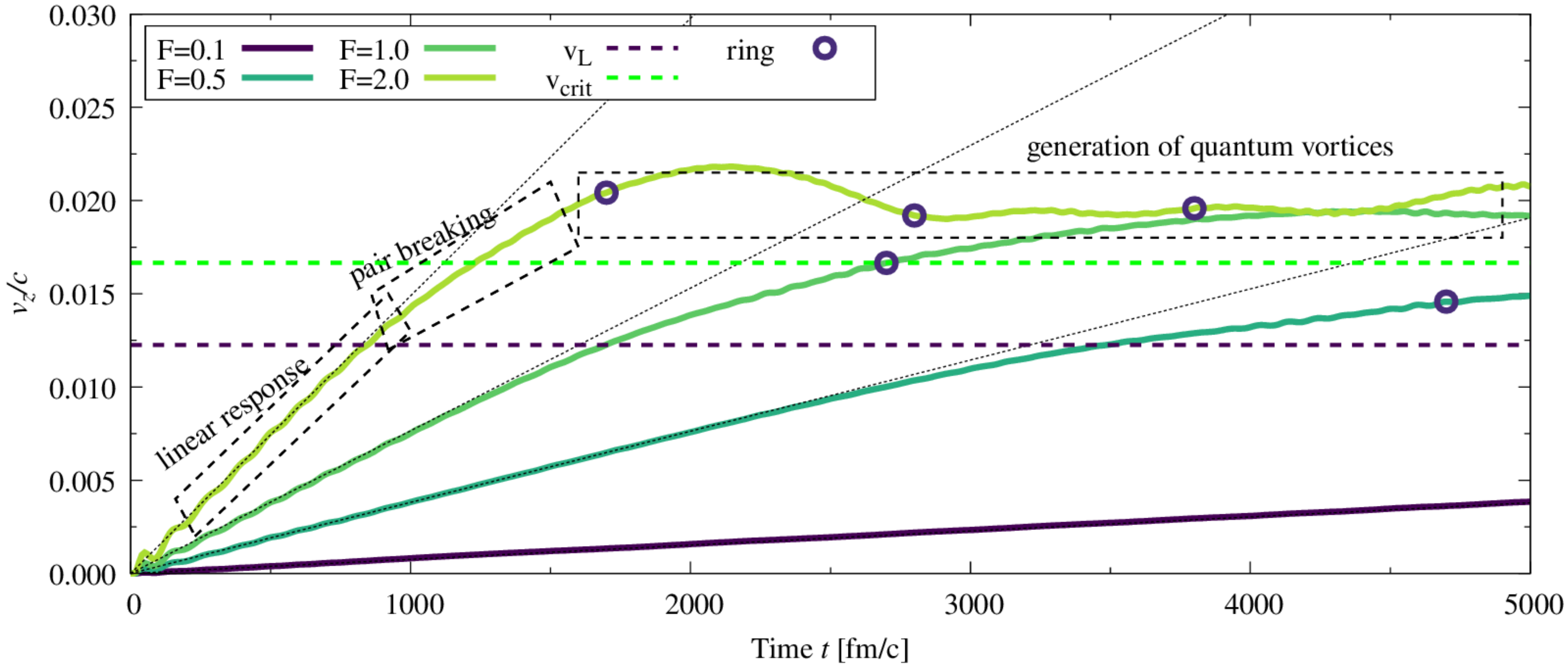
$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$



$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$



$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$

$$v_L = \frac{\Delta}{\hbar k_F}$$

$$v_{\rm crit} = \frac{e}{2}\frac{\Delta}{\hbar k_F}$$

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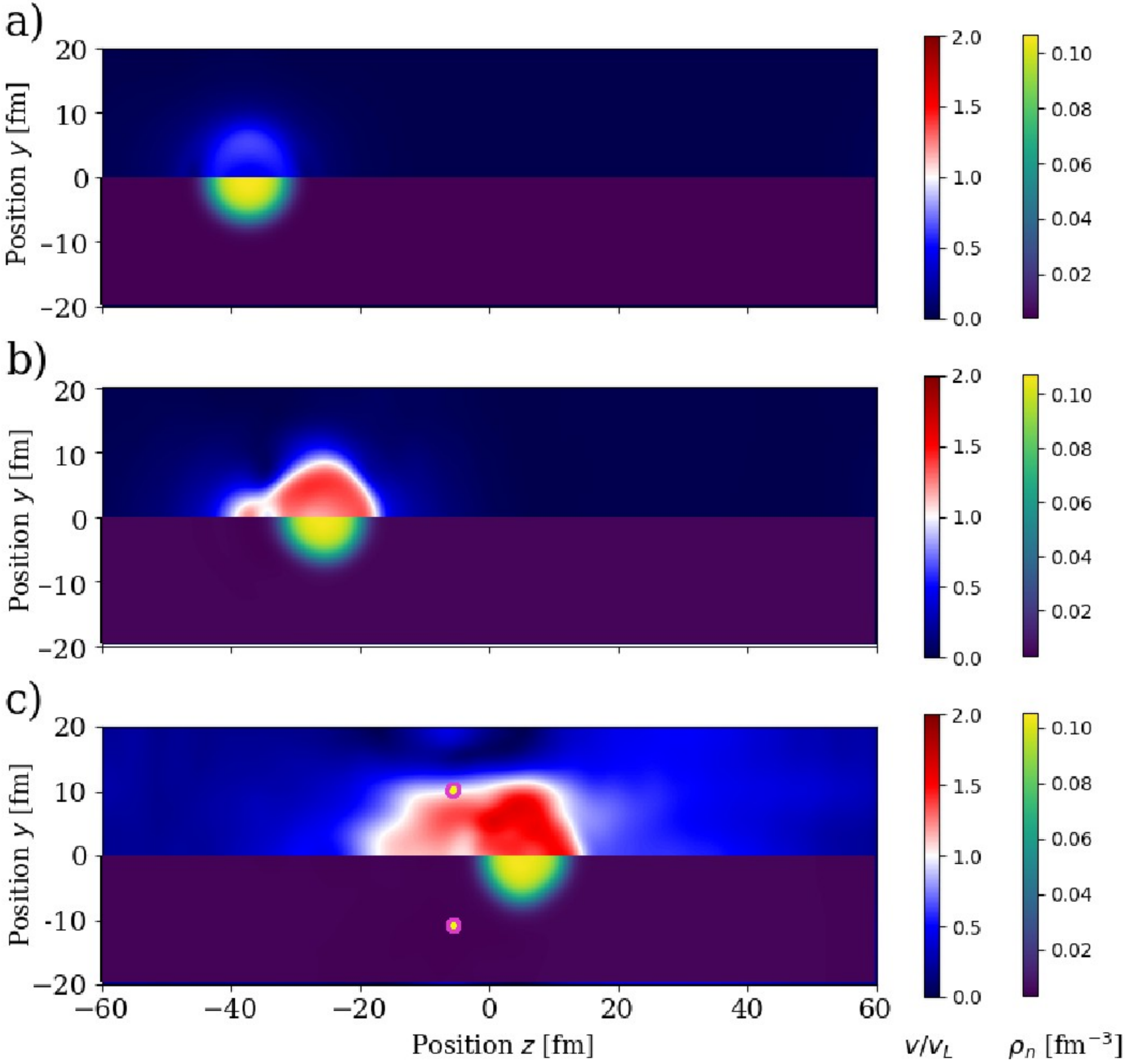
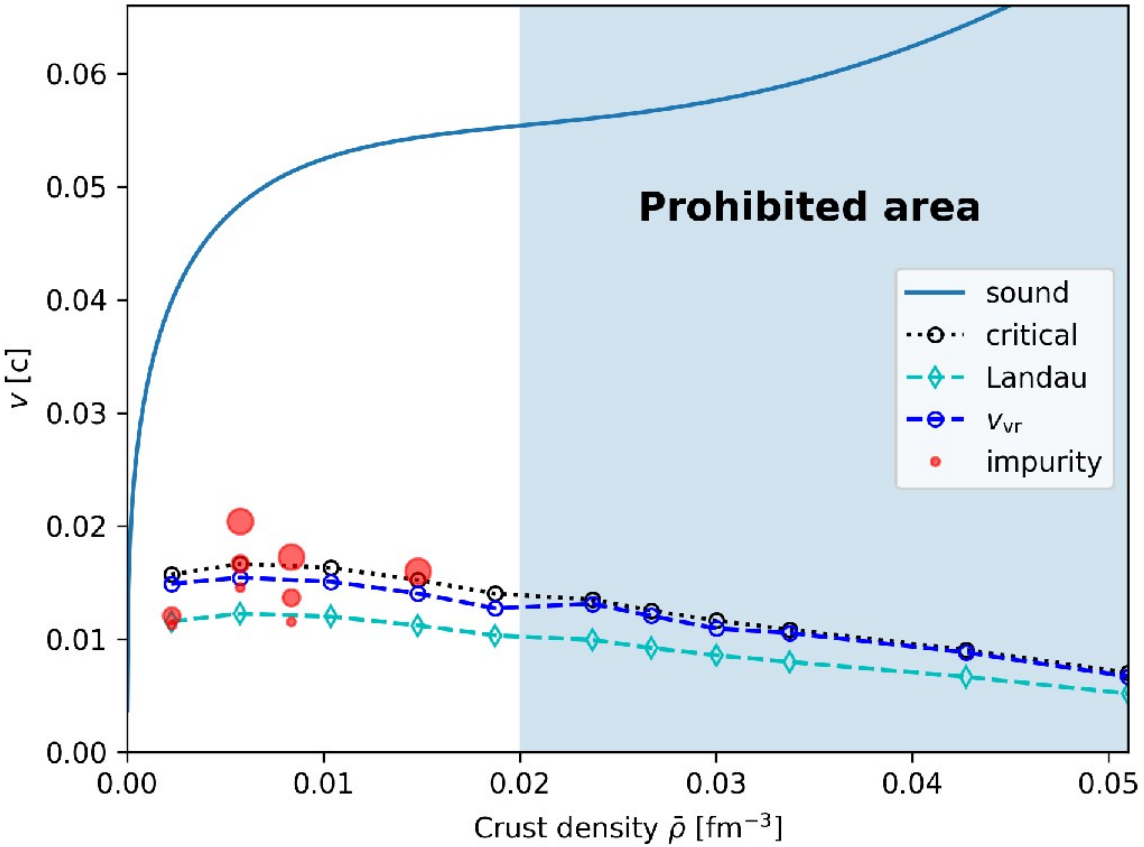
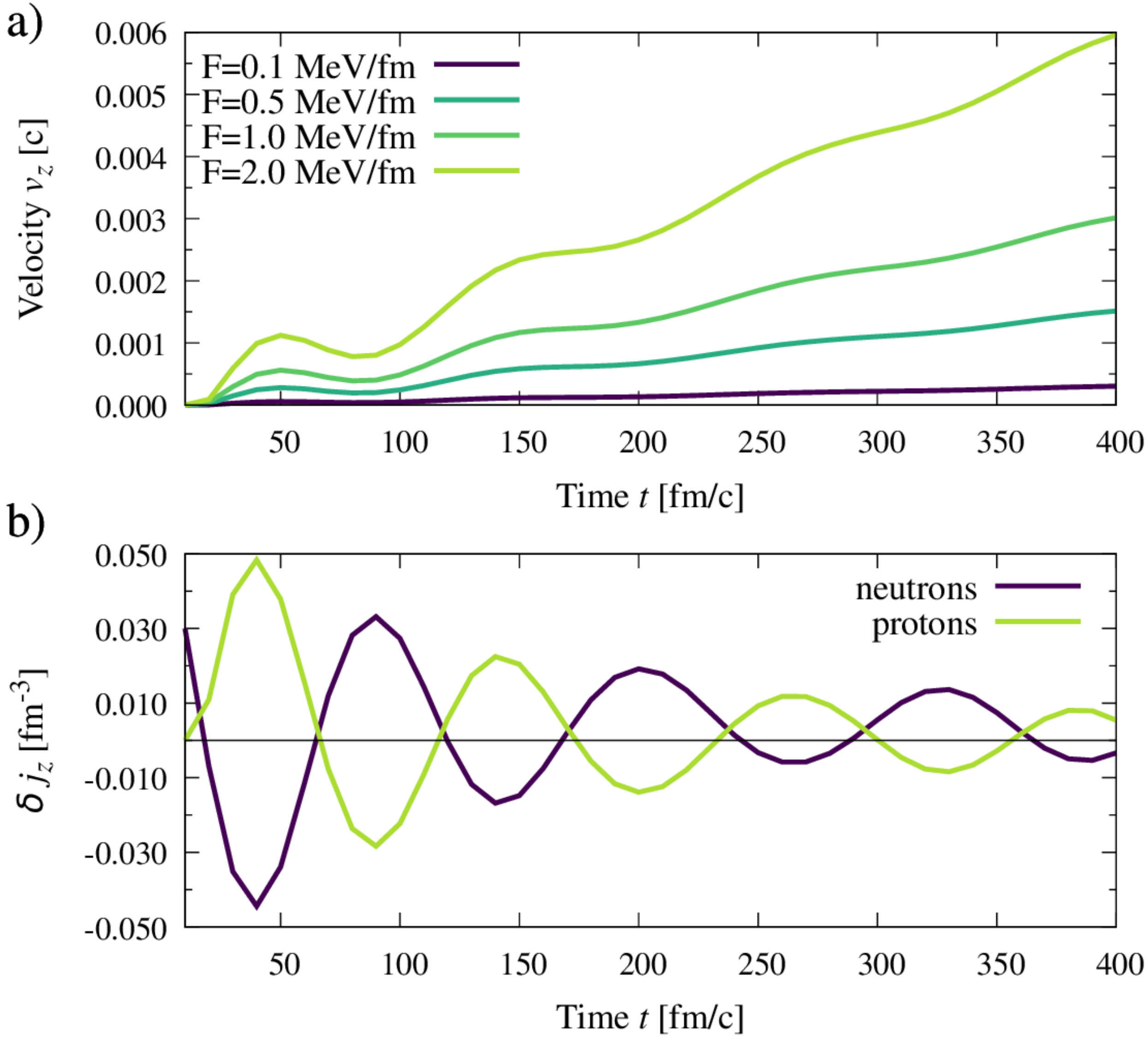
By Robert D Anderson - Own work, CC BY-SA 3.0



a)
b)
c)
Position y [fm]
Position z [fm]
v/v_L
ρ_n [fm^{-3}]

$$v_{vr} = \frac{1}{2\pi R}\frac{\hbar c}{M_n c^2}\left(\ln\frac{8R}{a_{core}} - \alpha\right)$$





a)
0.006
0.005
0.004
0.003
0.002
0.001
0.000
Velocity v_z [c]
F=0.1 MeV/fm
F=0.5 MeV/fm
F=1.0 MeV/fm
F=2.0 MeV/fm
50
100
150
200
250
300
350
400
Time t [fm/c]
b)
0.050
0.030
0.010
-0.010
-0.030
-0.050
δj_z [fm^{-3}]
neutrons
protons
50
100
150
200
250
300
350
400
Time t [fm/c]

Warsaw University of Technology | W-SLDA Toolkit W-BSk Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r}) - \mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

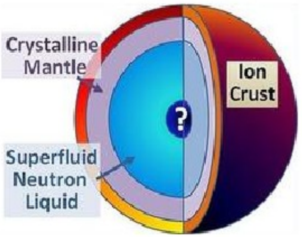
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) - \mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

Extension to nuclear matter in neutron stars

Unified solvers for static and time-dependent problems

Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

ALL FUNCTIONALITIES →

Integration with VisIt: visualization, animation and analysis tool

Speed-up calculations by exploiting Hig
Computing

Functionals fo
BCS and unitar

Getting the code

DOWNLOAD

The W-SLDA & W-BSk Toolkits are free to download. It is published as open source under GNU GPL License. In order to get W-SLDA or W-BSk Toolkit click "Read more" and follow instructions.

READ MORE →

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CA24139 - Superfluid Condensates in Astrophysics and Laboratory Experiments (SCALES)

Downloads

Description

Superfluidity is a striking phenomenon observed in many quantum fluids, which can flow without viscosity when cooled to low temperatures. Recent experimental advances allow us to study and visualise the complex flows of these systems in the presence of vorticity, and track the dynamics of quantum vortices far from equilibrium, both for bosonic and fermionic superfluids. Experiments with helium-4 and helium-3 allow the analysis of quantum turbulence on different scales, while cold atomic gases allow exquisite studies of single vortex dynamics in a variety of regimes, spanning the entire crossover from molecular Bose-Einstein condensates (BEC) to Bardeen-Cooper-Schrieffer (BCS) superfluids. Moreover, a growing body of theoretical and observational evidence suggests that nucleons in neutron stars are paired and form a fermionic superfluid, the dynamics of which is thought to be at the origin of the observed radio pulsar glitches.

The next years will bring a wealth of data on neutron star dynamics, from new radio observatories such as the Square Kilometer Array (SKA), but especially from gravitational wave observations with next-generation detectors, like the planned European Einstein Telescope (ET). Direct laboratory analogues of superfluid neutron stars are now being studied with superfluids on rotating platforms, but this connection is neither systematically explored nor disseminated among the relevant scientific communities.

SCALES will bring together novel laboratory experiments, emerging massive parallel simulations, and neutron star experts to kickstart this new

Action Details

- MoU - 048/25
- CSO Approval date - 19/05/2025
- Start date - 26/09/2025
- End date - 25/09/2029

How can I participate?

- Read the Action Description MoU
- Inform the Main Proposer/Chair of your interest (email)
- Apply to join your Working Groups of interest

Summary

- fully self-consistent 3D (TD)HFB calculations
- BSk31 Energy Density Functional
- effective parameters can be extracted
- effective mass
- dissipation channels
- creating vortex rings
- giant dipole resonance

More details

PRX 14, 041054 (2024)

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Thank you!