Théau DIVERRES

Study of the crust of neutron stars (at finite temperature) using the compressible liquid drop model (CLDM)

1st year PhD supervised by
Anthea Fantina

Outline

- Overview on Neutron stars and CLDM
- Comparison between CLDM and Extended Thomas Fermi (ETF)
- Bayesian Analysis of elastic properties (T=0)

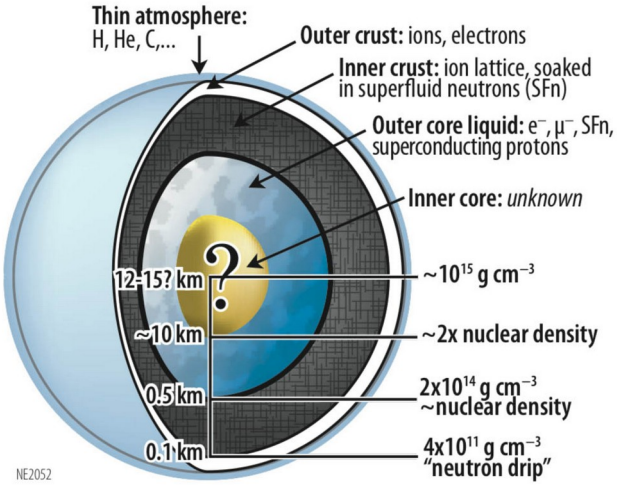
Neutron Stars

Neutron Stars are the remnant of supernovae explosions M>8-10 M_{sun}

Extreme objects : -A mass around 1-2 solar masses
-A radius around 10 km
-A density which can be several times the one in nucleus

Different layers :

- The atmosphere
- The outer crust
- The inner crust
- The core



From https://heasarc.gsfc.nasa.gov/docs/nicer/nicer_about.html

The crust

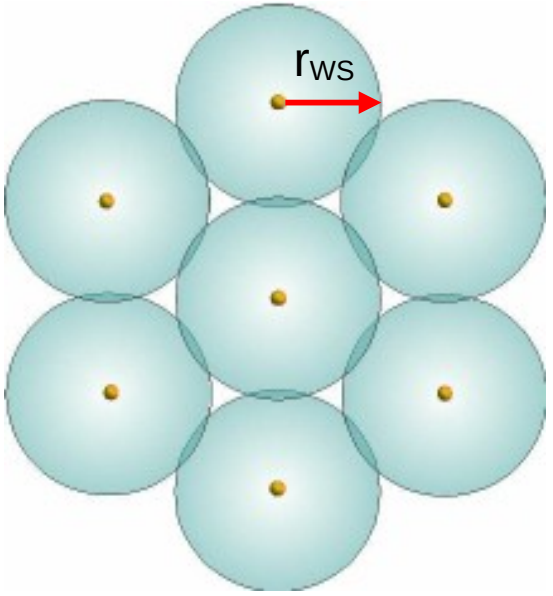
Baryonic density : - to ~1 x 10^{-4} fm^{-3} in the outer crust
- to ~0.08 fm^{-3} in the inner crust

At zero temperature, the crust is composed of ions on a lattice surrounded by a gas of electrons, and neutrons

→ Wigner-Seitz approximation : identical spherical cells

One component approximation

Non homogeneous many-body system



N. Chamel, “Neutron star crust beyond the Wigner-Seitz approximation” (2007)

The Compressible Liquid Drop Model

Sharp separation between the nucleus and the gas

$$\mathcal{F}_{\mathrm{CLDM}} = \mathcal{F}_e + \mathcal{F}_{\mathrm{g}}(1 - \frac{V_N}{V_{\mathrm{WS}}}) + \frac{F_i}{V_{\mathrm{WS}}}$$

with

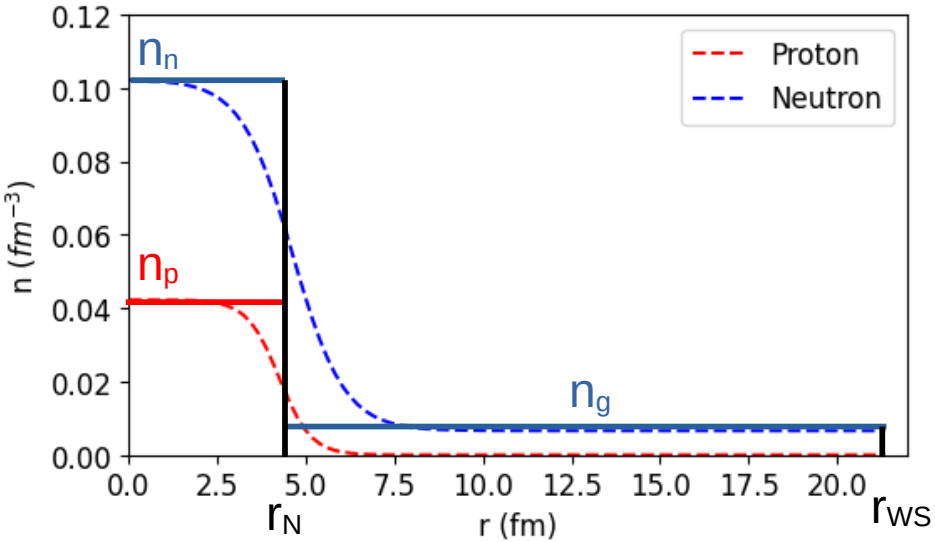
$$F_i = M_i c^2 + F_{\mathrm{bulk}} + F_{\mathrm{Coul}} + F_{\mathrm{surf+curv}}$$

- F_{bulk} : meta model approach (Taylor expansion around n_{sat})

$$\mathcal{F}_B(n,\delta,T) = \mathcal{F}_{\mathrm{kin}}(n,\delta,T) + \mathcal{V}_{\mathrm{MM}}(n,\delta)$$

$$\mathcal{V}^N_{\mathrm{MM}}(n,\delta) = \sum_{k=0}^{N} \frac{n}{k!}(v_k^{\mathrm{is}} + v_k^{\mathrm{iv}}\delta^2)x^k u_k^N(x)$$

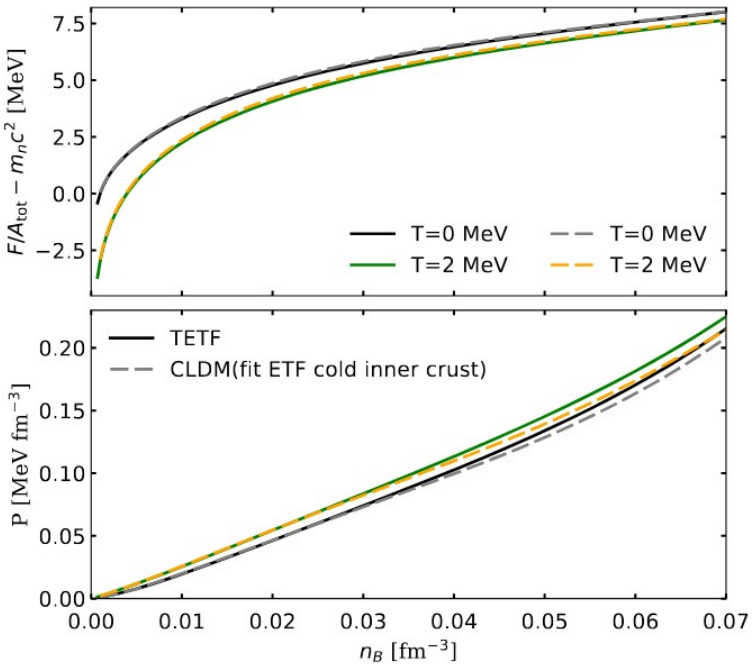
E_{sat}, K_{sat}, $Q_{sat...}$ E_{sym}, L_{sym}, $K_{sym...}$



Margueron, Hoffmann, Guilminelli, PRC (2018)

- $F_{surf+curv}$: parametrised with parameters fitted on exp/ETF results

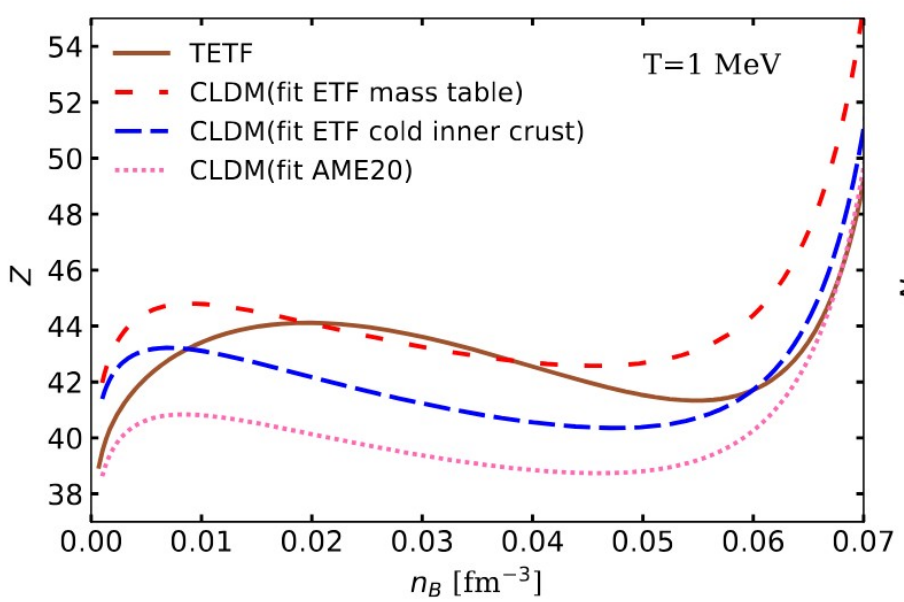
Comparison ETF/CLDM : thermodynamical quantities

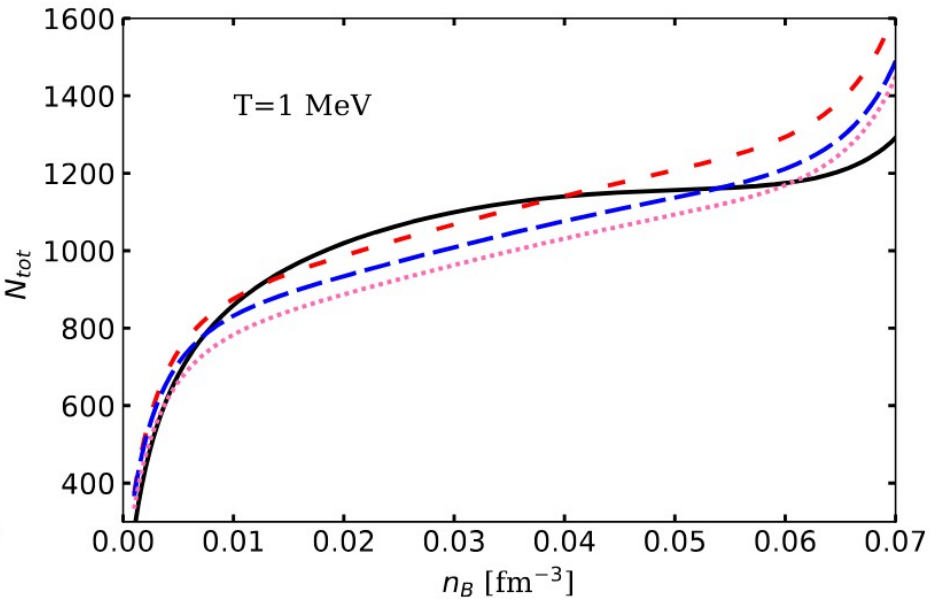


Very good agreement between the ETF and CLDM on thermodynamical quantities in the inner crust

Grams, Shchechilin, Diverres, et al., Universe (2025)

Comparison ETF/CLDM : composition





Similar trend but the composition is more sensitive to the model

Grams, Shchechilin, Diverres, et al., Universe (2025)

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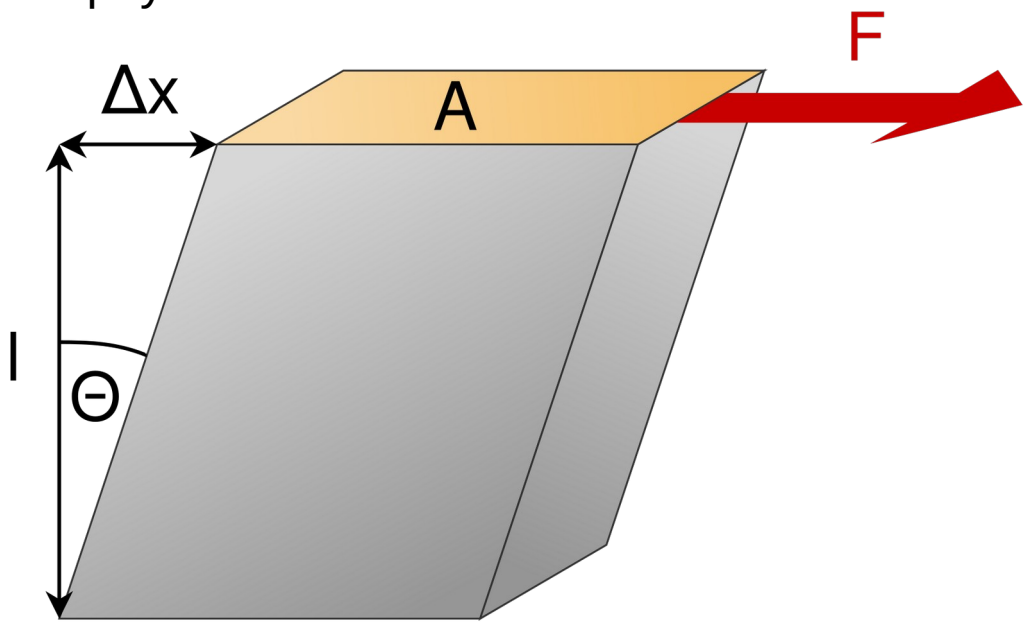
- Comparison between CLDM and Extended Thomas Fermi (ETF)
- Bayesian Analysis of elastic properties (T=0)

The shear modulus

The shear modulus is an input for different astrophysical applications (modes, glitches)

The shear modulus is an intrinsic property of the material related to its ability to be deformed under such forces F



Fit to Monte Carlo calculations :

$$\mu = 0.1194 \frac{(Ze)^2}{\frac{4}{3}\pi r_{\mathrm{ws}}^4}$$

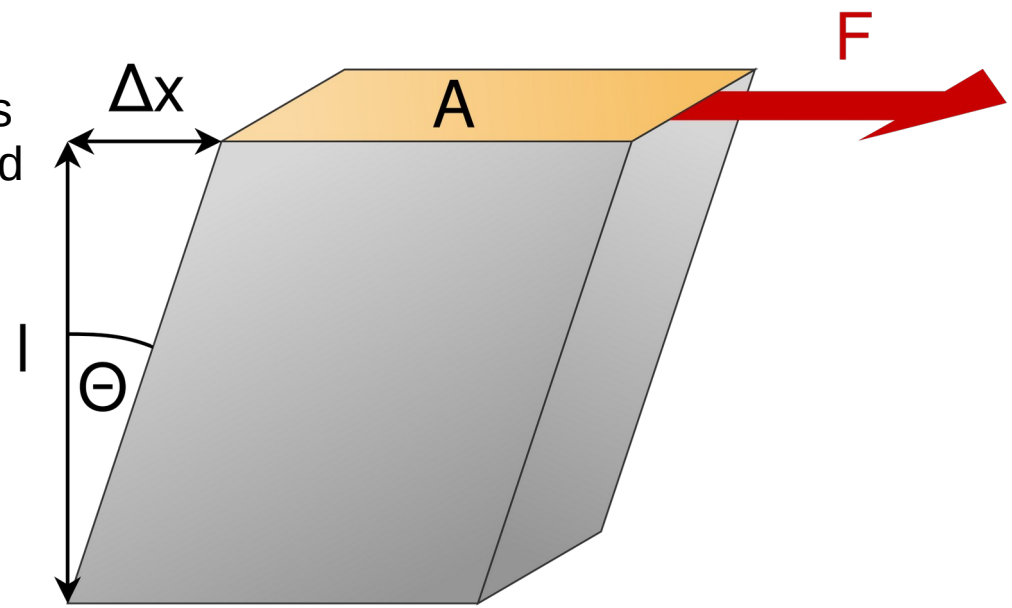
Stromayer, The Astrophysics Journal (1991)
Condensed Matter Physics, Marder (2000)

The shear velocity

The shear deformations propagate at a speed V_S

Considering the shear deformation propagates as a plane wave, the shear speed is expressed

$$V_s = \sqrt{\frac{\mu}{\rho}}$$



ρ is the mass density of nucleons moving with the lattice

$$\rho = \rho_B - \rho_g$$

Chamel PRC (2012) / Almirante & Urban arXiv:2503.21635 (2025)

Bayesian Analysis (1)

The simplicity and computation efficiency of the CLDM is adapted for bayesian analysis

Start from a set of empirical parameters

Going through several filters

$$\mathcal{L}(\mathrm{model}|\mathbf{c}) \propto \mathcal{L}(\mathrm{model}) \prod_{\mathrm{k}} \mathcal{L}(\mathrm{c_k}|\mathrm{model})$$

Replace the original Qs and Zs at densities > n_{sat}

Parameter	min	max
E_{sat} [MeV]	-17	-15
n_{sat} [fm^{-3}]	0.15	0.17
K_{sat} [MeV]	190	270
Q_{sat} [MeV]	-1000	1000
Z_{sat} [MeV]	-3000	3000
E_{sym} [MeV]	26	38
L_{sym} [MeV]	10	80
K_{sym} [MeV]	-400	200
Q_{sym} [MeV]	-2000	2000
Z_{sym} [MeV]	-5000	5000
$Q^{\star}_{\mathrm{sat}}$ [MeV]	-2000	2000
$Z^{\star}_{\mathrm{sat}}$ [MeV]	-5000	5000
$Q^{\star}_{\mathrm{sym}}$ [MeV]	-3000	3000
$Z^{\star}_{\mathrm{sym}}$ [MeV]	-5000	5000
$m^{\star}_{\mathrm{sat}}/m$	0.6	1.2
$\Delta m^{\star}_{\mathrm{sat}}/m$	0.0	2.0
b	1	10

Bayesian analysis (2)

Klausner et al. PRC (2025) / See presentation from Pietro Klausner

Fit a nuclear informed ETF distribution as a multivariate gaussian (Montefusco/Antonelli)

Create the prior models picking from this distribution

Parameter	min	max
$E_{\rm sat}$ [MeV]	~~-17~~	~~-15~~
$n_{\rm sat}$ [fm^{-3}]	~~0.15~~	~~0.17~~
$K_{\rm sat}$ [MeV]	~~190~~	~~270~~
$Q_{\rm sat}$ [MeV]	~~-1000~~	~~1000~~
$Z_{\rm sat}$ [MeV]	~~-3000~~	~~3000~~
$E_{\rm sym}$ [MeV]	~~26~~	~~38~~
$L_{\rm sym}$ [MeV]	~~10~~	~~80~~
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$Z_{\rm sym}$ [MeV]	~~-5000~~	~~5000~~
$Q^{\star}_{\rm sat}$ [MeV]	-2000	2000
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$m^{\star}_{\rm sat}/m$	~~0.6~~	~~1.2~~
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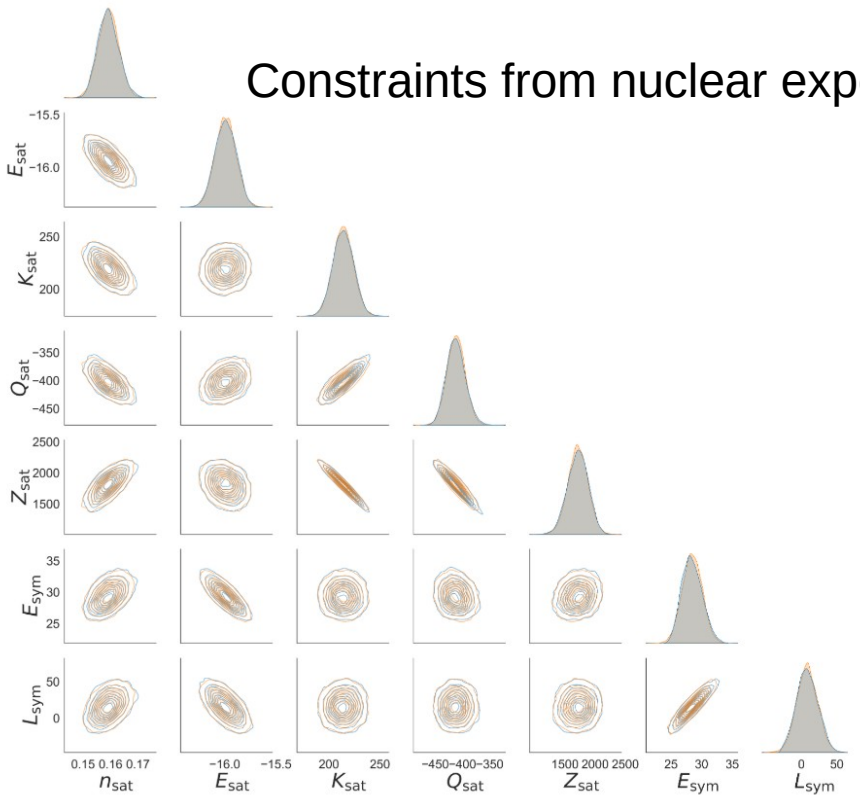
b	1	10

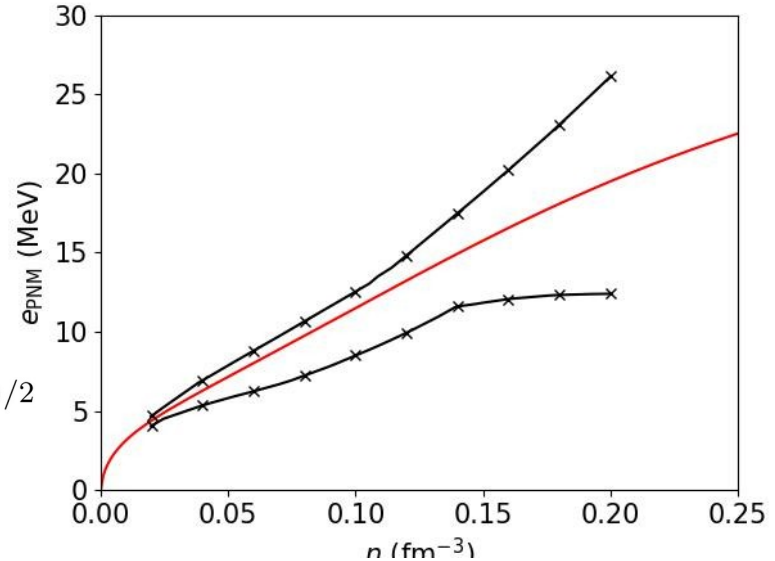
12 parameters constrained by nuclear physics experiments

Constraints from nuclear experiments



Bayesian filters

$$\mathcal{L}(\text{model}) = \prod_{\text{filter}} \mathcal{L}_{\text{filter}}(\text{model})$$



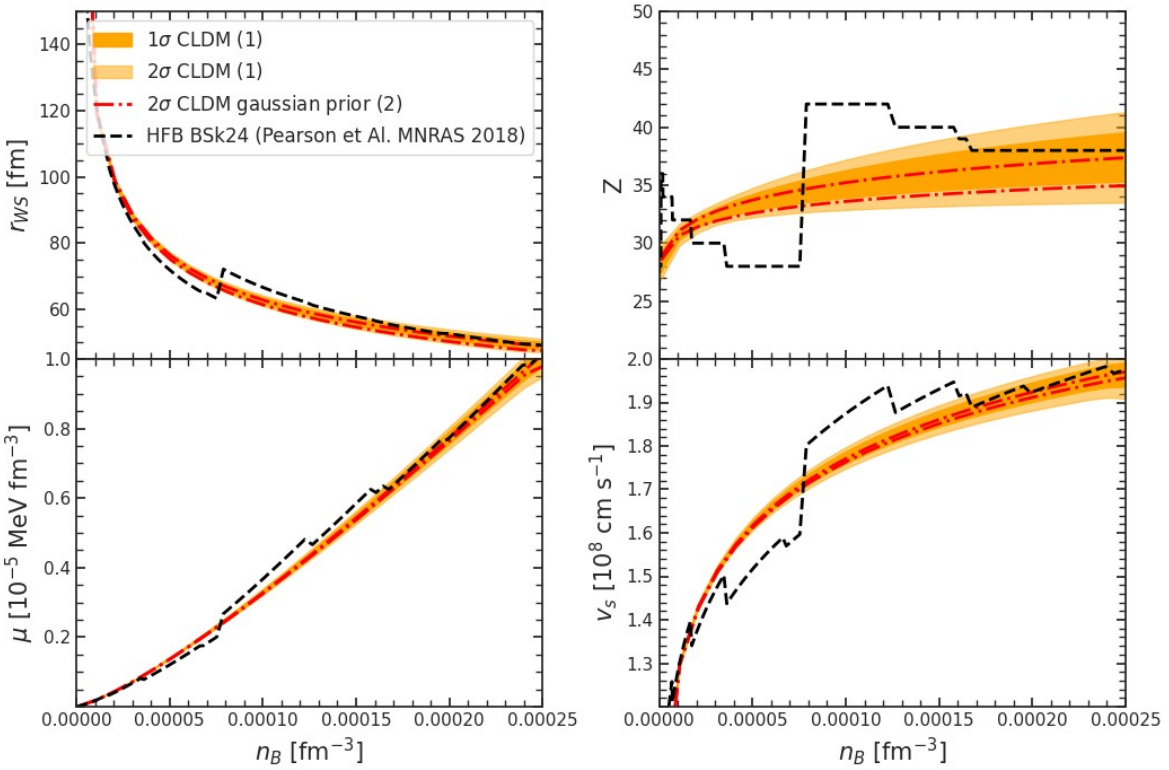
The filters:

- Surface energy fitting (AME2020) $\mathcal{L}_{\text{fit}}(\text{model}) \propto \mathrm{e}^{-\chi^2(\text{model})/2}$
- low density chiral EFT
- causality, thermodynamic stability, non-negative symmetry energy at all densities
- capability of producing a neutron star with a mass > 2 solar masses
- constraints from tidal deformability (GW170817) and NICER

Bayesian Analysis of the shear modulus

Outer Crust :

Differences from microscopic calculation due to shell effects

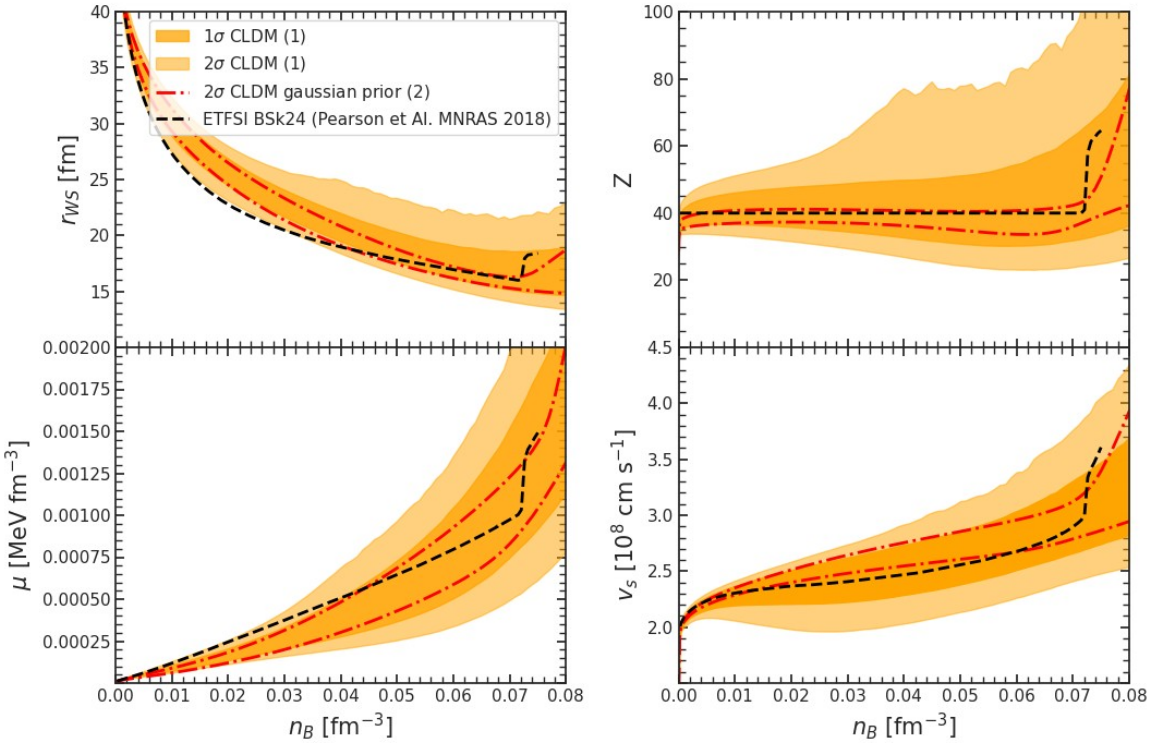


(Diverres et al. in preparation)

Bayesian Analysis of the shear modulus

Inner Crust :

Inclusion of nuclear physics experimental data constrains the crust predictions



(Diverres et al. in preparation)

Conclusions

- Good agreement between the CLDM and ETF even at finite temperatures
- Provide uncertainties on the calculation of elastic properties
- Crust properties constrained by nuclear physics experiments

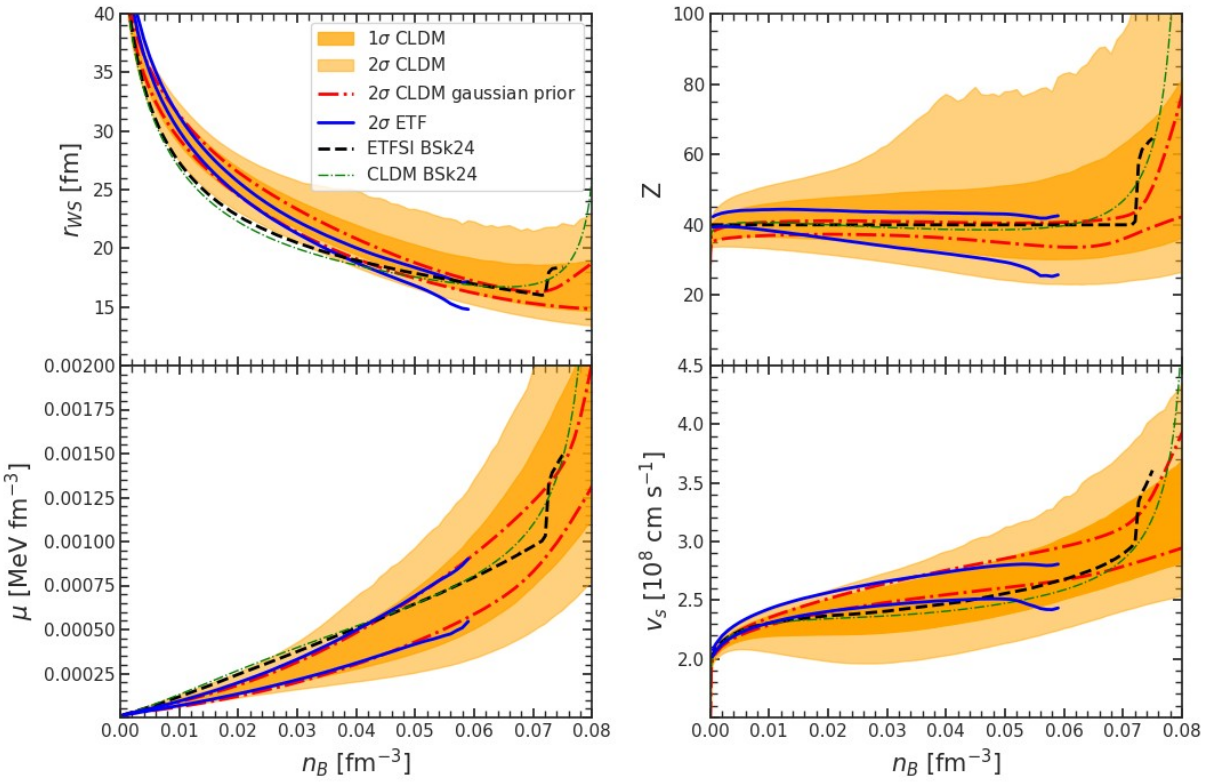
Merci !

Bayesian Analysis of the shear modulus

Inner Crust :

Comparison to ETF results from Klausner et al. arXiv:2505.16929 (2025)

- Inclusion of nuclear physics experimental data constrains the crust predictions
- Difference from the many-body method (Grams et al. Universe (2025))

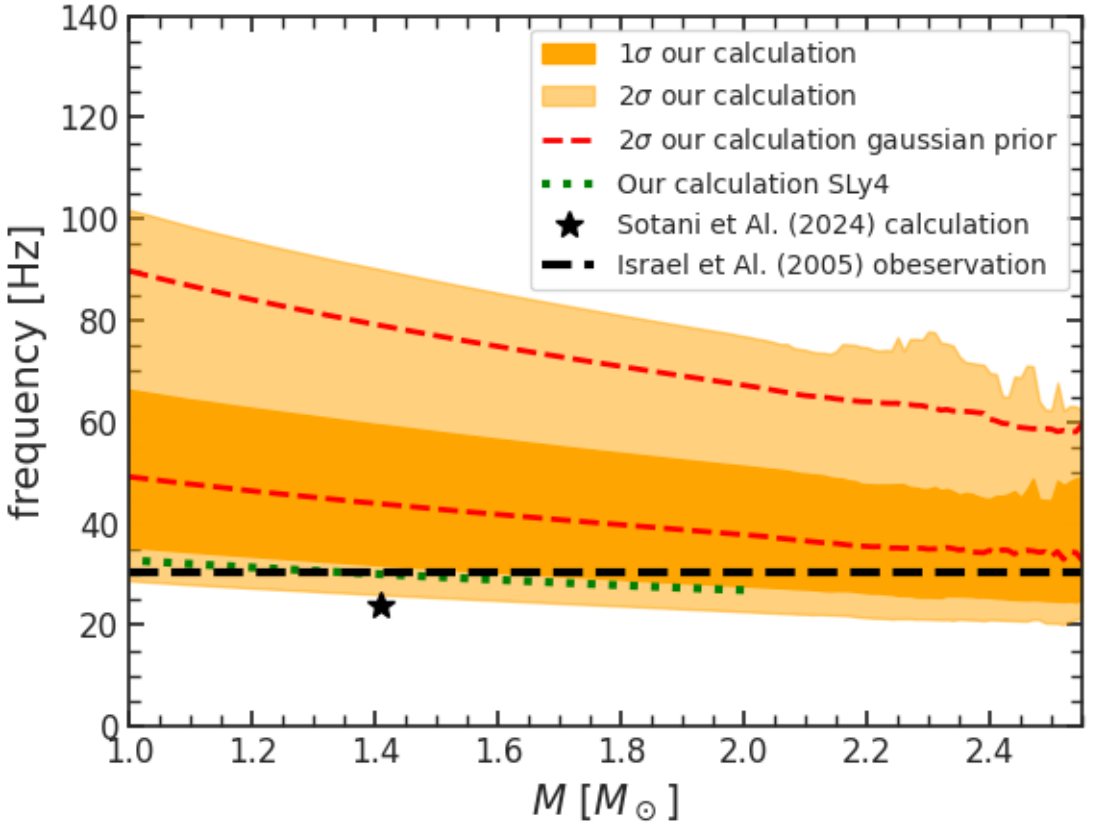


ETFSI results from Pearson et al. MNRAS (2018)

(Diverres et al. in preparation)

$$\omega_0 \approx \frac{e^{2\nu} v_s^2 (l-1)(l+2)}{2RR_{\rm cc}}$$

Samuelsson & Andersson MNRAS (2007)



The Compressible Liquid Drop Model

Sharp separation between the nucleus and the gas

$$\mathcal{F}_{\mathrm{CLDM}} = \mathcal{F}_e + \mathcal{F}_{\mathrm{g}}(1 - \frac{V_N}{V_{\mathrm{WS}}}) + \frac{F_i}{V_{\mathrm{WS}}}$$

with

$$F_i = M_i c^2 + F_{\mathrm{bulk}} + F_{\mathrm{Coul}} + F_{\mathrm{surf+curv}}$$

$$F_{\mathrm{bulk}} = V_N \mathcal{F}_B(n = n_N, \delta = \delta_N, T)$$

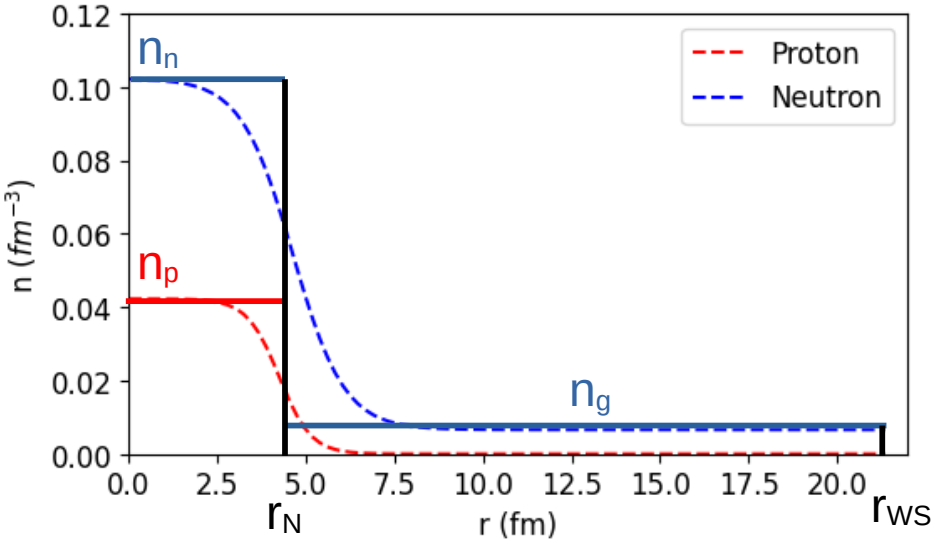
$$\mathcal{F}_{\mathrm{g}} = \mathcal{F}_B(n = n_{\mathrm{g}}, \delta = 1, T) + m_n c^2 n_{\mathrm{g}}$$

with

$$\mathcal{F}_B(n, \delta, T) = \mathcal{F}_{\mathrm{kin}}(n, \delta, T) + \mathcal{V}_{\mathrm{MM}}(n, \delta)$$

$$\mathcal{V}^N_{\mathrm{MM}}(n, \delta) = \sum_{k=0}^{N} \frac{n}{k!}(v_k^{\mathrm{is}} + v_k^{\mathrm{iv}}\delta^2) x^k u_k^N(x)$$

E_{sat}, K_{sat}, Q_{sat}... E_{sym}, L_{sym}, K_{sym}...



$$n_N = n_n + n_p$$

$$\delta_N = \frac{n_n - n_p}{n_n + n_p}$$

$$x = \frac{n - n_{\mathrm{sat}}}{3n_{\mathrm{sat}}}$$

Margueron, Hoffmann, Guilminelli, PRC (2018)

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Sharp separation between the nucleus and the gas

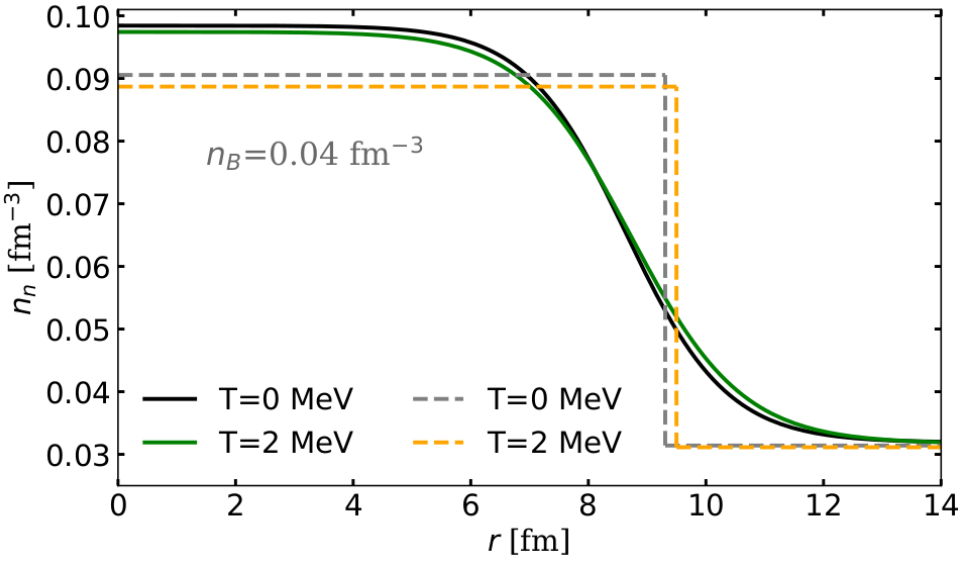
$$\mathcal{F}_{\mathrm{CLDM}} = \mathcal{F}_e + \mathcal{F}_{\mathrm{g}}\left(1 - \frac{V_N}{V_{\mathrm{WS}}}\right) + \frac{F_i}{V_{\mathrm{WS}}}$$

with

$$F_i = M_i c^2 + F_{\mathrm{bulk}} + F_{\mathrm{Coul}} + F_{\mathrm{surf+curv}}$$

The surface energy accounts for the missed interface energy and can be fitted to experimental data or ETF results

$$F_{\mathrm{surf+curv}} = 4\pi\sigma_s r_N^2 + 8\pi\sigma_c r_N$$



Grams et al, Universe (2025)

$$\sigma_{\mathrm{s}} = \sigma_0 \frac{2^{p+1} + b_s}{y_{\mathrm{p}}^{-p} + b_s + (1 - y_{\mathrm{p}})^{-p}}$$

$$\sigma_{\mathrm{c}} = \frac{\sigma_{0c}}{\sigma_0} \alpha \left(\beta - y_{\mathrm{p}}\right) \sigma_{\mathrm{s}}.$$

$$y_p = n_p/(n_n + n_p)$$

J. M. Lattimer and F. Douglas Swesty, NPA (1991)