Hybrid Star Properties with NJL and MFTQCD Model:

A Bayesian Approach

Milena Bastos Albino
Advisor: Prof. Dr. Constança Providência
Co-advisors: Prof. Dr. Márcio Ferreira and Prof. Dr. Tuhin Malik

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Universidade de Coimbra

Why are Neutron Stars cool?

$M \approx 1.4 M_\odot$

$R \approx 14$ km

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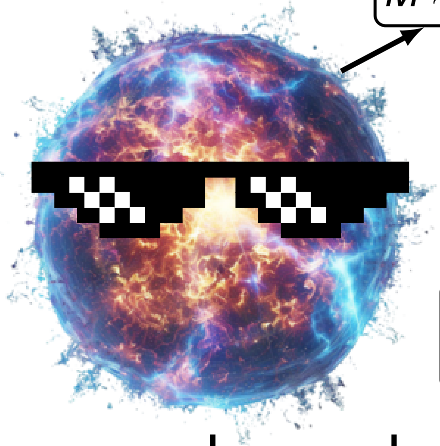


$R \approx 14$ km

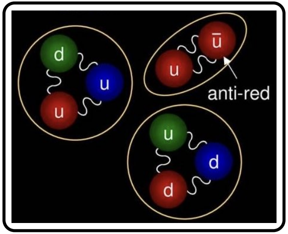
1.000.000.000 kg!

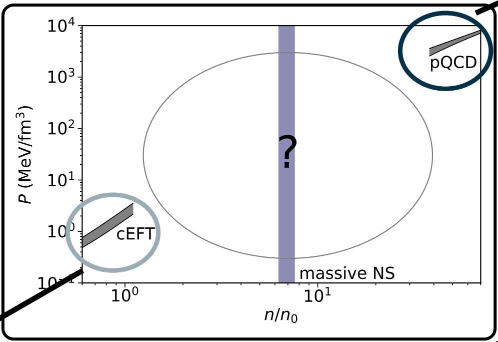
No more H fusion
+
neutrino loss

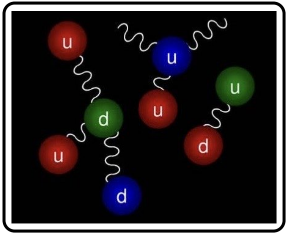
low temperature
$T \lesssim 1$ MeV

Matter at extreme densities

Quark phase



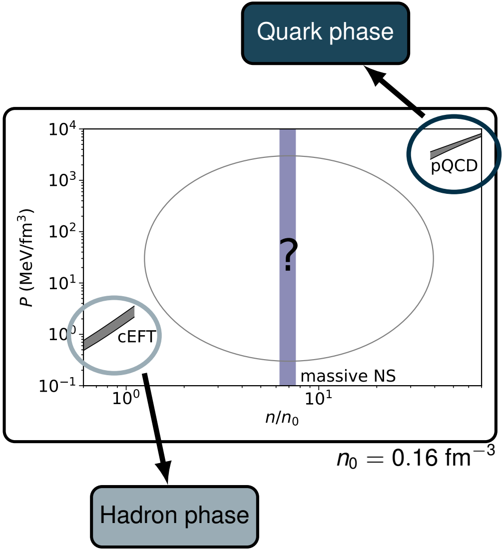




$n_0 = 0.16\ \text{fm}^{-3}$

Hadron phase

A question to be answered



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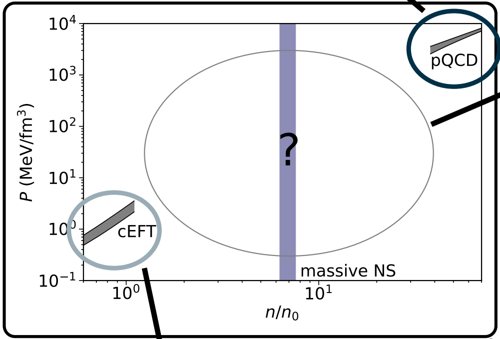
Quark phase

There is a phase transition in this unknown region



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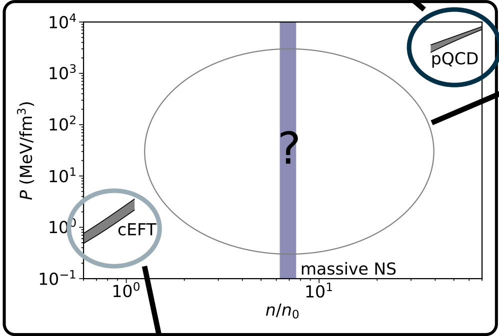

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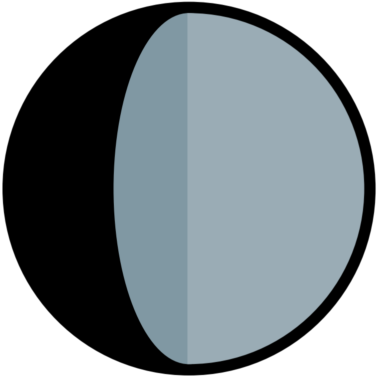

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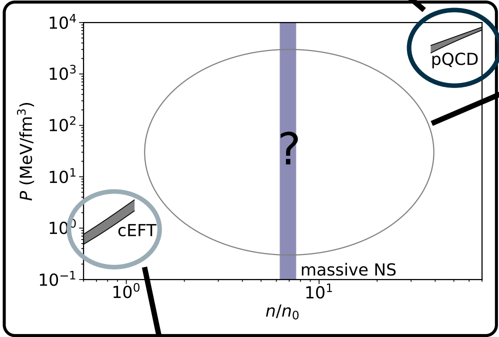

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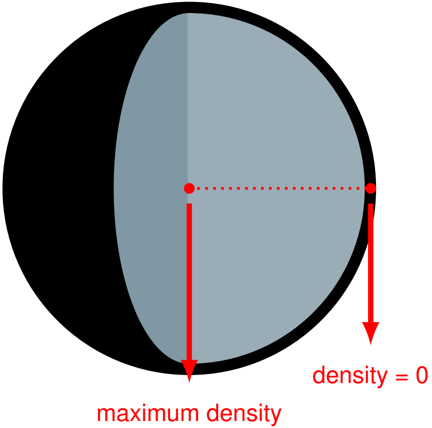

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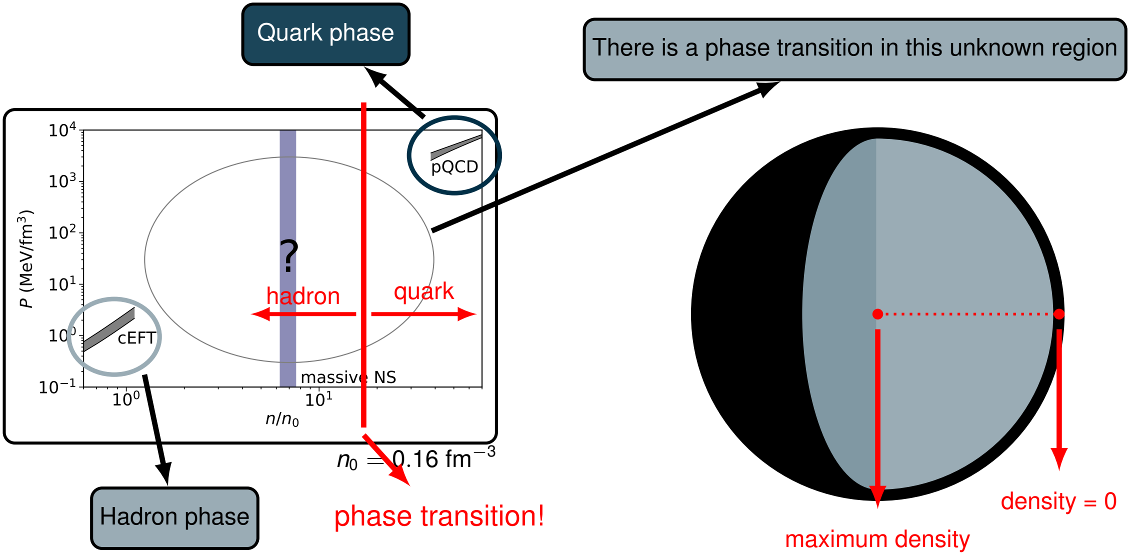

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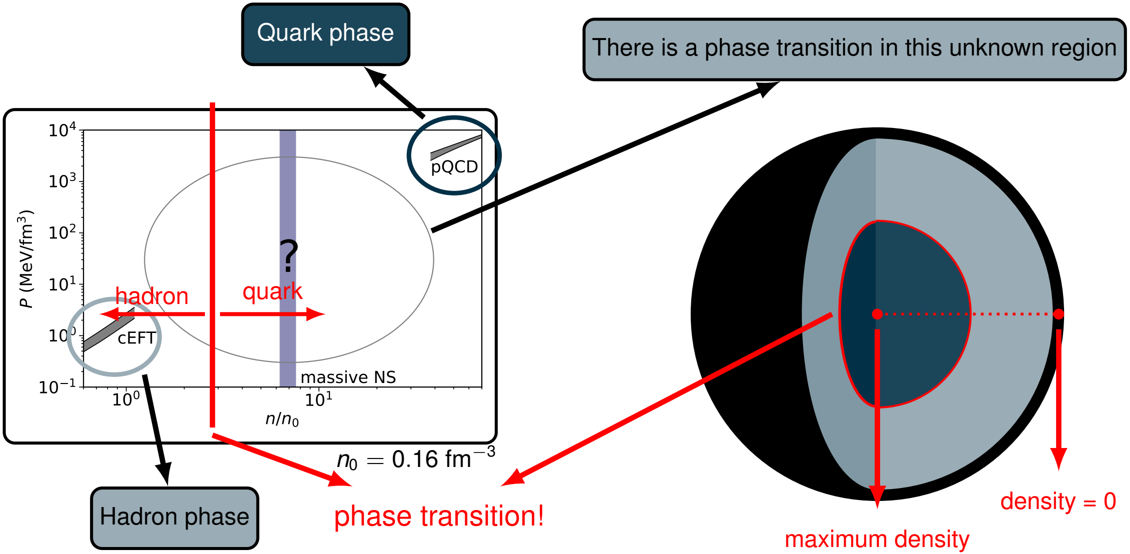
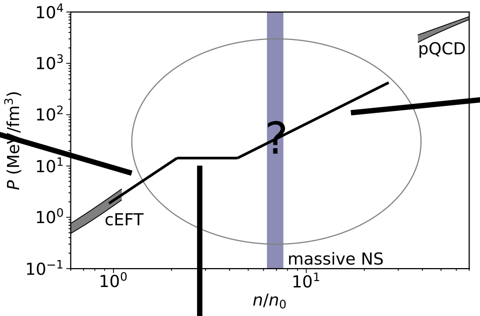


Objective:
explore the possibility of
quark matter inside neutron stars

Exploring the unknown region

We must to assume models for the hadron and quark phases

Hadron equation



Quark equation

Phase transition

$n_0 = 0.16 \text{ fm}^{-3}$

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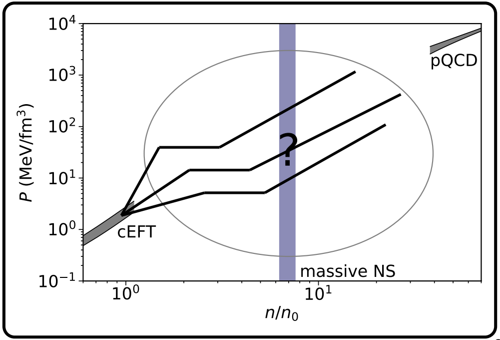
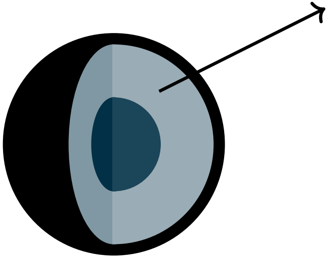


$n_0 = 0.16$ fm^{-3}

By changing some parameters, we get more equations

Objective: explore the possibility of quark matter inside neutron stars



Hadron phase

Relativistic Mean Field (RMF)[1]

$$\mathcal{L}_N = \bar{\Psi}\left[\gamma^{\mu}\left(i\partial_{\mu} - g_{\omega}\omega_{\mu} - g_{\rho}\boldsymbol{t}\cdot\boldsymbol{\rho}_{\mu}\right) - \left(m - g_{\sigma}\phi\right)\right]\Psi,$$

$$\mathcal{L}_M = \frac{1}{2}\left[\partial_{\mu}\sigma\partial^{\mu}\sigma - m_{\sigma}^2\sigma^2\right] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu} - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{2}m_{\varrho}^2\boldsymbol{\varrho}_{\mu}\cdot\boldsymbol{\varrho}^{\mu},$$

$$\mathcal{L}_{NL} = -\frac{1}{3}bmg_{\sigma}^3(\sigma)^3 - \frac{1}{4}cg_{\sigma}^4(\sigma)^4 + \frac{\xi}{4!}g_{\omega}^4(\omega_{\mu}\omega^{\mu})^2 + \Lambda_{\omega}g_{\varrho}^2\boldsymbol{\varrho}_{\mu}\cdot\boldsymbol{\varrho}^{\mu}g_{\omega}^2\omega_{\mu}\omega^{\mu},$$

[1] Tuhin Malik et al. **"Spanning the full range of neutron star properties within a microscopic description".** In: *Phys. Rev. D* 107.10 (2023), p. 103018. DOI: 10.1103/PhysRevD.107.103018. arXiv: 2301.08169 [nucl-th].

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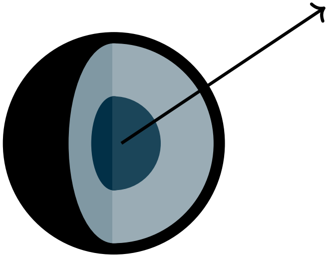


Quark phase

NJL
(Nambu-Jona-Lasinio)

- t' Hooft term;
- 5 different types of interactions.

MFTQCD
(Mean Field Theory of QCD)[2]

- from the QCD Lagrangian
- decomposition of gluon field in soft and hard momentum components.

[2] D. A. Fogaca and F. S. Navarra. **"Gluon condensates in a cold quark–gluon plasma".** In: *Phys. Lett. B* 700 (2011), pp. 236–242. DOI: 10.1016/j.physletb.2011.05.011. arXiv: 1012.5266 [hep-ph].

Mean Field Approximation of QCD (MFTQCD)

- Start with the QCD lagrangian;
- Assume we can decompose the gluon field in[3]
  - low momentum components (soft gluons);
  - high momentum components (hard gluons);

$$\tilde{G}^{a\mu}(k) = \tilde{A}^{a\mu}(k) + \tilde{\alpha}^{a\mu}(k) \qquad (1)$$

[3] Fogaca and Navarra, "Gluon condensates in a cold quark–gluon plasma".

Nambu-Jona-Lasinio (NJL)

We use the following SU(3) NJL lagrangian:

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + \mu\gamma^0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma^5\lambda_a\psi\right)^2\right] + \mathcal{L}_{\text{'t Hooft}} + \mathcal{L}_I$$

- standard NJL term: $\frac{G}{2}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma^5\lambda_a\psi\right)^2\right]$
- 't Hooft term: $\mathcal{L}_{\text{'t Hooft}}$
- added interation terms: $\mathcal{L}_I$

NJL

$$\mathcal{L}_I = -G_\omega \left[(\bar\psi\gamma^\mu\lambda_0\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\lambda_0\psi)^2 \right]$$
$$- G_\rho \sum_{a=1}^{8} \left[(\bar\psi\gamma^\mu\lambda^a\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\lambda^a\psi)^2 \right]$$
$$- G_{\omega\omega} \left[(\bar\psi\gamma^\mu\lambda_0\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\lambda_0\psi)^2 \right]^2$$
$$- G_{\sigma\omega} \sum_{a=0}^{8} \left[(\bar\psi\lambda_a\psi)^2 + (\bar\psi i\gamma^5\lambda_a\psi)^2 \right]$$
$$\times \left[(\bar\psi\gamma^\mu\lambda_0\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\lambda_0\psi)^2 \right]$$
$$- G_{\rho\omega} \sum_{a=1}^{8} \left[(\bar\psi\gamma^\mu\lambda_0\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\lambda_0\psi)^2 \right]$$
$$\times \left[(\bar\psi\gamma^\mu\lambda_a\psi)^2 + (\bar\psi\gamma^\mu\gamma_5\lambda_a\psi)^2 \right]. \quad (2)$$

$$P \to P + B. \quad (3)$$

MFTQCD[4]

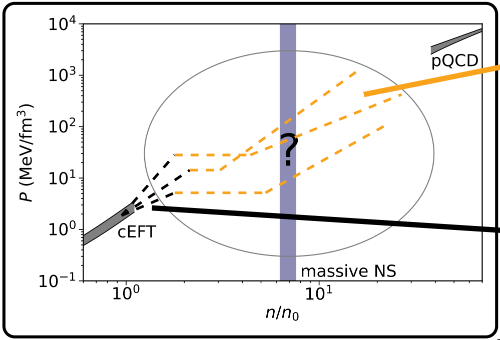
$$P = \frac{27}{2}\xi^2\rho_B^2 - B + P_F, \quad (4)$$
$$\epsilon = \frac{27}{2}\xi^2\rho_B^2 + B + \epsilon_F, \quad (5)$$

where

- $\xi = g/m_G$ and $m_G = \frac{9}{32}g^2\mu_0^2$;
- $B = \frac{9}{4(34)}g^2\phi_0^4$;
- P_F and ϵ_F are the pressure and energy density of a non-interacting Fermi gas of quarks and electrons.

[4] Fogaca and Navarra, "Gluon condensates in a cold quark–gluon plasma".

NJL Set

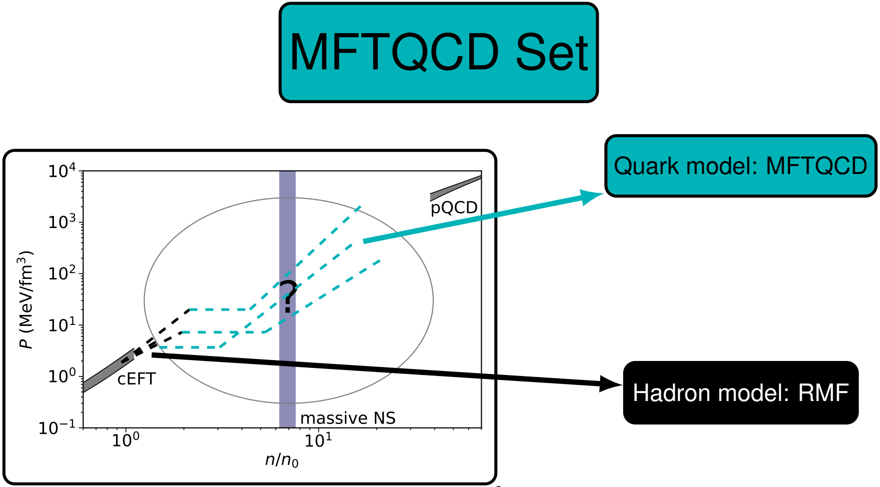


Quark model: NJL

Hadron model: RMF

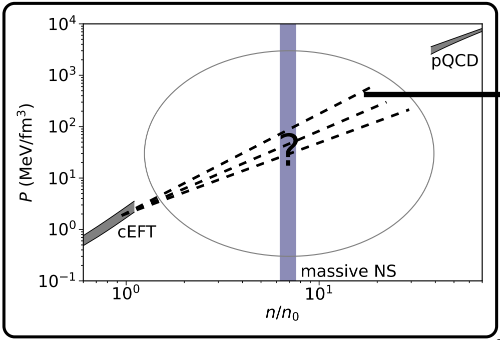
$n_0 = 0.16\ \mathrm{fm}^{-3}$

What we have done



$n_0 = 0.16\ \mathrm{fm}^{-3}$

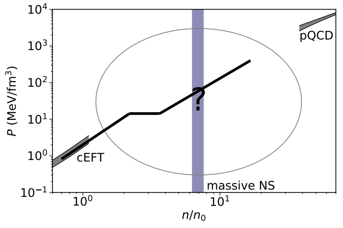
RMF Set



Hadron model: RMF

$n_0 = 0.16$ fm^{-3}

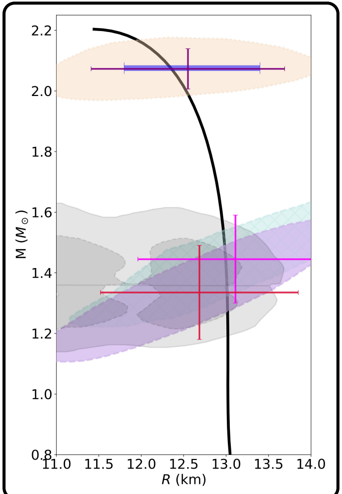
How we test the models



TOV equations

$$\frac{dP}{dr} = -\frac{[\epsilon(r)+P(r)]\left[M(r)+4\pi r^3 P(r)\right]}{r^2\left[1-\frac{2GM(r)}{r}\right]}$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$



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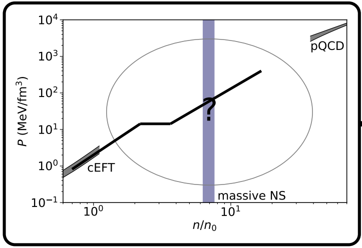

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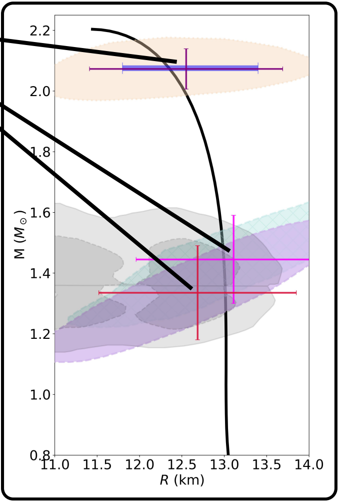
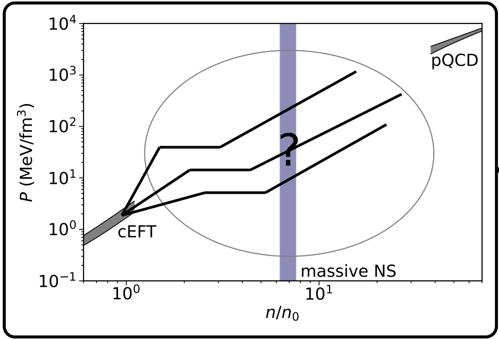
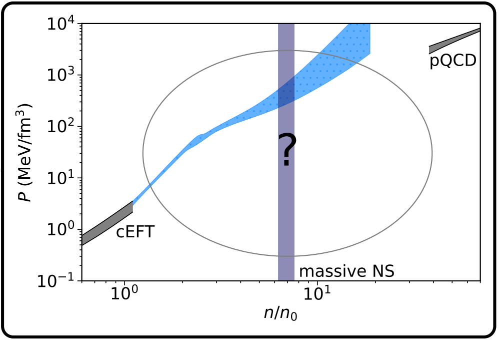
observational data



Building the hybrid sets of EoS





Bayesian inference

Bayesian constraint

Theorical constaint

- ⋆ Pure nuclear matter (cEFT);
- ⋆ perturbative QCD (pQCD);
- ⋆ Causality

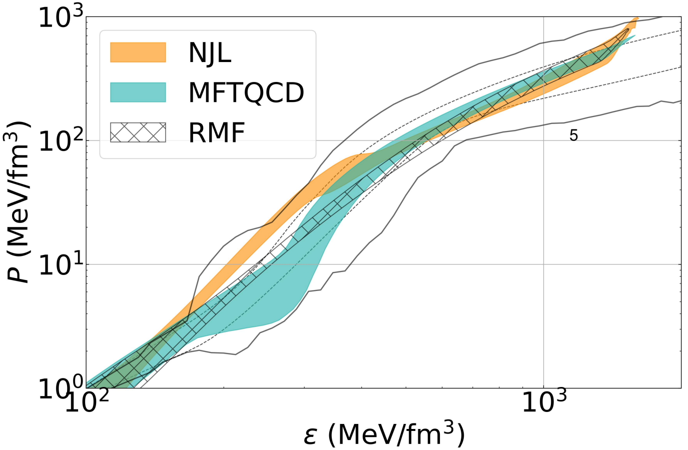
Obs./exp. data

- ⋆ Nuclear matter properties;
- ⋆ X-ray Nicer data

Hybrid structure

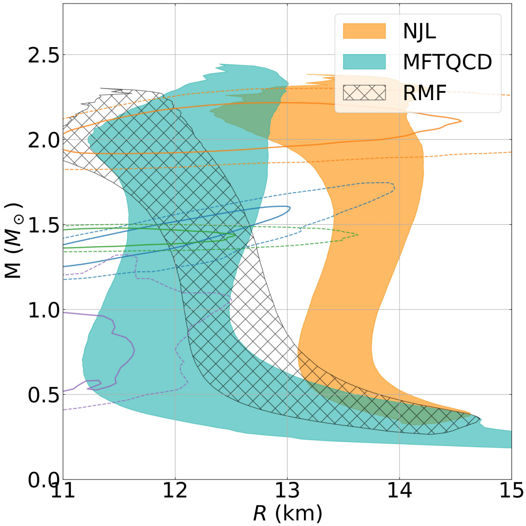
- ⋆ Intersection point;
- ⋆ phT from hadron to quark phase;
- ⋆ $0.16 < \rho_{\text{trans}} < 0.40\ \text{fm}^{-3}$;
- ⋆ must have quarks in maximum NS

EOS Results



[5] Sinan Altiparmak, Christian Ecker, and Luciano Rezzolla. **"On the Sound Speed in Neutron Stars".** In: *Astrophys. J. Lett.* 939.2 (2022), p. L34. DOI: 10.3847/2041-8213/ac9b2a. arXiv: 2203.14974 [astro-ph.HE].

Mass-radius diagram



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- ⋆ GW170817 data

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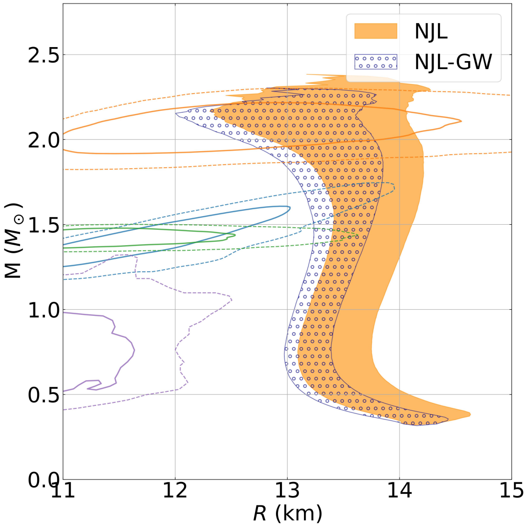
NJL-GW results



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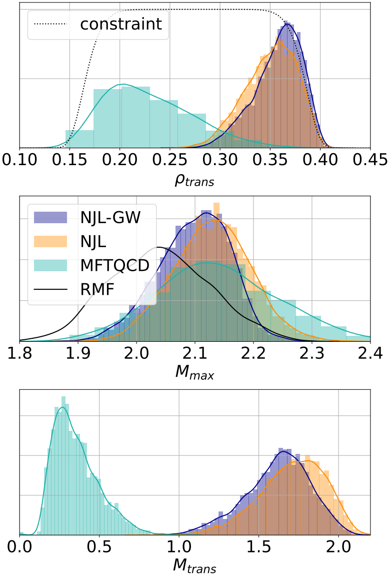

Conclusion

Objective: explore the possibility of quark matter inside neutron stars

Bayesian inference

- NJL
- NJL-GW
- MFTQCD
- RMF

Results

- $\star$ $\rho_{\mathrm{trans}} \gtrsim 0.30$ (NJL), $0.16\ \mathrm{fm}^{-3}$ (MFTQCD)
- $\star$ $1.4M_\odot$
  - MFTQCD $\rightarrow$ quarks
  - NJL $\rightarrow$ no quarks
- $\star$ $2M_\odot$ $\longrightarrow$ NJL and MFTQCD $\rightarrow$ quarks

Thank you!

milena.albino@student.uc.pt

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References

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[2] L. S. Celenza and C. M. Shakin. **“Description of the gluon condensate”.** In: *Phys. Rev. D* 34 (5 Sept. 1986), pp. 1591–1600. DOI: 10.1103/PhysRevD.34.1591.

[3] D. A. Fogaca and F. S. Navarra. **“Gluon condensates in a cold quark–gluon plasma”.** In: *Phys. Lett. B* 700 (2011), pp. 236–242. DOI: 10.1016/j.physletb.2011.05.011. arXiv: 1012.5266 [hep-ph].

[4] Oleg Komoltsev and Aleksi Kurkela. **“How Perturbative QCD Constrains the Equation of State at Neutron-Star Densities”.** In: *Phys. Rev. Lett.* 128.20 (2022), p. 202701. DOI: 10.1103/PhysRevLett.128.202701. arXiv: 2111.05350 [nucl-th].

[5] Xiangdong Li and C. M. Shakin. **“Description of gluon propagation in the presence of an A^2 condensate”.** In: *Phys. Rev. D* 71 (7 Apr. 2005), p. 074007. DOI: 10.1103/PhysRevD.71.074007.

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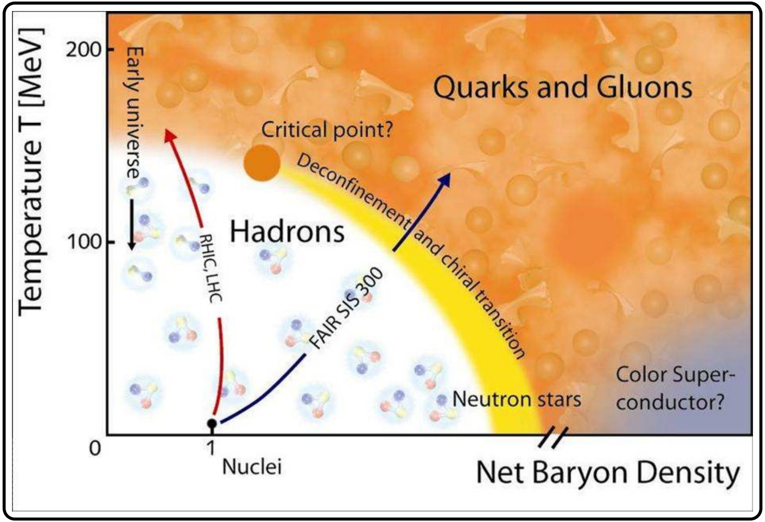
QCD phase diagram



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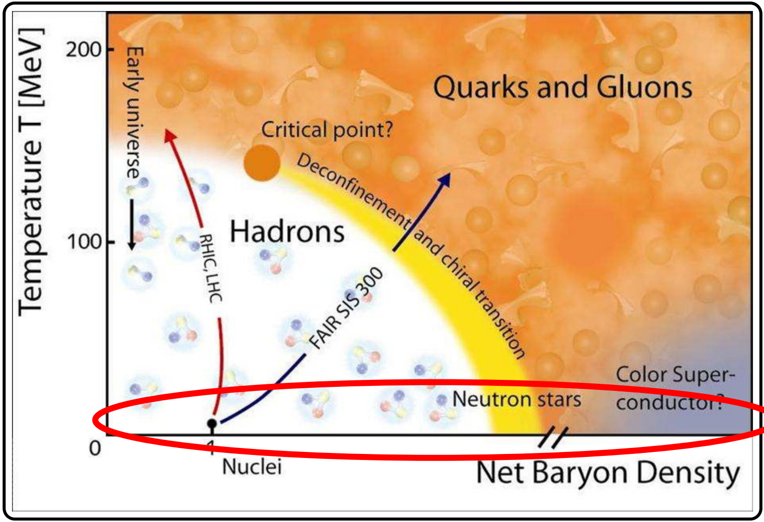

How we test the models

General Relativity

$$G^{\mu}_{\nu} = -8\pi G T^{\mu}_{\nu}$$

+

assume NSs are:

- ⋆ spherical
- ⋆ static
- ⋆ ideal, isotropic fluid

→

TOV equations

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2 eqs for 3 variables (P, ϵ, M)!

How we test the models

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EoS
$$P = P(\epsilon)$$

+

TOV equations
$$\frac{dP}{dr} = -\frac{[\epsilon(r)+P(r)]\left[M(r)+4\pi r^3 P(r)\right]}{r^2\left[1-\frac{2GM(r)}{r}\right]}$$
$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$

→

results in
- ⋆ mass
- ⋆ radius

Bayesian constraints

Constraints imposed by
Bayesian inference:

1. X-ray observations (NICER)

2. phase transition

3. pQCD EOS

Bayesian constraints

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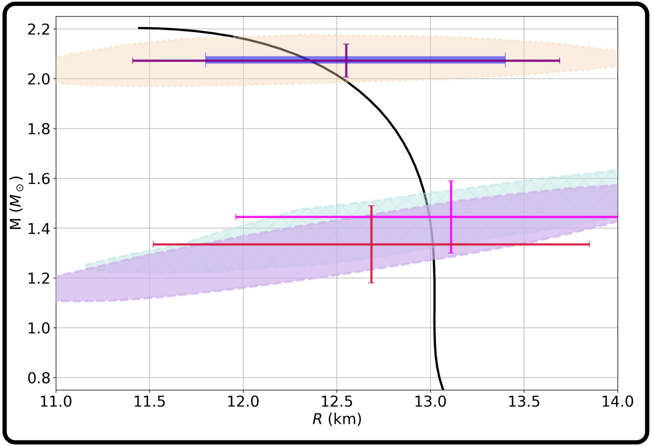

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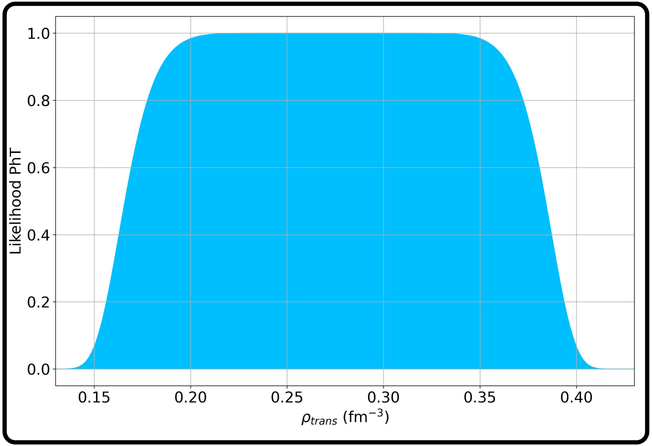

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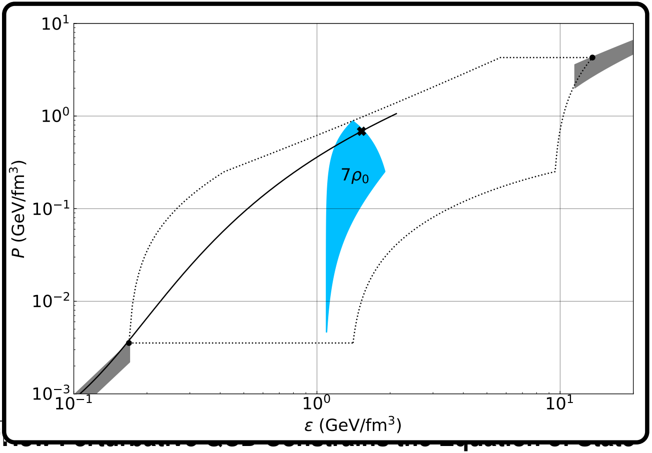

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Nambu-Jona-Lasinio (NJL)

We add 4- and 8-quark interaction terms:

$$\begin{aligned}
\mathcal{L}_I = &-G_\omega \left[\left(\bar{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right] \\
&- G_\rho \sum_{a=1}^{8} \left[\left(\bar{\psi}\gamma^\mu\lambda^a\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda^a\psi\right)^2\right] \\
&- G_{\omega\omega} \left[\left(\bar{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right]^2 \\
&- G_{\sigma\omega} \sum_{a=0}^{8} \left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma^5\lambda_a\psi\right)^2\right] \left[\left(\bar{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right] \\
&- G_{\rho\omega} \sum_{a=1}^{8} \left[\left(\bar{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right] \left[\left(\bar{\psi}\gamma^\mu\lambda_a\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\lambda_a\psi\right)^2\right]. \qquad (6)
\end{aligned}$$

EOS can be obtained through the Mean Field Approximation (MFA).

Mean Field Approximation of QCD (MFTQCD)

Apply an approximation to the fields

Hard gluons

- large occupation numbers at all energy levels;
- consider hard gluons as **classical field**:

$$\alpha_\mu^a \rightarrow \langle \alpha_\mu^a \rangle = \alpha_0^a \delta_{\mu 0}. \qquad (7)$$

Soft gluons

- approximate the momentum of the soft gluons to be zero;
- replace the fields with their expected value in the vacuum[7,8]:

$$\langle A^{a\mu} A^{b\nu} \rangle = -\frac{\delta^{ab}}{8} \frac{g^{\mu\nu}}{4} \mu_0^2, \qquad (8)$$

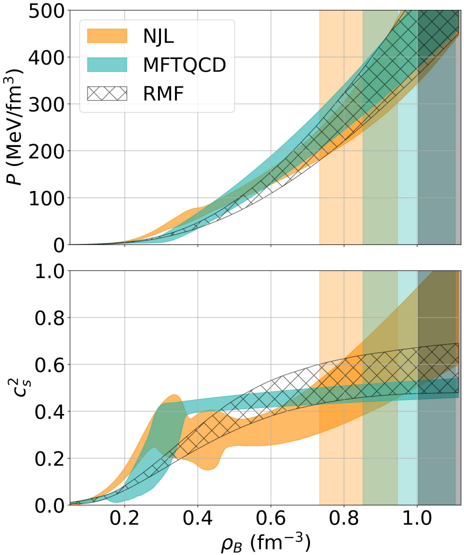
$$\langle A^{a\mu} A^{b\nu} A^{c\rho} A^{d\eta} \rangle = \frac{\phi_0^4}{(32)(34)} \left[g_{\mu\nu} g^{\rho\eta} \delta^{ab} \delta^{cd} + g_\mu^\rho g_\nu^\eta \delta^{ac} \delta^{bd} + g_\mu^\eta g_\nu^\rho \delta^{ad} \delta^{bc} \right], \qquad (9)$$

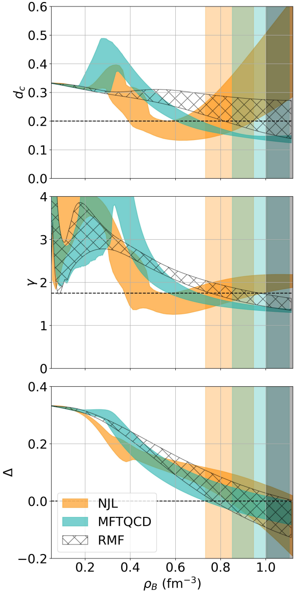
- ϕ_0 and μ_0 are the energy scale to be determined.

[7] L. S. Celenza and C. M. Shakin. **"Description of the gluon condensate".** In: *Phys. Rev. D* 34 (5 Sept. 1986), pp. 1591–1600. DOI: 10.1103/PhysRevD.34.1591.

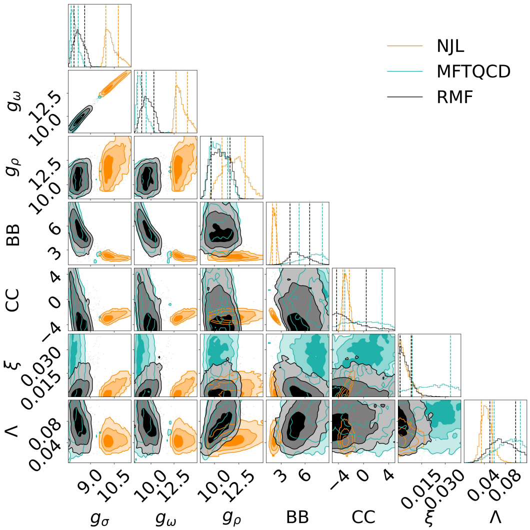
[8] Xiangdong Li and C. M. Shakin. **"Description of gluon propagation in the presence of an A^2 condensate".** In: *Phys. Rev. D* 71 (7 Apr. 2005), p. 074007. DOI: 10.1103/PhysRevD.71.074007.

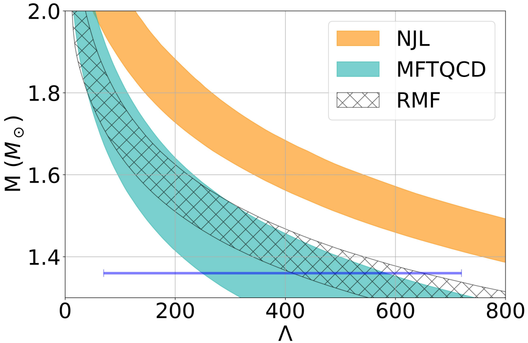
More Results





More Results





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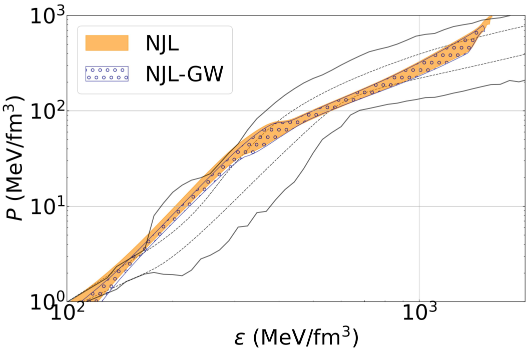

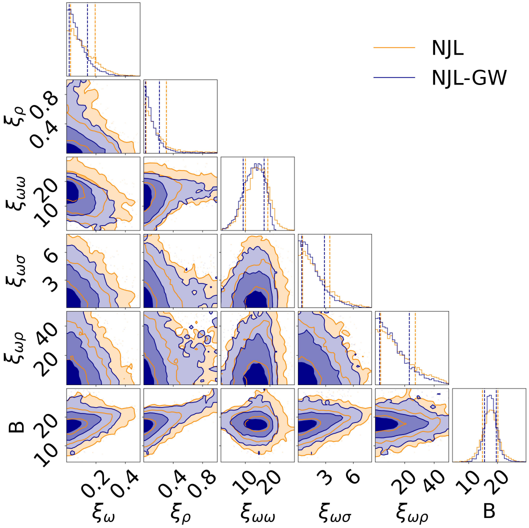

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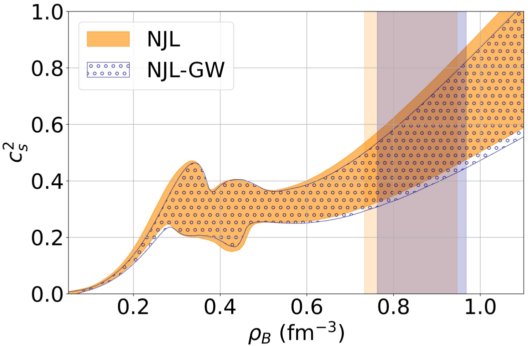

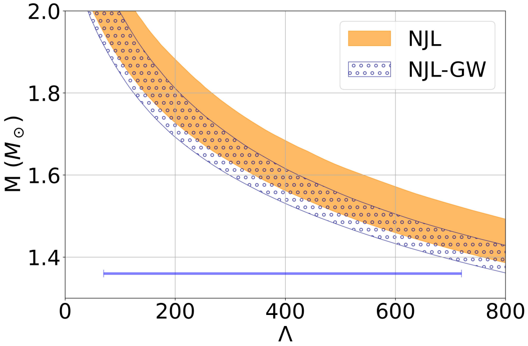

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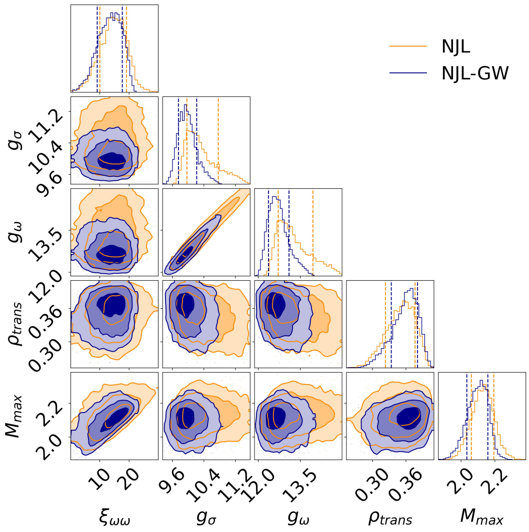
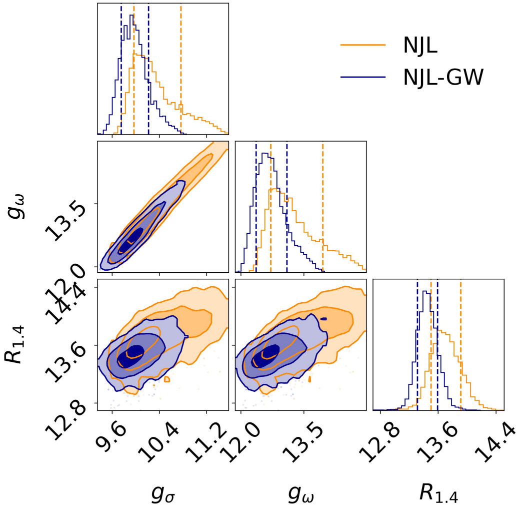
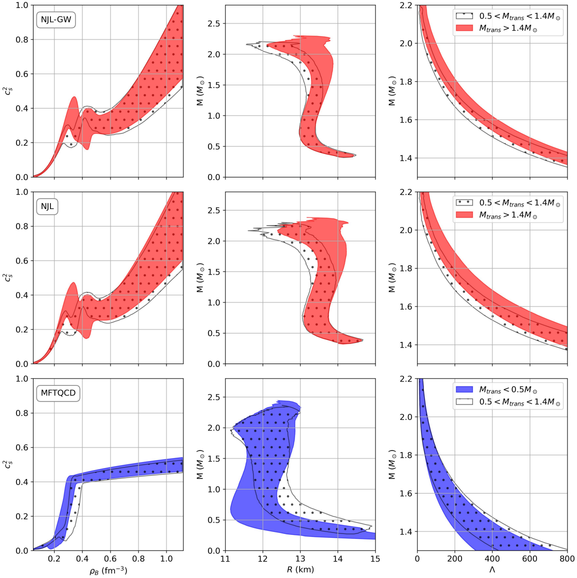
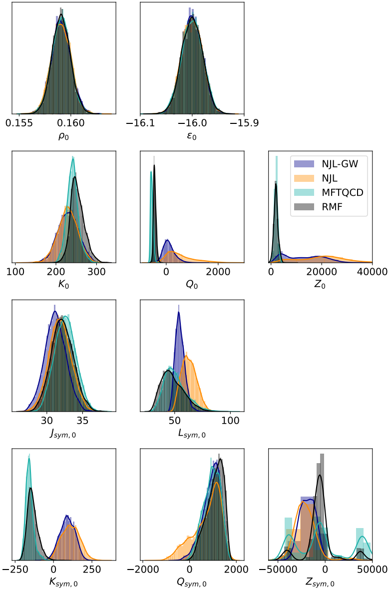




More Results





Priors

Set NJL					
NJL			RMF		
Parameters	min	max	Parameters	min	max
ξ_ω	0	0.5	g_σ	9	12
ξ_ρ	0	1	g_ω	11	15
$\xi_{\omega\omega}$	0	30	g_ρ	9.546	15.000
$\xi_{\sigma\omega}$	0	8	B	1.500	3.500
$\xi_{\rho\omega}$	0	50	C	-4.627	-1.500
B (MeV/fm^3)	0	30	ξ	0	0.016
			Λ_ω	0	0.103

Priors

Set MFTQCD					
MFTQCD			RMF		
Parameters	min	max	Parameters	min	max
ξ_Q (MeV^{-1})	0	0.0018	g_σ	7	10
B (MeV/fm^3)	50	180	g_ω	8	13
			g_ρ	8.000	15.000
			B	1.000	9.000
			C	-5.000	5.000
			ξ	0	0.040
			Λ_ω	0	0.120

Priors

Set RMF		
Parameters	min	max
g_σ	6.5	13
g_ω	6.5	15.5
g_ρ	6.5	16.5
B	0.500	9.000
C	-5.000	5.000
ξ	0	0.040
Λ_ω	0	0.120

Priors

Set NJL-GW					
NJL			RMF		
Parameters	min	max	Parameters	min	max
ξ_ω	0	0.5	g_σ	8	11
ξ_ρ	0	1	g_ω	10	14
$\xi_{\omega\omega}$	0	30	g_ρ	9.546	15
$\xi_{\sigma\omega}$	0	8	B	1.500	3.500
$\xi_{\rho\omega}$	0	50	C	-4.627	-1.500
B (MeV/fm^3)	0	30	ξ	0	0.016
			Λ_ω	0	0.103

Properties

	Set NJL			Set NJL-GW		
quant	median	min	max	median	min	max
ρ_{trans}	0.353	0.304	0.388	0.361	0.313	0.391
P_{trans}	57.466	35.143	78.623	56.044	33.319	76.506
M_{max}	2.130	2.018	2.236	2.103	1.989	2.190
R_{max}	12.329	10.738	12.874	12.121	10.816	12.580
ρ_{max}	0.829	0.733	0.947	0.864	0.763	0.968
$M_{Q,max}$	1.102	0.755	1.512	8.007	7.019	8.933
$R_{Q,max}$	7.954	6.950	8.934	0.455	0.385	0.500
$R_{1.4M_\odot}$	13.695	13.390	14.051	13.450	13.231	13.699
M_{trans}	1.728	1.341	2.008	1.638	1.241	1.926

Properties

	Set MFTQCD			Set RMF		
quant	median	min	max	median	min	max
ρ_{trans}	0.222	0.170	0.308	-	-	-
P_{trans}	5.752	2.451	15.851	-	-	-
M_{max}	2.133	1.970	2.315	2.039	1.905	2.185
R_{max}	11.150	10.509	11.689	10.749	10.264	11.192
ρ_{max}	0.976	0.852	1.104	1.103	1.003	1.212
$M_{Q,max}$	2.018	1.789	2.223	-	-	-
$R_{Q,max}$	10.280	9.428	11.121	-	-	-
$R_{1.4M_\odot}$	12.111	11.571	12.649	12.297	11.923	12.714
M_{trans}	0.327	0.174	0.630	-	-	-

NMP and PNM constraints

NMP		PNM	
Quantity	value/band	Quantity	value/band
$\frac{dEA}{d\rho}$	0	$EA_{PNM}(\rho = 0.05)$	6.8 ± 1.02
EA_0	-16 ± 0.02	$EA_{PNM}(\rho = 0.10)$	10.5 ± 1.97
K_0	230 ± 30	$EA_{PNM}(\rho = 0.15)$	15.3 ± 3.44
$J_{sym,0}$	32.5 ± 1.8		