

SHELL-MODEL : RECENT ADVANCES FROM MID-MASS TO SUPERHEAVY ELEMENTS

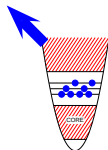
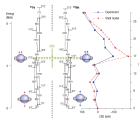
Duy-Duc Dao, F. Nowacki



SM-CI approaches through the nuclear chart

Fundamental interactions and collective excitations

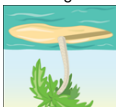
- Deformation Dipole/M1 resonances
- Superfluidity, Symmetries
- Isospin symmetry breaking



- define effective interaction
- $\mathcal{H}_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}}$
- build and diagonalize energy matrix

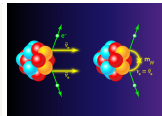
Nuclear structure far from stability

- New magic numbers
- Vanishing of shell closures



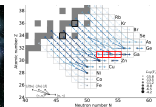
Weak processes

- β decay $\longleftrightarrow$ fundamental interactions
 - $\beta\beta$ decay $\longleftrightarrow$ nature of neutrinos
- $$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2$$



Astrophysics and nucleosynthesis

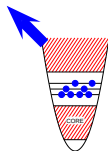
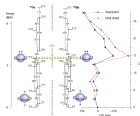
- rp process
- r process



SM-CI approaches through the nuclear chart

Fundamental interactions and collective excitations

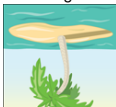
- Deformation Dipole/M1 resonances
- Superfluidity, Symmetries
- Isospin symmetry breaking
- **Superformation, Superheavy nuclei**



- define effective interaction
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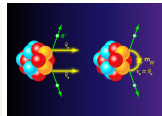
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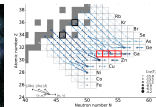
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Astrophysics and nucleosynthesis

- rp process
- r process



Diagonalization versus Variational Methods

Shell Model Exact Diagonalization

- Exponential growth of basis dimensions :

$$D \sim \begin{pmatrix} d_\pi \\ p \end{pmatrix} \cdot \begin{pmatrix} d_\nu \\ n \end{pmatrix}$$

In *pf* shell :

^{48}Cr 1,963,461

^{56}Ni 1,087,455,228

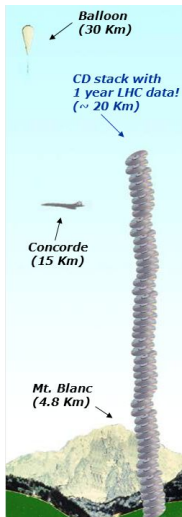
In *pf-sdg* space :

^{78}Ni 210,046,691,518

ANTOINE CODE (1989)

- Actual limits in giant diagonalizations :
 $\sim 10^{12}, \sim 10^{15} \neq 0$ matrix elements
- Some of the largest diagonalizations ever are performed in Strasbourg with relatively modest computational resources :
E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427

- m-scheme ANTOINE code
BIGSTICK
KHELL
- coupled scheme NATHAN code



Variational Approximation

$$\mathcal{H}_{\text{eff}} |\Psi_\alpha^{JM}\rangle = E_\alpha^{(J)} |\Psi_\alpha^{JM}\rangle \rightarrow \delta \frac{\langle \Psi_\alpha^{JM} | \mathcal{H}_{\text{eff}} | \Psi_\alpha^{JM} \rangle}{\langle \Psi_\alpha^{JM} | \Psi_\alpha^{JM} \rangle} = 0$$

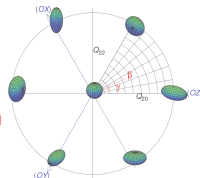
Mixing of shapes :

$$|\Psi_{\text{eff}}\rangle = |\text{shape 1}\rangle + |\text{shape 2}\rangle + \dots$$

Restoration of the rotational symmetry

Degree of freedom

- Multipole: $\beta, \gamma, Q_{30}, \dots$
- Cranking: J_x, J_z
- Pairing constraints
- Variation-After-Projection



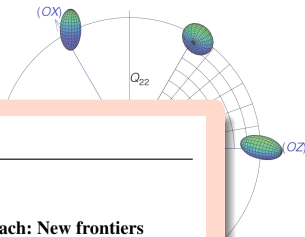
Different implementations

- MCSM Tokyo group
- DNO-SM Strasbourg group
- PGCM Madrid group

Discrete Non-Orthogonal Shell Model (DNO-SM)

◆ Generator coordinate method :

- Deformed Hartree-Fock Slater determinants
- Contribution of $q = (\beta, \gamma)$ in the correlated state J_α
- Rotational symmetry restoration



PHYSICAL REVIEW C **105**, 054314 (2022)

Nuclear structure within a discrete nonorthogonal shell model approach: New frontiers

D. D. Dao and F. Nowacki

Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

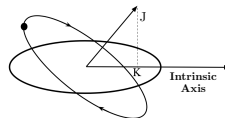
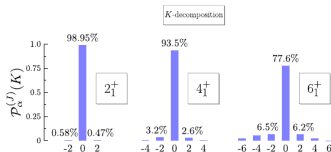


(Received 8 March 2022; accepted 6 May 2022; published 23 May 2022)

● K : Projection of total angular momentum J onto the intrinsic axis

- Contribution of K -components in the correlated state J_ν

$$P_\alpha^{(J)}(K) = \sum_q |M_\alpha^{(J)}(q, K)|^2$$



Discrete Non-Orthogonal Shell Model (DNO-SM)

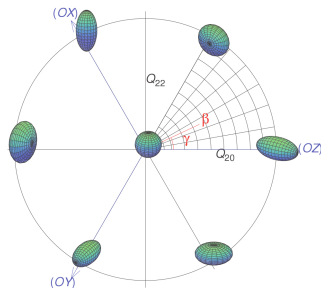
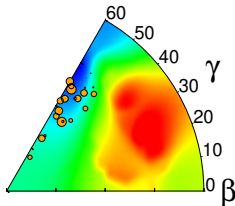
◆ Generator coordinate method :

- Deformed Hartree-Fock Slater determinants
- Contribution of $q = (\beta, \gamma)$ in the correlated state J_α
- Rotational symmetry restoration

$$|\Psi_{\text{eff}}\rangle = \text{[deformed HF Slater determinant 1]} + \text{[deformed HF Slater determinant 2]} + \text{[deformed HF Slater determinant 3]} \dots$$

$$P_\alpha^{(J)}(q) = \sum_K |M_\alpha^{(J)}(q, K)|^2$$

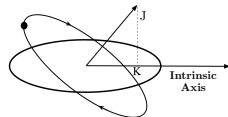
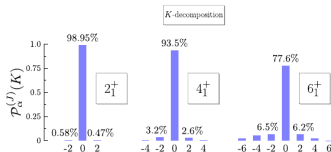
(D.D. Dao and F. Nowacki, PRC 105, 054314 (2022))



◆ K -mixing contents of the wave functions in the intrinsic frame

- K : Projection of total angular momentum J onto the intrinsic axis
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$$P_\alpha^{(J)}(K) = \sum_q |M_\alpha^{(J)}(q, K)|^2$$



Discrete Non-Orthogonal Shell Model (DNO-SM)

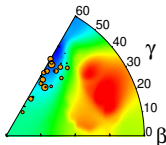
◇ Generator Coordinate Method :

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$$|\Psi_{\text{eff}}\rangle = |\text{shape 1}\rangle + |\text{shape 2}\rangle + |\text{shape 3}\rangle + \dots$$

$$P_{\alpha}^{(J)}(q) = \sum_K |M_{\alpha}^{(J)}(q, K)|^2$$

(D.D. Dao and F. Nowacki, PRC 105, 054314 (2022))



◇ Problems :

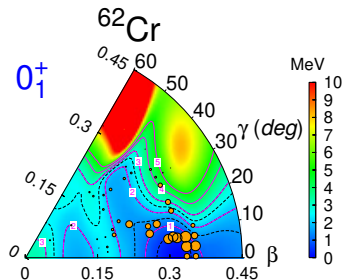
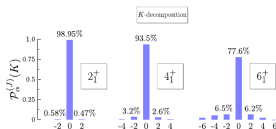
- Underbinding energies
- Compressed spectra, large BE2
- Missing correlations

LNPS effective interaction		
⁶² Cr	DNO-SM(β, γ)	SM
$E(0_1^+)$ (MeV)	-212.426	-214.138
$B(E2, 2_1^+ \rightarrow 0_1^+)$ ($e^2\text{fm}^4$)	458	362

◇ K -mixing contents of the wave functions in the intrinsic frame

- K : Intrinsic quantum number
- Contribution of K -mixing

$$P_{\alpha}^{(J)}(K) = \sum_q |M_{\alpha}^{(J)}(q, K)|^2$$



Variation After Projection = Incorporation of $NpNh$ excitations

◇ Trial symmetry-projected Slater determinants :

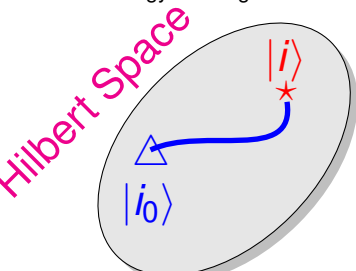
$$\mathcal{H}_{\text{eff}}|\Psi_{\alpha}^{JM}\rangle = E_{\alpha}^{(J)}|\Psi_{\alpha}^{JM}\rangle \implies \delta \frac{\langle \Psi_{\alpha}^{JM} | \mathcal{H}_{\text{eff}} | \Psi_{\alpha}^{JM} \rangle}{\langle \Psi_{\alpha}^{JM} | \Psi_{\alpha}^{JM} \rangle} = 0, \quad |\Psi_{\alpha}^{JM}\rangle = \sum_{q,K} \underbrace{f_{\alpha}^{(J)}(q, K)}_{\text{Mixing coefficient}} \mathcal{P}_{MK}^J \underbrace{|\Phi(q)\rangle}_{\text{Slater state}}$$

Double variation AFTER Angular Momentum Projection : **Mixing coefficient**

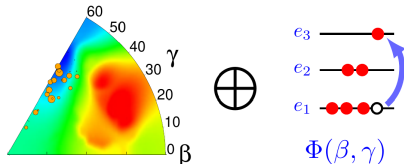
Slater state

◇ DNO-SM(VAP)

- $q = 1, 2, 3, \dots$
- $J_{\alpha} = 0_1, \dots$
- Best energy-favoring Slater states



◇ DNO-SM(PAV) : $(\beta, \gamma) + NpNh$



$$|\Psi_{\text{eff}}\rangle = \text{[orbital 1]} + \text{[orbital 2]} + \text{[orbital 3]} + \dots$$

Variation After Projection = Incorporation of $NpNh$ excitations

◇ Trial symmetry-projected Slater determinants :

$$\frac{1}{\sqrt{2J+1}} \langle JM | \hat{O} | JM \rangle$$

Exact solutions of the nuclear shell-model secular problem: Discrete Non-Orthogonal Shell Model within a Variation After Projection approach

Duy-Duc Dao^a, Frédéric Nowacki^a

^a Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

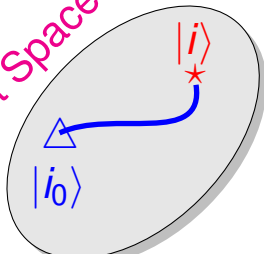
arXiv :2507.09073

Do

◇ DM

- $q = 1, 2, 3, \dots$
- $J_\alpha = 0, 1, \dots$
- Best energy-favoring Slater states

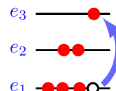
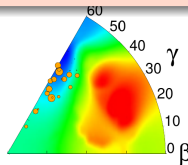
Hilbert Space



$$| \Phi(q) \rangle$$

water state

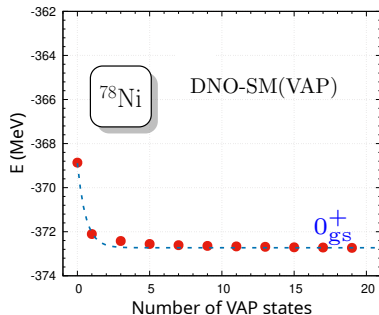
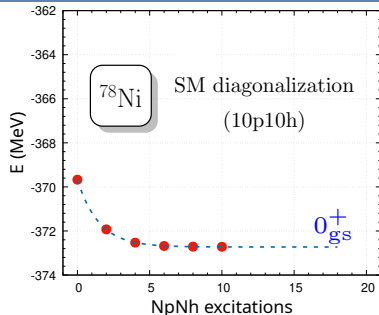
$NpNh$



$$\Phi(\beta, \gamma)$$

$$| \Psi_{\text{eff}} \rangle = \text{[orbital 1]} + \text{[orbital 2]} + \text{[orbital 3]} \dots$$

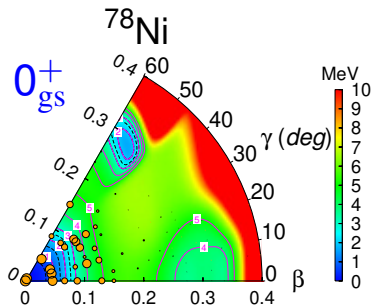
Coexistence in ^{78}Ni



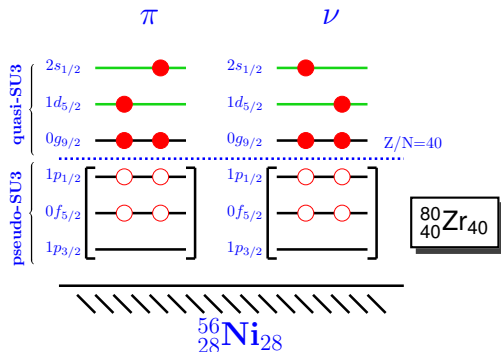
(D.D. Dao, F. Nowacki, in preparation)

PFSDG-U effective interaction
Ground State Energy in MeV

(β, γ)	VAP	SM (10p-10h)
81	19	$\sim 2 \times 10^{12}$
-370.630	-372.733	-372.717



N=40 at N=Z



- p shell : ^{16}O
spherical/doubly
magic

PRL 21 (1968) 39

ZBM1

- sd shell : ^{40}Ca
spherical/doubly magic

PLB 522 (2001) 240

ZBM2

- pf shell : ^{80}Zr deformed
nucleus

...

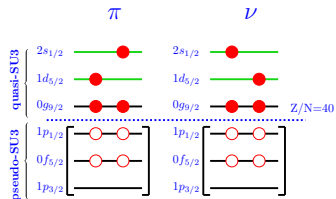
ZBM3

- Low-lying states in H.O. $N=Z=8$: **CS** , 4p4h, 8p8h
- Low-lying states in H.O. $N=Z=20$: **CS** , 4p4h, 8p8h
- Low-lying states in H.O. $N=Z=40$: 4p4h ? 8p8h ? 12p12h ?

Island of Inversion at the $N=Z$ line

Strongly deformed states at $N = Z$:

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr , ^{80}Zr
- New spectroscopy for ^{84}Mo and ^{86}Mo
NSCL/GRETINA Experiment



R.D.O. Llewellyn *et al.*, PRL **124**, 152501 (2020)

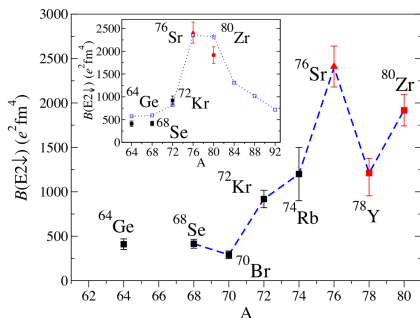


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei

- ZBM3 valence space :
extension of JUN45
to pseudo-SU3 + Quasi-SU3
- New effective interactions :
 - Realistic TBME + Monopole “3N” constraints”
 - ab-initio N3LO (2N) interaction
 - ongoing ab-initio N3LO (2N) + 3N (Inl) interaction
- SM + DNO-SM for most **deformed cases**

Island of Inversion at the N=Z line

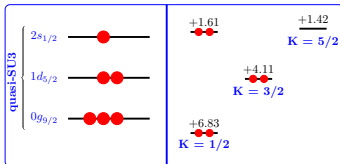
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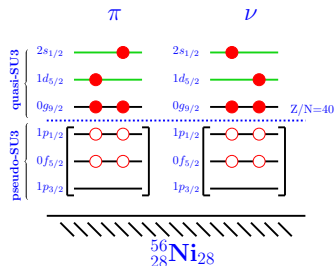
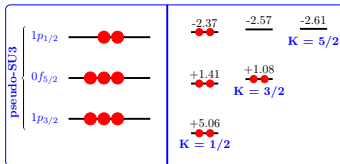
s. p. energy levels

s. p. quadrupole moments



s. p. energy levels

s. p. quadrupole moments



nucleus	Np-Nh*	B(E2)(e ² .fm ⁴)				
		ZRP	PHF	Exp.	DNO-SM*	SM
^{84}Mo	4p-4h	1104	1193	1740 ⁺⁵⁸⁰ ₋₄₃₀	1512	-
	8p-8h	1891	1732			
^{86}Mo	0p-0h	542	196	707(71)	893	731
	2p-2h	1030	871			
	4p-4h	1416	1179			
	6p-6h	1858	1655			

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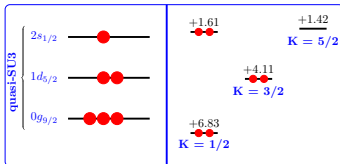
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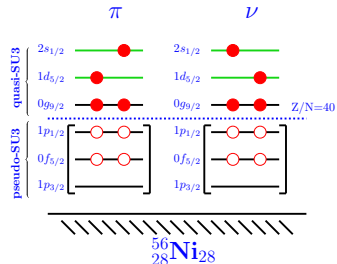
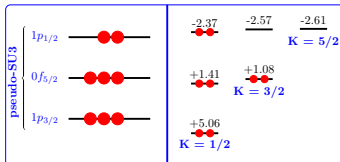
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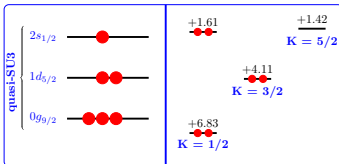
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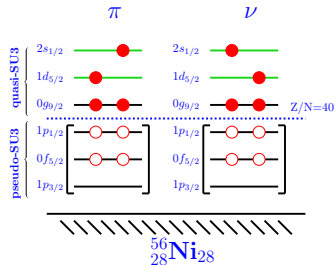
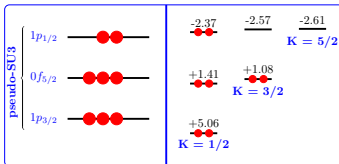
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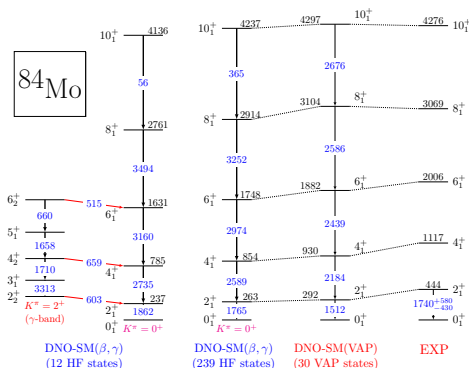
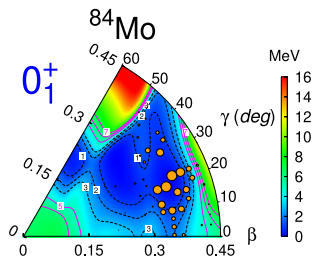
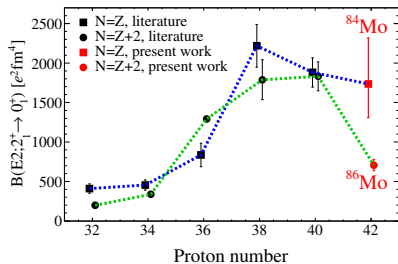
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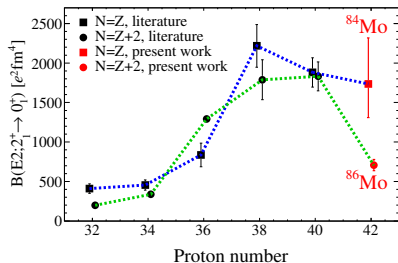
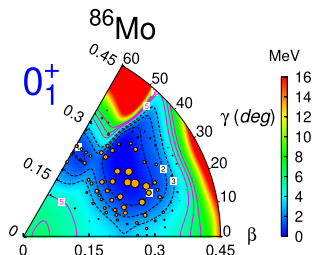


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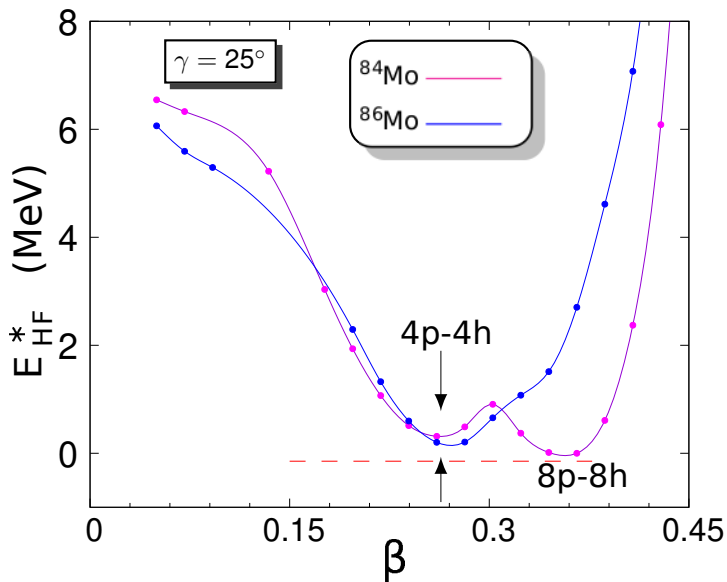
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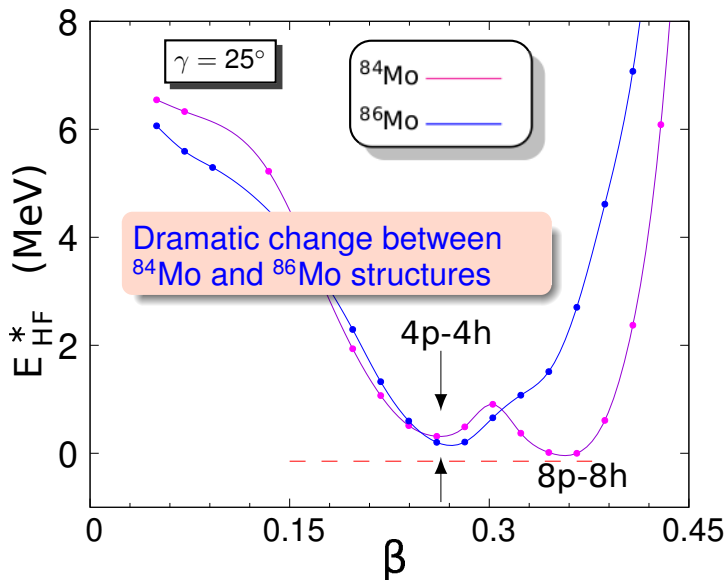


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	4p-4h	1416	1179			
	6p-6h	1858	1655			

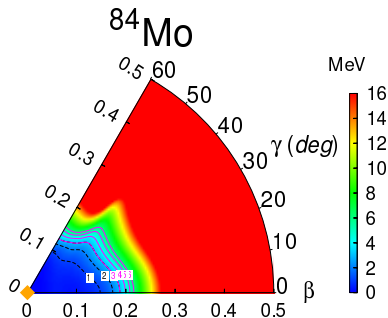
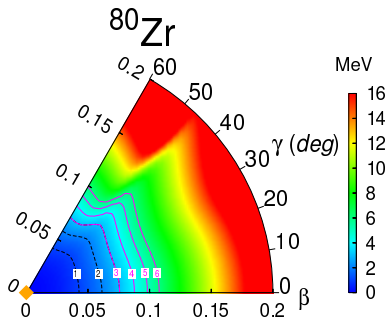
3N forces in light nuclei



3N forces in light nuclei

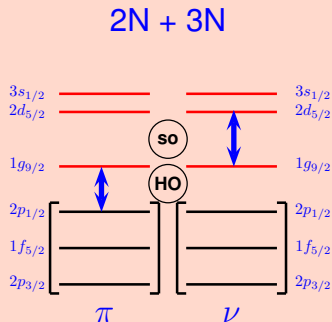
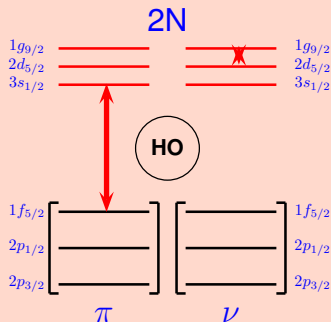


N3LO NN calculations



nucleus	NpNh*	ZRP	PHF	B(E2)(e ² .fm ⁴)		N3LO
				Exp.	DNO-SM	
^{80}Zr	4p-4h	587	637			
	8p-8h	1713	1509	1910(180)	2325	0.03
	12p-12h	2663	2396			
^{84}Mo	4p-4h	1104	1193	1740 ⁺⁵⁸⁰ ₋₄₃₀	1740	174
	8p-8h	1891	1732			

N3LO NN calculations



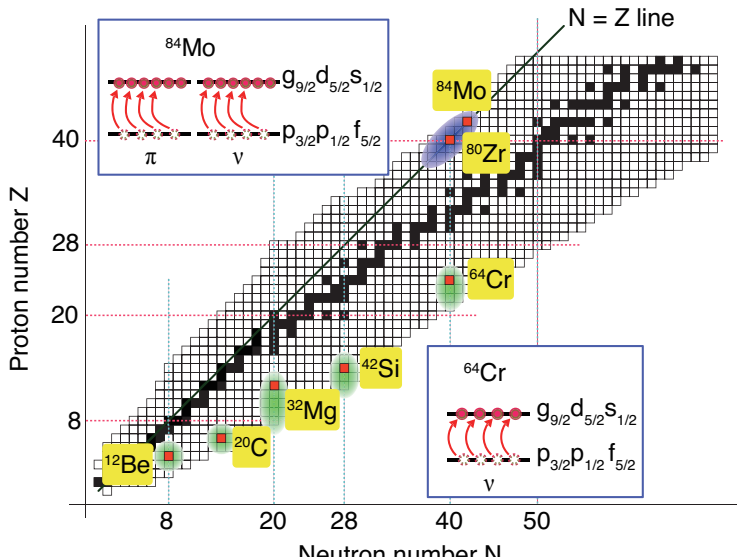
Three body forces and persistence of spin-orbit shell gaps in medium-mass nuclei : Towards the doubly magic ^{78}Ni ,

K. Sieja, F. Nowacki

Phys. Rev. **C85**, 051301(R) (2012)

^{80}Zr	4p-4h	587	637	1910(180)	2325	0.03
	8p-8h	1713	1509			
	12p-12h	2663	2396			
^{84}Mo	4p-4h	1104	1193	1740⁺⁵⁸⁰₋₄₃₀	1740	174
	8p-8h	1891	1732			

Isospin Symmetric Island of Inversion



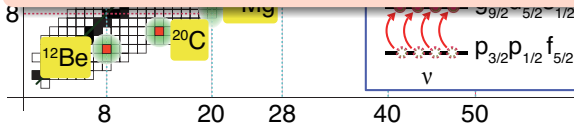
Isospin Symmetric Island of Inversion

Abrupt Structural Transition in Exotic Molybdenum Isotopes unveils an Isospin-Symmetric Island of Inversion

J. Ha^{1,2,3,4}, F. Recchia^{2,3*}, S. M. Lenzi^{2,3}, H. Iwasaki^{5,6}, D. D. Dao⁷, F. Nowacki⁷, A. Revel^{5,6}, P. Aguilera^{2,3}, G. de Angelis⁸, J. Ash^{5,6}, D. Bazin^{5,6}, M. A. Bentley⁹, S. Biswas⁵, S. Carollo², M. L. Cortes⁸, R. Elder^{5,6}, R. Escudeiro^{2,3,10}, P. Farris^{5,6}, A. Gade^{5,6}, T. Ginter⁵, M. Grindler^{5,6}, J. Li⁵, D. R. Napoli⁸, S. Noji⁵, J. Pereira⁵, S. Pigliapoco^{2,3}, A. Pompermaier², A. Poves¹¹, K. Rezyunkina^{2,3}, A. Sanchez^{5,6}, R. Wadsworth⁹ and D. Weisshaar⁵

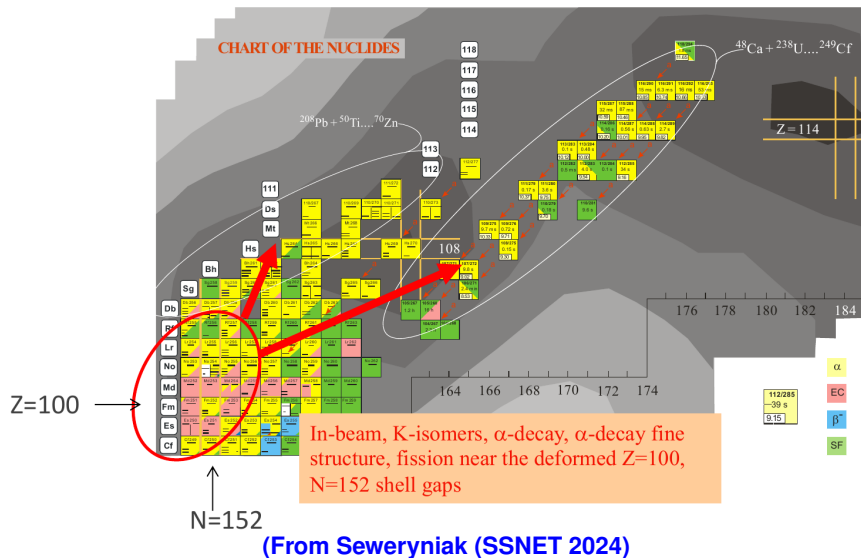
J. Ha et al., Nature Communications accepted

Proton number Z



Neutron number N

Landscape of superheavy nuclei



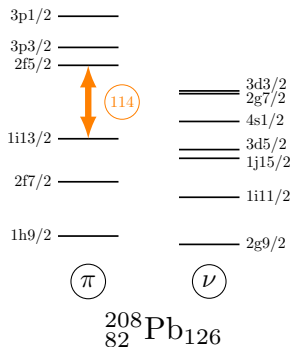
Applications to superheavy nuclei

Kuo-Herling interaction :

- $^{208}_{82}\text{Pb}_{126}$ core, realistic TBMEs
- $82 \leq Z \leq 126$ shells for proton and $126 \leq N \leq 184$ for neutrons
- monopole corrections (3N force)

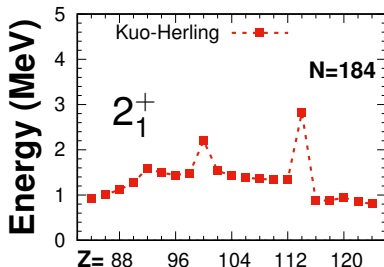
E. Caurier and F. Nowacki,

PRL 87 (2001),072511



Calculations : NATHAN & CARINA codes

- ◊ Diagonalization within the seniority scheme along the chains of $N = 126$ and $N = 184$
- ◊ Variation After Projection calculations : $^{253,254,255}\text{Es}$, $^{249,251,253}\text{Cf}$, $^{253,254}\text{No}$, ^{256}Fm
- ◊ Comparison of spectra and electromagnetic moments



Yrast systematics : odd-even $^{251}_{98}\text{Cf}_{153}$

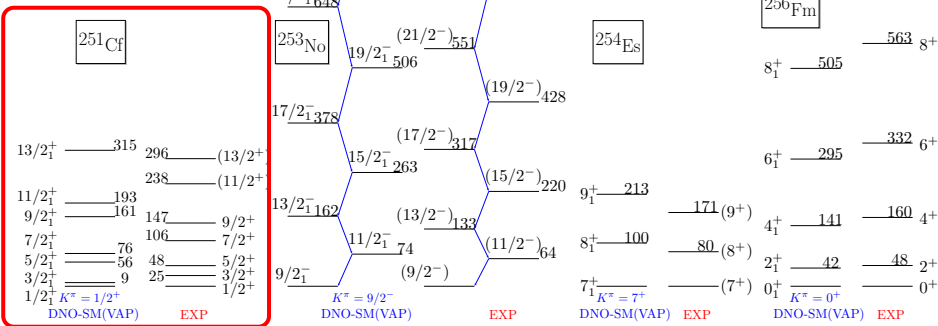
First complete description of low-lying spectroscopy in ^{254}No

Duy Duc Dao¹ and Frédéric Nowacki¹

¹ Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

Systematic comparison of Yrast bands

arXiv :2409.08210



Yrast systematics : odd-even $^{253}_{102}\text{No}_{151}$

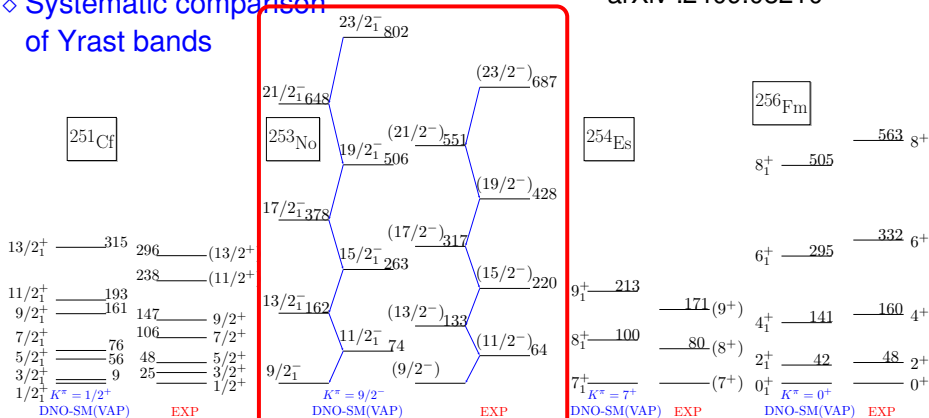
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arXiv :2409.08210



Yrast systematics : odd-odd $^{254}_{99}\text{Es}_{155}$

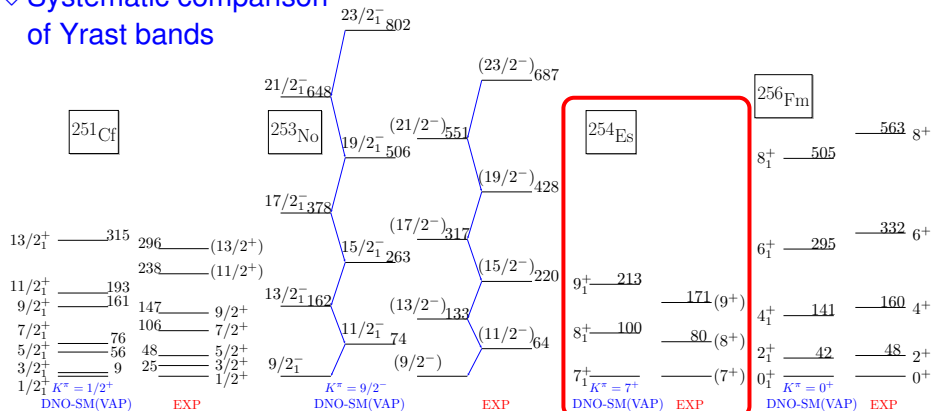
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Systematic comparison of Yrast bands

arXiv :2409.08210



Yrast systematics : even-even $^{256}_{100}\text{Fm}_{156}$

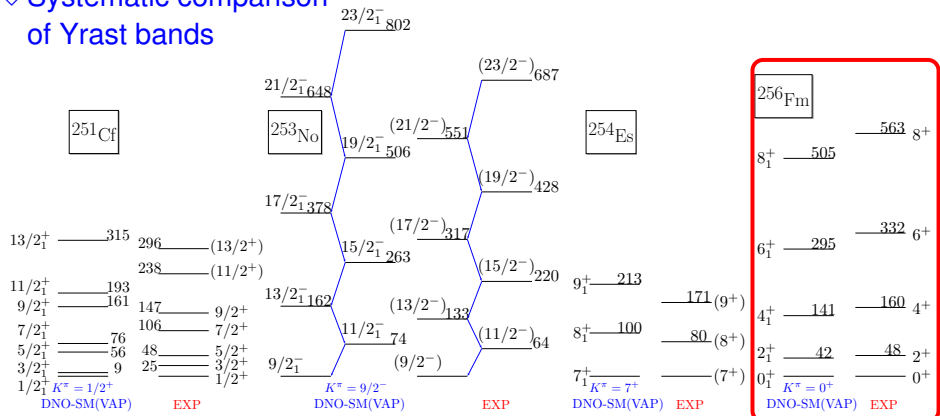
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arXiv :2409.08210

Systematic comparison of Yrast bands



Applications to superheavy nuclei : Electromagnetic moments

First complete description of low-lying spectroscopy in ^{254}No

Duy Duc Dao¹ and Frédéric Nowacki¹

¹ Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

arXiv :2409.08210

◇ Comparison of magnetic and quadrupole moments

Effective charges : $e_p = 1.72$, $e_n = 0.75$

	J_{gs}^π	E (MeV)		PHF	$\mu(\mu_N)$		PHF	Q_s (eb)	
		PHF	VAP		VAP	EXP		VAP	EXP
^{253}No	$9/2^-$	-241.818	-242.816	+0.591	-0.493	-0.527(33)(75)	-3.5	+7.2	+5.9(1.4)(0.9)
^{253}Cf	$7/2^+$	-253.402	-253.818	-0.677	-0.556	-0.731(35)	+5.77	+5.78	+5.53(51)
^{251}Cf	$1/2^+$	-241.321	-241.724	-0.727	-0.610	-0.571(24)	-	-	-
^{249}Cf	$9/2^-$	-229.021	-229.381	-0.480	-0.461	-0.395(17)	+6.62	+6.63	+6.27(33)
^{255}Es	$7/2^+$	-263.512	-264.695	-1.10	+3.94	+4.14(10)	+6.0	+5.8	+5.1(1.7)
^{254}Es	7^+	-257.492	-258.441	+0.778	+3.36	+3.42(7)	+1.8	+8.4	+9.6(1.2)
^{253}Es	$7/2^+$	-251.837	-252.280	+3.63	+3.93	+4.10(7)	+5.87	+5.9	+6.7(8)
^{256}Fm	0^+	-268.999	-269.717	+0.87	+0.89	-	-3.57	-3.60	-
^{254}No	0^+	-249.568	-250.187	+0.87	+0.91	-	-3.78	-3.75	-

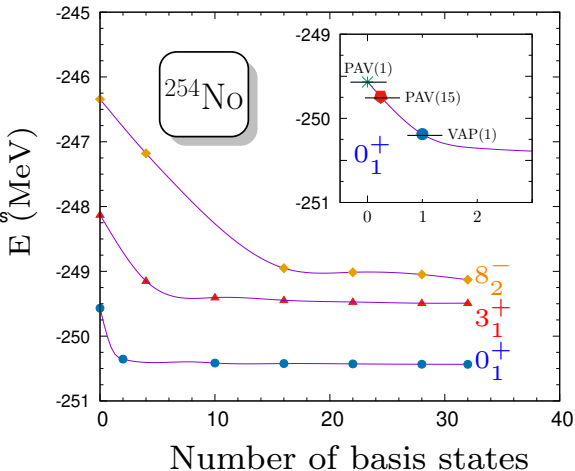
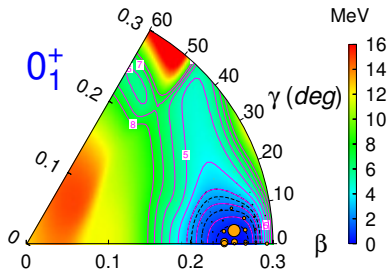
Applications to superheavy nuclei : $^{254}_{102}\text{No}_{152}$

Known spectroscopy :

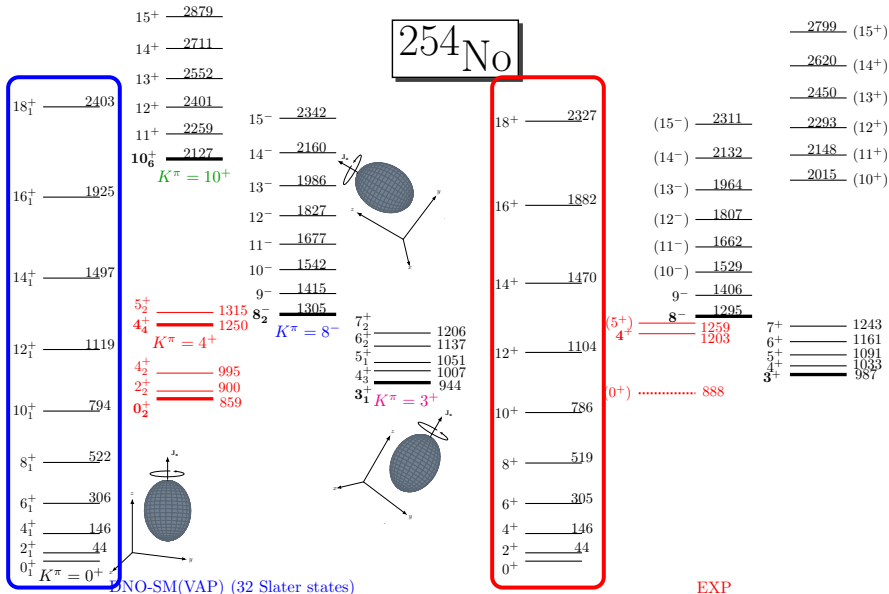
- Axially well-deformed nucleus
- Ground-state rotational band
- Several isomeric states (8^- , 3^+ , 10^+)

Calculations :

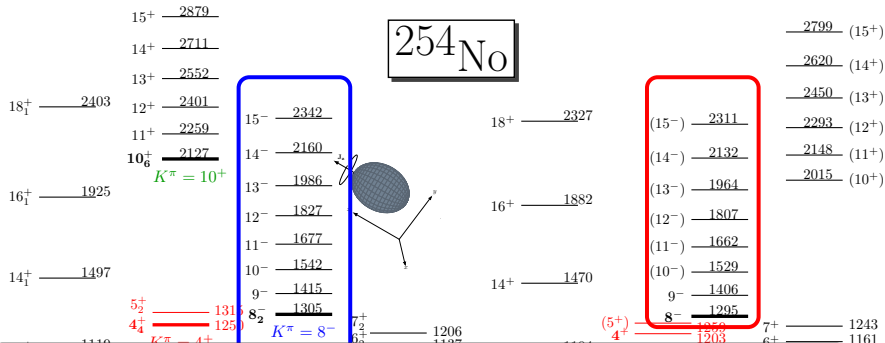
- Deformed Axial Minimum at $\beta = 0.25$
- Mixing of triaxial configurations (β, γ) (PAV(15))
- VAP calculations for band-head structures



Reproduction of spectra : Yrast rotational band



Isomeric 8^- ($K = 8$) band



8_2^-	Calc.	EXP.
E^* (keV)	1305	1295 ^a

(^a M. Forge *et al.*, Physical Review, under review)

Single-particle configurations :

~ Two-neutron excitations

$$\nu j15/2 \otimes \nu g9/2$$

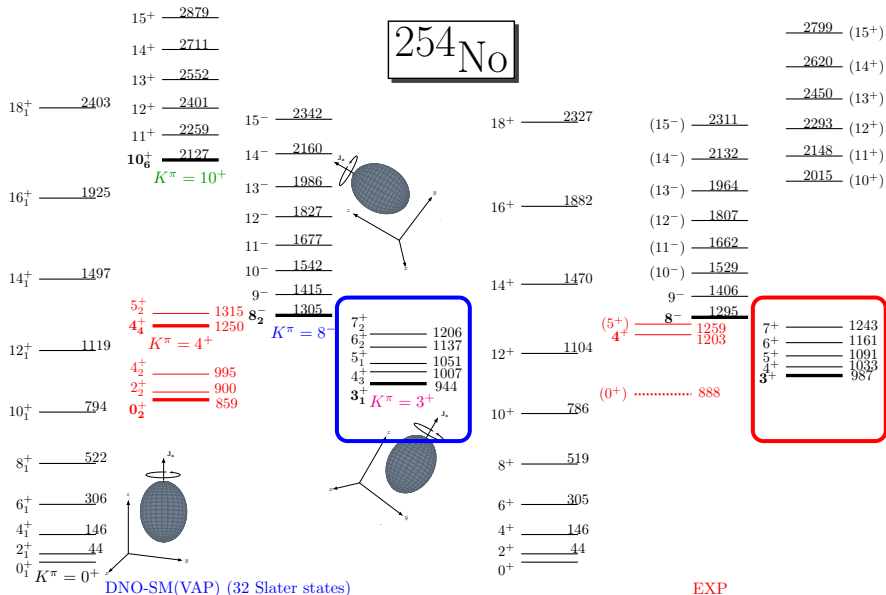
2_1^+ 44
 0_1^+ $K^\pi = 0^+$

DNO-SM(VAP) (32 Slater states)

2^+ 44
 0^+

EXP

Isomeric $3^+(K=3)$ band



Isomeric $3^+(K=3)$ band : Gallagher-Moszkowski partner $4^+(K=4)$

^{254}No	Calc.	EXP.
$E^*(3^+)$ (keV)	944	987 ^a
$E^*(4^+)$ (keV)	1250	1203 ^a
$\Delta E_{GM} = E_{\uparrow\downarrow} - E_{\uparrow\uparrow}$	306	216 ^a

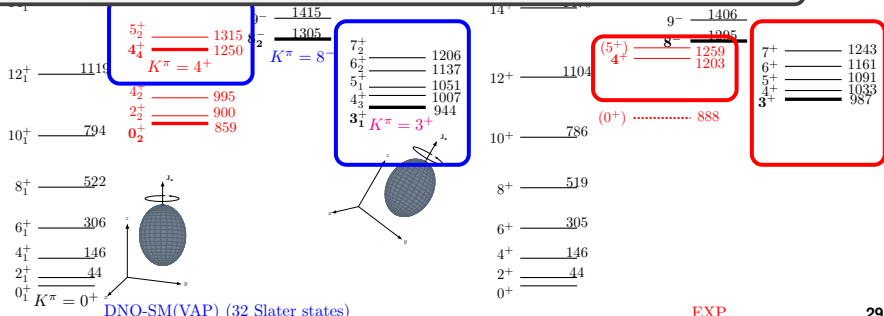
(^a M. Forge *et al.*, Physical Review, under review)

$$K = |\Omega_n \pm \Omega_p|$$

$K=3$ (spin-aligned)

lies lower than

$K=4$ (spin-anti-aligned)



Single-particle Occupations : $3^+(K=3)$, $4^+(K=4)$

- $3_1^+, 4_4^+$: similar structure involving a $\pi h9/2$ recoupled with others orbitals

Ground- and excited-states occupations

Proton orbits	$0h_{9/2}$	$0i_{13/2}$	$1f_{7/2}$	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$		
0_1^+	6.03	7.75	3.43	1.49	0.77	0.52		
0_2^+	7.08	7.91	3.23	0.89	0.69	0.21		
$3_1^+(K=3)$	6.47	7.98	3.34	1.15	0.72	0.34		
$4_4^+(K=4)$	6.50	7.83	3.41	1.18	0.72	0.36		
$8_2^-(K=8)$	6.48	7.90	3.36	1.19	0.70	0.37		
$10_6^+(K=10)$	6.55	7.03	3.48	1.56	0.79	0.58		
Neutron orbits	$0i_{11/2}$	$0j_{15/2}$	$1g_{9/2}$	$1g_{7/2}$	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$	
0_1^+	7.30	9.91	5.43	1.00	1.09	0.84	0.43	
0_2^+	7.36	9.95	5.45	0.96	1.05	0.80	0.42	
$3_1^+(K=3)$	7.32	9.94	5.46	0.97	1.07	0.81	0.42	
$4_4^+(K=4)$	7.34	9.79	5.48	1.03	1.11	0.81	0.44	
$8_2^-(K=8)$	7.42	9.00	6.30	0.98	1.06	0.81	0.43	
$10_6^+(K=10)$	7.23	8.89	5.72	1.40	1.34	0.97	0.45	

Branching Ratio : $3^+(K = 3)$ band

15+ — 2879

14+ — 2711

13+ — 2552

^{254}No

— 2799 (15+)

— 2620 (14+)

— 2450 (13+)

— 2293 (12+)

— 2148 (11+)

— 2015 (10+)

$$\Delta E_{GM} = E_{\uparrow\downarrow} - E_{\uparrow\uparrow} \quad \text{Calc.} \quad \text{EXP.}$$

$$\Delta E_{GM} \text{ (keV)} \quad 306 \quad 216^a$$

(^a M. Forge *et al.*, Physical Review, under review)

$$K = |\Omega_n \pm \Omega_p|$$

$K = 3$ (spin-aligned)

lies lower

$K = 4$ (spin-anti-aligned)

14+ — 1497

10- — 1342

1470

(10-) — 1529

^{254}No

$$B(E2, 7_2^+ \rightarrow 5_1^+)$$

$$23971 \text{ e}^2\text{fm}^4$$

$$B(M1, 7_2^+ \rightarrow 6_2^+)$$

$$0.228 \mu_N^2$$

Branching Ratio

Calc.

EXP.^a

0.94

1.1(4)

(^a S. K. Tandel *et al.*, Phys. Rev. Lett. **97**, 082502 (2006))

1243
1161
1091
1033
987

DNO-SM(VAP) (32 Slater states)

EXP

Low-lying excited 0_2^+ state

15+ — 2879
14+ — 2711
13+ — 2552

^{254}No

— 2799 (15+)
— 2620 (14+)

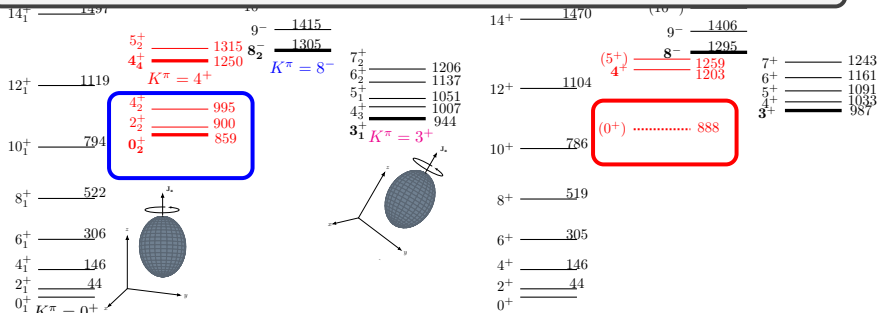
(13+)
(12+)
(11+)
(10+)

Structure :

Involving essentially proton excitations

0_2^+	Calc.	EXP.
E^* (keV)	859	888 ^a

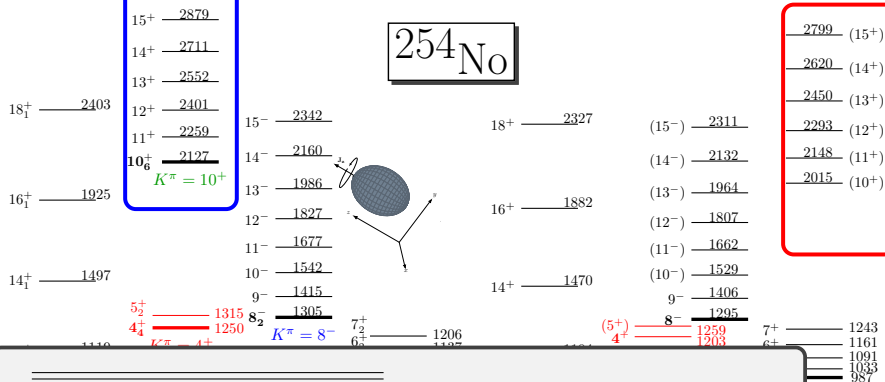
(^a M. Forge *et al.*, Physical Review, under review)



DNO-SM(VAP) (32 Slater states)

EXP

Isomeric $10^+(K=10)$ band



10^+

Calc.

EXP.

^a S. G. Wahid *et al.*,

E^* (keV)

2127

2015^a

Phys. Rev. C 111, 034320 (2025)

0_1^+ — 0
 4_1^+ — 146
 2_1^+ — 44
 0_1^+ $K\pi = 0^+$

DNO-SM(VAP) (32 Slater states)

0^+ — 0
 4^+ — 146
 2^+ — 44
 0^+ — 0

EXP

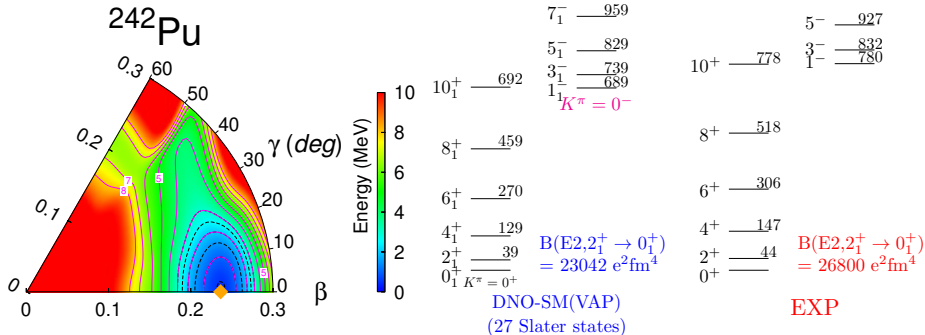
Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

Known spectroscopy :

- Axially well-deformed nucleus
- Ground-state rotational band
- Several side bands :
 $1^-(K=0), 3^-(K=3), 7^-$

Calculations :

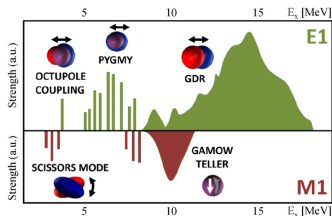
- Deformed Axial Minimum at $\beta = 0.24$
- VAP calculations for band-head structures



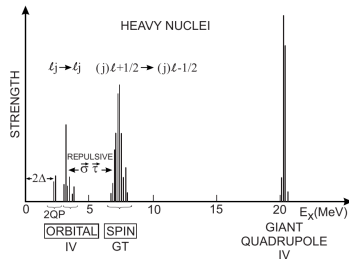
Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

◇ Strength function description of $M1$, $E1$:

- Scissors mode in deformed nuclei
- DNO-SM(VAP) : capture correlations



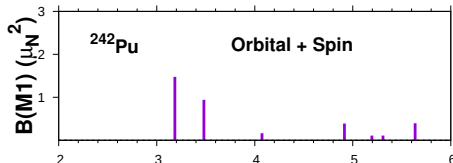
D. Budker *et al.*, Ann. Phys. **2022**, 534, 2100284.



K. Heyde *et al.*, Rev. Mod. Phys. **104**, 045801 (2021).

◇ Open problems :

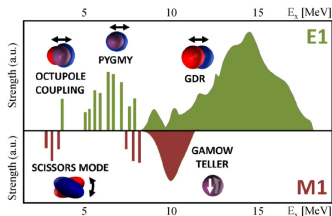
- Generate easily structure function for many final states
- $E1$ transitions forbidden in the natural valence space
→ add QSU3 higher orbitals



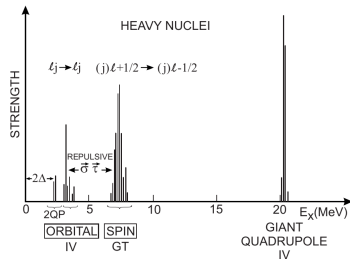
Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

◇ Strength function description of $M1$, $E1$:

- Scissors mode in deformed nuclei
- DNO-SM(VAP) : capture correlations



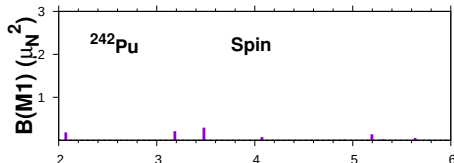
D. Budker *et al.*, Ann. Phys. **2022**, 534, 2100284.



K. Heyde *et al.*, Rev. Mod. Phys. **104**, 045801 (2021).

◇ Open questions :

- Generate easily structure function for many final states
- $E1$ transitions forbidden in the natural valence space
→ add QSU3 higher orbitals



Summary

- Non-Orthogonal Multi-Slater Determinants give an exact representation of shell-model wave functions
- Pairing correlations are fully captured by the variation after projection where the rotational symmetry is consistently restored
- Isospin symmetric Island of Inversion around $A \sim 80$
- First shell-model description of superheavy nuclei : $^{253,254,255}\text{Es}$, $^{249,251,253}\text{Cf}$, $^{253,254}\text{No}$, ^{256}Fm
- Complete reproduction of known spectroscopy of ^{254}No : yrast and excited isomeric structures
- Ongoing study of dipole excitations in ^{242}Pu

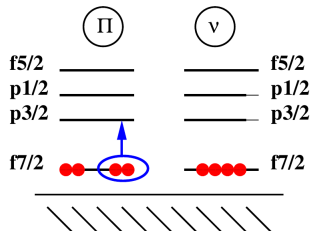
Special thanks to :

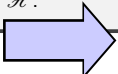
- G. Martinez-Pinedo, A. Poves, S. Lenzi
- A. Gade, O. Sorlin, A. Obertelli

Discrete Non-Orthogonal Shell Model (DNO-SM)

- Broeckhove-Deumens Theorem : Z. Phys. A292, 243 (1979)

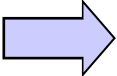
Given a separable Hilbert space $\mathcal{H}$ spanned by a continuous family of states $\Gamma = \{\psi(\alpha)|\alpha \in \mathbb{R}\}$, there exists a countable subset $\Gamma_0 = \{\phi(\alpha_i)|i \in \mathbb{N}\} \subset \Gamma$ with the property $\mathcal{H} = \overline{\text{span}\Gamma_0}$, i.e. Γ_0 is a skew or non-orthogonal basis in $\mathcal{H}$.




$$\psi = c_1\phi_1 + c_2\phi_2 + \dots + c_i\phi_i + \dots$$

IMPLICATIONS :

- Non-Orthogonal Slater Determinants spans the full shell-model space
- The number of Slater Determinants is FINITE
- “Labels” for eigenstates : $|\alpha, J^\pi M, TT_z\rangle$
 - symmetries : $(J, \pi, \dots)$
 - intrinsic label : α related to the dynamics of the Hamiltonian

- 
- VARIATIONAL PROJECTED SLATER DETERMINANT
SOLVES THE SHELL-MODEL DIAGONALIZATION PROBLEM

Variation After Projection : DNO-SM(VAP)

◇ Trial symmetry-projected Slater determinants :

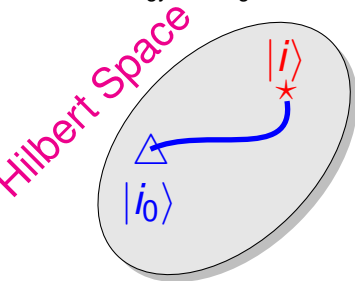
$$\mathcal{H}_{\text{eff}}|\Psi_{\alpha}^{JM}\rangle = E_{\alpha}^{(J)}|\Psi_{\alpha}^{JM}\rangle \implies \delta \frac{\langle \Psi_{\alpha}^{JM} | \mathcal{H}_{\text{eff}} | \Psi_{\alpha}^{JM} \rangle}{\langle \Psi_{\alpha}^{JM} | \Psi_{\alpha}^{JM} \rangle} = 0, \quad |\Psi_{\alpha}^{JM}\rangle = \sum_{q,K} \underbrace{f_{\alpha}^{(J)}(q, K)}_{\text{Mixing coefficient}} \mathcal{P}_{MK}^J \underbrace{|\Phi(q)\rangle}_{\text{Slater state}}$$

Double variation AFTER Angular Momentum Projection : Mixing coefficient

Slater state

◇ DNO-SM(VAP)

- $q = 1, 2, 3, \dots$
- $J_{\alpha} = 0_1, \dots$
- Best energy-favoring Slater states



◇ Energy Minimization :

$$|\Phi\rangle = e^{\sum_{ij} Z_{ij} c_i^{\dagger} c_j} |\Phi_0\rangle$$



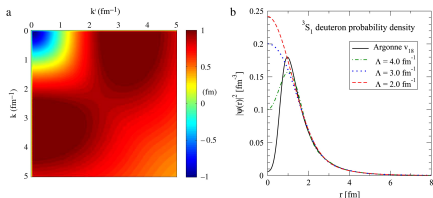
$$\frac{\partial E_{\alpha}^{(J)}}{\partial Z}, |\Phi\rangle = |\Phi_0\rangle + \eta \frac{\partial E_{\alpha}^{(J)}}{\partial Z}$$



$$(\mathcal{H}^{(J)} - E_{\alpha}^{(J)} \mathcal{N}^{(J)}) f_{\alpha}^{(J)} = 0, \{E_{\alpha}^{(J)}, |\Psi_{\alpha}^{JM}\rangle\}$$

Monopole-corrected effective interactions

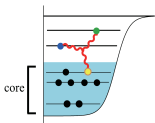
△ V-low k derived from NN interactions



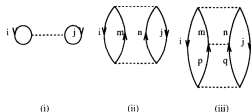
(S.K. Bogner et al., Prog. Part. Nucl. Phys. **65**, 1 (2010) 94-147)



△ Valence-space MBPT renormalization



(H.-W. Hammer et al. RMP **85**, 197 (2013))

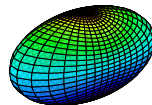
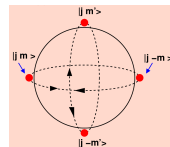
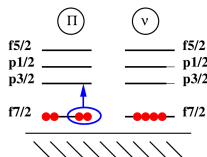


(M. Hjorth-Jensen et al. Phys. Rep. **261** (1995) 125-270)

△ Multipole decomposition

$$H = H_{monopole} + H_{multipole}$$

(M. Dufour, A. P. Zuker, Phys. Rev. C **54**, 1641 (1996))



$H_{monopole}$: spherical mean-field

■ responsible for the global saturation properties and for the evolution of the spherical single particle levels.

$H_{multipole}$: correlator

■ pairing, quadrupole, octupole...

Three-Body Forces and the Limit of Oxygen Isotopes

Takaharu Otsuka,^{1,2,3} Toshio Suzuki,⁴ Jason D. Holt,⁵ Achim Schwenk,⁵ and Yoshinori Akaishi⁶

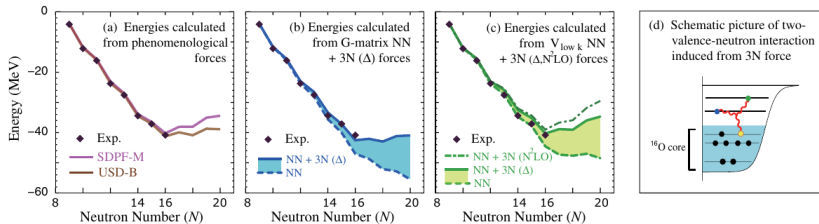


FIG. 4 (color online). Ground-state energies of oxygen isotopes measured from ^{16}O , including experimental values of the bound 16–24 O. Energies obtained from (a) phenomenological forces SDPF-M [13] and USD-B [14], (b) a G matrix and including FM 3N forces due to Δ excitations, and (c) from low-momentum interactions $V_{\text{low } k}$ and including chiral EFT 3N interactions at $N^2\text{LO}$ as well as only due to Δ excitations [25]. The changes due to 3N forces based on Δ excitations are highlighted by the shaded areas. (d) Schematic illustration of a two-valence-neutron interaction generated by 3N forces with a nucleon in the ^{16}O core.

Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,^{1,2} M. Hjorth-Jensen,^{3,4} G. R. Jansen,³ R. Machleidt,⁵ and T. Papenbrock^{1,2}

¹*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*

²*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*

³*Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway*

⁴*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy,
Michigan State University, East Lansing, Michigan 48824, USA*

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(Received 16 April 2012; published 17 July 2012)

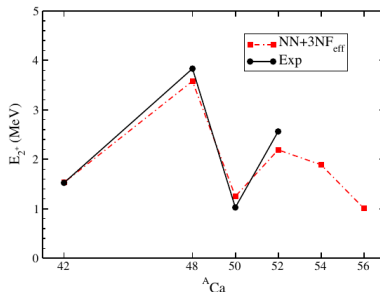


FIG. 2 (color online). (Excitation energies of $J^\pi = 2^+$ states in the isotopes $^{42,48,50,52,54,56}\text{Ca}$ (experiment: black circles,

Contribution of chiral three-body forces to the monopole component of the effective shell-model Hamiltonian

Y. Z. Ma,¹ L. Coraggio,² L. De Angelis,² T. Fukui,² A. Gargano,² N. Itaco,^{2,3} and F. R. Xu¹

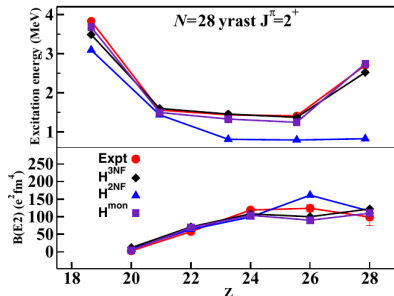


FIG. 22. Experimental and calculated excitation energies of the yrast $J^\pi = 2^+$ states and $B(E2; 2_1^+ \rightarrow 0_1^+)$ transition rates for the $N = 28$ isotones. See text for details.

Shell closures and 2N forces only

PHYSICAL REVIEW C **74**, 061302(R) (2006)

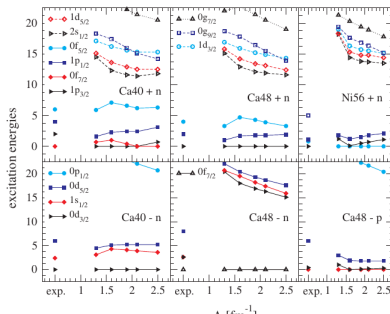
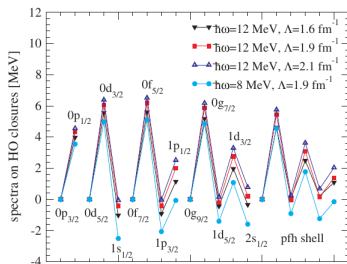
Shell-model phenomenology of low-momentum interactions

Achim Schwenk^{1,*} and Andrés P. Zuker^{2,†}

¹Nuclear Theory Center, Indiana University, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47408, USA

²Institut de Recherches Subatomiques, IN2P3-CNRS, Université Louis Pasteur, F-67037 Strasbourg, France

(Received 14 January 2005; revised manuscript received 20 September 2006; published 12 December 2006)



- **no Spin-orbit** shell closures in ^{12}C , ^{22}O , ^{48}Ca , ^{56}Ni
- too **strong** H. O. shell closures ^{16}O , ^{40}Ca , ... and ^{80}Zr !!!

Yrast systematics : odd-even $^{251}_{98}\text{Cf}_{153}$

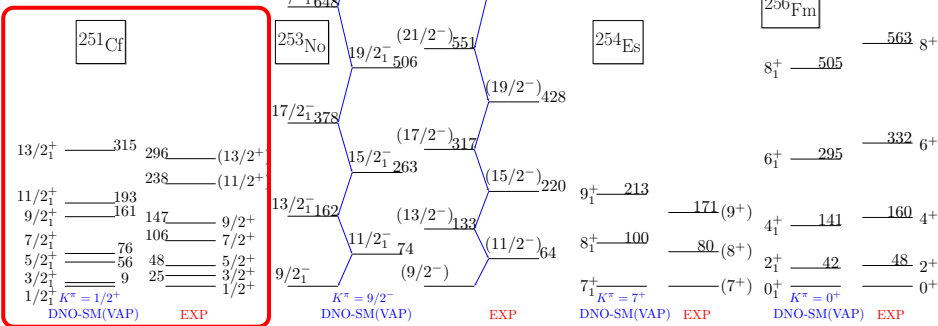
First complete description of low-lying spectroscopy in ^{254}No

Duy Duc Dao¹ and Frédéric Nowacki¹

¹ Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

Systematic comparison of Yrast bands

arXiv :2409.08210



Yrast systematics : odd-even $^{253}_{102}\text{No}_{151}$

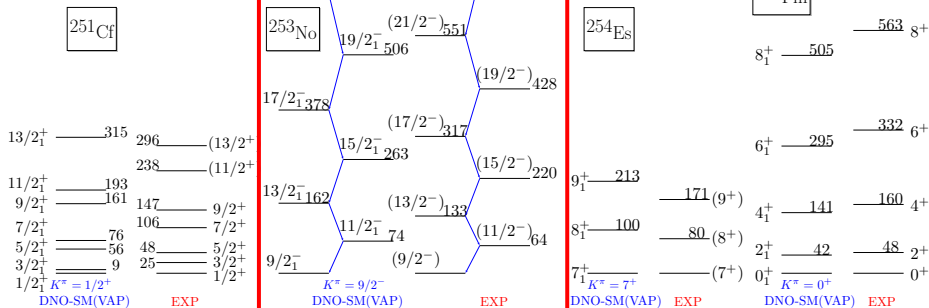
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Systematic comparison of Yrast bands

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Yrast systematics : odd-odd $^{254}_{99}\text{Es}_{155}$

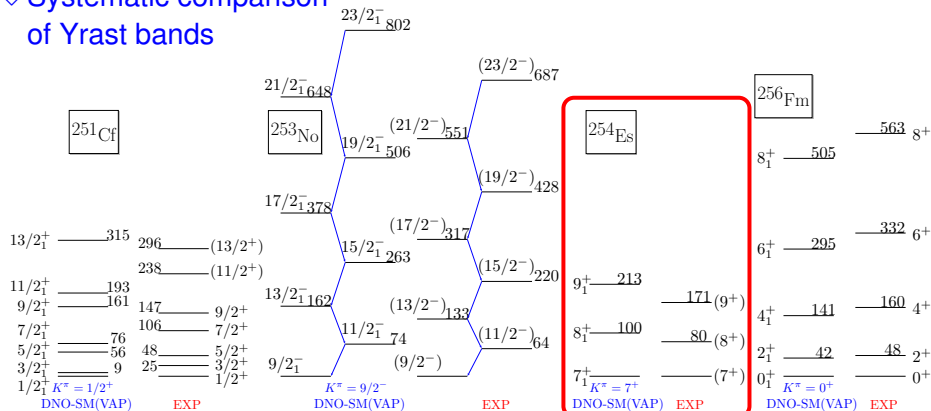
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Systematic comparison of Yrast bands

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Yrast systematics : even-even $^{256}_{100}\text{Fm}_{156}$

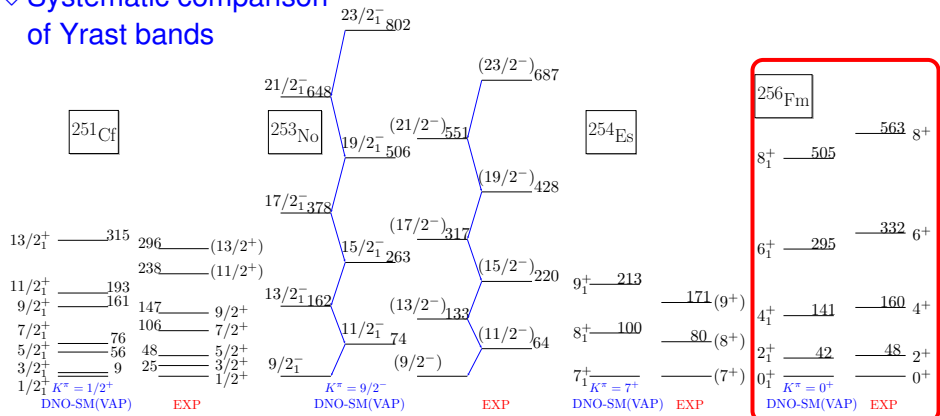
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Systematic comparison of Yrast bands



Applications to superheavy nuclei : Electromagnetic moments

First complete description of low-lying spectroscopy in ^{254}No

Duy Duc Dao¹ and Frédéric Nowacki¹

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arXiv :2409.08210

◇ Comparison of magnetic and quadrupole moments

Effective charges : $e_p = 1.72$, $e_n = 0.75$

J^π_{gs}	E (MeV)			$\mu(\mu_N)$			Q_s (eb)		
		PHF	VAP	PHF	VAP	EXP	PHF	VAP	EXP
^{253}No	$9/2^-$	-241.818	-242.816	+0.591	-0.493	-0.527(33)(75)	-3.5	+7.2	+5.9(1.4)(0.9)
^{253}Cf	$7/2^+$	-253.402	-253.818	-0.677	-0.556	-0.731(35)	+5.77	+5.78	+5.53(51)
^{251}Cf	$1/2^+$	-241.321	-241.724	-0.727	-0.610	-0.571(24)	-	-	-
^{249}Cf	$9/2^-$	-229.021	-229.381	-0.480	-0.461	-0.395(17)	+6.62	+6.63	+6.27(33)
^{255}Es	$7/2^+$	-263.512	-264.695	-1.10	+3.94	+4.14(10)	+6.0	+5.8	+5.1(1.7)
^{254}Es	7^+	-257.492	-258.441	+0.778	+3.36	+3.42(7)	+1.8	+8.4	+9.6(1.2)
^{253}Es	$7/2^+$	-251.837	-252.280	+3.63	+3.93	+4.10(7)	+5.87	+5.9	+6.7(8)
^{256}Fm	0^+	-268.999	-269.717	+0.87	+0.89	-	-3.57	-3.60	-
^{254}No	0^+	-249.568	-250.187	+0.87	+0.91	-	-3.78	-3.75	-

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^{253}No	PHF	VAP	EXP
$J^\pi = 9/2^-$	+0.591	-0.493	-0.527(33)(75)
$K = 9/2$	100.0%	99.6%	
$K = 7/2$	0.0 %	0.4 %	

- K-mixing is necessary to reproduce the magnetic moment.

Single-particle Occupations : $8^- (K = 8)$

Isomeric structures :

● 8_2^- : two-neutron coupling

$$\nu j15/2 \otimes \nu g9/2$$

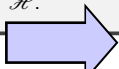
Ground- and excited-states occupations

Proton orbits	$0h_{9/2}$	$0i_{13/2}$	$1f_{7/2}$	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$	
0_1^+	6.03	7.75	3.43	1.49	0.77	0.52	
0_2^+	7.08	7.91	3.23	0.89	0.69	0.21	
$3_1^+ (K = 3)$	6.47	7.98	3.34	1.15	0.72	0.34	
$4_4^+ (K = 4)$	6.50	7.83	3.41	1.18	0.72	0.36	
$8_2^- (K = 8)$	6.48	7.90	3.36	1.19	0.70	0.37	
$10_6^+ (K = 10)$	6.55	7.03	3.48	1.56	0.79	0.58	
Neutron orbits	$0i_{11/2}$	$0j_{15/2}$	$1g_{9/2}$	$1g_{7/2}$	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$
0_1^+	7.30	9.91	5.43	1.00	1.09	0.84	0.43
0_2^+	7.36	9.95	5.45	0.96	1.05	0.80	0.42
$3_1^+ (K = 3)$	7.32	9.94	5.46	0.97	1.07	0.81	0.42
$4_4^+ (K = 4)$	7.34	9.79	5.48	1.03	1.11	0.81	0.44
$8_2^- (K = 8)$	7.42	9.00	6.30	0.98	1.06	0.81	0.43
$10_6^+ (K = 10)$	7.23	8.89	5.72	1.40	1.34	0.97	0.45

Discrete Non-Orthogonal Shell Model (DNO-SM)

- Broeckhove-Deumens Theorem : Z. Phys. A**292**, 243 (1979)

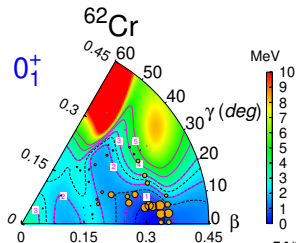
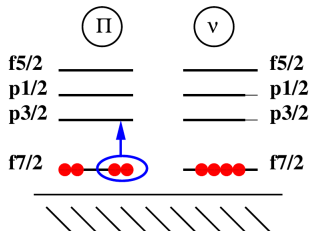
Given a separable Hilbert space $\mathcal{H}$ spanned by a continuous family of states $\Gamma = \{\psi(\alpha) | \alpha \in \mathbb{R}\}$, there exists a countable subset $\Gamma_0 = \{\phi(\alpha_i) | i \in \mathbb{N}\} \subset \Gamma$ with the property $\mathcal{H} = \overline{\text{span}\Gamma_0}$, i.e. Γ_0 is a skew or non-orthogonal basis in $\mathcal{H}$.



$$\psi = c_1\phi_1 + c_2\phi_2 + \dots + c_i\phi_i + \dots$$

IMPLICATIONS :

- Non-Orthogonal Slater Determinants spans the full shell-model space
- The number of Slater Determinants is FINITE
- Heuristic technique to select triaxial (β, γ) configurations



Isomeric single-particle configurations

Ground- and excited-states occupations

Proton orbits	$0h_{9/2}$	$0i_{13/2}$	$1f_{7/2}$	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$		
0_1^+	6.03	7.75	3.43	1.49	0.77	0.52		
0_2^+	7.08	7.91	3.23	0.89	0.69	0.21		
$3_1^+(K=3)$	6.47	7.98	3.34	1.15	0.72	0.34		
$4_4^+(K=4)$	6.50	7.83	3.41	1.18	0.72	0.36		
$8_2^-(K=8)$	6.48	7.90	3.36	1.19	0.70	0.37		
$10_6^+(K=10)$	6.55	7.03	3.48	1.56	0.79	0.58		
Neutron orbits	$0i_{11/2}$	$0j_{15/2}$	$1g_{9/2}$	$1g_{7/2}$	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$	
0_1^+	7.30	9.91	5.43	1.00	1.09	0.84	0.43	
0_2^+	7.36	9.95	5.45	0.96	1.05	0.80	0.42	
$3_1^+(K=3)$	7.32	9.94	5.46	0.97	1.07	0.81	0.42	
$4_4^+(K=4)$	7.34	9.79	5.48	1.03	1.11	0.81	0.44	
$8_2^-(K=8)$	7.42	9.00	6.30	0.98	1.06	0.81	0.43	
$10_6^+(K=10)$	7.23	8.89	5.72	1.40	1.34	0.97	0.45	

Isomeric structures :

- 8_2^- : two-neutron coupling
 $\nu j15/2 \otimes \nu g9/2$
- $3_1^+, 4_4^+$: involving a $\pi h9/2$

Monopole-corrected effective interactions

Multipole Universality :

△ Multipole decomposition

$$H = H_{\text{monopole}} + H_{\text{multipole}}$$

F. Nowacki et al., Prog. Part. Nucl. Phys. **120**, 103866 (2021)

Table 2

Dominant terms of several effective interactions for the *sd*-shell in MeV (see text). To the left particle–particle channels, isovector, isoscalar and quadrupole pairing, to the right particle–hole channels, quadrupole, hexadecapole, spin, and spin–isospin.

	pp(JT)			ph($\lambda\tau$)			
	10	01	21	20	40	10	11
KB	−5.83	−4.96	−3.21	−3.53	−1.38	+1.61	+3.00
USD−A	−5.62	−5.50	−3.17	−3.24	−1.60	+1.56	+2.99
CCEI	−6.79	−4.68	−2.93	−3.40	−1.39	+1.21	+2.83
NN+NNN−MBPT	−6.40	−4.36	−2.91	−3.28	−1.23	+1.10	+2.43
NN−MBPT	−6.06	−4.38	−2.92	−3.35	−1.31	+1.03	+2.49

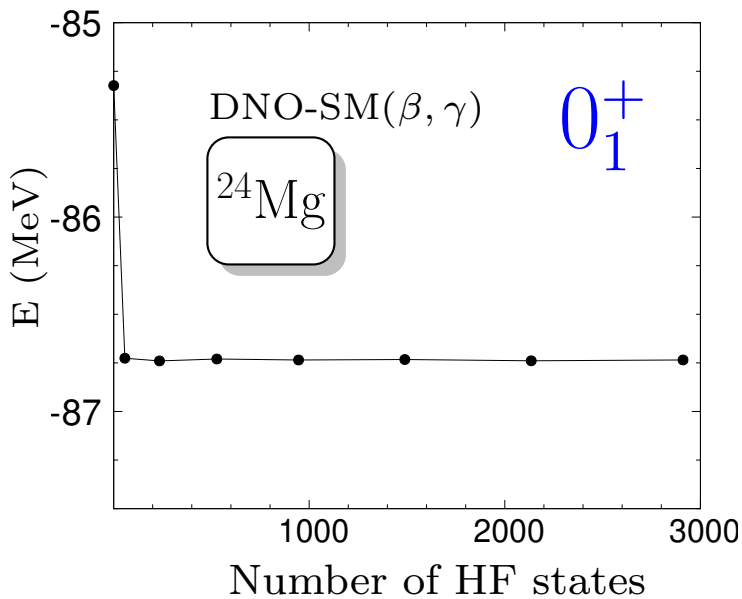
Pairing Components

Quadrupole

Table 3

Strengths of the coherent multipole components of different interactions for the *pf*-shell.

Interaction	particle-particle		particle-hole		
	JT = 01	JT = 10	$\lambda\tau = 20$	$\lambda\tau = 0$	$\lambda\tau = 1$
KB3	−4.75	−4.46	−2.79	−1.39	+2.46
FPD6	−5.06	−5.08	−3.11	−1.67	+3.17
GOGNY	−4.07	−5.74	−3.23	−1.77	+2.46
GXPFI	−4.18	−5.07	−2.92	−1.39	+2.47
BONNC	−4.20	−5.60	−3.33	−1.29	+2.70



$J_i^\pi \rightarrow J_f^\pi$	$B(E2) \text{ (e}^2 \cdot \text{fm}^4)$	$B(M1) \text{ (}\mu_N^2)$
$2_1^+ \rightarrow 0_1^+$	28666	-
$4_1^+ \rightarrow 2_1^+$	41021	-
$0_2^+ \rightarrow 2_1^+$	98.236	-
$2_2^+ \rightarrow 0_2^+$	25327	-
4_1^+	60.738	-
2_1^+	25.227	3.980×10^{-6}
$3_1^+ \rightarrow 2_2^+$	2.5959×10^{-2}	8.020×10^{-6}
4_1^+	2.7359×10^{-4}	2.957×10^{-7}
2_1^+	9.6147×10^{-4}	6.050×10^{-6}
$4_2^+ \rightarrow 2_2^+$	36147	-
3_1^+	5.4204×10^{-1}	4.200×10^{-7}
4_1^+	23.783	8.360×10^{-6}
2_1^+	18.569	-
$4_3^+ \rightarrow 3_1^+$	45803	1.3852×10^{-1}
6_1^+	6.3712×10^{-4}	-
4_1^+	6.5482×10^{-3}	3.634×10^{-5}
2_1^+	8.2100×10^{-6}	-
$4_4^+ \rightarrow 3_1^+$	1.1354×10^3	1.977×10^{-3}
4_3^+	9.6847×10^2	1.737×10^{-2}
2_2^+	4.438×10^{-4}	-
2_1^+	1.398×10^{-4}	-

Pairing correlations in ^{254}No

^{48}Cr	DNO-SM(β, γ)	DNO-SM(VAP)	Exact SM
dimension	22	50	1 963 461
$E(0_{gs}^+) \text{ (MeV)}$	-31.873	-32.953	-32.953

◇ Approaches :

- AMP Projected Hartree-Fock :

~ Pure Rotor $J(J+1)$ law

- “Well-deformed” heavy/superheavy nuclei :

Impact of pairing is PERTURBATIVE

- Variation After Projection :

~ Fully capturing pairing

(Slater Determinants

preserving Particle Numbers)

