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PIANO NAZIONALE
DI RIPRESA E RESILIENZA



UNIVERSITÀ
DEL SALENTO

Three-nucleon systems and three-nucleon interactions

Luca Girlanda



Istituto Nazionale di Fisica Nucleare
SEZIONE DI LECCE



with E. Filandri (ECT* Trento), A. Kievsky and M. Viviani (INFN Pisa), L.E. Marcucci (Univ. Pisa)



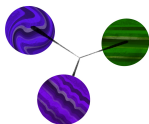
European Nuclear Physics Conference 2025

Outline

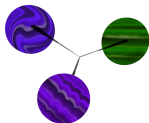
- ▶ Historical perspective on the 3N interaction
- ▶ Modern understanding based on ChEFT
- ▶ Present discrepancies in 3N systems
- ▶ Possible ways out
 1. role of higher order contact terms
 2. a forgotten interaction: freedom in 3NF at N3LO
- ▶ A closer look at the strictures of relativity
- ▶ Towards the resolution of the A_y puzzle at N3LO
- ▶ Conclusions

The 3N interaction

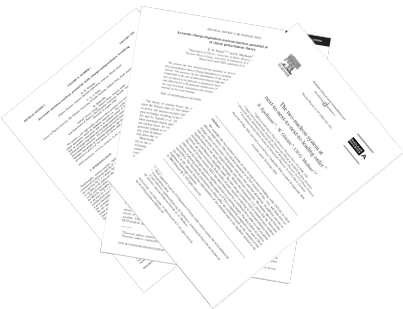
an important component
of the nuclear interaction



The 3N interaction

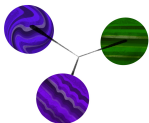


an important component
of the nuclear interaction
standing up thanks to

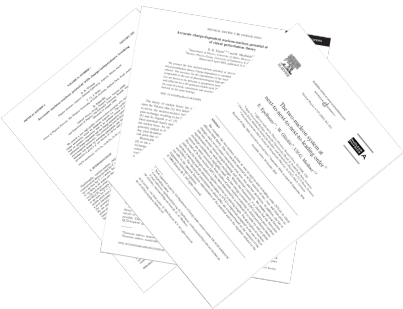


high-accuracy *NN* potentials

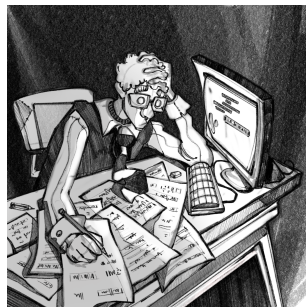
The 3N interaction



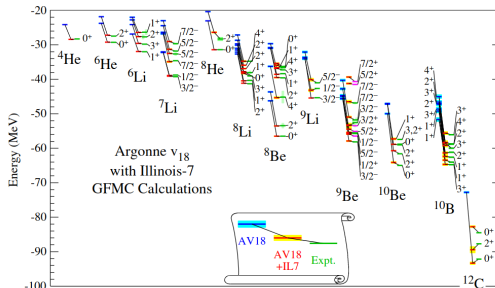
an important component
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high-accuracy NN potentials



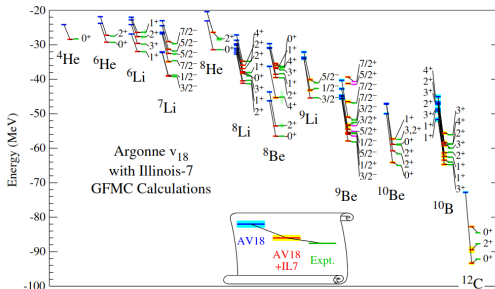
accurate numerical methods



provide
missing attraction

J. Carlson *et al.*

Rev. Mod. Phys 87 (2015) 1067



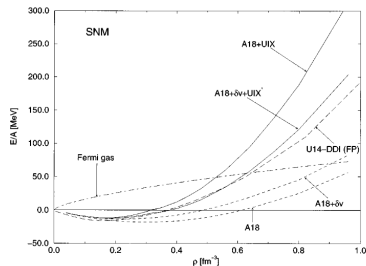
provide
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J. Carlson et al.

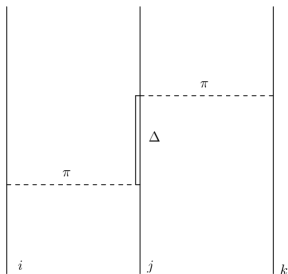
Rev. Mod. Phys 87 (2015) 1067

... and repulsion

A. Akmal et al., Phys. Rev. C 58 (1998) 1804



Modeling the 3N interaction



360

Progress of Theoretical Physics, Vol. 17, No. 3, March 1957

Pion Theory of Three-Body Forces

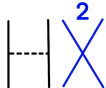
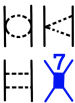
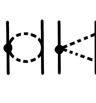
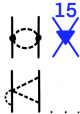
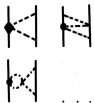
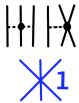
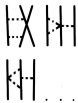
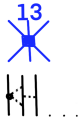
Jun-ichi FUJITA and Hironari MIYAZAWA

Department of Physics, University of Tokyo, Tokyo

(Received October 27, 1956)

$$V = \frac{-g_A^2 h_A^2}{18(m_\Delta - m_N)} \frac{\sigma_i \cdot \mathbf{q}_i \sigma_k \cdot \mathbf{q}_k}{\omega_{\mathbf{q}_i}^2 \omega_{\mathbf{q}_k}^2} \left[\mathbf{q}_i \cdot \mathbf{q}_k \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k - \frac{1}{4} \boldsymbol{\sigma}_j \cdot \mathbf{q}_i \times \mathbf{q}_k \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_i \times \boldsymbol{\tau}_k \right]$$

Modern understanding within ChEFT

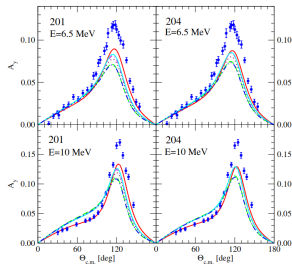
	LO	NLO	N2LO	N3LO	N4LO
2N					
3N					
4N					

Long-standing discrepancies

Eur. Phys. J. A (2014) 50: 177

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The $N - d$ A_y puzzle



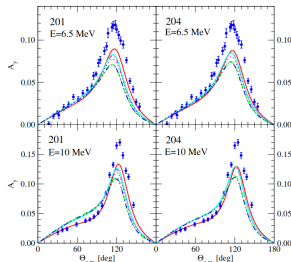
J. Golak *et al.* Eur. Phys. J. A 50 (2014) 177

Long-standing discrepancies

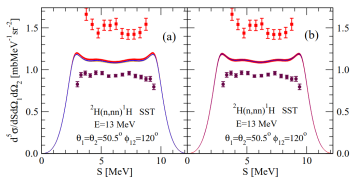
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The $N - d$ A_y puzzle



J. Golak *et al.* Eur. Phys. J. A 50 (2014) 177



H. Witala *et al.* Phys. Rev. C 104 (2021) 014002

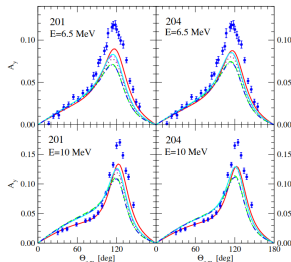
and the Space Star anomaly

Long-standing discrepancies

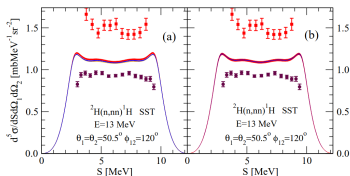
Eur. Phys. J. A (2014) 50: 177

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The $N - d$ A_y puzzle



J. Golak *et al.* Eur. Phys. J. A 50 (2014) 177



H. Witala *et al.* Phys. Rev. C 104 (2021) 014002

and the Space Star anomaly
still there at N3LO...

Possible ways out: beyond N3LO?

	LO	NLO	N2LO	N3LO	N4LO
2N					
3N					
4N					

→ new free parameters at N4LO

The new LECs at N4LO

- ▶ in [LG *et al.* PRC78 (2011) 014001] we classified all possible $3N$ contact terms with two powers of momenta

The new LECs at N4LO

- ▶ in [LG *et al.* PRC78 (2011) 014001] we classified all possible $3N$ contact terms with two powers of momenta
- ▶ and derived a local $3N$ potential

$$\begin{aligned}
 V = & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
 & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
 & + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
 & + (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\
 & + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
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 \end{aligned}$$

Spin-orbit terms suitable for the A_y puzzle [Kievsky PRC60 (1999) 034001]

Fitting the new parameters

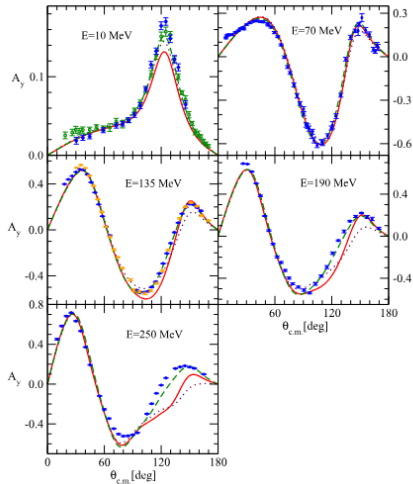
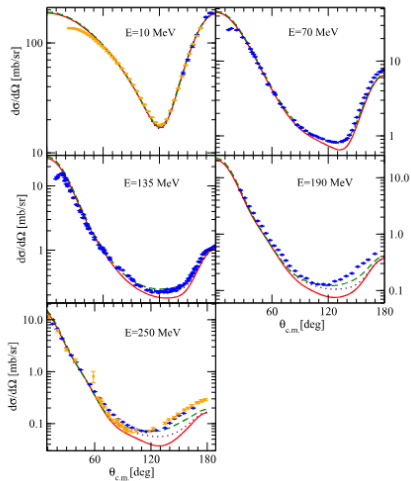
[Skibinski et al. FBS65(2024)]

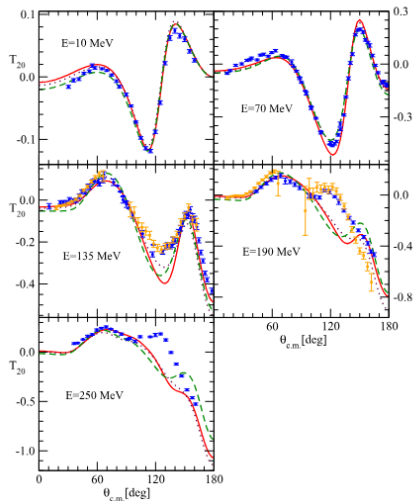
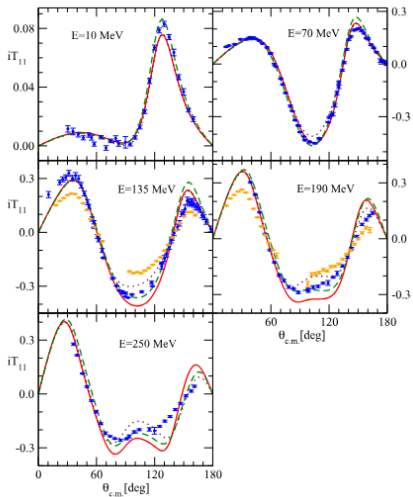
Emulator for Nd scattering – fit to the true data at 10,70, and 135 MeV (786 data points)

TABLE III. The values of strengths c_i found in the least squares fit to the data from Table II at the three energies $E = 10, 70$, and 135 MeV.

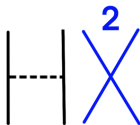
	c_D	c_E	c_{E_1}	c_{E_2}	c_{E_3}	c_{E_4}	c_{E_5}	c_{E_6}	c_{E_7}	c_{E_8}	c_{E_9}	$c_{E_{10}}$	$c_{E_{11}}$
c_D	1.000	-0.122	0.068	0.202	0.040	0.107	-0.237	-0.566	0.124	0.094	-0.052	0.038	-0.066
c_E	-0.122	1.000	0.048	-0.166	0.067	-0.084	0.059	-0.144	-0.284	-0.298	-0.127	0.128	0.191
c_{E_1}	0.068	0.048	1.000	0.945	0.982	-0.843	0.087	-0.291	0.551	0.500	0.176	0.228	0.027
c_{E_2}	0.202	-0.166	0.945	1.000	0.934	-0.770	-0.112	-0.356	0.629	0.581	-0.007	0.041	-0.184
c_{E_3}	0.040	0.067	0.982	0.934	1.000	-0.917	0.098	-0.214	0.574	0.528	0.111	0.093	-0.012
c_{E_4}	0.107	-0.084	-0.843	-0.770	-0.917	1.000	-0.317	-0.114	-0.583	-0.556	-0.172	0.053	-0.035
c_{E_5}	-0.237	0.059	0.087	-0.112	0.098	-0.317	1.000	0.540	0.113	0.126	0.898	0.521	0.719
c_{E_6}	-0.566	-0.144	-0.291	-0.356	-0.214	-0.114	0.540	1.000	-0.243	-0.197	0.253	-0.175	0.167
c_{E_7}	0.124	-0.284	0.551	0.629	0.574	-0.583	0.113	-0.243	1.000	0.995	0.108	-0.044	-0.205
c_{E_8}	0.094	-0.298	0.500	0.581	0.528	-0.556	0.126	-0.197	0.995	1.000	0.102	-0.068	-0.203
c_{E_9}	-0.052	-0.127	0.176	-0.007	0.111	-0.172	0.898	0.253	0.108	0.102	1.000	0.795	0.787
$c_{E_{10}}$	0.038	0.128	0.228	0.041	0.093	0.053	0.521	-0.175	-0.044	-0.068	0.795	1.000	0.746
$c_{E_{11}}$	-0.066	0.191	0.027	-0.184	-0.012	-0.035	0.719	0.167	-0.205	-0.203	0.787	0.746	1.000

- Big values of $c_{E_1}, c_{E_2}, c_{E_3}, c_{E_7}, c_{E_9}$
- Correlation coefficients close to ± 1 : $\rho(E_1, E_2), \rho(E_2, E_3), \rho(E_1, E_3), \rho(E_3, E_4), \rho(E_7, E_8)$
- Correlation coefficients close to 0: $(c_D, c_E), (c_D, c_{E_i} \text{ beside } E_6), (c_E, c_{E_i})$
- $\chi^2/\text{data} \approx 35$





A closer look at the strictures of symmetry: Leading Order



- ▶ OPE constrained by chiral symmetry (Goldberger-Treiman)
- ▶ unconstrained contact terms encode complicated short-distance dynamics
- ▶ minimal contact Lagrangian at LO

$$\mathcal{L} = -\frac{1}{2}C_S N^\dagger N N^\dagger N - \frac{1}{2}C_T N^\dagger \sigma_i N N^\dagger \sigma_i N$$

Next-to-Leading Order

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$
O_1	$(N^\dagger \overleftrightarrow{\nabla} N)^2 + \text{h.c.}$
O_2	$(N^\dagger \overleftrightarrow{\nabla} N) \cdot (N^\dagger \overleftrightarrow{\nabla} N)$
O_3	$(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla}^2 N) + \text{h.c.}$
O_4	$i(N^\dagger \overleftrightarrow{\nabla} N) \cdot (N^\dagger \overleftrightarrow{\nabla} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} \times \overleftrightarrow{\nabla} N)$
O_6	$i(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overleftrightarrow{\nabla} \times \overleftrightarrow{\nabla} N)$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^k \overleftrightarrow{\nabla}^j N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N)(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \overleftrightarrow{\nabla}^k \sigma^j N)$
O_{13}	$(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} \overleftrightarrow{\nabla}^j N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2(N^\dagger \overleftrightarrow{\nabla} \sigma^j \cdot \overleftrightarrow{\nabla} N)(N^\dagger \sigma^j N)$

[Ordoñez *et al.* PRC 53 (1996)]

Next-to-Leading Order

► only 12 independent operators since

$$O_7 + 2O_{10} = O_8 + 2O_{11} \text{ and } O_4 + O_5 = O_6$$

O_S	$(N^\dagger N)(N^\dagger N)$
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O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^k \overleftrightarrow{\nabla}^j N) + \text{h.c.}$
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[Ordoñez *et al.* PRC 53 (1996)]

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O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \vec{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \vec{\nabla} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \vec{\nabla}^k N)(N^\dagger \sigma^k \vec{\nabla}^j N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \vec{\nabla}^k N)(N^\dagger \sigma^j \vec{\nabla}^k N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \vec{\nabla} N)(N^\dagger \vec{\nabla} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \vec{\nabla}^k N)(N^\dagger \vec{\nabla}^j \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \vec{\nabla}^k N)(N^\dagger \vec{\nabla}^k \sigma^j N)$
O_{13}	$(N^\dagger \vec{\nabla} \cdot \boldsymbol{\sigma} \vec{\nabla}^j N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2(N^\dagger \vec{\nabla} \sigma^j \cdot \vec{\nabla} N)(N^\dagger \sigma^j N)$

- ▶ only 12 independent operators since

$$O_7 + 2O_{10} = O_8 + 2O_{11} \text{ and } O_4 + O_5 = O_6$$

- ▶ NN potential in the c.o.m frame is given in terms of 7 LECs



$$\mathbf{k} = \mathbf{p} - \mathbf{p}', \quad \mathbf{Q} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$$

$$v(\mathbf{k}, \mathbf{Q}) = C_1 k^2 + C_2 Q^2 + (C_3 k^2 + C_4 Q^2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ + i C_5 \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{Q} \times \mathbf{k} + C_6 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} + C_7 \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$$

[Ordoñez *et al.* PRC 53 (1996)]

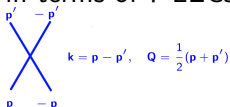
Next-to-Leading Order

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$
O_1	$(N^\dagger \overleftrightarrow{\nabla} N)^2 + \text{h.c.}$
O_2	$(N^\dagger \overleftrightarrow{\nabla} N) \cdot (N^\dagger \overleftrightarrow{\nabla} N)$
O_3	$(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla}^2 N) + \text{h.c.}$
O_4	$i(N^\dagger \overleftrightarrow{\nabla} N) \cdot (N^\dagger \overleftrightarrow{\nabla} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} \times \overleftrightarrow{\nabla} N)$
O_6	$i(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overleftrightarrow{\nabla} \times \overleftrightarrow{\nabla} N)$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^k \overleftrightarrow{\nabla}^j N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N)(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \overleftrightarrow{\nabla}^k \sigma^j N)$
O_{13}	$(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} \overleftrightarrow{\nabla}^j N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2(N^\dagger \overleftrightarrow{\nabla} \sigma^j \cdot \overleftrightarrow{\nabla} N)(N^\dagger \sigma^j N)$

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- ▶ significance of the 5 remaining LECs?

[Ordoñez *et al.* PRC 53 (1996)]

Next-to-Leading Order

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$
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O_2	$(N^\dagger \overleftrightarrow{\nabla} N) \cdot (N^\dagger \overleftrightarrow{\nabla} N)$
O_3	$(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla}^2 N) + \text{h.c.}$
O_4	$i(N^\dagger \overleftrightarrow{\nabla} N) \cdot (N^\dagger \overleftrightarrow{\nabla} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla} \cdot \boldsymbol{\sigma} \times \overleftrightarrow{\nabla} N)$
O_6	$i(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \overleftrightarrow{\nabla} \times \overleftrightarrow{\nabla} N)$
O_7	$(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N)(N^\dagger \boldsymbol{\sigma} \cdot \overleftrightarrow{\nabla} N) + \text{h.c.}$
O_8	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^k \overleftrightarrow{\nabla}^j N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N)(N^\dagger \sigma^j \overleftrightarrow{\nabla}^k N) + \text{h.c.}$
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- ▶ significance of the 5 remaining LECs? ($\vec{P}$ -dependent NN interaction?)

The relativistic answer

1 ⊗ 1	$\hat{O}_1$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_2$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_3$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_4$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
1 ⊗ 7	$\hat{O}_5$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_6$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
1 ⊗ 7	$\hat{O}_7$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_8$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_9$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{10}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{11}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{12}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{13}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{14}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{15}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{16}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{17}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{18}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{19}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{20}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{21}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{22}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{23}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{24}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{25}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{26}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{27}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{28}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{29}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{30}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{31}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{32}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{33}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{34}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{35}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{36}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{37}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{38}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{39}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{40}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{41}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{42}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{43}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{44}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{45}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{46}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{47}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{48}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{49}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{50}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
7 ⊗ 7	$\hat{O}_{51}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{52}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{53}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$
	$\hat{O}_{54}$	$\frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$

- ▶ build the *NN* contact Lagrangian in terms of relativistic nucleon fields
- ▶ many more operators due to the large nucleon mass
- ▶ non-relativistic reduction

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m} \begin{pmatrix} 0 \\ \sigma \cdot \nabla \end{pmatrix} + \frac{1}{8m^2} \begin{pmatrix} \nabla^2 \\ 0 \end{pmatrix} \right] N(x)$$

- ▶ only 9 combinations of non-relativistic operators up to $O(p^2)$

$$O_5 + \frac{1}{4m^2}(O_1 + O_3 + O_5 + O_6)$$

$$O_7 - \frac{1}{4m^2}(O_5 + O_6 - O_7 + O_8 + 2O_{12} + O_{14})$$

+ 7 “c.o.m operators”

[LG + M. Viviani, FBS 49 (2011)]

The algebraic answer

$$\begin{aligned} [J^i, J^j] &= i\epsilon^{ijk} J^k, & [K^i, K^j] &= -i\epsilon^{ijk} J^k, & [J^i, K^j] &= i\epsilon^{ijk} K^k, & [P^\mu, P^\nu] &= 0, \\ [K^i, P^j] &= -i\delta^{ij} H, & [J^i, P^j] &= i\epsilon^{ijk} P^k, & [K^i, H] &= -iP^i, & [J^i, H] &= 0. \end{aligned}$$

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in the instant-form $H = H_0 + H_I$, $\vec{K} = \vec{K}_0 + \vec{K}_I$

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in the instant-form $H = H_0 + H_I$, $\vec{K} = \vec{K}_0 + \vec{K}_I$

$$\vec{K}_I = \int d^3\mathbf{x} \vec{x} \mathcal{H}_I(\mathbf{x}) + \delta \vec{W}$$

The algebraic answer

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expand in soft momenta

$$H_0 = H_0^{(0)} + H_0^{(2)} + H_0^{(4)} + \dots, \quad K_0 = K_0^{(-1)} + K_0^{(1)} + K_0^{(3)} + \dots,$$

and solve order by order for $H_I = H_{NN} + H_{3N} + \dots$, and $\delta \vec{W}$

The algebraic answer

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and solve order by order for $H_I = H_{NN} + H_{3N} + \dots$, and $\delta \vec{W}$

$$H_{NN} = H_{NN}^{(3)} + H_{NN}^{(5)} + H_{NN}^{(7)} + \dots$$

$$H_{3N} = H_{3N}^{(6)} + H_{3N}^{(8)} + \dots$$

$$\delta \vec{W} = \delta \vec{W}^{(4)} + \dots$$

Resulting *drift interaction* at NLO

genuine NN interaction

$$\begin{aligned} v(\mathbf{k}, \mathbf{Q}) = & C_1 k^2 + C_2 Q^2 + (C_3 k^2 + C_4 Q^2) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + i C_5 \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{Q} \times \mathbf{k} \\ & + C_6 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} + C_7 \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q} , \end{aligned}$$

Resulting *drift interaction* at NLO

genuine NN interaction

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$\vec{P}$ -dependent interaction

$$\begin{aligned} v_{\mathbf{P}}(\mathbf{k}, \mathbf{Q}) = & i C_1^* \frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k} + C_2^* (\boldsymbol{\sigma}_1 \cdot \mathbf{P} \boldsymbol{\sigma}_2 \cdot \mathbf{Q} - \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{P}) \\ & + (C_3^* + C_4^* \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) P^2 + C_5^* \boldsymbol{\sigma}_1 \cdot \mathbf{P} \boldsymbol{\sigma}_2 \cdot \mathbf{P} , \end{aligned}$$

with *fixed* LECs as $1/m^2$ boost corrections to the LO interaction

$$C_1^* = \frac{C_S - C_T}{4m^2} , \quad C_2^* = \frac{C_T}{2m^2} , \quad C_3^* = -\frac{C_S}{4m^2} , \quad C_4^* = -\frac{C_T}{4m^2} , \quad C_5^* = 0 ,$$

[LG *et al.* PRC81 034005 (2010)]

Drift effects

PHYSICAL REVIEW C

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Variational Monte Carlo calculations of ^3H and ^4He with a relativistic Hamiltonian

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In relativistic Hamiltonian action in the $\mathbf{P}_{ij} = 0$ rest total momentum $\mathbf{P}_{ij}$ and terms: δv_{RE} , δv_{LC} , δv_{TP} , a relation, Lorentz contraction of δv_{RE} and δv_{LC} for ^3H and ^4He . In this paper δv_{TP} and δv_{QM} . These are

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Nucleon-nucleon potential: Drift effects

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In the rest frame of a many-body system, used in the calculation of its static and scattering properties, the center of mass of a two-body subsystem is allowed to drift. We show, in a model-independent way, that drift corrections to the nucleon-nucleon potential are relatively large and arise from both one- and two-pion exchange processes. As far as chiral symmetry is concerned, corrections to these processes begin, respectively, at $\mathcal{O}(q^2)$ and $\mathcal{O}(q^4)$. The two-pion exchange interaction also yields a new spin structure, which promotes the presence of P waves in trinuclei and is associated with profile functions that do not coincide with either central or spin-orbit ones. In principle, the new spin terms should be smaller than the $\mathcal{O}(q^3)$ spin-orbit components. However, in the isospin-even channel, a large contribution defies this expectation and gives rise to the prediction of important drift effects.

(Next-to)³ Leading Order

The “c.o.m. interaction” (depending on 15 LECs)

(Next-to)³ Leading Order

The “c.o.m. interaction” (depending on 15 LECs)

$$\begin{aligned} v(\mathbf{k}, \mathbf{Q}) = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k} \times \mathbf{Q})^2 \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + \left(D_9 k^2 + D_{10} Q^2 \right) (-i \mathbf{S} \cdot (\mathbf{k} \times \mathbf{Q})) \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\ & + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) D_{15} (\boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})) . \end{aligned}$$

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$$\begin{aligned}v(\mathbf{k}, \mathbf{Q}) = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k} \times \mathbf{Q})^2 \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\& + \left(D_9 k^2 + D_{10} Q^2 \right) (-i \mathbf{S} \cdot (\mathbf{k} \times \mathbf{Q})) \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\& + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) D_{15} (\boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})).\end{aligned}$$

what about the $\vec{P}$ -dependent interaction?

(Next-to)³ Leading Order

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$$\begin{aligned} v(\mathbf{k}, \mathbf{Q}) = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k} \times \mathbf{Q})^2 \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + \left(D_9 k^2 + D_{10} Q^2 \right) (-i \mathbf{S} \cdot (\mathbf{k} \times \mathbf{Q})) \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\ & + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) D_{15} (\boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})) . \end{aligned}$$

what about the $\vec{P}$ -dependent interaction?

- identify a basis of non relativistic operators at $O(p^4)$

(Next-to)³ Leading Order

The “c.o.m. interaction” (depending on 15 LECs)

$$\begin{aligned} v(\mathbf{k}, \mathbf{Q}) = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k} \times \mathbf{Q})^2 \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + \left(D_9 k^2 + D_{10} Q^2 \right) (-i \mathbf{S} \cdot (\mathbf{k} \times \mathbf{Q})) \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\ & + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) D_{15} (\boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})). \end{aligned}$$

what about the $\vec{P}$ -dependent interaction?

- ▶ identify a basis of non relativistic operators at $O(p^4)$
- ▶ write down an overcomplete set of relativistic operators

(Next-to)³ Leading Order

The “c.o.m. interaction” (depending on 15 LECs)

$$\begin{aligned}
 v(\mathbf{k}, \mathbf{Q}) = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k} \times \mathbf{Q})^2 \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
 & + \left(D_9 k^2 + D_{10} Q^2 \right) (-i \mathbf{S} \cdot (\mathbf{k} \times \mathbf{Q})) \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
 & + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) D_{15} (\boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})) .
 \end{aligned}$$

what about the $\vec{P}$ -dependent interaction?

- ▶ identify a basis of non relativistic operators at $O(p^4)$
- ▶ write down an overcomplete set of relativistic operators
- ▶ express the latter in the chosen basis using the non-relativistic expansion

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m} \begin{pmatrix} 0 \\ \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2} \begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3} \begin{pmatrix} 0 \\ \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \boldsymbol{\nabla}^2 \end{pmatrix} + \frac{11}{128m^4} \begin{pmatrix} \boldsymbol{\nabla}^4 \\ 0 \end{pmatrix} \right] N(x) + O(p^5) .$$

(Next-to)³ Leading Order

The “c.o.m. interaction” (depending on 15 LECs)

$$\begin{aligned}
 v(\mathbf{k}, \mathbf{Q}) = & D_1 k^4 + D_2 Q^4 + D_3 k^2 Q^2 + D_4 (\mathbf{k} \times \mathbf{Q})^2 \left(D_5 k^4 + D_6 Q^4 + D_7 k^2 Q^2 + D_8 (\mathbf{k} \times \mathbf{Q})^2 \right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
 & + \left(D_9 k^2 + D_{10} Q^2 \right) (-i \mathbf{S} \cdot (\mathbf{k} \times \mathbf{Q})) \left(D_{11} k^2 + D_{12} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \\
 & + \left(D_{13} k^2 + D_{14} Q^2 \right) (\boldsymbol{\sigma}_1 \cdot \mathbf{Q}) (\boldsymbol{\sigma}_2 \cdot \mathbf{Q}) D_{15} (\boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})) .
 \end{aligned}$$

what about the $\vec{P}$ -dependent interaction?

- ▶ identify a basis of non relativistic operators at $O(p^4)$
- ▶ write down an overcomplete set of relativistic operators
- ▶ express the latter in the chosen basis using the non-relativistic expansion

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m} \begin{pmatrix} 0 \\ \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2} \begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3} \begin{pmatrix} 0 \\ \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \boldsymbol{\nabla}^2 \end{pmatrix} + \frac{11}{128m^4} \begin{pmatrix} \boldsymbol{\nabla}^4 \\ 0 \end{pmatrix} \right] N(x) + O(p^5) .$$

- ▶ extract the linearly independent combinations

Non relativistic basis

Operator	Operator
$O_S = 1$	$O_1^{*''} = ik^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_T = \sigma_1 \cdot \sigma_2$	$O_2^{*''} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 - \mathbf{k} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_1^I = k^2$	$O_3^{*''} = k^2(\mathbf{Q} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 - \mathbf{Q} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_2^I = Q^2$	$O_4^{*''} = k^2 P^2$
$O_3^I = k^2 \sigma_1 \cdot \sigma_2$	$O_5^{*''} = k^2 P^2 \sigma_1 \cdot \sigma_2$
$O_4^I = Q^2 \sigma_1 \cdot \sigma_2$	$O_6^{*''} = (\mathbf{k} \cdot \mathbf{P})^2$
$O_5^I = i \frac{\sigma_1 + \sigma_2}{2} \cdot \mathbf{Q} \times \mathbf{k}$	$O_7^{*''} = (\mathbf{k} \cdot \mathbf{P})^2 \sigma_1 \cdot \sigma_2$
$O_6^I = \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_8^{*''} = P^2 \mathbf{k} \cdot \sigma_1 \mathbf{k} \cdot \sigma_2$
$O_7^I = \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_9^{*''} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 + \mathbf{k} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_8^I = i \frac{\sigma_1 - \sigma_2}{2} \cdot \mathbf{P} \times \mathbf{k}$	$O_{10}^{*''} = k^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_2^{*'} = \sigma_1 \cdot \mathbf{P} \sigma_2 \cdot \mathbf{Q} - \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{P}$	$O_{11}^{*''} = i Q^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_3^{*'} = P^2$	$O_{12}^{*''} = i \mathbf{Q} \cdot \mathbf{P}(\mathbf{Q} \times \mathbf{k} \cdot (\sigma_1 - \sigma_2))$
$O_4^{*'} = \sigma_1 \cdot \sigma_2 P^2$	$O_{13}^{*''} = iP^2(\mathbf{k} \times \mathbf{Q} \cdot (\sigma_1 + \sigma_2))$
$O_5^{*'} = \sigma_1 \cdot \mathbf{P} \sigma_2 \cdot \mathbf{P}$	$O_{14}^{*''} = i \mathbf{P} \cdot \mathbf{k}(\mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 + \sigma_2))$
$O_1^{*''} = k^4$	$O_{15}^{*''} = i \mathbf{Q} \cdot \mathbf{P}(\mathbf{P} \times \mathbf{k} \cdot (\sigma_1 + \sigma_2))$
$O_2^{*''} = Q^4$	$O_{16}^{*''} = iP^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_3^{*''} = k^2 Q^2$	$O_{17}^{*''} = Q^2(\mathbf{Q} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 - \mathbf{Q} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_4^{*''} = (\mathbf{k} \times \mathbf{Q})^2$	$O_{18}^{*''} = P^2 Q^2$
$O_5^{*''} = k^4 \sigma_1 \cdot \sigma_2$	$O_{19}^{*''} = (\mathbf{P} \cdot \mathbf{Q})^2$
$O_6^{*''} = Q^4 \sigma_1 \cdot \sigma_2$	$O_{20}^{*''} = P^2 Q^2 \sigma_1 \cdot \sigma_2$
$O_7^{*''} = k^2 Q^2 \sigma_1 \cdot \sigma_2$	$O_{21}^{*''} = (\mathbf{P} \cdot \mathbf{Q})^2 \sigma_1 \cdot \sigma_2$
$O_8^{*''} = (\mathbf{k} \times \mathbf{Q})^2 \sigma_1 \cdot \sigma_2$	$O_{22}^{*''} = Q^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_9^{*''} = \frac{ik^2}{2}(\sigma_1 + \sigma_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{23}^{*''} = \mathbf{P} \cdot \mathbf{Q}(\mathbf{P} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 + \mathbf{P} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_{10}^{*''} = \frac{iQ^2}{2}(\sigma_1 + \sigma_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{24}^{*''} = P^2 \mathbf{Q} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2$
$O_{11}^{*''} = k^4 \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_{25}^{*''} = P^2(\mathbf{P} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 - \mathbf{P} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_{12}^{*''} = Q^2 \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_{26}^{*''} = P^4$
$O_{13}^{*''} = k^2 \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_{27}^{*''} = P^4 \sigma_1 \cdot \sigma_2$
$O_{14}^{*''} = Q^2 \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_{28}^{*''} = P^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_{15}^{*''} = \sigma_1 \cdot (\mathbf{k} \times \mathbf{Q}) \sigma_2 \cdot (\mathbf{k} \times \mathbf{Q})$	
$O_{16}^{*''} = i \mathbf{k} \cdot \mathbf{Q} \mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2)$	
$O_{17}^{*''} = \mathbf{k} \cdot \mathbf{Q}(\mathbf{k} \times \mathbf{P}) \cdot (\sigma_1 \times \sigma_2)$	

Redundant set of relativistic operators

$\Gamma_A \otimes \Gamma_B$	Operators	Gradient structures
$1 \otimes 1$	$\tilde{O}_{1-6} = \bar{\psi}\psi\bar{\psi}\psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2, (\vec{d}_A \cdot \vec{d}_B)^2, (\vec{d}_A \cdot \vec{d}_B)d^2, d^4\}$
$1 \otimes \gamma$	$\tilde{O}_{7-9} = \frac{i}{2m} \bar{\psi} \overleftrightarrow{\partial}^\mu \psi \bar{\psi} \gamma_\mu \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$1 \otimes \gamma\gamma_5$	$\tilde{O}_{10-12} = \frac{-i}{8m^3} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \overleftrightarrow{\partial}_\mu \psi \partial_\nu \bar{\psi} \gamma_\alpha \gamma_5 \overleftrightarrow{\partial}_\beta \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$\gamma_5 \otimes \gamma_5$	$\tilde{O}_{13-15} = -\bar{\psi} \gamma_5 \psi \bar{\psi} \gamma_5 \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$\gamma_5 \otimes \sigma$	$\tilde{O}_{16} = \frac{1}{16m^4} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_5 \overleftrightarrow{\partial}_\gamma \overleftrightarrow{\partial}_\mu \psi \partial_\nu \bar{\psi} \sigma_{\alpha\gamma} \overleftrightarrow{\partial}_\beta \psi$	1
$\gamma \otimes \gamma$	$\tilde{O}_{17-19} = \bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma^\mu \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
	$\tilde{O}_{20-22} = \frac{-i}{4m^2} \bar{\psi} \gamma_\mu \overleftrightarrow{\partial}_\nu \psi \bar{\psi} \gamma^\nu \overleftrightarrow{\partial}^\mu \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$\gamma \otimes \gamma\gamma_5$	$\tilde{O}_{23} = \frac{-i}{16m^4} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\mu \overleftrightarrow{\partial}_\gamma \overleftrightarrow{\partial}_\nu \psi \partial_\alpha \bar{\psi} \gamma_\gamma \gamma_5 \overleftrightarrow{\partial}_\beta \psi$	1
	$\tilde{O}_{24} = \frac{-i}{16m^4} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\gamma \overleftrightarrow{\partial}_\mu \psi \partial_\nu \bar{\psi} \gamma_\alpha \gamma_5 \overleftrightarrow{\partial}_\gamma \overleftrightarrow{\partial}_\beta \psi$	1
	$\tilde{O}_{25-27} = \frac{i}{4m^2} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\mu \overleftrightarrow{\partial}_\nu \psi \partial_\alpha \bar{\psi} \gamma_\beta \gamma_5 \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
	$\tilde{O}_{28-30} = \frac{i}{4m^2} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\mu \psi \partial_\nu \bar{\psi} \gamma_\alpha \gamma_5 \overleftrightarrow{\partial}_\beta \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \gamma\gamma_5$	$\tilde{O}_{31-36} = \bar{\psi} \gamma^\mu \gamma_5 \psi \bar{\psi} \gamma_\mu \gamma_5 \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2, (\vec{d}_A \cdot \vec{d}_B)^2, (\vec{d}_A \cdot \vec{d}_B)d^2, d^4\}$
	$\tilde{O}_{37-39} = \frac{-i}{4m^2} \bar{\psi} \gamma^\mu \gamma_5 \overleftrightarrow{\partial}_\nu \psi \bar{\psi} \gamma_\nu \gamma_5 \overleftrightarrow{\partial}_\mu \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \sigma$	$\tilde{O}_{40-42} = \frac{-i}{8m^3} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\mu \gamma_5 \overleftrightarrow{\partial}_\nu \overleftrightarrow{\partial}_\gamma \psi \bar{\psi} \sigma_{\alpha\gamma} \overleftrightarrow{\partial}_\beta \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
	$\tilde{O}_{43-45} = \frac{i}{2m} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\mu \gamma_5 \psi \bar{\psi} \sigma_{\nu\alpha} \overleftrightarrow{\partial}_\beta \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
	$\tilde{O}_{46-48} = \frac{i}{2m} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} \gamma_\mu \gamma_5 \overleftrightarrow{\partial}_\nu \psi \bar{\psi} \sigma_{\alpha\beta} \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
$\sigma \otimes \sigma$	$\tilde{O}_{49-51} = \bar{\psi} \sigma_{\mu\nu} \psi \bar{\psi} \sigma^{\mu\nu} \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$
	$\tilde{O}_{52-54} = \frac{-i}{4m^2} \bar{\psi} \sigma^{\mu\alpha} \overleftrightarrow{\partial}^\beta \psi \bar{\psi} \sigma_{\mu\beta} \overleftrightarrow{\partial}_\alpha \psi$	$\{1, \vec{d}_A \cdot \vec{d}_B, d^2\}$

to be expanded non-relativistically

[E. Filandri + LG, PLB 841 137957 (2023)]

26 independent operators

Operator	Operator
$O_5 = 1$	$O_1^{*''} = ik^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_7 = \sigma_1 \cdot \sigma_2$	$O_2^{*''} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 - \mathbf{k} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_1' = k^2$	$O_3^{*''} = k^2(\mathbf{Q} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 - \mathbf{Q} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_2' = Q^2$	$O_4^{*''} = k^2 P^2$
$O_3' = k^2 \sigma_1 \cdot \sigma_2$	$O_5^{*''} = k^2 P^2 \sigma_1 \cdot \sigma_2$
$O_4' = Q^2 \sigma_1 \cdot \sigma_2$	$O_6^{*''} = (\mathbf{k} \cdot \mathbf{P})^2$
$O_5' = i \frac{\sigma_1 \times \sigma_2}{2} \cdot \mathbf{Q} \times \mathbf{k}$	$O_7^{*''} = (\mathbf{k} \cdot \mathbf{P})^2 \sigma_1 \cdot \sigma_2$
$O_6' = \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_8^{*''} = P^2 \mathbf{k} \cdot \sigma_1 \mathbf{k} \cdot \sigma_2$
$O_7' = \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_9^{*''} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 + \mathbf{k} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_8^{*'} = i \frac{\sigma_1 - \sigma_2}{2} \cdot \mathbf{P} \times \mathbf{k}$	$O_{10}^{*''} = k^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_2^{*'} = \sigma_1 \cdot \mathbf{P} \sigma_2 \cdot \mathbf{Q} - \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{P}$	$O_{11}^{*''} = i Q^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_3^{*'} = P^2$	$O_{12}^{*''} = i \mathbf{Q} \cdot \mathbf{P}(\mathbf{Q} \times \mathbf{k} \cdot (\sigma_1 - \sigma_2))$
$O_4^{*'} = \sigma_1 \cdot \sigma_2 P^2$	$O_{13}^{*''} = i P^2(\mathbf{k} \times \mathbf{Q} \cdot (\sigma_1 + \sigma_2))$
$O_5^{*'} = \sigma_1 \cdot \mathbf{P} \sigma_2 \cdot \mathbf{P}$	$O_{14}^{*''} = i \mathbf{P} \cdot \mathbf{k}(\mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 + \sigma_2))$
$O_1'' = k^4$	$O_{15}^{*''} = i \mathbf{Q} \cdot \mathbf{P}(\mathbf{P} \times \mathbf{k} \cdot (\sigma_1 + \sigma_2))$
$O_2'' = Q^4$	$O_{16}^{*''} = i P^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_3'' = k^2 Q^2$	$O_{17}^{*''} = Q^2(\mathbf{Q} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 - \mathbf{Q} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_4'' = (\mathbf{k} \times \mathbf{Q})^2$	$O_{18}^{*''} = P^2 Q^2$
$O_5'' = k^4 \sigma_1 \cdot \sigma_2$	$O_{19}^{*''} = (\mathbf{P} \cdot \mathbf{Q})^2$
$O_6'' = Q^4 \sigma_1 \cdot \sigma_2$	$O_{20}^{*''} = P^2 Q^2 \sigma_1 \cdot \sigma_2$
$O_7'' = k^2 Q^2 \sigma_1 \cdot \sigma_2$	$O_{21}^{*''} = (\mathbf{P} \cdot \mathbf{Q})^2 \sigma_1 \cdot \sigma_2$
$O_8'' = (\mathbf{k} \times \mathbf{Q})^2 \sigma_1 \cdot \sigma_2$	$O_{22}^{*''} = Q^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_9'' = i \frac{k^2}{2} (\sigma_1 + \sigma_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{23}^{*''} = \mathbf{P} \cdot \mathbf{Q}(\mathbf{P} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 + \mathbf{P} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_{10}'' = i \frac{Q^2}{2} (\sigma_1 + \sigma_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{24}^{*''} = P^2 \mathbf{Q} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2$
$O_{11}'' = k^2 \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_{25}^{*''} = P^2(\mathbf{P} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 - \mathbf{P} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_{12}'' = Q^2 \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_{26}^{*''} = P^4$
$O_{13}'' = k^2 \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_{27}^{*''} = P^4 \sigma_1 \cdot \sigma_2$
$O_{14}'' = Q^2 \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_{28}^{*''} = P^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_{15}'' = \sigma_1 \cdot (\mathbf{k} \times \mathbf{Q}) \sigma_2 \cdot (\mathbf{k} \times \mathbf{Q})$	
$O_{16}'' = i \mathbf{k} \cdot \mathbf{Q} \mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2)$	
$O_{17}'' = \mathbf{k} \cdot \mathbf{Q}(\mathbf{k} \times \mathbf{P}) \cdot (\sigma_1 \times \sigma_2)$	

26 independent operators

Operator	Operator
$O_S = 1$	$O_1^{*'''} = ik^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_T = \sigma_1 \cdot \sigma_2$	$O_3^{*'''} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 - \mathbf{k} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_1' = k^2$	$O_3^{*'''} = k^2(\mathbf{Q} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 - \mathbf{Q} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_2' = Q^2$	$O_4^{*'''} = k^2 P^2$
$O_3' = k^2 \sigma_1 \cdot \sigma_2$	$O_5^{*'''} = k^2 P^2 \sigma_1 \cdot \sigma_2$
$O_4' = Q^2 \sigma_1 \cdot \sigma_2$	$O_6^{*'''} = (\mathbf{k} \cdot \mathbf{P})^2$
$O_5' = i \frac{\sigma_1 + \sigma_2}{2} \cdot \mathbf{Q} \times \mathbf{k}$	$O_7^{*'''} = (\mathbf{k} \cdot \mathbf{P})^2 \sigma_1 \cdot \sigma_2$
$O_6' = \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_8^{*'''} = P^2 \mathbf{k} \cdot \sigma_1 \mathbf{k} \cdot \sigma_2$
$O_7' = \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_9^{*'''} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 + \mathbf{k} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_1^{*'} = i \frac{\sigma_1 - \sigma_2}{2} \cdot \mathbf{P} \times \mathbf{k}$	$O_{10}^{*'''} = k^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_2^{*'} = \sigma_1 \cdot \mathbf{P} \sigma_2 \cdot \mathbf{Q} - \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{P}$	$O_{11}^{*'''} = iQ^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_3^{*'} = P^2$	$O_{12}^{*'''} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{Q} \times \mathbf{k} \cdot (\sigma_1 - \sigma_2))$
$O_4^{*'} = \sigma_1 \cdot \sigma_2 P^2$	$O_{13}^{*'''} = iP^2(\mathbf{k} \times \mathbf{Q} \cdot (\sigma_1 + \sigma_2))$
$O_5^{*'} = \sigma_1 \cdot \mathbf{P} \sigma_2 \cdot \mathbf{P}$	$O_{14}^{*'''} = i\mathbf{P} \cdot \mathbf{k}(\mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 + \sigma_2))$
$O_1^{*''} = k^4$	$O_{15}^{*'''} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{P} \times \mathbf{k} \cdot (\sigma_1 + \sigma_2))$
$O_2^{*''} = Q^4$	$O_{16}^{*'''} = iP^2(\mathbf{k} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2))$
$O_3^{*''} = k^2 Q^2$	$O_{17}^{*'''} = Q^2(\mathbf{Q} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2 - \mathbf{Q} \cdot \sigma_2 \mathbf{P} \cdot \sigma_1)$
$O_4^{*''} = (\mathbf{k} \times \mathbf{Q})^2$	$O_{18}^{*'''} = P^2 Q^2$
$O_5^{*''} = k^4 \sigma_1 \cdot \sigma_2$	$O_{19}^{*'''} = (\mathbf{P} \cdot \mathbf{Q})^2$
$O_6^{*''} = Q^4 \sigma_1 \cdot \sigma_2$	$O_{20}^{*'''} = P^2 Q^2 \sigma_1 \cdot \sigma_2$
$O_7^{*''} = k^2 Q^2 \sigma_1 \cdot \sigma_2$	$O_{21}^{*'''} = (\mathbf{P} \cdot \mathbf{Q})^2 \sigma_1 \cdot \sigma_2$
$O_8^{*''} = (\mathbf{k} \times \mathbf{Q})^2 \sigma_1 \cdot \sigma_2$	$O_{22}^{*'''} = Q^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_9^{*''} = i \frac{k^2}{2} (\sigma_1 + \sigma_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{23}^{*'''} = \mathbf{P} \cdot \mathbf{Q}(\mathbf{P} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 + \mathbf{P} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_{10}^{*''} = i \frac{Q^2}{2} (\sigma_1 + \sigma_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{24}^{*'''} = P^2 \mathbf{Q} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2$
$O_{11}^{*''} = k^2 \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_{25}^{*'''} = P^2(\mathbf{P} \cdot \sigma_1 \mathbf{Q} \cdot \sigma_2 - \mathbf{P} \cdot \sigma_2 \mathbf{Q} \cdot \sigma_1)$
$O_{12}^{*''} = Q^2 \sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}$	$O_{26}^{*'''} = P^4$
$O_{13}^{*''} = k^2 \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_{27}^{*'''} = P^4 \sigma_1 \cdot \sigma_2$
$O_{14}^{*''} = Q^2 \sigma_1 \cdot \mathbf{Q} \sigma_2 \cdot \mathbf{Q}$	$O_{28}^{*'''} = P^2 \mathbf{P} \cdot \sigma_1 \mathbf{P} \cdot \sigma_2$
$O_{15}^{*''} = \sigma_1 \cdot (\mathbf{k} \times \mathbf{Q}) \sigma_2 \cdot (\mathbf{k} \times \mathbf{Q})$	
$O_{16}^{*''} = i\mathbf{k} \cdot \mathbf{Q} \mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2)$	
$O_{17}^{*''} = \mathbf{k} \cdot \mathbf{Q}(\mathbf{k} \times \mathbf{P}) \cdot (\sigma_1 \times \sigma_2)$	

$$v_P(\mathbf{k}, \mathbf{Q}) = \text{drift terms} + iD_{16}\mathbf{k} \cdot \mathbf{Q} \mathbf{Q} \times \mathbf{P} \cdot (\sigma_1 - \sigma_2) + D_{17}\mathbf{k} \cdot \mathbf{Q}(\mathbf{k} \times \mathbf{P}) \cdot (\sigma_1 \times \sigma_2)$$

→ 2 “forgotten” LECs @N3LO contributing in larger systems than 2N

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- ▶ yes, it is *new*

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It can be absorbed with a suitable unitary transformation of the 1-body kinetic energy H_0

$$U^\dagger H_0 U \longrightarrow H_0 + [H_0, \alpha_4 T_4 + \alpha_5 T_5] = H_0 - V_{NN}(\textcolor{red}{P})$$

with

$$U = e^{\alpha_4 T_4 + \alpha_5 T_5}, \quad \alpha_4 = -\frac{m}{8} D_{16}, \quad \alpha_5 = -\frac{m}{4} D_{17}$$

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and

$$T_4 = i\epsilon^{ijk} \int d^3x N^\dagger \nabla^i N N^\dagger \nabla^j \sigma^k N \sim i\mathbf{P} \times \mathbf{Q} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$$

$$T_5 = \int d^3x \left[N^\dagger \nabla^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) - N^\dagger \nabla^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \sim (\mathbf{P} \cdot \boldsymbol{\sigma}_1 \mathbf{k} \cdot \boldsymbol{\sigma}_2 - \mathbf{P} \cdot \boldsymbol{\sigma}_2 \mathbf{k} \cdot \boldsymbol{\sigma}_1)/2$$

[LG *et al.* PRC 102 064003 (2020)]

Further unitary transformations

$$U = e^{\alpha_i T_i} \longrightarrow U^\dagger H_0 U = \sum_i \alpha_i$$



[P. Reinert *et al.* Eur. Phys. J. A 54 (2018) 86]

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$$T_4 = i\epsilon^{ijk} \int d^3x N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N \sim i \mathbf{P} \times \mathbf{Q} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$$

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Induced 3N forces

The same unitary transformation induces 3N interactions

$$U^\dagger \left[C_{S/T} \text{ (diagram with 2 lines crossing)} \right] U = \sum_i m D_i C_{S/T} \text{ (diagram with 13 lines meeting at a central square)}$$

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$$\begin{aligned} V = & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\ & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\ & + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\ & + (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\ & + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \end{aligned}$$

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$$U^\dagger \left[C_{S/T} \text{ (diagram with 2 lines crossing)} \right] U = \sum_i m D_i C_{S/T} \text{ (diagram with 3 lines meeting at a central square labeled 13)}$$

$$\begin{aligned} V = & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(\mathbf{r}_{ik}) \\ & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(\mathbf{r}_{ik}) \\ & + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(\mathbf{r}_{ik}) \\ & + (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(\mathbf{r}_{ik}) \\ & + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(\mathbf{r}_{ik}) \end{aligned}$$

Induced 3N forces

The same unitary transformation induces 3N interactions

$$U^\dagger \left[C_{S/T} \begin{array}{c} \text{2} \\ \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \right] U = \sum_i m D_i C_{S/T} \begin{array}{c} \text{13} \\ \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array}$$

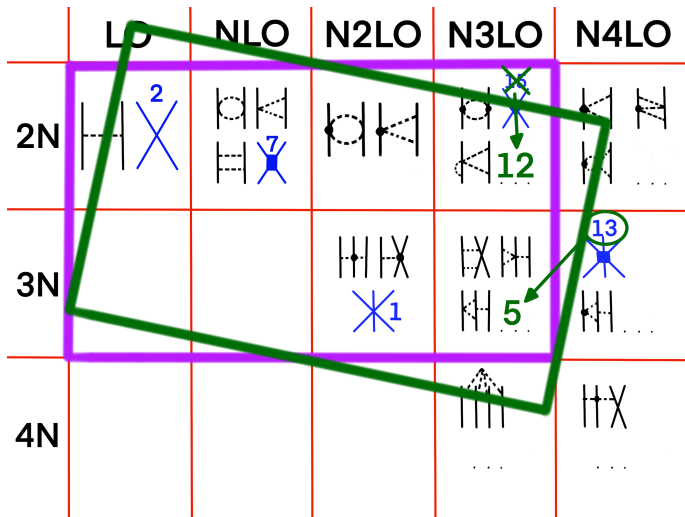
$$\begin{aligned} V = & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\ & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\ & + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\ & + (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\ & + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \end{aligned}$$

$$\begin{aligned}
E_1 &= C_5(\alpha_1 + \alpha_2) + C_7(\alpha_1 - 2\alpha_2), \\
E_2 &= C_7(3\alpha_2 + 2\alpha_3 - 8\alpha_4 + 2\alpha_5), \\
E_3 &= 2C_5(\alpha_2 + 2\alpha_3/3) + C_7(2\alpha_1 - \alpha_2 - 2\alpha_3/3 + 8\alpha_4 - 2\alpha_5), \\
E_4 &= 2C_5\alpha_2/3 + C_7(2\alpha_1 - 7\alpha_2 - 2\alpha_3 + 8\alpha_4 - 2\alpha_5)/3, \\
E_5 &= 2C_5(\alpha_2 + 2\alpha_3/3) + 2C_7(\alpha_1 - 2\alpha_2 - \alpha_3/3 + 4\alpha_4 - \alpha_5), \\
E_6 &= 2C_5\alpha_2/3 + 2C_7(\alpha_1 - 2\alpha_2 - \alpha_3 + 4\alpha_4 - \alpha_5)/3, \\
E_7 &= 24C_7\alpha_4, \\
E_8 &= 8C_7\alpha_4, \\
E_9 &= C_5(3\alpha_2 + 2\alpha_3 - \alpha_4 + 2\alpha_5) + C_7(3\alpha_1 - 6\alpha_2 - 4\alpha_3 + 11\alpha_4 - 4\alpha_5), \\
E_{10} &= C_5(\alpha_2 - \alpha_4) + C_7(\alpha_1 - 2\alpha_2 + 5\alpha_4), \\
E_{11} &= C_5(3\alpha_2 + 2\alpha_3 + \alpha_4 - 2\alpha_5) + C_7(3\alpha_1 - 6\alpha_2 - 4\alpha_3 - 11\alpha_4 + 4\alpha_5), \\
E_{12} &= C_5(\alpha_2 + \alpha_4) + C_7(\alpha_1 - 2\alpha_2 - 5\alpha_4), \\
E_{13} &= -4C_7(4\alpha_4 - \alpha_5).
\end{aligned}$$

with $\alpha_i \sim O(m) \sim O(p^{-1})$ enhanced (Weinberg counting)

$$\begin{aligned}
\alpha_1 &= \frac{m}{16} (16D_1 + D_2 + 4D_3), \quad \alpha_2 = \frac{m}{16} (16D_5 + D_6 + 4D_7), \quad \alpha_3 = \frac{m}{32} (D_{14} + 16D_{11} + 4D_{12} + 4D_{13}), \\
\alpha_4 &= \frac{m}{2} D_{16}, \quad \alpha_5 = \frac{m}{16} (8D_{17} - D_{14} - 16D_{11} - 4D_{12} - 4D_{13}),
\end{aligned}$$

Reshuffling ChEFT low-energy expansion



Further topologies

induced from one-pion-exchange

$$U^\dagger \left[\text{diagram of two vertical lines connected by two horizontal dashed lines} \right] U = \text{diagram of two vertical lines connected by one horizontal dashed line, with a vertex on the right vertical line crossed by two diagonal lines}$$

$$\begin{aligned}
 V_{3NF} = & -\frac{g_A}{F_\pi} \sum_{i \neq j \neq k} \frac{\mathbf{k}_k \cdot \boldsymbol{\sigma}_k \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k}{k_k^2 + m_\pi^2} \{ \alpha_1 \mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \\
 & + \alpha_2 [\mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i \mathbf{k}_j \cdot (\mathbf{Q}_i - \mathbf{Q}_j) \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j] \\
 & + (\alpha_3 + \alpha_5) [k_k^2 \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - 2i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{Q}_i \times \boldsymbol{\sigma}_i + 2i \mathbf{Q}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i] \\
 & + (\alpha_3 - \alpha_5) [\mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_i - 2i \mathbf{Q}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i] \\
 & - 2\alpha_4 [\mathbf{k}_k \cdot \boldsymbol{\sigma}_i \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_j - 2i \mathbf{k}_k \cdot \mathbf{Q}_i (\mathbf{k}_k \cdot \mathbf{Q}_i \mathbf{Q}_j \cdot \boldsymbol{\sigma}_i - \mathbf{k}_k \cdot \mathbf{Q}_j \mathbf{Q}_i \cdot \boldsymbol{\sigma}_i)] \}
 \end{aligned}$$

Further topologies

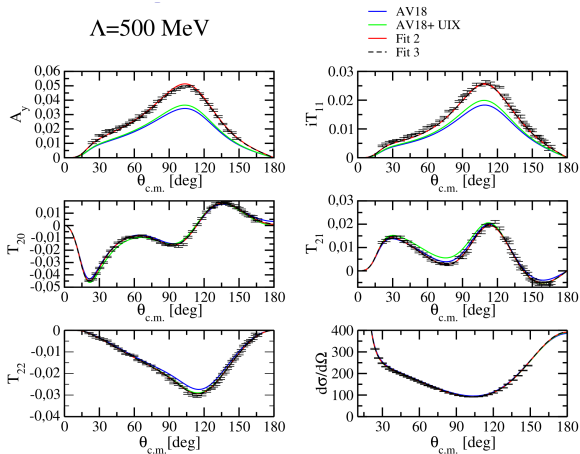
induced from one-pion-exchange

$$\begin{aligned}
 V_{3NF} = & -\frac{g_A}{F_\pi} \sum_{i \neq j \neq k} \frac{\mathbf{k}_k \cdot \boldsymbol{\sigma}_k \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k}{k_k^2 + m_\pi^2} \{ \alpha_1 \mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \\
 & + \alpha_2 [\mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i \mathbf{k}_j \cdot (\mathbf{Q}_i - \mathbf{Q}_j) \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j] \\
 & + (\alpha_3 + \alpha_5) [k_k^2 \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - 2i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{Q}_i \times \boldsymbol{\sigma}_i + 2i \mathbf{Q}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i] \\
 & + (\alpha_3 - \alpha_5) [\mathbf{k}_j \cdot \mathbf{k}_k \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_i - 2i \mathbf{Q}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i] \\
 & - 2\alpha_4 [\mathbf{k}_k \cdot \boldsymbol{\sigma}_i \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_j - 2i \mathbf{k}_k \cdot \mathbf{Q}_i (\mathbf{k}_k \cdot \mathbf{Q}_i \mathbf{Q}_j \cdot \boldsymbol{\sigma}_i - \mathbf{k}_k \cdot \mathbf{Q}_j \mathbf{Q}_i \cdot \boldsymbol{\sigma}_i)] \}
 \end{aligned}$$

$$O(p^{-1}) \times O(p^5) \sim O(p^4) = \text{N3LO}$$

Impact of the forgotten interaction

$\Lambda=500$ MeV



Fitting procedure	2-param.	3-param.
$\chi^2/\text{d.o.f.}$	2.1	1.9
e_0	-	0.459
$\tilde{\alpha}_4 C_S$	1.751	1.894
$\tilde{\alpha}_5 C_S$	-0.495	-1.175
$^2a_{nd}$ (fm)	0.573	0.599

[LG *et al.* Phys. Rev. C 107 (2023) L061001]

Conclusions

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- ▶ N3LO can be the order where accuracy is met in both 2N and 3N systems
- ▶ there are 5 free LECs in the 3N force at N3LO to improve the description of scattering data
- ▶ $N - d$ A_y puzzle can be solved in this way

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thank you