QCD x QED on the lattice

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Motivations

Why are QED effects in hadronic processes relevant?

- Precision physics: for many hadronic observables the current accuracy of lattice QCD calculations is at the level of $\lesssim 1\%$. At this level of precision, $\mathcal{O}(\alpha_{\rm em})$ and strong isospin-breaking (SIB) $\mathcal{O}(\frac{\delta m_{ud}}{\Lambda_{\rm QCD}})$ corrections must be included. E.g.:

$$\text{Hadron spectroscopy}, \quad \pi^{\pm}(K^{\pm}) \to \ell^{\pm}\nu_{\ell}, \quad \pi(K) \to \pi\ell\nu_{\ell}, \quad g_{\mu}-2, \quad \ldots$$

- Assessing weak decays of hadrons involving photons in the final states and/or mediated by virtual photons. Many phenomenologically relevant rare decays fall in this category:

$$[\text{NON-FCNC}] \qquad (D^{\pm}_{(s)}, B^{\pm}) \to \ell^{\pm}\nu_{\ell}\gamma, \quad K^{\pm} \to \bar{\ell}'\ell'\ell^{\pm}\nu_{\ell}, \quad \ldots$$

$$[\text{FCNC}] \qquad B_{(s)} \to \mu^{+}\mu^{-}\gamma, \qquad B_{(s)} \to \nu\bar{\nu}\gamma, \quad \ldots$$

Signals of New Physics must be searched in processes that are either suppressed in the SM or very precisely predicted (and measured).

Challenges in lattice QCD+QED calculations

$$\mathcal{S}_{\mathrm{QCD+QED}} = \underbrace{\mathcal{S}_{\mathrm{iso}}}_{\alpha_{\mathrm{em}} = \delta m_{ud} = 0} + S_{\mathrm{photon}}[A] + \underbrace{\mathcal{S}^{I}_{\mathrm{QED}}}_{\propto \sqrt{\alpha_{\mathrm{em}}}} + \underbrace{\mathcal{S}_{\mathrm{SIB}}}_{\propto \delta m_{ud}}$$

Monte Carlo simulations typically performed in isosymmetric pure QCD (i.e. with S_{iso} only). Inclusion of QED + SIB effects can be done by:

- Direct approach: perform new MC simulations with non-zero α_{em} and δm_{ud}.
- MC generation of configurations needs to be modified/generalized to include the additional photon field. ✗
- Diagrams appearing in the expansion method, all obtained at once. ✓

- RM123-SOTON method: expand observables at LO around $\alpha_{\mathrm{em}} = \delta m_{ud} = 0$.
- $\langle \mathcal{O} \rangle_{\mathrm{QCD+QED}} \simeq \langle \mathcal{O} \rangle_{\mathrm{iso}} + \delta \langle \mathcal{O} \rangle$:

$$\delta \langle \mathcal{O} \rangle = \left\langle \left\langle \mathcal{O} \left[\frac{1}{2} \left(\mathcal{S}^{I}_{QED} \right)^2 - \mathcal{S}_{IB} \right] \right\rangle_{\mathrm{iso}} \right\rangle_A$$

- No new simulations need to be performed. ✓
- Insertions of perturbations generate many diagrams to be computed. ✗

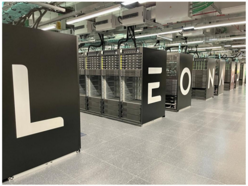
- Additional UV divergences when switching on $\alpha_{\mathrm{em}} \implies$ needed retuning of Lagrangian counterterms.
- Finite size effects (FSEs) only power suppressed due to the long-rangeness of Coulomb force. IR divergences in electroweak amplitudes when virtual γ exchanged [solved through BN mechanism, Bloch and Nordsieck PR 53 (1937)].

More than a decade of improvements

A series of seminal papers paved the way towards inclusion and control of QED+SIB effects in lattice calculations. . .

- *Leading isospin breaking effects on the lattice* [RM123-SOTON, PRD 87 (2013)].
- *QED Corrections to Hadronic Processes in Lattice QCD* [RM123-SOTON, PRD 91 (2015)].
- *QED in finite volume and finite size scaling effect on electromagnetic properties of hadrons*, [Hayakawa and Uno, PTP 120 (2008)].
- *Finite-volume electromagnetic corrections to the masses of mesons, baryons, and nuclei* [Davoudi and Savage, PRD 90 (2014)].
- *Quantum electrodynamics in finite volume and nonrelativistic effective field theories* [Fodor et al., PLB 755 (2015)].
- *QED* self energies from lattice QCD without power-law finite-volume errors [Feng et al., PRD 100 (2019)]. And many other. . .

. . . and exceptional machines get us through



Results in hadron spectroscopy

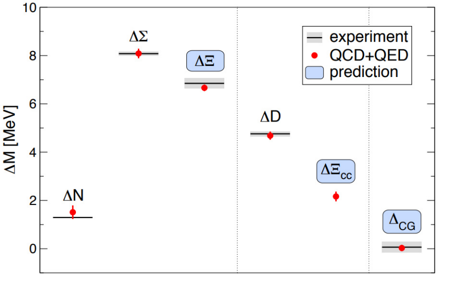
REPORTS

NUCLEAR PHYSICS

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi,[1] S. Durr,[1,2] Z. Fodor,[1,2,3*] C. Hoelbling,[1] S. D. Katz,[3,4] S. Krieg,[1,2] L. Lellouch,[5] T. Lippert,[1,2] A. Portelli,[5,6] K. K. Szabo,[1,2] B. C. Toth[1]

$$\Delta N = M_n - M_p = 1.51(16)(23)\ \mathrm{MeV}$$



BMWc, Science vol. 347, (2015).

PHYSICAL REVIEW D **106,** 014502 (2022)

Lattice calculation of the pion mass difference $M_{\pi^+} - M_{\pi^0}$ at order $\mathcal{O}(\alpha_{em})$

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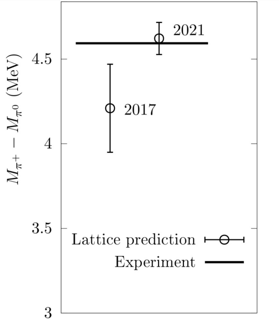
$$M_{\pi^+} - M_{\pi^0} = 4.622\ (95)\ \mathrm{MeV}$$



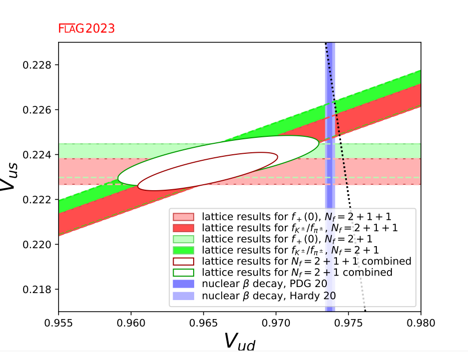
Also: Xu Feng et al., PRL 128 (2022).

Splitting of hadron multiplets well reproduced by lattice QCD+QED calculations

Determination of CKM matrix elements

The unitarity of the top row of the CKM matrix is under scrutiny

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \ (?)$$



$$\frac{\Gamma[K \to \ell\nu_\ell(\gamma)]}{\Gamma[\pi \to \ell\nu_\ell(\gamma)]} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2}\left(1 + \delta R_{K\pi}\right)$$

$$\Gamma[K \to \pi\ell\nu_\ell(\gamma)] \propto |V_{us}|^2 |f_+(0)|^2 \left(1 + \delta R^\ell_{K\pi}\right)$$

FLAG '23 $N_f = 2+1+1$ average:

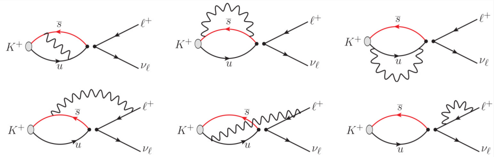
$$\boxed{\frac{f_{K^+}}{f_{\pi^+}} = 1.1934(19) \quad f_+(0) = 0.9698(17)}$$

- $\delta R_{K\pi}$ and $\delta R^\ell_{K\pi}$ are QED and SIB corrections.
- They can be evaluated in χPT [V.Cirigliano and H.Neufeld, PLB 700 (2011)], but...

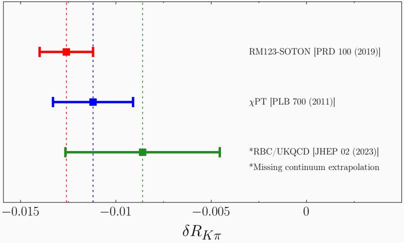
Leptonic/semileptonic decay rates from lattice QCD+QED

...recently a major progress has been achieved in the lattice calculation of QED+SIB corrections to $(K/\pi)_{\ell 2}$ exploiting the expansion (RM123-SOTON) method.

E.g. for $K_{\ell 2}$:



$+ \; \ldots + K \to \ell\nu\gamma$ [to remove IR div. through BN mech.]



Lattice estimates of $\delta R_{K\pi}$ compatible with χPT, and already more precise!

Progresses also in the evaluation of QED corrections to semileptonic decays:

- γW-box contribution to $\delta R^{\ell}_{K\pi}$ computed in [Ma et al, PRD 103 (2021)]...
- ...and for $n \to p\, e\, \bar{\nu}_e$ and superallowed $\beta-$decay in [Ma et al, arXiv:2308.16755].

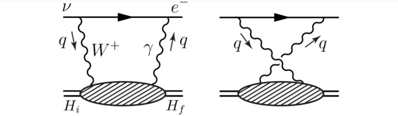
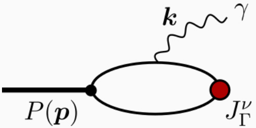


Figure 1. The γW-box diagrams for the semileptonic decay process $H_i \to H_f e \bar{\nu}_e$.

Decays with real photons in external states

$P(\boldsymbol{p})$ is a meson with momentum $\boldsymbol{p}$, J_Γ a quark-bilinear operator (stemming e.g. from $H_{\text{eff}}^{\text{weak}}$)



$$A_\Gamma^\nu(\boldsymbol{k},\boldsymbol{p}) \equiv \langle \gamma(\boldsymbol{k},\varepsilon)|J_\Gamma^\nu(0)|P(\boldsymbol{p})\rangle_{\text{QCD+QED}}$$

Matrix elements of this kind are encountered in many processes: radiative leptonic decays, $B_{(s)} \to \mu^+\mu^-\gamma,\ \ldots$

At leading-order in α_{em} such process are however factorizable:

$$A_\Gamma^\nu(\boldsymbol{k},\boldsymbol{p}) = -ie\int d^4x\,\langle\gamma(\boldsymbol{k},\varepsilon)|A_\mu(x)|0\rangle_{\text{QED}} \times \langle 0|\mathrm{T}\left\{J_{\text{em}}^\mu(x)J_\Gamma^\nu(0)\right\}|P(\boldsymbol{p})\rangle_{\text{QCD}}$$

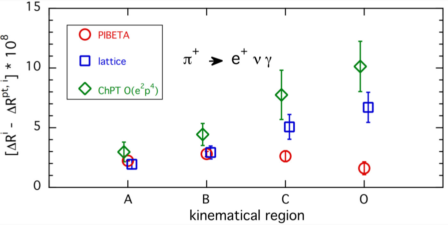
$$= -ie\varepsilon_\mu\int d^4x\, e^{ikx}\langle 0|\mathrm{T}\left\{J_{\text{em}}^\mu(x)J_\Gamma^\nu(0)\right\}|P(\boldsymbol{p})\rangle_{\text{QCD}} = -ie\,\varepsilon_\mu\ \underbrace{H_\Gamma^{\mu\nu}(\boldsymbol{k},\boldsymbol{p})}_{\text{pure QCD input}}$$

- Complications due to γ^* exchange in quark lines are absent at LO: no additional UV and IR (if $|\boldsymbol{k}|\neq 0$) divergences, no power-law QED-induced FSEs.
- However, we do need to compute higher-order correlation functions for different momenta to explore the whole phase space. . .

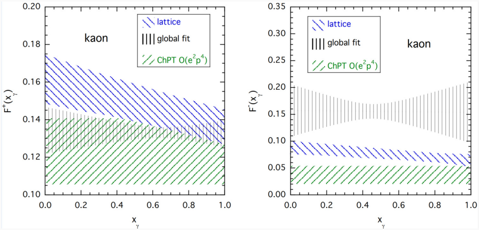
Recent results on $K/\pi \to \ell \bar{\nu}_\ell \gamma$

Based on: *Desiderio et al. [PRD 103(2021)], **Frezzotti et al. [PRD 103(2021), POS-LAT21]

- For $P \to \ell \nu_\ell \gamma$ the hadronic tensor $H_W^{\mu\nu}$, $\Gamma^\nu = W^\nu \equiv \gamma^\nu(1-\gamma^5)$, parametrized by two structure-dependent form factors F_V (vector) and F_A (axial), and by decay constant f_P.
- The f_P part of $H_W^{\mu\nu}$ together with the *bremsstrahlung* contribution gives the so-called point-like part of the amplitude.
- Form factors $F^{\pm} \equiv F_A \pm F_V$ computed in $\star$ for $P = K, \pi$, over full phase space as a function of $x_\gamma \equiv 2E_\gamma/m_P$, and compared in $\star\star$ with experiments.



Difference between full and point-like contribution to the rate for $\pi \to e \nu_e \gamma$ for the four kinematical regions probed by the PIBETA experiment.



Comparison for $F^{\pm}$ of the kaon between lattice, χPT and a global fit of KLOE, E787, ISTRA+ and OKA experimental results.

Radiative decays of heavy mesons: the case of the D_s

For heavy-mesons little is known experimentally, only upper-bounds available:

- $\mathrm{Br}[D \to e\nu_e\gamma](E_\gamma > 10\ \mathrm{MeV}) < 3 \times 10^{-5}$ [BESIII arXiv:1702.05837]
- $\mathrm{Br}[D_s \to e\nu_e\gamma](E_\gamma > 10\ \mathrm{MeV}) < 1.3 \times 10^{-4}$ [BESIII arXiv:1902.03351]
- $\mathrm{Br}[B \to e\nu_e\gamma](E_\gamma > 1\ \mathrm{GeV}) < 4.3 \times 10^{-6}$ [Belle arXiv:1810.12976]
- $\mathrm{Br}[B \to \mu\nu_\mu\gamma](E_\gamma > 1\ \mathrm{GeV}) < 3.4 \times 10^{-6}$ [Belle arXiv:1810.12976]

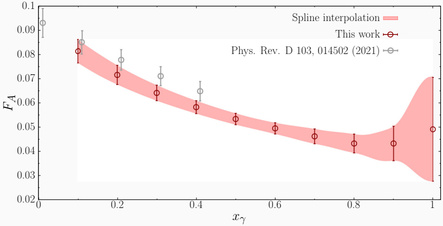
Providing a first-principle determination of the branching fractions may however

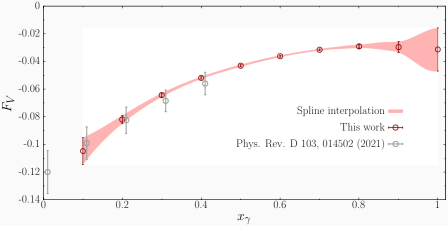
encourage further experimental searches!

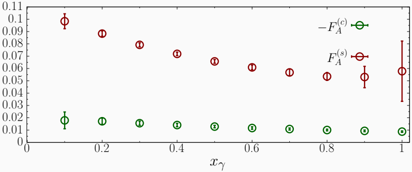
- Heavy-meson $P \to \ell\nu\gamma$ decays are interesting mainly because the point-like contribution ($\propto f_P^2$) to the rate is (helicity) suppressed w.r.t. the structure dependent (SD) contribution ($\propto |F_A|^2, |F_V|^2, F_A F_V$) by $r_\ell^2 = (m_\ell/m_P)^2$.
- $\Longrightarrow$ for heavy-meson decays, the electron mode is very sensitive to SD contributions. E.g. : for $P = D_s$ and $\ell = e$, $r_e^2 \simeq 6 \times 10^{-8}$.
- χPT cannot be used here, only way to obtain model-independent predictions is given by lattice QCD.

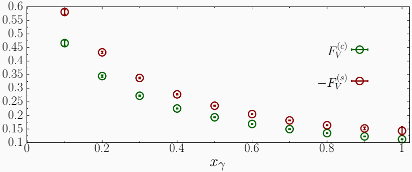
Recent results on F_V and F_A of the D_s meson

Based on our recent paper: Frezzotti et al. PRD 108 (2023)







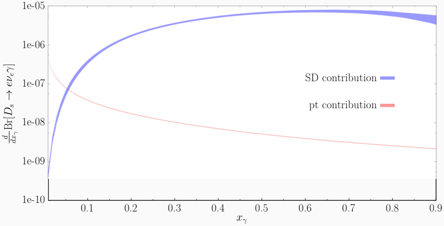


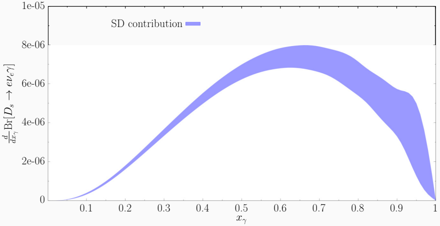
$\Longleftarrow$ Single-flavour contributions (photon emitted from strange or charm quark-line).

- $F_{V/A} \equiv F_{V/A}^{(s)} + F_{V/A}^{(c)}$.
- Strong cancellation between strange- and charm-quark contributions to vector form factor F_V.

The differential branching fraction for $D_s \to e\nu_e\gamma$

The most interesting decay-channel to probe the internal structure of D_s meson is $D_s \to e\nu_e\gamma$ $\quad [r_e^2 \simeq 6 \times 10^{-8}]$



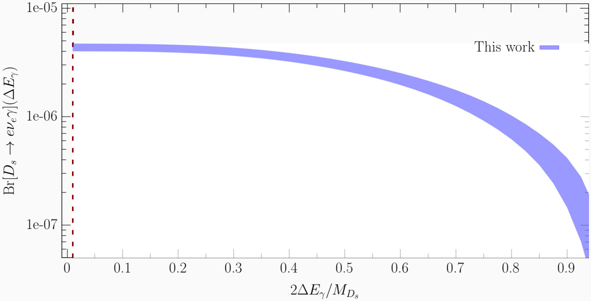


- The pt contribution to the differential branching is suppressed w.r.t. SD one for $x_\gamma \gtrsim 0.06$. SD contribution maximum at $x_\gamma \simeq 0.6 - 0.7$.
- The total decay rate

$$\Gamma_e(\Delta E_\gamma) \equiv \int_{\frac{2\Delta E_\gamma}{m_{D_s}}}^{1-r_e^2} dx_\gamma \, \frac{d\Gamma(D_s \to e\nu_e\gamma)}{dx_\gamma}$$

turns out to be dominated by SD contribution for photon-energy cuts ΔE_γ as low as 10 MeV.

The total branching for $D_s \to e\nu_e\gamma$



- BESIII has recently "measured" the branching fraction for $D_s \to e\nu_e\gamma$ employing a lower-cut $\Delta E_\gamma = 10$ MeV finding

$$\mathrm{Br}[D_s \to e\nu_e\gamma](10\ \mathrm{MeV}) \equiv \frac{\Gamma_e(10\ \mathrm{MeV})}{\Gamma_{\mathrm{tot}}} < 1.3 \times 10^{-4} \quad \text{at } 90\%\ \text{C.L.}$$

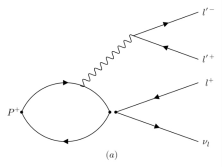
- Quark-model [hep-ph:0012066, hep-ph/0212363] and HQET+pQCD [hep-ph/9911427] calculations predicted a branching of order $10^{-4} - 10^{-5}$ and 10^{-3} respectively.

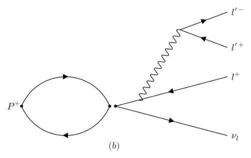
- Our value $\mathrm{Br}[D_s \to e\nu_e\gamma](10\ \mathrm{MeV}) \simeq 4.4(3) \times 10^{-6}$ is lower and well within the BESIII bound. Boring, but it's life...

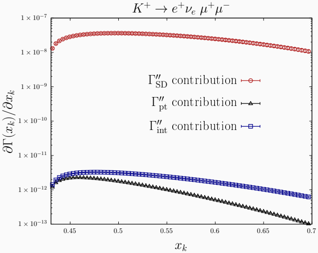
A flash on $P \to \bar{\ell}'\ell'\ell\nu_\ell$ [Gagliardi et at. PRD 105 (2022)]

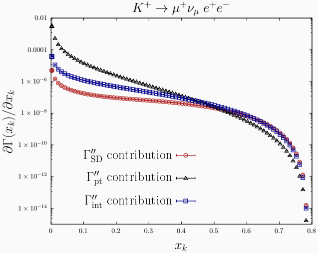
Exploratory calculation for unphysical pion masses ($m_\pi \simeq 320$ MeV) of the rare $K \to \bar{\ell}'\ell'\ell\nu_\ell$ decay ($x_k = m_{\ell'\ell'}/m_K$)

This work	point-like	E865 exp.
$\mathrm{Br}\,[K^+ \to e^+\,\nu_e\,\mu^+\,\mu^-]$		
$0.762(49) \times 10^{-8}$	3.0×10^{-13}	$1.72(45) \times 10^{-8}$
$\mathrm{Br}\,[K^+ \to \mu^+\,\nu_\mu\,e^+\,e^-]$ for $x_k > 0.284$		
$8.26(13) \times 10^{-8}$	4.8×10^{-8}	$7.93(33) \times 10^{-8}$









- Unknown systematic due to $2m_\pi > m_K$ . . . ✗
- Reach the physical point $m_\pi \simeq 140$ MeV is extremely challenging. . .

. . . because for dilepton invariant masses $m_{\ell'\ell'} > 2m_\pi$ the relevant correlation functions do not admit analytic continuation from Minkowskian to Euclidean spacetime (where we simulate).

Future perspectives for $K \to \bar{\ell}'\ell'\ell\nu_\ell$

We now have a strategy to evade the problem of analytic continuation, using spectral density methods

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Spectral-function determination of complex electroweak amplitudes with lattice QCD

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We present a novel method to determine on the lattice both the real and imaginary parts of complex electroweak amplitudes involving two external currents and a single hadron or the QCD vacuum in the external states. The method is based on the spectral representation of the relevant time-dependent correlation functions and, by extending the range of applicability of other recent proposals built on the same techniques, overcomes the difficulties related to the analytic continuation from Minkowskian to Euclidean time, arising when intermediate states with energies smaller than the external states contribute to the amplitude. In its simplest form, the method relies on the standard $i\varepsilon$ prescription to regularize the Feynman integrals and at finite ε it requires to verify the condition $1/L \ll \varepsilon \ll \Delta(E)$, where L is the spatial extent of the lattice and, for any given energy E, $\Delta(E)$ represents the typical size of the interval around E in which the hadronic amplitude is significantly varying. In order to illustrate the effectiveness of this approach in a realistic case, we apply the method to evaluate nonperturbatively the hadronic amplitude contributing to the radiative leptonic decay $D_s \to \ell\nu_\ell\gamma^*$, working for simplicity with a single lattice ensemble at fixed volume and lattice spacing.

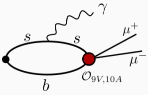
Stay tuned!

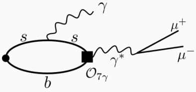
Work in progress for FCNC radiative decays

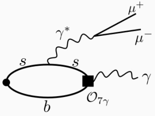
We are currently working to give a lattice QCD prediction for $B_s \to \mu^+\mu^-\gamma$ at high $q^2 \gtrsim (4.2\ \mathrm{GeV})^2$

$$H_{\mathrm{eff}}^{b\to s} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i(\mu)\mathcal{O}_i(\mu)$$

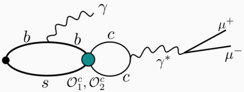
- Many operators from the effective $b \to s$ weak-Hamiltonian. We compute (γ from b-quark not shown):

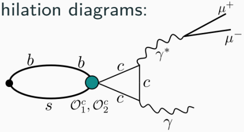






- but neglect charming-penguins and weak-annihilation diagrams:

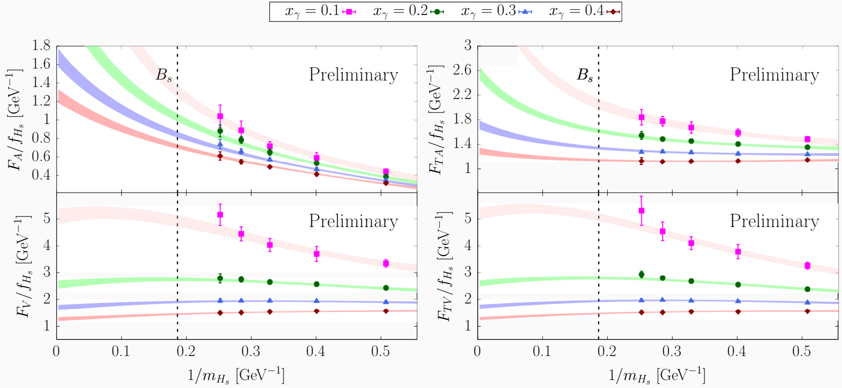




- Their contribution suppressed at large q^2 (actually the reason why one looks at $B_s \to \mu^+\mu^-\gamma$ at large q^2 [Guadagnoli et al, JHEP07(2023)]). Will be estimated using phenom. parametrization.
- A tremendously expensive project: four lattice spacings to control cut-off effects, five heavy-light meson masses H_s to extrapolate the results to the B_s (too heavy to be directly simulated on current lattices), many form factors...
- Draft in preparation. Results will be out hopefully by the end of the year.

Very preliminary results for tensor and vector form factors

Extrapolation of the form factors F_V, F_A, F_{TV} and F_{TA} entering the $B_s \to \mu^+\mu^-\gamma$ decay rate, to the physical B_s meson mass, for different values of $x_\gamma = 2E_\gamma/m_{H_s}$.



Conclusions

- Lattice QCD has entered the precision era, and QED and SIB corrections to hadronic processes MUST be taken into account at target precision ($\lesssim 1\%$).

We already have a quite long list of achievements:

- QED+SIB-induced mass-splitting of hadronic multiplets.
- QED+SIB corrections to leptonic decay rates of light mesons have been computed, and results on semileptonic decays start to appear.
- Radiative leptonic decay rates computed for light and some heavy mesons (in the next years we will have results also on $B \to \ell \nu_\ell \gamma$).
- Many new decay channels are starting to be explored: $K_{\ell 4}$, FCNC radiative decays such as $B_s \to \mu^+ \mu^- \gamma$, ...
- The take-away message is that we now have good control on the theoretical aspects of QED+SIB calculations, and the advances in HPC allow us to perform such calculations.

Stay tuned, and thank you for the attention!