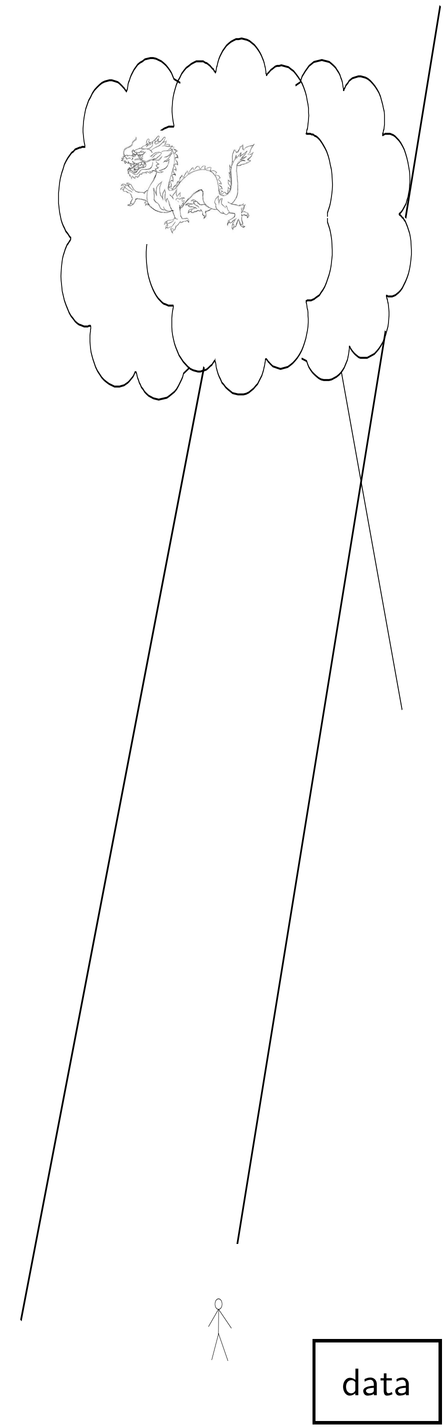
of τs and μs

An LFV review from an EFT perspective

Sacha Davidson
IN2P3/CNRS, France



1: intro:
why is LFV interesting?
what do we know?

2: comparing $\mu \leftrightarrow e$ and $\tau \leftrightarrow l$
(expt and) theory interpretation

(**3:** can $\mu \leftrightarrow e$ data rules models out?)

LFV $\equiv$ FCNC of charged leptons, $eg\ \mu \to e\gamma$
EFT $\Rightarrow$ NP heavy, so neglect $\mu \to ea$, etc

Reasons to like LFV

3. leptons do not have strong interactions

2. leptons can generate the baryon asym. without proton decay(via non-perturbative SM $\not{B}+\not{L}$)

1. $[m_\nu]$ says there is NP in lepton sector, that must give LFV:

calculate loops with m_ν and EW bosons, $\textit{predict } LFV > 0$ — *yippee!*

Reasons to like LFV

3. leptons do not have strong interactions

2. leptons can generate the baryon asym. without proton decay(via non-perturbative SM $\not{B}+\not{L}$)

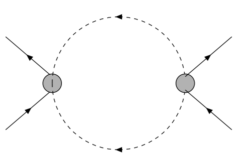
1. $[m_\nu]$ says there is NP in lepton sector, that must give LFV:

calculate loops with m_ν and EW bosons, *predict* $LFV > 0$ — *yippee!*

But **ugh**: get GIM for m_ν renorm, and for non-renorm. EFT gives result ≈ GIM:

Petcov76
DLeakGorbahn

$$\mathcal{A}_{LFV} \sim \frac{m_\nu^2}{16\pi^2 v^2} \quad , \quad BR_{LFV} \sim 10^{-55}$$



... lets hope LFV $\not\propto m_\nu^2$, because at diff scales or have diff couplings

("logGIM" occurs for quarks, but masses/charges not allow log(m_ν/m_W))

1807.06050
1912.09862

What we know is bounds: categories of LFV data

Heeck

$\boldsymbol{\Delta LF = 1, \Delta QF = 0}$

$\mu A \to eA$, $\tau \to 3l$, $h \to \tau^{\pm} l^{\mp}$... $(l \in \{e, \mu\})$

$\boldsymbol{\Delta LF = 2}$

$\mu\bar{e} \to e\bar{\mu}, \quad \tau \to ee\bar{\mu}$...

$\Lambda \gtrsim 25v|_{\mu}, \quad \Lambda \gtrsim 60v|_{\tau}$

$\boldsymbol{\Delta LF = \Delta QF = 1}$

$K \to \mu\bar{e}, B_s \to \tau\bar{\mu}, ...$

categories $\approx$ independent below $\Lambda_{\rm LFV}$

ArduDavidson

what we know about LFV : bounds/upcoming reach

$\Delta LF = 1, \Delta QF = 0 \quad (\Delta LF = \Delta QF = 1)\ ,\ \ (\Delta LF = 2)$

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEGII) $\to$ ...
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (202x, Mu3e)
$\mu Ti \to eTi$	$< 6 \times 10^{-13}$, (SINDRUMII)	$10^{-(16 \to ?)}$ (Mu2e,COMET)
$\mu Au \to eAu$	$< 7 \times 10^{-13}$, (SINDRUMII)	$?10^{-(18 \to ?)}$ (PRISM/PRIME/ENIGMA
$(\mu \to e\gamma\gamma$	$< 7.2 \times 10^{-11})$ (CrystalBox)	
$\tau \to \{e, \mu\}\gamma$	$< 3.3, 4.4 \times 10^{-8}$	$\sim 10^{-8}$ (Belle-II)
$\tau \to e\bar{e}e, \mu\bar{\mu}\mu, e\bar{\mu}\mu...$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-10}$(Belle-II, LHCb?)
$\tau \to \left\{ \begin{matrix} e \\ \mu \end{matrix} \right\} \{\pi, \rho, \phi, ...\}$	$\lesssim$ few $\times 10^{-8}$	$\sim 10^{-9}$ (Belle-II)
$h \to \tau^{\pm}\ell^{\mp}$	$< 1.5, 2.2 \times 10^{-3}$(ATLAS/CMS)	$< 2 \times 10^{-4}$ (ILC)
$h \to \mu^{\pm}e^{\mp}$	$< 6.1 \times 10^{-5}$ (ATLAS/CMS)	2×10^{-5} (ILC)
$Z \to e^{\pm}\mu^{\mp}$	$< 2.6 \times 10^{-7}$(ATLAS)	
$Z \to l^{\pm}\tau^{\mp}$	$< ... \times 10^{-7}$(ATLAS)	
$K^+ \to \pi^+\bar{\mu}e$	$< 4.7 \times 10^{-12}$ (E865)	$\sim 10^{-12}$ (NA62)
...		
muonium	$P_{M\bar{M}} < 8.2 \times 10^{-11}$ (PSI)	2×10^{-14} (MACE)

Recall that: τ and μ bounds are restrictive...

...because μ and τ decay weakly : $\quad BR(\mu \to e\bar{e}e) = \dfrac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)}$

$$\text{if write} \quad \delta\mathcal{L} = \frac{1}{v^2}(\overline{e}\gamma P_L \mu)(\overline{\nu}\gamma P_L \nu) + \frac{1}{\Lambda^2_{\rm LFV}}(\overline{e}\gamma P_L \mu)(\overline{e}\gamma P_L e)$$

$$\text{so } BR(\mu \to e\bar{e}e) \approx \frac{v^4}{\Lambda^4_{LFV}} \quad \Rightarrow \quad BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{cases}$$

Recall that: τ and μ bounds are restrictive...

...because μ and τ decay weakly : $BR(\mu \to e\bar{e}e) = \dfrac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)}$

if write $\delta\mathcal{L} = \dfrac{1}{v^2}(\overline{e}\gamma P_L\mu)(\overline{\nu}\gamma P_L\nu) + \dfrac{1}{\Lambda^2_{\rm LFV}}(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_L e)$

so $BR(\mu \to e\bar{e}e) \approx \dfrac{v^4}{\Lambda^4_{LFV}} \quad \Rightarrow \quad BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{cases}$

(Compare to decays of $\Upsilon(2s)$ electromagnetically decaying $\bar{b}b$ state

$$BR(\Upsilon \to \tau^\pm\mu^\mp) = \frac{\Gamma(\Upsilon \to \tau^\pm\mu^\mp)}{\Gamma(\Upsilon \to \mu\bar{\mu})} \sim \left|\frac{m^2_\Upsilon/\Lambda^2_{LFV}}{e^2 Q_b}\right|^2 \leq \frac{6\times 10^{-6}}{2.5\times 10^{-2}}$$

which excludes $\Lambda_{LFV} \lesssim 100$ GeV.
Similarly, $BR(\pi^0 \to e^\pm\mu^\mp) < 3.6\times 10^{-10}$ gives $\Lambda_{LFV} \lesssim$ few TeV.)

$\star$ Also recall $BR(\tau \to 3l) = \dfrac{\Gamma(\tau \to 3l)}{\Gamma(\tau \to \mu\bar{\nu}\nu)}\dfrac{\Gamma(\tau \to \mu\bar{\nu}\nu)}{\Gamma(\tau \to \text{all})} \simeq 0.18\dfrac{\Gamma(\tau \to 3l)}{\Gamma(\tau \to \mu\bar{\nu}\nu)}$

Lets compare $\tau \leftrightarrow l$ *and* $\mu \leftrightarrow e$

reach: more restrictive BRs for $\mu \leftrightarrow e$ than $\tau \leftrightarrow l$

constraints: exptal bounds on more $\tau \leftrightarrow l$ processes

Compare reach in Λ_{LFV} — at tree level

$$\tau \leftrightarrow l \qquad 5 \times BR(\tau \leftrightarrow l) \sim 10^{-7} \to 10^{-9} \quad \Rightarrow \quad \Lambda_{\rm LFV} \sim (50 \to 200)v$$

$$\mu \leftrightarrow e \qquad BR(\mu \leftrightarrow e) \lesssim 10^{-12} \to 10^{-16} \quad \Rightarrow \quad \Lambda_{\rm LFV} \sim (10^3 \to 10^4)v$$

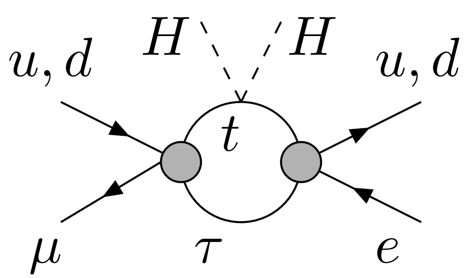
$\Rightarrow$ $\mu \leftrightarrow e$ can probe to higher scales at tree level...(?)promising for discovery(?)

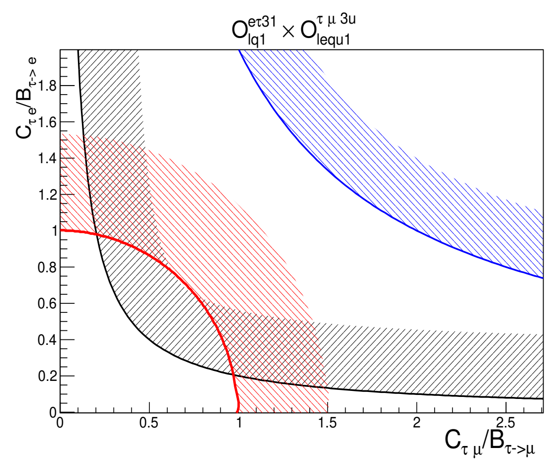
($\mu \leftrightarrow e$ can have interesting sensitivity to some $(\mu \to \tau) \times (\tau \to e)$ operators:

ArduDGorbahn

$$BR(\mu \leftrightarrow e) \propto |... + \frac{C^{\tau\mu Qq} C'^{e\tau qQ}}{16\pi^2}|^2 \lesssim 10^{-12} \to 10^{-16}$$

can have complementary sensitivity to direct bounds $|C^{\tau l Qq}|^2 \leq ...$ for 3rd Gen Q, 1st gen q.)





Count number of constraints from $\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA$

(want to compare # constraints in $\mu \leftrightarrow e$ vs $\tau \leftrightarrow l$)

$\mu A \to eA$ = $\mu \to e$ conversion on nucleus A: (talk:C Carloganu)

(can obtain some μ polarisation: KunoNagamineYamazaki)

- μ^- captured by Al nucleus, tumbles down to $1s$. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM: muon capture $\mu + p \to \nu + n$

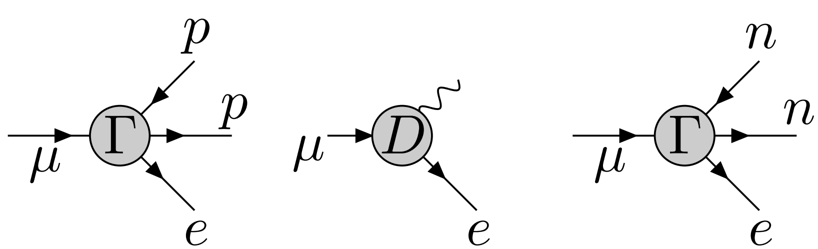
Count number of constraints from $\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA$

(want to compare # constraints in $\mu \leftrightarrow e$ vs $\tau \leftrightarrow l$)

$\mu A \to eA = \mu \to e$ conversion on nucleus A:

(can obtain some μ polarisation: KunoNagamineYamazaki)

- μ^- captured by Al nucleus, tumbles down to $1s$. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM: muon capture $\mu + p \to \nu + n$
- bound μ interacts with nucleus, converts to e ($E_e \approx m_\mu$)



$$\Gamma = \{I, \gamma_5, \gamma, \gamma\gamma_5, \sigma\}$$

$\approx$ WIMP scattering on nuclei

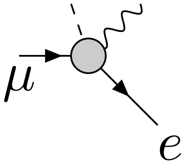
Czarnecki,MarcianoEtal
KitanoKoikeOkada10
...
HaxtonEtal22

1) "Spin Independent" rate,, grows with A (amplitude $\propto \sum_N \sim A$)

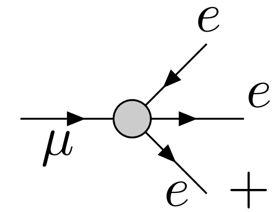
$$BR \propto |\sum C_D, C_{SI}^{nn}, C_{SI}^{pp}|^2$$

..) subdominant("Spin Dependent"...), neglected

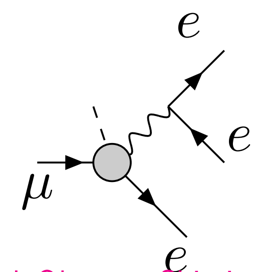
count 4 constraints: exptal bounds on $\left\{ \begin{matrix} \text{light}(\text{Al}, \text{Ti}) \\ \text{heavy}(\text{Au}) \end{matrix} \right\}$ **targets** $\times \{\mu_L, \mu_R\}$



count $\mu \to e$ constraints, ctd



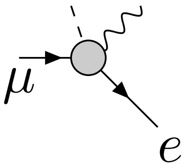
- $\mu \to e\gamma$: chirality-flip, only **two** dipole operators contribute



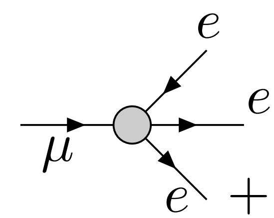
- $\mu \to e\bar{e}e$: 4lepton(4*V+2S)+dipole ops.
  angular distributions$\Rightarrow$ indep constraints on **6**$\to$**8** coeffs.

OkadaOkumuraShimizu

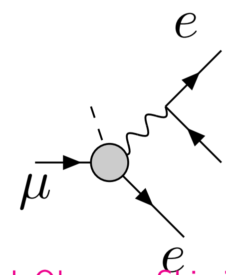
- $\Rightarrow \mu \to e_L(e_R)$ processes, at exptal scale $\left\{ \begin{array}{l} \text{described by} \\ \textbf{constrain} \end{array} \right\}$ **6(+6)** operators:



count $\mu \to e$ constraints, ctd



- $\mu \to e\gamma$: chirality-flip, only **two** dipole operators contribute



- $\mu \to e\bar{e}e$: 4lepton(4*V+2S)+dipole ops.
  angular distributions⇒ indep constraints on **6→8** coeffs.

OkadaOkumuraShimizu

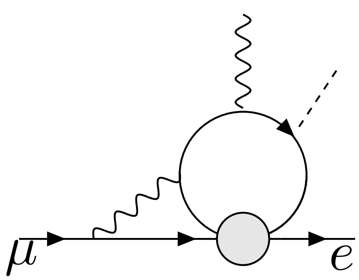
- $\Rightarrow \mu \to e_L(e_R)$ processes, at exptal scale $\left\{ \begin{array}{c} \text{described by} \\ \textbf{constrain} \end{array} \right\}$ **6(+6)** operators:

Ack! but∼100$\mu \leftrightarrow e$ operators...if $\mu \leftrightarrow e$ is there, will it be seen?

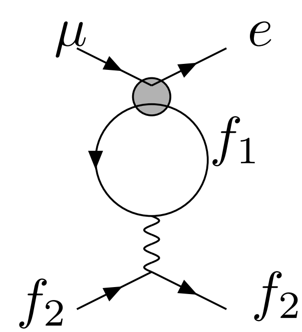
Probably yes : (modulo cancellations) SM loops ensure almost every $\Delta QF = 0$, $\mu \to e$ interaction with ≤ 4 legs, contributes $\gtrsim \mathcal{O}(10^{-3})$ to amplitudes $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and/or $\mu A \to eA$

2010.00317

that is: current bounds sensitive to $\Lambda_{\rm LFV} \lesssim \left\{ \begin{array}{ll} \sim 10^3 v & \text{at tree} \\ \lesssim 100 v & \text{at loop} \end{array} \right.$



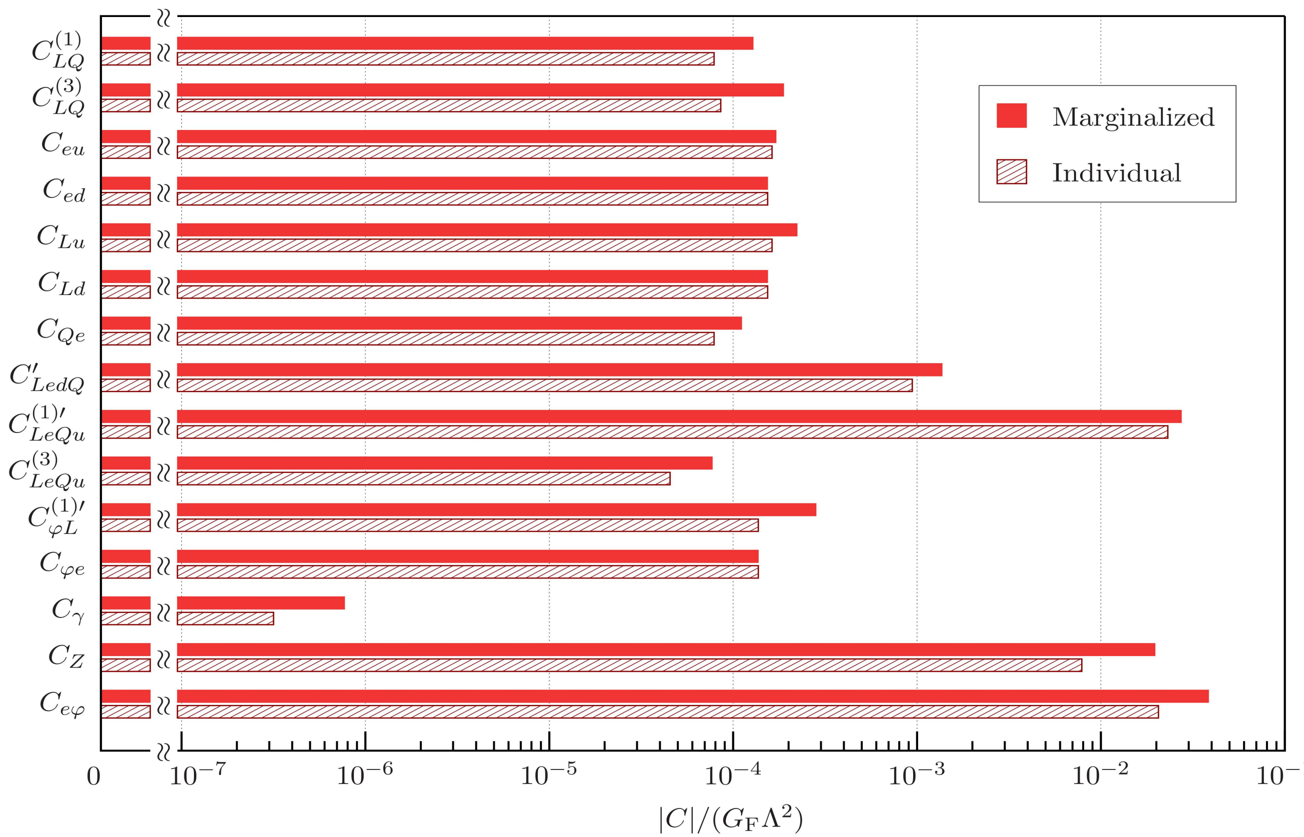
+...



The $\tau \leftrightarrow l$ sector : *marvellous place to observe LFV*

many processes: current data give indep bounds on magnitude of (almost) all operator coeffs, with $\Lambda_{\rm LFV} \sim 10$ TeV
$\Rightarrow$ promising for distinguishing models (+insensitive to most loops≈theoretically simple)

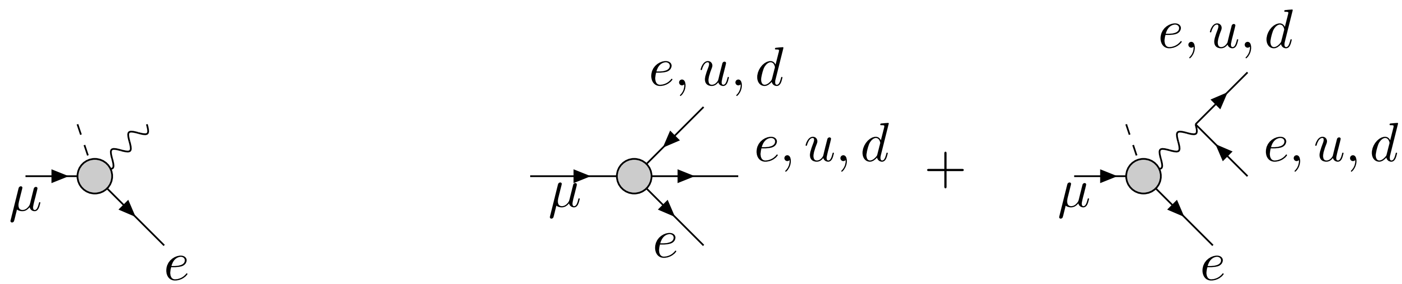
expected sensitivity of BelleII: BR$\lesssim 10^{-9} \to 10^{-10} \Leftrightarrow \Lambda_{\rm LFV} \sim 30$ TeV.



(taken from BanerjeeEtal, Snowmass WPaper 2203.14919). Axis$=v^2/(2\sqrt{2}\Lambda_{\rm LFV}^2)$ dipole as $C_\gamma v \mathcal{O}_D$

If see $\mu \leftrightarrow e$, can we learn something about the model?

recall three processes: $\mu \to e$, $\Delta F_Q = 0$



$\mu \to e\gamma$ $\qquad$ $\mu \to e\bar{e}e$ and $\mu A \to eA$

- $\Rightarrow \mu \to e_L(e_R)$ processes, at exptal scale $\left\{ \begin{array}{l} \text{described by} \\ \text{constrain} \end{array} \right\}$ **6(+6)** operators:

$$\delta\mathcal{L} = \frac{1}{v^2}\Big[C_D(m_\mu \bar{e}\sigma^{\alpha\beta}P_R\mu)F_{\alpha\beta} + C_S(\bar{e}P_R\mu)(\bar{e}P_R e) + C_{VR}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_R e)$$
$$\Big[+ C_{VL}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_L e) + C_{Alight}\mathcal{O}_{Alight} + C_{Aheavy\perp}\mathcal{O}_{Aheavy\perp}\Big]$$

$\{C\}$ are $\mathcal{O}(1)$ dimless numbers that can be measured ($\exists$ more info than just rates)
O_{Alight} =combo of 4fermion operators probed by light targets (Al,Ti)
$O_{Aheavy\perp}$= indep. combo of 4fermion ops probed by heavy targets (Au)

if see $\mu \to e\gamma$, $\mu \to e\bar{e}e$, or $\mu A \to eA$...?can distinguish models?

...model predictions studied for decades...

EFT recipe to study this: (not scan model space—no measure)

- data is a "12-d" ellipse/box in coefficient-space (in an ideal theorist's world)

- with RGEs, can take ellipse to $\Lambda_{\rm LFV}$

- are there parts of ellipse that a model *cannot* fill?

If yes, model can be distinguished/ruled out by $\mu \leftrightarrow e$ data.

Apply recipe to three models around TeV:

1) type II seesaw
2) singlet LQ for R_D^*
3) inverse seesaw

None of the models fills the ellipse:

- seesaws: not generate LFV among singlet (RH) leptons (NP talks to doublets)
- typeII predicts dipole $<$ at least one four-fermion coeff.
- LQ cannot generate scale 4-lepton operators...

Summary

LFV is NP that exists... but we don't see it yet

$\mu \leftrightarrow e$ is an EFT playground: a few processes constrain coefficients of a few operators to be very small: $\Lambda_{\rm LFV} \gtrsim 10^3 v$ now, $\Lambda_{\rm LFV} \sim 10^4 v$ upcoming.
$\star$ include RGEs at leading order, to give *sensitivity* to almost every $\mu \leftrightarrow e$ operator (in chiral basis) with ≤ 4 legs (at $\Lambda_{\rm LFV} \lesssim 10^2 v$)
$\star$ matching a few models onto this exptally accessibly subspace, shows $\mu \leftrightarrow e$ observations could rule out models

$\tau \leftrightarrow l$ processes currently set moderate *constraints* on (almost) all $\tau \leftrightarrow l$ operator coefficients :$\Lambda_{\rm LFV} \gtrsim 50v$.
Marvellous place to reconstruct NP model of lepton sector

BackUp

But to reconstruct $\mu \to e$ bottom-up, need all data?

eg $BR(\pi^0 \to e^\pm \mu^\mp) < 3.6 \times 10^{-10}$, or $BR(\Upsilon \to l_1 \bar{l}_2) \lesssim 10^{-6}$?

Ummm: μ decays weakly $\Leftrightarrow \tau_\mu \sim 10^{-6}$ sec.

vs $\tau_{\pi^0} \sim 10^{-16}$ sec (loop-suppressed QED), or $\tau_\Upsilon \sim 10^{-20}$ sec (tree QED/QCD)

Compare *weak* μ decays to *anomalous QED* π_0 decay

(write $\delta\mathcal{L} \sim \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\mu)(\bar{q}q) + \frac{1}{\Lambda^2_{\rm LFV}}(\bar{e}\gamma\mu)(\bar{e}\gamma e)$) :

$$BR(\mu \to e\bar{e}e) = \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \sim \left|\frac{m_\mu^2/\Lambda^2_{\rm LFV}}{m_\mu^2 G_F}\right|^2 \sim \frac{v^4}{\Lambda^4_{\rm LFV}} \lesssim 10^{-12} \Rightarrow \Lambda_{\rm LFV} \gtrsim 10^5 {\rm GeV}$$

$$BR(\pi_0 \to \bar{e}\mu) = \frac{\Gamma(\pi_0 \to \bar{e}\mu)}{\Gamma(\pi_0 \to \gamma\gamma)} \sim \left|\frac{m_\pi^2/\Lambda^2_{LFV}}{\alpha/4\pi}\right|^2 \sim \left(\sqrt{\frac{4\pi}{\alpha}}\frac{m_\pi}{\Lambda_{LFV}}\right)^4 \Rightarrow \Lambda_{\rm LFV} \gtrsim {\rm TeV}$$

... rare μ processes have exceptional *sensitivity*, because μ decay weak.
Other $\mu \to e$ processes constrain "orthogonal" operator coefficients, less well.

...but: uncertainties in matching to quarks

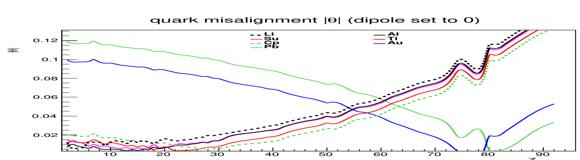
suppose measure coefficients of LFV ops with vector and scalar currents of n or p, from $\mu A \to eA$ on different targets
Then match to quarks:

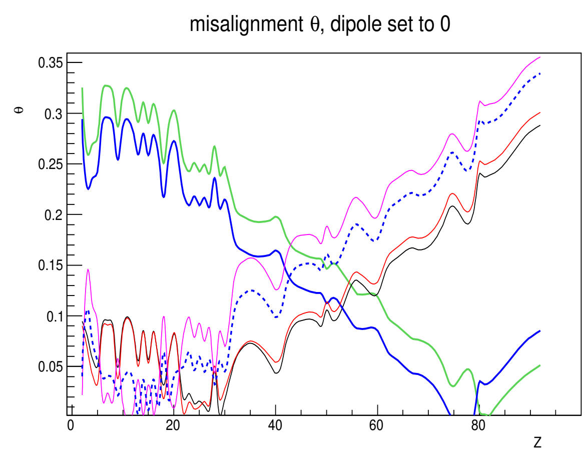
$$\begin{pmatrix} C^{pp}_{V,L} \\ C^{nn}_{V,L} \\ C^{pp}_{S,R} \\ C^{nn}_{S,R} \end{pmatrix} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & G^{pu}_S & G^{pd}_S \\ 0 & 0 & G^{nu}_S & G^{nd}_S \end{bmatrix} \begin{pmatrix} C^{uu}_{V,L} \\ C^{dd}_{V,L} \\ C^{uu}_{S,R} \\ C^{dd}_{S,R} \end{pmatrix}$$

- But for scalar ops, $G^{p,u}_S = G^{n,d}_S \simeq G^{p,d}_S \simeq G^{n,u}_S$

so need great precision to differentiate LFV ops with scalar currents of u or d :(

- and...curent determinations of Gs from lattice and pions disagree by 50%





Climbing the mountain for $\mu \to e$: EFT

Renormalisation Group Eqns/matching/scheme-dep./...

(conceptually simple, technically involved)

Can't we do without RGEs, etc?

in discovery mode for LFV+electroweak loops are small...include later?

counterex: $\mu A \to eA$ in model giving tensor $2\sqrt{2}G_F C_T^{uu}(\overline{e}\sigma P_R \mu)(\overline{u}\sigma u)$ at weak scale

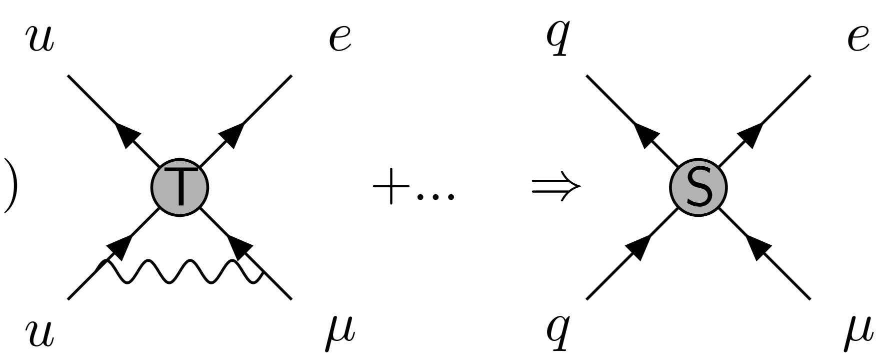
1: forget loops quark tensor matches to nucleon spin $\bar{N}\gamma\gamma_5 N$: $(N \in \{n, p\})$

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \tfrac{1}{2}|C_T^{uu}|^2}$$

(CiriglianoDKuno)
Hoferichter etal

2: include QED loops $m_W \to 2$ GeV:

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$



$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$$

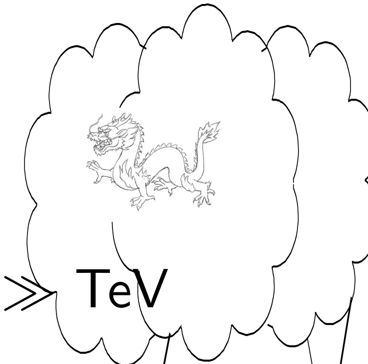
$$\Delta C_S^{uu}(m_\tau) \sim \tfrac{1}{7}C_T^{uu}(m_W)$$

Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained. Important for $\mu \to e$. (?not $\tau \to l$?)

need operators+bases for 3 EFTs?



$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$SU(3) \times SU(2) \times U(1)$

$m_W \sim m_h \sim m_t$

$\{\gamma, g, f\}$

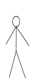
$QCD \times QED$

2 GeV$\sim m_c, m_b, m_\tau$

NB: $\frac{2\text{GeV}}{m_\mu} \sim 20$

$\{n, p, \pi, \gamma, e, \mu\}$ QED $+ \chi PT$

data ($\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA$)

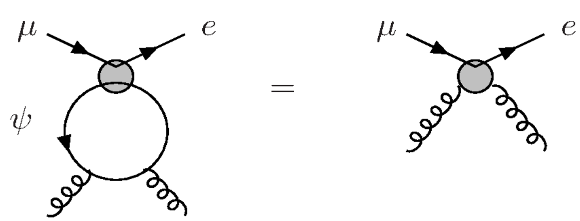


operators + RGEs: *everything to which data could be sensitive*

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.
add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$
(not dim6: bottom-up perspective/ operator dim. not preserved in matching)
above m_W: dim 6 + selected dim 8 (guess by powercounting)
ArduDavidson

ex: $(\bar{e}\mu)G_{\alpha\beta}G^{\alpha\beta}$ is dim7 $< m_W$, dim8 in SMEFT. But

- dim6 heavy quark scalar ops $(\bar{e}\mu)(\bar{Q}Q)$ match to $(\bar{e}\mu)GG$ at m_Q (coef.$C_{QQ}/(m_Q\Lambda^2_{\rm LFV})$):



- gluons contribute most of the mass of the nucleon ShifmanVainshteinZahkarov

$$\langle N|m_N\overline{N}N|N\rangle = \sum_{q\in\{u,d,s\}}\langle N|m_q\overline{q}q|N\rangle - \frac{\alpha_s}{8\pi}\beta_0\langle N|GG|N\rangle$$

$\Rightarrow$ dim7 $(\bar{e}\mu)GG$ contributes significantly to $\mu A \to eA$ via scalar $\mu \to e$ interactions with nucleons N.
CiriglianoKitanoOkadaTuscon

operators + RGEs: *everything to which data could be sensitive*

operator basis: below m_W, all gauge invariant operators with $\leq$ 4 legs$\approx$ 100 ops.

add to $\mathcal{L}_{SM}$ as $\delta\mathcal{L} = 2\sqrt{2}G_F C^{e\mu ee}_{V,LL}(\overline{e}\gamma\mu)(\overline{e}\gamma e) + ...$

(not dim6: bottom-up perspective/ operator dim. not preserved in matching)

above m_W: dim 6 + selected dim 8 (guess by powercounting)

ArduDavidson

RGEs+matching: at "leading order"$\equiv$ largest contribution of each operator to each observable. (2GeV$\to m_W$:resum LL QCD, $\alpha_e \log$, some $\alpha_e^2 \log^2$, $\alpha_e^2 \log$)

why not just 1-loop RGEs?

- expand in loops, hierarchical Yukawas, $1/\Lambda^2_{\rm LFV}$,... largest effect maybe not 1-loop (ex: Barr-Zee)
- sometimes 1-loop vanishes...eg: 2-loop $\Delta a_\mu|_{EW} \simeq$ 1-loop $\Delta a_\mu|_{EW}$.
  or 2-loop log-enhanced
  $=$ mixing vector ops to dipole in 2-loop RGEs.

What can one learn

in bottom-up EFT?

.

But 3 processes, ∼100 operators ⇒ zoo of flat directions?

DKunoYamanaka

Count constraints: (write $\delta\mathcal{L} = C^{flavour}_{Lorentz,XY}/v^n \; \mathcal{O}^{flav}_{Lorentz,XY}$, $X, Y \in \{L, R\}$)

$\boldsymbol{\mu \to e\gamma}$ **:** $BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ ⇒ **2 constraints**

$\boldsymbol{\mu \to e\bar{e}e}$ **:** (e relativistic ≈ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64 \ln \frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

⇒ **6 more constraints**

$\boldsymbol{\mu A \to eA}$ **:**(S^N_A,V^N_A=integral over nucleus A of N distribution×lepton wavefns, **different** for diff. A)

$$BR_{SI} \sim Z^2|V^p_A\tilde{C}^p_{V,L} + S^p_A\tilde{C}^p_{S,R} + V^n_A\tilde{C}^n_{V,L} + S^b_A\tilde{C}^n_{S,R} + D_A C_{D,R}|^2 + |L \leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) ⇒ **$4+2$ more constraints**

future: improved theory, 3SI+2SD targets ⇒ **6+4** constraints

is 12-20 constraints on ~ 100 operators a problem?

many operators+few constraints=using inconvenient basis

Have 6 (+6) constraints on e_L (e_R) operator coefficients. Focus on e_L.
Want to change basis to *scale -dependent* **basis of constrained 6-d subspace.**

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$
Have RGEs for coefficients (arranged in row vector)

$$\mu \frac{\partial}{\partial \mu} \vec{C}(\mu) = \vec{C}(\mu) \mathbf{\Gamma}(\mu, g_s(\mu), ...) \quad \Rightarrow \quad \vec{C}(m_\mu) = \vec{C}(m_W) \boldsymbol{G}(m_\mu, m_W)$$

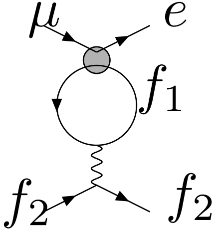
solved as scale-ordered exponential (resummed QCD, $\alpha \log$, some $\alpha^2 \log^2$, $\alpha^2 \log$)
$\Rightarrow$ define scale-dep $\vec{v}_{\mu \to e\gamma}(\Lambda)$, column of **G** such that: $C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu \to e\gamma}(\Lambda)$

$\boldsymbol{\vec{v}_{\mu \to e\gamma}(\Lambda)}$ **is scale-dep basis vector for constrainable subspace**

2-6. repeat for other independent constraints. So obtain scale-dep basis vectors for the subspace, defined from the observables.
The "flat directions" (experimentally inaccessible) are orthogonal, and therefore irrelevant.

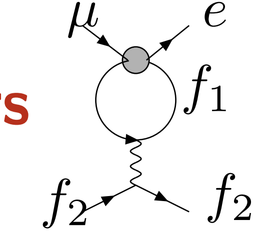
Basis should span the finite-eigenvalue subspace of the correlation matrix.

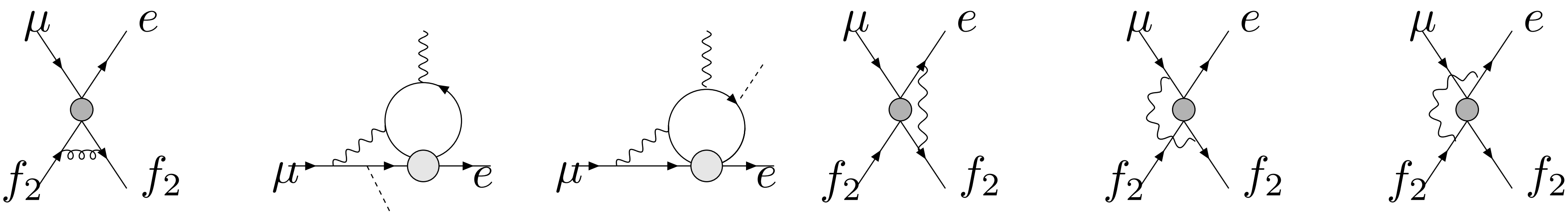
what to do with this basis?



Including SM loop corrections to operators

ex: 1-loop QED + QCD (+2-loop QED V→D)





$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_s}{4\pi} \vec{C} \mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

solve (analytically/numerically):

$$\vec{C}(m_\mu) = \vec{C}(\Lambda_{\rm LFV}) \boldsymbol{G} \quad , \quad \boldsymbol{G} = \text{fn of SM parameters}, \log(\Lambda_{\rm LFV}/\Lambda_{exp})$$

For ex: $BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2) < 4.1 \times 10^{-13} \Rightarrow C_{D,X} \lesssim 10^{-8}$

$$C_{D,X}(m_\mu) = C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) - \frac{\alpha_e}{4\pi e}\left(C^{\mu\mu}_{S,XX} - 8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C_{2loop}\right)\ln\frac{m_W}{m_\mu}$$

$$+16\frac{\alpha_e^2}{2e(4\pi)^2}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right)\ln^2\frac{m_W}{m_\mu} - 8\lambda^{a_T} f_{TD}\frac{\alpha_e}{4\pi e}\left(\frac{2m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_s}{m_\mu}C^{ss}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)\ln\frac{m}{2}$$

$$+16\frac{\alpha_e^2}{3e(4\pi)^2}\left(\sum_{u,c} 4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)\ln^2\frac{m_W}{2\text{GeV}}$$

$C^{\zeta}_{Lor}(m_W)$ on right. $\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

operator list:Kuno-Okada, +CiriglianoKitanoOTuzon +BowmanChengLiMatis

Operator basis $m_\tau \to m_W$: ~ 90 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad dim\ 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad dim\ 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad dim\ 7$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

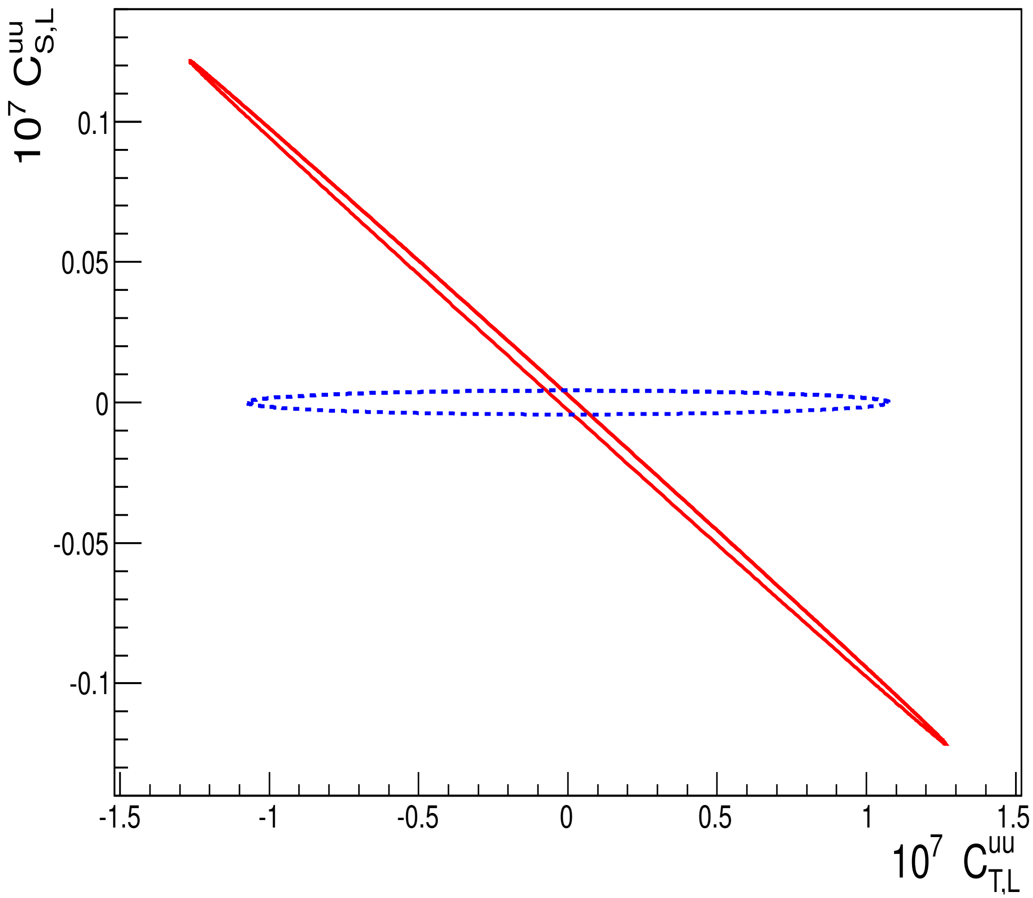
...zzz...but ~ 90 coeffs!

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

sensitivity *vs* constraint

Suppose that $BR(\mu Al \to eAl) \lesssim 10^{-14}$, and :

$$\delta\mathcal{L}(m_W) = C_T^{uu}(\overline{e}\sigma P_Y \mu)(\overline{u}\sigma u) + C_S^{uu}(\overline{e}P_Y\mu)(\overline{u}u)$$



C_T^{uu}, C_S^{uu} constrained to live inside blue (red) ellipse at exptal scale (at m_W): sensitivity to C_S^{uu} = cut ellipse @ $C_T^{uu} = 0$; constraint = live in projection of ellipse onto C_S^{uu} axis.