Noise characterization in LISA data analysis for Stochastic Gravitational Wave Backgrounds

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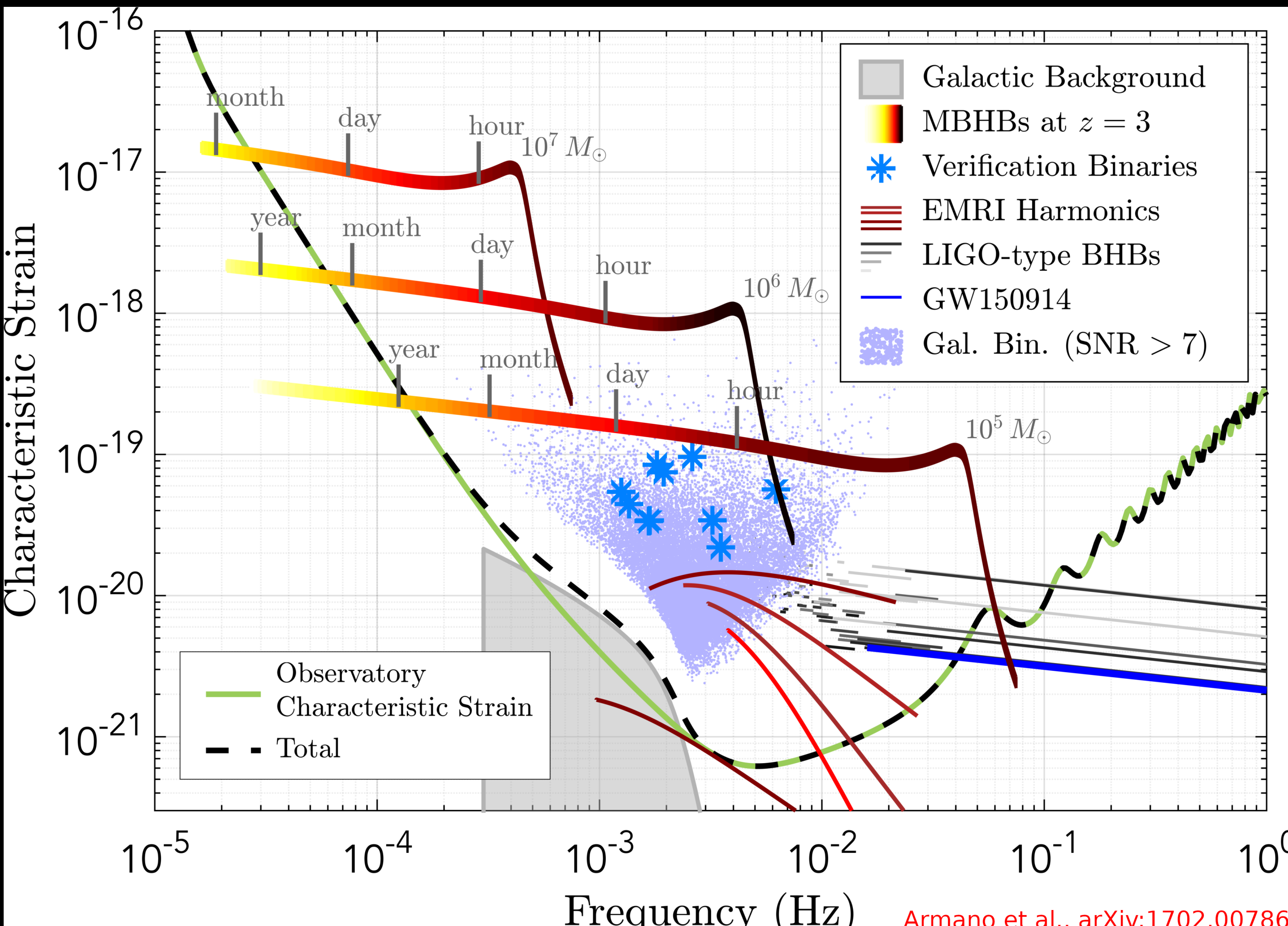
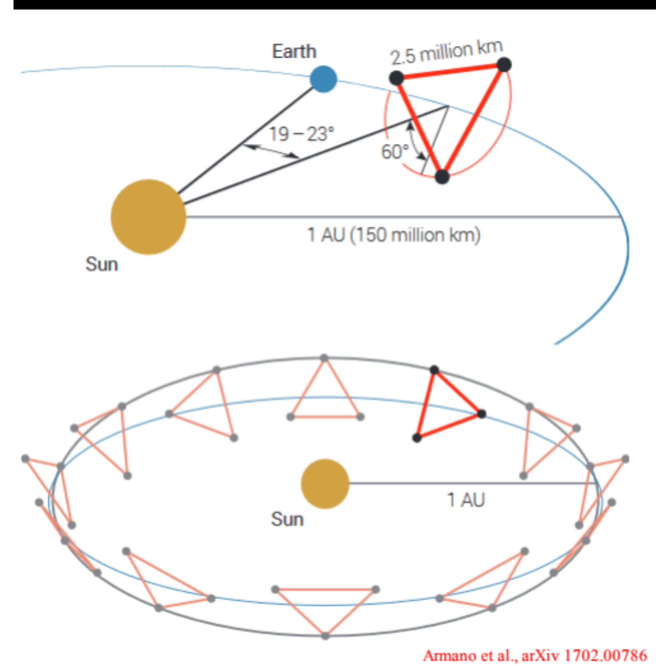
Background credits: Simon Barke

Introduction

- Laser Interferometry Space Antenna (LISA)
  - **space-based** gravitational wave (GW) detector, from mid 2030s
  - sensitive frequency band: **0.1 mHz to 1 Hz**
  - able to observe astrophysical and cosmological GW sources

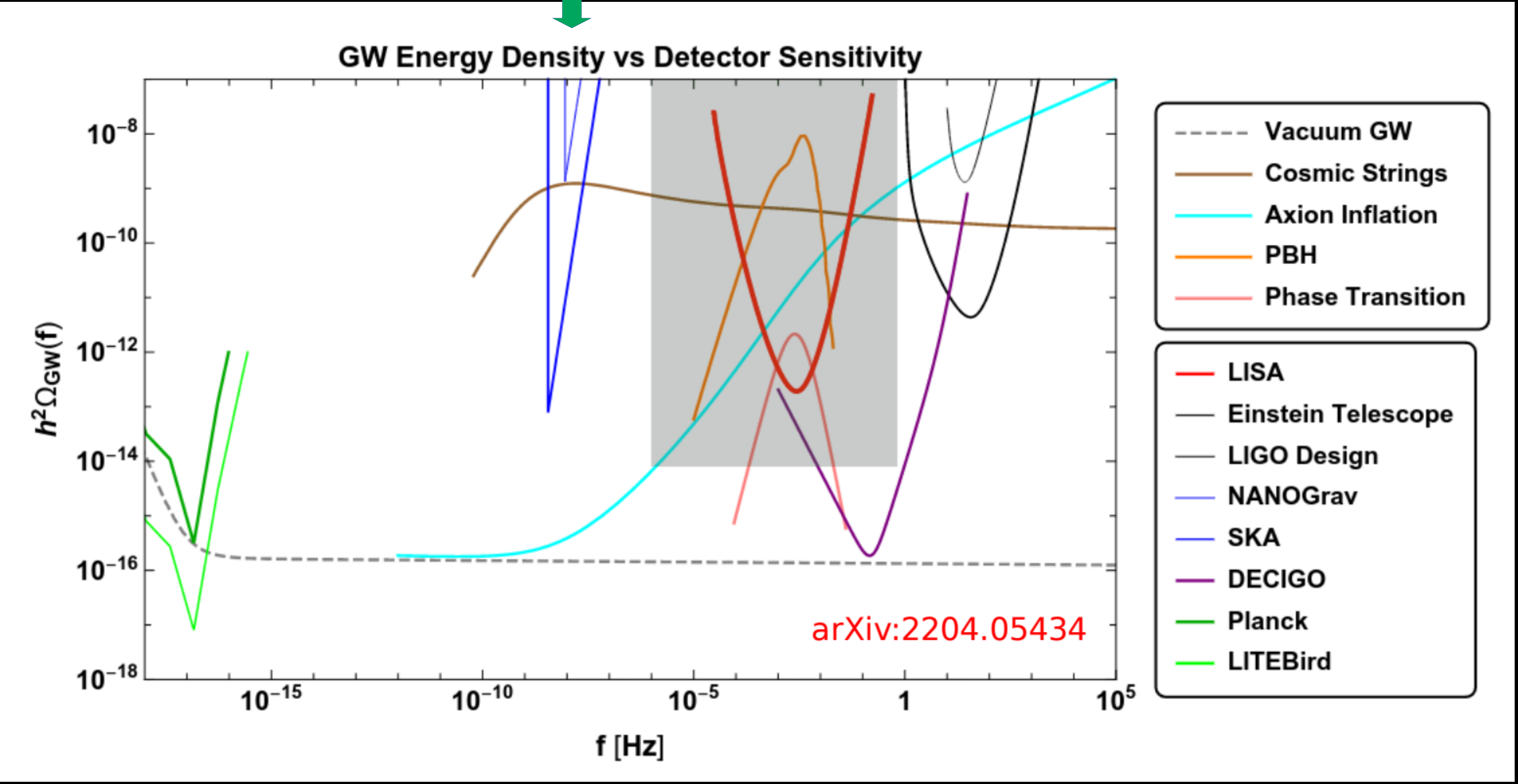


Armano et al., arXiv:1702.00786



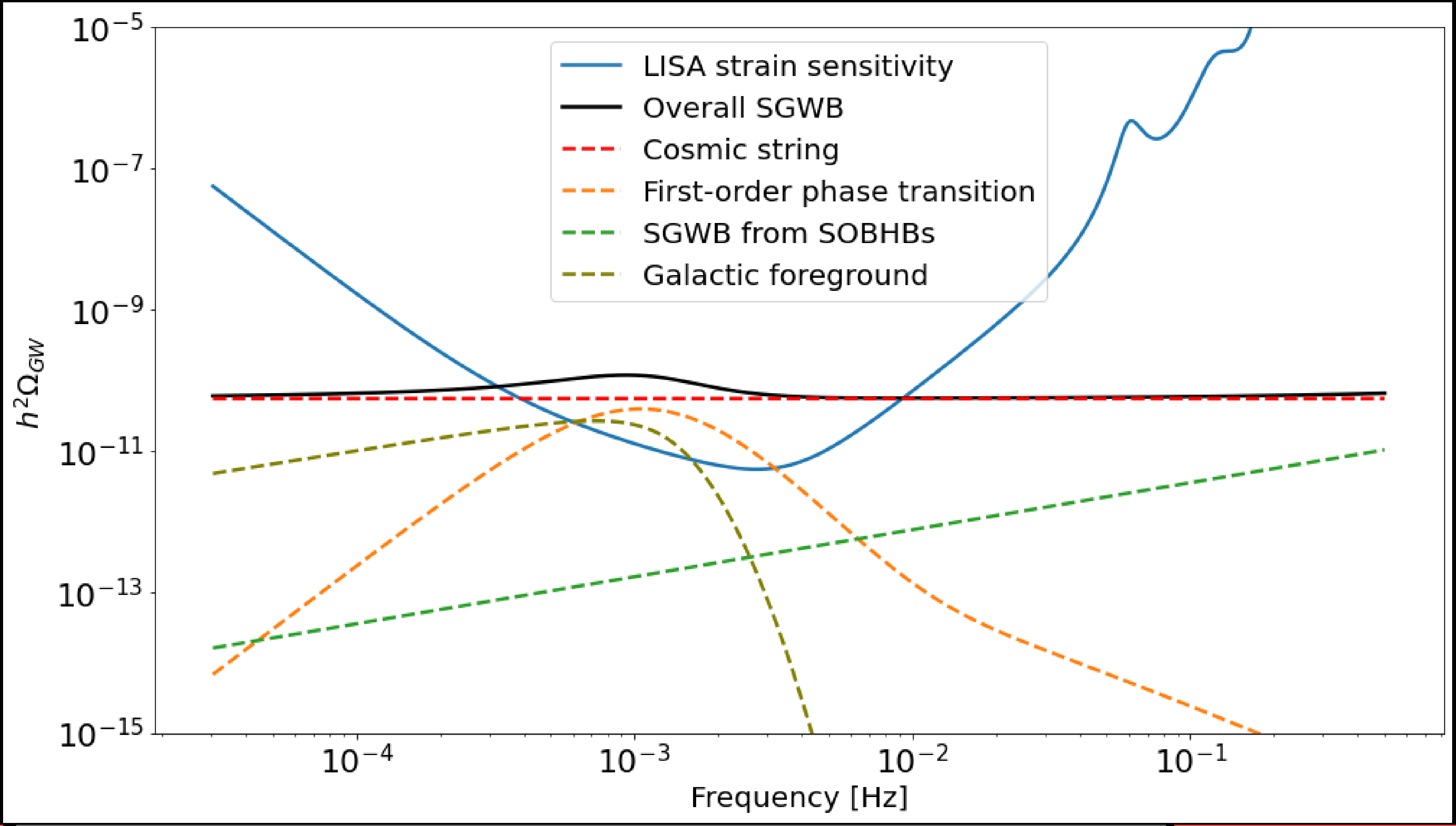
Introduction

- Stochastic gravitational wave backgrounds (SGWBs)
  - ➢ large number of **weak/unresolved**, independent GW signals → challenge to distinguish from noises
  - ➢ either **astrophysical** or **cosmological** origin (see H. Inchauspé's talk, A. Roper Pol's talk)

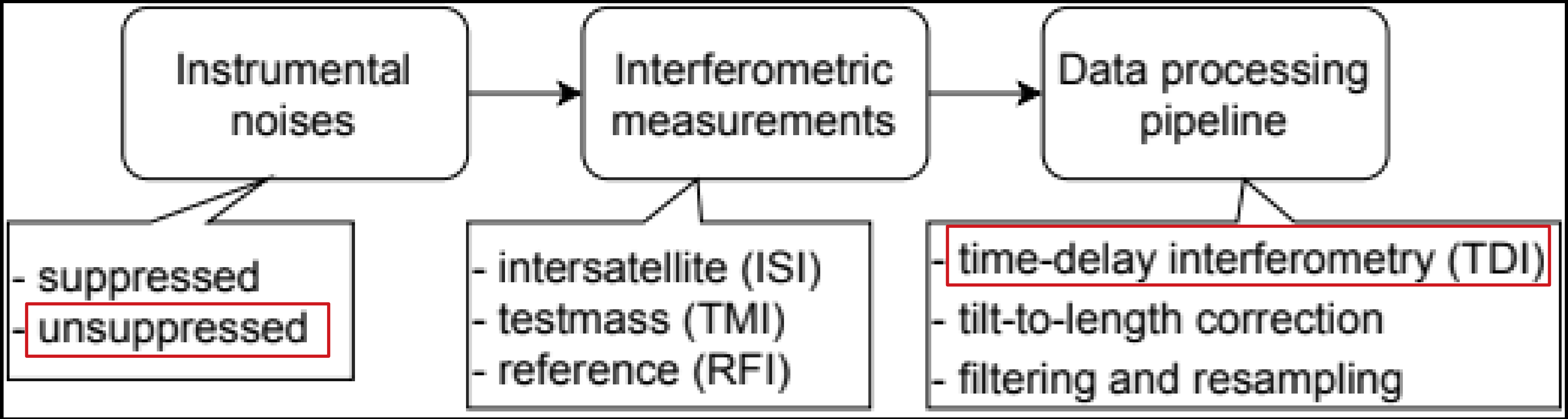


Introduction

- Goals of SGWB data analysis:
  - the **overall shape** of SGWB
  - **decompose** into **individual contributions**
- Detection methods:
  - **Cross-correlation** with **multiple** detectors, which have uncorrelated instrument noises
  - **Exceed power** in data comparing to **well-characterized** noise model



LISA noise model



- The noise covariance matrix of the data decomposed of
  - Noise **input spectral shape** → could not be provided precisely in general
  - Data pipeline (measurements and processing) **transfer function** → "fully" accessible

=> In this study, we focus on the **TDI transfer functions of unsuppressed noises**

- The noise covariance matrix will be **more complex in more realistic** configurations (unequal arms, different noise levels, non-stationarity, noise correlation, etc.)

LISA noise model

- With knowledge of TDI transfer functions (see arXiv:2211.02539), **new noise model** has

  1) Test-mass acceleration noise overall amplitude: A

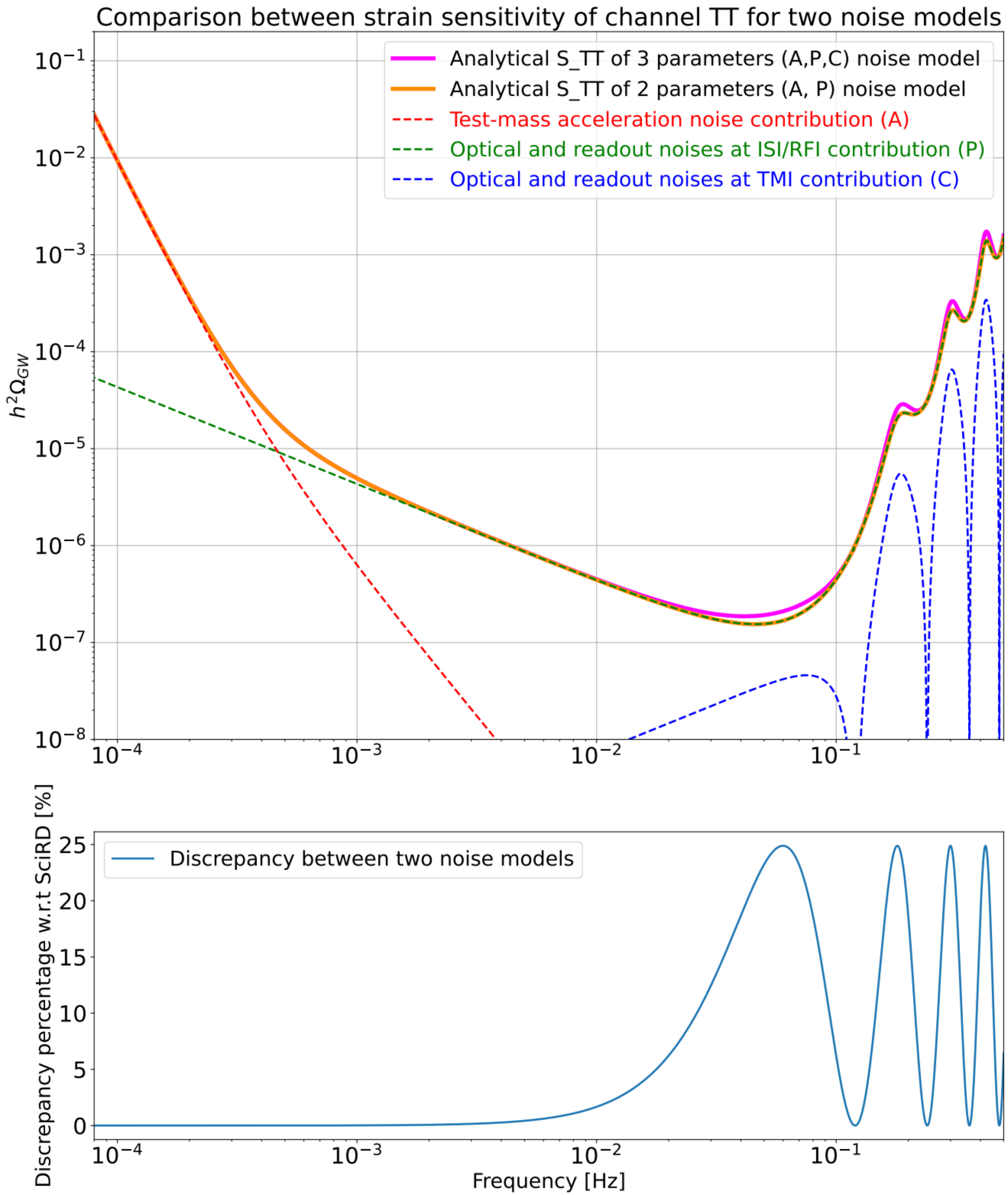
  2) Optical and readout noises **at ISI and RFI** overall amplitude: P

  3) Optical and readout noises **at TMI** overall amplitude: C

- Compare to **standard LISA noise model** (SciRD), parameterized by **2 parameters**

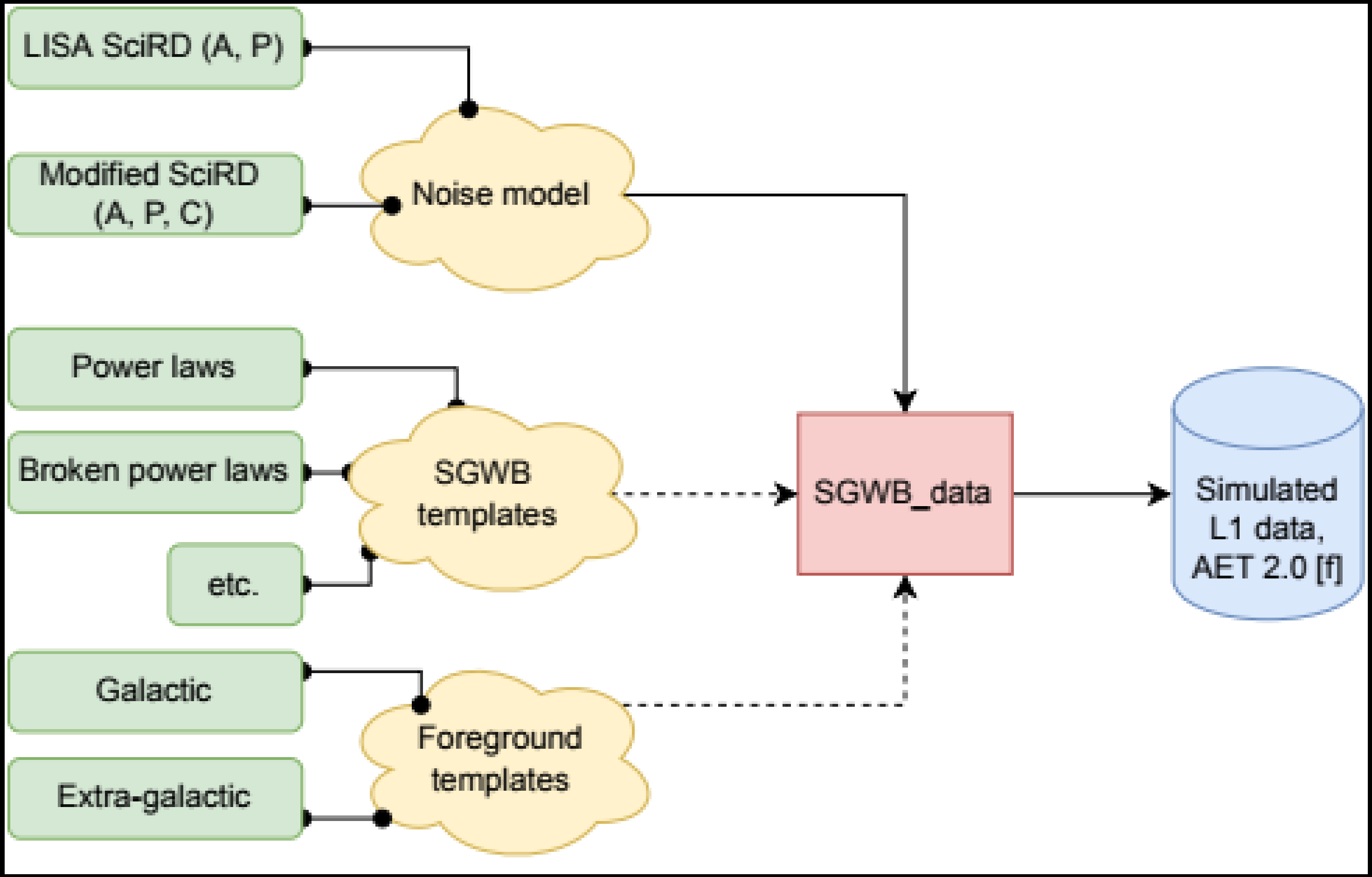
  1) Test-mass acceleration noise overall amplitude: A

  2) Optical and readout noises overall amplitude: P



SGWB data generation

- **Assumptions:** all resolvable GW signals, dominant noises are subtracted from the data

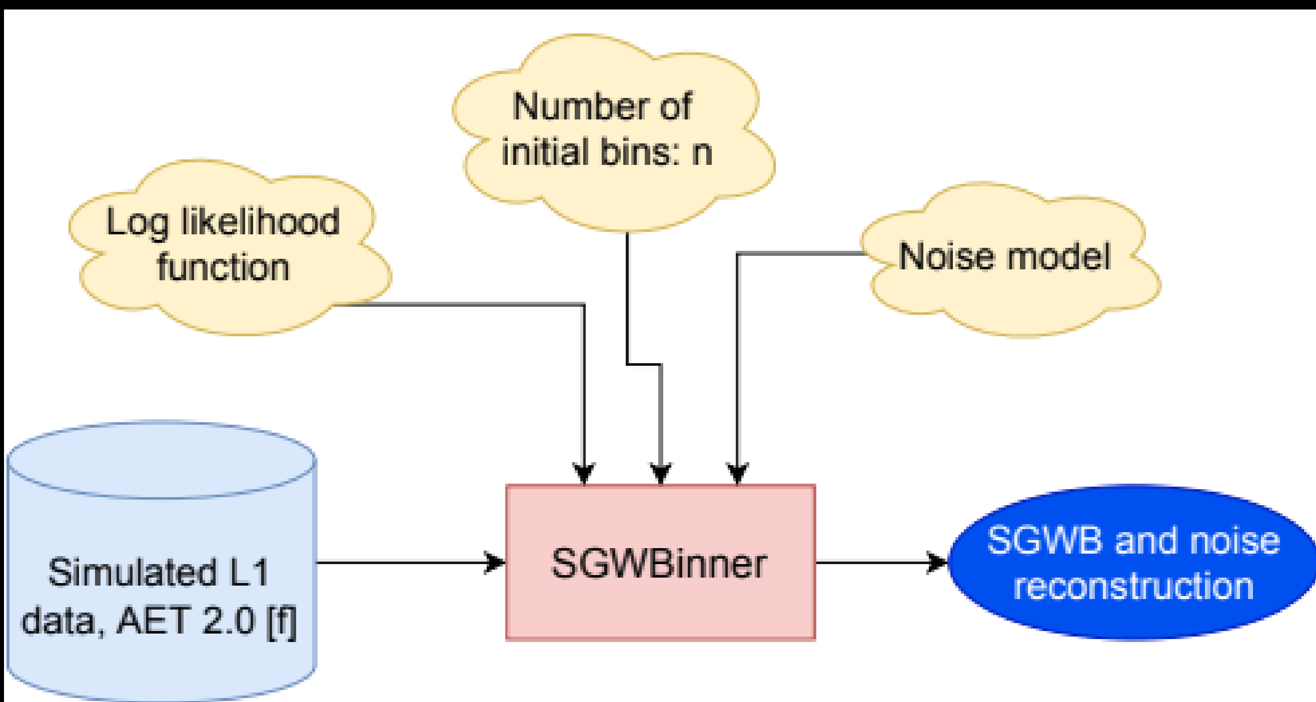


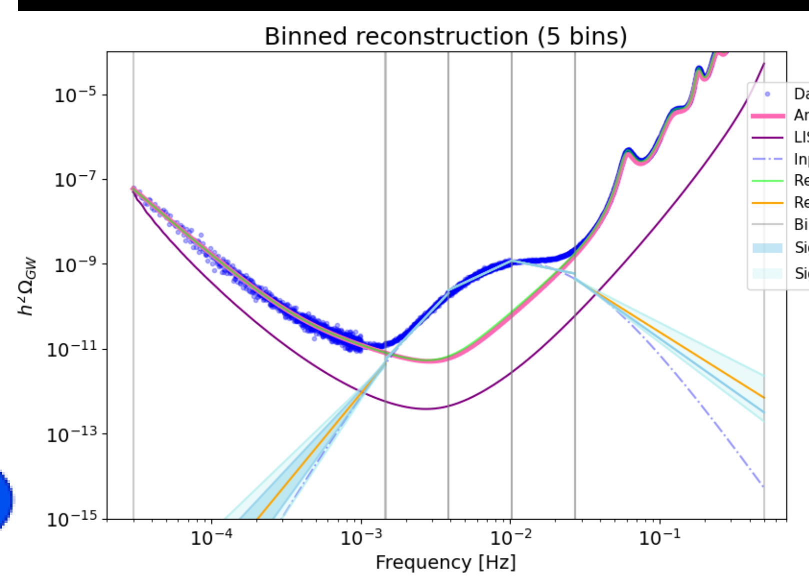
- An alternative pipeline generating time-series data is working in progress (see J-B. Bayle's talk)

[1] Caprini, C., Figueroa, D. G., Flauger, R., Nardini, G., Peloso, M., Pieroni, M., ... & Tasinato, G. (2019). Reconstructing the spectral shape of a stochastic gravitational wave background with LISA. Journal of Cosmology and Astroparticle Physics, 2019(11), 017.

SGWB data analysis

- use **SGWBinner** for SGWB data analysis [2]
  - search **SGWB signal** for each bin (small equal log-frequency intervals) as a **single power law**, assuming they are **independent** for every bins. Collection of power-law signals gives the overall SGWB → SGWB template-free (blind) reconstruction
  - parameter estimation based on **Bayesian analysis**
  - some nearby bins can be merged to avoid over-fitting

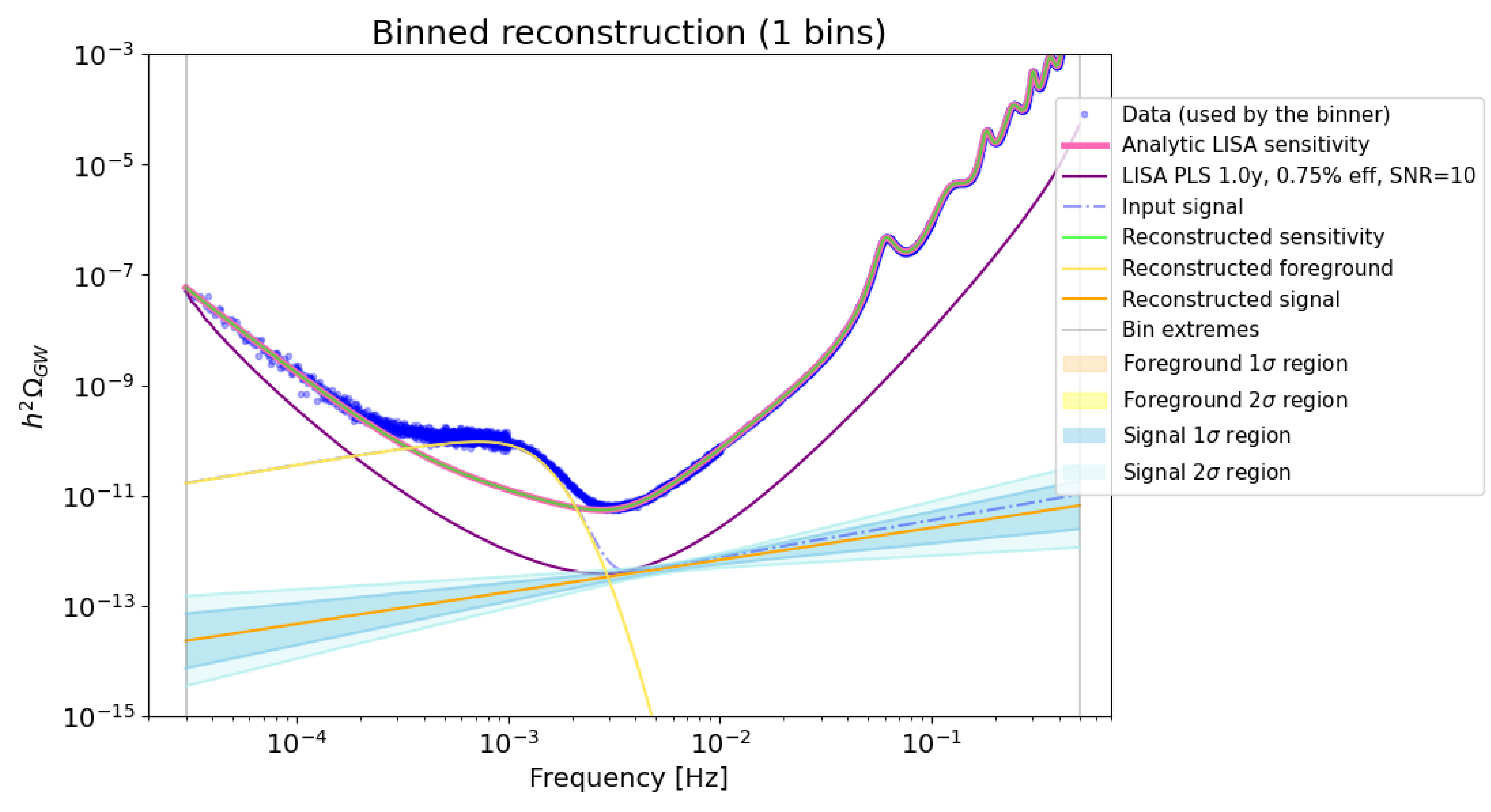


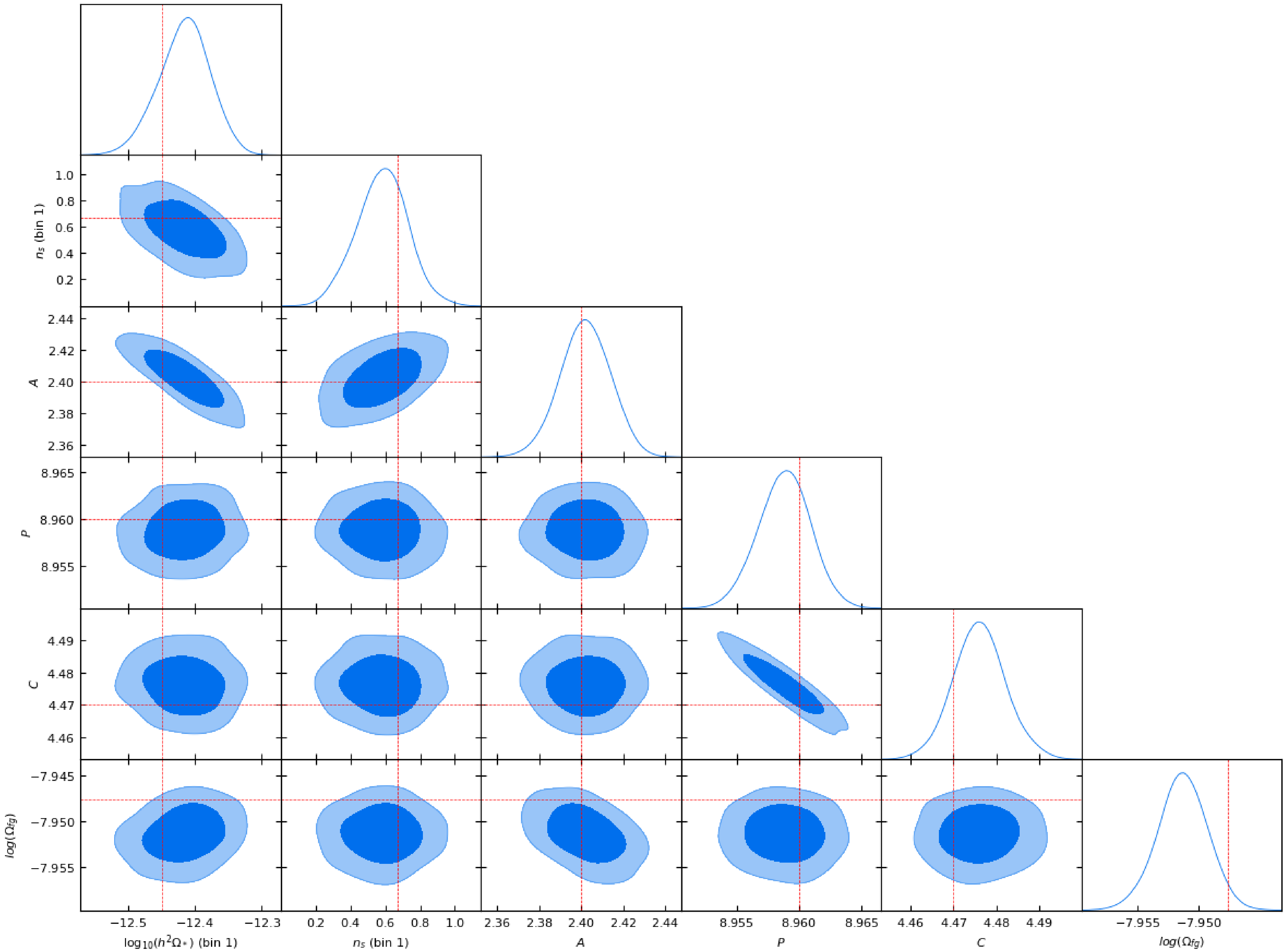


[2] Flauger, R., Karnesis, N., Nardini, G., Pieroni, M., Ricciardone, A., & Torrado, J. (2021). Improved reconstruction of a stochastic gravitational wave background with LISA. Journal of Cosmology and Astroparticle Physics, 2021(01), 059.

SGWB data analysis: Result

- use the **same noise model** in data analysis w.r.t. one used for data generation (3 noise parameters model), **with single power law SGWB** and **galactic foreground**

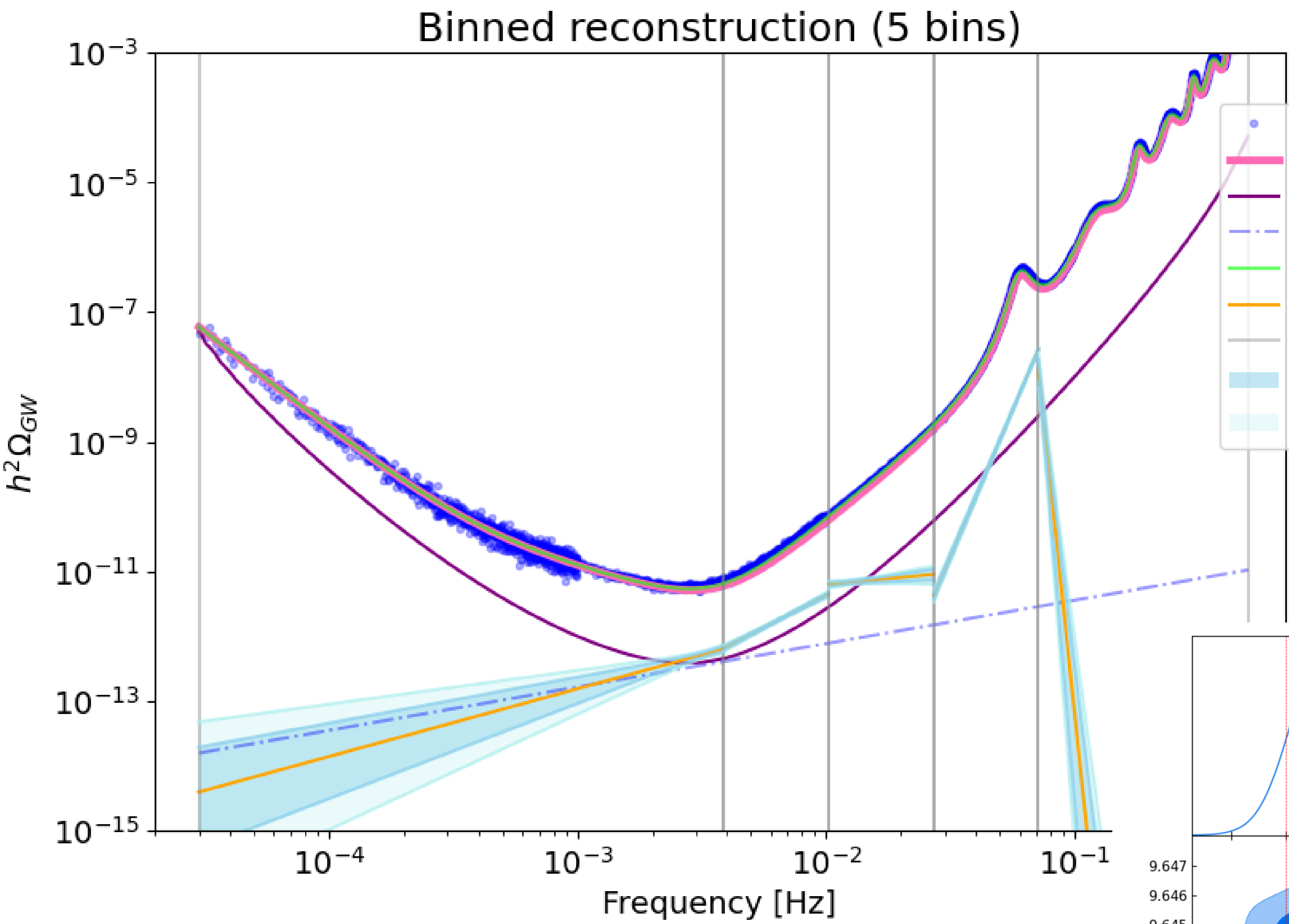


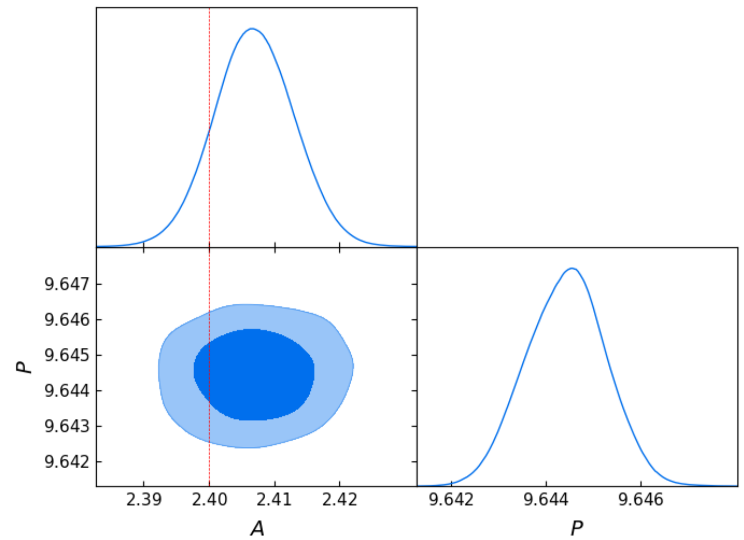


n_s (bin 1)
A
P
C
$log(\Omega_{fg})$
$\log_{10}(h^2\Omega_*)$ (bin 1)
n_s (bin 1)
A
P
C
$log(\Omega_{fg})$

SGWB data analysis: Result

- If the **noise model** used in data analysis (2 noise parameters) **disagrees** with the one used to generated data (3 noise parameters)





Take away messages

- Noise model for LISA might not fully characterized
- The TDI transfer functions plays important role to distinguish different group of noises, and GWs
- The more accurate knowledge of noise characterizations (input shape and transfer function), the more reliable the **agnostic SGWB reconstruction**
- Outlooks:
  - the time domain data generation
  - more realistic LISA instrumental configuration (such as different arm-lengths/ noise levels, noise correlations, non-stationarity, etc.)
  - reconstruct SGWB signal and noise with both agnostic noise and signal templates

Thank you for your attentions!

LISA noise model

- The TDI transfer functions for **unsuppressed noises** summarized in Q.N. Dam et al. (2023)
- The **quasi-orthogonal TDI** combination **AET** proposed to **minimize the cross-correlation** of instrumental noises in final TDI variables
- Some noises have the same TDI transfer functions, and if they have same input noise shape => group to one for fitting one overall parameter

Noise type	Correlation	PSD A & E	PSD T
test-mass acceleration	None	$4C_{XX}(\omega)\,[3+2\cos(\omega L)+\cos(2\omega L)]$	$32C_{XX}(\omega)\sin^4(\frac{\omega L}{2})$
	Fully-correlated noises at the same S/C	$4C_{XX}(\omega)\,[1+2\cos(\omega L)]^2$	$64C_{XX}(\omega)\sin^4(\frac{\omega L}{2})$
	Anti-correlated at the same S/C	$12C_{XX}(\omega)$	0
Readout (TMI) and Optical Pathlength (TMI)	None	$C_{XX}(\omega)\,[3+2\cos(\omega L)+\cos(2\omega L)]$	$8C_{XX}(\omega)\sin^4(\frac{\omega L}{2})$
	Correlated adjacent TMI noise	$3C_{XX}(\omega)$	0
	Anti-correlated adjacent TMI noise	$C_{XX}(\omega)\,[1+2\cos(\omega L)]^2$	$16C_{XX}(\omega)\sin^4(\frac{\omega L}{2})$
Backlink (TMI)	None	$C_{XX}(\omega)\,[3+2\cos(\omega L)+\cos(2\omega L)]$	$8C_{XX}(\omega)\sin^4(\frac{\omega L}{2})$
Readout (ISI and RFI) and Optical Pathlength (ISI and RFI)	None	$2C_{XX}(\omega)\,[2+\cos(\omega L)]$	$4C_{XX}(\omega)\,[1-\cos(\omega L)]$
	Correlated adjacent IFO noise	$3C_{XX}(\omega)$	0
	Anti-correlated adjacent IFO noise	$C_{XX}(\omega)\,[5+4\cos(\omega L)]$	$-8C_{XX}(\omega)\,[-1+\cos(\omega L)]$
	Fully correlated at the same telescope	$4C_{XX}(\omega)\,[3+2\cos(\omega L)+\cos(2\omega L)]$	$32C_{XX}(\omega)\sin^4(\frac{\omega L}{2})$
Backlink (RFI)	None	$2C_{XX}(\omega)\,[2+\cos(\omega L)]$	$4C_{XX}(\omega)\,[1-\cos(\omega L)]$

$$C_{XX}(\omega) = 16\sin^2(\omega L)\sin^2(2\omega L),$$
$$C_{XY}(\omega) = -16\sin(\omega L)\sin^3(2\omega L).$$

TDI noise transfer functions

$$
\begin{aligned}
S^{\mathrm{tot}}_{AA}(f, A, P, C) &= S^{\mathrm{tot}}_{EE}(f, A, P, C) \\
&= 32 \sin^2\left(\frac{2\pi f L}{c}\right) \sin^2\left(\frac{4\pi f L}{c}\right) \\
&\left\{4\left[1 + \cos\left(\frac{2\pi f L}{c}\right) + \cos^2\left(\frac{2\pi f L}{c}\right)\right] S_{\mathrm{acc}}(f, A)\right. \\
&+ \left[2 + \cos\left(\frac{2\pi f L}{c}\right)\right] S_{\mathrm{OMS,\ isi/rfi}}(f, P) \\
&+ \left.\left[1 + \cos\left(\frac{2\pi f L}{c}\right) + \cos^2\left(\frac{2\pi f L}{c}\right)\right] S_{\mathrm{OMS,\ tmi}}(f, C)\right\}, \qquad (58)
\end{aligned}
$$

$$
\begin{aligned}
S^{\mathrm{tot}}_{TT}(f, A, P, C) &= 32 \sin^2\left(\frac{2\pi f L}{c}\right) \sin^2\left(\frac{4\pi f L}{c}\right) \\
&\left\{4\left[1 - \cos\left(\frac{2\pi f L}{c}\right)\right]^2 S_{\mathrm{acc}}(f, A)\right. \\
&+ 2\left[1 - \cos\left(\frac{2\pi f L}{c}\right)\right] S_{\mathrm{OMS,\ isi/rfi}}(f, P) \\
&+ \left.\left[1 - \cos\left(\frac{2\pi f L}{c}\right)\right]^2 S_{\mathrm{OMS,\ tmi}}(f, C)\right\}. \qquad (59)
\end{aligned}
$$

Noise shapes

$$S_{\rm acc}(f, A) = A^2 \frac{\rm fm^2}{s^4 \rm Hz} \left[1 + \left(\frac{0.4\rm mHz}{f}\right)^4\right] \left(\frac{1}{2\pi f}\right)^4 \left(\frac{2\pi f}{c}\right)^2$$

$$S_{\rm OMS,\ isi/rfi}(f, P) = P^2 \left[1 + \left(\frac{2\rm mHz}{f}\right)^4\right] \frac{\rm pm^2}{\rm Hz} \left(\frac{2\pi f}{c}\right)^2$$

$$S_{\rm OMS,\ tmi}(f, P) = C^2 \left[1 + \left(\frac{2\rm mHz}{f}\right)^4\right] \frac{\rm pm^2}{\rm Hz} \left(\frac{2\pi f}{c}\right)^2$$

Parameter	A	P	C
True (injected) value	2.4	8.96	4.47
Units	$\frac{\rm fm}{\rm s^2\sqrt{Hz}}$	$\frac{\rm pm}{\rm \sqrt{Hz}}$	$\frac{\rm pm}{\rm \sqrt{Hz}}$

SGWB model template

- Power law:

$$h^2\Omega_{\rm gw}(f) = 10^A \left(\frac{f}{f_p}\right)^n , \tag{32}$$

where $A = -12.45$, $n = 2/3$. This template represents the **foreground from Solar-mass Black Hole Binaries**.

- Double power law:

$$h^2\Omega_{\rm gw}(f) = 10^{A_1} \left(\frac{f}{f_p}\right)^{n_1} + 10^{A_2} \left(\frac{f}{f_p}\right)^{n_2} , \tag{33}$$

where $A_1 = -15.5$, $n_1 = -4$, $A_2 = -13$, $n_2 = 3$ and $f_p = 3.2 \times 10^{-3}$. This template is the reverse shape of the SGWB modelled from the **first order electroweak phase transition**.

- Galactic foreground:

$$S_{\rm galactic\_fg}(f, A_{\rm fg}) = 10^{A_{\rm fg}} f^{2/3} \exp\left[-f^\alpha - \beta f \sin(\kappa f)\right] \{1 + \tanh\left[\gamma(f_k - f)\right]\} . \tag{34}$$

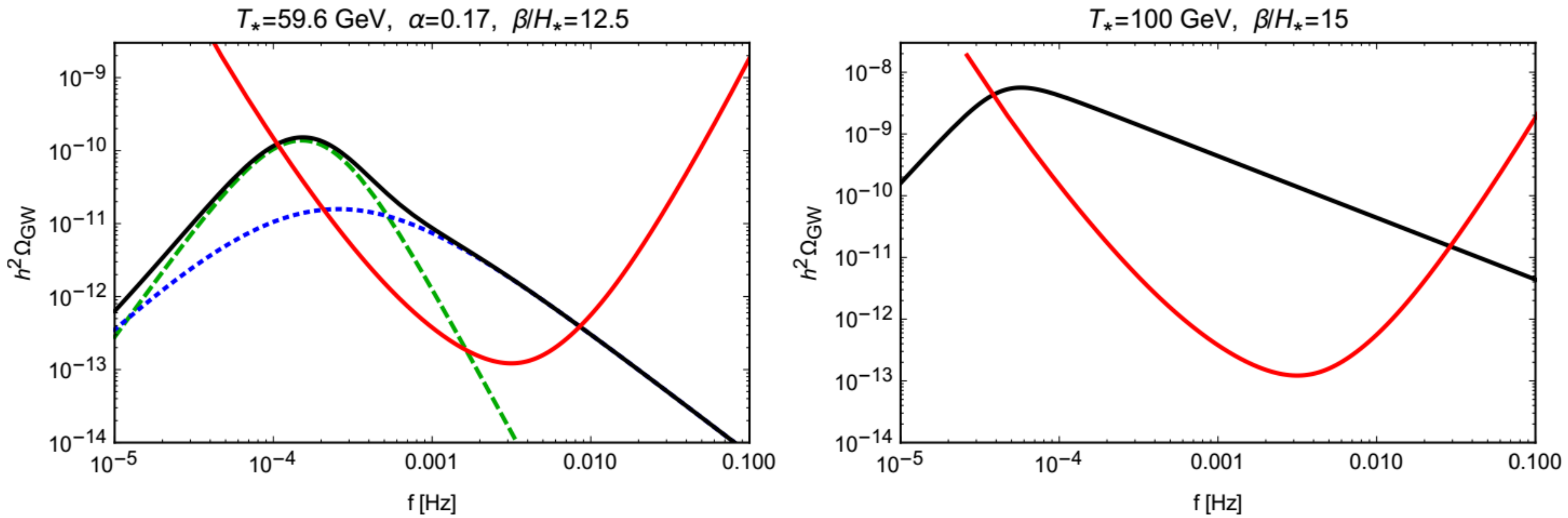
We fit data with this model for only **one free parameter**, the **log amplitude** $A_{\rm fg}$. The other parameters are set to be fixed at the values corresponding to 4 years of LISA observation.

SGWB model template

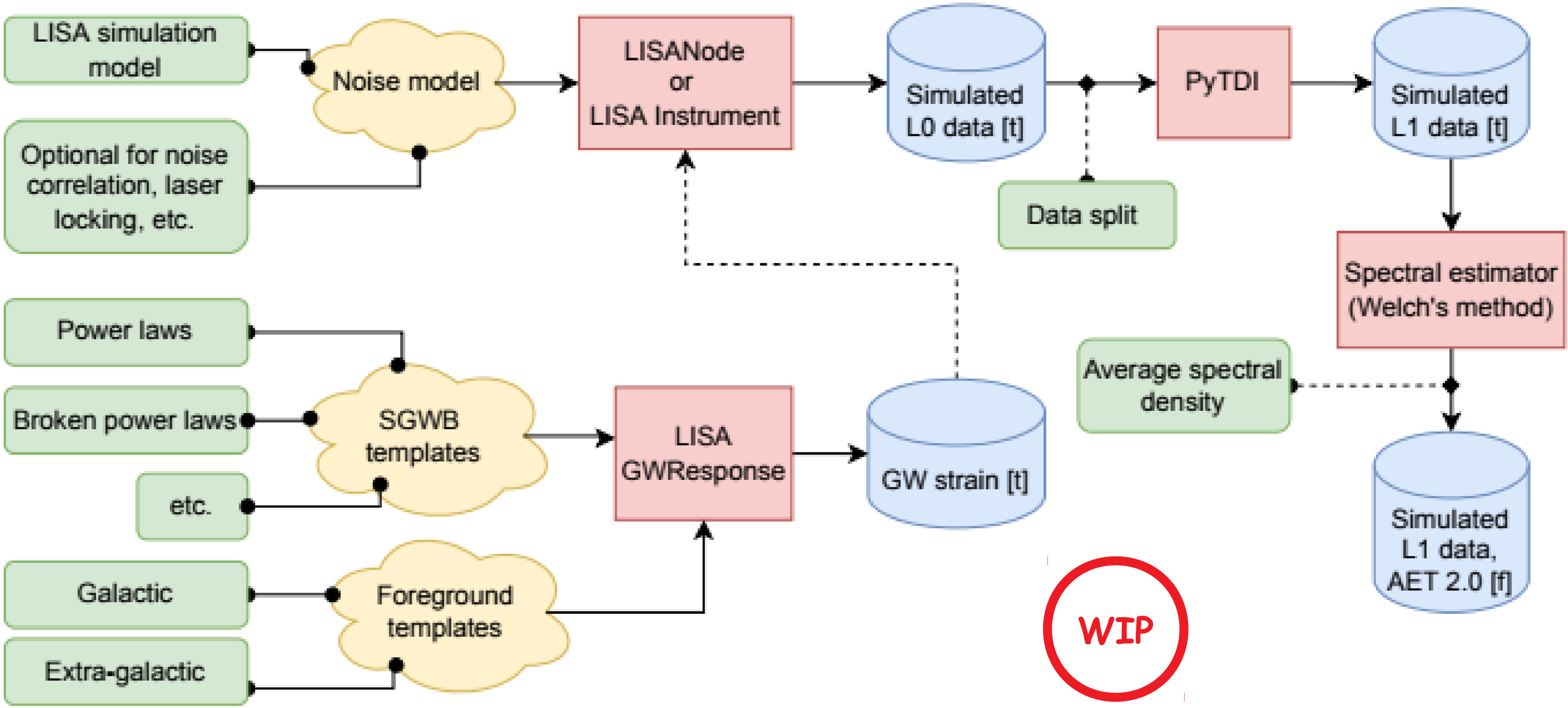
- Double power law:

$$h^2\Omega_{\rm gw}(f) = 10^{A_1}\left(\frac{f}{f_p}\right)^{n_1} + 10^{A_2}\left(\frac{f}{f_p}\right)^{n_2}, \qquad (33)$$

where $A_1 = -15.5$, $n_1 = -4$, $A_2 = -13$, $n_2 = 3$ and $f_p = 3.2 \times 10^{-3}$. This template is the reverse shape of the SGWB modelled from the **first order electroweak phase transition**.



Data generation (time-series)



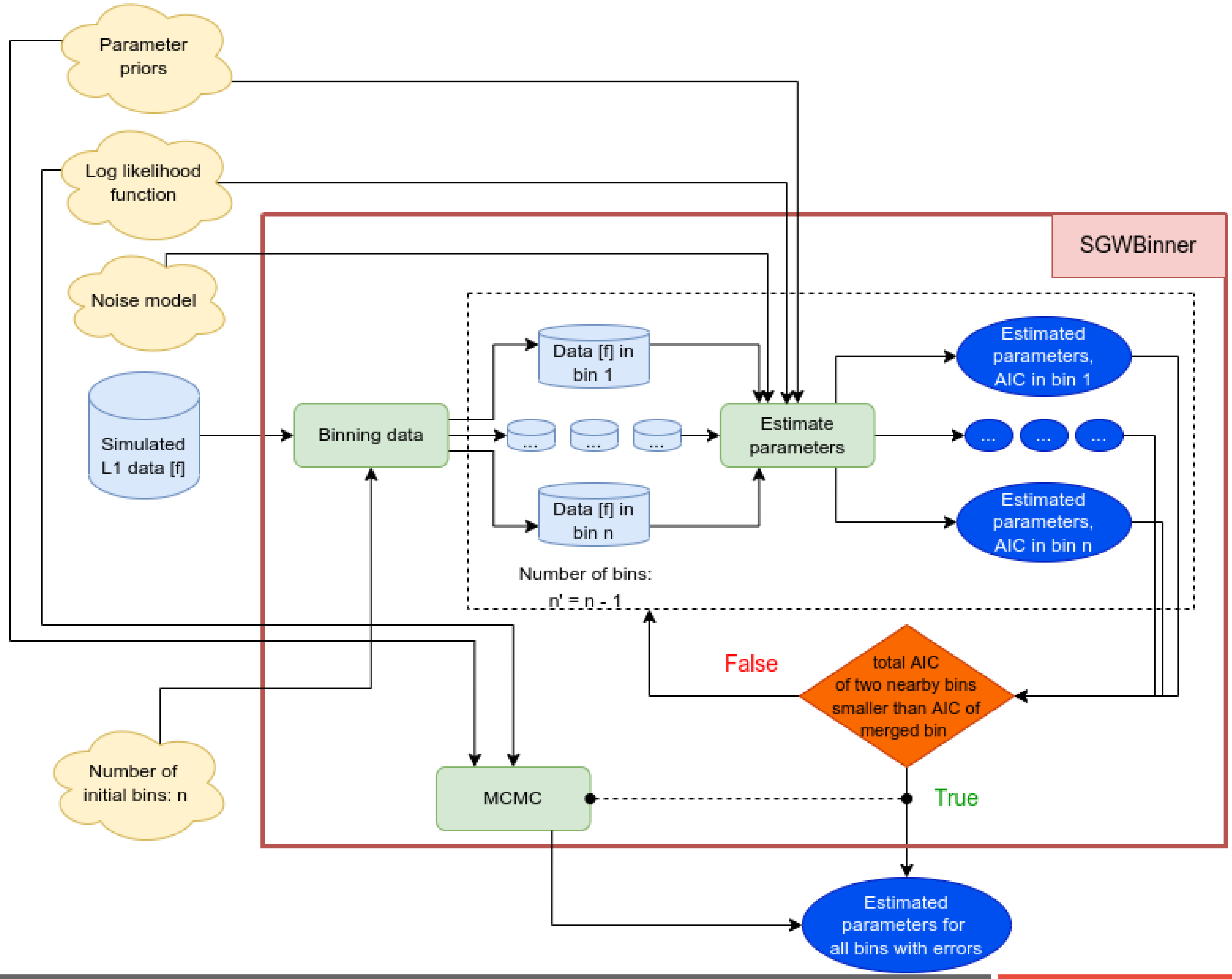
SGWB data analysis: SGWBinner and DA pipeline

- use **SGWBinner** for SGWB data analysis [2]
  - **2 or 3 noise parameters** model and could be different from the model used in data generation
  - search **SGWB signal** for each bin (smaller equal log-frequency intervals) as a **single power law**, assuming they are **independent** for every bins → SGWB template-free (blind) reconstruction
  - **noise and foreground** are **fit globally** in the whole frequency band
    - **TT** channel → constraint **prior for noise** parameters (**not well for TM accel noise**)
    - **AA/EE** channel → **estimate signal and noise parameters** using noise prior from TT
  - parameter estimation based on **Bayesian inference**
  - some nearby bins are merged to avoid over-fitting if **Akaike Information Criterion** [3] is fulfilled
  - after finish merging bins, estimate the uncertainty of the reconstruction
  - **MCMC** could be applied on total posterior for all data merged bins (the last step above) → more accurate estimation of the uncertainty of the reconstruction

[2] Flauger, R., Karnesis, N., Nardini, G., Pieroni, M., Ricciardone, A., & Torrado, J. (2021). Improved reconstruction of a stochastic gravitational wave background with LISA. Journal of Cosmology and Astroparticle Physics, 2021(01), 059.

[3] Akaike, H. (1974). A new look at the statistical model identification. IEEE transactions on automatic control, 19(6), 716-723.

SGWB data analysis: SGWBinner and DA pipeline



SGWB data analysis: SGWBinner and DA pipeline

- Bayesian data analysis based on theorem:

$$P(\vec{\theta}, \vec{n}|D) = \frac{\pi_S(\vec{\theta})\pi_N(\vec{n})\mathcal{L}\left(D|\vec{\theta}, \vec{n}\right)}{p(D)} \tag{47}$$

$$\Rightarrow \ln P(\vec{\theta}, \vec{n}) = \ln \pi_S(\vec{\theta}) + \ln \pi_N(\vec{n}) + \ln \mathcal{L}(\vec{\theta}, \vec{n}) \tag{48}$$

- Signal prior attributes to the total log posterior in uniform distribution:

$$\ln \pi_S(\vec{\theta}) = \sum_i \frac{1}{(\mathrm{max}_i - \mathrm{min}_i)}, \tag{49}$$

- Noise priors are normal (Gaussian) distributed

$$\ln \pi_N(\vec{n}) = -\sum_i \frac{1}{2}\left[\ln(2\pi\sigma_i^2) + \left(\frac{n_i - \mu_i}{\sigma_i}\right)^2\right] \quad ; \quad \Sigma = \vec{\sigma^2} = \mathrm{diag}(0.2\vec{\mu})^2 \tag{50}$$

- The likelihood function is built by combining a Gaussian likelihood and a log-normal one:

$$\ln \mathcal{L} = \frac{1}{3} \ln \mathcal{L}_G + \frac{2}{3} \ln \mathcal{L}_{LN}, \tag{51}$$

$$\ln \mathcal{L}_G \left(D | \vec{\theta}, \vec{n} \right) = -\frac{N_c}{2} \sum_{i,j} \sum_k n_{ij}^{(k)} \left[\frac{\mathcal{D}_{ij}^{theory}(f_{ij}^k, \vec{\theta}, \vec{n}) - \mathcal{D}_{ij}^{(k)}}{\mathcal{D}_{ij}^{theory}(f_{ij}^k, \vec{\theta}, \vec{n})} \right]^2, \tag{52}$$

$$\ln \mathcal{L}_{LN} \left(D | \vec{\theta}, \vec{n} \right) = -\frac{N_c}{2} \sum_{i,j} \sum_k n_{ij}^{(k)} \ln^2 \left[\frac{\mathcal{D}_{ij}^{theory}(f_{ij}^k, \vec{\theta}, \vec{n})}{\mathcal{D}_{ij}^{(k)}(f_{ij}^k, \vec{\theta}, \vec{n})} \right]. \tag{53}$$

SGWB data analysis: SGWBinner and DA pipeline

- We compute the covariance matrix of estimated parameter from Fisher information matrix.

$$\mathcal{I}_{ij} \equiv -\,\partial_i \partial_j \ln P(\vec{\theta}, \vec{n})\Big|_{\vec{\theta}_b, \vec{n}_b} \tag{54}$$

$$C_{ij} = \mathcal{I}_{ij}^{-1}. \tag{55}$$

- In the case of SGWB signal attributes insignificantly in the data and noises are uncorrelated, that matrix is diagonal.

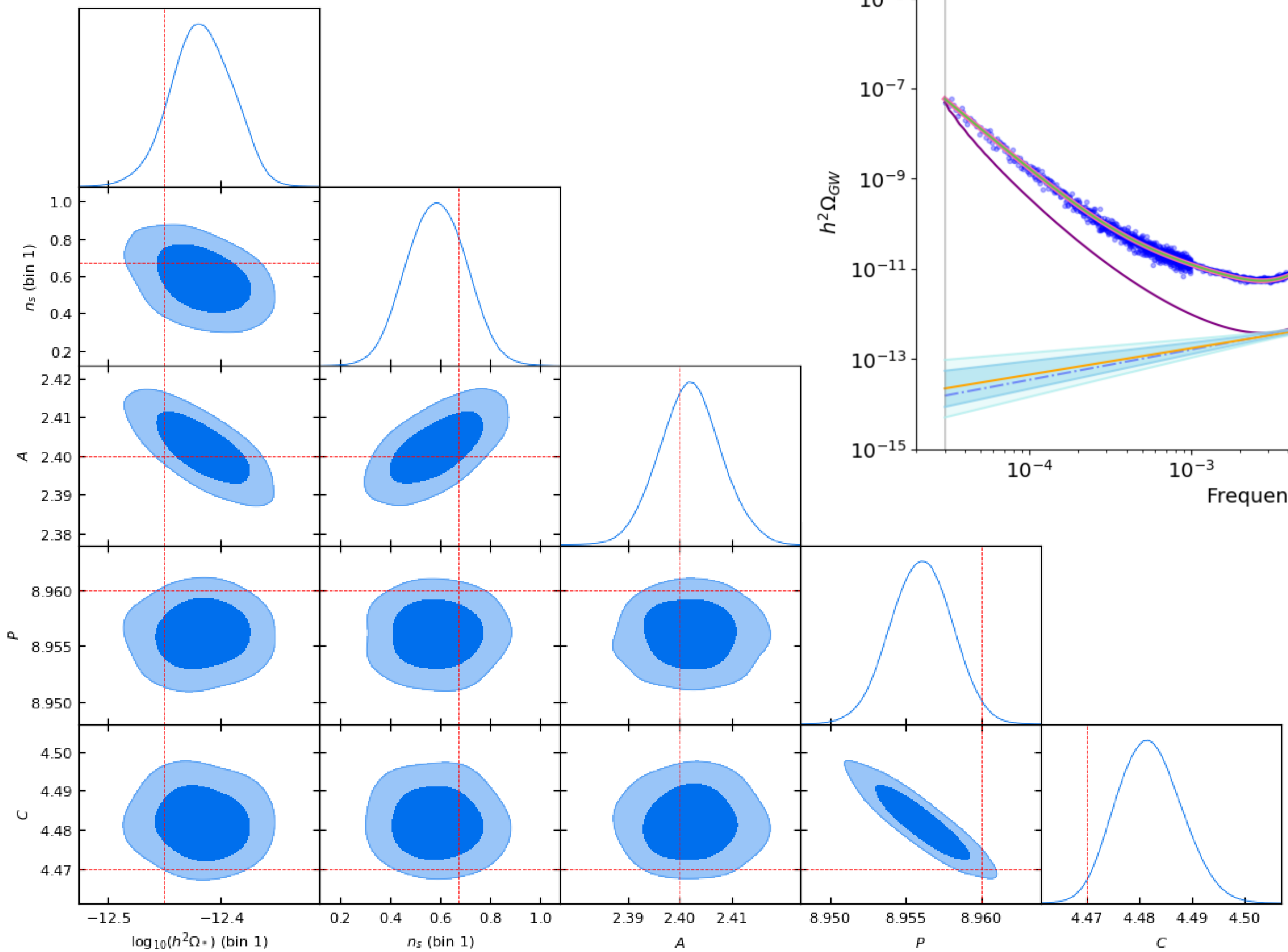
$$C_{ab} \approx \mathrm{diag}(\vec{\sigma^2}) \tag{56}$$

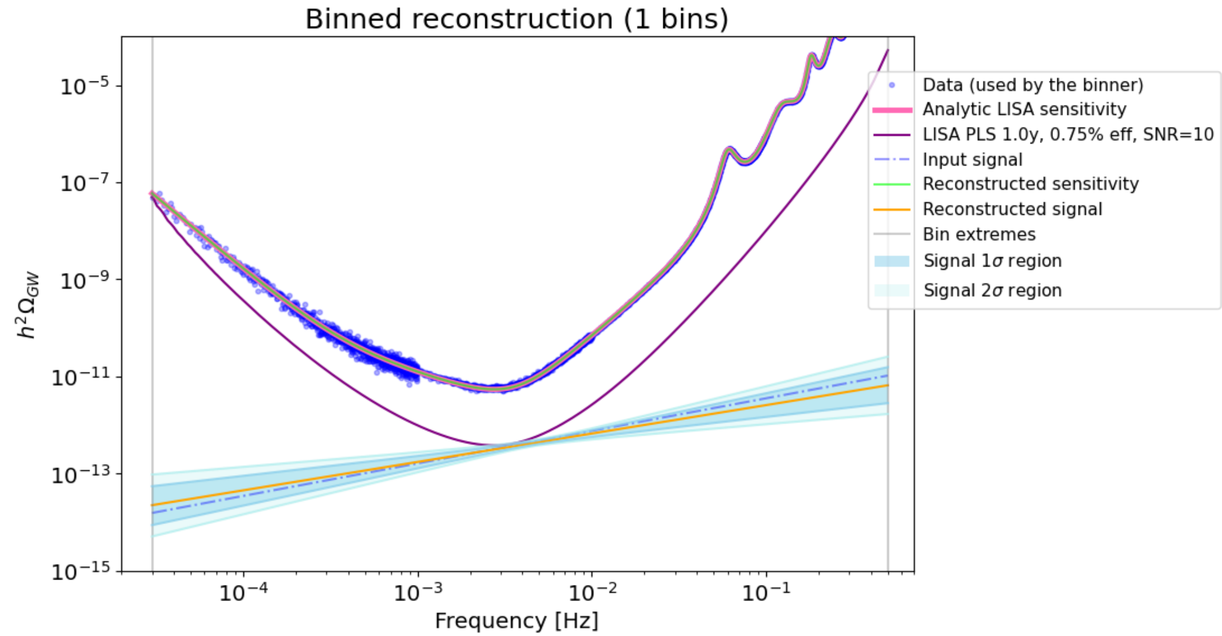
- AIC:

$$\mathrm{AIC} := 2K - 2 \ln \mathcal{L}|_{\vec{\theta}_b, \vec{n}_b} \tag{57}$$

SGWB data analysis: Result

- with the **correct noise model** in data analysis w.r.t. one used for data generation, for example using 3 noise parameters model for both, data includes signal from a **single power law SGWB** template [4]

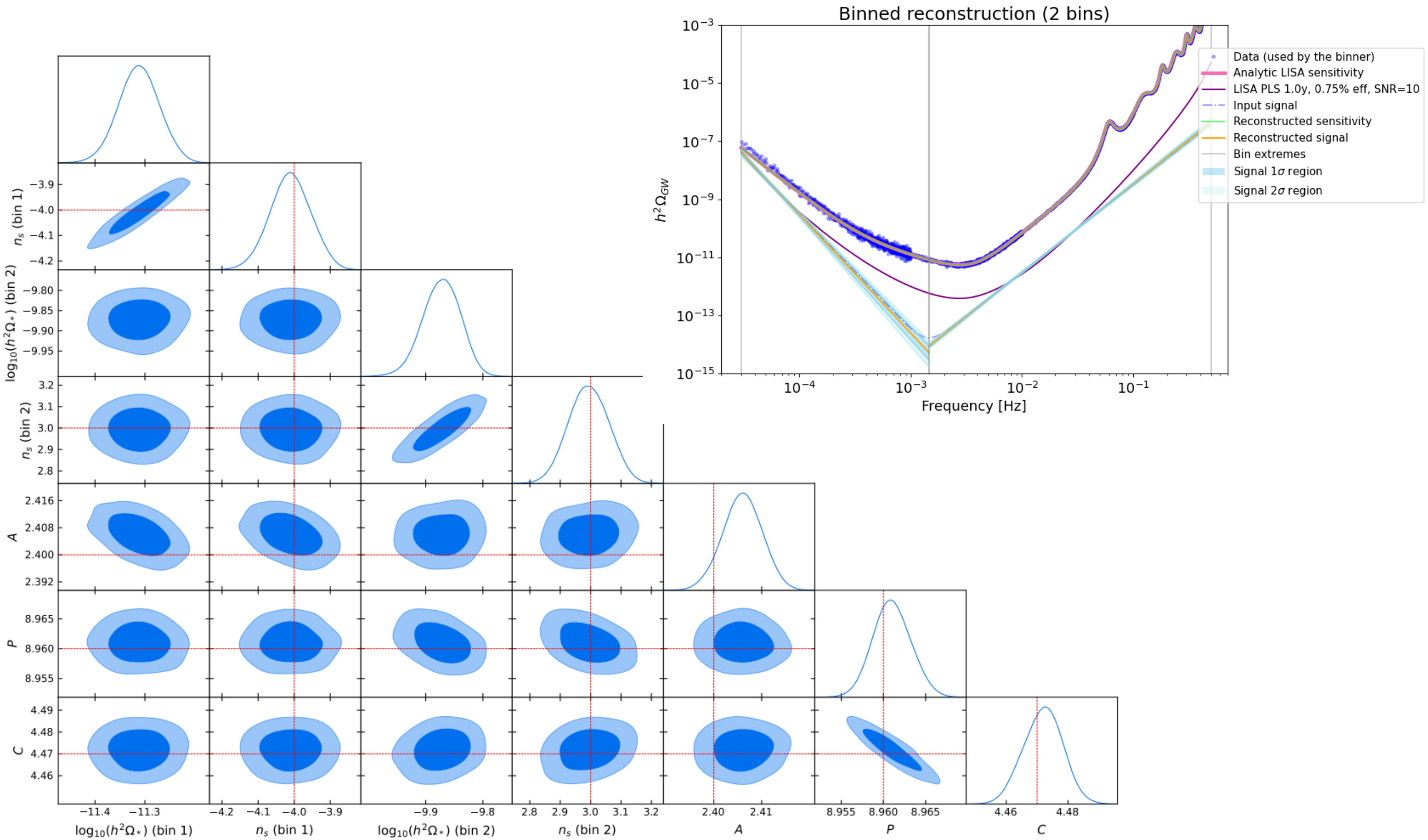




[4] This power law is the SGWB foreground from the stellar-origin blackhole binaries in LISA frequency band, predicted by using LIGO-Virgo-KAGRA estimates

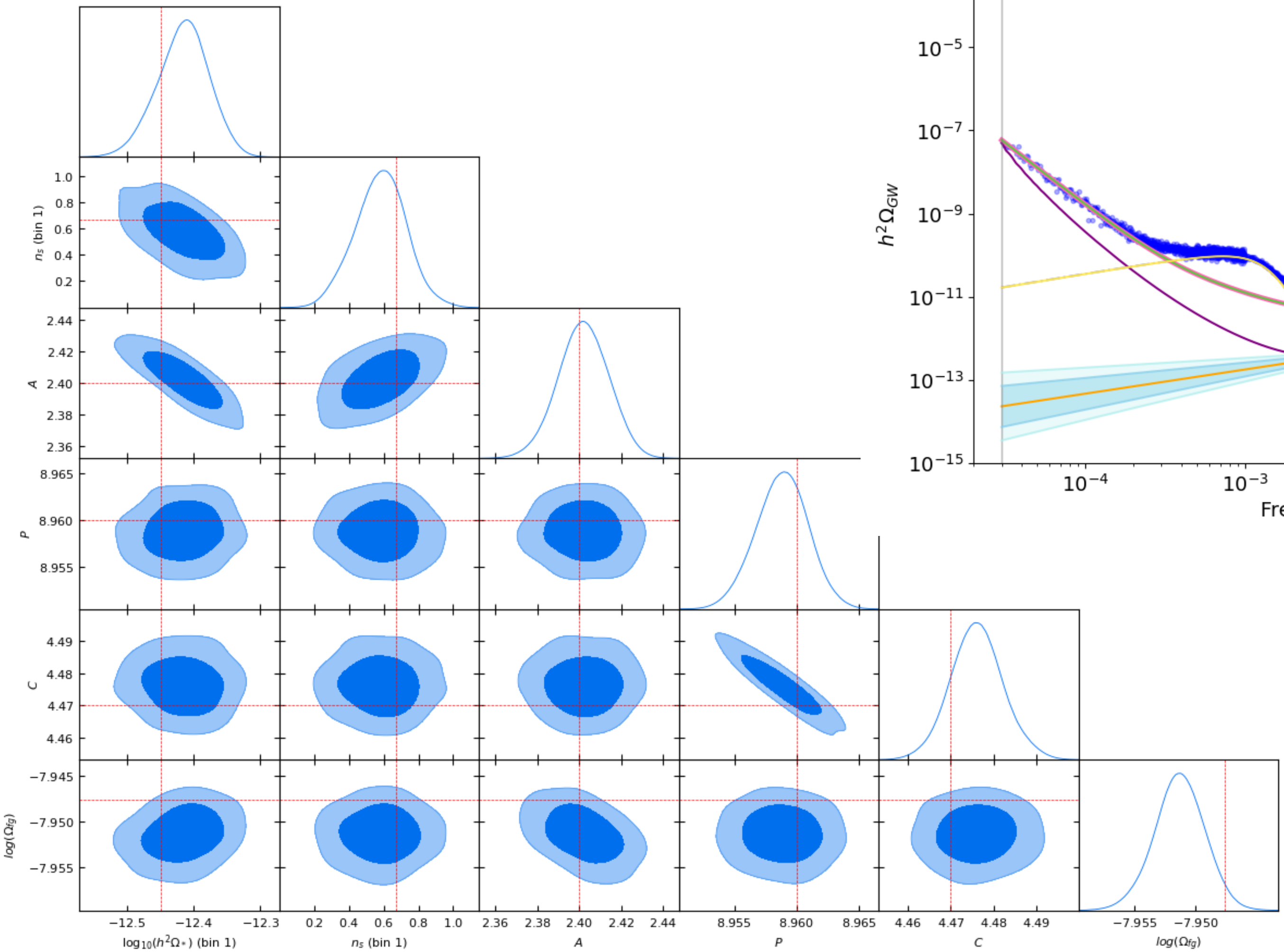
SGWB data analysis: Result

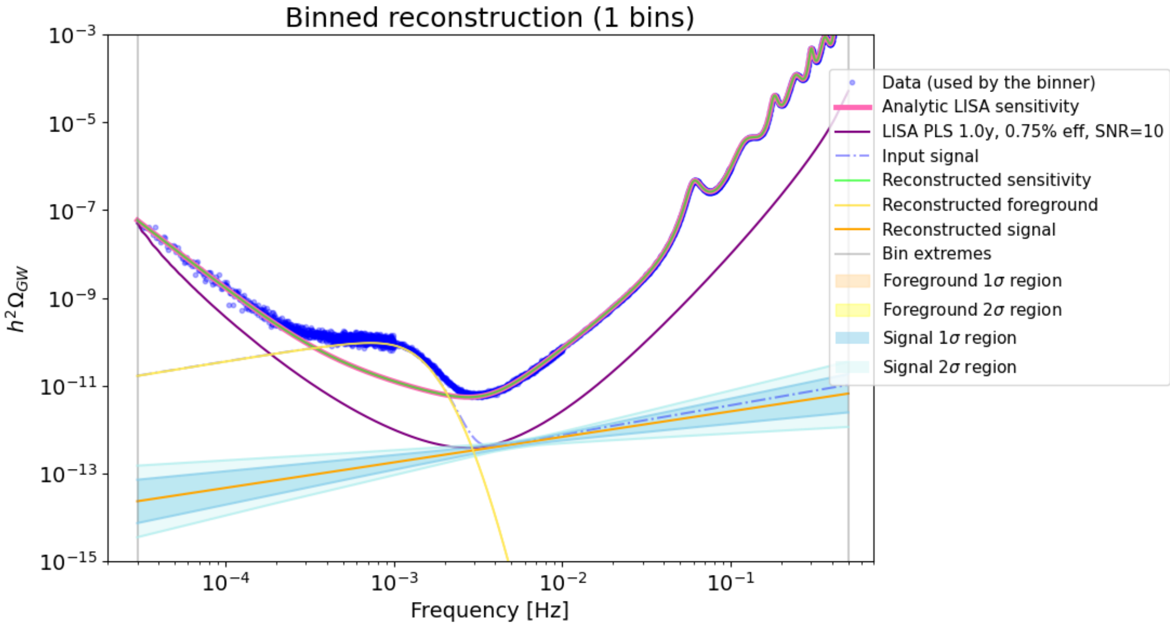
- with the **correct noise model** in data analysis w.r.t. one used for data generation, for example using 3 noise parameters model for both**, with double power laws SGWB template**



SGWB data analysis: Result

- with the **correct noise model** in data analysis w.r.t. one used for data generation (3 noise parameters model)**, with single power law SGWB** (foreground from SOBHs in LISA frequency band predicted by using LVK estimates) and **galactic foreground** (template from equation 14 in [5])

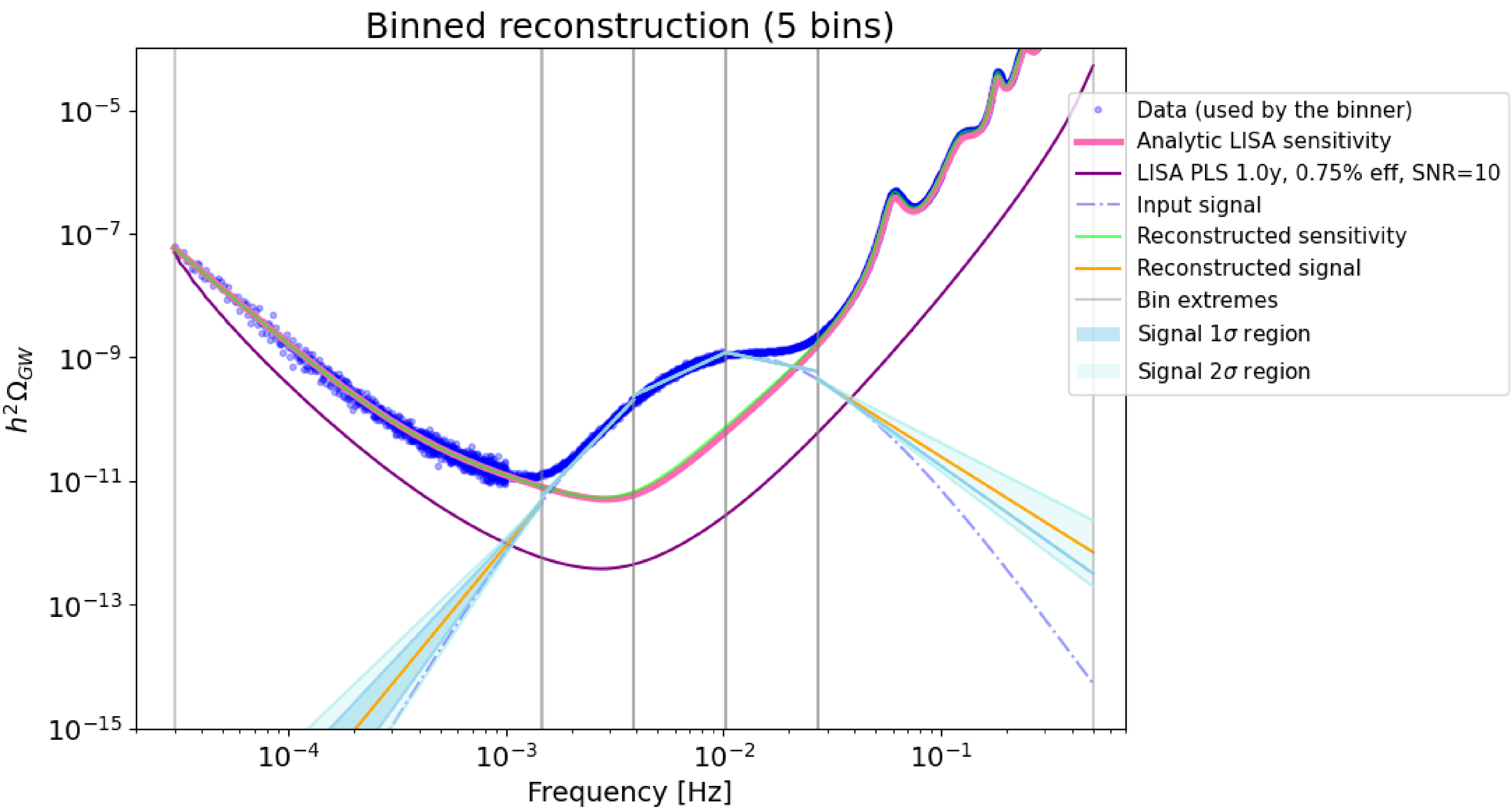




[5] Robson, T., Cornish, N. J., & Liu, C. (2019). The construction and use of LISA sensitivity curves. Classical and Quantum Gravity, 36(10), 105011.

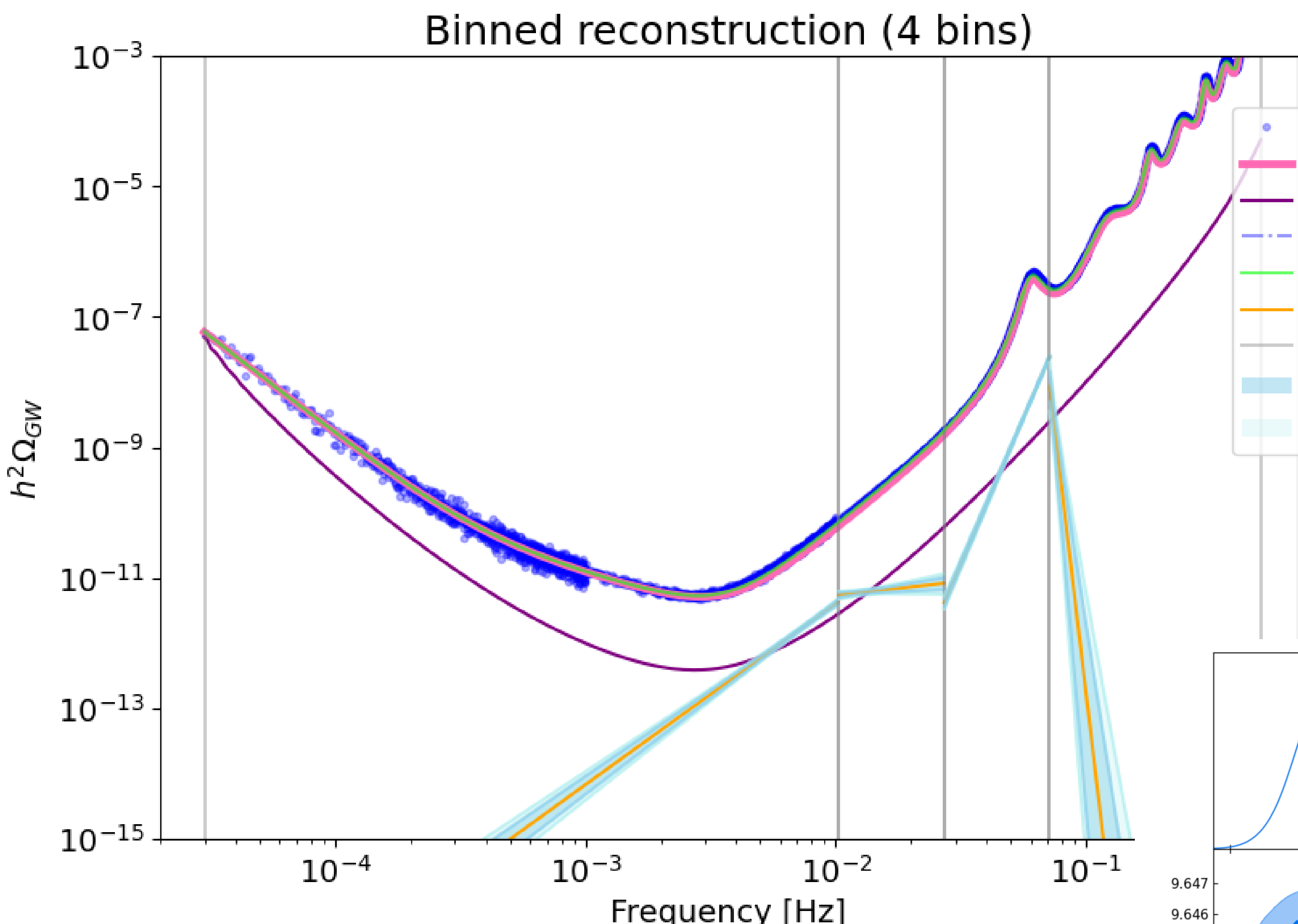
SGWB data analysis: Result

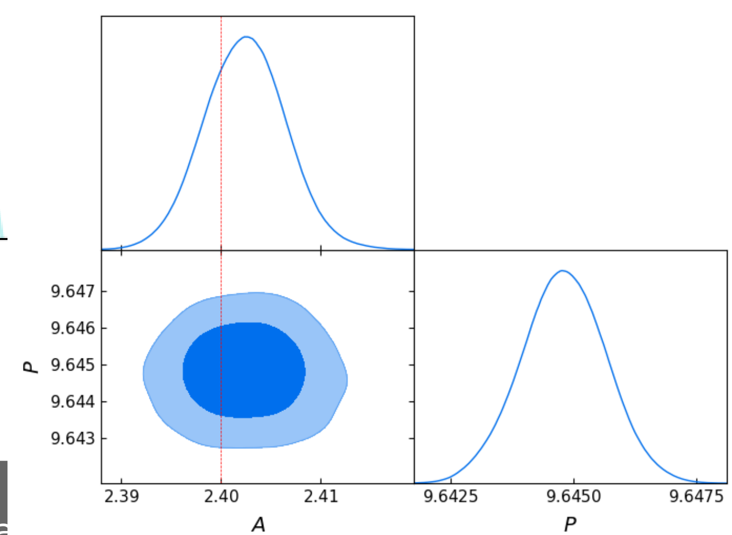
- with the **correct noise model** in data analysis w.r.t. one used for data generation (3 noise parameters model)**, with broken power law SGWB**



SGWB data analysis: Result

- for **noise-only data** with the **disagreed noise models** in data analysis (2 noise parameters) and in data generation (3 noise parameters)





SGWB data analysis: Result

- Using time-series data generated by LISANode, we get some **singularities** in the data (when converting to strain sensitivity), perhaps the spectral estimation leads some errors
- Exclusive those points by re-weighting the data point did not work, the analysis on noise-only data gives unexpected signal