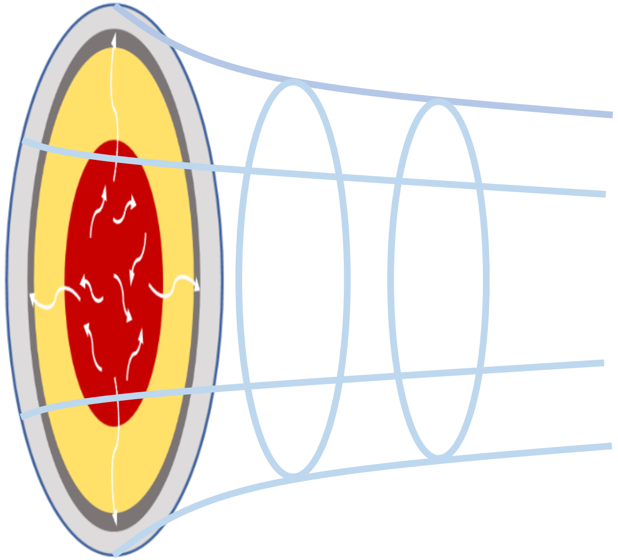
Neutrino Transport in Holography



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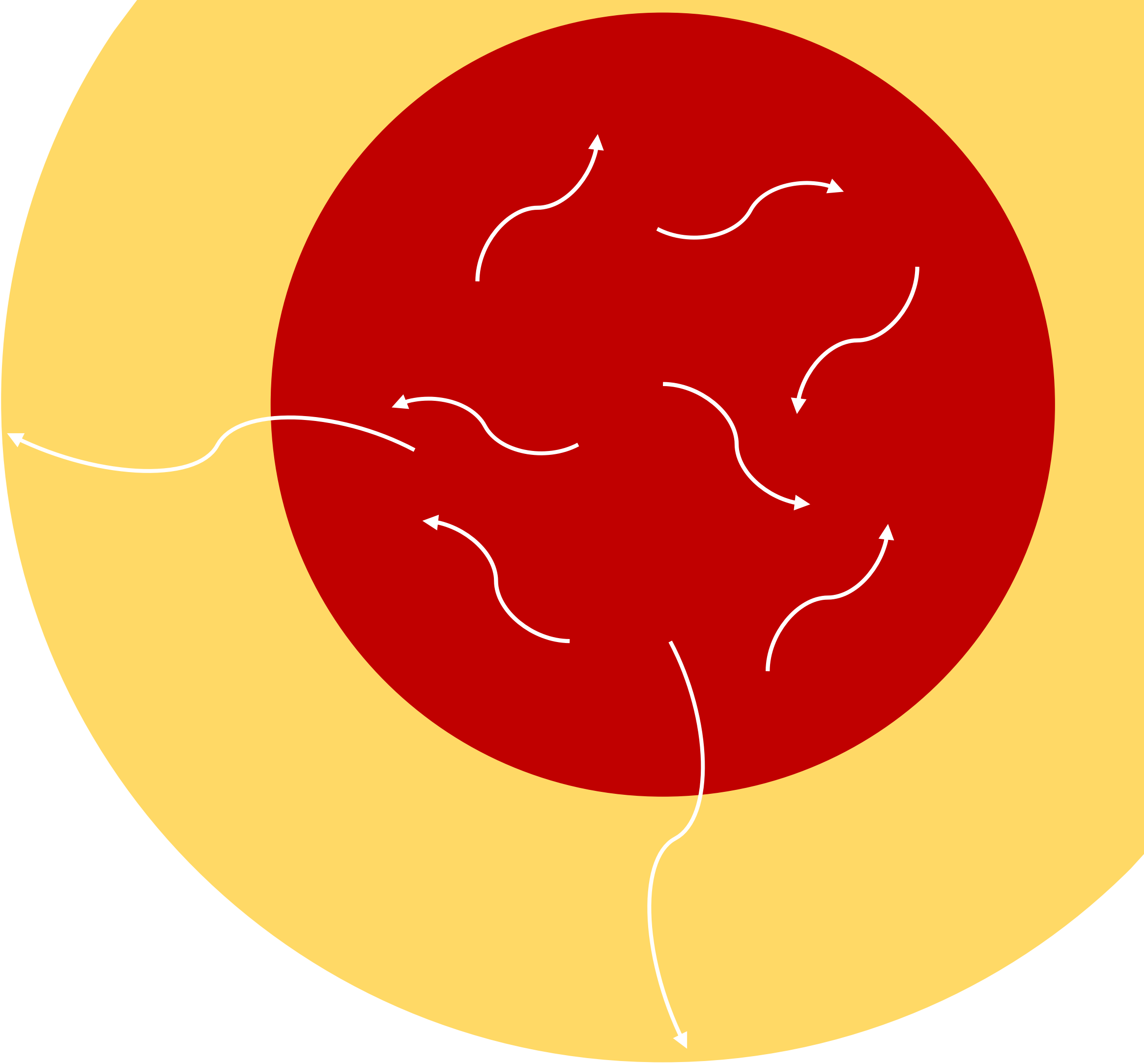
arXiv:2306.00192

Motivation

- **Neutrino (ν)** radiation is the main mechanism for **Neutron Star (NS) cooling**

- Requires the knowledge of ν interaction with **dense QCD matter** in the core :

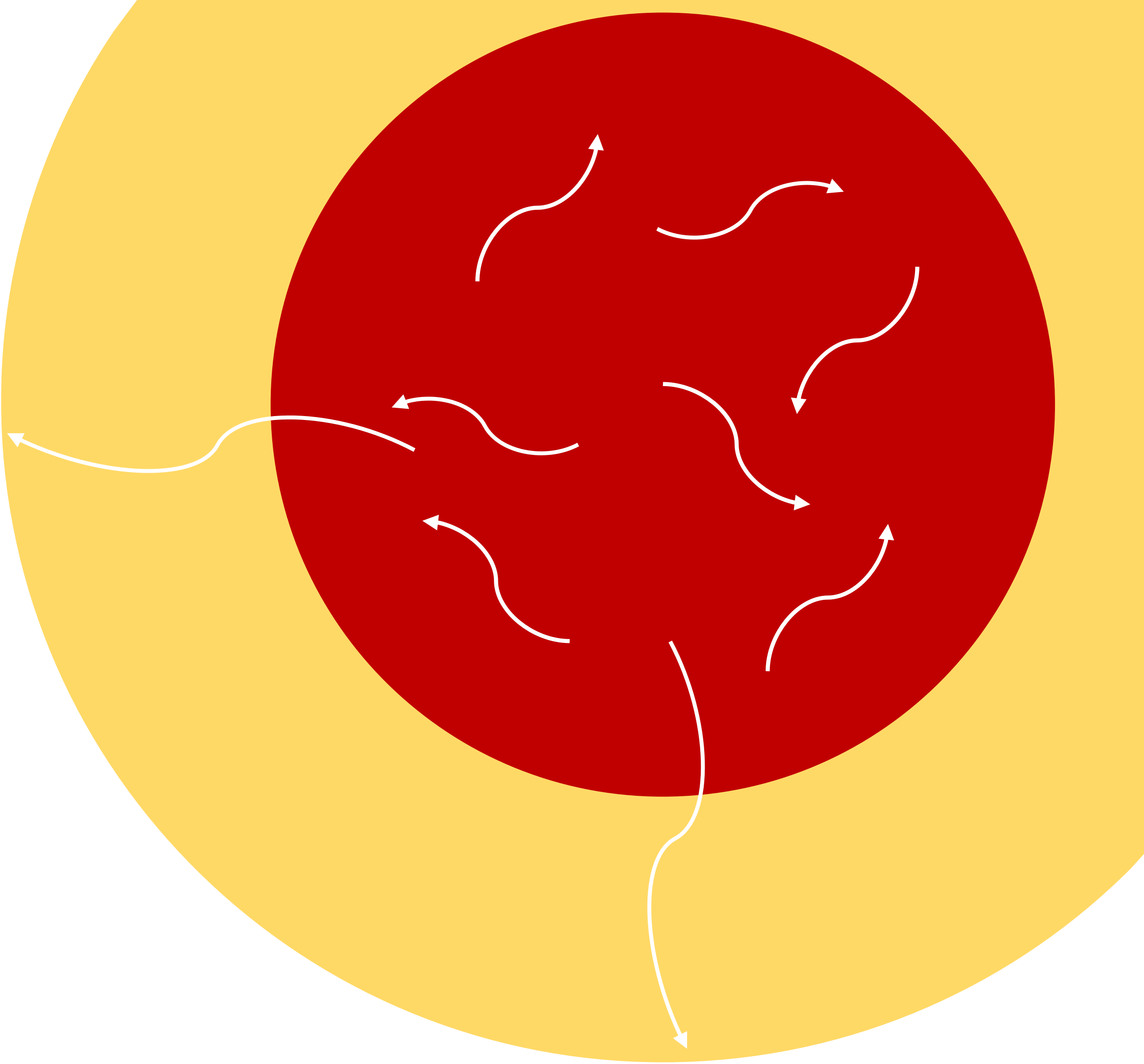
$$j \ \& \ \lambda \leftrightarrow \langle J_{L/R} J_{L/R} \rangle^R$$



Motivation

- Computing $\langle J_{L/R} J_{L/R} \rangle^R$ in the **dense strongly-coupled** QCD matter is a difficult problem
- We consider the **holographic** approach

Problem : compute $\langle J_{L/R} J_{L/R} \rangle^R$ in **holographic QCD** at finite T and n_B
→ This work : **simplest** toy model (quark matter in $\mathcal{N} = 4$ SYM)



Formalism for neutrino transport

The transport of neutrinos is described by the **Boltzmann equation** obeyed by the **ν distribution function $f_\nu(\vec{x}, t; k_\nu)$**

$$(k_\nu \cdot \partial) f_\nu \equiv j(E_\nu)(1 - f_\nu) - \frac{1}{\lambda(E_\nu)} f_\nu \, .$$

Emissivity

Mean Free Path

We focused on **charged current interactions** : $\nu_e + d \leftrightarrow e^- + u$

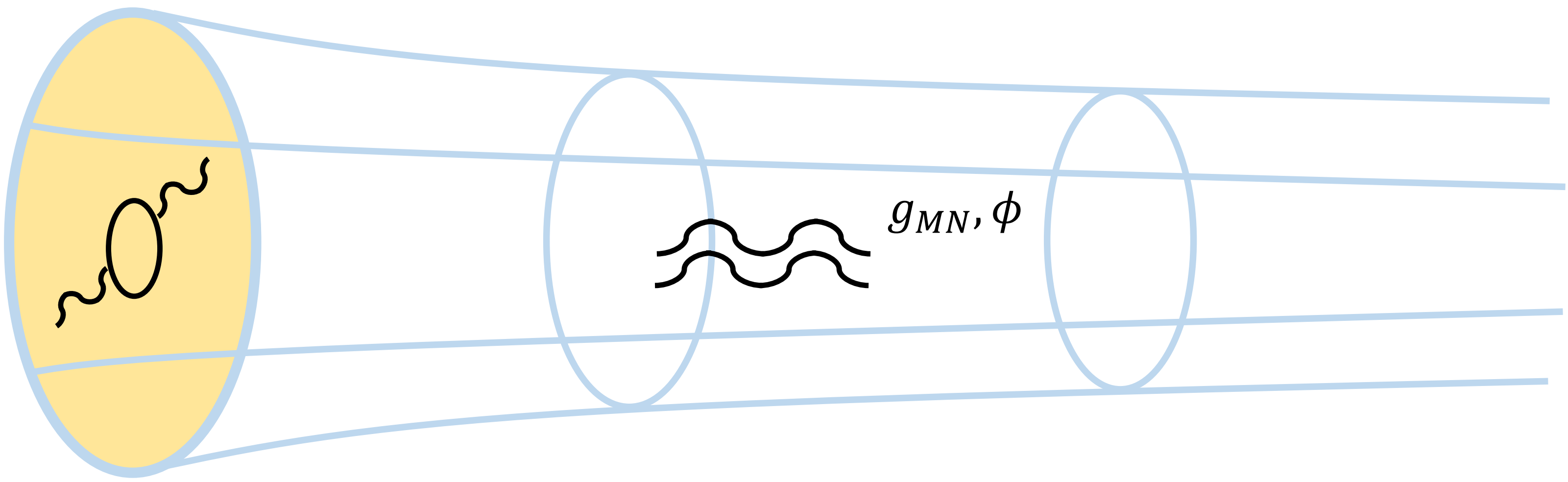
At order $\mathcal{O}(G_F^2)$ in the electro-weak interaction

$$j_c(E_\nu) = G_F^2 \int \frac{\mathrm{d}\vec{\mathrm{k}}_e^3}{(2\pi)^3} \underbrace{(\mathrm{kins})^{\lambda\,\sigma}}_{\vec{k}_e, \vec{k}_\nu} \times \underbrace{(\mathrm{stats})}_{f_e, f_W} \times \underbrace{\mathrm{Im}(i\langle J_\lambda^- J_\sigma^+ \rangle^R)}_{\text{Dense QCD}} \, ,$$

Dense QCD $\sim \langle J_\lambda^L J_\sigma^L \rangle^R$

The holographic set-up

The holographic correspondence



Strongly-coupled quantum field theory in 4D ⟷ **Weakly**-curved classical gravitational theory in 5D

The **boundary** of the 5D space (**bulk**) is the **4D space-time** on which the quantum theory is defined

The Holographic Dictionary

Every **quantum operator** has a **dual field** in the bulk of same quantum numbers

$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}
O	$\leftrightarrow$	φ
G : $\partial_\mu J^\mu = 0$	$\leftrightarrow$	G : A^M

The Holographic Set-up

Simplest bottom-up holographic toy model with **chiral currents $J^{\mu}_{L/R}$**

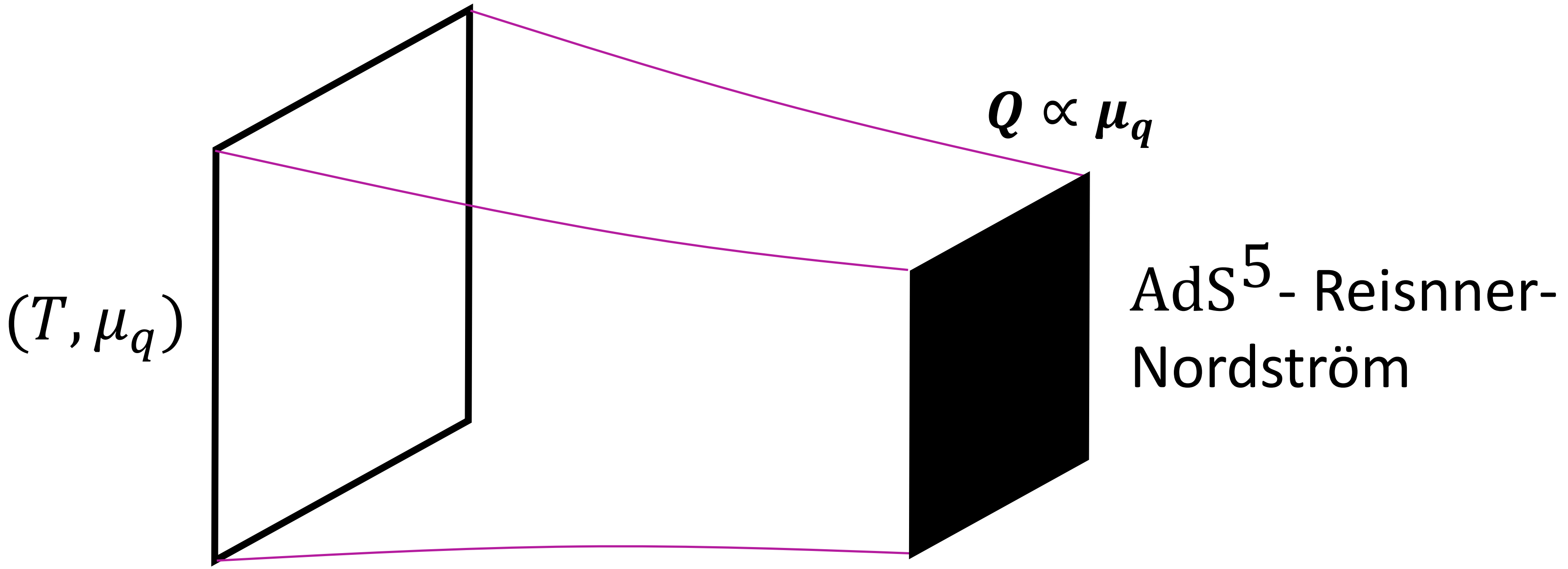
$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}
$U(N_f)_L \times U(N_f)_R : \partial_\mu J^{\mu}_{L/R} = 0$	$\leftrightarrow$	$U(N_f)_L \times U(N_f)_R : A^{M}_{L/R}$

$$S = M_{Pl}^3 \int dx^5 \sqrt{-g}\left(R + \frac{12}{\ell^2} - \kappa\, \mathrm{Tr}\left\{F^{(L)}_{MN} F^{MN}_{(L)} + F^{(R)}_{MN} F^{MN}_{(R)}\right\}\right),$$

Background solution

We want to compute $\langle J_\lambda^- J_\sigma^+ \rangle^R$ in an equilibrium state at **finite (T, μ_q)** = dense strongly-coupled **quark matter**

→ Charged AdS **black hole**, with charge $Q \propto \mu_q$



Summary of parameters

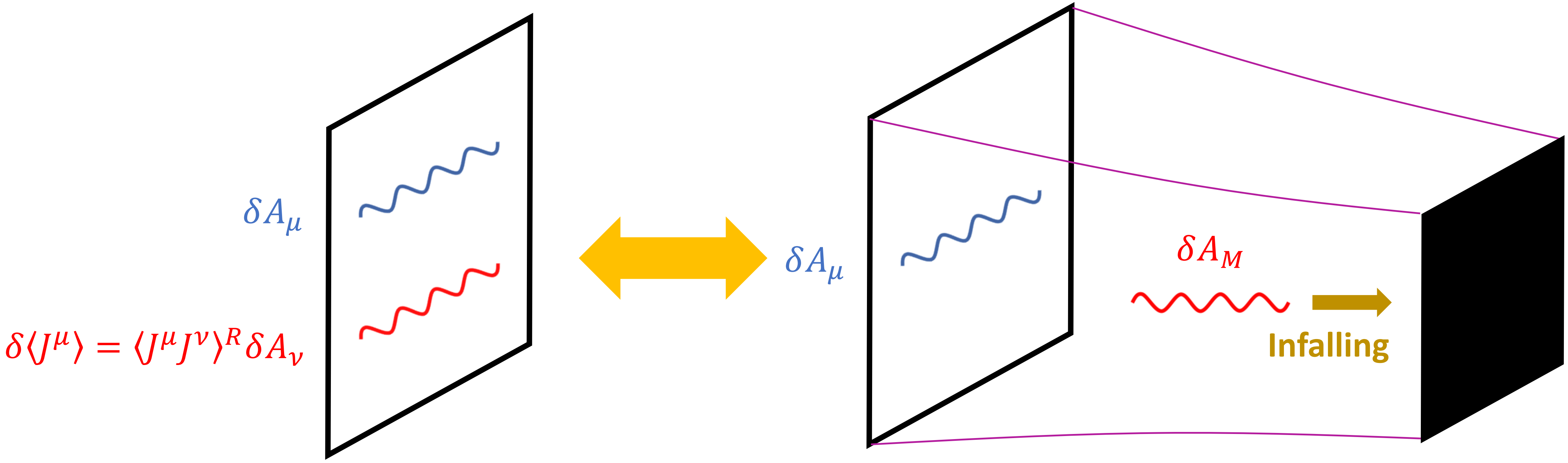
Parameters of the **model**	$M_{Pl}\ell$	Fitted to free quark-gluon thermodynamics
	κ	
Environmental parameters	$\frac{\mu_q}{T}$	Varied
Neutrino properties	$\frac{E_\nu}{T}$	Varied

Holographic calculation of the chiral current 2-point function

Perturbations of AdS-RN

[Son & Starinets '02]

[Skenderis & van Rees '08]



Linear response theory

Perturbations of AdS-RN

Hydrodynamic approximation

Hydrodynamics describes the **long-range** dynamics of the system :

→ **Expansion** in $(\omega/T, k/T)$, with **transport coefficients**

$$\langle J_\lambda^- J_\sigma^+ \rangle^R \left(\omega, \vec{k}\right) = \boldsymbol{\sigma} \left(P_{\lambda\sigma}^{\perp}\, \omega + P_{\lambda\sigma}^{\parallel} \frac{\omega^2 - k^2}{\omega + i\boldsymbol{D} k^2} \right) \left(1 + \mathcal{O}\left(\frac{\omega}{T}, \frac{k^2}{T^2} \right) \right),$$

Conductivity

$\partial_t J^0 = D \Delta J^0$

AdS-RN : the **hydro** approximation remains valid at $\boldsymbol{T \ll \omega, k \ll \mu_q}$

→ $\boldsymbol{\nu}$ **transport** in a NS: $\boldsymbol{E_\nu, \mu_e, \mu_\nu \ll \mu_q}$

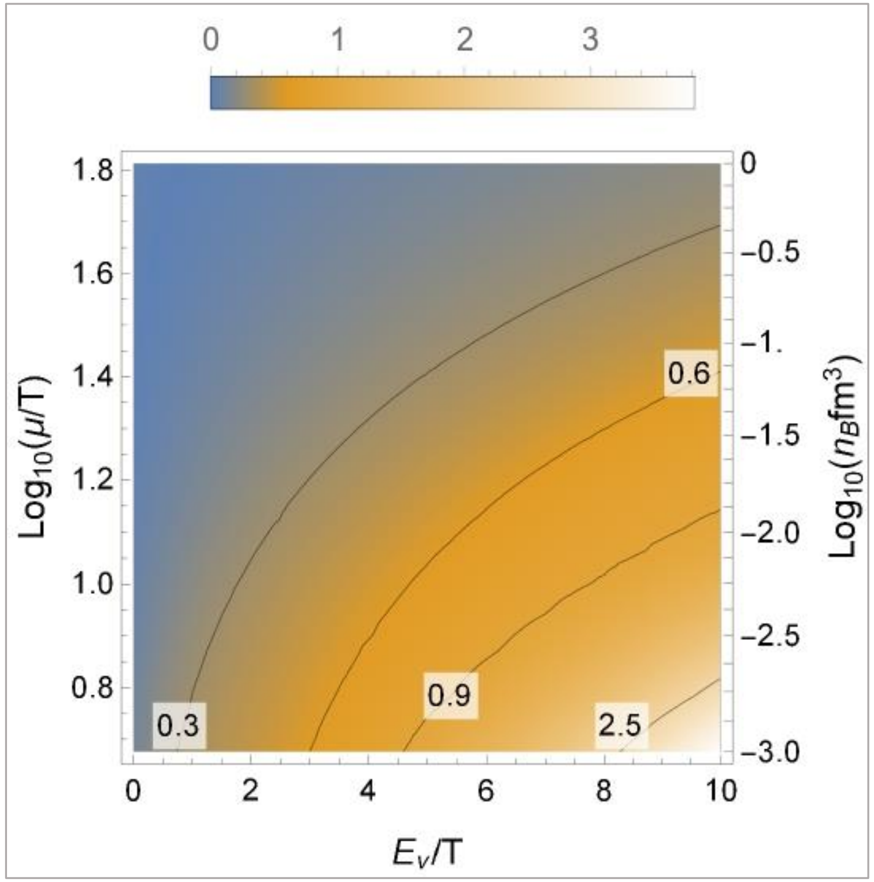
[Davison & Parnachev ‘13]
[Moitra, Sake & Trivedi ‘21]

At $\boldsymbol{\mu_q \gg T}$, we have $\mu_e, \mu_\nu \simeq 0.7\, \mu_q$

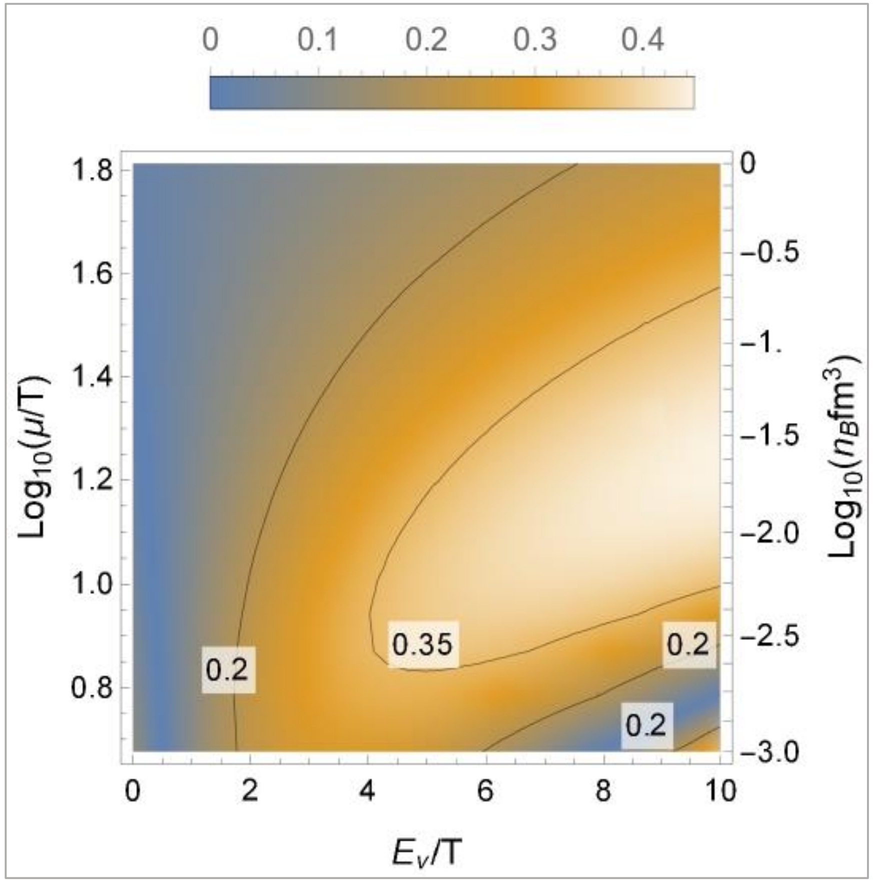
Numerical results

Opacities : comparison with hydro

$T = 10$ MeV



ν

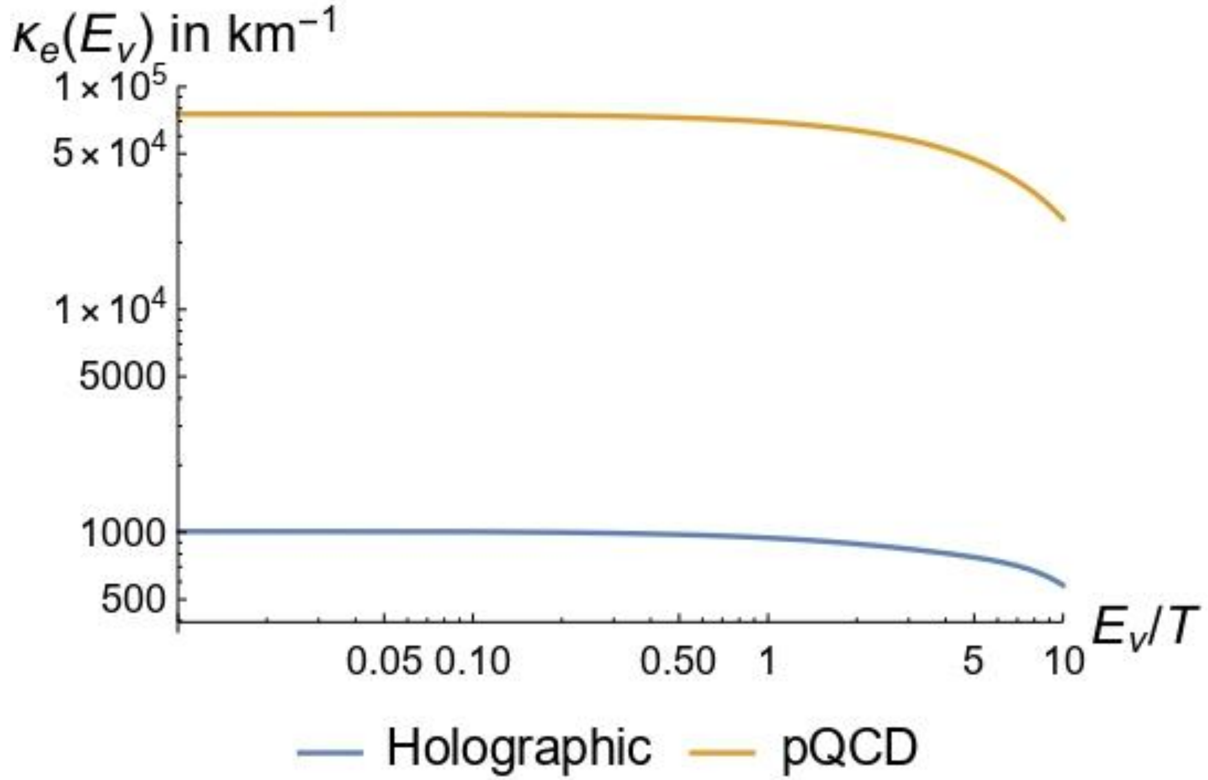


$\bar{\nu}$

$$\kappa(E_\nu) = j(E_\nu) + \frac{1}{\lambda(E_\nu)}$$

Comparison with weak coupling

[Iwamoto ‘82]



$$T = 10\ \text{MeV}, \quad n_B = 0.11\ \text{fm}^{-3}$$

Summary and outlook

First step towards the description of holographic **neutrino transport** : toy model of **strongly-coupled quark matter**

- **Hydrodynamic** behavior
- **Opacity suppressed** compared with the weak coupling result
- More work is needed to **corroborate** these results

Several directions of improvement :

- **Neutrino rates** from **neutral current** interactions
- Transport in an **isospin asymmetric** medium
- **More realistic model** of holographic QCD

Thank you !

Appendix

A. Details about the formalism for neutrino transport

Formalism for neutrino transport

Exercice : compute the **exact propagator $G_\nu(\vec{x}_1, t_1; \vec{x}_2, t_2)$** of ν's in a **dense QCD medium**

Quasi-particle approximation :

G_ν is described by the **ν distribution function $f_\nu(\vec{x}, t; k_\nu)$**

The transport of neutrinos is described by the **Boltzmann equation** obeyed by f_ν

$$(k_\nu \cdot \partial) f_\nu \equiv j(E_\nu)(1 - f_\nu) - \frac{1}{\lambda(E_\nu)} f_\nu \,.$$

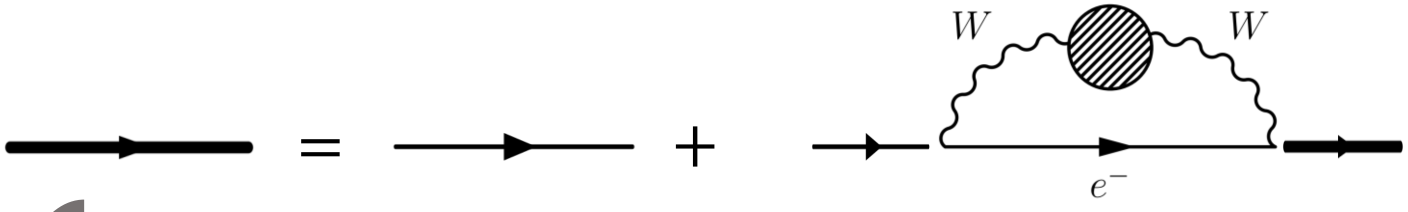
Emissivity

Mean Free Path

Schwinger-Dyson equation

The kinetic equation can be derived from the finite temperature **Schwinger-Dyson equation, at order $\mathcal{O}(G_F^2)$** in the electro-weak interaction

$\boldsymbol{\nu + n \leftrightarrow e^- + p}$



+ *Neutral current*

Dirac equation

$$j(E_\nu) = G_F^2 \int \frac{\mathrm{d}\vec{k}_e^3}{(2\pi)^3} \underbrace{(\text{kins})^{\lambda\sigma}}_{\vec{k}_e,\, \vec{k}_\nu} \times \underbrace{(\text{stats})}_{f_e,\, f_W} \times \mathrm{Im}(i\langle J_\lambda^- J_\sigma^+\rangle^R),$$

Dense QCD

$\sim \langle J_\lambda^L J_\sigma^L \rangle^R$

B. Large N

The Holographic Set-up

Simplest bottom-up holographic toy model with **chiral currents $J^{\mu}_{L/R}$**

$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}
$U(N_f)_L \times U(N_f)_R : \partial_\mu J^\mu_{L/R} = 0$	$\leftrightarrow$	$U(N_f)_L \times U(N_f)_R : A^M_{L/R}$

$N_c \to \infty \, , \frac{N_f}{N_c}$ finite

$$S = M_{Pl}^3 N_c^2 \int dx^5 \sqrt{-g} \left(R + \frac{12}{\ell^2} - \frac{\kappa}{N_c} \mathbf{Tr} \left\{ F^{(L)}_{MN} F^{MN}_{(L)} + F^{(R)}_{MN} F^{MN}_{(R)} \right\} \right) ,$$

C. Details about the perturbations of AdS-RN

Perturbations of AdS-RN

[Son & Starinets '02]
[Skenderis & van Rees '08]

$\langle J_\lambda J_\sigma \rangle^R$ is obtained by considering **perturbations** of the fields on top of **AdS-RN**

$$A^M_{L/R} \to \bar{A}^M_{L/R} + \delta A^M_{L/R}\,, \qquad g_{MN} \to \bar{g}_{MN} + \delta g_{MN}\,,$$

$$\forall \boldsymbol{\varphi},\ \delta\varphi = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{ik.x} C_k(z)\delta\varphi_0(k)\,, \quad \text{At } z \sim z_H : \mathrm{C_k}(z) \sim (z_H - z)^{-\frac{ik^0 z_H}{4}}$$

Infalling boundary condition

- Only $\boldsymbol{\delta T_{MN}} \propto \delta A_B$ **couples to** $\boldsymbol{\delta g}$
- The **charged current** gauge fields **decouple** from δg

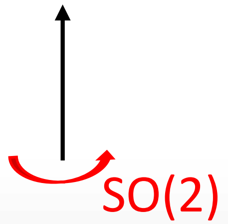
Perturbations : Symmetries

The boundary plasma has an **SO(3) rotational invariance**

$$\langle J_\lambda J_\sigma \rangle^R \left(\omega, \vec{k}\right) = P^\perp \left(\omega, \vec{k}\right)_{\lambda\sigma} i\boldsymbol{\Pi}^\perp(\boldsymbol{\omega}, \mathbf{k}) + P^\parallel \left(\omega, \vec{k}\right)_{\lambda\sigma} i\boldsymbol{\Pi}^\parallel(\boldsymbol{\omega}, \mathbf{k})$$

$\vec{k} = k\vec{e}_3$

For a given **mode $(\omega, \vec{k})$**, it reduces to an **SO(2) subgroup**



The perturbations are divided into **helicity sectors** that decouple

Helicity	Gauge field	Metric
$h = 0$	$\delta A_0 \,, \delta A_3$	$\delta g_0^0 \,, \delta g_0^3 \,, \delta g_3^3 \,, \delta g_1^1 + \delta g_2^2$
$h = 1$	$\delta A_{1,2}$	$\delta g_0^{1,2} \,, \delta g_3^{1,2}$
$h = 2$	$-$	$\delta g_2^1 \,, \delta g_1^1 - \delta g_2^2$

Sector decoupled from the metric

Consider δA_μ that decouples from $\delta g_{\mu\nu}$

The modes are organized in terms of the **gauge-invariants** under

$$\mathrm{U}(1) : \delta A \to \delta A + \mathrm{d}\delta\lambda$$

$\boldsymbol{h = 1}$	$\boldsymbol{h = 0}$
$\delta A_1\,, \delta A_2$	$E^{\parallel} \equiv \omega\delta A_3 + k\delta A_0$

The linearized **Maxwell equations** in each helicity sector can be written in terms of the gauge-invariants

The Π's are extracted from the **solutions near the boundary** $(z \to 0)$

$$\Pi^{\perp} \propto -\frac{\ell}{z}\frac{\partial_z \delta A_1}{\delta A_1}\bigg|_{z\to 0}\,, \qquad \Pi^{\parallel} \propto -\frac{\ell}{z}\frac{\partial_z \delta E^{\parallel}}{\delta E^{\parallel}}\bigg|_{z\to 0}\,.$$

Sector coupled to the metric

$\delta X \equiv \delta A_B$ couples to $\delta g_{\mu\nu}$

Again, organize the modes in terms of the **gauge-invariants** under :

- $U(1) : \delta X \to \delta X + \mathrm{d}\delta\lambda$
- **Diffeomorphisms** :

$$\delta X_M \to \delta X_M + \delta\xi^N \partial_N \bar{X}_M + \bar{X}_N \partial_M \delta\xi^N$$

$$\delta g_{MN} \to \delta g_{MN} + \nabla_M \delta\xi_N + \nabla_N \delta\xi_M$$

$\boldsymbol{h = 1}$	$\boldsymbol{h = 0}$
$\delta X_{1,2}$	$\delta S_1 \equiv \omega\delta X_3 + k\delta X_0 + a(z)\mu\, k(\delta g_1^1 + \delta g_2^2)$
$\delta Y^{1,2} \equiv k\delta g_0^{1,2} + \omega\delta g_3^{1,2}$	δS_2 $\equiv 2\omega k\delta g_0^3 + \omega^2\delta g_z^z - f(z)k^2\delta g_0^0 + b(z,\omega/k)k^2(\delta g_1^1 + \delta g_2^2)$

Sector coupled to the metric

The linearized **Einstein-Maxwell equations** in each helicity sector can be written in terms of the **gauge-invariants** :

- $\boldsymbol{h=1}$: 2 coupled 2nd order ODE's for $\delta X_{1,2}$ and $\delta Y^{1,2}$
- $\boldsymbol{h=0}$: 2 coupled 2nd order ODE's for δS_1 and δS_2

The Π's are extracted from the **solutions near the boundary** $(z \to 0)$

$$\boldsymbol{h=1:} \qquad \delta X_1 = \delta\hat{X}_1 + z^2 \delta\Pi_{X_1} + \cdots , \qquad \delta\Pi_{X_1} \equiv \boldsymbol{\Pi^{\perp}_{XX}} \delta\hat{X}_1 + \Pi^{\perp}_{XY} \delta\hat{Y}^1 ,$$

Compute **2 solutions** and invert the linear relation

$$\left(\boldsymbol{\Pi^{\perp}_{XX}} \; \Pi^{\perp}_{XY} \right) = \left(\delta\Pi^{(1)}_{X_1} \; \delta\Pi^{(2)}_{X_1} \right) \begin{pmatrix} \delta\hat{X}^{(1)}_1 & \delta\hat{X}^{(2)}_1 \\ \delta\hat{Y}^1_{(1)} & \delta\hat{Y}^1_{(2)} \end{pmatrix}^{-1}$$

Hydrodynamic approximation

The **long-range** behavior of a system **near equilibrium** is described by **hydrodynamics**

→ Equilibrium **correlators** follow a **universal** long-range structure :

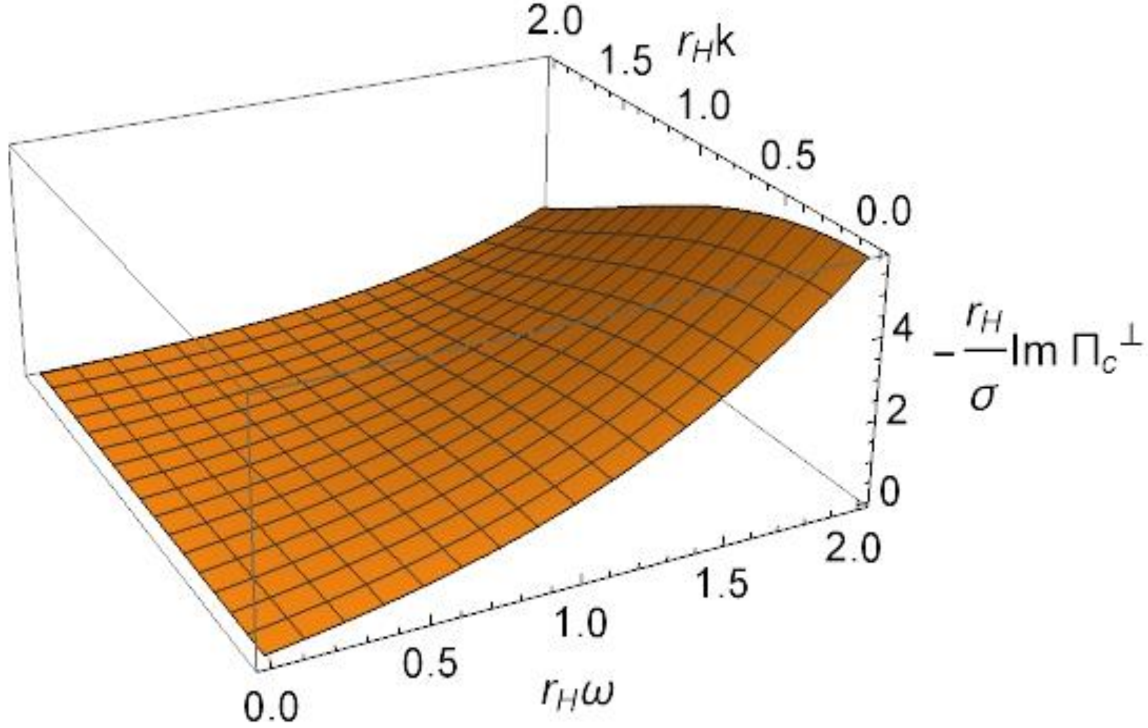
- **Expansion** in $(\omega/T, k/T)$, with **transport coefficients**
- The **hydro modes** appear as **poles** at leading order

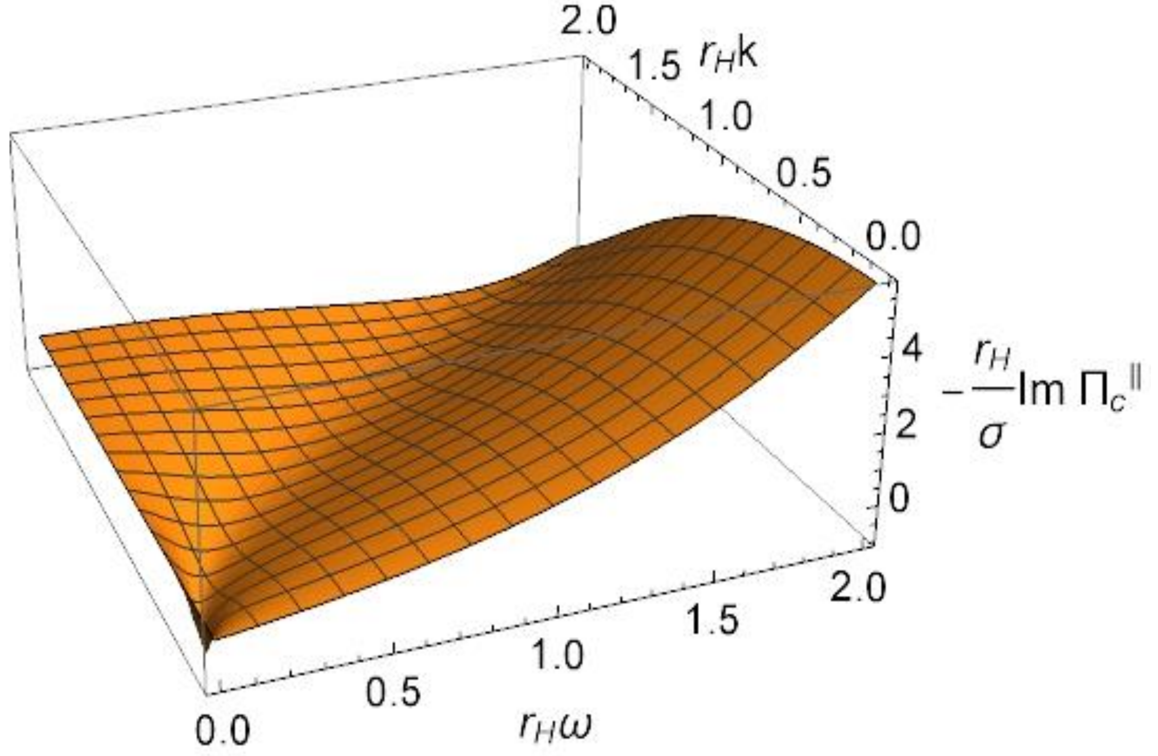
$$\langle J_\lambda J_\sigma \rangle^R \left(\omega, \vec{k}\right) = \boldsymbol{\sigma} \left(P^{\perp}_{\lambda\sigma}\, \omega + P^{\parallel}_{\lambda\sigma} \frac{\omega^2 - k^2}{\omega + i\boldsymbol{D}k^2} \right) \left(1 + \mathcal{O}\left(\frac{\omega}{T}, \frac{k^2}{T^2} \right) \right),$$

Conductivity

$\partial_t J^0 = D \Delta J^0$

Charged current correlators





$$\frac{\mu_q}{T} \simeq 65$$