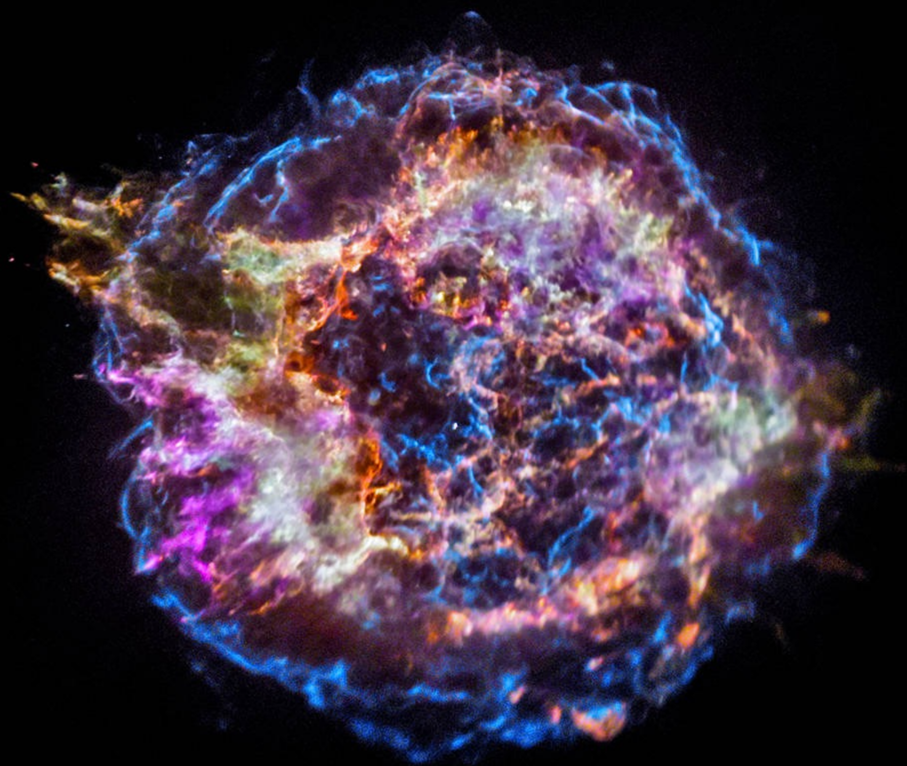
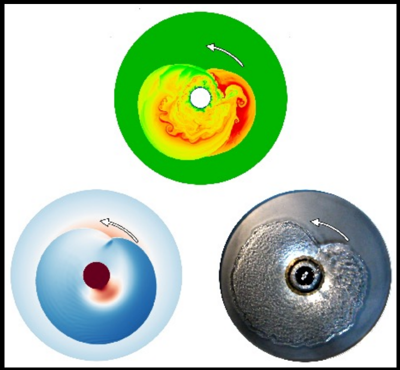
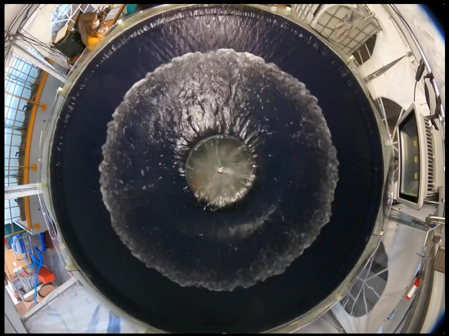


The impact of stellar rotation (and turbulence) on the gravitational wave signal from supernovae

Thierry Foglizzo

CEA Saclay





G. Durand, J. Guilet, M. Bugli, M. González,
A-C Buellet, F. Masset, L. Walk, I. Tamborra,

Outline

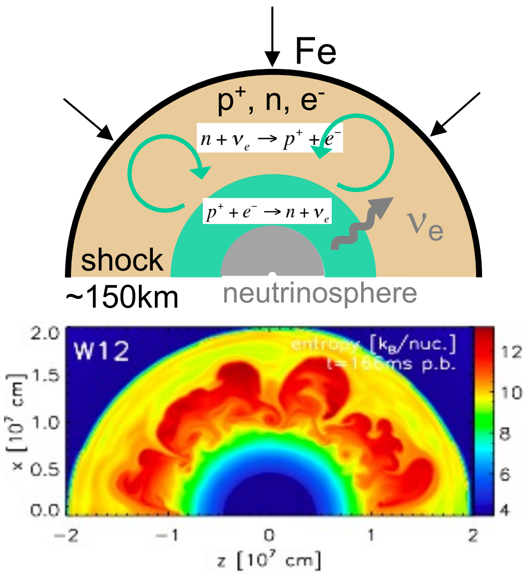
I. Bounce, low T/W, spiral SASI, ν-driven convection → see talk by Matteo Bugli

II. SASI puzzles

III. A new adiabatic framework for SASI: mechanism with rotation clarified

IV. Viscous stabilisation: unexpected impact of turbulence

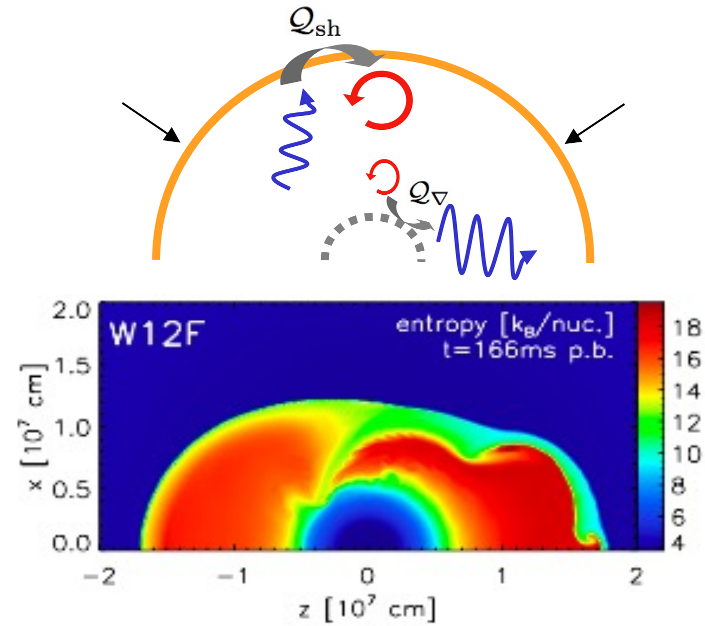
Instabilities during the phase of stalled accretion shock



Neutrino-driven convection

(Herant+92)

- entropy gradient
- angular scale l=5,6



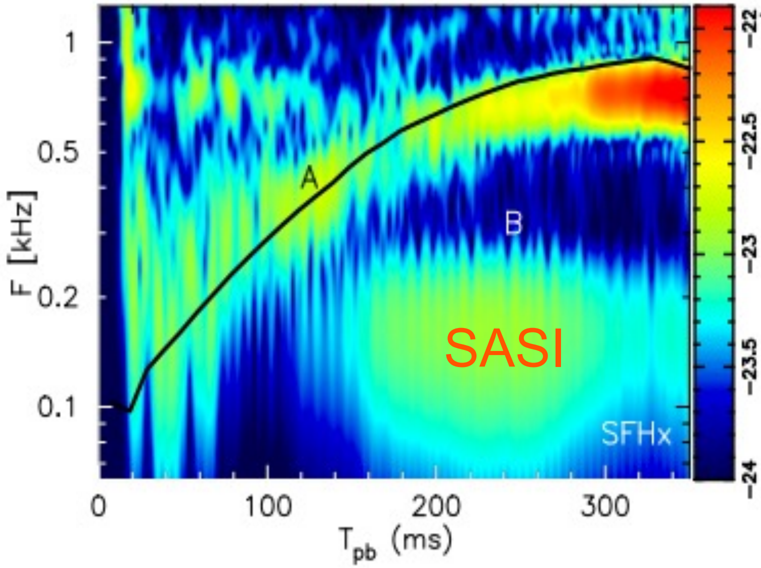
SASI: Standing Accretion Shock Instability

(Blondin+03)

- advective-acoustic cycle
- oscillatory, large angular scale l=1,2

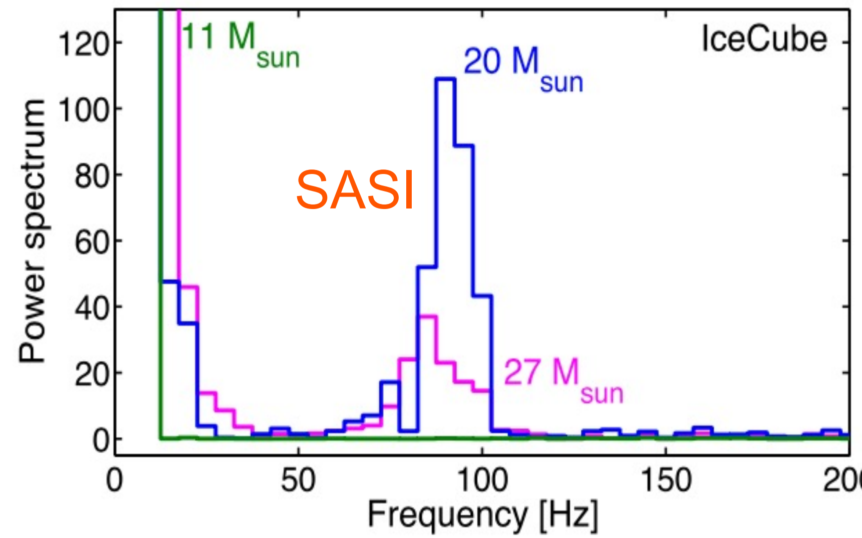
SASI oscillations can leave a direct imprint on the gravitational wave and neutrino signals: reverse engineering?

Gravitational Waves



Kuroda+16

Oscillations of the neutrino flux

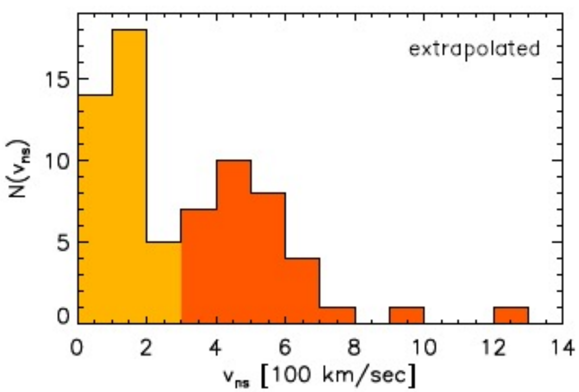


Tamborra+13

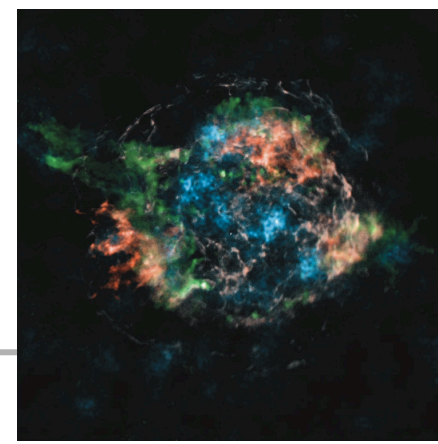
Indirect information can also be learnt from

-the kick & spin of the compact object

-the chemical composition of the remnant



Scheck+06



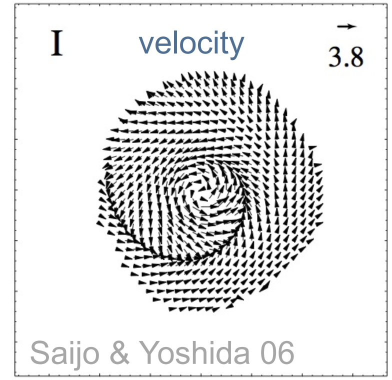
Grefenstette+14

Additional instabilities induced by moderate rotation

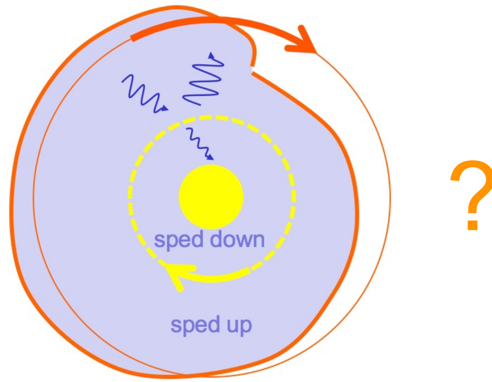
-low T/|W| instability?

(Shibata+02, Watts+05, Passamonti & Andersson 15, Takiwaki+21, Bugli+23)

- corotation radius
- vorticity gradient? mid-latitude Rossby waves?



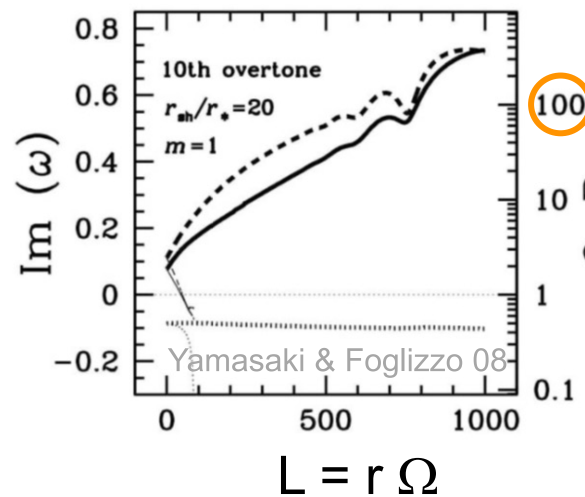
Saijo & Yoshida 06



-spiral mode of SASI?

(Blondin & Mezzacappa 07, Yamasaki & Foglizzo 08, Walk+23)

- rotation-enhanced advective-acoustic cycle?
- why such as strong impact of rotation on the prograde mode?



Yamasaki & Foglizzo 08

stellar parameters:
progenitor mass,
compactness,
angular momentum,
inhomogeneities

\+

puzzling dynamics:
SASI
ν-driven convection
low T/|W|
PNS dynamo

\+

uncertain physics:
reaction rates,
EOS,
neutrino interactions,
magnetic fields

\+

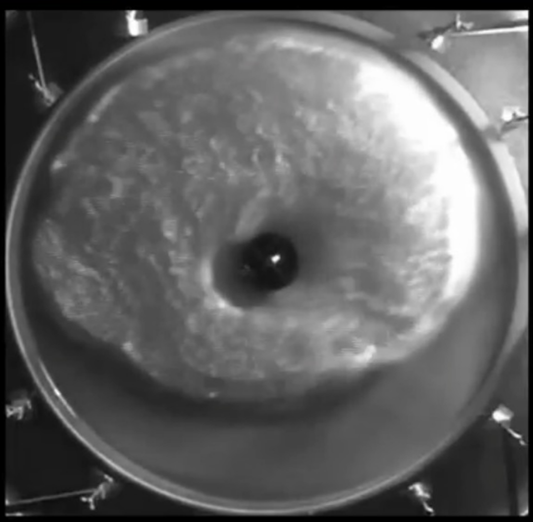
numerical approximations:
neutrino transport,
2D vs 3D,
turbulence

Can gravitational waves and neutrino signatures disentangle so many processes ?

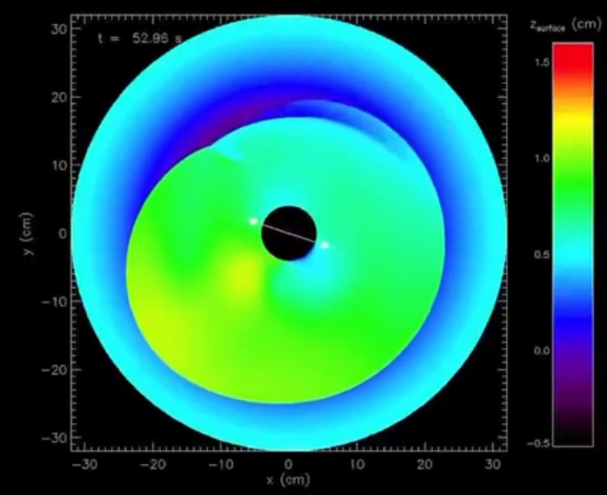
Dynamics of water
in the fountain

Dynamics of the gas
in the supernova core

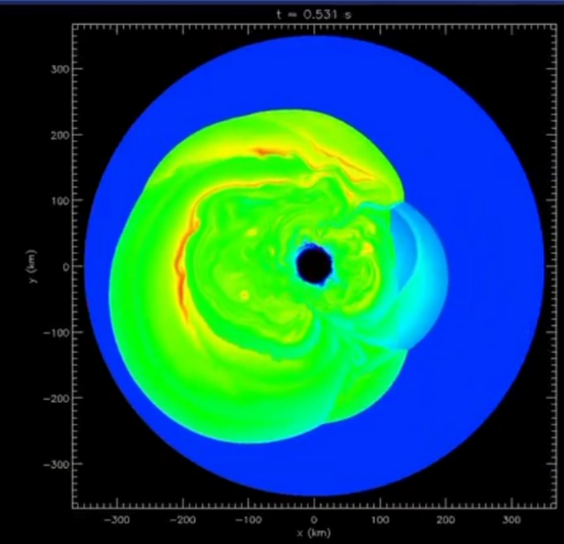
diameter 40cm ← 1 000 000 x bigger → diameter 400km
3s/oscillation ← 100 x faster → 0.03s/oscillation



Expérience hydraulique



Simulation numérique de l'expérience hydraulique



Simulation numérique de l'onde de choc dans le coeur de la supernova

SASI dynamics seems to be adiabatic

adiabatic approximation

inspired by the shallow water experiment

Stellar SASI:

- non adiabatic cooling/heating (ν-processes)

$$\mathcal{L} = A_{\rm cool}\rho^{\beta-\alpha}p^{\alpha}$$

- 4th order differential system

$$\delta w_\perp \equiv r(\nabla\times\delta w)_r$$

$$\delta K \equiv rv\delta w_\perp + l(l+1)\frac{c^2}{\gamma}\delta S$$

$$\frac{\partial\delta f}{\partial r} = \frac{i\omega v}{1-\mathcal{M}^2}\left[\delta h - \frac{\delta f}{c^2} + \left(\gamma - 1 + \frac{1}{\mathcal{M}^2}\right)\frac{\delta S}{\gamma}\right] + \delta\left(\frac{\mathcal{L}}{\rho v}\right), \quad \text{(B1)}$$

$$\frac{\partial\delta h}{\partial r} = \frac{i\omega}{v(1-\mathcal{M}^2)}\left(\frac{\mu^2}{c^2}\delta f - \mathcal{M}^2\delta h - \delta S\right) + \frac{i\delta K}{\omega r^2 v}, \quad \text{(B2)}$$

$$\frac{\partial\delta S}{\partial r} = \frac{i\omega}{v}\delta S + \delta\left(\frac{\mathcal{L}}{pv}\right), \quad \text{(B3)}$$

$$\frac{\partial\delta K}{\partial r} = \frac{i\omega}{v}\delta K + l(l+1)\delta\left(\frac{\mathcal{L}}{\rho v}\right). \quad \text{(B4)}$$

Foglizzo+07

Adiabatic approximation:

- linear conservation of entropy δS and baroclinic vorticity δK

- 2nd order differential system

$$\mathrm{dX} \equiv \frac{v}{1-\mathcal{M}^2}\mathrm{d}r$$

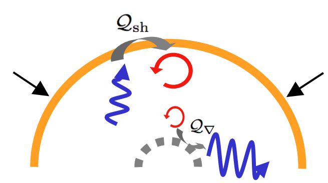
$$\left[\left(\frac{\partial}{\partial X} + \frac{i\omega}{c^2}\right)^2 + \frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} = \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$

perturbed specific angular momentum

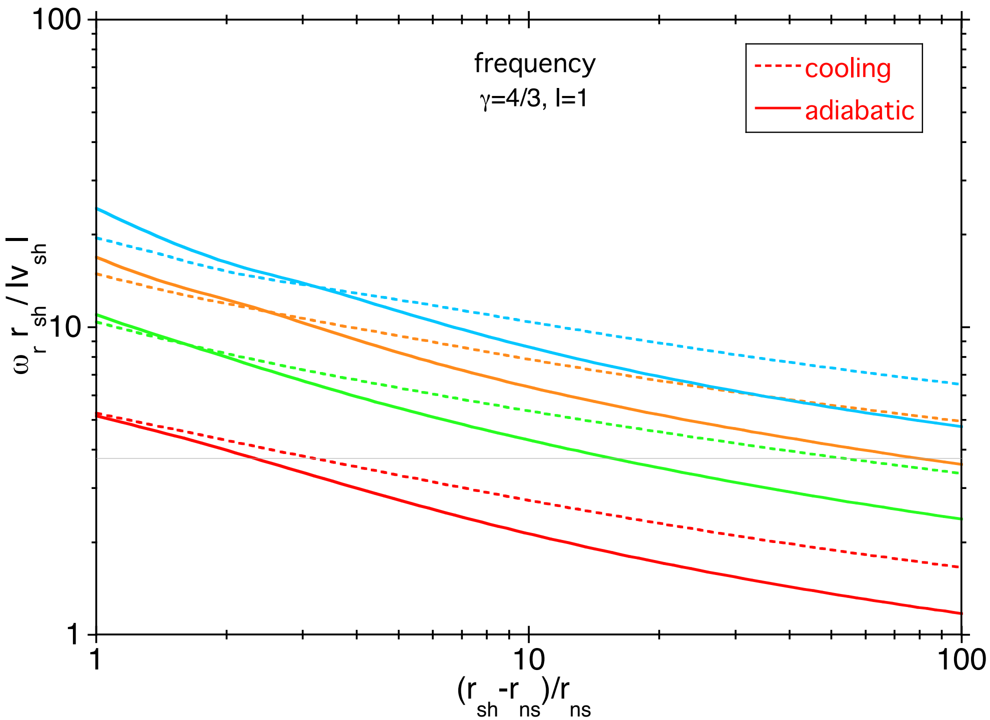
$$\delta\mathbf{L} \equiv \mathbf{r}\times\delta\mathbf{v}$$

perturbed vorticity

$$\delta\mathbf{w} \equiv \nabla\times\delta\mathbf{v}$$



- acoustic oscillator forced by the advection of vorticity



Comparison of SASI eigenfrequencies with/without a cooling function

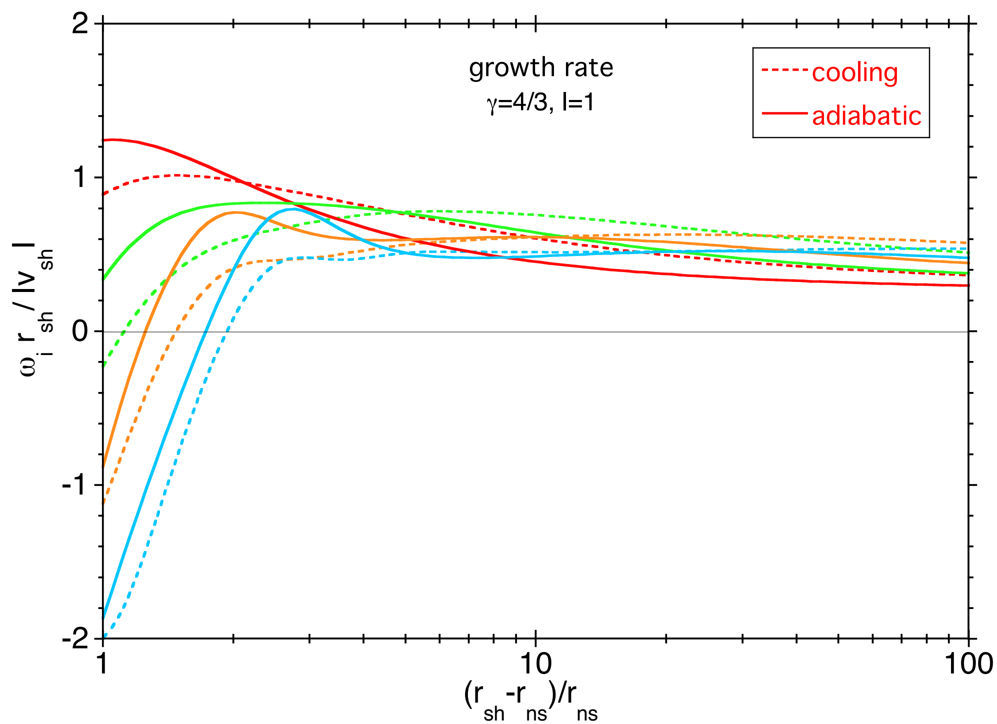
$$\left[\left(\frac{\partial}{\partial X}+\frac{i\omega}{c^2}\right)^2+\frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} = \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$

fundamental mode

1st, 2nd, 3rd harmonics

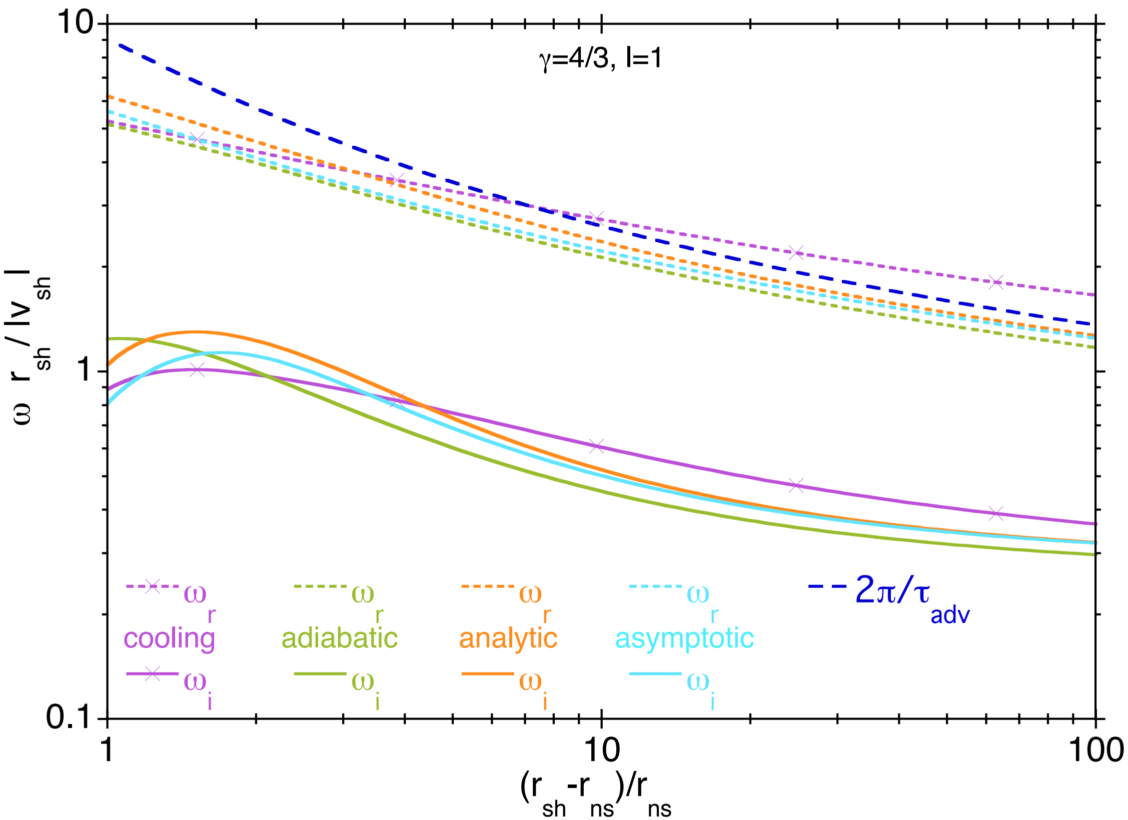
→general trends are captured by the adiabatic approximation

→the physical mechanism of SASI is approximately adiabatic



Analytical estimate of the SASI growth rate and frequency

state of the art = plane parallel model (Foglizzo 2009)



$$\left[\left(\frac{\partial}{\partial X}+\frac{i\omega}{c^2}\right)^2+\frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} = \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$

Forced oscillator + shock & pns boundary conditions

$$\left\{\frac{\partial^2}{\partial X^2}+\frac{\omega^2-\omega_{\rm Lamb}^2}{v^2c^2}\right\}Y_0=0$$ acoustic solution

→integral equation defining the eigenfrequencies

$$a_1' Y_0^{\rm sh} + a_2' r_{\rm sh}\left(\frac{\partial Y_0}{\partial r}\right)_{\rm sh} = -\mathcal{M}_{\rm sh}^2 {\rm e}^{\int_{\rm sh}^{\rm ns}\frac{i\omega}{v}\frac{{\rm d}r}{1-\mathcal{M}^2}} Y_0^{\rm ns} - \int_{\rm ns}^{\rm sh}\frac{\partial}{\partial r}\left(Y_0 {\rm e}^{\int_{\rm sh}\frac{i\omega\mathcal{M}^2}{1-\mathcal{M}^2}\frac{{\rm d}r}{v}}\right)\frac{\mathcal{M}_{\rm sh}^2}{\mathcal{M}^2}{\rm e}^{\int_{\rm sh}\frac{i\omega}{v}{\rm d}r}{\rm d}r,$$

with a_1', a_2' defined by:

$$a_1' \equiv (\gamma-1)\mathcal{M}_{\rm sh}^2 + \frac{\frac{i\omega r_{\rm sh}}{v_{\rm sh}}\frac{v_{\rm sh}}{v_1}}{\frac{v_1}{v_{\rm sh}}\frac{1}{2\eta^2}-2-\left(1-\frac{v_{\rm sh}}{v_1}\right)\frac{i\omega r_{\rm sh}}{v_{\rm sh}}},$$

$$a_2' \equiv \frac{1-\mathcal{M}_{\rm sh}^2}{\frac{v_1}{v_{\rm sh}}\frac{1}{2\eta^2}-2-\left(1-\frac{v_{\rm sh}}{v_1}\right)\frac{i\omega r_{\rm sh}}{v_{\rm sh}}}.$$

→asymptotic approximation

$$\frac{i\omega r_{\rm sh}}{|v_{\rm sh}|} = b-2+\frac{2ni\pi}{\zeta-d_1}+\mathcal{O}\left(\frac{1}{\zeta^3}\right),$$

$$\mathcal{Q}\left(\frac{i\omega r_{\rm sh}}{|v_{\rm sh}|}\right) = \frac{\left(\frac{r_{\rm sh}}{r_{\rm ns}}\right)^{2-b}}{1+\frac{2ni\pi d_1}{\zeta-d_1}-\frac{4n^2\pi^2 d_2}{b(\zeta-d_1)^2}+\mathcal{O}\left(\frac{1}{\zeta^3}\right)},$$

$$|\mathcal{Q}| = \left(\frac{r_{\rm sh}}{r_{\rm ns}}\right)^{2-[1+l(l+1)]^{\frac{1}{2}}}+\mathcal{O}\left(\frac{1}{\zeta^2}\right),$$

$$\zeta \equiv \log\frac{r_{\rm sh}}{r_{\rm ns}},$$

$$\omega_i^{(0)} = (2-b)\frac{|v_{\rm sh}|}{r_{\rm sh}},$$

$$\omega_i^{(k)} = \frac{1}{\tau_{\rm adv}^{\rm ns}}\log\left|Q\left(\frac{2n\pi}{\zeta-d_1}+\frac{i\omega_i^{(k-1)}r_{\rm sh}}{|v_{\rm sh}|}\right)\right|$$

→analytic approximation

$$\mathcal{Q}(Z) \equiv \frac{2b\left(\frac{r_{\rm sh}}{r_{\rm ns}}\right)^{2-b}\left\{1+\left[(Z+2)^2-b^2\right]\frac{\mathcal{M}_{\rm sh}^2}{l(l+1)x_{\rm sh}^3}\right\}}{\left[1-(Z+2-b)N\right](Z+2+b)-\frac{Z+2-b}{x_{\rm sh}^{2b}}},$$

$$\mathcal{Q}\left(\frac{i\omega r_{\rm sh}}{|v_{\rm sh}|}\right){\rm e}^{i\omega\tau_{\rm adv}^{\rm ns}} = 1,$$

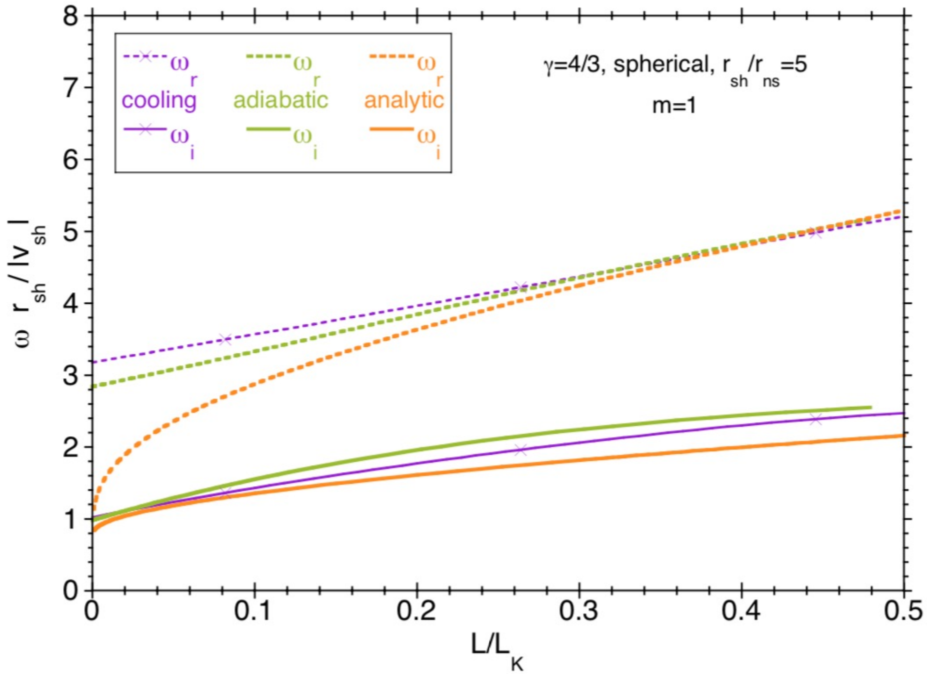
→ practical use for multi-messenger analysis

Physical insight on the impact of rotation on SASI

→adiabatic approximation

$$\left\{\left(\frac{\partial}{\partial X}+\frac{i\omega'}{c^2}\right)^2+\frac{\omega'^2\mu'^2}{v_r^2c^2}\right\}(r\delta v_\varphi)=-\frac{\partial}{\partial X}\left(\frac{r\delta w_\theta}{v_r}\right)$$

$$\omega'\equiv\omega-\frac{mL}{r^2}$$



Modest rotation: differential rotation $\Omega \sim L/r^2$ at small radius increases the radial wavelength $\lambda_r \sim 2\pi v/(\omega - mL/r^2)$ of advected perturbations

→increases the match between the acoustic oscillator and the advected forcing = "un-mixing" of the phase

Strong rotation: corotation radius r_{co} where $\omega'=0$

stationary phase approximation

$$\int_{\rm ns}^{\rm sh}\frac{\partial Y_0}{\partial r}\frac{1}{\mathcal{M}^2}{\rm e}^{\int_{\rm sh}\frac{i\omega'}{v_r}{\rm d}r}\frac{{\rm d}r}{r_{\rm sh}}\sim {\rm e}^{i\Psi_{\rm co}}\int_{\rm ns}^{\rm sh}\frac{\partial Y_0}{\partial r}\frac{{\rm e}^{-\omega_i\tau_{\rm adv}(r)}}{\mathcal{M}^2}{\rm e}^{-i\left(\frac{r-r_{\rm co}}{\Delta r}\right)^2}\frac{{\rm d}r}{r_{\rm sh}}$$

$$\sim {\rm e}^{i\Psi_{\rm co}}\pi^{\frac{1}{2}}{\rm e}^{-i\frac{\pi}{4}}\left(\frac{\partial Y_0}{\partial r}\right)_{\rm co}\frac{{\rm e}^{-\omega_i\tau_{\rm adv}^{\rm co}}}{\mathcal{M}_{\rm co}^2}\frac{\Delta r}{r_{\rm sh}}$$

→spiral SASI is produced by an advective-acoustic cycle with a coupling at the the corotation radius

→analytic approximation

$$\mathcal{Q}{\rm e}^{-\omega_i\tau_{\rm adv}^{\rm co}}=1$$

$$\mathcal{Q}\equiv\frac{\pi^{\frac{1}{2}}\left(\frac{r_{\rm sh}}{r_{\rm co}}\right)^{2a-b}{\rm e}^{i\left(\Psi_{\rm co}-\frac{5\pi}{4}\right)}}{\left(\frac{\omega_r r_{\rm sh}}{|v_{\rm sh}|}\right)^{\frac{1}{2}}\left[N\left(\frac{i\omega'_{\rm sh}r_{\rm sh}}{|v_{\rm sh}|}\right)+\frac{2b}{m_l^2}\frac{\mathcal{M}_{\rm sh}^2}{x_{\rm sh}^{a+b}}{\rm e}^{i\omega\tau_{\rm adv}^{\rm sh}}\right]}$$

Impact of viscosity ν and thermal diffusivity κ on SASI?

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0,$$

$$\frac{\partial v}{\partial t} + (v\cdot\nabla)v + \nabla\Phi = -\frac{\nabla p}{\rho} + \nu\left[\nabla^2 v + \frac{1}{3}\nabla(\nabla\cdot v)\right]$$

$$\frac{\partial S}{\partial t} + (v\cdot\nabla)S = \frac{\gamma\kappa}{\gamma-1}\frac{\nabla^2 c^2}{c^2} + \frac{1}{p}\tau : \nabla v.$$

$$\tau : \nabla v = 2\nu\rho\left[\frac{1}{2}(\partial_j v_i + \partial_i v_j) - \frac{1}{3}(\nabla\cdot v)\delta_{ij}\right]^2$$

in a plane parallel uniform flow:

$$\omega_i^{\rm visc} = -k^2\nu$$

$$\omega_i^{\rm diff} = -k^2\kappa$$

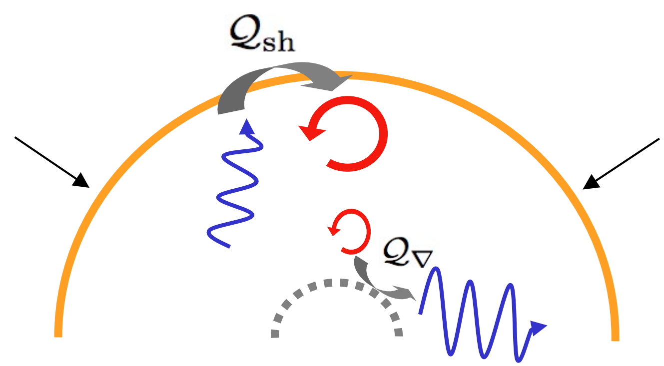
$$\omega_i^{\rm ac} = -\left[\frac{2\nu}{3} + \frac{\kappa}{2}(\gamma-1)\right]k^2$$

$$\left[\left(\frac{\partial}{\partial X} + \frac{i\omega}{c^2}\right)^2 + \frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} = \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$

$$\left[\left(\frac{\partial}{\partial X} + \frac{i\omega}{c^2}\right)^2 + \frac{\omega^2\mu^2}{v^2c^2}\right]\frac{\delta f}{i\omega} = e^{i\omega\int_{\rm sh}\frac{dX}{c^2}}\frac{\partial}{\partial X}\frac{\delta S}{\gamma\mathcal{M}^2}$$

$$Q_{\rm damp} \equiv \exp\left(-\int_{\rm sh}^{\rm eff}\omega_i^{\rm damp}\frac{dr}{v}\right)$$

→vorticity perturbations are damped by viscosity
→entropy perturbations are damped by thermal diffusivity



Impact of viscosity ν and thermal diffusivity κ on SASI?

perturbative calculation

$$\rightarrow \frac{\partial w_i}{\partial \nu} \sim -\frac{4\pi^2}{r_{\rm sh}^2}$$

as expected from

$\omega_i^{\rm SASI} \sim -\omega_i^{\rm visc}$

$$w_i^{\rm SASI} \sim \frac{|v_{\rm sh}|}{r_{\rm sh}} \qquad \omega_i^{\rm visc} = -k^2\nu$$

$$k \sim \frac{\omega_r}{|v_{\rm sh}|} \sim \frac{2\pi}{r_{\rm sh}}$$

$$\frac{\delta w_i}{\omega_i} \sim -4\pi^2 \frac{\nu}{r_{\rm sh}|v_{\rm sh}|}$$

stabilization by turbulence

$$\frac{v_{\rm turb}}{v_{\rm sh}} \sim \frac{\nu_{\rm stab}}{r_{\rm sh}|v_{\rm sh}|} \sim \frac{1}{4\pi^2} \sim 3\%$$

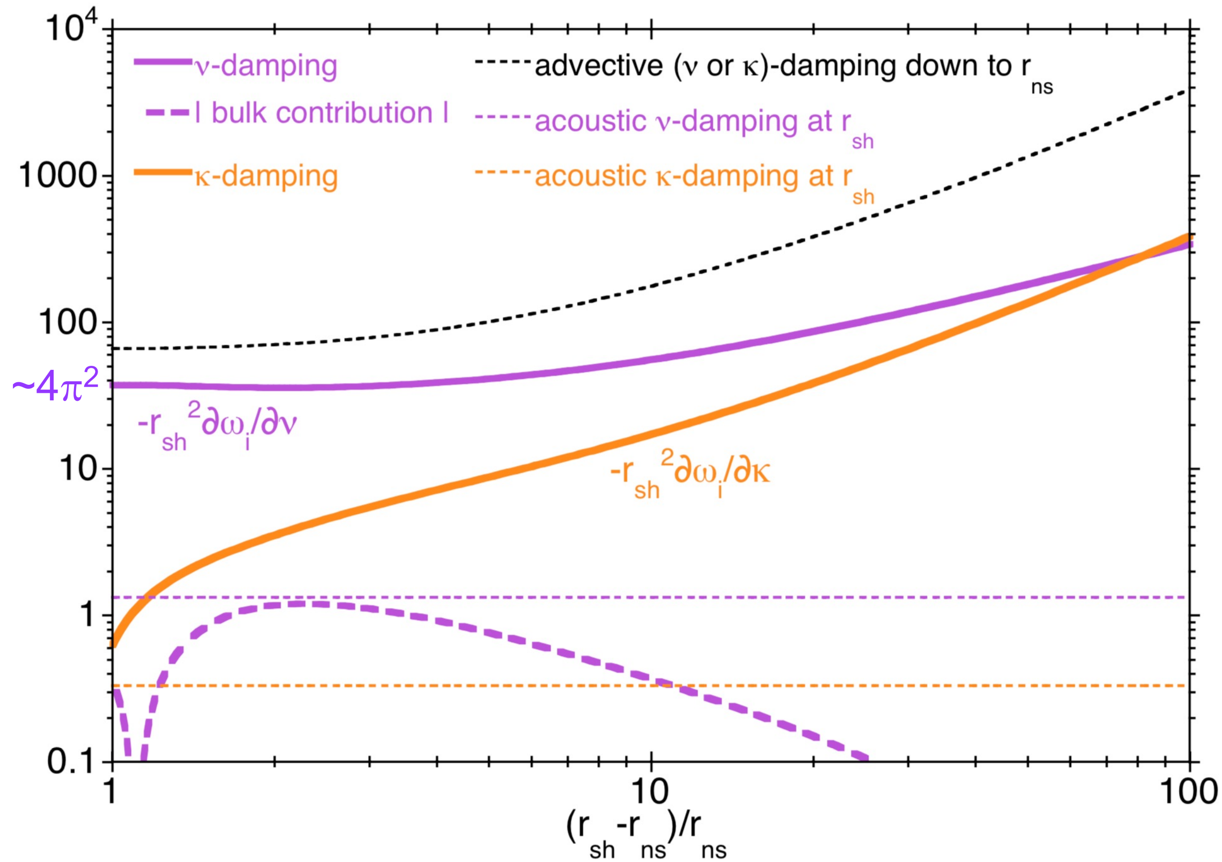
stabilization by numerical viscosity

$$\nu_{\rm num} \sim \left(\frac{1-C_{\rm CFL}}{2}\right) v\Delta r$$

$$\frac{\delta w_i}{\omega_i} \sim -\frac{4\pi^2}{N_r}\left(\frac{1-C_{\rm CFL}}{2}\right)\frac{r_{\rm sh}-r_{\rm ns}}{r_{\rm sh}}$$

$$\frac{\delta w_i}{\omega_i} \sim -26\% \left(\frac{30}{N_{\rm pns}^{\rm sh}}\right)$$

$C_{\rm CFL}\sim 0.4$



viscous damping is too strong to be compatible with an acoustic mechanism

→ confirms the advective-acoustic interpretation of SASI mechanism

→ 30 grid points from r_{pns}=50km to r_{sh}=150km may be insufficient

Conclusion

Rotation effect clarified using the adiabatic approximation

-acoustic oscillations of the post-shock cavity are forced by advected vorticity perturbation
-the Doppler shift induced by rotation increases the radial wavelength of advected vorticity waves
→rotation lessens the innermost phase mixing
-the spiral mode of SASI is driven by an advective-acoustic cycle forced at the corotation radius

First analytical estimates of SASI growth rate and frequency in radial and equatorial accretion

Unexpectedly large stabilization effect of viscosity

-turbulent velocities ≥ 3% $|v_{sh}|$ can stabilize SASI
-warning on the damping effect of numerical viscosity

What's next? → towards reverse engineering of multimessenger signatures

- low-T/W mechanism and its interaction with SASI and convection
- complementarity of neutrino and GW signatures for each instability (collab AIM-APC-IJCLab)