Black holes with primary scalar hair

Nicolas Lecœur

Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France

based on A. Bakopoulos, C. Charmousis, P. Kanti, N. Lecœur and T. Nakas, arXiv:2310.xxxxx.

October 16

Outline

Hairy black holes in Scalar-Tensor theories

Primary hair solution

Conclusions

No-hair theorem in General Relativity

- General Relativity coupled to electrodynamics: a stationary black hole is completely characterized by its mass M, angular momentum J and electric charge Q (only M if spherical symmetry and without Maxwell)

- Two such black holes with identical M, J and Q are described by the exact same Kerr-Newman metric

- They have **no hair**, i.e., no independent integration constant other than M, J and Q [P. O. Mazur, hep-th/0101012]

Scalar-tensor gravity

- Modifying General Relativity is motivated from both theoretical and observational considerations [E. J. Copeland, M. Sami, S. Tsujikawa, Int.J.Mod.Phys.D, 2006] [T. Clifton, P. G. Ferreira, A. Padilla, C. Skordis, Phys.Rept., 2012]
- Scalar-tensor gravity: modification of gravity which includes, in addition to the usual metric **tensor** field $g_{\mu\nu}$, a non-minimally coupled **scalar** field ϕ
- Adds a unique degree of freedom $\rightsquigarrow$ both simple and general [T. Chiba, Phys.Lett.B, 2003]
- Most general scalar-tensor theory with second-order field equations: Horndeski theory. Healthy generalizations beyond Horndeski. [J. Gleyzes, D. Langlois, F. Piazza, F. Vernizzi, Phys.Rev.Lett., 2015] [D. Langlois, K. Noui, JCAP, 2016]

Previous scalar-tensor solutions

- **Stealth** solutions, e.g.: [E. Babichev, C. Charmousis, JHEP, 2014]

$$S\left[g_{\mu\nu},\phi\right]=\int \mathrm{d}^4x\sqrt{-g}\Big\{R+\beta G^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\Big\},$$

$$\mathrm{d}s^2=\text{Schwarzschild},\quad \phi=qt+q\int\frac{\sqrt{2Mr}}{r-2M}\mathrm{d}r$$

- **Non-stealth** solutions, e.g. coming from dimensional reduction of higher-dimensional theory: [P. G. S. Fernandes, P. Carrilho, T. Clifton, D. J. Mulryne, Phys.Rev.D, 2020]

$$\mathrm{d}s^2=-f\left(r\right)\mathrm{d}t^2+\frac{\mathrm{d}r^2}{f\left(r\right)}+r^2\mathrm{d}\Omega^2,\quad f\left(r\right)=1+\frac{r^2}{2\alpha}\left(1-\sqrt{1+\frac{8\alpha M}{r^3}}\right),$$

$$\phi=qt+\int\frac{\sqrt{q^2r^2+f(r)}-f(r)}{r\,f(r)}\,\mathrm{d}r$$ [C. Charmousis, A. Lehébel, E. Smyrniotis, N. Stergioulas, JCAP, 2022]

Previous scalar-tensor solutions

- Both solutions are **hairy** because ϕ is non-trivial
- In both cases, q is an arbitrary integration constant, but it appears only in the scalar field, not in the metric. The metric is fully determined by its mass $\rightsquigarrow$ **secondary hair** solutions
- In this talk: present a black hole solution with **primary hair**, i.e., the metric is parameterized by two independent integration constants, its mass M and the primary hair q (which is not the angular momentum J nor the electric charge Q, since we work in spherical symmetry and without Maxwell)

Hairy black holes in Scalar-Tensor theories

Primary hair solution

Conclusions

Framework

Beyond Horndeski action ($\phi_\mu = \partial_\mu \phi$, $\phi_{\mu\nu} = \nabla_\mu \partial_\nu \phi$, $X = -\phi^\mu \phi_\mu / 2$):

$$S = \int \mathrm{d}^4 x \sqrt{-g} \Big\{ G_2(X) + G_4(X) R + G_{4X} \left[(\Box \phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu} \right] + F_4(X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma}{}_\sigma \phi_\mu \phi_\alpha \phi_{\nu\beta} \phi_{\rho\gamma} \Big\}$$

Static, spherically-symmetric solution:

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\Omega^2,$$

$$\phi = qt + \psi(r)$$

Linear time-dependence in ϕ is compatible with the symmetries because the theory is invariant under $\phi \to \phi +$ cst. (shift-symmetric)

Primary hair solution

Theory functionals depend on two couplings λ (> 0) and η:

$$G_2 = -\frac{8\eta}{3\lambda^2}X^2, \quad G_4 = 1 - \frac{4\eta}{3}X^2, \quad F_4 = \eta$$

This theory is solved by the following metric function with two independent integration constants M and q,

$$f(r) = 1 - \frac{2M}{r} + \eta q^4 \left(\frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1 + (r/\lambda)^2} \right),$$

while the scalar field is

$$\phi = qt + \psi(r), \quad \psi'(r)^2 = \frac{q^2}{f^2(r)} \left[1 - \frac{f(r)}{1 + (r/\lambda)^2} \right].$$

$q = 0$: Schwarzschild, $q \neq 0$: departure from Schwarzschild

$$f(r) = 1 - \frac{2M}{r} + \eta q^4 \left(\frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$$
$$= 1 - \frac{2M}{r} + 2\lambda^2 \frac{\eta q^4}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right), \quad r \to \infty$$

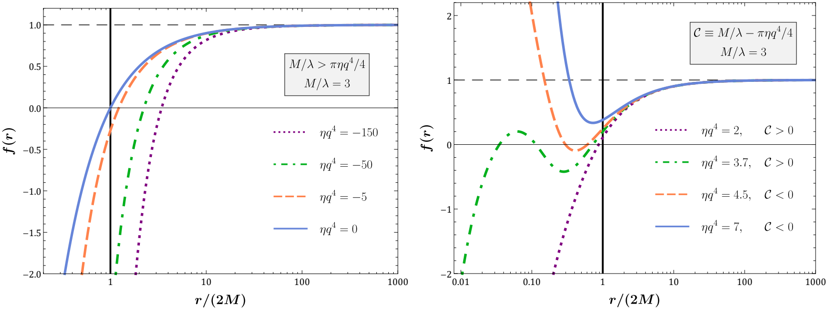

Figure 1: Left: $\eta < 0$, unique horizon greater than the Schwarzschild radius $r_S = 2M$. Right: $\eta > 0$, one, two, three or zero horizons, horizon smaller than Schwarzschild.

Regular spacetime (black hole or soliton)

For $M = \pi\eta q^4\lambda/4$, the central singularity disappears and all curvature invariants become infinitely regular:

$$f(r) = 1 - \frac{4M}{\pi\lambda}\left(\frac{\arctan(r/\lambda)}{r/\lambda} - \frac{1}{1+(r/\lambda)^2}\right)$$

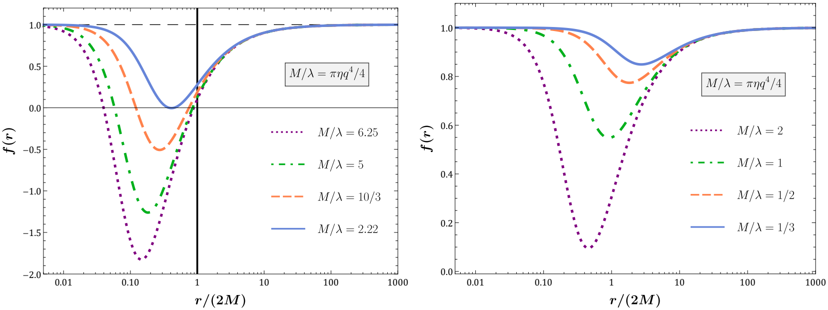

Figure 2: Left: Regular BH solutions. Right: regular solitonic solutions.

Conclusions

- General Relativity black holes are completely characterized by their mass M, electromagnetic charge Q, angular momentum J (so only M in vacuum and spherical symmetry)
- Found a black hole with primary hair q in a simple scalar-tensor theory
- For a particular relation between the mass M and the primary hair q, the central singularity disappears (regular spacetime)
- Perspective: better understanding of q. Observational constraints, thermodynamics, ...

Thank you for your attention!

Case of canonical kinetic term

Theory:

$$G_2 = \frac{2\eta}{\lambda^2}X, \quad G_4 = 1 + \eta X, \quad F_4 = -\frac{\eta}{4X}$$

Metric function:

$$f\left(r\right) = 1 + \eta q^2 - \frac{2M}{r} + \eta q^2 \frac{\pi/2 - \arctan\left(r/\lambda\right)}{r/\lambda}$$

Scalar field:

$$\phi = qt + \psi\left(r\right), \quad \psi'\left(r\right)^2 = \frac{q^2}{f^2\left(r\right)}\left[1 - \frac{f\left(r\right)}{1 + \left(r/\lambda\right)^2}\right]$$

Asymptotic behaviour:

$$f\left(r\right) = 1 + \eta q^2 - \frac{2M}{r} + \eta q^2 \frac{\lambda^2}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right)$$

Expansion near $r = 0$

$$f(r) = 1 - \frac{2M - \pi\eta q^4\lambda/2}{r} - \frac{2\eta q^4 r^2}{3\lambda^2} + \mathcal{O}\left(r^4\right)$$