RELATING POST-MINKOWSKIAN AND BONDI-SACHS FORMALISMS

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Based on 2011.10000, 2206.12597 and 2303.07732

Collaborators: Blanchet, Compère, Faye and Seraj

MOTIVATIONS

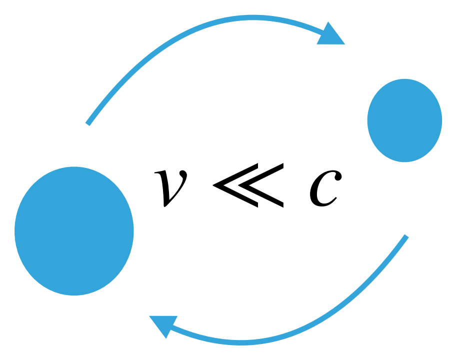
d: characteristic scale of the source

λ: characteristic scale of the wave

$d \ll \lambda$ separation of scales

0 d λ r

near-zone: $0 < r \ll \lambda$



exterior-zone (vacuum): $d < r < \infty$

radiation-zone $\lambda \ll r < \infty$

PN/multipolar radiation-reaction

multipolar PM expansion

Bondi-Sachs

Matching of PN/MPM solutions

[Epstein-Wagoner-Thorne et al., since '70s]

[Blanchet-Damour-Iyer et al., since '80s]

[Will-Wiseman et al., since '90s]

[Bondi-van der Burg-Metzner, 60s]

[Sachs, '60s]

PN & MPM ADOPT IN <u>HARMONIC GAUGE</u>

VS

BONDI-SACHS ADOPTS IN <u>RADIATIVE GAUGES</u>

MPM METRIC

$$h^{\alpha\beta}_{\rm ext} = \sum_{n=1}^{+\infty} G^n \, h^{\alpha\beta}_{(n)}$$

PERTURBATIVE IN G
VALID FOR ANY $r > d$

VS

BONDI-SACHS METRIC

$$g_{uu} = \sum_{n=0}^{\infty} \frac{1}{r^n} g^{(n)}_{uu},$$

$$g_{ua} = \sum_{n=0}^{\infty} \frac{1}{r^n} g^{(n)}_{ua},$$

$$g_{ab} = \sum_{n=-2}^{\infty} \frac{1}{r^n} g^{(n)}_{ab}$$

VALID FOR ANY G
PERTURBATIVE IN $1/r$

PART I

Multipolar post-Minkowskian approach

BASICS OF THE MPM ALGORITHM

Main assumptions:

- We solve the field equations in the vacuum region outside the isolated matter system;
- The metric is asymptotically flat and stationary in the far past / no incoming radiation.

$$\Box h^{\alpha\beta} = \Lambda^{\alpha\beta}(h, \partial h, \partial^2 h)$$

(relaxed Einstein's equations)

$$\partial_\alpha h^{\alpha\beta} = 0$$

(harmonic gauge condition)

$$h^{\alpha\beta} = \sqrt{g} g^{\mu\nu} - \eta^{\mu\nu} = \sum_{n=1}^{\infty} G^n h_n^{\alpha\beta}$$

(MPM metric)

$\rightarrow$ functionals of canonical moments $\{M_L, S_L\}$

$\uparrow$ Related to source moments $\{I_L, J_L; \ldots\}$

$\downarrow$ Related to radiative moments $\{U_L, V_L\}$

For $n = 1$

$\Box h_1^{\alpha\beta} = 0$ and $\partial_\alpha h_1^{\alpha\beta} = 0$ with [Thorne, 1980]

$$h_1^{\alpha\beta} \propto \sum_{l=0}^{\infty} \partial_L \left(\frac{K_L^{\alpha\beta}(t-r)}{r} \right)$$

(retarded multipolar waves)

[Notation: $\partial_L \equiv \partial_{i_1 i_2 \ldots i_\ell}$]

For $n \geq 2$

$\Box h_n^{\alpha\beta} = \Lambda_n^{\alpha\beta}$ and $\partial_\alpha h_n^{\alpha\beta} = 0$ with [Blanchet-Damour, 1986]

$$h_n^{\alpha\beta} = p_n^{\alpha\beta} + q_n^{\alpha\beta}$$

$$p_n^{\alpha\beta} = \mathrm{FP}_{B=0} \Box_{ret}^{-1} \left(r^B \Lambda_n^{\alpha\beta} \right)$$

$$\Box q_n^{\alpha\beta} = 0 \quad \text{and} \quad \partial_\beta q_n^{\alpha\beta} = -\partial_\beta p_n^{\alpha\beta}$$

LINEAR ORDER IN THE MPM FORMALISM

The most general retarded solution (modulo infinitesimal gauge transformation) reads as [Thorne, 1980]

$$h_1^{00} = -4\sum_{\ell=0}^{+\infty}\frac{(-)^\ell}{\ell!}\tilde{\partial}_L\left(\frac{M_L(\tilde{u})}{\tilde{r}}\right),$$

$$h_1^{0i} = 4\sum_{\ell=1}^{+\infty}\frac{(-)^\ell}{\ell!}\left[\tilde{\partial}_{L-1}\left(\frac{\overset{(1)}{M}_{iL-1}(\tilde{u})}{\tilde{r}}\right) + \frac{\ell}{\ell+1}\tilde{\partial}_{pL-1}\left(\frac{\varepsilon_{ipq}S_{qL-1}(\tilde{u})}{\tilde{r}}\right)\right],$$

$$h_1^{ij} = -4\sum_{\ell=2}^{+\infty}\frac{(-)^\ell}{\ell!}\left[\tilde{\partial}_{L-2}\left(\frac{\overset{(2)}{M}_{ijL-2}(\tilde{u})}{\tilde{r}}\right) + \frac{2\ell}{\ell+1}\tilde{\partial}_{pL-2}\left(\frac{\varepsilon_{pq(i}\overset{(1)}{S}_{j)qL-2}(\tilde{u})}{\tilde{r}}\right)\right].$$

$\tilde{u} = \tilde{t} - \tilde{r}$ (harmonic) retarded time

$h_1^{\mu\nu}$ contains only instantaneous terms: $M_L^{(k)}, S_L^{(k)}$

To know where *tail and memory terms* are located in $h_2^{\mu\nu}$ [recall: $\Box h_2^{\mu\nu} = \Lambda_2^{\mu\nu}(h_1, \partial h_1, \partial^2 h_1)$], we need the *asymptotic behaviour* of $h_1^{\mu\nu}$

$$h_1^{\mu\nu} = \frac{1}{\tilde{r}}\begin{pmatrix} -4\big(M + \tilde{n}_i P^i\big) + z_1^{00}(\tilde{u}, \tilde{\boldsymbol{n}}) \\ -4P^i + z_1^{0i}(\tilde{u}, \tilde{\boldsymbol{n}}) \\ z_1^{ij}(\tilde{u}, \tilde{\boldsymbol{n}}) \end{pmatrix} + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

$$z_1^{00} = -4\sum_{\ell=2}^{+\infty}\frac{1}{\ell!}\tilde{n}_L\overset{(\ell)}{M}_L(\tilde{u}),$$

→ Connected with radiative moments/shear

$$z_1^{0i} = -4\sum_{\ell=2}^{+\infty}\frac{1}{\ell!}\left(\tilde{n}_{L-1}\overset{(\ell)}{M}_{iL-1}(\tilde{u}) + \frac{\ell}{\ell+1}\tilde{n}_{pL-1}\varepsilon_{ipq}\overset{(\ell)}{S}_{qL-1}(\tilde{u})\right),$$

$$z_1^{ij} = -4\sum_{\ell=2}^{+\infty}\frac{1}{\ell!}\left(\tilde{n}_{L-2}\overset{(\ell)}{M}_{ijL-2}(\tilde{u}) + \frac{2\ell}{\ell+1}\tilde{n}_{pL-2}\varepsilon_{pq(i}\overset{(\ell)}{S}_{j)qL-2}(\tilde{u})\right)$$

QUADRATIC ORDER IN THE MPM FORMALISM: TAILS AND MEMORY TERMS

Hereditary terms are generated by the $1/\tilde{r}^2$ term in the source $\Lambda_2^{\mu\nu}$ [Blanchet-Damour, 1992]

$$\Box h_2^{\mu\nu} = \Lambda_2^{\mu\nu}(h_1, \partial h_1, \partial^2 h_1)$$

$$\Lambda_2^{\mu\nu} = \frac{1}{\tilde{r}^2}\left[4(M+\tilde{n}_i P_i)\frac{d^2 z^{\mu\nu}}{d\tilde{u}^2} + \tilde{k}^\mu \tilde{k}^\nu\ \Pi(\tilde{u},\tilde{\mathbf{n}})\right] + \mathcal{O}\left(\frac{1}{\tilde{r}^3}\right)$$

$$\tilde{k} = (1,\tilde{\mathbf{n}})$$

$$\Pi = \frac{1}{2}\frac{\mathrm{d}z^{\mu\nu}}{\mathrm{d}\tilde{u}}\frac{\mathrm{d}z_{\mu\nu}}{\mathrm{d}\tilde{u}} - \frac{1}{4}\frac{\mathrm{d}z^\mu_\mu}{\mathrm{d}\tilde{u}}\frac{\mathrm{d}z^\nu_\nu}{\mathrm{d}\tilde{u}}$$

$$\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\tilde{u}\mathrm{d}\tilde{\Omega}} = \frac{G}{16\pi}\Pi(\tilde{u},\tilde{\boldsymbol{n}}) + \mathcal{O}(G^2)$$

generates tail terms

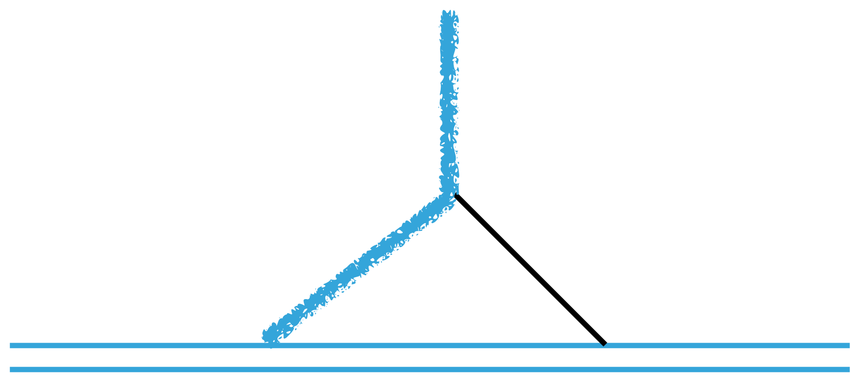
Tail terms are GW scattered off the curvature of the spacetime

generates memory terms (and losses of energy, linear/angular momenta, ...)

Memory terms are GW scattered off GW

Tails $\propto M \times \int_{-\infty}^{\tilde{u}}$ (kernel) M_L

$$h_2^{\mu\nu}\Big|_{\mathrm{tail}} = \frac{2(M+\tilde{n}_i P_i)}{\tilde{r}}\int_{-\infty}^{\tilde{u}} \mathrm{d}v \ln\left(\frac{\tilde{u}-v}{2b_0}\right)\frac{\mathrm{d}^2 z^{\mu\nu}}{\mathrm{d}\tilde{u}^2}(v,\tilde{\boldsymbol{n}}) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

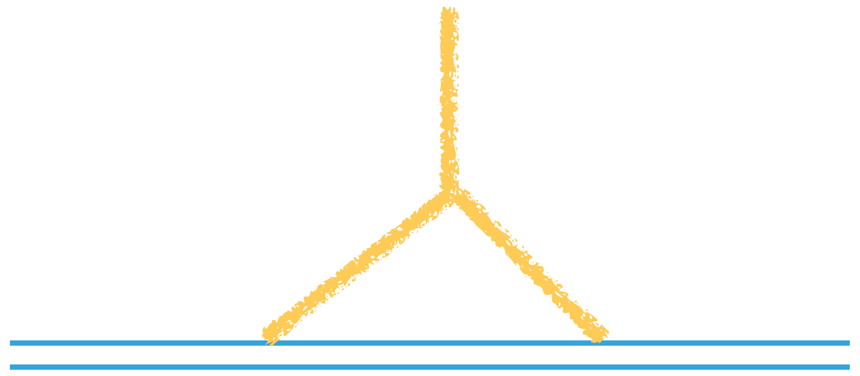


Memory $\propto \int_{-\infty}^{\tilde{u}} M_L \times M_L$

$K^{\mu\nu} \propto \Pi$

Secular losses

$$h_2^{\mu\nu}\Big|_{\mathrm{mem}} = \frac{1}{\tilde{r}}\int_{-\infty}^{\tilde{u}} \mathrm{d}v\, K^{\mu\nu}(v,\tilde{\boldsymbol{n}}) + \left(\frac{1}{\tilde{r}^2}\right)(\dots)$$



PART II

From harmonic to Newman-Unti (NU)/Bondi gauge

SETTING UP THE GAUGE TRANSFORMATION

Consistently with the MPM formalism, we assume

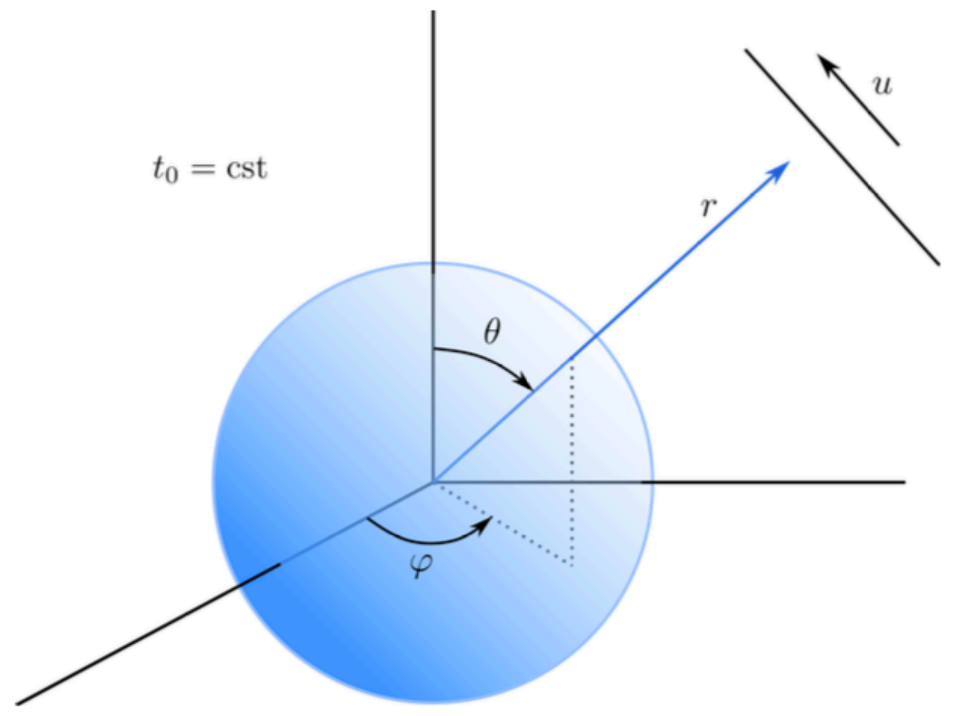
credit: Guillaume Faye

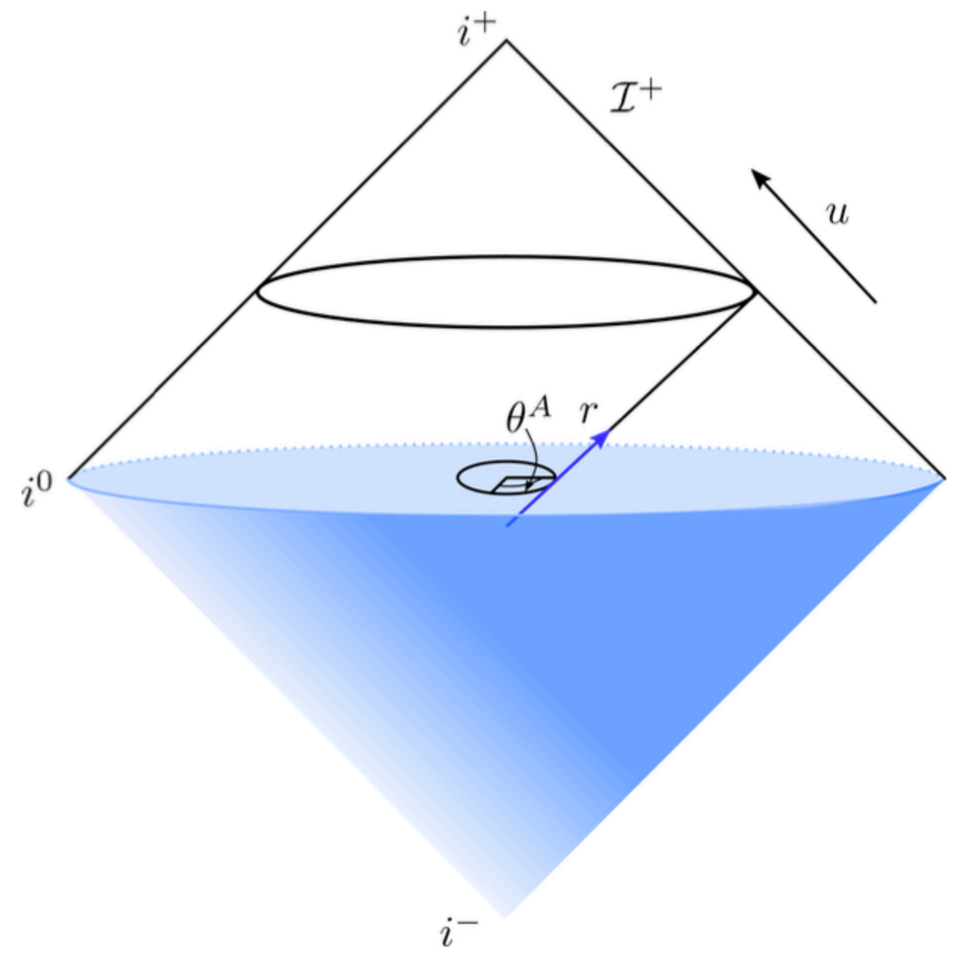
NU coordinates:

$$u = \tilde{u} + \sum_{n=1}^{+\infty} G^n U_n(\tilde{u}, \tilde{r}, \tilde{\theta}^a)\,,$$

$$r = \tilde{r} + \sum_{n=1}^{+\infty} G^n R_n(\tilde{u}, \tilde{r}, \tilde{\theta}^a)\,,$$

$$\theta^a = \tilde{\theta}^a + \sum_{n=1}^{+\infty} G^n \Theta_n^a(\tilde{u}, \tilde{r}, \tilde{\theta}^b)\,,$$





NU gauge conditions: $g^{uu} = 0\,, \quad g^{ur} = -1\,, \quad g^{ua} = 0$

Asymptotic boundary conditions: $g_{uu} = \mathcal{O}(1), \quad g_{ua} = \mathcal{O}(1), \quad \det(g_{ab}) = r^4 \sin^2\theta + \mathcal{O}(r^2)$

$$\tilde{g}^{\mu\nu}(\tilde{x}) \frac{\partial u}{\partial \tilde{x}^\mu} \frac{\partial u}{\partial \tilde{x}^\nu} = 0\,,$$
$$\tilde{g}^{\mu\nu}(\tilde{x}) \frac{\partial u}{\partial \tilde{x}^\mu} \frac{\partial r}{\partial \tilde{x}^\nu} = -1\,,$$
$$\tilde{g}^{\mu\nu}(\tilde{x}) \frac{\partial u}{\partial \tilde{x}^\mu} \frac{\partial \theta^a}{\partial \tilde{x}^\nu} = 0\,.$$

We obtain:

i. NU gauge conditions imply a system of coupled PDE for $\{U_n, R_n, \Theta_n\}$;

ii. "constants of integration" for $\{U_n, R_n, \Theta_n\}$ give the BMS generators;

iii. g_{uu}, g_{ua}, g_{ab} give Bondi aspects (mass and angular momentum) and shear in terms of the multipoles.

LINEAR ORDER: NU METRIC AND BMS TRANSFORMATIONS

Explicit expressions in linear theory of the Bondi fields are in [Blanchet, Compère, Faye, RO, Seraj, 2010:10000]

$$g_{uu} = -1 - G(\Delta + 2)\dot{f} + 2G\left(\frac{m}{r} + \sum_{n=2}^{+\infty} \frac{1}{r^n} \underset{(n)}{K}\right) + \mathcal{O}(G^2)\,,$$

$$g_{ua} = G\left(\frac{1}{2} D_b C^b{}_a + \frac{2}{3}\frac{N_a}{r} + e^i_a \sum_{n=2}^{+\infty} \frac{1}{r^n} \underset{(n)}{P^i}\right] + \mathcal{O}(G^2)\,,$$

$$g_{ab} = r^2\left[\gamma_{ab} + 2G D_{\langle a} Y_{b\rangle} + G\left(\frac{C_{ab}}{r} + e^i_{\langle a} e^j_{b\rangle} \sum_{n=2}^{+\infty} \frac{1}{r^n} \underset{(n)}{E^{ij}}\right)\right] + \mathcal{O}(G^2)\,.$$

MAIN ADVANTAGE:

WE GAINED THE SUB-LEADING TERMS IN $1/r$

FULL $1/r$ EXPANSION IN NU GAUGE !

To compute "celestial charges" ! [Compère, RO, Seraj, 2206:12597]

More explicitly, the Bondi mass aspect reads as

$$m = \sum_{\ell=0}^{+\infty} \frac{(\ell+1)(\ell+2)}{2\ell!} n_L M_L^{(\ell)} + \mathcal{O}(G)\,,$$

where tails and memory are !

the Bondi angular momentum aspect is

$$N_a = e^i_a \sum_{\ell=1}^{+\infty} \frac{(\ell+1)(\ell+2)}{2(\ell-1)!} n_{L-1}\left[M_{iL-1}^{(\ell-1)} + \frac{2\ell}{\ell+1}\varepsilon_{ipq} n_p S_{qL-1}^{(\ell-1)}\right] + \mathcal{O}(G)$$

and finally the Bondi shear is given by

$$C_{ab} = e^i_{\langle a} e^j_{b\rangle} H^{ij}_{\mathrm{TT}} = 4 e^i_{\langle a} e^j_{b\rangle} \perp^{ijkl}_{\mathrm{TT}} \sum_{\ell=2}^{+\infty} \frac{n_{L-2}}{\ell!}\left[M_{klL-2}^{(\ell)} - \frac{2\ell}{\ell+1}\varepsilon_{kpq} n_p S_{lqL-2}^{(\ell)}\right] + \mathcal{O}(G)$$

$$\delta_{BMS} C_{ab} = -2 D_{\langle a} D_{b\rangle} f$$

$$\perp^{ij} = \delta^{ij} - n^i n^j$$

$$\perp^{ijkl}_{\mathrm{TT}} = \perp^{k(i}\perp^{j)l} - \frac{1}{2}\perp^{ij}\perp^{kl}$$

PART III

Bondi aspects, charges and flux-balance laws

TWO QUALITATIVE DIFFERENT SETS OF LOCAL FLUX-BALANCE LAWS

In Bondi gauge, Einstein's equations reduce to a countable infinite set of local flux-balance equations on future null infinity

$$
\begin{aligned}
n=0 &: \quad \frac{1}{4} D_b D_c N^{bc} = -\mathcal{F}(u) + \partial_u m, \\
n=1 &: \quad -\frac{u}{2} D_c D_{\langle a} D_{b\rangle} N^{bc} = -\mathcal{F}_a(u) + \partial_u \mathcal{N}_a, \\
n=2 &: \quad \frac{u^2}{12} \mathrm{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(2)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(2)}{\mathcal{E}_{ab}}, \\
n \geq 3 &: \quad \frac{(-u)^n}{6\,n!} \mathcal{D}_{n-3} \cdots \mathcal{D}_0\, \mathrm{STF}_{ab}[D_a D_c D_{\langle b} D_{d\rangle} N^{cd}] = -\underset{(n)}{\mathcal{F}_{ab}}(u) + \partial_u \underset{(n)}{\mathcal{E}_{ab}}\,.
\end{aligned}
$$

$$
\mathcal{Q}^+_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}}\, D_a D_b \hat{n}_L
$$

$$
\partial_u \mathcal{Q}^+_{n,L}(u) = \oint_S \underset{(n)}{\mathcal{F}^{ab}}\, D_a D_b \hat{n}_L + \frac{(-u)^n}{6\ n!} \oint_S \hat{n}_L D^{\langle b} D^{a\rangle} \mathcal{D}_{n-3} \cdots \mathcal{D}_0 D_a D_c D_{\langle b} D_{d\rangle} N^{cd},
$$

$\mathcal{Q}^{\pm}_{n,L}$ proportional to $Lw_{1+\infty}$ charges in [Freidel-Pranzetti-Raclariu, 2112.15573]

[Compère, RO, Seraj, 2206:12597]

MEMORY-LESS FLUX-BALANCE LAWS

$n=0 : \ \ell = 0$ ENERGY LOSS FORMULA AND $\ell = 1$ MOMENTUM LOSS FORMULA
$n=1 : \ \ell = 1$ ANGULAR AND CENTER-OF-MASS LOSS FORMULAE

$n = 2 : \varnothing$
$n \geq 3 : 2 \leq \ell \leq n-1$ (GENERALISED) NEWMAN-PENROSE CHARGES

MEMORY-FULL FLUX-BALANCE LAWS

$n = 0, \ell \geq 2$, DISPLACEMENT MEMORY EFFECT
$n = 1, \ell \geq 2$, SPIN AND CENTER-OF-MASS MEMORY EFFECTS

$n \geq 2, \ell \geq n$, SUBLEADING PERMANENT EFFECTS

CELESTIAL CHARGES – EXPLICIT EXPRESSIONS IN LINEARISED THEORY

We wish to compute $\mathcal{Q}^+_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}} D_a D_b \hat{n}_L$ in terms of multipole moments.

In the linear theory:

$$\mathcal{Q}^+_{n,L}(u) \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}} D_a D_b \hat{n}_L = \begin{cases} \displaystyle\sum_{p=n-l-1}^{n-3} q_{n,\ell,p}\, u^{p+1} M_L^{(\ell-n+p+1)} + b_{n,\ell}\, u^{n-1}\left(1 - \frac{u}{n}\partial_u\right) M_L^{(\ell-1)} + \mathcal{O}(G) & 2 \leq \ell \leq n-1 \\ a_{n,\ell}\, M_L^{(\ell-n)} + \displaystyle\sum_{p=0}^{n-3} q_{n,\ell,p}\, u^{p+1} M_L^{(\ell-n+p+1)} + b_{n,\ell}\, u^{n-1}\left(1 - \frac{u}{n}\partial_u\right) M_L^{(\ell-1)} + \mathcal{O}(G) & \ell \geq n \end{cases}$$

[Compère, RO, Seraj, 2206:12597]

$\mathcal{Q}^-_{n,L}(u)$ same expression with $M_L \to \frac{2l}{l+1} S_L$

By explicit computation: $\mathcal{Q}^+_{3,ij}(u) = 0 + \mathcal{O}(G^2)$

NEWMAN-PENROSE CHARGES IN THE MULTIPOLAR EXPANSION

NP charges are defined as

$$Q_m \equiv \oint_S {}_2\overline{Y}_{2m}\Psi_0^1\,, \qquad \Psi_0 = -C_{\mu\nu\alpha\beta}\,\ell^\mu m^\nu \ell^\alpha m^\beta = \frac{\psi_0^0}{r^5} + \frac{\psi_0^1}{r^6} + \dots$$

> ity. But it turns out that, quite unexpectedly, there is a set of 10 geometrical quantities, defined for asymptotically flat space-times, which have a quadrupole structure, and whose values cannot be altered in any way by gravitational radiation.
>
> [Newman-Penrose, 1965]

Explicitly, using the NP null tetrad $\ell = \partial_r \qquad m = \frac{1}{r}\left(\zeta^a - \frac{1}{2r}C^a{}_b\zeta^b + O(r^{-2})\right)\partial_a + \omega\partial_r$

$$\Psi_0^1 = 6\,\underset{(3)}{E_{ab}}\,\zeta^a\zeta^b \qquad E_{ij} = e_i^a e_j^b\,\underset{(3)}{E_{ab}} = G^2\,(\text{inst. terms } M_{ij}\times M_{ij}) + 5G^2M\perp^{\mathrm{TT}}_{ijkl} M_{kl}(-\mathcal{T})$$

It comes from the leading expansion of the tail term

$$g_{ab}|_{tail} \propto \int_0^{+\infty} \mathrm{d}z\, \frac{18 + \frac{8z}{r} + \frac{z^2}{r^2}}{(1+\frac{z}{2r})^4} M_{ij}(u-z)$$

$$Q_{ij} = 5G^2M\oint_S\left[2M_{ij}(-\mathcal{T}) - 2M_{il}(-\mathcal{T})n_jn_l - 2M_{jl}(-\mathcal{T})n_{il} + n_{ikjl}M_{kl}(-\mathcal{T})\right]$$
$$= 4G^2MM_{ij}(-\mathcal{T})\,.$$
$$= \mathcal{Q}^+_{3,ij}$$

NP charges = ADM mass x quadrupole moment (at early time!)

ASYMPTOTIC CHARGES IN THE MULTIPOLAR EXPANSION

"Celestial charges"

$$\mathcal{Q}^+_{n,L} \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}} \, D_a D_b \hat{n}_L \,, \qquad \mathcal{Q}^-_{n,L} \equiv \oint_S \underset{(n)}{\mathcal{E}^{ab}} \, \varepsilon_{ac} D_b D^c \hat{n}_L$$

(because we set $S_L = 0$)

WITHOUT MEMORY TERMS $n \geq 3$ *and* $2 \leq \ell \leq n-1$

$$\mathcal{Q}^+_{3,ij} = 4G^2 M M_{ij}(-\mathcal{T})\,,$$

(NP conserved charges)

$$\mathcal{Q}^+_{n,L} = \,?$$

(new conserved charges?)

WITH MEMORY TERMS $n \geq 2$, *and* $\ell \geq n$

$$\mathcal{Q}^+_{2,ij} = \frac{8}{5}G\left(M^{\text{rad}}_{ij} - uM^{\text{rad}(1)}_{ij} + \frac{1}{2}u^2 M^{\text{rad}(2)}_{ij}\right) + \frac{8}{35}G^2\left[-7M^{(1)}_{ik}M^{(2)}_{jk} + M_{ik}M^{(3)}_{jk} + u\left(7M^{(2)}_{ik}M^{(2)}_{jk} + 6M^{(1)}_{ik}M^{(3)}_{jk} - M_{ik}M^{(4)}_{jk}\right)\right.$$

$$\mathcal{Q}^+_{2,ijkl} = G^2\left[-4\left(M^{(1)}_{ij}M^{(2)}_{kl} + M_{ij}M^{(3)}_{kl}\right) + 4u\left(M^{(2)}_{ij}M^{(2)}_{kl} + 2M^{(1)}_{ij}M^{(3)}_{kl} + M_{ij}M^{(4)}_{kl}\right) + u^2\left(-\frac{68}{7}M^{(2)}_{ij}M^{(3)}_{kl} - 6M^{(1)}_{ij}M^{(4)}_{kl} - 2M_{ij}M^{(5)}_{kl}\right)\right]^{\text{STF}},$$

$$\mathcal{Q}^+_{3,ijkl} = G^2\left[\frac{8}{3}M^{(1)}_{ij}M^{(1)}_{kl} - 14M_{ij}M^{(2)}_{kl} + u\left(\frac{26}{3}M^{(1)}_{ij}M^{(2)}_{kl} + 14M_{ij}M^{(3)}_{kl}\right) + u^2\left(-\frac{13}{3}M^{(2)}_{ij}M^{(2)}_{kl} - 7(2M^{(1)}_{ij}M^{(3)}_{kl} + M_{ij}M^{(4)}_{kl})\right) + u^3\left(\frac{34}{3}M^{(2)}_{ij}M^{(3)}_{kl} + 7M^{(1)}_{ij}M^{(4)}_{kl} + \frac{7}{3}M_{ij}M^{(5)}_{kl}\right)\right]^{\text{STF}}.$$

Each of them obey a flux-balance law!

CONCLUSIONS AND OUTLOOK

Summary:

i. Powerful interplay between MPM and asymptotic analysis

ii. Explicit NU/Bondi metric with inst., mass x quadrupole and quadrupole x quadrupole interactions

iii. Connection of the asymptotic charges with the multipoles of the gravitational field

iv. Investigation of sub-leading coefficients in the asymptotic metric

Next steps:

i. Systematic extension of the algorithm (incl. spin x quadrupole) to complete the NU/Bondi metric at 2PM

ii. Investigation of late time behaviour of asymptotic metric / charges

iii. Generalisation of NP charges $\mathcal{Q}^{\pm}_{n,L}$ with $n > 3$ and $2 \leq \ell \leq n-1$. Are they conserved such as NP charges ?

iv. Peeling property / log-in-r <–––> assumption in the MPM formalism (no incoming radiation/stationarity in the past)

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?