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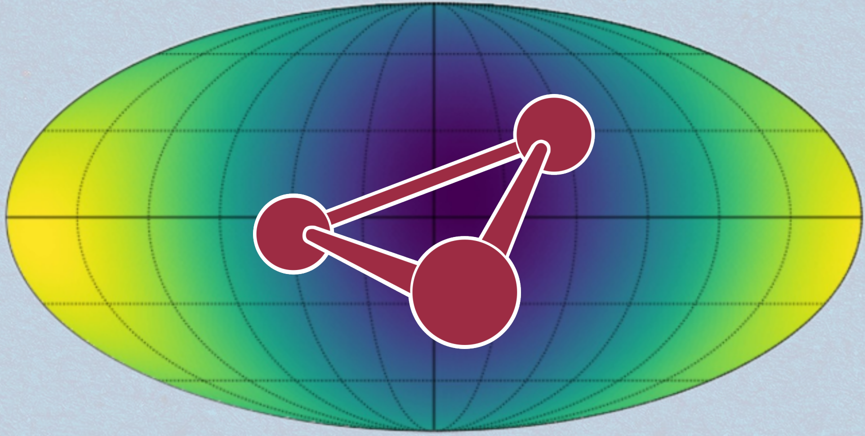

Observing Kinematic Anisotropies of a Stochastic Gravitational Waves Background with LISA

Speaker: Dr. Henri Inchauspé
Co-authors: David Maibach, Prof. Lavinia Heisenberg
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7ème Assemblée GdR Ondes Gravitationnelles
Meudon, Octobre 2023

Scope and objectives

- Main challenge of the search for SGWB signal with LISA:

  How do we distinguish a potential cosmological signal from instrumental noise (with a single interferometer in space)

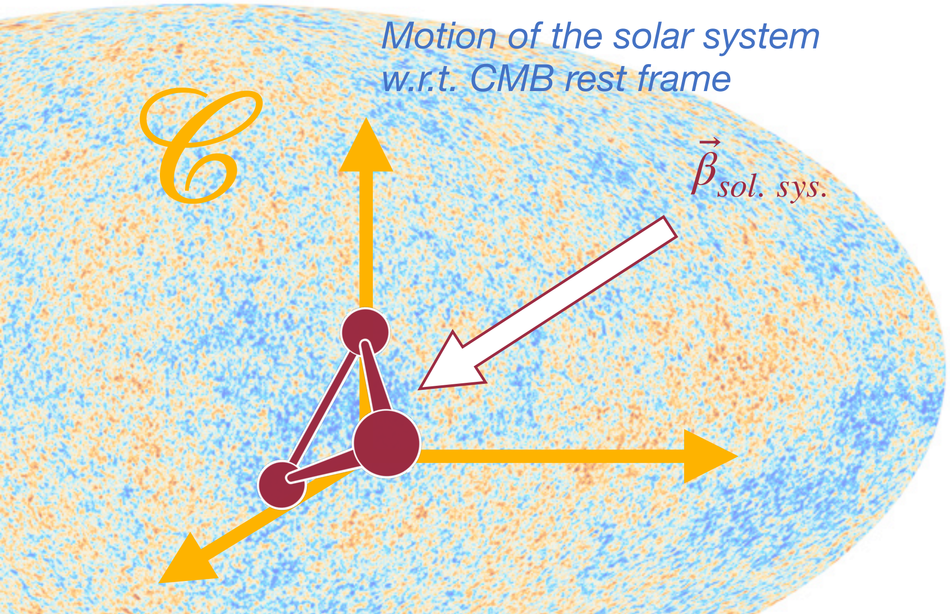
  (+ from galactic confusion noise, astrophysical background..)

  On what kind of evidence can we claim an apparent excess of power is cosmological ?

1. The instrument response projects differently noise and signal on data.
2. The signal has distinctive features not shared with the noise (anisotropy)

➡ **Kinematic anisotropy is a signature of an extragalactic origin**

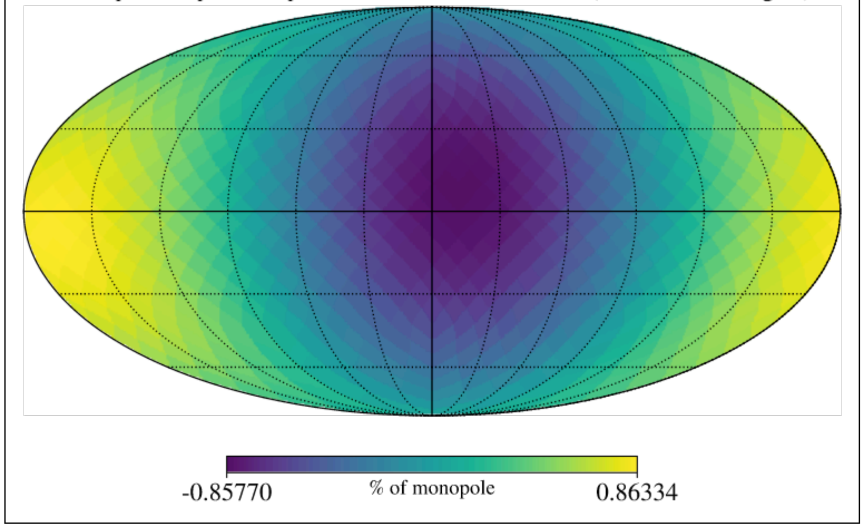
Principle: Doppler boosting of the SGWB



Solar system velocity $\approx 369 \mathrm{km\,s^{-1}}$ w.r.t to CMB rest frame

$$\rightarrow \beta = 1.23 \times 10^{-3}$$

Subsequent amplitude dipolar modulation in SSB frame (for a scale-free signal)

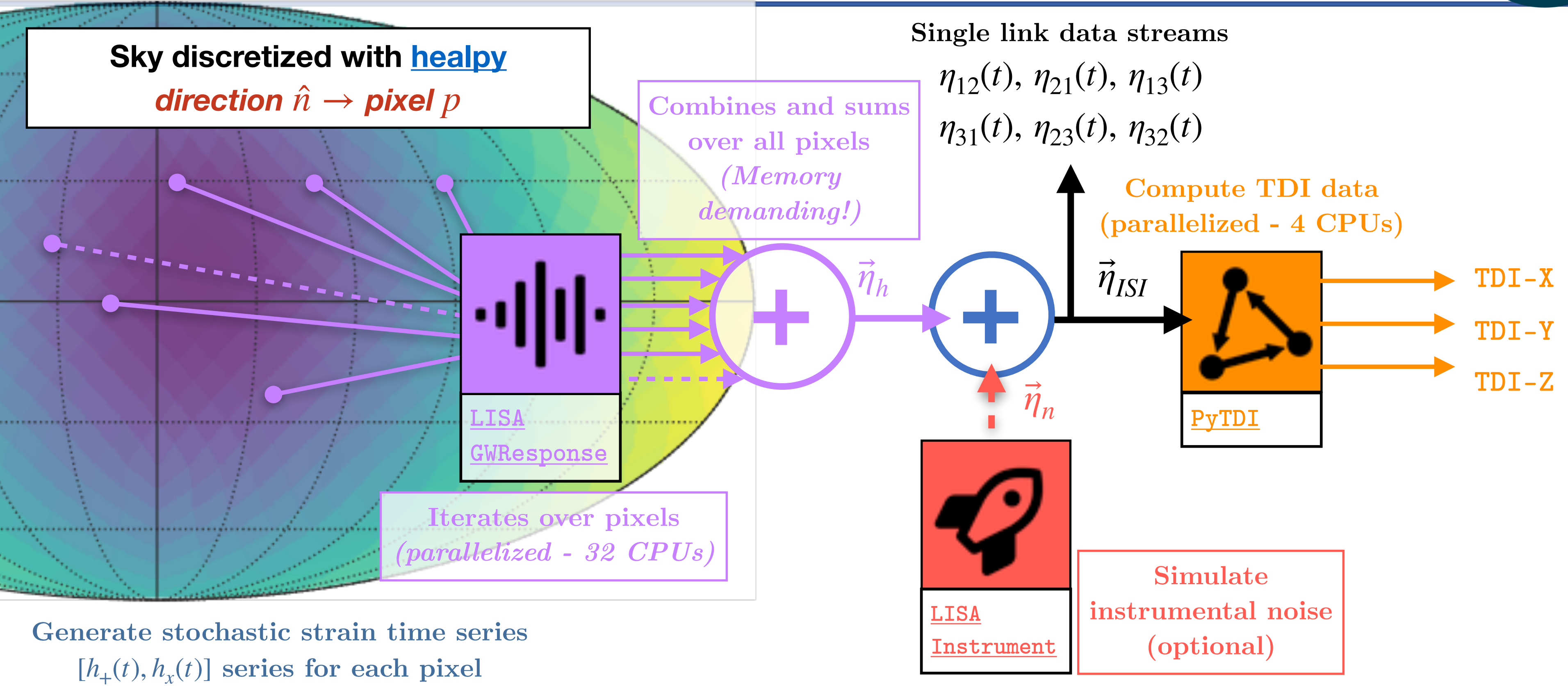


$$\mathscr{D} = \frac{1 - \beta^2}{1 - \beta\ \hat{n} \cdot \hat{v}}$$

$$\Omega_{GW}(f, \hat{\mathbf{n}}) = \mathscr{D}^4\ \Omega'_{GW}(f)\big(\mathscr{D}^{-1} f\ \big)$$

Cusin et al. 2022, "Doppler boosting the stochastic gravitational wave background"

DATA generation: full **time-domain** simulation of GW anisotropic sky, via `LISA Simulation Suite`



Sky discretized with healpy

direction $\hat{n} \rightarrow$ pixel p

Combines and sums over all pixels *(Memory demanding!)*

Single link data streams

$\eta_{12}(t),\ \eta_{21}(t),\ \eta_{13}(t)$

$\eta_{31}(t),\ \eta_{23}(t),\ \eta_{32}(t)$

Compute TDI data (parallelized - 4 CPUs)

Iterates over pixels *(parallelized - 32 CPUs)*

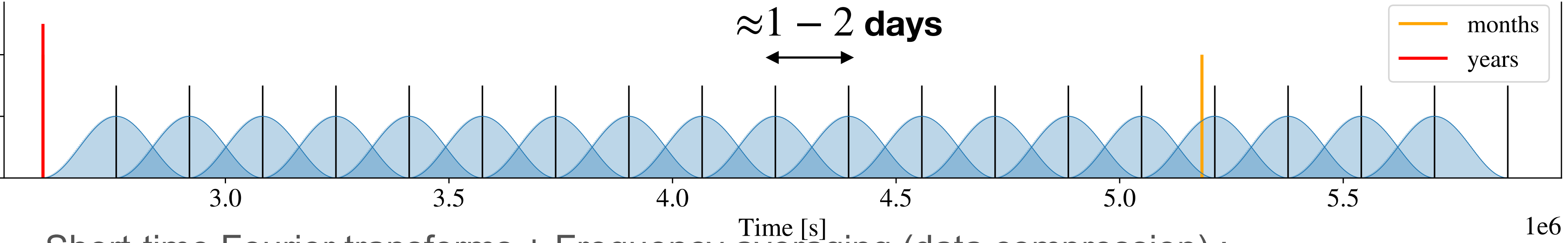
Simulate instrumental noise (optional)

Generate stochastic strain time series $[h_+(t), h_x(t)]$ series for each pixel

Map-making strategy: pre-processing the **DATA**

- Time-splitting the 3 years long TDI data streams
  Hanning window, 50% overlapping segments

Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"



- Short-time Fourier transforms + Frequency averaging (data compression) :

$$\bar{\mathbf{D}}(t_i, f_j) \equiv \frac{1}{n_j} \sum_{k=j-\frac{n_j}{2}}^{j+\frac{n_j}{2}} \tilde{\mathbf{d}}(t_i, f_k)\tilde{\mathbf{d}}(t_i, f_k)^\dagger \, .$$

Baghi et al. 2023, "Uncovering gravitational-wave backgrounds from noises of unknown shape with LISA"

TDI X, Y, Z data streams

- DA problem: we're solving for the covariance C_d of the signal $\tilde{\mathbf{d}}$.

Covariance **MODEL** and max likelihood map-making strategy

- Covariance model:

[Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"]

$$\mathbf{C}_d(t_i, f_j) = \mathbf{A}(t_i, f_j, p)\ I(p)\ + \mathbf{N}(t_i, f_j)$$

$$I(f, \hat{n}) = \Omega_{\mathrm{GW}}(f, \hat{n}) \frac{3H_0^2}{4\pi^2 f^3}$$

LISA (+TDI) quadratic response (Freq. domain model)

Pixel Map to solve for

Instrumental Noise

Sky discretized with healpy

direction $\hat{n}$ → ***pixel*** p

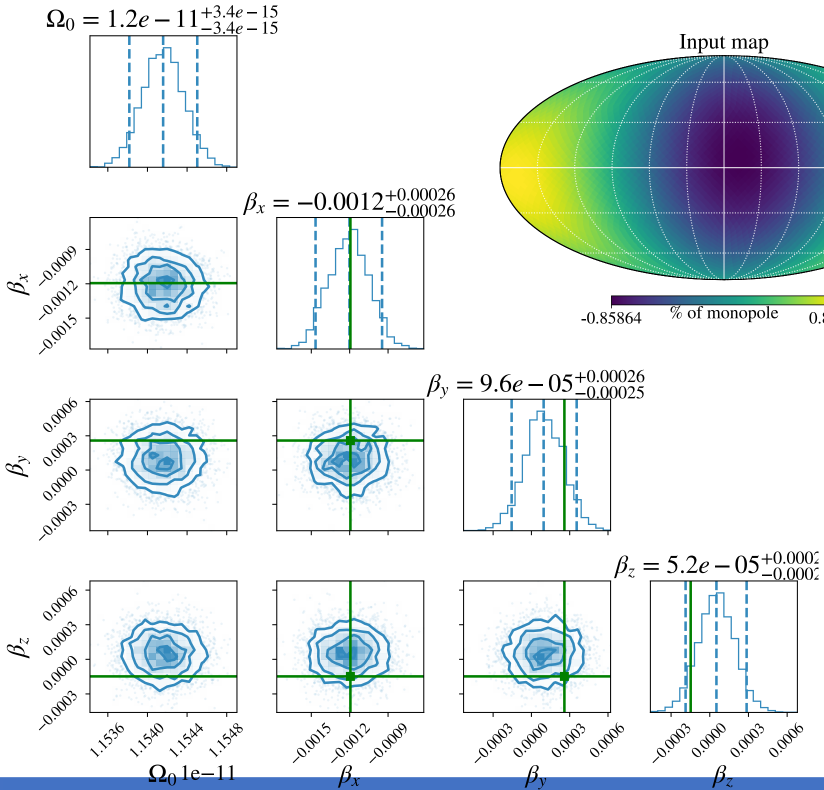
- log-Likelihood, Wishart statistics:

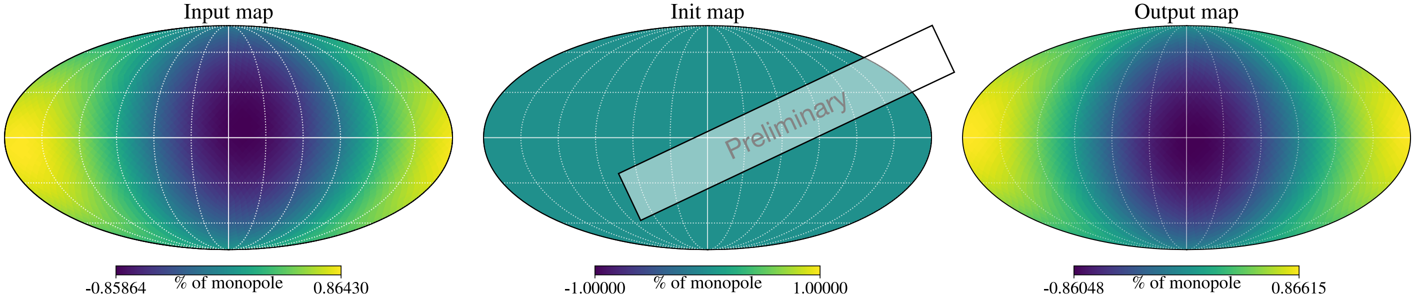
$$\log \mathscr{L} = \sum_{t_i} \sum_{f_j} \left[-\mathrm{tr}(\mathbf{C}_d^{-1} \mathbf{D}(t_i, f_j)) - \nu \log |\, \mathbf{C}_d(t_i, f_j) \,| \right]$$

[Baghi et al. 2023]

MCMC sampling the velocity: recovered sky maps

Building from Contaldi et al. 2020, extending to time-domain sim, spectrum averaging, and MCMC sampling





$$I(f,p) = \mathscr{D}\; I'(f)\left(\mathscr{D}^{-1}f\;,\;p\right)$$

$$\mathscr{D} = \frac{1-\beta^2}{1-\beta\;\hat{n}\cdot\hat{v}}$$

Fitting for $\vec{\beta}$ and $I'(f_0)$

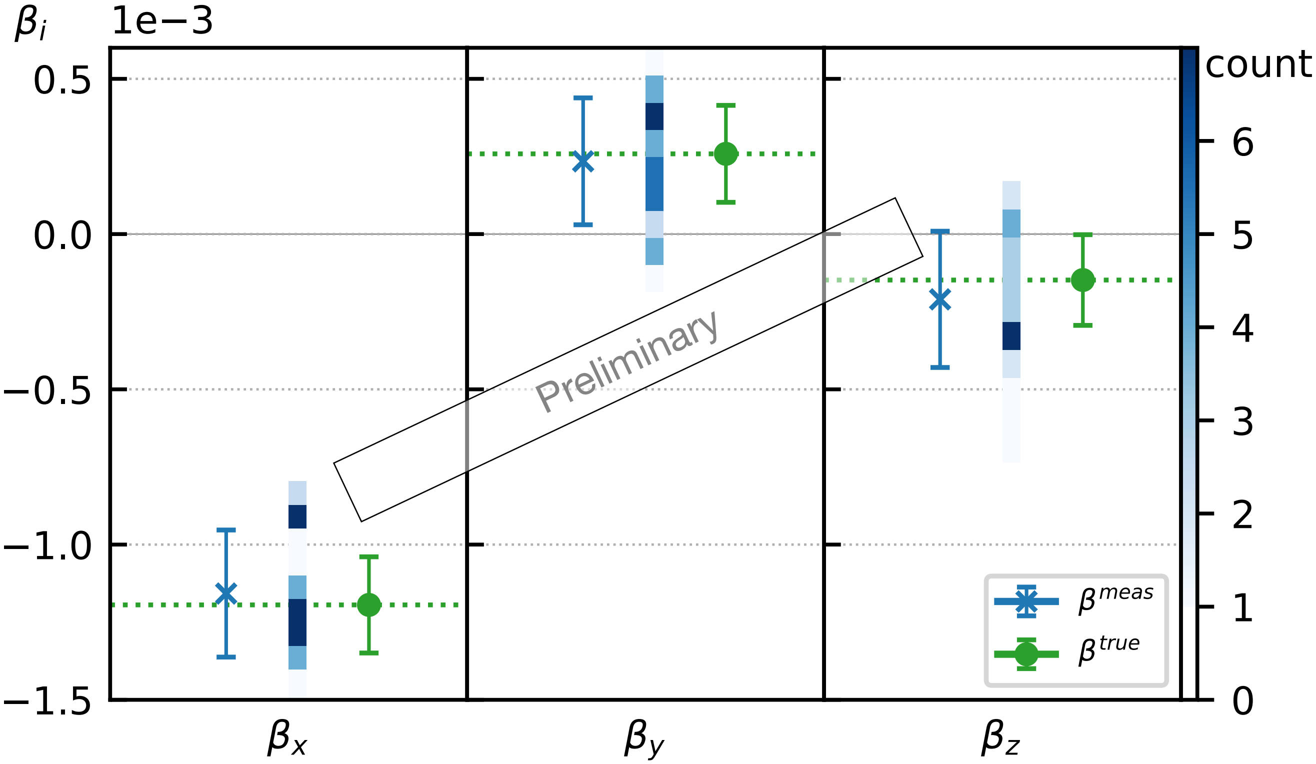
Could also Taylor expand in power of β:

$$\Omega_{GW}(f,\mathbf{n}) = \Omega'_{GW}(f)\left[1+M(f)+\mathbf{v}\cdot\mathbf{n}\;D(f)+\left((\mathbf{v}\cdot\mathbf{n})\right)^2+\frac{1}{3}Q(f)\right]$$

30 sky realization statistical test - velocity $\vec{\beta}$

- 30 measured $\vec{\beta}^{meas}$: mean values and standard deviations
- $\vec{\beta}^{true}$ with theoretical error bars from MCMC
- Histogram of $\vec{\beta}^{meas}$ values

$\vec{\beta}_x$ **resolved !**

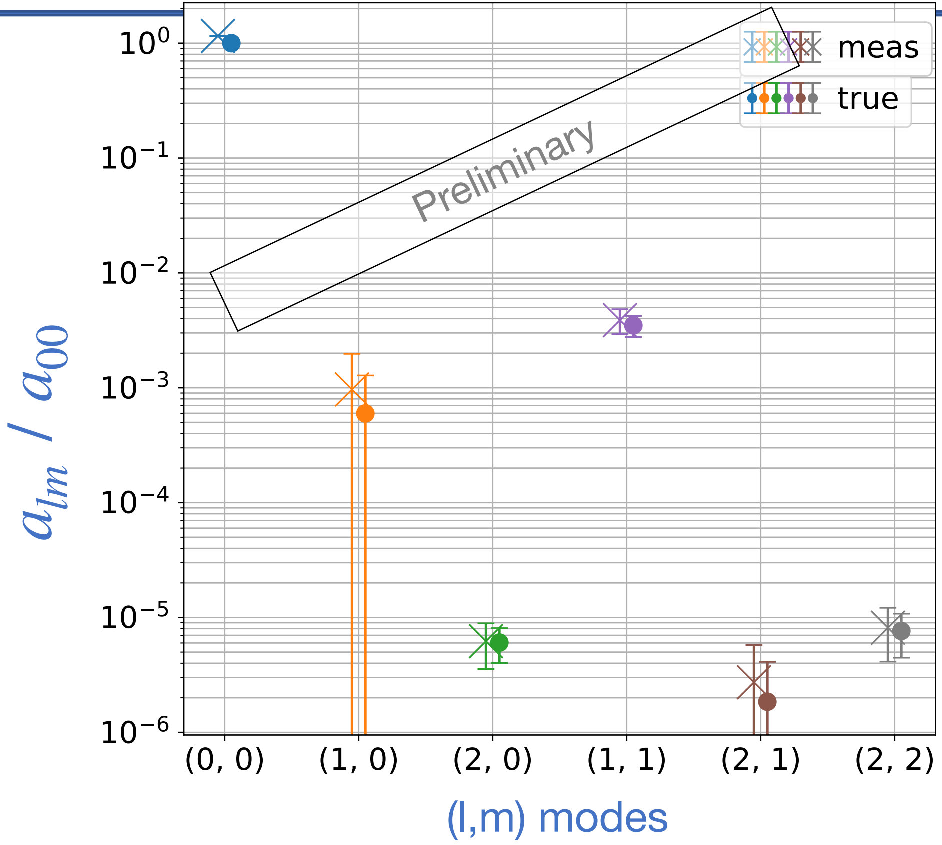


30 sky realization statistical test - $a_{\ell m}$

- Conversion of $\vec{\beta}^{meas}$ statistics to $a_{\ell m}$ counterpart
- Main mode resolved (1,1)
- Start to be sensitive to (2,0) and (2,2) !

Dipole: higher signal, but less responsive

Quadrupole: reduced signal, but more responsive



Conclusion & Perspectives

- End-to-end simulation and analysis of an anisotropic GW sky with LISA.
- With up-to-date and most complete simulation tools of the consortium to date (LISA GWResponse, LISA Instrument, PyTDI)
- Validation of the method to recover kinematic anisotropy, induced on scale-free SGWB signal (spectral index $\alpha = 0$), **for noiseless instrument.**
- What's next ?

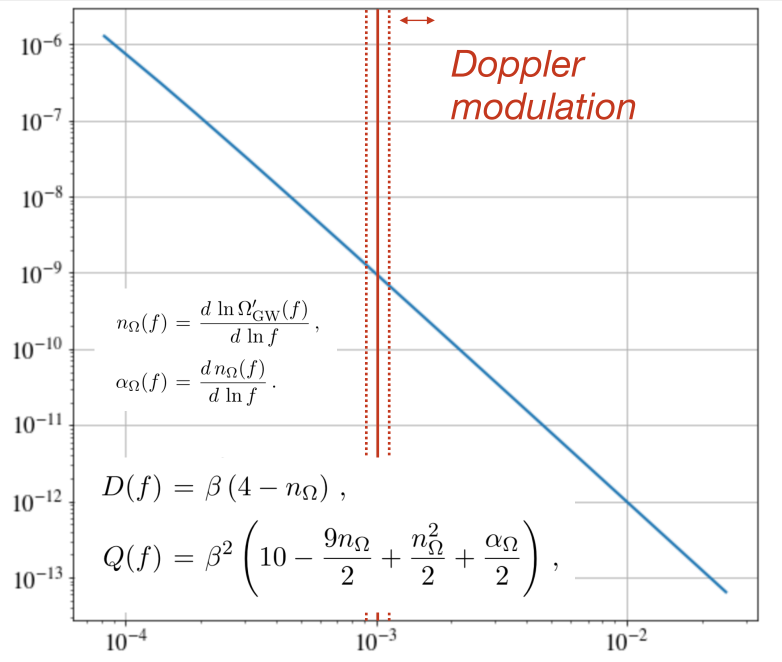
1. **Investigate time-frequency data representation (wavelets).**
2. Apply the method to SGWB with **richer spectrum profiles** (broken power laws, peaks)
   Sharp spectrum transition, breaks, peaks…
   → can boost the SNR a lot (dipole AND quadrupole)

$$D(f) = \beta\,(4 - n_\Omega)\ ,$$
$$Q(f) = \beta^2 \left(10 - \frac{9n_\Omega}{2} + \frac{n_\Omega^2}{2} + \frac{\alpha_\Omega}{2}\right)\ ,$$

3. Apply the method to **the mapping of the galactic foreground** (on LDC data!)

Back slides

Principle: Doppler boosting of the SGWB



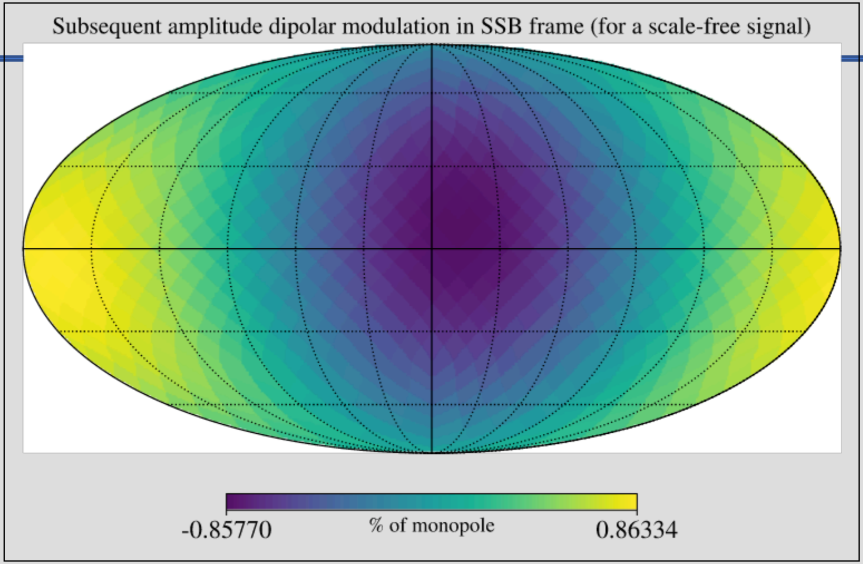
Doppler modulation

$$n_{\Omega}(f) = \frac{d\,\ln\Omega'_{\rm GW}(f)}{d\,\ln f}\,,$$

$$\alpha_{\Omega}(f) = \frac{d\,n_{\Omega}(f)}{d\,\ln f}\,.$$

$$D(f) = \beta\,(4 - n_{\Omega})\,,$$

$$Q(f) = \beta^2\left(10 - \frac{9n_{\Omega}}{2} + \frac{n_{\Omega}^2}{2} + \frac{\alpha_{\Omega}}{2}\right)\,,$$



Subsequent amplitude dipolar modulation in SSB frame (for a scale-free signal)

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1. Cusin et al. 2022, "Doppler boosting the stochastic gravitational wave background"

2. Bartolo et al. 2022, « Probing anisotropies of the Stochastic Gravitational Wave Background with LISA »

DATA generation: full **time-domain** simulation of GW anisotropic sky, via `LISA Simulation Suite`

- Physical assumptions:
  - ✓Pixel stochastic strain time series uncorrelated
  - ✓*Equal arm* or *keplerian* orbits.
  - ✓TDI 2.0
  - ✓**Arm propag. delays in TCB time**
  - ✓Secondary noise only (when noise on)

- Simulation settings
  - 3 years, sampling frequency = [0.05 Hz / 0.2 Hz]
  - Number of pixels: [12288 / 3072]
  - Cosmological signal: $\alpha = 0, \Omega = [10^{-12}, 10^{-7}]$

LISA response

$$G_{lm,p}(f',t,\hat{\mathbf{k}}) = \frac{\xi_p\big(\hat{\mathbf{u}}_k, \hat{\mathbf{v}}_k, \hat{\mathbf{n}}_{lm}\big)}{2\Big(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}_{lm}(t)\Big)}\left[e^{-\frac{2\pi i f'}{c}\left(L_{lm}(t)+\hat{\mathbf{k}}\cdot\mathbf{x}_m(t)\right)} - e^{-\frac{2\pi i f'}{c}\hat{\mathbf{k}}\cdot\mathbf{x}_l(t)}\right]. \qquad \text{(B.5)}$$

From Baghi et al. 2023

$$\begin{aligned} X_2 = X_1 &+ \mathbf{D}_{13121}y_{12} + \mathbf{D}_{131212}y_{21} + \mathbf{D}_{1312121}y_{13} + \mathbf{D}_{13121213}y_{31} \\ &- \left[\mathbf{D}_{12131}y_{13} + \mathbf{D}_{121313}y_{31} + \mathbf{D}_{1213131}y_{12} + \mathbf{D}_{12131312}y_{21}\right], \end{aligned}$$

$$\mathbf{D}_{ij}\tilde{x}(f) \approx \tilde{x}(f)e^{-2\pi i f L_{ij}}.$$

Covariance **MODEL** and max likelihood map-making strategy

Contaldi et al. 2020, "Maximum likelihood map making with the Laser Interferometer Space Antenna"

- Covariance model:

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LISA quadratic response

Pixel Map to solve for

Instrumental Noise

$$I(f, \hat{n}) = \Omega_{\mathrm{GW}}(f, \hat{n}) \frac{3H_0^2}{4\pi^2 f^3}$$

Sky discretized with healpy

direction $\hat{n} \rightarrow$ ***pixel*** p

- LISA quadratic response:

$$A(t_i, f_j, p) = R_+(t_i, f_j, p) \otimes R_+(t_i, f_j, p)^* \ + \ R_\times(t_i, f_j, p) \otimes R_\times(t_i, f_j, p)^*$$

$$R_P(t_i, f_j, p) = M_{TDI}(t_i, f_j)\ G_P(t_i, f_j, p)\ M_{TDI}(t_i, f_j)^\dagger$$

TDI matrix (phasing operators)

single link response (freq. domain, at time t_i)

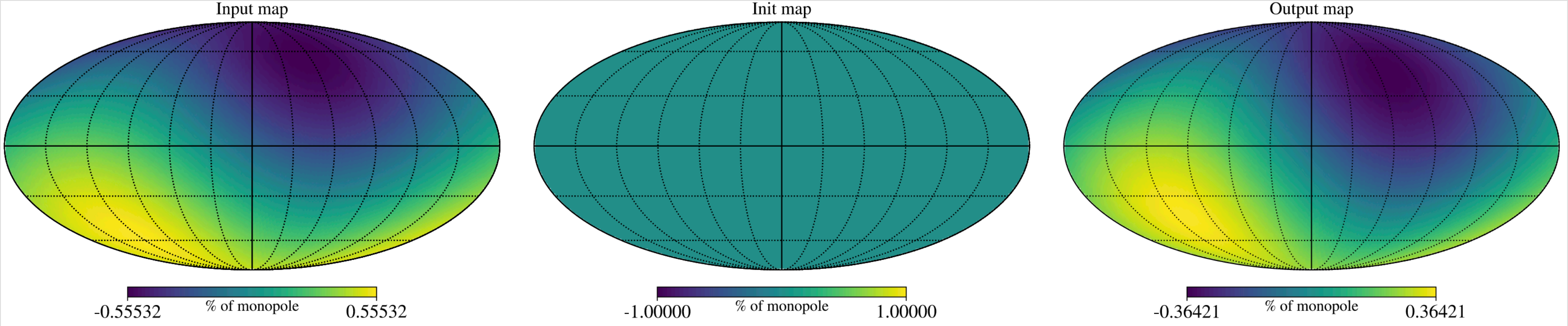
TDI matrix (phasing operators)

- log-Likelihood, Wishart statistics:

Baghi et al. 2023

$$\log \mathscr{L} = \sum_{t_i} \sum_{f_j} \left[-\mathrm{tr}(\mathbf{C}_d^{-1} \mathbf{D}(t_i, f_j)) - \nu \log | \mathbf{C}_d(t_i, f_j) | \right]$$

MCMC sampling the alms: **artificially rotated input** - *Sanity check*



Work from D. Maibach (Heidelberg Univ.)