

GPDs: Combining Experimental and Simulation Data

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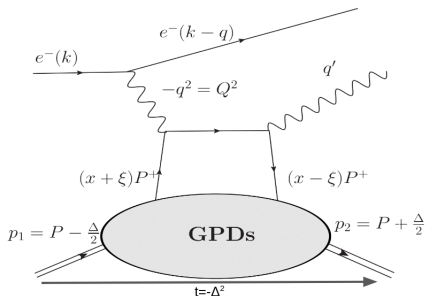


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Admit a 3D probabilistic interpretation

Forward Limit

$x = \frac{k^+}{p^+}$ is the average momentum fraction of the struck quark

$= \frac{\xi^+}{2p^+}$ is the skewness, or lightcone "kick"

$t = -Q^2$ is the mandelstam variable, which we do not treat here and simply set to 0

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$$\lim_{t \rightarrow 0} \lim_{\xi \rightarrow 0} \text{GPD}(x; \xi; t) = \text{PDF}(x_{BJ})$$

This is because the GPDs are a generalization of PDFs from matrix elements diagonal in momentum space to analogous matrix elements which are **o-diagonal in momentum space**

GPD Modeling

GPDs contribute to DVCS cross sections via Compton Form Factors via a convolution in x . At leading order in the strong coupling

$$H = \int_0^1 dx H(x; ; t) \left(\frac{1}{x - i} - \frac{1}{+x - i} \right) \quad (1)$$

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Enter: GPD Modeling using artificial neural networks to

- | Fulfill some theoretical constraints at the level of network architecture
- | Assess systematic uncertainties inherent to this uninvertible problem

Generation

The set of GPD replicas as a function of α and β (neglecting dependence) was produced by fitting a set of GK model GPD pseudodata using a set of ANNs [H. Dutrieux et al., 2022]

Generation

The set of GPD replicas as a function of μ and σ (neglecting dependence) was produced by fitting a set of GK model GPD pseudodata using a set of ANNs [H. Dutrieux et al., 2022]

Modeling took place in double distribution space in order to ensure both polynomiality

$$\int_{\mathbb{R}} dx x^n H^q(x; \mu, \sigma) = \sum_{i=0}^{\lfloor n/2 \rfloor} \binom{n}{2i} (2\sigma)^{2i} A_{n+1;2i}^q + \text{mod}(2; n) (2\sigma)^{n+1} C_{n+1}^q$$

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Positivity was enforced numerically

Example Replica Set

= 0:1:

Some replicas deviate greatly from the central value when (No ERBL positivity constraint exists)

How might the replica band be further constrained?

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assigning each a weight w_k using a Bayesian reweighting procedure based on the introduction of mock lattice data

assessing the reduction of uncertainty in both l_0 and \bar{x} spaces by using the weights w_k to calculate "Reweighted" central values and error bars

Weights are robust against transformations of replicas

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The blocks correspond to the three regions in We choose to reweight using such blocks as:

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- | 2:2 | 4, = 0:2
- | 4:4 | 6, = 0:4

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- that is the region in which lattice data may be provided given the current state of the arts
- the lattice signal vanishes around $\beta = 10$ [Egerer et al., 2021]

Why Reweight in Blocks at Low ?

Goal

Reweighting at **low** values of β may then be used to constrain them in the **high** region

Procedure

1: Calculate the central value μ_j of the set of replicas at each value μ_j

Procedure

2: Assign a corresponding standard deviation to each mock lattice point defined as $\sigma_i = \sigma_i f(x_i; b)$ where b determines the base of an exponential function f constrained by $f(0; b) = 0.05$, $f(10; b) = 1$

Mock Lattice Data Fabrication: An Example

Bayesian Reweighting

Relevant Metrics

Effective Fraction of Replicas retained after reweighting(f_k)

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Central Values: Raw: \bar{x} ; Reweighted: \bar{x}_w

Standard Deviations: Raw: σ_x ; Reweighted: σ_{x_w}

Local Uncertainty Retainment:

$$x: \left(\bar{x} \pm \frac{\sigma_x}{\sqrt{N}} \right); \quad x_w: \left(\bar{x}_w \pm \frac{\sigma_{x_w}}{\sqrt{N}} \right)$$

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Standard Deviations: Raw: σ_x ; Reweighted: σ_{x_w}

Local Uncertainty Retainment:

$$x: \left(\frac{\sigma_x}{x} \right); \quad : \left(\frac{\sigma_{x_w}}{x_w} \right)$$

Global Uncertainty Retainment:

$$x: r_{\ln x} = \frac{R}{\log(D)} \frac{dx}{x}; \quad : r = \frac{R}{d} \frac{d}{D} \left(\frac{\sigma_x}{x} \right)$$

GPD Replicas and Bands: Shown $\rho = 0.1$; Used: $j = 2$ f $0.1g$

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GPD Replicas and Bands: Shown $\beta=0.1$; Used: $j \geq 2$ f 0.1g

Low correlation; High precisionh Extremely Constraining

Results: $r_{\ln x} = 0.76$, $r = 0.28$, $r = 0.14$

() is at and r is low as replicas are coherent

(x) Peaks above 1 because mock lattice data is used to prioritize replicas based on their low behaviour, and highly weighted replicas may decohere at high (is relatively small)

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GPD Replicas and Bands: Shown $p = 0.5$; Used: $j = 2$ f $0.5g$
Low correlation; High precision \Rightarrow Extremely Constraining
Results: $r_{\ln x} = 0.47$, $r_{\ln y} = 0.12$, $r_{\ln z} = 0.23$

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space replicas are less coherent (pronouncedly at high g)
increased skewness implies less constraint from positivity as ERBL
support increases

GPD Replicas and Bands: Shown $p=0.5$; Used: $j \in [2, 5]$
Low correlation; High precision Extremely Constraining
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Local x space uncertainty retainment is decreased around $x=0.1$ due
to presence of mock lattice data, but is less drastic at high x due to
the positivity constraint

GPD Replicas and Bands: Shown $\rho = 0.5$; Used: $j = 2$ f 0.1, 0.2, 0.3, 0.4, 0.5g

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Approximate replication of uncertainty retainment of one highly
constraining data set by the current 5 sets of moderate constraining
ability data in 0:5

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Low precision, low correlation (top left) and high precision, high correlation (bottom right) reweightings yield similar uncertainty reductions with similar values of (! similar ANN replica generation costs)

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Lattice facility of each of these two compromisory options is to be further investigated

Conclusions

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We now have a consistent way to combine experimental and lattice data

- | Lattice data help to reduce the deconvolution uncertainties in momentum space by 25-50% at 0th order of the strong coupling.

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Calculus of the comparability of many low precision and few high precision reweightings at low correlation should be further investigated

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Lattice data would be more than welcome!

Thank You

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Replica Weights w_k & Effective Fraction of Replicas

A corresponding set of weights w_k are then calculated from the $\frac{2}{k}$ and the number of (mock) lattice data values introduced as

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We also define $f_{\text{eff}} = \frac{\sum_k w_k \ln(\binom{2}{k})}{N_{\text{rep}}}$ as the effective fraction of replicas retained after the reweighting is completed, where N_{rep} is the range of the index k

The Trouble with Outliers

We began by calculating the reweighted central value $\mu(\beta; x)$ and uncertainties $\sigma(\beta; x)$ as a function of β or x as

$$\mu(\beta; x) = \frac{\sum_k w_k R_k(\beta; x)}{\sum_k w_k}$$

$$\sigma(\beta; x) = \frac{1}{\sum_k w_k} \sqrt{\sum_k w_k (R_k(\beta; x) - \mu(\beta; x))^2}$$

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$$\mu(\hat{\mu}; x) = \frac{\sum_k w_k R_k(\hat{\mu}; x)}{\sum_k w_k}$$
$$R(\hat{\mu}; x) = \frac{1}{\sum_k w_k^2} \sum_k w_k (R_k(\hat{\mu}; x) - \mu(\hat{\mu}; x))^2$$

However, this method of estimation of the uncertainty associated with the reweighted central value was extremely sensitive to replicas far from the central value.

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We decided to locally employ the MAD (Median Absolute Deviation) to compute uncertainty bands

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At a value of q called q_{shown} use the weights to plot uncertainty bands. q_{shown} may or may not be present in the set q_{used}

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Nucleon Tomography

When $\lambda \neq 0$:

$|b_T|$ and p_T are Fourier Conjugates

One recovers a **Probabilistic Interpretation**

Figure: [Moutarde, Sznajder, and Wagner, 2018] Transverse position of quarks in an unpolarized proton as a function of the longitudinal momentum fraction x . Based on joint fit of CFFs to Hall A, CLAS, HERMES and COMPASS data.

Lattice Errors

Figure 9.a. of [Egerer et al., 2021]:

l o e Time

MAD (Median Absolute Deviation) Estimator

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We first calculate the central value $R(\theta; \mathbf{x})$ as the median of the set of replicas weighted by the weights w_k

We then estimate the uncertainty $R(\theta; \mathbf{x})$ as proportional to the median of a correspondingly weighted distribution given by

$$j \left| R(\theta; \mathbf{x}) - R_k(\theta; \mathbf{x}) \right|$$

- ! The longitudinal momentum fraction x , proportional to a quark's plus momentum is assigned a lightcone distance proportionality fraction z as a Fourier conjugate

$$\text{GPD}(;) = i \int_{-1}^1 dx \text{GPD}(x;) \sin(x z) \quad (2)$$

Results

j^0	j	c	b	r_{Inx}		r
0.1	0.1	0	1.1	0.82	0.47	0.25
0.1	0.1	0.5	1.1	1.02	0.83	0.85
0.1	0.1	0	2	0.78	0.3	0.16
0.1	0.1	0.5	2	0.82	0.46	0.23
0.5	0.5	0	1.1	0.67	0.36	0.44
0.5	0.5	0.5	1.1	0.64	0.52	0.58
0.5	0.5	0	2	0.54	0.11	0.25
0.5	0.5	0.5	2	0.77	0.37	0.51
0.5	0.1	0	1.1	1.24	0.47	0.92
0.5	0.1	0.5	1.1	1.15	0.83	0.93
0.5	0.1	0	2	1.08	0.3	0.9
0.5	0.1	0.5	2	1.23	0.46	0.91
0.5	0.1 0.2 0.3	0	1.1	0.95	0.3	0.62
0.5	0.1 0.2 0.3	0.5	1.1	1.0	0.77	0.82
0.5	0.1 0.2 0.3	0	2	0.54	0.1	0.34
0.5	0.1 0.2 0.3	0.5	2	0.73	0.3	0.61
0.5	0.1 0.2 0.3 0.4 0.5	0	1.1	0.66	0.16	0.19
0.5	0.1 0.2 0.3 0.4 0.5	0.5	1.1	0.75	0.57	0.65
0.5	0.1 0.2 0.3 0.4 0.5	0	2	0.45	0.03	0.13
0.5	0.1 0.2 0.3 0.4 0.5	0.5	2	0.77	0.18	0.25

Table: Results as a function of the reweighting parameters

$$f(\cdot; b) = \frac{0.05(b - b^{10}) + 1 - b}{1 - b^{10}} \quad (3)$$

Double Distribution Definition

$$H(x; \theta; t) = \int_{\Omega} F(\theta; \omega; t) d\Omega = \int_{\Omega} F(\theta; \omega; t) d\theta d\omega \quad (x \in \mathbb{R}^n; \theta \in \mathbb{R}^n; \omega \in \mathbb{R}^n)$$