

# Quarkonium quests with inclusive and exclusive productions

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# Background

- Integrated Masters (MPhys) Mathematical Physics  
2011-2016



- Ph.D. '*Exclusive Observables to NLO and low  $x$  PDF phenomenology at the LHC*'.

Thesis advisor: Prof. Thomas Teubner  
2016-2020



- Postdoc #1: QCD Theory group @ The University of Jyväskylä

2020-2022



JYVÄSKYLÄN YLIOPISTO  
UNIVERSITY OF JYVÄSKYLÄ

- Postdoc #2: Theory Department @ IJCLab, CNRS, Université Paris-Saclay

2022-present



# Research themes

## Exclusive heavy vector meson $V$ production

$$Q=V= J/\psi, \Upsilon, \psi(2S), \dots$$

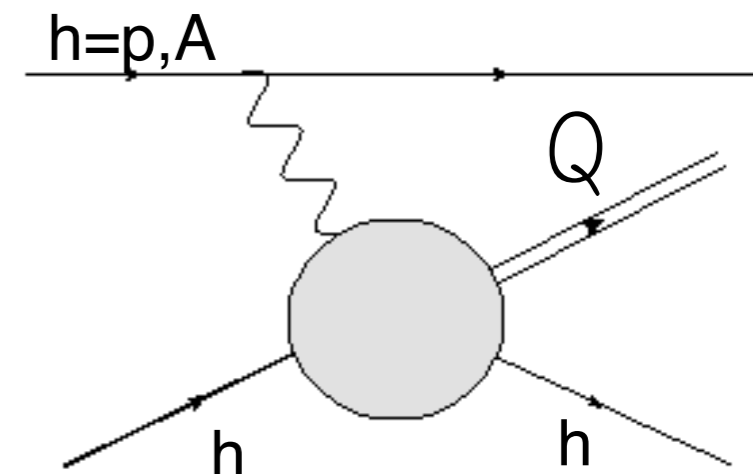
Exclusive

$$p + p \rightarrow p + V + p$$



$$p + A \rightarrow p + V + A$$

$$A + A \rightarrow A + V + A$$



automation

NLO

global fits

resummation

Quarkonia  $Q$

factorisation

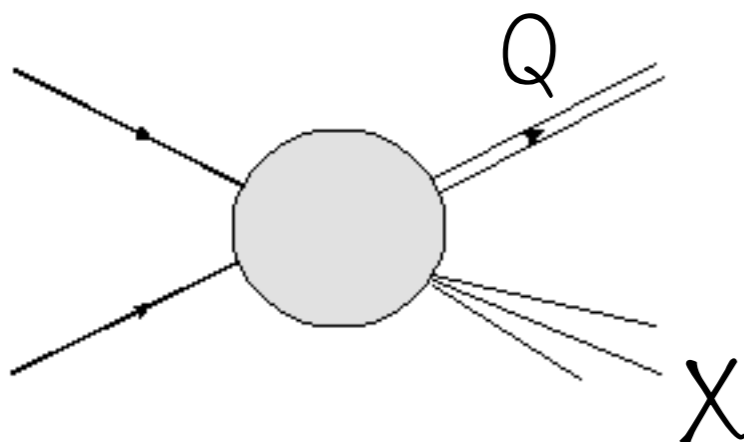
NRQCD

stability

## Inclusive quarkonium production

e.g.  $p + p \rightarrow Q + X$

-Madgraph5\_aMC@NLO



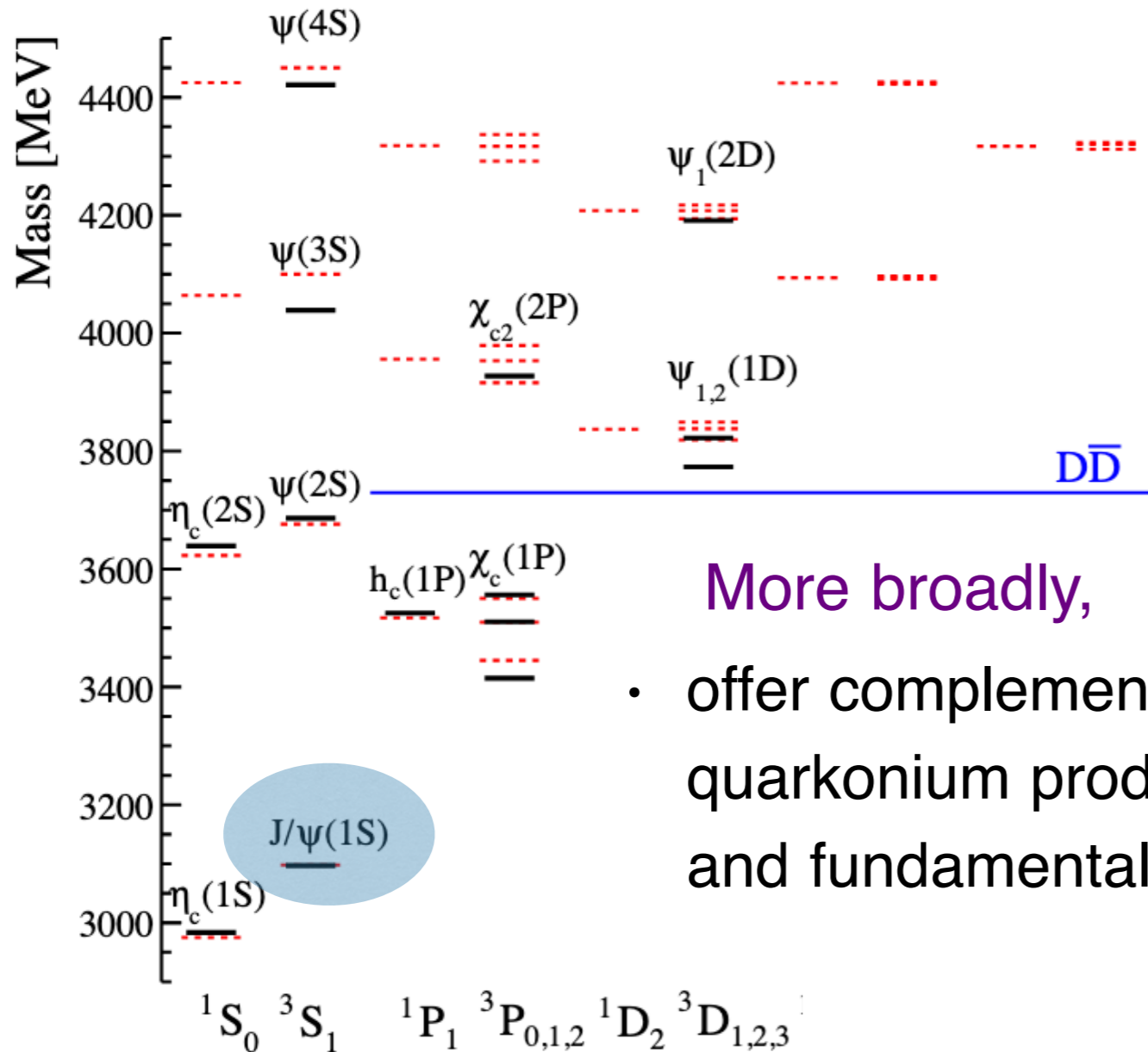
Inclusive



# Introduction - why quarkonia?

## Charmonium hierarchy

Rev.Mod.Phys. 90 (2018) 1, 015003



More broadly,

- offer complementary information on quarkonium production mechanisms and fundamentals of QCD

Quarkonia: bound states of heavy  $c, b$ , quarks<sup>1</sup>

$J/\psi$ :  $S=1$  (spin triplet),  
 $L=0$  (S wave),  $J=1$

e.g. gluon PDF constraints at low mom. fractions and res. scales

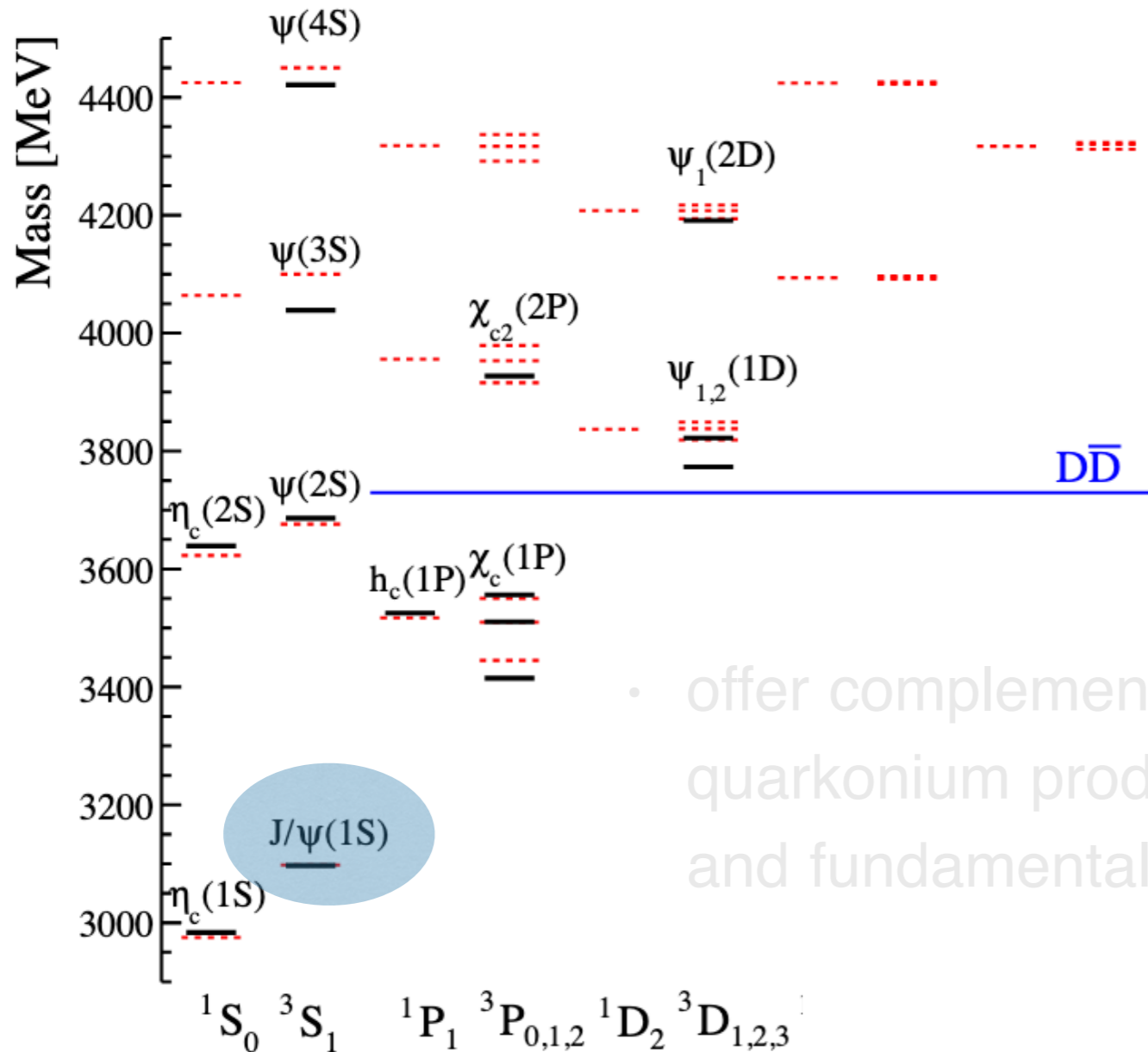
underpin the search for gluon saturation at the upcoming EIC and provide constraints on e.g. the QGP.

<sup>1</sup>bound states analogous to those of  $e^+e^-$  (positronium)

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- offer complementary information on quarkonium production and fundamentals of QCD
- are expected to underpin the search for gluon saturation at the upcoming EIC and provide constraints on e.g. the QGP.

<sup>1</sup>bound states analogous to those of  $e^+e^-$  (positronium)

# 'Inclusive' quests

# Quarkonium production

Factorisation:

$$\sigma(pp \rightarrow \mathcal{Q} + X) = \sum_{i,j,n} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \times \hat{\sigma}(ij \rightarrow Q\bar{Q}[n] + X) \langle \mathcal{O}_n^{\mathcal{Q}} \rangle.$$

short-distance matrix element (pert.)

long distance matrix element (non-pert.)

## Typical quarkonium production mechanisms

- Colour Singlet Model (CSM)<sup>1</sup>
- Colour Evaporation Model (CEM)
- Non-Relativistic QCD (NRQCD)

[Phys. Lett. B 390 \(1997\), pp. 323–328.](#)

[Phys. Rev. D 51 \(1995\). \[Erratum: Phys.Rev.D 55, 5853 \(1997\)\], pp. 1125–1171](#)

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<sup>1</sup>coincident with the LO term in the NRQCD expansion for  $S$  wave states

# Quarkonium production

## Factorisation:

$$\sigma(pp \rightarrow Q + X) = \sum_{i,j,n} \int dx_1 dx_2 f_{i/p}(x_1) f_{j/p}(x_2) \times \hat{\sigma}(ij \rightarrow Q\bar{Q}[n] + X) \langle \mathcal{O}_n^Q \rangle$$

proton PDFs (non-pert.)

short-distance matrix element (pert.)

long distance matrix element (non-pert.)

## NRQCD:

expansion in rel. velocity  $v$  of constituent heavy quarks allows one to systematically build up the quarkonium spectrum

e.g. zeroth order term  $\mathcal{O}(v^0)$  in expansion for  $J/\psi$  ( $= {}^3S_1$  state) couples to  $\chi^\dagger(0)\sigma_i\phi(0)$  and the corresponding LDME can be fixed by the QED di-leptonic decay width



# Automation of quarkonium cross sections

## Facilitates:

- Global data/theory comparisons
- Physics cases for future experimental facilities
- Global NRQCD fits

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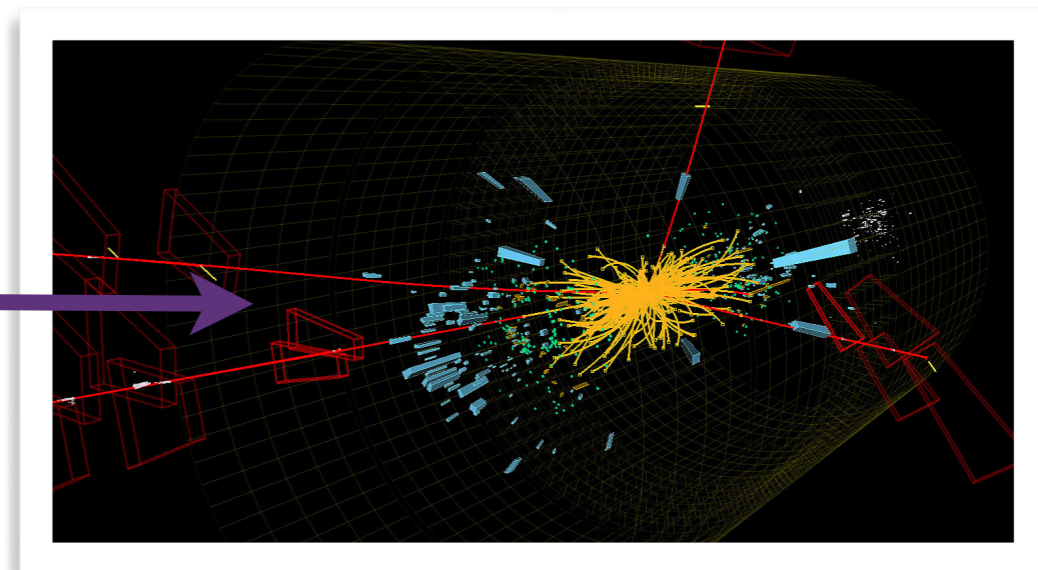
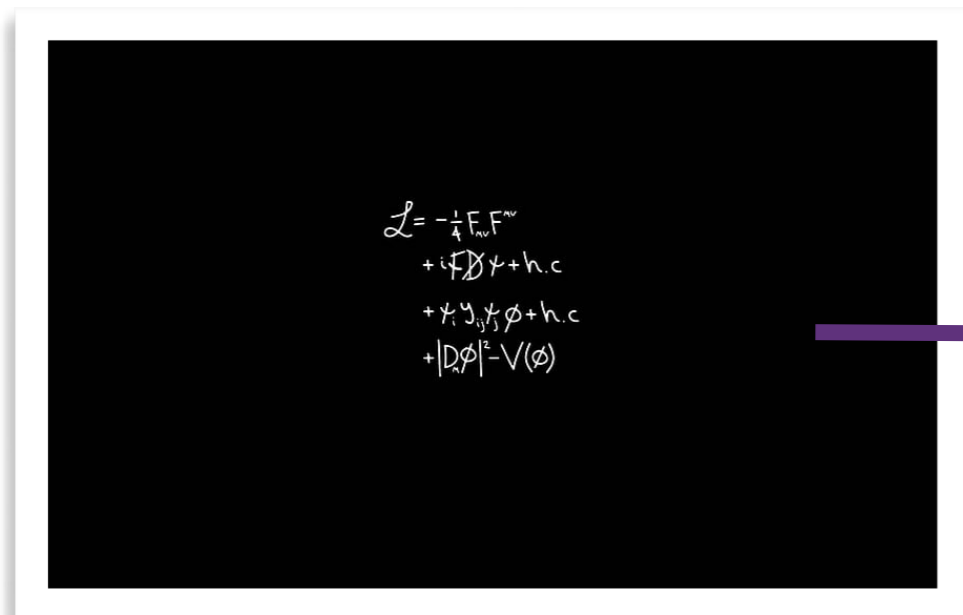
# Automation of quarkonium cross sections

## Facilitates:

- Global data/theory comparisons
- Physics cases for future experimental facilities
- Global NRQCD fits

MadGraph5\_aMC@NLO

- Only **automated** matrix element generator at LO and NLO + parton showering  
JHEP 07 (2014) 079
- Flexibility to support SM, BSM and large number of particle physics models



but..

- But no quarkonia final states -- Why? -- extra complexities arise

**Technical:** e.g. multi-channeling phase space adaptation needed for quarkonia

\*Final state IR divergence cancellation issues (different NRQCD Fock states contribute)

\*\*Feynman integral reduction to master integral basis using standard tools fails

## Aim:

Produce automation of LO quarkonium in MG5\_aMC with NLO in sight

## To date:

Towards single and multiple S-wave **inclusive** quarkonium (and associated) production at LO

- Colour projectors  $\mathcal{C}_1 = \delta_{ij} / \sqrt{N_c}$   $\mathcal{C}_8 = \sqrt{2} T_{ij}^c$
- Spin projectors
- Interface
- Phase space adaptation



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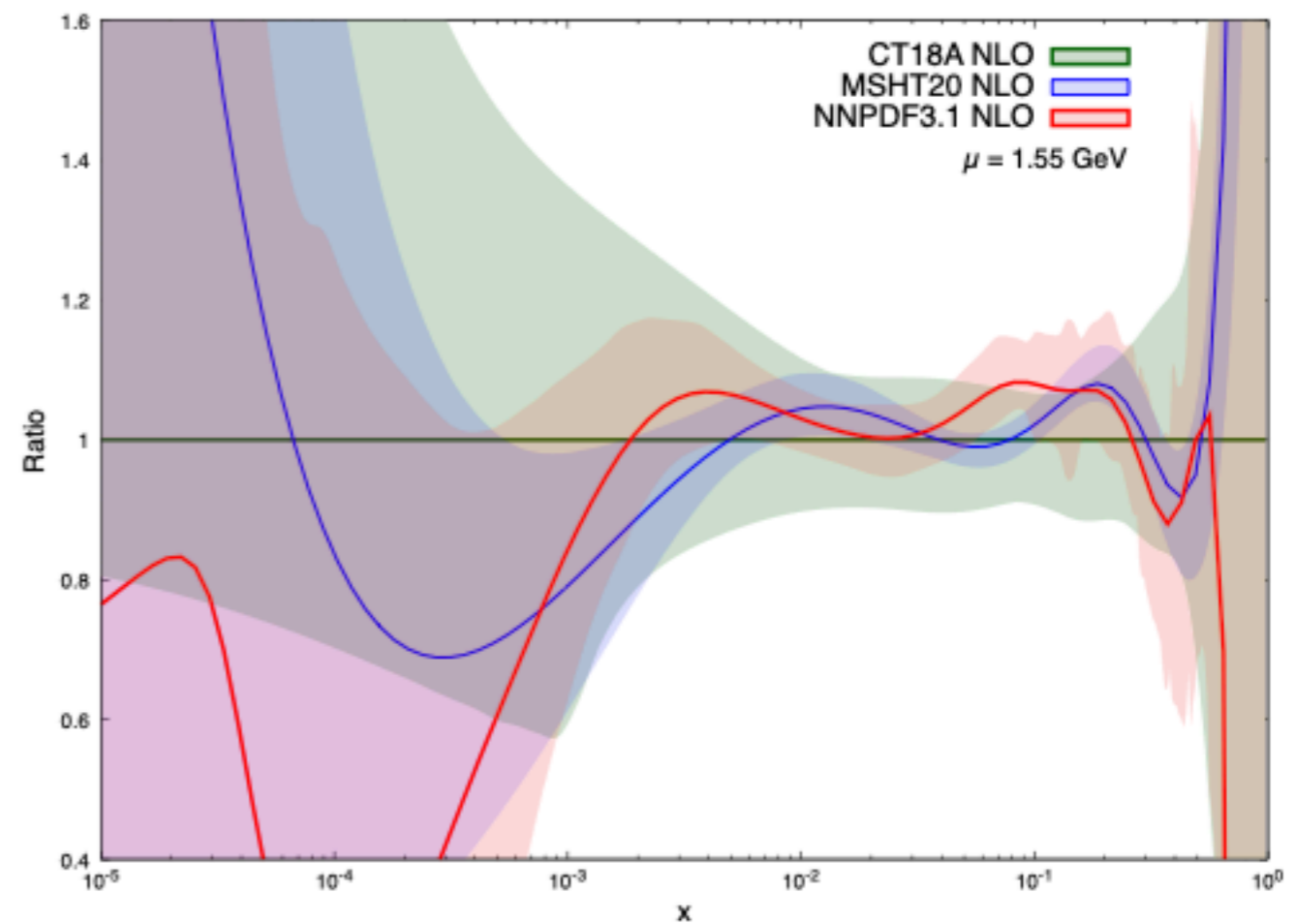
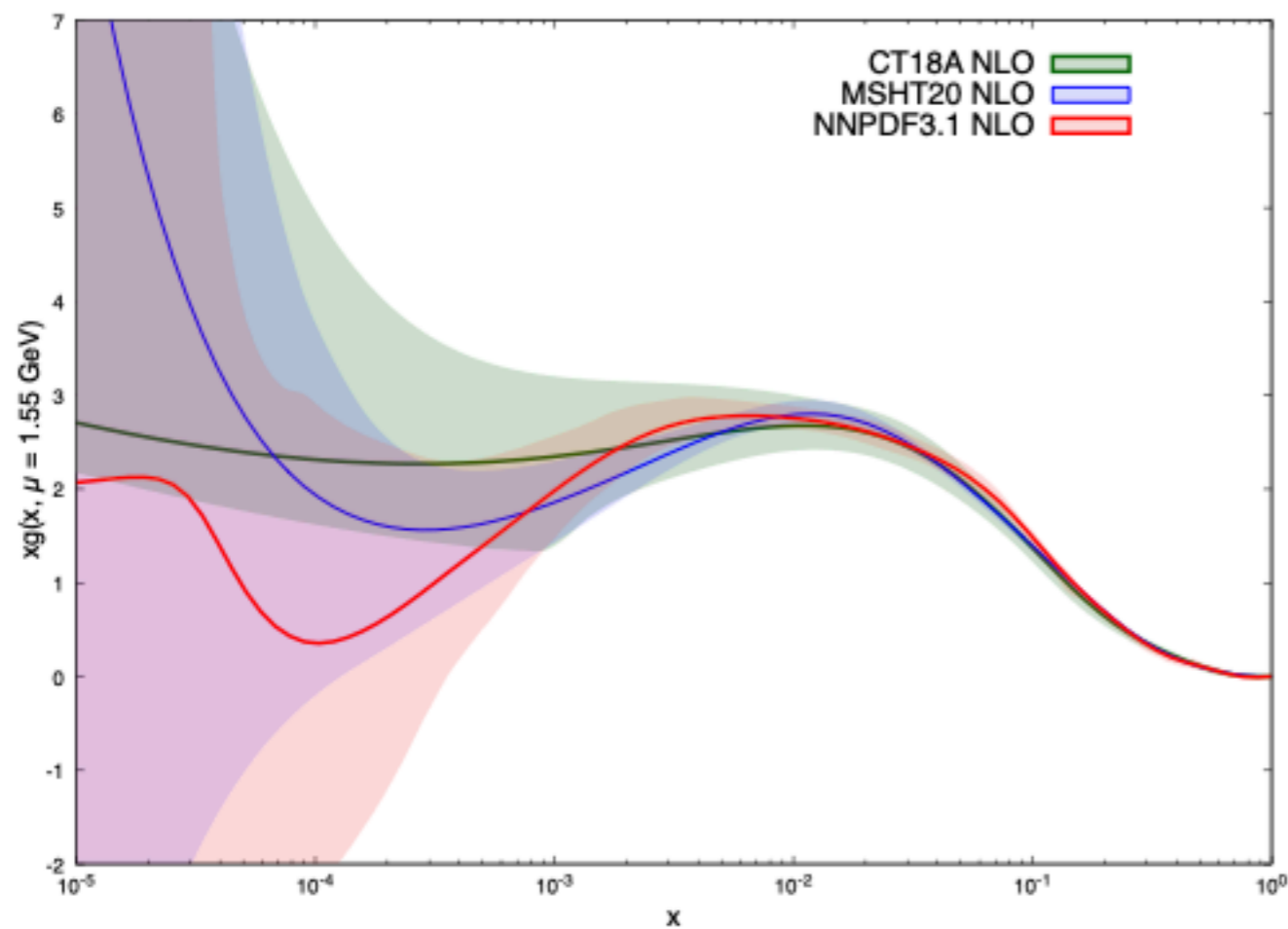
\*famous resolution of non-cancelling IR divergences through mixing

of P wave states with relevant S wave states at  $O(v^2)$

# 'Exclusive' quests

# Introduction

- Inclusive processes do not well constrain small  $x$ /Regge limit domain of PDFs
- Exclusive processes offer sensitive probe of this domain but as of yet not included in global analyses PDF determination – why?
  1. Off forward kinematics imply sensitivity to GPD over conventional PDFs
  2. Scale dependence and stability of theoretical predictions



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  2. Scale dependence and stability of theoretical predictions
- As higher CM energies are realised at LHC, pushed towards small  $x$  domain,  $W \sim 1/x$

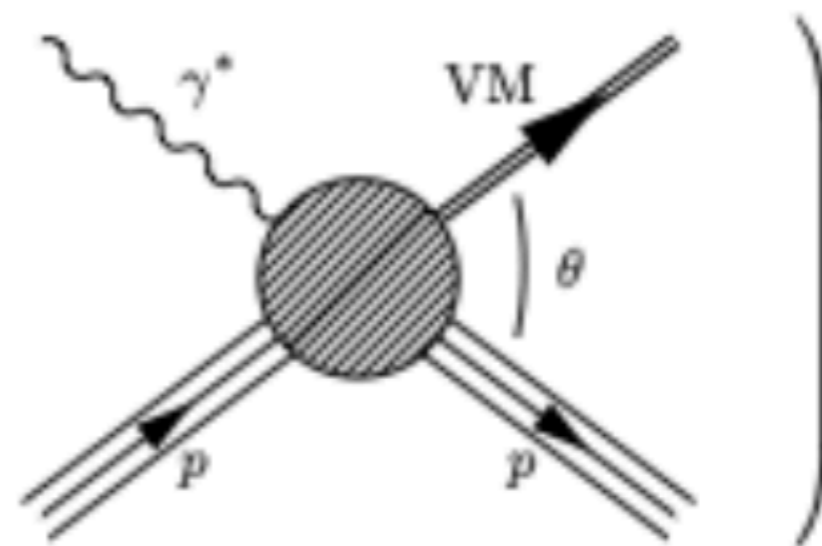
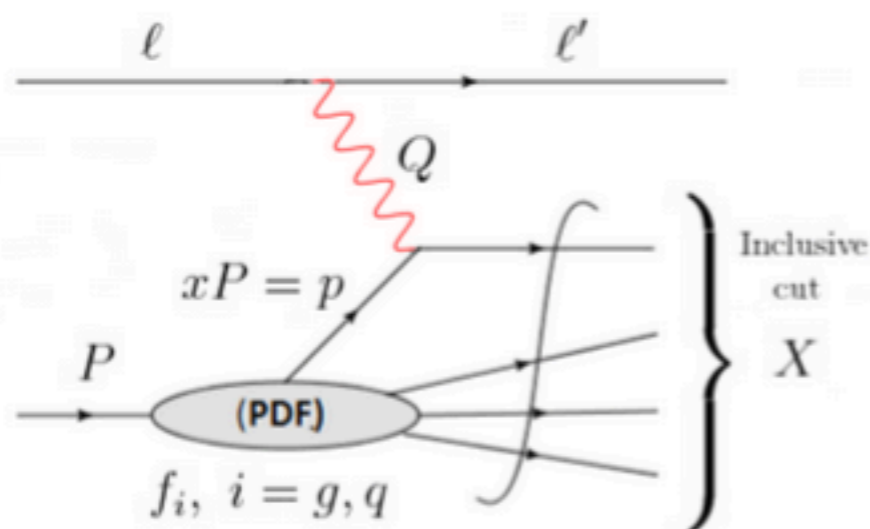
LLx exclusive J/psi production:

$$\left. \frac{d\sigma}{dt}(\gamma^* p \rightarrow J/\psi p) \right|_{t=0} = \frac{\Gamma_{ee}^{J/\psi} M_{J/\psi}^3 \pi^3}{48\alpha_{em}} \left[ \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} xg(x, \bar{Q}^2) \right]^2 \left( 1 + \frac{Q^2}{M_{J/\psi}^2} \right)$$

*Ryskin 1993*

**Inclusive** – e.g. DIS included in global parton analyses

**Exclusive** – can we use the data?



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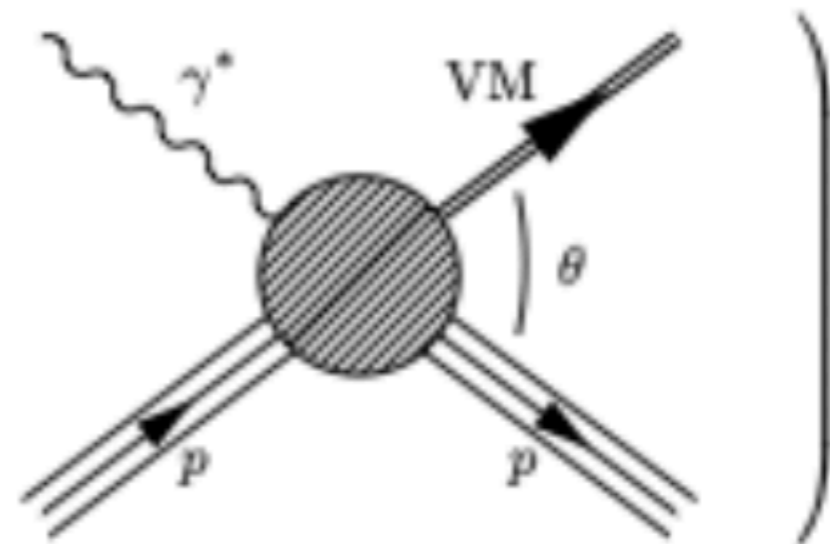
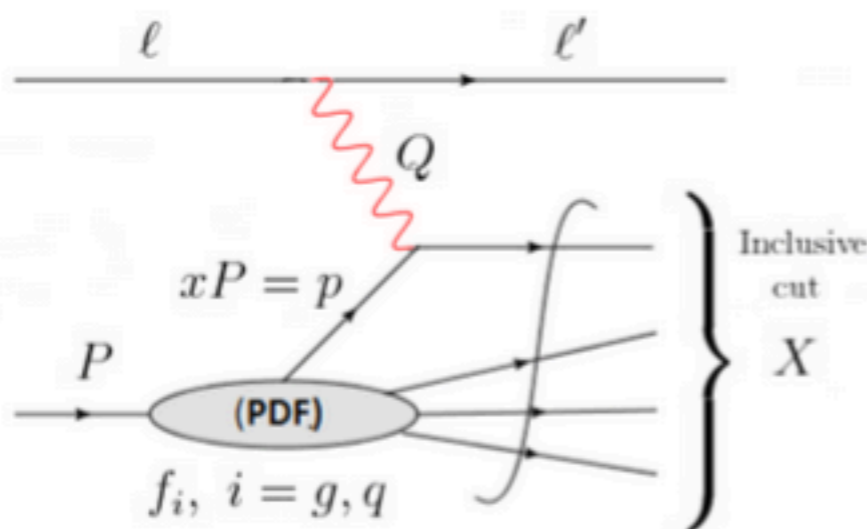
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DLLA exclusive  $J/\psi$  production:

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**Inclusive** – e.g. DIS included in global parton analyses

**Exclusive** – can we use the data?

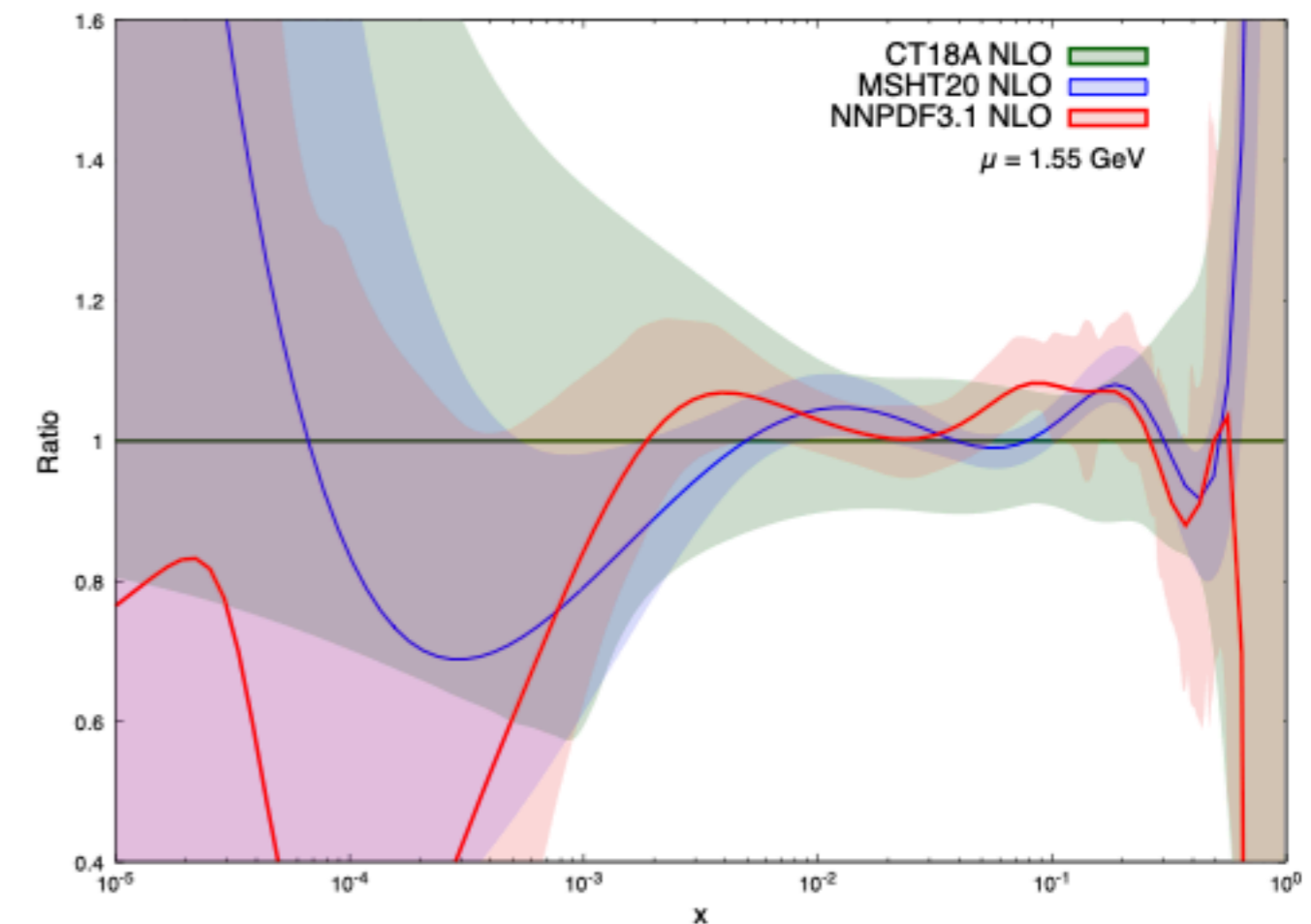
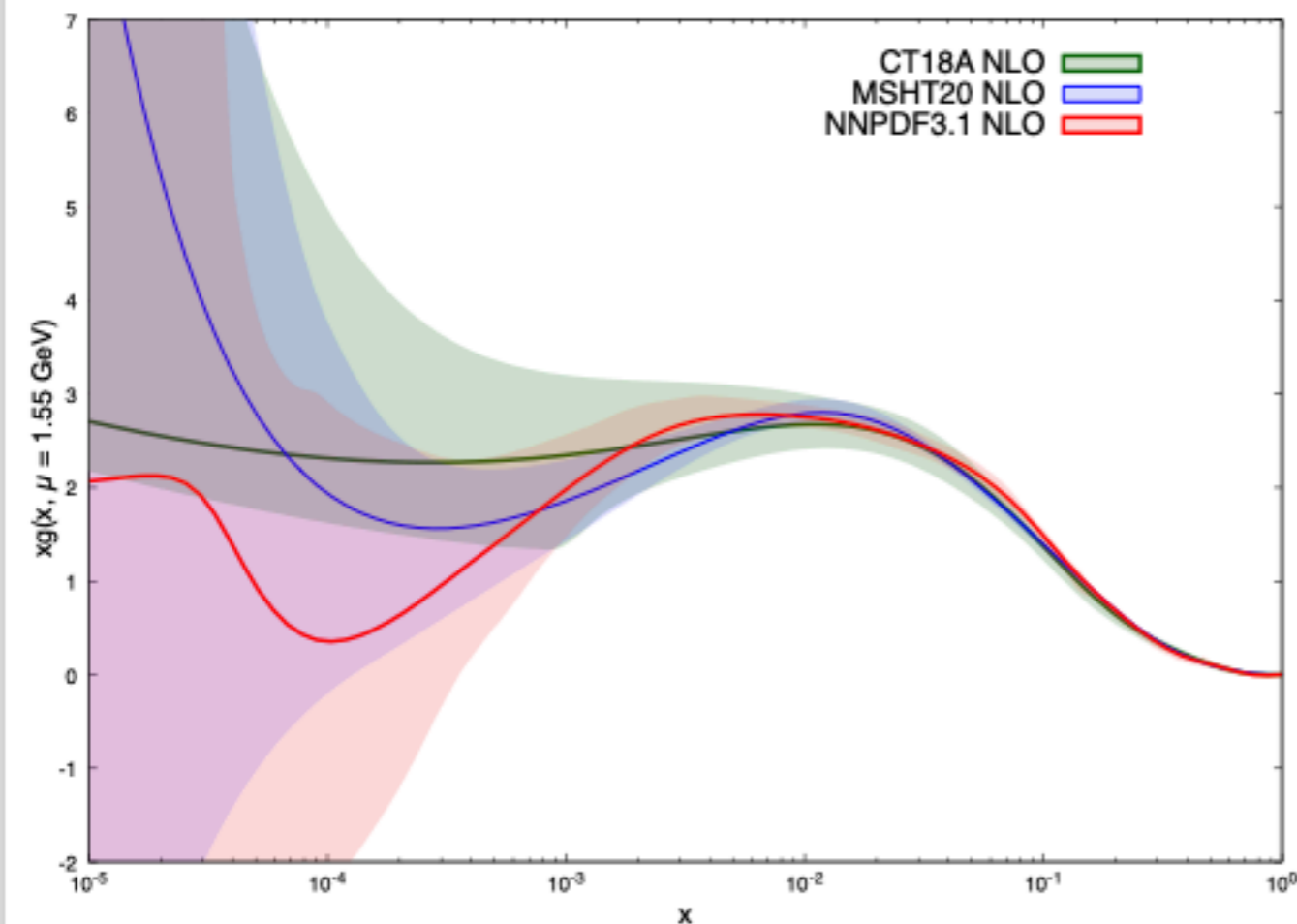




# Introduction

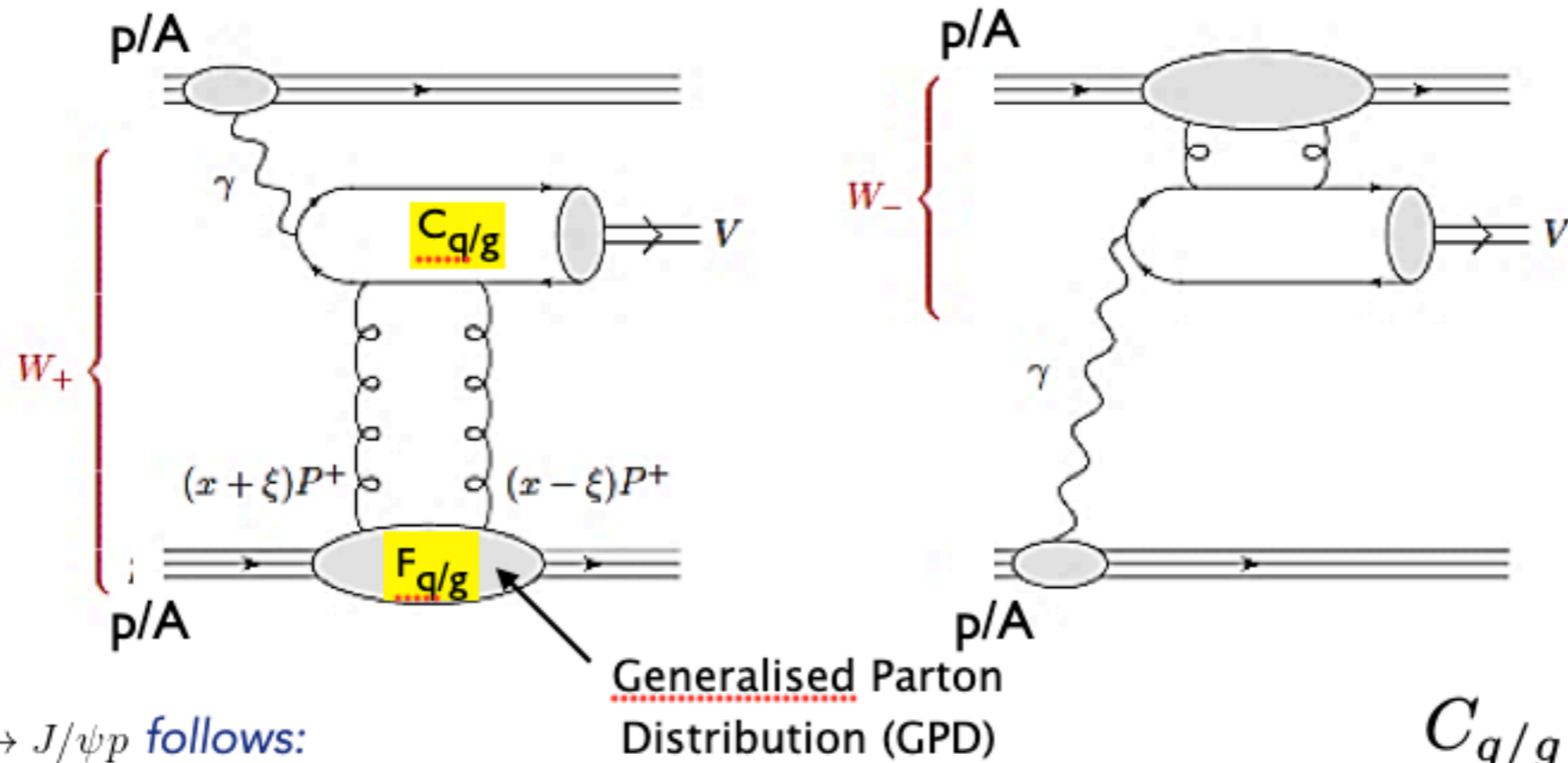
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Scope: how to counteract these problems and so allow exclusive  $J/\psi$  data to probe gluon PDF down to  $x \sim 3 \times 10^{-6}$  &  $\mu = O(M_{J/\psi}/2)$



# General Set up and Framework

Exclusive  $J/\psi$  photoproduction in  $p+p$  ( $A+A$ ) UPC collisions in collinear factorisation



Setup for  $\gamma p \rightarrow J/\psi p$  follows:

Ivanov, Schäfer, Szymanowski, Krasnikov, 04

- Factorisation:  $F_{q/g} \otimes C_{q/g} \otimes \phi_{Q\bar{Q}}^V$
- Leading zeroth order term in rel. velocity (NRQCD)
- Colour singlet exchange between hard and soft sectors

$$A \propto \int_{-1}^1 dx \left[ C_g(x, \xi) F_g(x, \xi) + \sum_{q=u,d,s} C_q(x, \xi) F_q(x, \xi) \right]$$

$C_{q/g}$

Photoproduction:

- hep-ph/0401131

Ivanov, Schäfer, Szymanowski, Krasnikov, 04

Electroproduction:

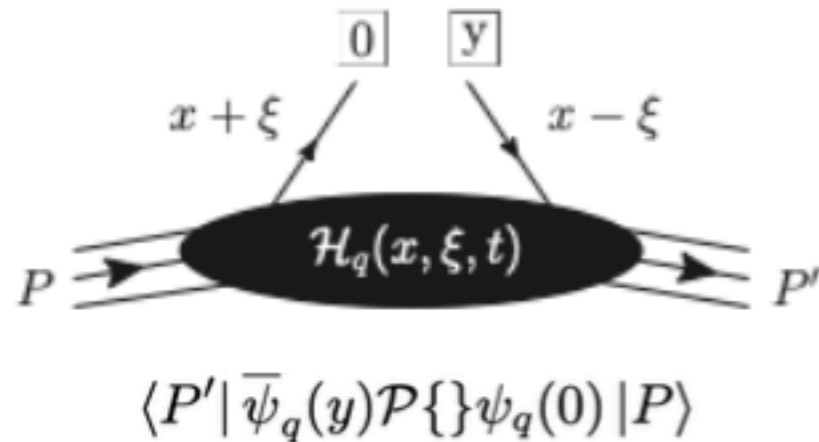
- arXiv:1903.00171
- arXiv:2105.07657

Chen, Qiao, 19

CAF, Gracey, Jones, Teubner, 21

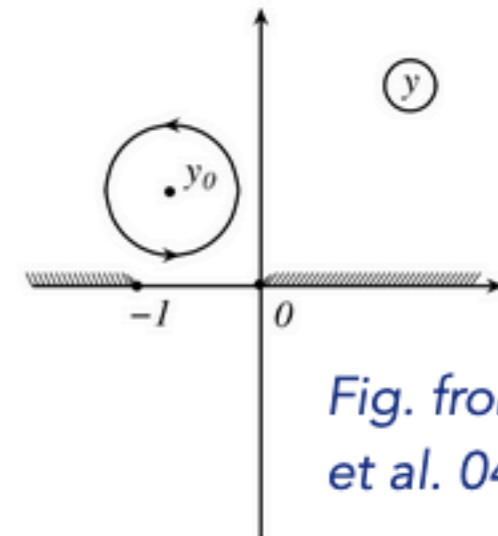
# GPDs and the Shuvaev transform

GPDs generalise PDFs: outgoing/incoming partons carry different momentum fractions *Müller 94; Radyushkin 97; Ji 97*



**Shuvaev:** Relates GPDs to PDFs at small  $x$  under physically motivated assumptions c.f analyticity

*Shuvaev 99 Martin et al. 09*



*Fig. from Ivanov et al. 04*

Idea: Conformal moments of GPDs = Mellin moments of PDFs  
(up to corrections of  $O(x^2)$  @ LO and  $O(x)$  @ NLO)

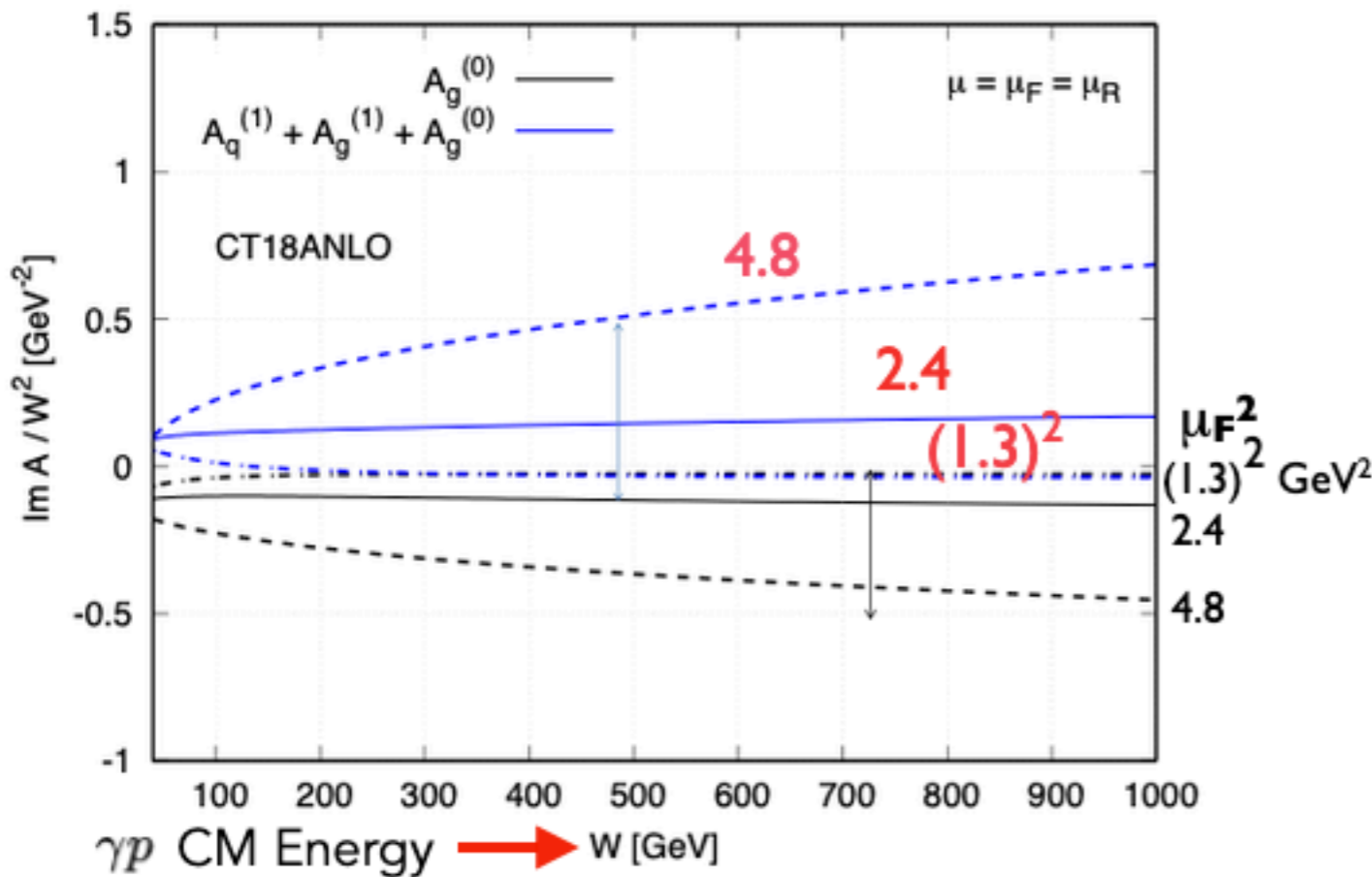
- Construct GPD grids in multidimensional parameter space  $x, \xi/x, qsq$  with forward PDFs from LHAPDF
- Costly computationally due to slowly converging double integral transform
- Regge theory considerations => Shuvaev transform valid in space-like (DGLAP) region only. In time-like (ERBL) region imaginary part of coefficient function is zero

# Stability of NLO prediction I+II

## NLO in $\overline{\text{MS}}$ scheme

hep-ph/0401131

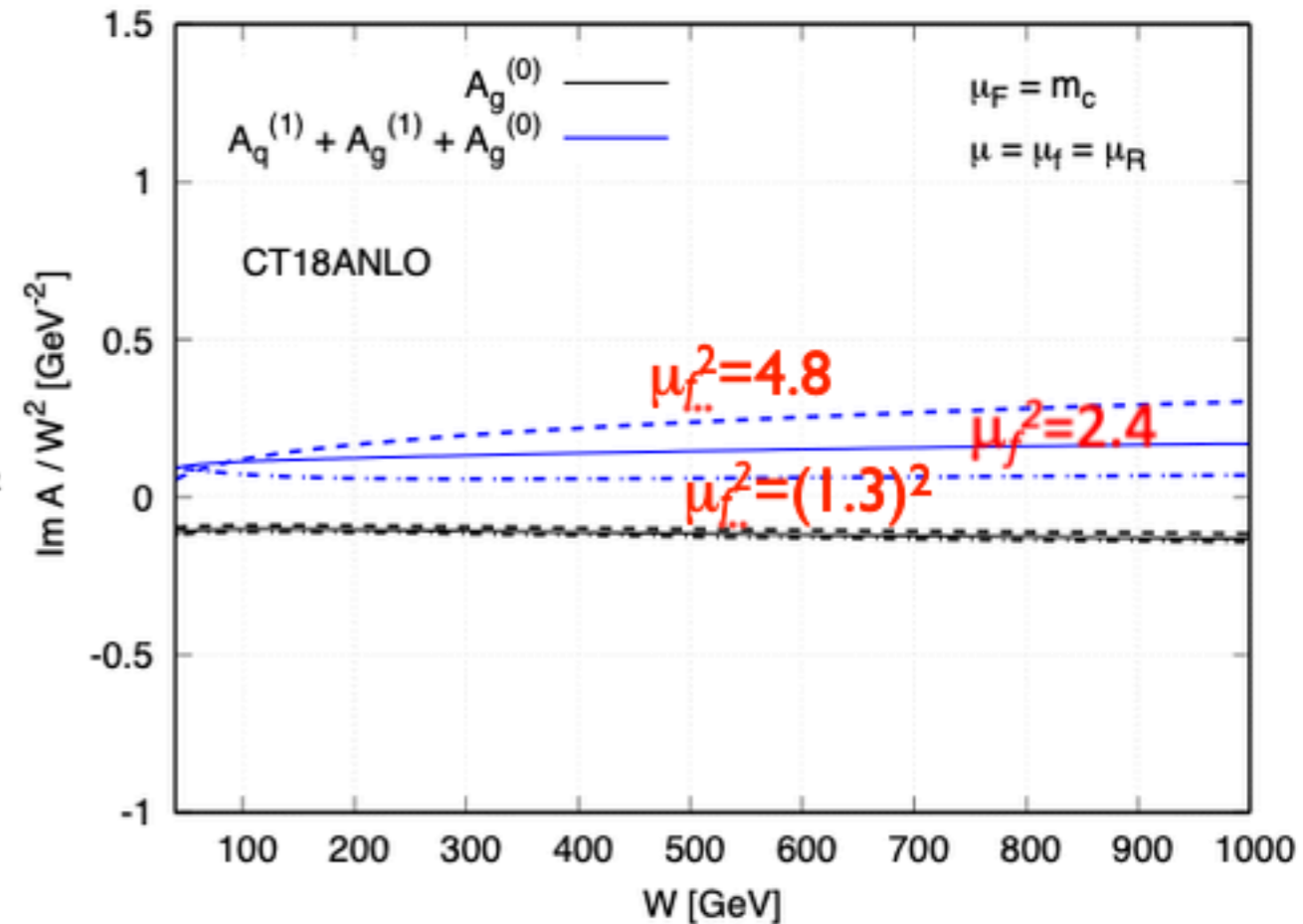
- A. Bad perturbative convergence  $|\text{NLO}_{\text{correctn.}}| > |\text{LO}|$  and
- B. Strong dependence on scale  $\mu_F$  opp. sign



## 'Effective' small-x resummation

1507.06942

Resummation of  $(\alpha_s \ln(1/\xi) \ln(\mu_F/m))^n$

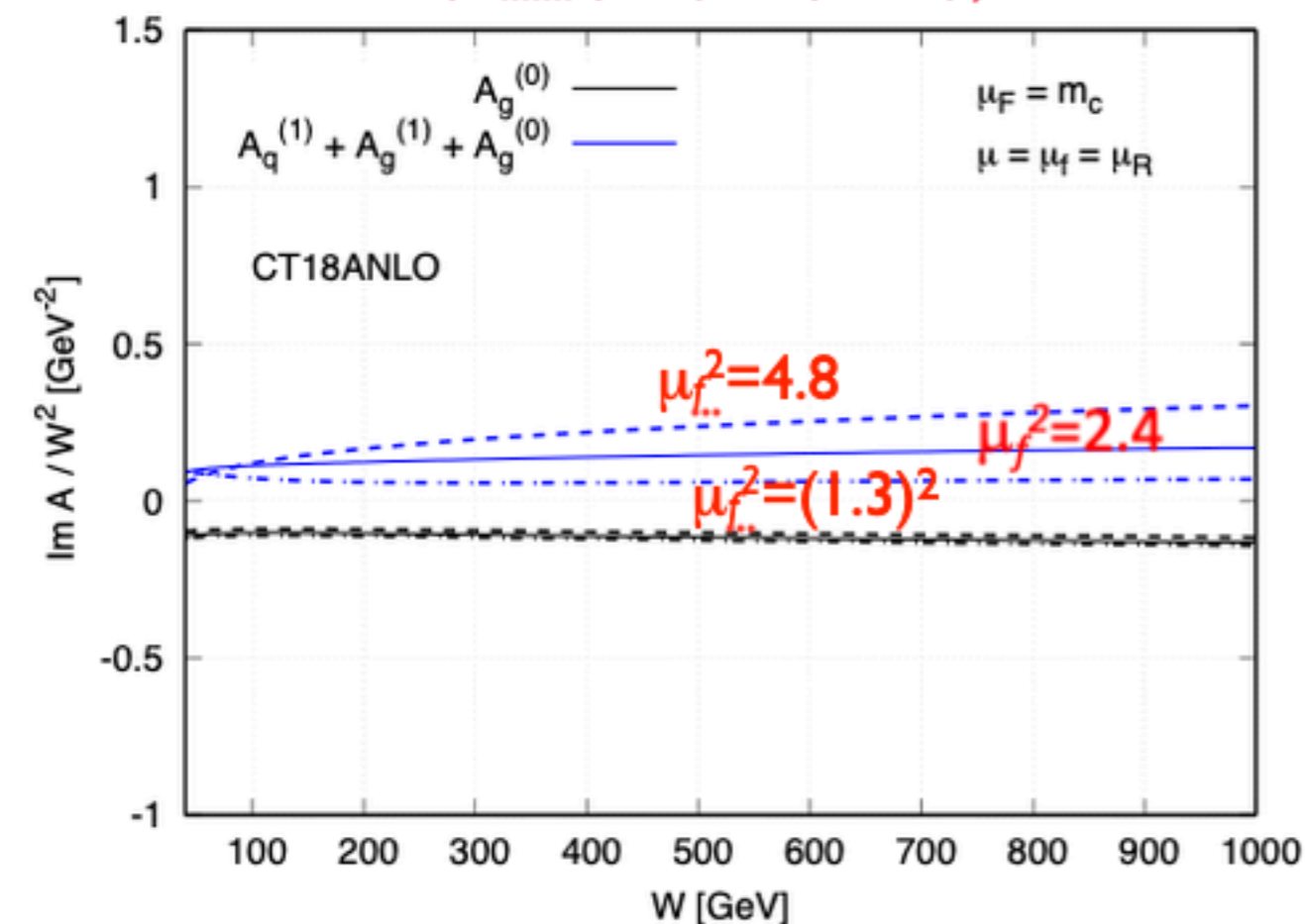


# Stability of NLO prediction II+III

'Effective' small- $x$  resummation

1507.06942

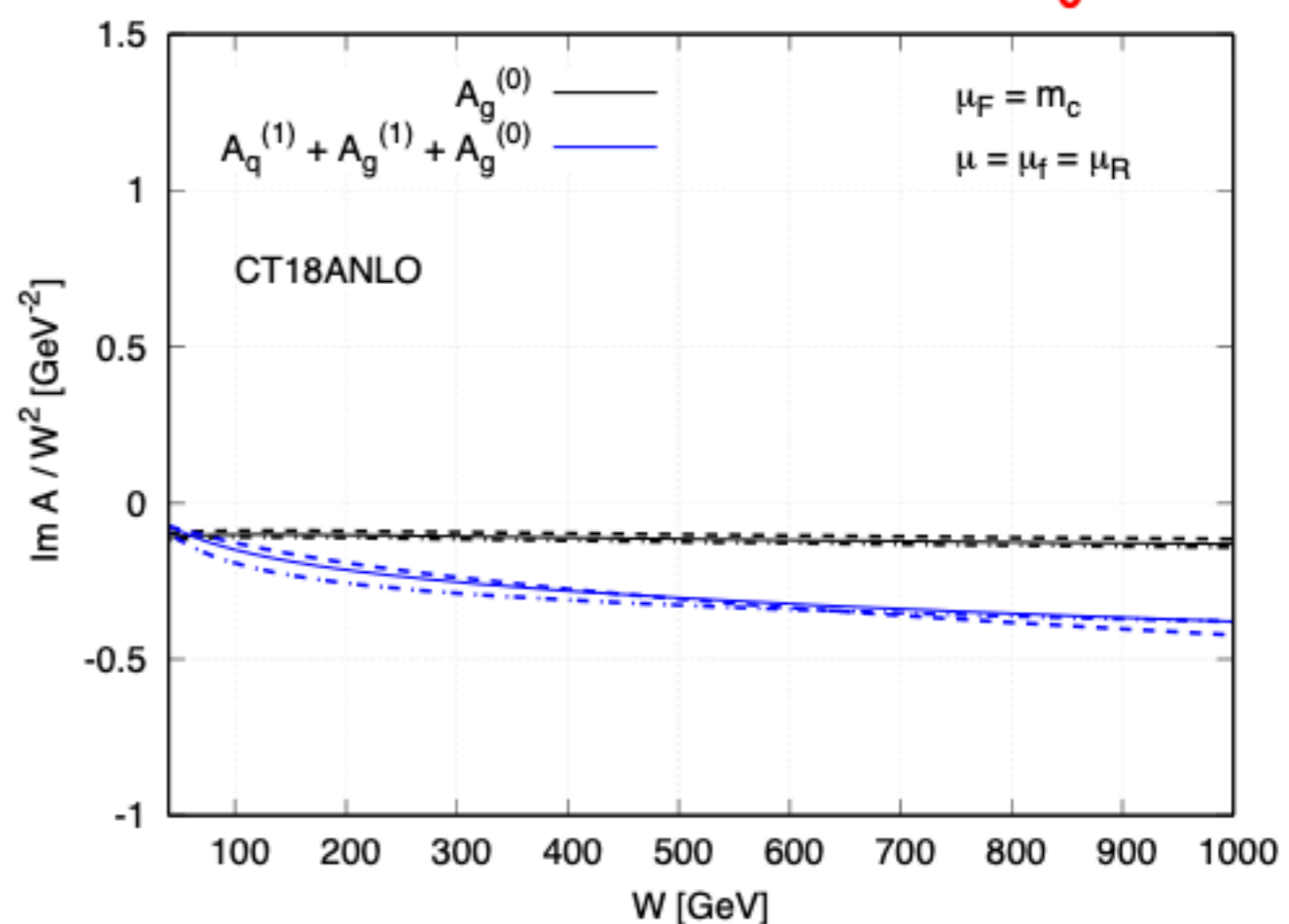
Resummation of  
 $(\alpha_s \ln(1/\xi) \ln(\mu_F/m))^n$



Low  $l_t < Q_0$  subtraction

1610.02272

Subtract DGLAP contribution NLO ( $|\ell^2| < Q_0^2$ )  
 from known NLO MSbar coefficient function to avoid a  
 double counting with input GPD at  $Q_0$ .



Predictions based on three global PDF analyses differ dramatically in large energy LHC region but are compatible in lower energy HERA region\*

# Extraction of low x gluon PDF via exclusive J/psi

**Left Approach 1:** Fit a low x gluon PDF ansatz to the data

**Right Approach 2:** Bayesian reweight current global PDF analyses

|          | $\lambda$ | $n$   | $\chi^2_{\min}$ | $\chi^2_{\min}/\text{d.o.f}$ |
|----------|-----------|-------|-----------------|------------------------------|
| NNPDF3.0 | 0.136     | 0.966 | 44.51           | 1.04                         |
| MMHT14   | 0.136     | 1.082 | 47.00           | 1.09                         |
| CT14     | 0.132     | 0.946 | 48.25           | 1.12                         |

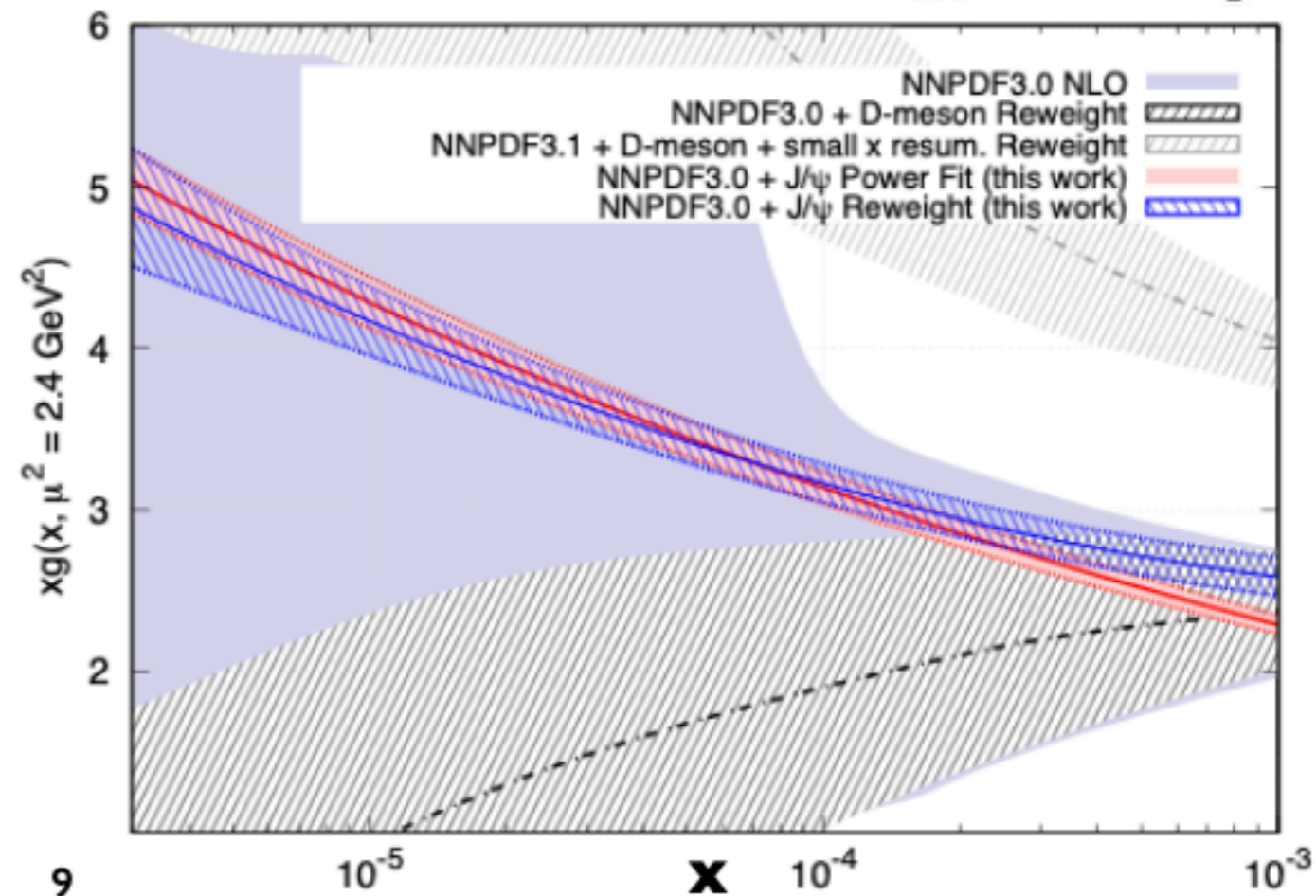
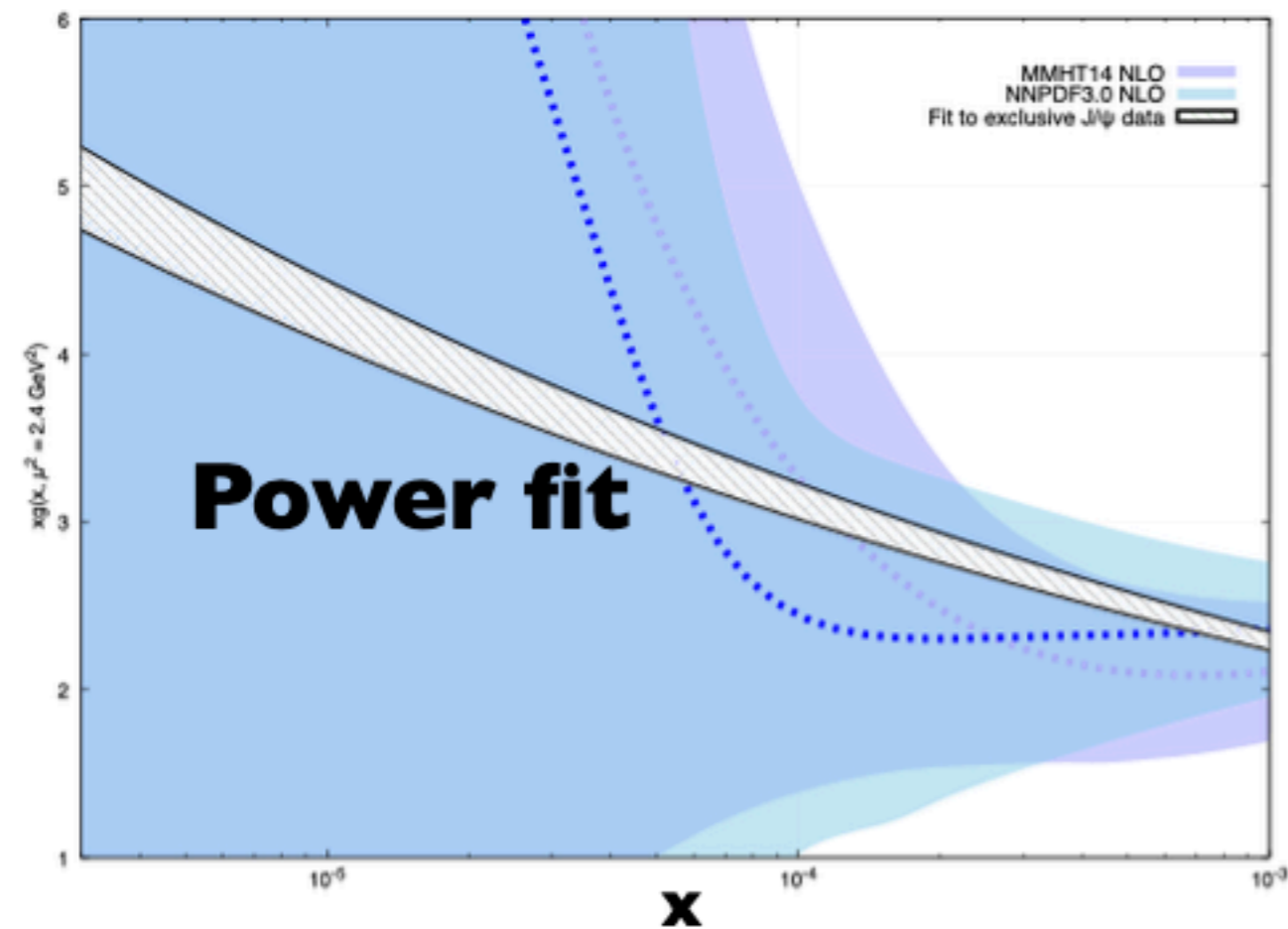
$$xg^{\text{new}}(x, \mu_0^2) = nN_0 (1-x) x^{-\lambda}$$

$$\lambda = 0.136 \pm 0.006$$

$$n = 0.966 \pm 0.025$$

CAF, Martin, Ryskin, Teubner 2006. 13857

$$N_{\text{eff}} \ll N_{\text{rep}}$$

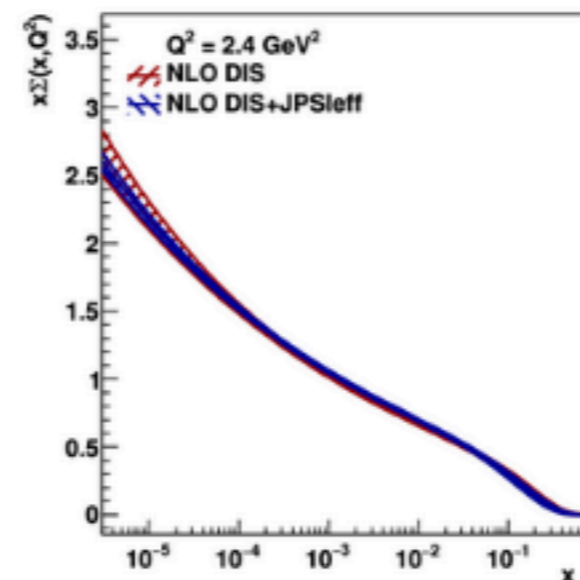
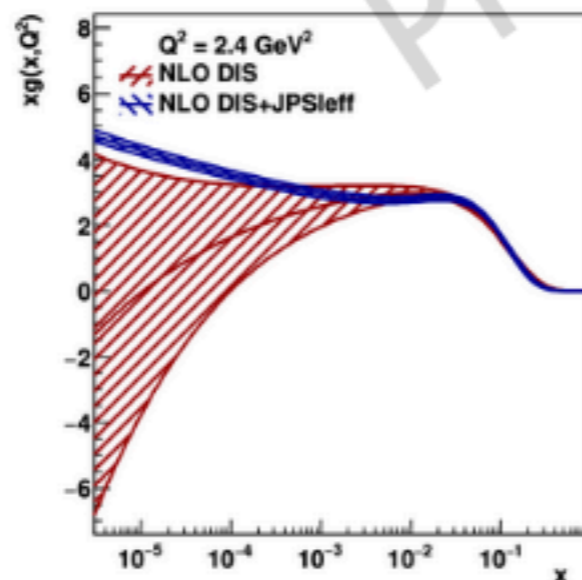
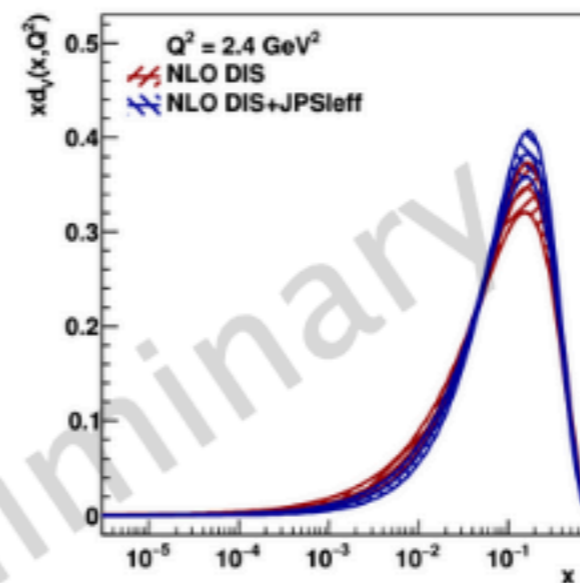
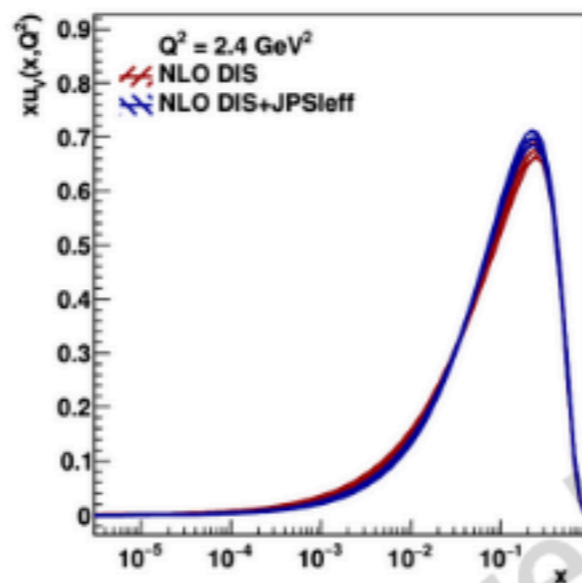


# xFitter implementation and next steps

- Incorporate new 'JPSI' reaction via xFitter's ReactionTheory class
  - i) PDF profiling w/ exclusive J/psi data and
  - ii) PDF fitting w/ exclusive J/psi+HERA DIS RunI+II datasets
  - iii) PDF fitting with generated gluon PDF pseudo-data

default DIS:  
chi2=1357/1131  
~1.19

DIS+JPSIeff:  
chi2=1400/1151  
~1.22



# Summary & Outlook

## Summary

- Towards full implementation of LO inclusive quarkonium + associated production for **S wave** Fock states in MG5. Incorporation into EU virtual access project **NLOAccess**
- First fitting results using **xFitter** integrated with framework with tamed collinear factorisation at NLO for exclusive J/psi photoproduction

## Outlook

- Extension to states with leading **P wave** Fock states --> global NRQCD picture, and/or BSM
- Ultimately NLO in mind with few **caveats**.
- Predictions for exclusive J/psi photoproduction employing HEF-CF **matching** formulae and **full** GPD evolution

Thank you



# Automation of quarkonium cross sections

## Facilitates:

- Global data/theory comparisons
- Physics cases for future experimental facilities
- Global NRQCD fits

## In public matrix element generators/event generators:

- Interfacing of e.g. HERWIG or PYTHIA with e.g. MG5\_aMC<sup>1</sup>

Facilitates complete computation  $\longrightarrow$

Versatility and enhanced physics simulation capabilities...

...but integration complexity, computational overhead, code compatibility and increased learning requirements.

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<sup>1</sup>e.g. cross sections with quarkonium in jets

# Automation of quarkonium cross sections cont.

## Motivated:

| Tool  | Features  |
|---|---|
| <ul style="list-style-type: none"><li>• MadOnia<br/>Artoisenet, Maltoni, Stelzer<br/>JHEP 02 (2008) 102</li></ul>                                   | <p>(Deprecated) module within MadGraph4 -<br/>was not ported to current version (v5)</p> <p>Single quarkonium production phenomenology</p>  |
| <ul style="list-style-type: none"><li>• Helac-Onia<br/>Shao<br/>Comput.Phys.Commun.<br/>184 (2013)<br/>Comput.Phys.Commun.<br/>198 (2016)</li></ul> | <p>One or more S-wave and/or P-wave heavy<br/>quarkonia production based on tree-level helicity<br/>amplitudes</p> <p>Limited to LO, not immediately extendable to NLO<br/>(no NLO matrix element or no phase space<br/>integrator for NLO)</p> |

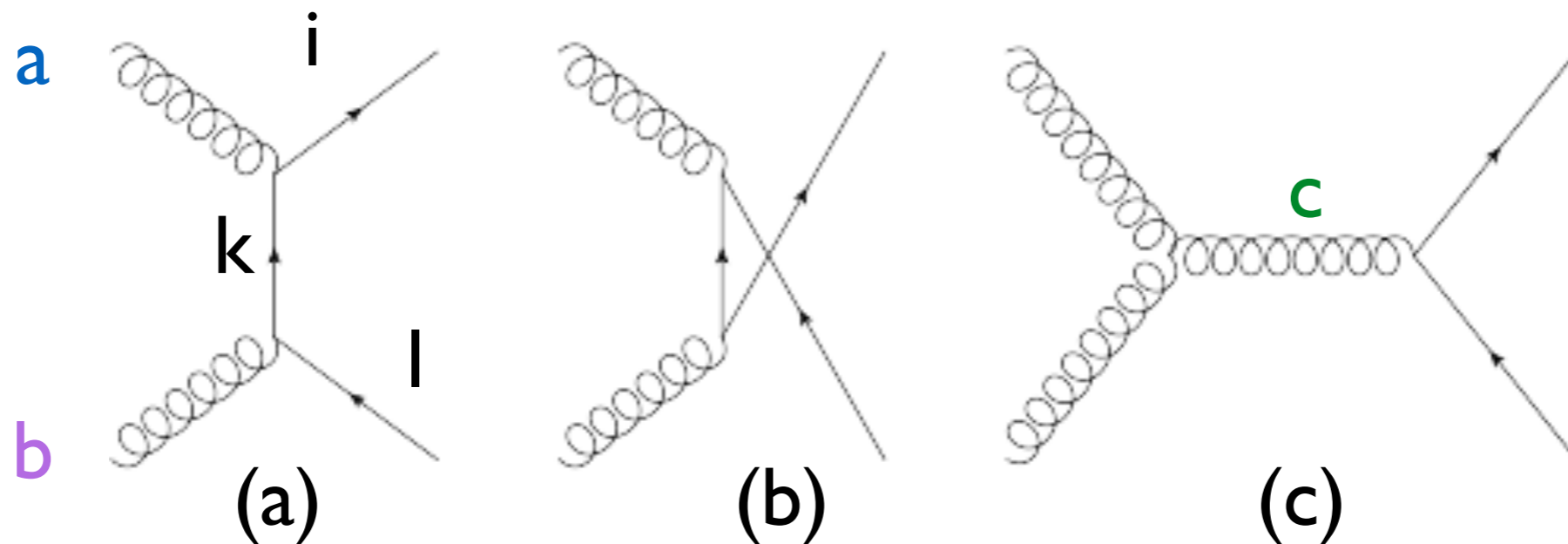


# Amplitude generation & spin projectors

- MG5 organises amplitude into colour basis '**JAMPs**'

Efficiency: For given process, may have large # of diagrams but colour basis will be much smaller

*E.g. generate LO  $g g \rightarrow c c$  ~ colour singlet (CS) and colour octet (CO)*



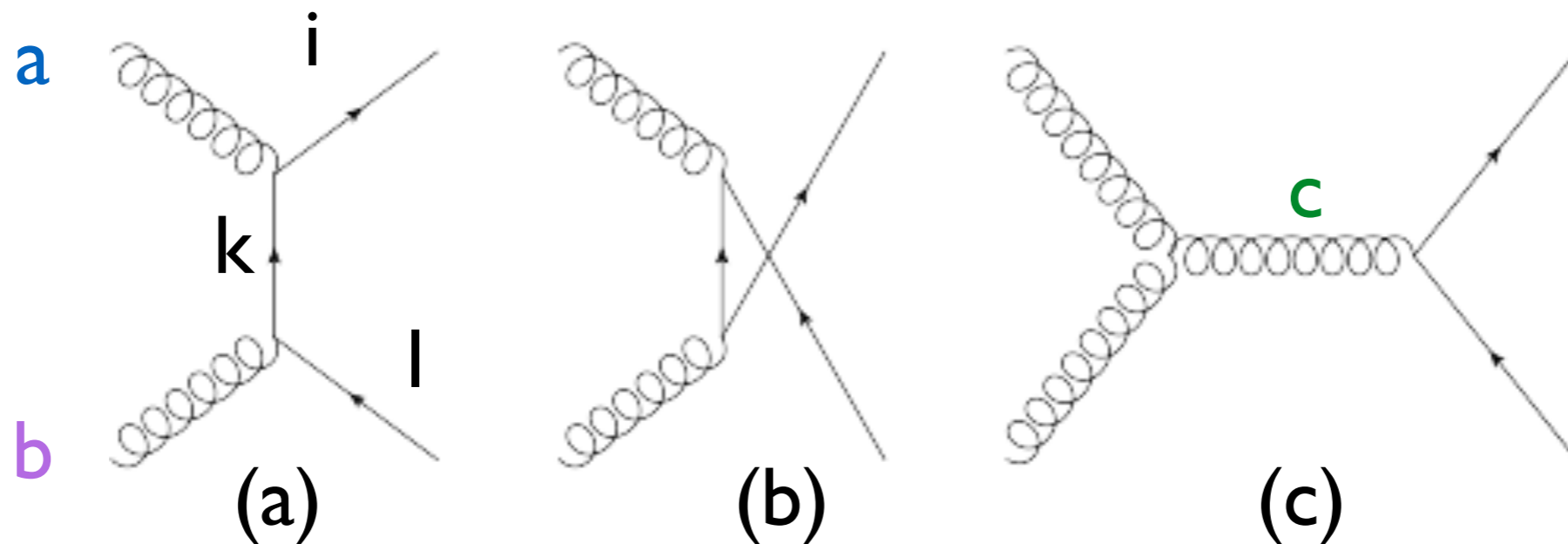
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$$\mathcal{A} = A_a + A_b + A_c$$

$$A_a = c_1 A_{11}$$

$$A_b = c_2 A_{22}$$

$$A_c = c_1 A_{31} + c_2 A_{32}$$

$$\text{JAMP}_1 = A_{11} + A_{31} \propto c_1$$

$$\text{JAMP}_2 = A_{22} + A_{32} \propto c_2$$

$$\text{CO} : c_1 = \text{Tr}(t^a t^b t^c)$$

$$c_2 = \text{Tr}(t^b t^a t^c)$$

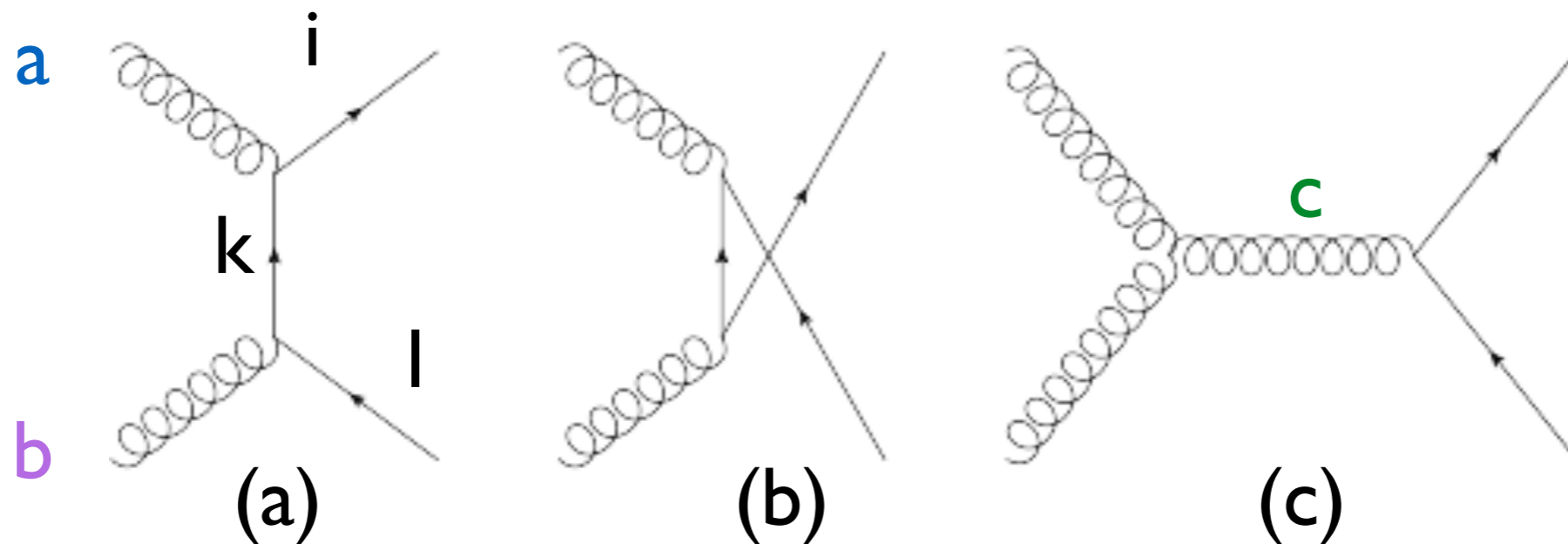
$$|\mathcal{A}|^2 = \sum_{i,j=1,2} \text{JAMP}_i^* \langle c_i | c_j \rangle \text{JAMP}_j$$

# Amplitude generation & spin projectors

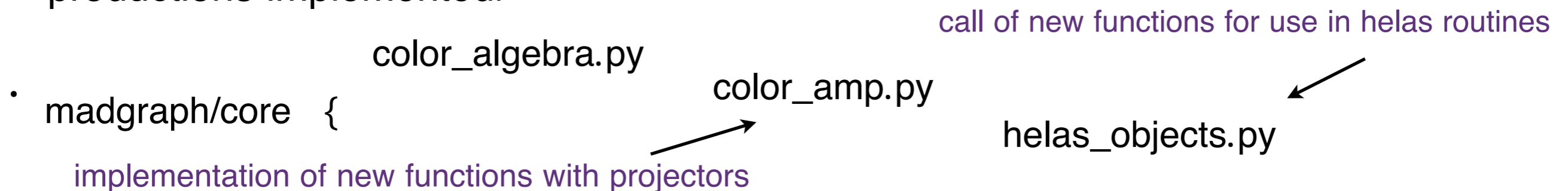
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- Colour projectors for  $m$  colour singlet and octet quarkonia production and associated productions implemented.



# Amplitude generation & spin projectors

Generic quarkonium spin projector:

$$\sum_{\lambda_1, \lambda_2 = -1/2}^{1/2} \bar{v}(p_2, \lambda_2) \Gamma_S \frac{\not{P} + M}{2M} u(p_1, \lambda_1) \quad *$$

$$S = 0, \gamma_5; 1, \not{\epsilon}(P)$$

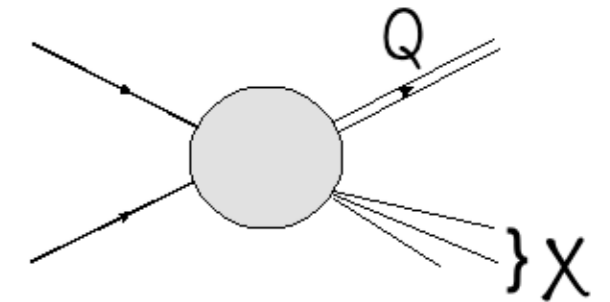
$$P = p_1 + p_2$$

$$M^2 = P^2$$

Generic fermion line:

$$\bar{u}(p_1, \lambda_1) \Gamma_1 \dots \Gamma_2 \nu(p_2, \lambda_2) \quad **$$

Contract fermion lines (\*\*\*) with projector (\*):



$$\sum_{\lambda_2 = -1/2}^{1/2} \bar{v}(p_2, \lambda_2) \Gamma_S \frac{\not{P} + M}{2M} (\not{p}_1 - m_1) \Gamma_1 \dots \Gamma_2 \nu(p_2, \lambda_2)$$

$$\sim \sum_{\lambda = -1/2}^{1/2} \Gamma_2 (\not{p}_2 - m_2) \frac{\Gamma_S}{2M} u(P, \lambda) \bar{u}(P, \lambda) (\not{p}_1 - m_1) \Gamma_1$$

$$\bar{v}_{\text{eff}}(P, \lambda)$$

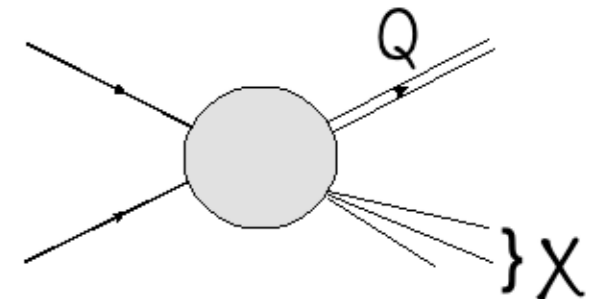
$$u_{\text{eff}}(P, \lambda)$$

# Amplitude generation & spin projectors

$$\sim \sum_{\lambda=-1/2}^{1/2} \underbrace{\Gamma_2 (\not{p}_2 - m_2) \frac{\Gamma_S}{2M} u(P, \lambda)}_{\bar{v}_{\text{eff}}(P, \lambda)} \underbrace{\bar{u}(P, \lambda) (\not{p}_1 - m_1) \Gamma_1}_{u_{\text{eff}}(P, \lambda)}$$

**Motivation:** MG5 recursively generates diagrams by carefully merging legs to avoid a double counting

-- need a workaround after spin projector applied because it 'glues' the two external quarkonium wfs.



**Counteract:** we introduce new effective wavefunctions

Declaration of new effective spinors in `aloha/template_files/aloha_functions.f`

Call to these new functions in `madgraph/iolib/helas_calls_objects.py`