Towards η_c GPDs from lattice QCD

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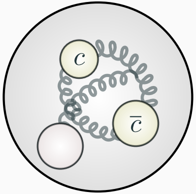
η_c meson

Composition:	$c\bar{c}$
J^{PC}:	0^{-+}
Mass:	2983.9 ± 0.4 MeV
Width:	32.0 ± 0.7 MeV



Introduction: η_c mesons

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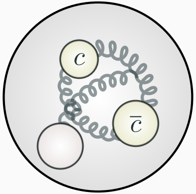

η_c-hadron structure

- How does it emerge from the bounding of a pair $c\bar{c}$?
- Comparison with lighter 0^{-} mesons: Assess effect of the Higgs mechanism on hadron structure.

Introduction: Hadron structure

Hadron structure

How do quarks and gluons combine to make hadrons up?

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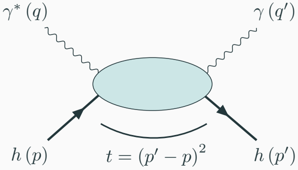
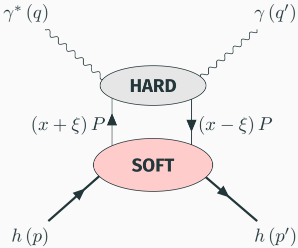


Generalized Bjorken limit
$Q^2 \to \infty$ with $Q^2 >> t$
and ξ fixed.

Factorization
[Phys.Rev.D59(1999)074009]



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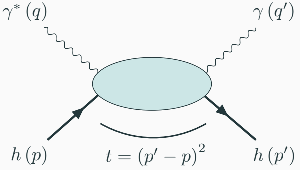

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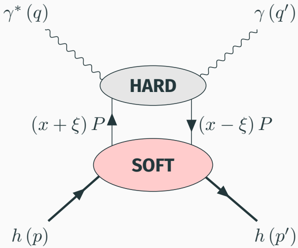

DVCS: $$\mathcal{H}\left(\xi, t; Q^2\right) = \sum_{p=q,g} \int_{-1}^{1} \frac{dx}{\xi} \mathcal{K}^p \left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s\left(\mu_F^2\right)\right) H^p\left(x, \xi, t; \mu_F^2\right)$$

Hard kernel $\mathcal{K}^p$: Perturbative information

Generalized Parton distributions H^p: Non perturbative QCD

Introduction: Generalized parton distributions

(GPD) – Generalized parton distributions:
Non-local, light-like separated quark and gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light front.
[Fortsch.Phys.:42(1994)101, Phys.Lett.B:380(1996)417, Phys.Rev.D:55(1997)7114]

Example: Twist-two chiral-even quark GPD of a spinless hadron.

$$H_{q/h}\left(x,\xi,t;\mu\right) = \frac{1}{2}\int \frac{d\nu}{2\pi} e^{-i\nu x} \left\langle h\left(p'\right)\right| \overline{\psi}_q\left(\nu n/2\right)\gamma^{\mu}\hat{\mathcal{W}}\left[\nu n/2,-\nu n/2\right]\psi_q\left(-\nu n/2\right)\left|h\left(p\right)\right\rangle n_{\mu}$$

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Properties:

- Universality *i.e.* hadron-specific objects.
- Contain parton distribution functions (PDFs) and form factors.
- Non-perturbative description of hadron structure: Tomography.

Introduction: Lattice QCD (I)

(Continuum) Quantum field theory

$$\langle\Omega|\,\mathcal{O}\,|\Omega\rangle \propto \int \mathcal{D}\left[A_\mu,\overline{\psi},\psi\right](x)\,\mathcal{O}\left[A_\mu,\overline{\psi},\psi\right](x)\,e^{iS\left[A_\mu,\overline{\psi},\psi\right](x)}$$

Extremely hard to assess beyond perturbation theory

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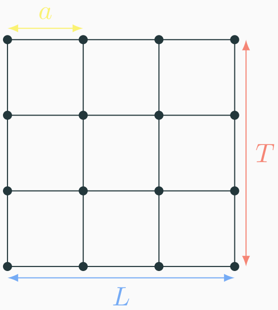
(Lattice) Quantum field theory

- Analytic continuation: $t \to -it_E \Rightarrow e^{iS} \to e^{-S_E}$
- Space time discretization:
  - a (lattice spacing): UV cut-off.
  - $L^3 \times T$ (finite box): Finite number of degrees of freedom.



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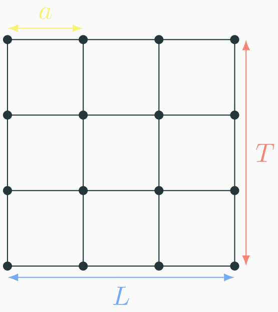

Amenable for numerical evaluation of the path integral:
Non-perturbative calculations!

Introduction: Lattice QCD (II)

In Lattice field theory, the expectation value of an observable $\langle \mathcal{O} \rangle$ is computed through Monte Carlo sampling of the Euclidean path integral, which is interpreted as a Boltzmann probability distribution.

$$\langle \mathcal{O} \rangle \propto \int \mathcal{D}\left[U, \overline{\psi}, \psi\right] \mathcal{O}\left[\overline{\psi}, \psi, U\right] e^{-S_E\left[U, \overline{\psi}, \psi\right]}$$

Computed expectation values are connected to (Euclidean) correlation functions
[Comm.Math.Phys.:42(1975)281, Comm.Math.Phys.:54(1977)283]

Generalized parton distributions and pseudodistributions (I)

Question: How can we compute GPDs in Lattice field theory?

Generalized parton distributions and pseudodistributions (I)

(ITGPD) – Ioffe time Generalized parton distributions:
Equal light-cone time, non-local quark and gluon operators, evaluated between hadron states in non-forward kinematics.
[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036,Phys.Rev.D:100(2019)116011]

$$\tilde{H}_{q/h}(\nu,\xi,t;\mu) = 2P^+ \int_{-1}^{1} dx e^{i\nu x} H_{q/h}(x,\xi,t;\mu)$$

Example: Twist-two chiral-even quark ITGPD of a spinless hadron.

$$\tilde{H}_{q/h}(\nu,\xi,t;\mu) = \langle h(p')| \overline{\psi}_q(-z/2)\gamma^+ \hat{\mathcal{W}}[-z/2;z/2]\psi_q(z/2)|h(p)\rangle\Big|_{\substack{z^+=0\\ \boldsymbol{z}_\perp=\boldsymbol{0}}}$$

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Generalized parton distributions and pseudodistributions (II)

Consider a **generic matrix element** with $z \in \mathbb{R}^{3,1}$ or even $z \in \mathbb{R}^4$
[Phys.Rev.D:96(2017)034025, Phys.Rev.D:96(2017)094503, Phys.Rev.D:100(2019)116011]

$$M^{\mu}_{q/h}(P,\Delta,z) = \langle h(p')| \overline{\psi}_q(-z/2)\gamma^{\mu}\hat{\mathcal{W}}[-z/2;z/2]\psi_q(z/2)|h(p)\rangle = 2P^{\mu}\mathcal{M}_P - \Delta^{\mu}\mathcal{M}_{\Delta} + z^{\mu}\mathcal{M}_z$$

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If we take $\mu = +$ and $z = z^-$

$$\begin{aligned} M^{+}_{q/h}(P,\Delta,z) &= \langle h(p')|\,\overline{\psi}_q\left(-z^-/2\right)\gamma^{+}\hat{\mathcal{W}}\left[-z^-/2;z^-/2\right]\psi_q\left(z^-/2\right)|h(p)\rangle \\ &= 2P^{+}\mathcal{M}_P(\nu,\xi,t,0) - \Delta^{+}\mathcal{M}_{\Delta}(\nu,\xi,t,0) = \tilde{H}_{q/h}(\nu,\xi,t;\mu) \end{aligned}$$

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which thus allows to compute GPDs as:

$$H_{q/h}\left(x,\xi,t\right) = \frac{1}{2}\int\frac{d\nu}{2\pi}e^{-i\nu x}\left[\mathcal{M}_P\left(\nu,\xi,t,0\right) + \xi\mathcal{M}_{\Delta}\left(\nu,\xi,t,0\right)\right]$$

Computing Generalized parton distributions is equivalent to computing form factors of the generic matrix element $M^{\mu}\left(P,\Delta,z\right)$.

GPDs from lattice QCD: Computational strategy

$$H_{q/h}(x,\xi,t;\mu) = \frac{1}{2}\int \frac{d\nu}{2\pi} e^{-i\nu x}\left[\mathcal{M}_P(\nu,\xi,t,0) + \xi\mathcal{M}_\Delta(\nu,\xi,t,0)\right]$$

Step 1. Evaluate generic matrix element on the Lattice:

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Matrix elements

[Phys.Lett.B 815 (2021) 136158]

[Chin. Phys. C 46 (2022) 1, 013105]

[Phys. Rev. D 105 (2022) 9, 094012]

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Step 2. Match it onto the light front:

$$\mathcal{M}_i\left(\nu,\xi,t,z^2;a,m_\pi\right) \xrightarrow[m_\pi\to m_\pi^{\rm Phys}]{a\to 0} \mathcal{M}_i\left(\nu,\xi,t,z^2\right) \xrightarrow{\rm LC} \mathcal{M}_i\left(\nu,\xi,t,z^2=0\right)$$

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$+$

LC Matching

[Phys.Rev.D:98(2018)014019]

[Phys.Rev.D:98(2018)056004]

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Step 3. Fourier transform to momentum-space.

Matrix elements

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[Phys.Rev.D:98(2018)056004]

[Phys.Rev.D:97(2018)074508]

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Inverse FT

[JHEP:04(2019)057]

Lattice calculation

Numerical setup

Goal: $\langle \eta_c(\vec{p}_{out};1s)|\,\overline{\psi}_c(z)\,\gamma_\mu \hat{\mathcal{W}}[z,0]\,\psi_c(0)\,|\eta_c(\vec{p}_{in};1s)\rangle$

- $N_f = 2$ ensemble (CLS)

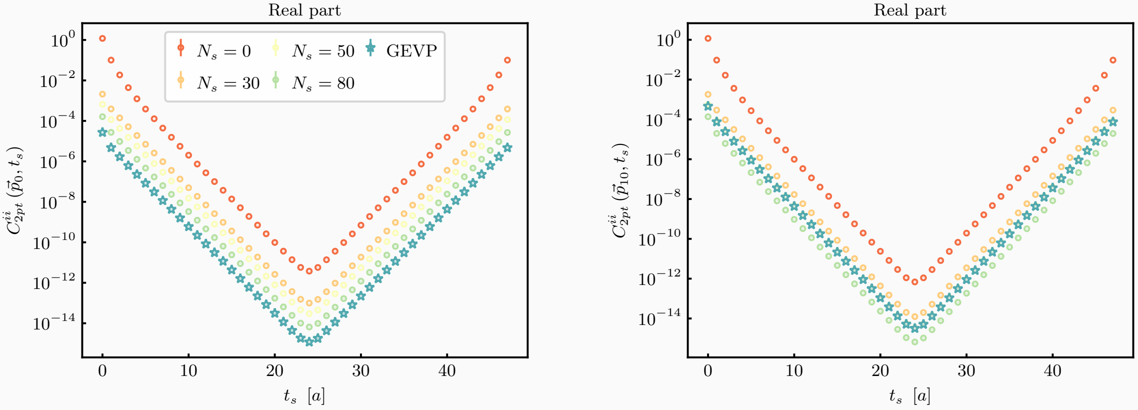
Name	β	a [fm]	$L^3 \times T$	N_f	m_π [MeV]
D5	5.3	0.06245	$24^3 \times 48$	u, d	476

- Twsited boundary conditions and a symmetric frame.
- 1 Interpolator and 4 smearings.

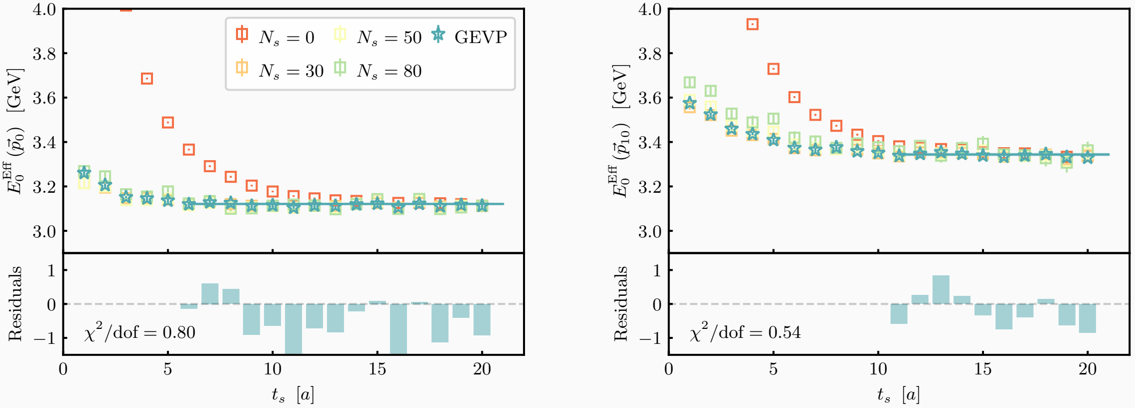
$$\eta_c^s(x) = \psi_c^s(x)\,\gamma_5 \psi_c^s(x) \quad , \quad J^{PC} = 0^{-+}$$
$$\psi_q^s(x) = (1 + 0.125\Delta_{\text{APE}})^{N_s}\,\psi_q(x) \quad , \quad N_s \in \{0, 30, 50, 80\}$$

- $2pt$ functions: $\langle \eta_c^i(\vec{x}', t_s)\,\overline{\eta}_c^j(\vec{x}, t)\rangle$
- $3pt$ functions: $\langle \eta_c^i(\vec{x}', t_s)\,\mathcal{O}_\mu^{ij}(\vec{z}, \tau)\,\overline{\eta}_c^j(\vec{x}, t)\rangle$, $\mathcal{O}_\mu^{ij}(\vec{z}, \tau) = \overline{\psi}_c^i(\vec{z}, \tau)\,\gamma_\mu \hat{\mathcal{W}}[\vec{z}, \vec{0}]\psi_c^j(\vec{0}, \tau)$

Data analysis: Two-point functions



Data analysis: Effective masses

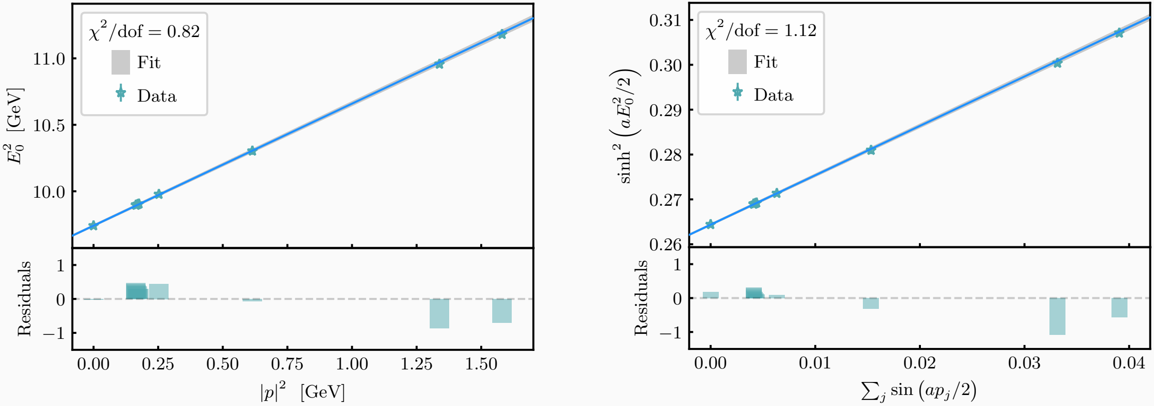


Energy spectrum compatible with expectation within finite volume and cut-off effects.

Systematics: - Fit range: Model averaging (AIC) [Phys.Rev.D.:103(2021)114502]

- Excited state: GEVP. [Nucl.Phys.B:259(1985)58, JHEP:04(2009)094]

Data analysis: Dispersion relations



Consistency check: Expected energy-momentum dispersion relations fulfilled.

Data analysis: Three-point functions

$$C_{3pt}^{ij}\left(\vec{p}_{out},\vec{p}_{in},t_s,\tau\right)=\sum_{\vec{x}',\vec{x}} e^{-i\vec{p}_{out}\cdot\vec{x}'+i\vec{p}_{in}\cdot\vec{x}}\left\langle \eta_c^i\left(\vec{x}',t_s\right)\mathcal{O}_\mu^{ij}(\vec{z},\tau)\overline{\eta}_c^j\left(\vec{x},t\right)\right\rangle,$$

with

$$\mathcal{O}_\mu^{ij}\left(\vec{z},\tau\right)=\overline{\psi}_c^i\left(\vec{z},\tau\right)\gamma_\mu\hat{\mathcal{W}}[\vec{z},\vec{0}]\psi_c(\vec{0},\tau)$$

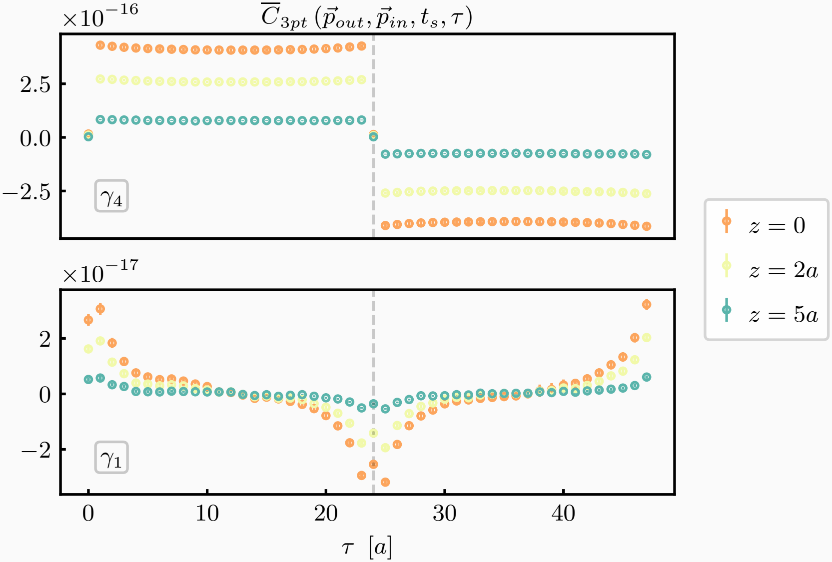
We choose

- $\gamma_\mu \in \{\gamma_1,\gamma_2,\gamma_4\}$
- $z_\mu=(0,0,z_3,0)$, with $z_3/a\in\mathbb{Z}\ \cap[0,10)$
- $\vec{p}_{out}$ and $\vec{p}_{in}$ chosen such that $p_{out}^{x,y}=-p_{in}^{x,y}$ and $\xi\equiv\frac{p_{out}^z-p_{in}^z}{p_{out}^z+p_{in}^z}=\text{Fixed}$

$$C_{3pt}^{ij}\left(\vec{p}_{out},\vec{p}_{in},t_s,\tau\right)=\sum_{n,m}\frac{e^{-E_n^{out}t_s+\tau\left(E_n^{out}-E_m^{in}\right)}}{4E_n^{out}E_m^{in}}\left\langle n\left(\vec{p}_{out}\right)\right|\hat{\mathcal{O}}_\mu^{ij}\left(\vec{z}\right)\left|m\left(\vec{p}_{in}\right)\right\rangle\mathcal{N}_n^i\left(\vec{p}_{out}\right)\mathcal{N}_m^{j,*}\left(\vec{p}_{in}\right)$$

$$\mathcal{N}_n^i\left(\vec{p}\right)\equiv\left\langle\Omega\right|\eta_c^i\left|n\left(\vec{p}\right)\right\rangle$$

Data analysis: Three-point functions



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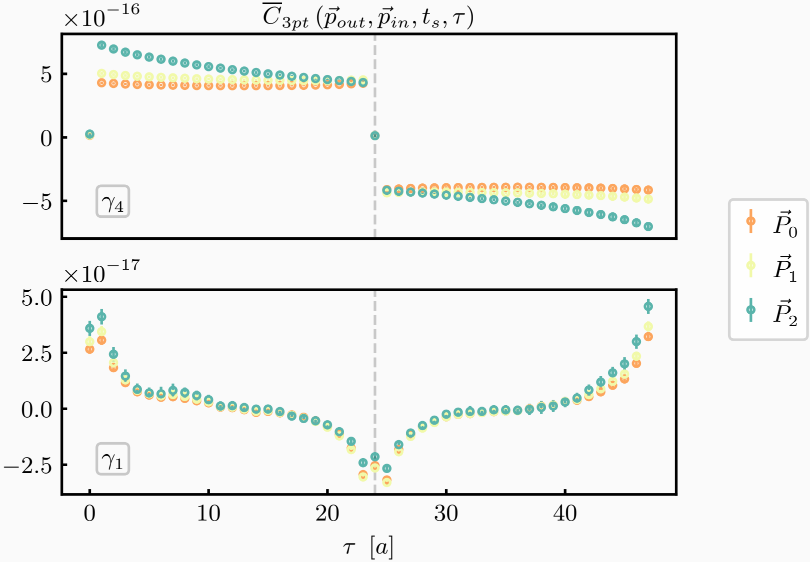

Data analysis: Ratios and matrix elements

The extraction of matrix elements is made easier through ratios of three- and two-point functions:

$$R_B\left(\tau, t_s\right)=\frac{\overline{C}_{3pt}\left(\vec{p}_{out}, \vec{p}_{in}, t_s, \tau\right)}{\overline{C}_{2pt}\left(\vec{p}_{out}, t_s\right)}\sqrt{\frac{\overline{C}_{2pt}\left(\vec{p}_{in}, t_s-\tau\right)\overline{C}_{2pt}\left(\vec{p}_{out}, \tau\right)\overline{C}_{2pt}\left(\vec{p}_{out}, t_s\right)}{\overline{C}_{2pt}\left(\vec{p}_{out}, t_s-\tau\right)\overline{C}_{2pt}\left(\vec{p}_{in}, \tau\right)\overline{C}_{2pt}\left(\vec{p}_{in}, t_s\right)}}$$

[PoS LAT2055(2006)360]

Under the assumption of ground state dominance ($0 << \tau << t_s$) and setting $t_s = T/2$

$$R_B\left(\tau, t_s=T/2\right)=\frac{\langle 0\left(\vec{p}_{out}\right)|\,\hat{\mathcal{O}}_\mu\,|0\left(\vec{p}_{in}\right)\rangle}{4\sqrt{E_0^{out}E_0^{in}}}\sqrt{\frac{\left(1+e^{-2\tau E_0^{in}}\right)\left(1+e^{-2(T/2-\tau)E_0^{out}}\right)}{\left(1+e^{-2\tau E_0^{in}}\right)\left(1+e^{-2(T/2-\tau)E_0^{out}}\right)}}$$

For $\tau - T/4 \equiv \delta \to 0$

$$R_B\left(\tau, t_s=T/2\right)\simeq\frac{\langle 0\left(\vec{p}_{out}\right)|\,\hat{\mathcal{O}}_\mu\left(\vec{z}\right)|0\left(\vec{p}_{in}\right)\rangle}{4\sqrt{E_0^{out}E_0^{in}}}\left[1+2\delta\mathcal{C}_\delta\left(E_0^{out}, E_0^{in}\right)+2\delta^2\mathcal{C}_\delta^2\left(E_0^{out}, E_0^{in}\right)\right]$$

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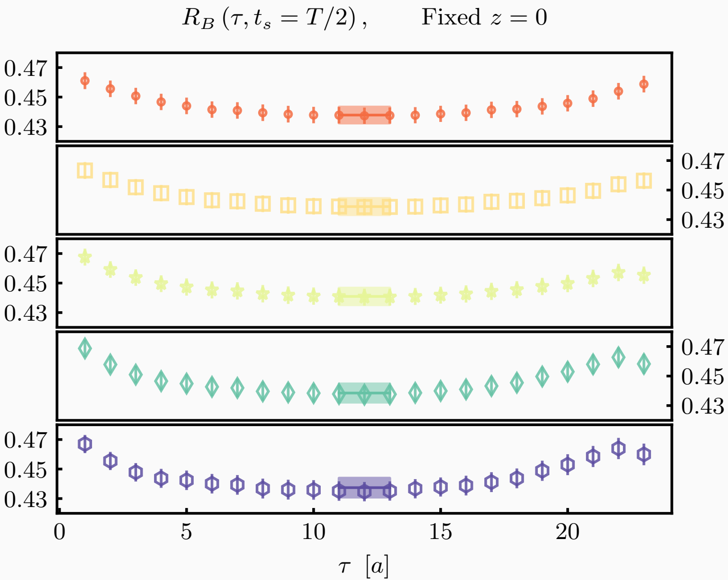
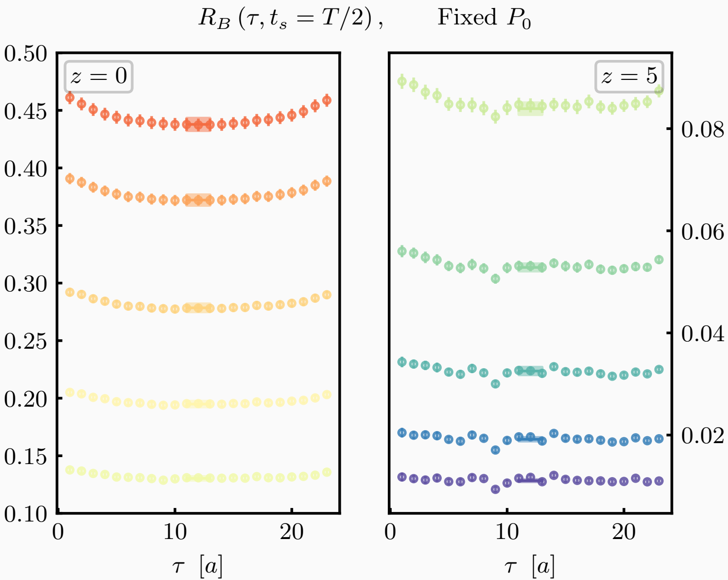


Fixed z changing P

$\chi^2/\mathrm{dof} \sim \mathcal{O}(1)$

3-points fits favored by AIC.

Data analysis: Matrix elements



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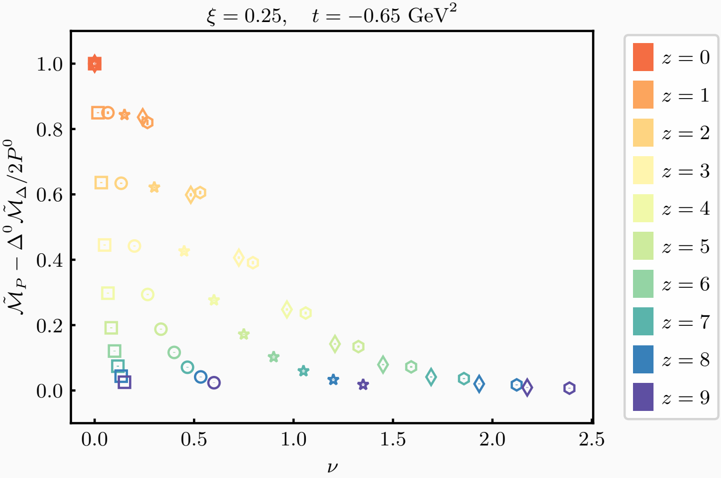
Data analysis: Three-point functions

$$\langle \eta_c (\vec{p}_{out}; 1s) | \hat{\mathcal{O}}_4 (\vec{z}) | \eta_c (\vec{p}_{in}; 1s) \rangle = \left[E_0^{out} + E_0^{in} \right] \mathcal{M}_P - \left[E_0^{out} - E_0^{in} \right] \mathcal{M}_\Delta$$



Conclusions and future steps

Summary

- Study of η_c-meson's structure through GPDs within lattice QCD.
- GPDs give a comprehensive picture about hadron structure.
- Ongoing effort for the extraction of form factors.

Future steps

- Handling excited state contamination.
- Extract form factors $\mathcal{M}_P$ and $\mathcal{M}_\Delta$.
- Matching to the light cone.
  - Continuum limit.
  - Infinite volume limit.
- Reconstruction of light-cone distribution ampltitudes.
  - Ill-posedness of the inverse Fourier transform.

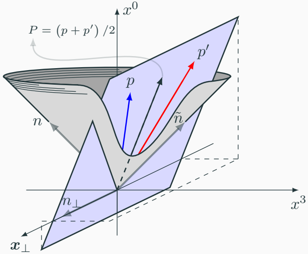
Thank you!

Back-up slides

Ioffe time

Question: How can we compute GPDs in Lattice field theory?

$$H_{q/h}\left(x,\xi,t;\mu\right)=\frac{1}{2}\int\frac{d\nu}{2\pi}e^{-i\nu x}\left\langle h\left(p'\right)\right|\overline{\psi}_q\left(\nu n/2\right)\gamma^\mu\hat{\mathcal{W}}\left[\nu n/2,-\nu n/2\right]\psi_q\left(-\nu n/2\right)\left|h\left(p\right)\right\rangle n_\mu$$



Hadron frame

$$P=\frac{p+p'}{2}\text{ such that }\boldsymbol{P}_\perp=0$$

trigger interpretation of ν as *Ioffe time*
[Phys.Lett:30B(1969)123, Phys.Rev.D:51(1995)6036]

$$z=-\nu n\Rightarrow\nu=-z^-P^+\equiv-z\cdot P$$

$$H_{q/h}\left(x,\xi,t;\mu\right)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle h\left(p'\right)\right|\overline{\psi}_q\left(-z/2\right)\gamma^+\hat{\mathcal{W}}\left[-z/2;z/2\right]\psi_q\left(z/2\right)\left|h\left(p\right)\right\rangle\Big|_{\substack{z^+=0\\ \boldsymbol{z}_\perp=\boldsymbol{0}}}$$

Momenta

j	p_x	p_y	p_z
0	0.0	0.0	0.0
1	−2.17075	−2.17075	0.2
2	−2.10085	−2.10085	0.8
3	−1.77431	−1.77431	1.8
4	−0.872673	−0.872673	2.9
5	−0.0526329	0.0526329	3.183
6	2.17075	2.17075	0.6
7	2.10085	2.10085	2.4
8	1.77431	1.77431	5.4
9	0.872673	0.872673	8.7
10	0.0526329	0.0526329	9.549

GEVP

Consider a matrix of correlators,

$$C_{ij}\left(t\right) \equiv \left\langle \mathcal{O}_i\left(t\right) \mathcal{O}_j\left(0\right)\right\rangle = \sum_{n=0}^{\infty} e^{-tE_n} \mathcal{N}_n^i \mathcal{N}_n^j$$

If N operators are available ($i.e.$ $i, j = 0, \ldots, N-1$), separate

$$C_{ij}^{(0)}\left(t\right) = \sum_{n=0}^{N-1} e^{-tE_n} \mathcal{N}_n^i \mathcal{N}_n^j, \qquad C_{ij}^{(1)}\left(t\right) = \sum_{n=N}^{\infty\}} e^{-tE_n} \mathcal{N}_n^i \mathcal{N}_n^j$$

GEVP

Lemma: The matrix $C^{(0)}(t)$ satisfies a Generalized Eigenvalue Problem (GEVP):
[Nucl.Phys.B:259(1985)58, JHEP:04(2009)094]

$$C^{(0)}(\vec{p},t)\, v_n^{(0)}(t,t_0) = \lambda_n^{(0)}(t,t_0)\, C^{(0)}(\vec{p},t_0)\, v_n^{(0)}(t,t_0)$$

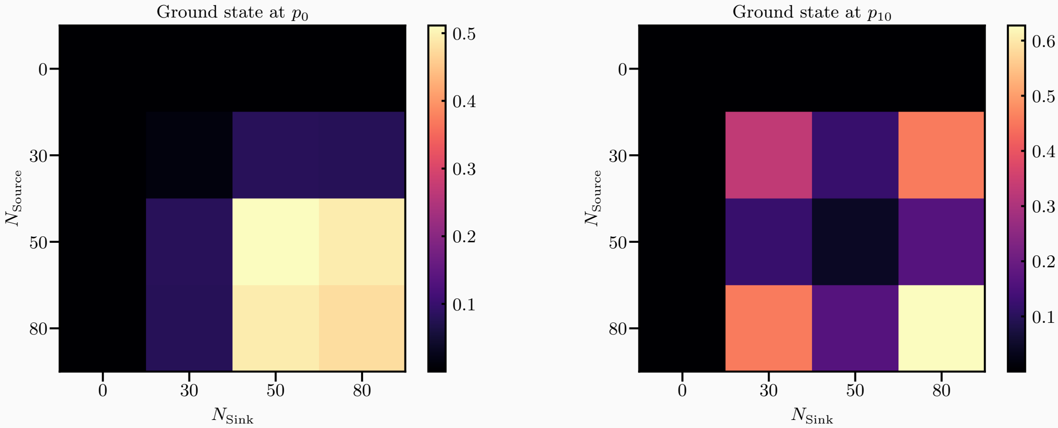
such that

$$\left(v_n^{(0)}, \mathcal{N}_m\right) = \delta_{nm}, \qquad \lambda_n^{(0)}(t,t_0) = e^{-E_n^{\mathrm{Eff}}(t-t_0)}, \qquad \left(v_m^{(0)}, C^{(0)} v_n^{(0)}\right) = \delta_{nm} e^{-tE_n^{\mathrm{Eff}}}$$

Theorem: If $t_0 \geq t/2$, the corrections to a state n of $C^{(0)}$ given by perturbations from $C^{(1)}$ vanish exponentially as:
[JHEP:04(2009)094]

$$E_n^{\mathrm{Eff}} = E_n + \mathcal{O}\left(e^{-t(E_N - E_n)}\right)$$

GEVP


Ground state at p_0
0
30
50
80
N_{Source}
N_{Sink}
0.1
0.2
0.3
0.4
0.5
Ground state at p_{10}
0
30
50
80
N_{Source}
N_{Sink}
0.1
0.2
0.3
0.4
0.5
0.6