Probing gluon saturation in semi-hard diffractive $\gamma^{(*)}$ + p/A processes

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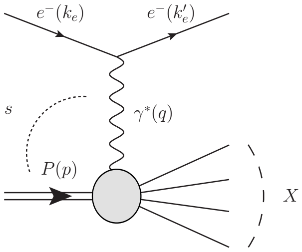
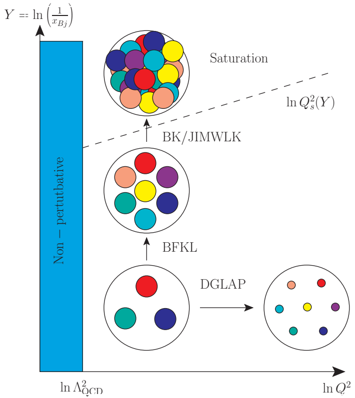
DIS



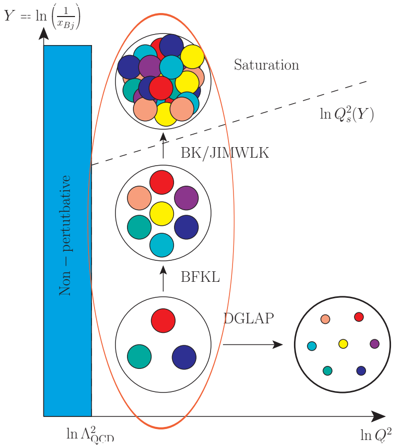
$$Q^2 = -q^2$$
$$s = (p+q)^2$$
$$x_{Bj} = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2 + s}$$



DIS in the high-energy limit

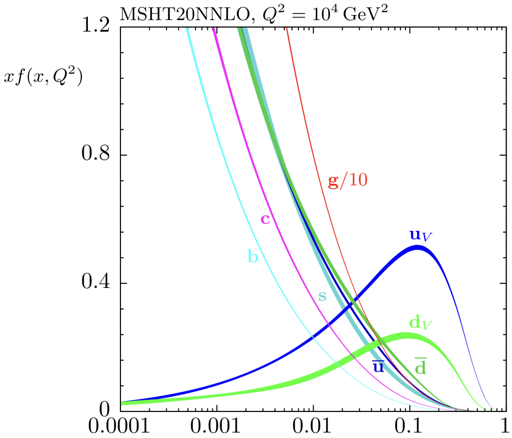
Regge-Gribov or semi-hard limit : $s \gg Q^2 \gg \Lambda^2_{\mathrm{QCD}}$

Equivalent to small-$x_{Bj} \sim \frac{Q^2}{s} \to 0$ limit



PDFs at small x

Gluons dominate at small x_{Bj}



[NNLO MSHT20 PDFs, taken from Eur.Phys.J.C 81 (2021) 4, 341]

DIS in the high-energy limit

- Resummation of $\alpha_s(Q^2)\ln\left(\frac{1}{x_{Bj}}\right) = \alpha_s(Q^2)\ln\left(\frac{s}{Q^2}\right) \sim 1$
- BFKL approach

$$\sigma^{\gamma^* p}(x) = \Phi_{\gamma^*\gamma^*}(\vec{k}\,) \otimes_{\vec{k}} \mathcal{F}(x,\vec{k}\,)$$

$$\downarrow$$

$$\sigma^{\gamma^* p}(x) \sim \left(\frac{s}{Q^2}\right)^{\omega_0}, \quad \omega_0 = \frac{4N_c\ln(2)}{\pi}\alpha_s$$

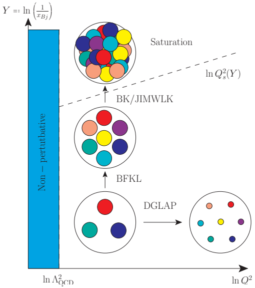
- Froissart-Martin bound

$$\sigma_{\text{tot}} < A\ln^2 s$$

- Non-linear effects become important $\Longrightarrow$ gluon saturation
- Saturation scale

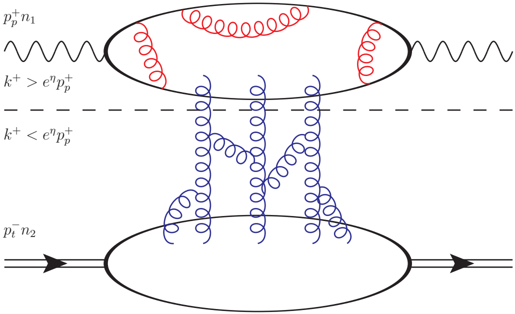
$$Q_s^2 \propto A^{1/3}x^{-\omega_0}$$

Saturation window or dense regime : $Q^2 < Q_s^2$



Shockwave formalism

- High-energy limit: $s = (p_p + p_t)^2 \gg Q^2, |p_p|^2, M_t^2, |t|, ...$
- n_1^μ, n_2^μ are light-like vectors (+/− directions)
- Working frame: IMF frame where $p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$



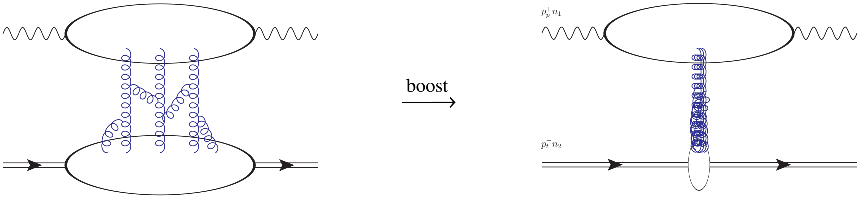
- Separation of gluon field into "fast" (quantum) and "slow " (classical) parts with a cut-off defined by an arbitrary rapidity parameter $\eta < 0$:

$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

Shockwave formalism

- Boost from target rest frame to the IMF frame with large $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{M_t}$:

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda z^+, \Lambda^{-1} z^-, \vec{z}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda z^+, \Lambda^{-1} z^-, \vec{z}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda z^+, \Lambda^{-1} z^-, \vec{z}) \end{cases}$$



$b_0^\mu(x)$

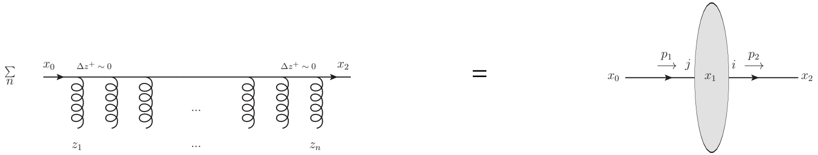
$$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+)\mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$$

Shockwave approximation

- Gauge choice: $A \cdot n_2 = 0$

  $A \cdot b = 0 \implies$ Simple effective Lagrangian

Quark line through the shockwave and Wilson line



- Interactions with the simple shockwave field:
  - Independence on x^- $\implies$ conservation of p^+ (eikonal approximation)
  - $\delta(x^+)$ $\implies$ interactions at a single transverse coordinate.
- Resummation of the multiple interactions into a Wilson line :

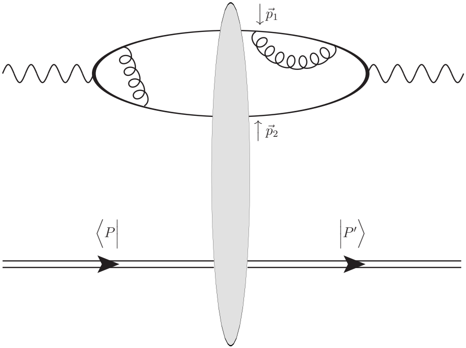
$$V(\vec{z}\,) = \mathcal{P} \exp\left[ig \int_{-\infty}^{+\infty} dz^+ b^- \left(z^+, \vec{z}\right)\right]$$

$$V(\vec{p}\,) = \int d^d\vec{z}\; e^{-i\vec{p}\cdot\vec{z}}\; V(\vec{z}\,)$$

- Quark line through the shockwave:

$$G_{ij}(x_2, x_0)|_{x_2^+>0>x_0^+} = \int d^D x_1\; \delta(x_1^+) G_0(x_{21}) V_{ij}(\vec{x}_1) \gamma^+ G_0(x_{10}) \theta(x_2^+) \theta(-x_0^+)$$

Scattering amplitude in the shockwave approximation



$$\mathcal{M}^\eta = \int d^d\vec{p}_1 d^d\vec{p}_2\,\Phi^\eta(\vec{p}_1,\vec{p}_2)\left\langle P'\left|\left[\mathrm{Tr}\left(V_1^\eta V_2^{\eta\dagger}\right)-N_c\right](\vec{p}_1,\vec{p}_2)\right|P\right\rangle$$

Dipole operator:

$$\mathcal{U}_{ij}^\eta = 1-\frac{1}{N_c}\mathrm{Tr}\left[V^\eta(\vec{z}_i)V^{\eta\dagger}(\vec{z}_j)\right]$$

$$\widetilde{\mathcal{U}}_{ij}^\eta(\vec{p}_i,\vec{p}_j) = \int d^d\vec{z}_i d^d\vec{z}_j\, e^{-i\vec{p}_i\cdot\vec{z}_i}\, e^{-i\vec{p}_j\cdot\vec{z}_j}\,\mathcal{U}_{ij}^\eta$$

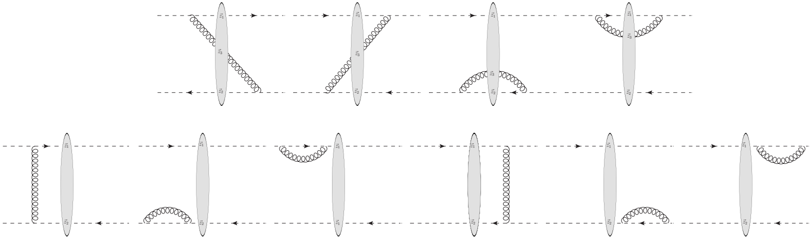
Balitsky-JIMWLK equations

- Balitsky-JIMWLK evolution equations [Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner] equations:

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2}\vec{z}_{31}^{\,2}} \right) \Big[\underbrace{\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta}}_{\text{BFKL}} - \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta} \Big]$$

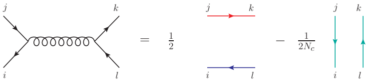
$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \cdots \qquad \longleftarrow \qquad \text{Balitsky hierarchy}$$

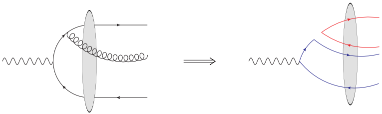
$$\vdots$$



Balitsky-Kovchegov evolution equation

- Large-N_c limit





- Mean-field approximation

$$\langle \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta} \rangle \to \langle \mathcal{U}_{13}^{\eta} \rangle \langle \mathcal{U}_{32}^{\eta} \rangle$$

- Hierarchy of equations broken $\longrightarrow$ closed non-linear BK equation

[Balitsky (1997)] [Kovchegov (1999)]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 \vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2} \vec{z}_{31}^{\,2}} \right) \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle - \langle \mathcal{U}_{13}^{\eta} \rangle \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

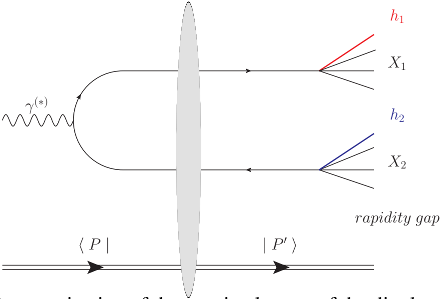
with $\langle \mathcal{U}_{12}^{\eta} \rangle \equiv \dfrac{\left\langle P \middle| \mathcal{U}_{12}^{\eta} \middle| P \right\rangle}{\langle P | P \rangle}$.

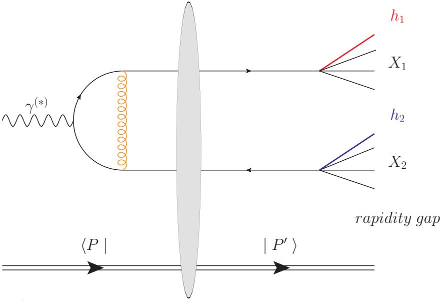
Diffractive di-hadron production

- Diffractive di-hadron production at NLO [Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)] JHEP 03 (2023) 159

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h_1(p_{h1}) + h_2(p_{h2}) + X + P(p_{0'}) \qquad (X = X_1 + X_2)$$

  Rapidity gap between $(h_1 h_2 X)$ and $P'(p_{0'})$.
- General kinematics (t, Q^2) and arbitrary photon polarization: process could be either photo-production or electro-production





Parametrization of the matrix element of the dipole operator:

$$\left\langle P'(p_{0'}) \left| \mathrm{Tr}\left[V\left(\frac{\vec{z}}{2}\right) V^\dagger\left(-\frac{\vec{z}}{2}\right)\right] - N_c \right| P(p_0) \right\rangle \equiv 2\pi\delta\left(p^-_{00'}\right) F(\vec{z})$$

$$\int d^d\vec{z}\, e^{-i\vec{z}\cdot\vec{p}} F(\vec{z}) \equiv \mathbf{F}(\vec{p})\,.$$

Hybrid factorisation

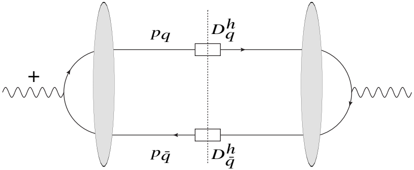
Hybrid factorization:

- Collinear factorization: Hard scale with $\Lambda^2_{\mathrm{QCD}} \ll \vec{p}^{\,2}_{h_1} \sim \vec{p}^{\,2}_{h_2}$.
  Constraint $\vec{p}^{\,2} \gg \vec{p}^{\,2}_{h_{1,2}}$ with $\vec{p}$, the relative transverse momentum of the two hadrons.
  $\Longrightarrow$ Use of single hadron fragmentation function (FF) only to describe hadronization.
- The high-energy factorisation :
  Partonic process at NLO from [Boussarie, Grabovsky, Szymanowski, Wallon (2016)]
- Saturation region : $\vec{p}^{\,2}_{h_1} \sim \vec{p}^{\,2}_{h_2} < Q_s^2$

LO cross-section

- Sudakov decomposition for the momenta: $p_i^\mu = x_i p_\gamma^+ n_1^\mu + \frac{\vec{p}^{\,2}}{2x_i p_\gamma^+} n_2^\mu + p_\perp^\mu$.

$$p_\gamma = \begin{array}{c} \quad + \quad\quad - \quad\quad \perp \\ \left(p_\gamma^+, -\frac{Q^2}{2p_\gamma^+}, \vec{0}\right) \end{array}$$

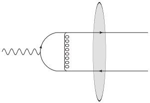


- Collinearity $(p_q^+, \vec{p}_q) = (x_q/x_{h_1})(p_{h_1}^+, \vec{p}_{h_1})$ and $(p_{\bar{q}}^+, \vec{p}_{\bar{q}}) = (x_{\bar{q}}/x_{h_2})(p_{h_2}^+, \vec{p}_{h_2})$:

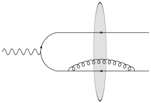
$$\frac{d\sigma_{0JI}^{h_1 h_2}}{dx_{h_1} dx_{h_2} d^d\vec{p}_{h_1} d^d\vec{p}_{h2}} = \sum_q \int_{x_{h_1}}^1 \frac{dx_q}{x_q} \int_{x_{h_2}}^1 \frac{dx_{\bar{q}}}{x_{\bar{q}}} \left(\frac{x_q}{x_{h1}}\right)^d \left(\frac{x_{\bar{q}}}{x_{h2}}\right)^d$$
$$D_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}\right) \frac{d\hat{\sigma}_{JI}}{dx_q dx_{\bar{q}} d^d\vec{p}_q d^d\vec{p}_{\bar{q}}} + (h_1 \leftrightarrow h_2)$$

J, I labels the photon polarization for respectively the complex conjugated amplitude and the amplitude.

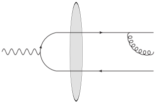
NLO impact factor and UV + rapidity divergences



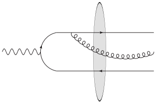
Φ_{V1}



Φ'_{V1}, Φ_{V2}



Φ_{R1}



Φ'_{R1}, Φ_{R2}

Rapidity divergences ($x_g \to 0$, $\vec{p}_g$ arbitrary)

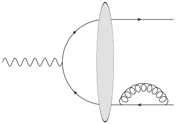
- Present in Φ_{V2}
- Of the form $\ln \alpha$, α longitudinal cut-off $p_g^+ = x_g p_\gamma^+ > \alpha p_\gamma^+$
- B-JIMWLK evolution equation of the dipole operator in the LO term for α to e^η

$$\widetilde{\mathcal{U}}_{12}^{\alpha} = \widetilde{\mathcal{U}}_{12}^{e^\eta} - \int_{\alpha}^{e^\eta} d\rho \, \frac{\partial \widetilde{\mathcal{U}}_{12}}{\partial \rho} \implies \Phi_{\mathrm{BK}} = \Phi_0 \otimes \mathcal{K}_{\text{B-JIMWLK}} \implies \tilde{\Phi}_{V2} = \Phi_{V2} + \Phi_{\mathrm{BK}} \text{ is finite}$$

UV-divergences ($\vec{p}_g^{\,2} \to \infty$)

- Just dressing of the external massless quark lines

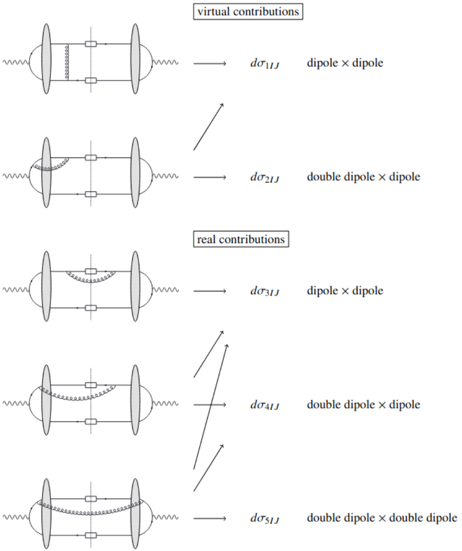
$$\Phi_{\text{dress}} \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$



- $\epsilon_{IR} = \epsilon_{UV} = \epsilon$ turns **UV** into **IR** divergences

NLO cross-section in a nutshell and divergences

Operator structure classification :



NLO cross-section in a nutshell and divergences

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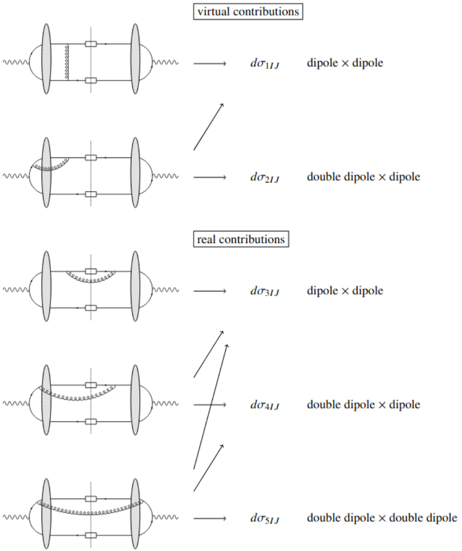
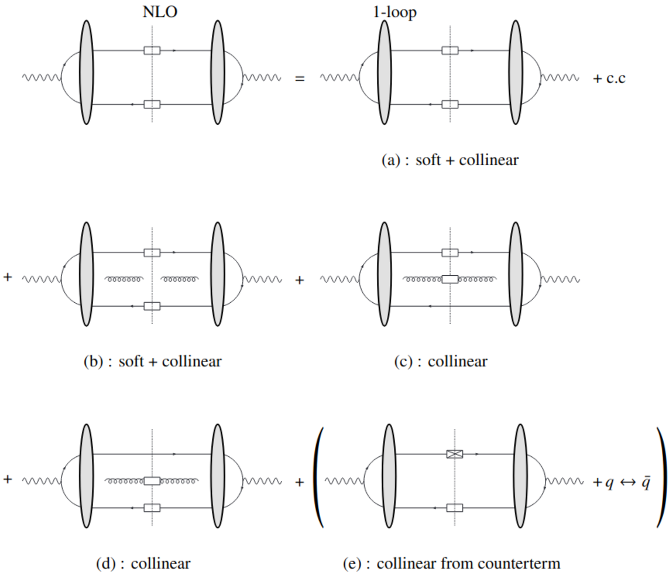


IR divergences

- Collinear divergences: $\vec{p}_g \propto \vec{p}_q$ or $\vec{p}_g \propto \vec{p}_{\bar{q}}$
- Soft divergences : $x_g \to 0$ and $\vec{p}_g = x_g \vec{u} \sim x_g \to 0$ $(p_g \to 0)$

Regularization with dimensional regularization $D = 2 + d = 4 + 2\epsilon$ and longitudinal cut-off $|x_g| > \alpha$.

NLO cross-section in a nutshell and divergences



IR singularities : counterterm

- Quark FF renormalization (same formula for the antiquark)

$$\underbrace{D_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right)}_{\text{bare}} = \underbrace{D_q^{h_1}\left(\frac{x_{h_1}}{x_q},\mu_F\right)}_{\text{dressed}} - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_{\frac{x_{h_1}}{x_q}}^{1}\frac{d\beta}{\beta}\left[P_{qq}(\beta)D_q^{h_1}\left(\frac{x_{h_1}}{x_q\beta},\mu_F\right) + P_{gq}(\beta)\,D_g^{h_1}\left(\frac{x_{h_1}}{\beta x_q},\mu_F\right)\right]$$

$$= D_q^{h_1}\left(\frac{x_{h_1}}{x_q},\mu_F\right) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\left\{\left[P_{qq}\otimes D_q^{h_1}\right]\left(\frac{x_{h_1}}{x_q},\mu_F\right) + \left[P_{gq}\otimes D_g^{h_1}\right]\left(\frac{x_{h_1}}{x_q},\mu_F\right)\right\}$$

$\downarrow$ in LO cross-section

- Counterterm

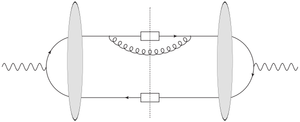
$$\left.\frac{d\sigma_{LL}}{dx_{h_1}\,dx_{h_2}\,d^d\vec{p}_{h_1}\,d^d\vec{p}_{h_2}}\right|_{\text{ct}} = \frac{4\alpha_{\text{em}}Q^2}{(2\pi)^{4(d-1)}N_c}\sum_q Q_q^2\int_{x_{h_1}}^1 dx_q\int_{x_{h_2}}^1 dx_{\bar{q}}\,x_q x_{\bar{q}}\left(\frac{x_q}{x_{h_1}}\right)^d\left(\frac{x_{\bar{q}}}{x_{h_2}}\right)^d\delta(1-x_q-x_{\bar{q}})\mathcal{F}_{LL}$$

$$\left(-\frac{\alpha_s}{2\pi}\right)\left(\frac{1}{\hat{\epsilon}} + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)\left\{\underbrace{\left[P_{qq}\otimes D_q^{h_1}\right]\left(\frac{x_{h_1}}{x_q},\mu_F\right)D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}},\mu_F\right)}_{(1)} + \underbrace{\left[P_{gq}\otimes D_g^{h_1}\right]\left(\frac{x_{h_1}}{x_q}\right)D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}},\mu_F\right)}_{(6)}\right.$$

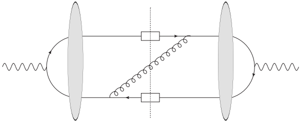
$$\left. + \underbrace{\left[P_{qq}\otimes D_{\bar{q}}^{h_2}\right]\left(\frac{x_{h_2}}{x_{\bar{q}}},\mu_f\right)D_q^{h_1}\left(\frac{x_{h_1}}{x_q},\mu_F\right)}_{(3)} + \underbrace{\left[P_{gq}\otimes D_g^{h_2}\right]\left(\frac{x_{h_2}}{x_{\bar{q}}},\mu_F\right)D_q^{h_1}\left(\frac{x_{h_1}}{x_q},\mu_F\right)}_{(5)}\right\} + (h_1 \leftrightarrow h_2)$$

IR singularities: quark-antiquark fragmentation

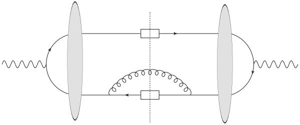
- Divergent diagrams in the case of quark-antiquark fragmentation



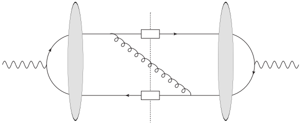
$(1):$ soft + collinear qg



$(2):$ soft



$(3):$ soft + collinear $\bar{q}g$



$(4):$ soft

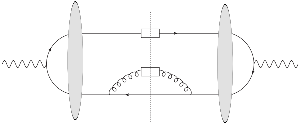
- Cancellation of divergences in a nutshell

$$d\sigma_3^{q\bar{q}}\Big|_{\text{soft}} + \underbrace{\left(d\sigma_3^{q\bar{q}}\Big|_{(1)} - d\sigma_3^{q\bar{q}}\Big|_{(1)\ \text{soft}}\right)}_{d\sigma_3^{q\bar{q}}\big|_{\text{coll.}\ qg}} + \left(d\sigma_3^{q\bar{q}}\Big|_{(2)} - d\sigma_3^{q\bar{q}}\Big|_{(2)\ \text{soft}}\right) + \underbrace{\left(d\sigma_3^{q\bar{q}}\Big|_{(3)} - d\sigma_3^{q\bar{q}}\Big|_{(3)\ \text{soft}}\right)}_{d\sigma_3^{q\bar{q}}\big|_{\text{coll.}\ \bar{q}g}} + \left(d\sigma_3^{q\bar{q}}\Big|_{(4)} - d\sigma_3^{q\bar{q}}\Big|_{(4)\ \text{soft}}\right)$$

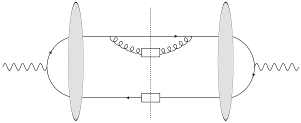
$$+ \, d\sigma_1 + d\sigma|_{\text{ct}\ (1)} + d\sigma|_{\text{ct}\ (3)}$$

IR singularities : Gluon fragmentation

- Divergent diagrams



(5) : collinear $\bar{q}g$



(6) : collinear qg

- Cancellation of divergences in a nutshell

$$d\sigma_3^{qg}\Big|_{\text{coll. } \bar{q}g} + d\sigma|_{\text{ct } (5)} + d\sigma_3^{\bar{q}g}\Big|_{\text{coll. } qg} + d\sigma|_{\text{ct } (6)}$$

- Finite parts of the cross-section

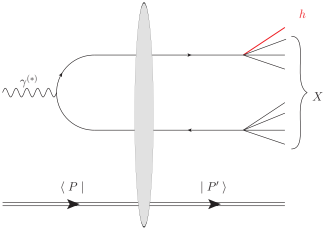
$$d\sigma_{JI}^{h_1 h_2} = \sum_{(a,b)} D_a^{h_1} \otimes D_b^{h_2} \otimes d\hat{\sigma}_{JI}^{ab} + (h_1 \leftrightarrow h_2) \qquad (a,b) = \{(q,\bar{q}),(q,g),(\bar{q},g)\}$$

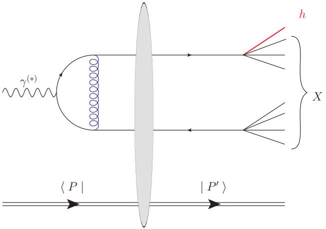
Diffractive production of a single hadron

- Diffractive production of a single hadron at LNO [Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)] (to appear)

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h(p_h) + X + P'(p_{0'})$$

Rapidity gap between (hX) and $P'(p_{0'})$.





- General kinematics (t, Q^2) and arbitrary photon polarization: process could be either photo-production or electro-production
- Hybrid factorization: high-energy factorization (shockwave formalism) and collinear factorization with th hard scale $\vec{p}_h^{\,2} \gg \Lambda^2_{\rm QCD}$
- Explicit cancellation of divergences and extraction of all the finite terms

$$d\sigma^h_{JI} = \sum_a D^h_a \otimes d\hat{\sigma}^a_{JI}$$

Summary

- Computation of the NLO cross-section of the diffractive production of double and single hadron with large $\vec{p}_h^{\,2} \gg \Lambda^2_{\mathrm{QCD}}$ in $\gamma^{(*)}$ +p/A collisions
- Full cancellation of divergences has been observed between real corrections, virtual ones, and counterterm from FFs renormalization.
- General kinematics (Q^2, t) and arbitrary photon polarization: process could be either photo-production or electro-production
- Results are applicable to ultra-peripheral collisions at the LHC and to the EIC.

Thank you for you attention!